

# Programming Languages: Imperative Program Construction

## Practicals 4: Hoare Logic and Weakest Precondition: Loop

Shin-Cheng Mu

Autumn Term, 2022

1. Prove the correctness of the following program:

```
con  $N : \text{Int} \{N \geq 0\}$   
var  $x, y : \text{Int}$   
 $x, y := 0, 1$   
do  $x \neq N \rightarrow x, y := x + 1, y + y$  od  
 $\{y = 2^N\}$ 
```

2. Prove the correctness of the following program:

```
con  $N : \text{Int} \{N \geq 0\}$   
var  $y : \text{Int}$   
 $y := 1$   
do  $y < N \rightarrow y := y + y$  od  
 $\{y \geq N \wedge \langle \exists k : k \geq 0 : y = 2^k \rangle\}$ 
```

3. Given integers  $N \geq 0$  and  $M > 0$ , the following program computes integral division  $N / M$ . Prove its correctness.

```
con  $N, M : \text{Int} \{N \geq 0 \wedge M > 0\}$   
var  $l, r : \text{Int}$   
 $l, r := 0, N + 1$   
do  $l + 1 \neq r \rightarrow$   
  if  $((l + r) / 2) \times M \leq N \rightarrow l := (l + r) / 2$   
  |  $((l + r) / 2) \times M > N \rightarrow r := (l + r) / 2$   
  fi  
od  
 $\{l \times M \leq N < (l + 1) \times M\}$ 
```

4. The following program non-deterministically computes  $x$  and  $y$  such that  $x \times y = N$ . Prove:

```
con  $N : \text{Int} \{N \geq 1\}$   
var  $p, x, y : \text{Int}$   
 $p, x, y := N - 1, 1, 1$   
 $\{N = x \times y + p \wedge \dots\}$   
do  $p \neq 0 \rightarrow$   
  if  $p \bmod x = 0 \rightarrow y, p := y + 1, p - x$   
  |  $p \bmod y = 0 \rightarrow x, p := x + 1, p - y$   
  fi  
od  
 $\{x \times y = N\}$ 
```

5. Prove the correctness of the following program:

```
con  $N : \text{Int} \{N \geq 0\}$   
var  $x, y : \text{Int}$   
 $x, y := 0, 0$   
do  $x \neq 0 \rightarrow x := x - 1$   
   $| \quad y \neq N \rightarrow x, y := x + 1, y + 1$   
od  
 $\{x = 0 \wedge y = N\}$ 
```

6. Prove the correctness of the following program:

```
con  $N : \text{Int} \{N \geq 0\}$   
var  $x, y : \text{Int}$   
 $x, y := 0, 0$   
do  $x \neq 0 \rightarrow x := x - 1$   
   $| \quad y \neq N \rightarrow x, y := N, y + 1$   
od  
 $\{x = 0 \wedge y = N\}$ 
```