Programming Languages: Imperative Program Construction
Practicals 1: Non-Looping Constructs and Weakest Precondition

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1. Determine the weakest P that satisfies
   (a) $\{P\}\, x := x + 1; x := x + 1\, \{x \geqslant 0\}$.
   (b) $\{P\}\, x := x + y; y := 2 \times x\, \{y \geqslant 0\}$.
   (c) $\{P\}\, x := y; y := x\, \{x = A \wedge y = B\}$.
   (d) $\{P\}\, x := E; x := E\, \{x = E\}$.
2. Assuming that x, y, and z are integers, prove the following
   (a) $\{True\}$ **if** $x \geqslant 1 \to x := x + 1 \mid x \leqslant 1 \to x := x - 1$ **fi** $\{x \neq 1\}$.
   (b) $\{True\}$ **if** $x \geqslant y \to skip \mid y \geqslant x \to x, y := y, x$ **fi** $\{x \geqslant y\}$.
   (c) $\{x = 0\}$ **if** $True \to x := 1 \mid True \to x := -1\, \{x = 1 \vee x = -1\}$.
   (d) $\{A = x \times y + z\}$ **if** $even\ x \to x, y := x \mathbin{/} 2, y \times 2 \mid True \to y, z := y - 1, z + y\, \{A = x \times y + z\}$.
   (e) $\{x \times y = 0 \wedge y \leqslant x\}$ **if** $y < 0 \to y := -y \mid y = 0 \to x := -1\, \{x < y\}$.
3. What is the weakest P such that the following holds?

   var x : Int
   $\{P\}$
   $x := x + 1$
   if $x > 0 \to x := x + 1$
   $\mid\ x < 0 \to x := x + 2$
   $\mid\ x = 1 \to skip$
   fi
   $\{x \geqslant 1\}$.

4. Two programs S_0 and S_1 are equivalent if, for all Q, $wp\ S_0\ Q = wp\ S_1\ Q$. Show that the two following programs are equivalent.

   if $B_0 \to S_0 \mid B_1 \to S_1$ **fi**; S
   if $B_0 \to S_0; S \mid B_1 \to S_1; S$ **fi**

5. Consider the two programs:

   IF_0 = **if** $B_0 \to S_0 \mid B_1 \to S_1$ **fi** ,
   IF_1 = **if** $B_0 \to S_0 \mid B_1 \wedge \neg\, B_0 \to S_1$ **fi** .

   Show that for all Q, $wp\ \mathsf{IF}_0\ Q \Rightarrow wp\ \mathsf{IF}_1\ Q$.