Programming Languages: Imperative Program Construction
Practicals 4: Hoare Logic and Weakest Precondition: Loop

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1. Prove the correctness of the following program:

$$
\begin{array}{l}
\textbf{con } N : \mathit{Int}\ \{N \geqslant 0\} \\
\textbf{var } x, y : \mathit{Int} \\
x, y := 0, 1 \\
\textbf{do } x \neq N \to x, y := x + 1, y + y \ \textbf{od} \\
\{y = 2^N\}
\end{array}
$$

2. Prove the correctness of the following program:

$$
\begin{array}{l}
\textbf{con } N : \mathit{Int}\ \{N \geqslant 0\} \\
\textbf{var } y : \mathit{Int} \\
y := 1 \\
\textbf{do } y < N \to y := y + y \ \textbf{od} \\
\{y \geqslant N \wedge \langle \exists k : k \geqslant 0 : y = 2^k \rangle\}
\end{array}
$$

3. Given integers $N \geqslant 0$ and $M > 0$, the following program computes integral division $N \mathbin{/} M$. Prove its correctness.

$$
\begin{array}{l}
\textbf{con } N, M : \mathit{Int}\ \{N \geqslant 0 \wedge M > 0\} \\
\textbf{var } l, r : \mathit{Int} \\
l, r := 0, N + 1 \\
\textbf{do } l + 1 \neq r \to \\
\quad \textbf{if } ((l + r) \mathbin{/} 2) \times M \leqslant N \to l := (l + r) \mathbin{/} 2 \\
\quad \ \mid ((l + r) \mathbin{/} 2) \times M > N \ \to r := (l + r) \mathbin{/} 2 \\
\quad \textbf{fi} \\
\textbf{od} \\
\{l \times M \leqslant N < (l + 1) \times M\}
\end{array}
$$

4. The following program non-deterministically computes x and y such that $x \times y = N$. Prove:

$$
\begin{array}{l}
\textbf{con } N : \mathit{Int}\ \{N \geqslant 1\} \\
\textbf{var } p, x, y : \mathit{Int} \\
p, x, y := N - 1, 1, 1 \\
\{N = x \times y + p \wedge \ldots\} \\
\textbf{do } p \neq 0 \to \\
\quad \textbf{if } \quad p \ \textbf{mod}\ x = 0 \to y, p := y + 1, p - x \\
\qquad \mid p \ \textbf{mod}\ y = 0 \to x, p := x + 1, p - y \\
\quad \textbf{fi} \\
\textbf{od} \\
\{x \times y = N\}
\end{array}
$$

5. Prove the correctness of the following program:

$$
\begin{array}{l}
\textbf{con } N : \mathit{Int}\ \{N \geqslant 0\} \\
\textbf{var } x, y : \mathit{Int} \\
x, y := 0, 0 \\
\textbf{do } x \neq 0 \ \rightarrow x := x - 1 \\
\ \ \mid \ \ y \neq N \rightarrow x, y := x + 1, y + 1 \\
\textbf{od} \\
\{x = 0 \wedge y = N\}
\end{array}
$$

6. Prove the correctness of the following program:

$$
\begin{array}{l}
\textbf{con } N : \mathit{Int}\ \{N \geqslant 0\} \\
\textbf{var } x, y : \mathit{Int} \\
x, y := 0, 0 \\
\textbf{do } x \neq 0 \ \rightarrow x := x - 1 \\
\ \ \mid \ \ y \neq N \rightarrow x, y := N, y + 1 \\
\textbf{od} \\
\{x = 0 \wedge y = N\}
\end{array}
$$