

# Programming Languages: Imperative Program Construction

## Practicals 1: Non-Looping Constructs and Weakest Precondition

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1. Determine the weakest  $P$  that satisfies

- (a)  $\{P\} x := x + 1; x := x + 1 \{x \geq 0\}$ .
- (b)  $\{P\} x := x + y; y := 2 \times x \{y \geq 0\}$ .
- (c)  $\{P\} x := y; y := x \{x = A \wedge y = B\}$ .
- (d)  $\{P\} x := E; x := E \{x = E\}$ .

2. Assuming that  $x$ ,  $y$ , and  $z$  are integers, prove the following

- (a)  $\{True\} \text{if } x \geq 1 \rightarrow x := x + 1 \mid x \leq 1 \rightarrow x := x - 1 \text{ fi } \{x \neq 1\}$ .
- (b)  $\{True\} \text{if } x \geq y \rightarrow \text{skip} \mid y \leq x \rightarrow x, y := y, x \text{ fi } \{x \geq y\}$ .
- (c)  $\{x = 0\} \text{if } True \rightarrow x := 1 \mid True \rightarrow x := -1 \{x = 1 \vee x = -1\}$ .
- (d)  $\{A = x \times y + z\} \text{if even } x \rightarrow x, y := x / 2, y \times 2 \mid True \rightarrow y, z := y - 1, z + y \{A = x \times y + z\}$ .
- (e)  $\{x \times y = 0 \wedge y \leq x\} \text{if } y < 0 \rightarrow y := -y \mid y = 0 \rightarrow x := -1 \{x < y\}$ .

3. What is the weakest  $P$  such that the following holds?

```

var  $x : Int$ 
 $\{P\}$ 
 $x := x + 1$ 
if  $x > 0 \rightarrow x := x + 1$ 
   $\mid x < 0 \rightarrow x := x + 2$ 
   $\mid x = 1 \rightarrow \text{skip}$ 
fi
 $\{x \geq 1\}$  .
  
```

4. Two programs  $S_0$  and  $S_1$  are equivalent if, for all  $Q$ ,  $wp S_0 Q = wp S_1 Q$ . Show that the two following programs are equivalent.

```

if  $B_0 \rightarrow S_0 \mid B_1 \rightarrow S_1 \text{ fi}; S$ 
if  $B_0 \rightarrow S_0; S \mid B_1 \rightarrow S_1; S \text{ fi}$ 
  
```

5. Consider the two programs:

```

 $IF_0 = \text{if } B_0 \rightarrow S_0 \mid B_1 \rightarrow S_1 \text{ fi}$  ,
 $IF_1 = \text{if } B_0 \rightarrow S_0 \mid B_1 \wedge \neg B_0 \rightarrow S_1 \text{ fi}$  .
  
```

Show that for all  $Q$ ,  $wp IF_0 Q \Rightarrow wp IF_1 Q$ .