

Programming Languages: Imperative Program Construction

Practicals 5: Loop Constuction I

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1. Derive a program for the computation of square root.

```

con  $N : Int \{0 \leq N\}$ 
var  $x : Int$ 
squareroot
 $\{x^2 \leq N < (x+1)^2\}$  .

```

2. Find substitutions (on variables) that satisfy the following implications. (As a convention, variables start with small letters while constants start with capital letters. We assume that all variables and constants are *Int*.)

- (a) $(x = 2 \times E)[? \setminus ?] \Leftarrow x = E$.
- (b) $(x = 2 \times E + A)[? \setminus ?] \Leftarrow x = E$.
- (c) $(x = f E)[? \setminus ?] \Leftarrow x = E$, for some function f .
- (d) $(x = A)[? \setminus ?] \Leftarrow x = 2 \times A + B \wedge \dots$ You may need to discover an additional side condition in place of ... to make the implication valid.
- (e) $(A = 2 \times b \times x + c)[? \setminus ?] \Leftarrow A = b \times x + c \wedge \dots$ Again you may need an additional condition in
- (f) $(A = B \times x + B + C)[? \setminus ?] \Leftarrow A = B \times x + C$.
- (g) $(A = B \times x / 2 + 2 \times C)[? \setminus ?] \Leftarrow A = B \times x + C \wedge \dots$

3. **The Zune problem.** Let D be the number of days since 1st January 1980. What is the current year? Assume that there exists a function $daysInYear : Int \rightarrow Int$ such that $daysInYear i$, with $i \geq 1980$, yields the number of days in year i , which is always a positive number. Derive a program having two variables y and d such that, upon termination, y is the current year, and d is the number of days since the beginning of this year.

- (a) How would you specify the problem? The specification may look like:

```

con  $D : Int \{0 \leq D\}$ 
var  $y, d : Int$ 
zune
 $\{???\}$ 

```

What would you put as the postcondition? In this postcondition, is 1st January 1980 day 0 or 1?

- (b) Derive the program.

4. Assuming that $-\infty$ is the identity element of (\uparrow) . Derive a solution for:

```

con  $N : Int \{N \geq 0\}$ 
con  $A : \text{array } [0..N) \text{ of } Int$ 
var  $r : Int$ 
S
 $\{r = \langle \uparrow i : 0 \leq i < N : A[i] \rangle\}$  .

```

5. Derive a solution for:

```
con  $N, X : Int \{0 \leq N\}$   
con  $A : \text{array}[0..N) \text{ of } Int$   
var  $r : Int$   
 $S$   
 $\{r = \langle \sum i : 0 \leq i < N : A[i] \times X^i \rangle\} .$ 
```