

# Programming Languages: Imperative Program Construction

## Practicals 2. Propositional Logic

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Autumn Term, 2022

Prove each of the following properties using only axioms or theorems established before it (for example, prove (3.11) using only (1.?) and (3.1) - (3.10)).

Note that there are more than one ways to prove a property. You may discover a proof that is better than the one given in the solution.

1. Prove (3.9):  $\neg(p \equiv q) \equiv \neg p \equiv q$ .
2. Prove (3.12):  $\neg\neg p \equiv p$ .
3. Prove (3.13):  $\neg\text{False} \equiv \text{True}$ .
4. Prove (3.29):  $p \vee \text{True} \equiv \text{True}$ .
5. Prove (3.32):  $p \vee q \equiv p \vee \neg q \equiv p$ .
6. Prove (3.42):  $p \wedge \neg p \equiv \text{False}$ .
7. Prove (3.43a):  $p \wedge (p \vee q) \equiv p$ .
8. Prove (3.44a):  $p \wedge (\neg p \vee q) \equiv p \wedge q$ .
9. Prove (3.65):  $p \wedge q \Rightarrow r \equiv p \Rightarrow (q \Rightarrow r)$ .
10. Prove (3.66):  $p \wedge (p \Rightarrow q) \equiv p \wedge q$ .
11. Prove (3.67):  $p \wedge (q \Rightarrow p) \equiv p$ .
12. Prove (3.68):  $p \vee (p \Rightarrow q) \equiv \text{True}$ .
13. Prove (3.69):  $p \vee (q \Rightarrow p) \equiv q \Rightarrow p$ .
14. Prove (3.78):  $(p \Rightarrow r) \wedge (q \Rightarrow r) \equiv (p \vee q \Rightarrow r)$ .
15. Prove that  $(p \Rightarrow q) \wedge (p \Rightarrow r) \equiv (p \Rightarrow q \wedge r)$ .
16. Prove that  $(r \Rightarrow)$  is monotonic with respect to implication. That is,  $(p \Rightarrow q) \Rightarrow ((r \Rightarrow p) \Rightarrow (r \Rightarrow q))$ .
17. Prove that  $(\Rightarrow r)$  is anti-monotonic with respect to implication. That is,  $(p \Rightarrow q) \Rightarrow ((q \Rightarrow r) \Rightarrow (p \Rightarrow r))$ .
18. Prove that conjunction is monotonic with respect to implication. That is,  $(p \Rightarrow q) \Rightarrow ((p \wedge r) \Rightarrow (q \wedge r))$ .
19. Prove (4.3)  $(p \Rightarrow q) \Rightarrow (p \wedge r \Rightarrow q \wedge r)$ . Start with the consequent, since it has more structure.
20. Prove (3.76d)  $p \vee (q \wedge r) \Rightarrow p \vee q$  using inequality reasoning. Start with the antecedent, since it has more structure, and use distributivity.
21. Prove  $(p \Rightarrow q) \wedge (r \Rightarrow s) \Rightarrow (p \vee r \Rightarrow q \vee s)$  using inequality reasoning. **Hint:** first remove the implication in the antecedent, distribute as much as possible, and use (3.76d) and an absorption theorem.

- 22. Prove (4.1)  $p \Rightarrow (q \Rightarrow p)$  by assuming the antecedent.
- 23. Prove  $(\neg p \Rightarrow q) \Rightarrow ((p \Rightarrow q) \Rightarrow q)$  by assuming the antecedent.
- 24. Prove  $(p \Rightarrow p') \wedge (q \Rightarrow q') \Rightarrow (p \vee q \Rightarrow p' \vee q')$  by assuming the antecedent.