Programming Languages: Imperative Program Construction
Practicals 6: Loop Constuction II

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1. Recall the maximum segment sum problem. What if we want to compute the maximum sum of *non-empty* segments?
   (a) How would you write the specification? Does the specification still make sense with N being constrained only by $0 \leqslant N$?
   (b) Derive a program solving the problem.

2. Recall the derivation of the maximum segment sum problem. Assuming that we had instead used the loop invariant $P_0 \wedge P_1 \wedge Q$, where

$$
\begin{aligned}
P_0 &\equiv r = \langle \uparrow p\ q : 0 \leqslant p \leqslant q \leqslant n : sum\ p\ q \rangle) \quad , \\
P_1 &\equiv s = \langle \uparrow p : 0 \leqslant p \leqslant n + 1 : sum\ p\ (n+1) \rangle \quad , \\
Q &\equiv 0 \leqslant n \leqslant N \quad .
\end{aligned}
$$

   Can you construct a program using the invariant above? What if the array is non-empty, that is, $1 \leqslant N$?

3. Derive a solution for:

$$
\begin{aligned}
&\textbf{con}\ N : Int\{N \geqslant 0\}; a : \textbf{array}\ [0..N)\ \textbf{of}\ Int \\
&\textbf{var}\ r : Int \\
&S \\
&\{r = \langle \uparrow i, j : 0 \leqslant i < j < N : a[i] - a[j] \rangle\} \quad .
\end{aligned}
$$

4. Derive a solution for:

$$
\begin{aligned}
&\textbf{con}\ N : Int\{N \geqslant 1\}; a : \textbf{array}\ [0..N)\ \textbf{of}\ Int \\
&\textbf{var}\ r : Int \\
&S \\
&\{r = \langle \#i, j : 0 \leqslant i < j < N : a[i] \times a[j] \geqslant 0 \rangle\} \quad .
\end{aligned}
$$