Programming Languages: Imperative Program Construction
Practicals 10: Swaps in Arrays

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1. Prove that

$\{h[0] = 0 \wedge h[1] = 1\}$ -- hence $h[h[0]] = 0$
swap h $(h[0])$ $(h[1])$
$\{h[h[1]] = 1\}$

Solution: Assume $h[0] = 0 \wedge h[1] = 1$, we have

$$\begin{aligned} & (h : h[0], h[1] \mapsto h[h[1]], h[h[0]]) \\ = \; & (h : 0, 1 \mapsto h[1], h[0]) \\ = \; & (h : 0, 1 \mapsto 1, 0) \ . \end{aligned}$$

Therefore, let $h' = (h : h[0], h[1] \mapsto h[h[1]], h[h[0]])$,

$$\begin{aligned} & \mathit{wp}\ (\mathit{swap}\ h\ (h[0])\ (h[1]))\ (h[h[1]] = 1)) \\ \equiv \; & h'[h'[1]] = 1 \\ \equiv \; & h'[0] = 1 \\ \equiv \; & 1 = 1 \\ \equiv \; & \mathit{True} \ . \end{aligned}$$