Programming Languages: Imperative Program Construction
Practicals 8: Case Studies

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1. Verify the following program.

```
$\{0 \leqslant r < b\}$
$b := b \,/\, 2$
**if** $r < b \rightarrow skip$
  $\mid\ b \leqslant r \rightarrow r := r - b$
**fi**
$\{0 \leqslant r < b\}$
```

2. Derive a $O(\log N)$ algorithm for computing square root:

```
**con** $N : Int\ \{0 \leqslant N\}$
**var** $x : Int$
?
$\{x^2 \leqslant N < (x+1)^2\}$ ,
```

   by introducing a variable y and use $P_0 \wedge P_1$ as the invariant, where

$$P_0 \equiv x^2 \leqslant N < (x+y)^2 \quad ,$$
$$P_1 \equiv 0 \leqslant k \wedge y = 2^k \quad .$$

3. Derive, again, a $O(\log N)$ algorithm for computing square root:

```
**con** $N : Int\ \{0 \leqslant N\}$
**var** $x : Int$
?
$\{x^2 \leqslant N < (x+1)^2\}$ .
```

   (a) This time, construct an algorithm using binary search. What Φ will you use? Does your program rely on the fact that x^2 is monotonic on x (that is, $x \geqslant y \Rightarrow x^2 \geqslant y^2$)?

   (b) Knowing that $x \geqslant y \Rightarrow x^2 \geqslant y^2$, after the loop terminates, what can you conclude in addition to $x^2 \leqslant N < (x+1)^2$?