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INFINITY AND METAPHYSICS

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Do space and time have limits, or do they go on infinitely? Are there smallest units of space, or time, or smallest divisions of objects, or can they be divided into infinitely many pieces? How are we to understand the infinities we find in mathematics? Metaphysicians since the Ancient Greeks have been fascinated by these questions and others about infinity.

Let us begin by considering a relatively familiar mathematical infinity. The counting numbers (1, 2, 3 …) go on infinitely: whenever you have such a number, you can always add 1. There is no greatest counting number, so the sequence has no end. When we consider the integers (… −2, −1, 0, 1, 2 …) we can see that this is a sequence that has no greatest member and also no least member – for any integer, no matter how low, you can always *subtract* 1 and get another integer. It is a sequence that is infinite in both directions.

There are some unusual features of these infinite series. One is that a part of the series can be the same size as the whole. Consider the sequence of even counting numbers (2, 4, 6 …) and the series of all counting numbers (1, 2, 3 …). The first, intuitively, only has "half" of the numbers of the second. However, they can be paired off one–one with each other. This is also true of the counting numbers and, say, the prime numbers – there are as many prime numbers as counting numbers. We can even assign a different counting number to each of the integers without leaving any out, if we order the integers in the right way. To illustrate,

Counting:	1	2	3	4	5	6	7 …
Even:	2	4	6	8	10	12	14 …
Primes:	2	3	5	7	11	13	17 …
Integers:	0	−1	1	−2	2	−3	3 …

Infinite collections can be split into non-overlapping infinite sub-collections, for example, by dividing the counting numbers into the odd and even numbers. The whole does not necessarily have more members than a part.

These results are curious, but it is easy to get used to them. Arithmetical infinities are not the only kind of mathematical infinity. In Euclidean geometry, lines (as opposed to line segments) go on in each direction without end, as do planes. Lines can be divided into line segments of greater and greater lengths, without end: and indeed there can be two divisions of a line, each adding up to an infinite length, which do not overlap.

As well as these sorts of infinities in the numbers and in geometry, we are used to infinities "in the small." Just as counting numbers can get larger and larger without end, so can fractions get smaller and smaller without end (just as there is no largest counting number n, there is no smallest fraction $1/n$). Between any two fractions there is another halfway between them, and there are infinitely many fractions between zero and one.

But what about infinities in the physical world? Even if mathematical infinities, of these familiar sorts, are well behaved, should we think the physical world can have those structures? Should we think that real space and time go on infinitely?

Space and time

The first issue of infinity about space and time that occurs to many is the issue about whether space and time extend infinitely. The consensus these days seems to be that this is an issue for cosmologists more than philosophers, though some philosophers still object to the idea that the past even *could* be infinite. (This is connected to the issue of an infinite regress of causes, discussed in the final section.) Of more philosophical interest is the question of whether space and time are infinitely divided: whether there are smaller and smaller regions or durations *ad infinitum*. This issue has been a traditional source of paradox.

Zeno of Elea is famous for a number of paradoxes about space and time, several of which use the infinite divisibility of space and time. Exactly what Zeno's arguments were and what they were intended to achieve remains a matter of controversy, so I will choose versions with more of an eye on illuminating issues of infinity than historical accuracy. Perhaps Zeno's most famous is the paradox of Achilles and the Tortoise. Swift Achilles and the very slow tortoise agree to have a race, and the tortoise is given a head start. Let us suppose the track is 100 metres long, and the tortoise has a 10-metre head start, and let us suppose Achilles travels 10 times faster than the tortoise. By the time Achilles has reached the tortoise's starting point, the tortoise has moved on to a new point (call it p_1). By the time Achilles reaches p_1, the tortoise has advanced again to a new point (p_2). And so on – no matter how many times Achilles reaches one of the ps, he has not yet caught the tortoise. He could only catch the tortoise, it seems, if he could reach the end of this infinite series: but the infinite series *has* no end. So Achilles can never catch up with the tortoise.

Obviously the above reasoning has gone wrong somewhere: fast runners *can* catch tortoises, even when the tortoises have a head start. We can even calculate when Achilles will catch the tortoise if we know their speeds; for example, if Achilles is travelling at 10 metres per second, and the tortoise 1 metre per second, Achilles will catch up with the tortoise after 1.1111111 … seconds. So what went wrong with the reasoning in the previous paragraph?

Zeno himself may have wanted to use the paradox to show that motion was an illusion, which would provide one "solution" to the puzzle. We could try denying that there is the infinite sequence of points p_1, p_2, p_3 and so on, and so deny a crucial assumption in Zeno's argument. But the most popular solution to the puzzle is to allow that, after all, Achilles can pass through the infinitely many p_n points in a finite time. Does that mean that we have to postulate an infinite series with a start and a finish? (Something I imagine Zeno would claim is just a contradiction in terms – a series with first and last members seems *finite*.) In one sense, yes, and in another sense, no. Since the infinity of points are all found in a line in a finite segment of space, we must be able to find a point after all of those points: the point where Achilles is adjacent to the tortoise comes after every point where he has not yet caught the tortoise up. In that sense the space where all the p_n are found has an end. However, we do not need a last p_n: *that* series of points will have no last member – it is just that as the time goes towards 1.11111 … seconds, the p_ns get closer and closer together. So the p_n series has no last member.

Another paradox of motion attributed to Zeno concerns the impossibility of an object starting motion in the first place. (Let me call this the "paradox of the arrow," though names for this paradox vary.) Consider an arrow fired at a target. Before it reaches the target, it must reach halfway. Before that, it must reach a quarter of the way, and before that, an eighth, a sixteenth, etc. Before the arrow can even get halfway to where it needs to go, it needs to travel through an infinite number of points. But that argument did not require us to focus on the target (or on arrows): anything that moves at all to another place would first have to make it to halfway, and before that a quarter … any mover has to complete infinitely many motions before it can get anywhere. So, Zeno concludes, motion is impossible.

One interesting thing about this second paradox is that it concerns an infinite series of points with a clear end (reaching the target, for example), but no beginning: no matter how far back we go towards the arrow's start, there are still midpoints it must reach before it can get any further. There is no first one of these points the arrow can reach – to have moved any distance the arrow must already have passed through many points (infinitely many, in fact). This matches the mathematical structure of the number line. There is no first fraction after zero, for example: pick any rational number to be the "first" and we can always find another one closer to zero by halving it. The application to physical space strikes some people as more troubling: how can an arrow start moving without there being a first place it moves to? But at least on the orthodox conception of space being made up of points at positive distances from each other, there is no such first place.

A third paradox of Zeno's (which I will call the "paradox of plurality") raises an interesting question about what space is like if it has infinitely many parts. Zeno invites us to consider what size the smallest parts are. If they are some positive, finite size, then when we add infinitely many of them together we will get an infinite magnitude. But if they are all of zero size (as points are often conceived of as being), then even adding together infinitely many of them will still give us zero. The first horn of the dilemma may not strike someone as particularly worrying: maybe space *is* infinite in size? The problem that makes it so uncomfortable is that the ordinary view of space has infinitely many points even in a finite region, such as the distance an arrow traverses to hit its target. Even if space as a whole is infinitely large, we do not want to say that the space on a typical archery range is infinitely large! Notice the same problem arises for time – time seems divisible into smaller and smaller intervals, so what are we to say about the smallest intervals, if any?

Perhaps this could motivate us to reject infinitely many parts of space after all. Indeed, some people believe that space is "granular" and only finitely many smallest pieces of space can be found in a space of finite magnitude. Another, slightly subtler response is to say that space has infinitely many parts, but no *smallest* parts – regions get smaller and smaller *ad infinitum*, just as we can find smaller and smaller distances from zero along the number line, but there are no zero-length points underneath. Or perhaps space has only a *potential* infinity of small parts – it *can* be divided smaller and smaller, but there are not already smaller and smaller regions corresponding to such divisions. Each of these alternatives to infinitely many zero-magnitude points will be discussed further below.

There is another answer available to someone who wants to maintain the orthodoxy of a space of points without positive finite size. This relies on the *kind* of infinity of points postulated in contemporary theories of space. According to standard theories of space, space is a *continuum*. It is dense (between any two points there is another) and complete, in a "no-gaps" sense (which has a variety of technical characterisations). Space has a structure like that of the real numbers, rather than just that of the rationals: there are lengths of $\sqrt{2}$ metres and π metres, as well as lengths expressible as ratios of integers.

Surprisingly, while the rational numbers and the real numbers are both infinite, there are strictly speaking more real numbers than rational numbers: the real numbers cannot be put in a one–one correspondence with the rational numbers. There are, in fact, many sizes of infinity in mathematics (infinitely many, in fact!). The size corresponding to the real numbers is often known as *continuum many*, while the size associated with the rationals (which is the same as for the counting numbers) is known as *countably many*.

When adding the sizes of finitely many non-overlapping regions together at once, the obvious thing to do is to assume the resulting larger region has the size which is the sum of the sizes of the smaller regions. A straightforward extension of this procedure covers adding countably many sizes together, and indeed on the standard picture the metric on space is *countably additive*. But it is less straightforward to extend this to the case of "adding" continuum-many regions – it cannot be done as limits of longer and longer finite sequences of addition, for example. When we want to know the size of a region made up of continuum-many distinct smaller regions, we cannot "add" the smaller sizes. Instead, the size of the large region is specified by a *measure* on the smaller regions, not by "addition."

Adolf Grünbaum (Grünbaum 1967) famously pointed out that this gives contemporaries another kind of response to Zeno's paradox of plurality. Zeno claimed that if we put together lots of things of size zero, the resulting size will be zero. But if we take a measure of a set of continuum-many size-zero points, the measure need not be zero. We can *agree* with Zeno that additions of zero (even countably infinite additions) always give zero, and nevertheless hold that a space made up of enough size-zero points is not itself of size zero.

The mathematical tools to make this distinction were not properly clarified until the twentieth century, so it is no surprise that Zeno did not consider this response. Suppose Zeno objected that this was mere mathematical trickery – we are still getting positive magnitude from putting together things of zero magnitude, even if the "putting together" is not addition. How is this any improvement? In response, we could say that the intuition that "putting together zeros" gives you zero is an intuition we have about addition – and we have conceded that addition does work this way. There are coherent models of measure theory that show us another way of "putting together." Of course, that by itself would not show that we can put together spatial points and get non-zero-sized spatial regions; but it does tend to undercut Zeno's argument that we *could not* get regions of non-zero finite size if we started from points.

A host of contemporary puzzles about the infinite divisibility of space and time, and what possibilities there are if actual infinities of objects are possible, have flowered in the second half of the twentieth century. These puzzles often go under the heading of "new Zeno." They include Thomson's Lamp (see the discussion reprinted in Salmon [1970]), Hilbert's Hotel (Gamow 1946), and many other fascinating scenarios introduced by José Benardete (Benardete 1964). Some have thought these raise new problems for the actual infinite (Craig 1979), but many have just drawn the conclusion that actual infinities can be used to generate surprising thought experiments.

Alternatives to continuous space and time

There are a number of alternatives to the view that space and time are made up of infinitely many points. You could believe that space and time are *granular*, and that there are minimum lengths and durations. If there are minimum lengths and times, the arguments of Zeno's given do not get off the ground: there is no guarantee that there will be any "halfway points" for the arrow to pass through, for example. (Consider an arrow that is only three spatial minima away from its target.) While the tortoise will have moved on from some places that it was when Achilles reaches him, there will be no guarantee that the tortoise will have moved on from all of them. (If Achilles moves 10 minima every time the tortoise moves 1, for example, if Achilles starts off 5 minima behind, the tortoise will not have moved when Achilles comes up level.)

Minimum spaces and times do bring with them puzzles of their own, however. Are speeds restricted to certain values, n minima of space per m minima of time? If someone moves 2 spatial minima per minimum of time, do they "jump"? After all, there is no instant of time for them to be in the middle space minimum. Shapes are also puzzling without continuous space. In standard geometry, the ratio of the hypotenuse of a right-angled triangle to the sides is often irrational: for example, when a right-angled triangle has the two sides of 1 metre, the hypotenuse will be $\sqrt{2}$ metres. But in a discrete space, it is hard to see how to get distances like $\sqrt{2}$ metres. Indeed, if we model discrete triangles by drawing a grid, and represent distances by the number of adjacent squares different squares are apart, then we get odd results if we draw a right-angled "triangle" with short sides of e.g. 5 squares long. If we allow squares that touch at a vertex to be adjacent, then the hypotenuse is 5 squares long! And if we insist that squares are adjacent only if they share a side, then the hypotenuse is 10 squares long! A model of space that treats $5\sqrt{2}$-metres as being either 5 metres or 10 metres comes with a serious cost. There are more sophisticated models of granular space: Forrest (1995) has an interesting though very technical discussion of a more plausible option.

What if space has smaller and smaller regions in it but it never grounds out in components of zero size (or parts without size at all, if you prefer to treat points as lacking volume altogether)? This picture of space is often called "gunky," connected to "gunk," the technical term for an object such that all of its proper parts have proper parts (i.e. parts other than themselves). The paradox of plurality, as presented, is blocked at the beginning: there are no "smallest parts" of space, on this view. The gunky view of space can allow that space has infinitely many parts of more than zero size, but they only add up to a finite amount because there are not infinitely many *non-overlapping* parts of the same finite size. In 1 metre of space, you can find 10 non-overlapping parts of 10 centimetres, or 100 of 1 centimetre or 1000 of 0.1 centimetre …, but when you add up non-overlapping parts of the same size, you never get more than 1 metre. Since there are infinitely many parts, the gunk theorist has to agree that the arrow passes through an infinite sequence of distances before it hits the target, and that Achilles must run through infinitely many distances to catch the tortoise. The gunk theorist is a friend of infinity. Gunky space has been less discussed than its main rivals, but one worry about it is discussed under "Infinite-regress arguments," below.

Another option is to say that the infinite divisibility of space is only a "potential infinity." Insisting that infinities in the world are only "potential" is a tradition that goes back to Aristotle, and it is safe to say that this was the dominant tradition in the West until the twentieth century. A "potential infinity" could mean one of two things. The first is that there is, in fact, only a finitude, but it is just that this finite collection *could be increased* or *could be extended*. For example, one way for the counting numbers to be potentially infinite in this way would be if only finitely many of them exist but we can always extend the collection of counting numbers by adding 1. Another way, less connected with us and our adding activities, would be to think that there were only as many numbers as objects but that the highest number *could have been larger*, without finite limit, if extra objects had existed.

The other way to think about potential infinities is to think that there is indeed an infinite collection (regions of space, counting numbers, etc.), but that many members of that collection only have *potential existence*: they are not yet "actualised." For example, an Aristotelian might think that a point only has actual existence when it is a boundary of a real thing (e.g. when it is at the tip of the arrow) – those points at places where boundaries could be, but are not at the moment, could be considered *potential existents*. This version of potential infinities is not very popular today, since most philosophers dislike drawing this sort of distinction between kinds of existence.

Insisting that the divisibility of time or space is only *potentially* infinite is often offered as a solution to Zeno's challenges. It deals with the paradox of plurality: if space is not *made up of* any merely potential parts, either because there aren't any potential parts (on the first view), or they lack the "actual" existence needed to make up actual space (on the second), then the question of how these small bits can *make up* something finite does not arise. How potential infinities are supposed to address the paradox of the arrow is less clear. Aristotle seemed to deny that the "halfway points" the arrow had to pass through had actual existence. But completing an infinite series of passing through points with potential existence seems just as problematic. And if we interpret the claim that the halfway points have only "potential existence" as the claim that there are no halfway points *at all* (though there could be), we have to answer awkward questions about why the arrow does not ever get halfway to its target, since there's no halfway for it to get to. Saying infinities are only potential still leaves plenty of problems to deal with.

One interesting philosophical issue is which of the alternatives correctly describe space and time, or spatial and temporal things. But another interesting issue is which of these alternatives are coherent. If only one of these alternatives is coherent, (e.g. some finitists have suggested that only the granular view is possible), then surprisingly we can tell, without having to investigate the world, some important information about the ultimate structure of space and time. Or at least the only investigation we need to do is to notice that some things move, and other elementary observations of our everyday world. To decide whether there is a scientific question about whether the world contains continuous space, for example, we need to be as clear as we can about what the alternatives would amount to.

Infinite regresses in metaphysics

Infinities are often discussed when "infinite-regress arguments" are deployed to try to prove metaphysical conclusions. Perhaps the most famous infinite-regress arguments are those for the existence of a "first cause" – arguments often employed in an attempt to prove the existence of God. It is doubtful that a proof that there is a "first cause" would help very much in making the case for the existence of God – why suppose a first cause would be intelligent, or powerful, or beneficent, or have any other attributes commonly attributed to deities? But leaving aside theological concerns, the question of whether there is a first cause is interesting in its own right, and it would be surprising if armchair reflection about causation could establish that much about the origin of the universe.

There are dozens, if not hundreds, of versions of cosmological arguments. A simple form of the regress argument might go like this:

(1) Every natural thing has a cause.
(2) The chain of causes cannot go back infinitely.
(3) There can be no causal loops.
(4) Therefore every chain of causation must have a first cause at its beginning.

Presumably this "first cause" must be something that does not fall under the generalisation at (1): God is often offered as a candidate first cause that does not itself need a cause. Both premises (1) and (3) can be disputed, of course. Premise (1) is probably disputed more than premise (3): the idea that the Big Bang, or for that matter random quantum fluctuations, could "just happen" in an uncaused way remains a popular view.

For our purposes, though, the most interesting premise is (2). Why not think that there is a succession of cause and effect stretching back into the past without end? Of course, we might have specific evidence from cosmology that the world had a beginning (e.g. the Big Bang). But is there any principled philosophical reason to reject an infinite regress of cause and effect?

Two have traditionally been offered. One is the belief that there cannot be an "actual infinity," but only a potential infinity (see above). If the past is "actual," and we had an infinite regress of causes, we would have an infinite series of actual causes and effects already in existence. Those who claim actual infinities are incoherent will reject an actual infinity of past causes. Of course, this reasoning is only as good as that which supports rejecting actual infinities. (Interestingly, Aristotle himself, despite his rejection of actual infinities, did not object to an infinity of past causes. Aristotle seems to have thought that the past was not "actual" any more in the relevant sense: only the present was.)

The second reason to reject an infinite regress of cause and effect is that there is something incoherent in this infinite chain of dependence. However, the intuition that "the buck must stop somewhere" is hard to argue for. A defender of the claim could suggest that when we explain an effect by reference to its cause, for example, that explanation is somehow incomplete unless we explain the cause's occurrence as well: and if that in turn is explained by reference to a further cause, a further explanation is called for, and so on, and somehow there is something unsatisfactory unless that sequence comes to an end.

Of course, it is hard to see how such a sequence *could* end if we do insist that each cause is an unsatisfactory explanation until it itself is explained: any "first cause" will have this problem, and postulating a god as a first cause will not make the problem go away. Indeed, the challenge of trying to explain a god or that god's existence can seem even more intractable than explaining the Big Bang. Theists interested in using this sort of argument usually have some special pleading about how God is his own cause or explanation in the way that ordinary events and things are not, or that God is a necessary being and necessary beings do not need a further cause or explanation.

Other metaphysical infinite-regress arguments also rely on the idea that chains of dependence must ground out somewhere. Some people think that wholes depend on their parts – once you have the parts and the relations between them, nothing more is needed for the whole, whether we are talking about heaps of sand, tables and chairs, or human bodies. Some of them are tempted by a regress argument against the view that there is *gunk* (see above). If an object (or a region of space or duration of time) is made up of parts, and they are made up of parts, and so on for ever, then the whole structure will not "ground out" in ultimate parts that themselves have no proper parts. (Objects without proper parts are often called *simples* or *atoms*, though this use of "atom" is to be distinguished from the word's use in chemistry.) It should be clear how the infinite-regress argument would go.

If there is gunk, then the parts of objects have (proper) parts, and they have (proper) parts, and so on *ad infinitum*, without ever reaching a level of ultimate parts.

But this chain of dependence must ground out – it cannot go on for ever.

Therefore there is no gunk.

Here it is not infinity *per se* that is the problem: many of those who think gunk is impossible do believe in the possibility of an object made up of infinitely many simples (and so "infinitely divisible" in at least one good sense). It is rather the never-ending chain of parthood that is supposed to be the problem.

Others will want to deny that there is dependence of a whole on its parts. For example, one might think that the whole and its parts are equally well existing objects, and while they stand in a particularly intimate relationship, it does not follow that either is *dependent* on the other. Others may even want to think that parts sometimes, or always, depend on their wholes, and not vice versa. (Those who think dependence runs both ways will at least believe in the sort of dependence necessary for the argument above.)

Still others may be happy to concede that wholes depend on their parts, but be happy with infinite chains of dependence that do not "ground out" in partless parts that do not depend, in this way, on anything else. In the "first-cause" case, many are happy with the idea that there could be never-ending chains of causal dependence that do not stop with an uncaused cause. In the gunk case, many are happy with chains of part–whole dependence that do not "ground out" in partless parts (i.e. parts with no proper parts). Getting beyond this unease to a consideration that will convince their opponents, however, is a goal that has remained elusive.

Other topics

There are a number of other places in metaphysics where infinities play an important role, though there will be space to do little more than mention these topics. Readers who find these tastes particularly interesting can follow up the further references given below. In no particular order, other philosophical issues involving infinity include the following.

Are there *infinitesimal* magnitudes? An infinitesimal is less than any positive real number, but greater than zero. (Consistent mathematical models of infinitesimals have been developed, but they are not part of classical mathematics.) If infinitesimals have applications in the real world (e.g. there are infinitesimal distances or masses or objective chances), what impact does that have on standard theories (e.g. of space, time or chance)?

Some philosophers want to defend *strict finitism* – the view that there are not, and could not be, infinitely many objects. Strict finitists sometimes want to argue that there is something wrong with the concept of infinity, others just that it is unnecessary. One interesting question pursued by strict finitists is how much of our ordinary mathematical practice can be recaptured if there are only finitely many numbers or mathematical sets.

There are issues about infinities and abstract objects. How many possible worlds and possibilities are there? How many propositions are there? Presumably there are at least infinitely many of each, if they exist at all. But coming up with principles about the cardinality of these things is not straightforward.

Infinities come in different sizes. What is the largest size like, if there is one? (If there is not one, why does everything all together not get a size?) Georg Cantor, who first proved that there are infinities of different sizes, believed that there is a greatest infinity, the Absolute Infinite, which is not entirely understandable by us. Some contemporary "class theorists" believe that there are *proper classes*, collections too big to be sets. If proper classes are all of the same size, this would be the largest size.

Is God infinite? If so, in what sense? In some relatively mundane sense (e.g. being located at infinitely many locations in space or time, or being able to employ forces of infinite magnitude, or to do infinitely many different kinds of things), or in a special sense, or in both? If God is infinite in some distinctive sense, how is that sense related to the other mathematical and physical senses of infinity discussed here?

References

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Further reading

General: G. Oppy, *Philosophical Perspectives on Infinity* (Cambridge: Cambridge University Press, 2006) is perhaps the best book-length treatment of the role discussions of infinity play in contemporary philosophy, and certainly the most up to date (at the time of writing). It is worth consulting on most of the issues discussed in this article. A. Moore, *The Infinite* (London: Routledge, 1990) is a useful introduction to the history of debates about infinity, and contains a discussion more sceptical of theories of the infinite than this article. Zeno's paradoxes: Chapter 1 of R. M. Sainsbury, *Paradoxes*, 2nd edn (Cambridge: Cambridge University Press, 1995) contains a very accessible introduction to Zeno's paradoxes. This classic collection, W. Salmon (ed.), *Zeno's Paradoxes* (Indianapolis, IN: Bobbs-Merrill, 1970) contains not only important papers on Zeno's paradox, but an exchange between James Thomson and Paul Benacerraf about Thomson's Lamp. Chapters 12 and 13 of J. Barnes, *The Presocratic Philosophers*, revised edn (London: Routledge & Kegan Paul, 1982) contain interesting discussions of Zeno's puzzles in their historical context. J. Benardete, *Infinity: An Essay in Metaphysics* (Oxford: Clarendon Press, 1964) is a fascinating book for those intrigued by "New Zeno" paradoxes. Alternatives to continuous time and space: A philosophical introduction to treatments of space that is not infinitely divisible is J. Van Bendegem, "Finitism in Geometry," in Edward N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy* (Summer 2002 edn) (available: http://plato.stanford.edu/archives/sum2002/entries/geometry-finitism/). See also P. Forrest, "Is Space–Time Discrete or Continuous?" *Synthese* 103, no. 3 (1995): 327–54. A good introduction to arguments about parts and wholes, including arguments about gunk, is H. Hudson, "Simples and Gunk," *Philosophy Compass* 2, no. 2 (2007): 291–302. Infinite-regress arguments: D. Nolan, "What's Wrong with Infinite Regresses?" *Metaphilosophy* 32, no. 5 (2001): 523–38. In this article I discuss infinite-regress arguments in general, and offer a reason to be suspicious of infinite regresses other than the reasons discussed above. Other topics: J. Bell introduces the history of infinitesimals and some of the debates about them in the history of philosophy in "Continuity and Infinitesimals," Edward N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy* (Fall 2005 edn) (available: http://plato.stanford.edu/archives/fall2005/entries/continuity/). P. Grim, *The Incomplete Universe* (Cambridge, MA: MIT Press, 1992) raises an interesting series of size-related paradoxes about totalities of propositions, possibilities, mathematical objects and so on. Grim's own view is that these abstract objects do not form totalities at all. M. Hallett, *Cantorian Set Theory and Limitation of Size* (Oxford: Clarendon Press, 1984) is an excellent introduction to Cantor's own views about sets and cardinality, and the Absolute Infinite.