Zeno 1

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Motion does not exist

1. Space is infinitely divisible or not infinitely divisible.
2. If space is infinitely divisible, motion is impossible.
3. If space is not infinitely divisible, motion is impossible.
4. Motion is impossible (From 1-3).

Premise 1 - the divisibility of space

- If x is infinitely divisible, x can be divided into ever smaller parts *ad infinitum*. In other words, x contains no indivisible parts, i.e. parts that cannot further be divided.
  - For example, suppose that a line, L, is infinitely divisible. Lines are divided into line segments. So every line segment of L can be divided into further smaller line segments - there is no smallest line segment.
  - Think of this process of dividing something out as merely conceptual. Don't worry whether or not we could literally do something to x to divide it in this way.
- If x is not infinitely divisible, x can be divided into a finite number of *smallest* parts, i.e. parts that cannot be divided into any smaller parts.
  - For example, suppose that a line, L, is not infinitely divisible. Then L contains a finite number of smallest line segments, i.e. line segments with some smallest extent that cannot be divided into any further line segments.

Premise 2

This handout will proceed by discussing Premise 2. See Handout 2 for discussion of Premise 3. I first outline Zeno's argument for Premise 2. I then examined 3 different

Strategy: Assume that space is infinitely divisible. Then argue that it is impossible to move from one place to another by showing that (a) doing so requires completing an infinite number of tasks, and (b) it is impossible to complete an infinite number of tasks.

Zeno argues for premise 2 by using a number of paradoxes. The first is called *Achilles and the Racecourse*:

Aristotle text

It is impossible to complete an arbitrary journey from A to B - to start at A, move to B, and then stop:

A. The distance between A and B is infinitely divisible (assumed).

B. A journey from A to B is a series of sub-journeys with no last member: from A to $\frac{1}{2}AB$, from $\frac{1}{2}AB$ to $\frac{3}{4}AB$, and so on.

C. It is impossible to complete a series of sub-journeys with no last member.

D. Completing a journey from A to B, requires completing the series of sub-journeys with no last member: from A to $\frac{1}{2}AB$, from $\frac{1}{2}AB$ to $\frac{3}{4}AB$, and so on.

E. It is impossible to complete a journey... (from A to D.)

Response 1: Reject C

There are two different ways we might reject Premise 3. The first is to claim that as we divide the distances of the journey, we should also divide the total time taken.

- It takes $\frac{1}{2}$ the time to run from A to $\frac{1}{2}AB$ as it does to run from A to B.
- It takes $\frac{1}{4}$ of the time to run from $\frac{1}{2}AB$ to $\frac{3}{4}AB$, and so on.
- The sum of these decreasing times is finite.
- Therefore, we can complete a infinite series of sub-journeys in a finite period of time.

This first attempt assumes that the argument for Premise 3 is the following:

a. Completing an infinite series of tasks would take an infinite amount of time.

b. It is not possible to spend an infinite amount of time completing some task(s).
c. Therefore, it is not possible to complete an infinite series of tasks.

The current response claims that this argument, while valid, is not sound because, it claims, (ii) is false. However, there is an alternative way of defending Premise 3 that is immune to this objection:

- Even if it takes less time to complete each sub-journey, don't I still need to first complete each sub-journey before completing the journey that comes after it? If so, how does this solution help?

Note that the response assumes that time is infinitely divisible, i.e. divisible into a infinite number of finite parts. Do you think that time is so divisible?

Response 3: Reject (C)

A second attempt at denying Premise C is to argue that infinite series can have finite sums. Consider this equation: $S = 1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000}$.

The value of S is $1\frac{1}{9}$.

Similarly, even though the journey from A to B contains an infinite number of finitely long sub-journeys, the journey from A to B still has only finite length. So you complete a series of sub-journeys with no last member by simply completing the journey from A to B.

- `Does this response speak to the problem?`

Response 3: Reject B

Aristotle claims that a journey from A to B is a series of *potential* and not *actual* sub-journeys, i.e. the full journey does not consist of actual parts each of which are sub-journeys.

(Text of Physics)

In order to evaluate this response, we need to investigate the nature of actions/activities and try to understand what is involved in completing them.