Zeno 1

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Motion does not exist

1. Space is infinitely divisible or not infinitely divisible.
2. If space is infinitely divisible, motion is impossible.
3. If space is not infinitely divisible, motion is impossible.
4. Motion is impossible (From 1-3).

Premise 1 - the divisibility of space

- If x is infinitely divisible, x can be divided into ever smaller parts *ad infinitum*. In other words, x contains no indivisible parts, i.e. parts that cannot further be divided.
  - For example, suppose that a line, L, is infinitely divisible. Lines are divided into line segments. So every line segment of L can be divided into further smaller line segments - there is no smallest line segment.
  - Think of this process of dividing something out as merely conceptual. Don't worry whether or not we could literally do something to x to divide it in this way.
- If x is not infinitely divisible, x can be divided into a finite number of *smallest* parts, i.e. parts that cannot be divided into any smaller parts.
  - For example, suppose that a line, L, is not infinitely divisible. Then L contains a finite number of smallest line segments, i.e., line segments with some smallest extent that cannot be divided into any further line segments.

Premise 2

Strategy: Assume that space is infinitely divisible. Then argue that it is impossible to move from one place to another by showing that (a) doing so requires completing an infinite number of tasks, and (b) it is impossible to complete an infinite number of tasks.

Zeno argues for premise 2 by using a number of paradoxes. The first is called *Racecourse*, which argues that it is impossible to complete an arbitrary journey from A to B - to start at A, move to B, and then stop.

> The first asserts the non-existence of motion on the ground that that which is in locomotion must arrive at the half-way stage before it arrives at the goal. (Aristotle Physics, 239b11)

A. The distance between A and B is infinitely divisible (assumed).

B. A journey from A to B is a series of sub-journeys with no last member: from A to $\frac{1}{2}AB$, from $\frac{1}{2}AB$ to $\frac{3}{4}AB$, and so on.

C. It is impossible to complete a series of sub-journeys with no last member.

D. Completing a journey from A to B, requires completing the series of sub-journeys with no last member: from A to $\frac{1}{2}AB$, from $\frac{1}{2}AB$ to $\frac{3}{4}AB$, and so on.

E. It is impossible to complete the journey from A to B.

- An inverted version of the paradox shows us that our traveler cannot begin to move.
- A different paradox, the Achilles paradox, shows us that in a race between Achilles and a tortoise, where the tortoise is given a head start, Achilles could never catch-up and pass the tortoise.

> The [second] argument was called "Achilles," accordingly, from the fact that Achilles was taken [as a character] in it, and the argument says that it is impossible for him to overtake the tortoise when pursuing it. For in fact it is necessary that what is to overtake [something], before overtaking [it], first reach the limit from which what is fleeing set forth. In [the time in] which what is pursuing arrives at this, what is fleeing will advance a certain interval, even if it is less than that which what is pursuing advanced . . . . And in the time again in which what is pursuing will traverse this [interval] which what is fleeing advanced, in this time again what is fleeing will traverse some amount . . . . And thus in every time in which what is pursuing will traverse the [interval] which what is fleeing, being slower, has already advanced, what is fleeing will also advance some amount. (Simplicius(b) On Aristotle's Physics, 1014.10)

Response 1: Reject C

Some deny premise C by claiming that as we divide the distances of the journey, we should also divide the total time taken, and, further, that the sum of these infinite series of decreasingly short time intervals is still equal to a finite period of time. These denials assume that the argument for C is the following:

- C1. Completing an infinite series of tasks would take an infinite amount of time.
- C2. It is not possible to spend an infinite amount of time completing some task(s).
- C. Therefore, it is not possible to complete an infinite series of tasks.

This argument for Premise C is valid, but some deny that it is not sound because C1 is false. They claim that it relies on the false assumption that completing an infinite series of tasks would take an infinite period of time. This seems false. As we divide the distances between the points we travel, we should also divide the time it takes to travel the ever smaller distances:

- It takes $\frac{1}{2}$ the time to run from A to $\frac{1}{2}AB$ as it does to run from A to B.
- It takes $\frac{1}{4}$ of the time to run from $\frac{1}{2}AB$ to $\frac{3}{4}AB$, and so on.
- The sum of these decreasing times is finite.[1]
- Therefore, we can complete an infinite series of sub-journeys in a finite period of time.

Let us grant that C1-C2 fail to establish Premise C. There is an alternative way of defending Premise C that is immune to our current objection:

- Even if it takes less time to complete each sub-journey, I still need to first complete each sub-journey before completing the journey that comes after it. If so, I always have one more sub-journey to complete before I can complete the final step.

Note that the response assumes that time is infinitely divisible, i.e. divisible into a infinite number of finite parts.

[1] See ‘A Contemporary Look at Zeno’s Paradoxes’, by Wesley Salmon

Response 2: Reject B

Aristotle claims that a journey from A to B is a series of *potential* and not *actual* sub-journeys, i.e. the full journey does not consist of actual parts each of which are sub-journeys. The point here is that Zeno is defining journeys in terms of the space over which you travel. Aristotle responds by denying that journeys are individuated in this way.

In order to evaluate this response, we need to investigate the nature of actions/activities and try to understand what is involved in completing them. In other words, if journeys are not individuated by the distance over which they occur, how are they individuated?

Premise 3

Recall that premise 3 states: If space is not infinitely divisible, motion is impossible. Zeno offers two distinct arguments for Premise 3 that again come in the form of paradoxes. The strategy for each is similar. We will fist assume that space is not infinitely divisible, then prove that certain absurdities follow. If an assumption leads to an absurdity, we know the assumption is false.

Stadium Paradox

> The fourth argument is that concerning equal bodies which move alongside equal bodies in the stadium from opposite directions—the ones from the end of the stadium, the others from the middle—at equal speeds, in which he thinks it follows that half the time is equal to its double.... (Aristotle Physics, 239b33)

Suppose these rows of blocks represent some chariots in a stadium. The B's are stationary. The A's are moving from left to right. The last block in that row is called D. The Cs are moving towards from right to left. The middle block in that row is called E.

[c]@lll@ & T1 && DAA & –>& BBB &<– & CEC &

[c]@rll@ & T3 &–> & DA & A& BBB & C & EC & <–

Compare Times 1 and 3. Suppose they are separated by a one minute interval. In this interval, D has passed one B block and two C blocks. Zeno thinks this is paradoxical. It's unclear why. For our purposes, let us assume the following:

1. There is smallest possible length, S
2. The length of each block is S.

3. There are no gaps between the blocks.
4. The blocks move with constant velocity.

It took 1 minute for D to pass two C blocks. It should take 30 seconds to pass one C block and become level with E. Suppose D passes one C block after 30 seconds. How many B blocks has it passed? Try filling out the diagram below to answer that question.

[c]@rll@ & T2 &–> & DAA && ? &   C & EC & <–

T3 This describes the moment that D and E are level. How does D relate to the B's at this moment?

We are stuck! Suppose that someone claims that D has passed *half of one B block.* Let this half be called *H*. What is H's length? You cannot, on pain of contradiction, claim that H has a length less than S. We have assumed that S is the smallest possible length, so H cannot be shorter than S.

This way of stating the paradox assumes that the length of time between T1 and T3 can be divided in two, i.e., 1 minute is divided into two 30 second intervals. Suppose that time is also atomic, that there is a smallest interval of time, a single quantum of time. Suppose also that the motion between T1 and T3 takes a single quantum of time. If this is correct, there is no T2 (which was half the interval between T1 and T3.)

Paradox still threatens. During a single quantum of time, D and E will have passed each other (as is seen in T3), but there is no moment at which they are level as is described in T2: since T1 & T3 are separated by the smallest possible time, there can be no instant between them—it would be a time smaller than the smallest time from the two moments we considered. Conversely, if one insisted that if they pass then there must be a moment when they are level, then it shows that cannot be a shortest finite interval.

The Arrow Paradox

> The third is ... that the flying arrow is at rest, which result follows from the assumption that time is composed of moments ... . he says that if everything when it occupies an equal space is at rest, and if that which is in locomotion is always in a now, the flying arrow is therefore motionless. (Aristotle Physics, 239b.30)

> Zeno abolishes motion, saying "What is in motion moves neither in the place it is nor in one in which it is not". (Diogenes Laertius Lives of Famous Philosophers, ix.72)

Outline of the Paradox

Assume the following claims:

1. Space is finitely divisible.
2. Time is composed of moment.

- P1. An arrow must occupy a space equal to itself at each moment that it exists.
- P2. If an arrow moves for, say, 1 minute, the arrow will occupy a space equal to itself at each moment that is it moving.
- P3. An arrow that occupies a space equal to itself at a specific moment is not moving in that moment in that space.
- P4. If an arrow is not moving

Consider an arrow, apparently in motion, at any instant. First, Zeno assumes that it travels no distance during that moment—'it occupies an equal space' for the whole instant. But the entire period of its motion contains only instants, all of which contain an arrow at rest, and so, Zeno concludes, the arrow cannot be moving.

" a moving arrow must occupy a space equal to itself during any moment. That is, during any moment it is at the place where it is. But places do not move. So, if in each moment, the arrow is occupying a space equal to itself, then the arrow is not moving in that moment because it has no time in which to move; it is simply there at the place. The same holds for any other moment during the so-called "flight" of the arrow. So, the arrow is never moving. Similarly, nothing else moves. The source for Zeno's argument is Aristotle (Physics, Book VI, chapter 5, 239b5-32).

The Standard Solution

The standard solution to the paradox uses the "at-at" theory of motion:

- (i) being in motion involve being at different places at different times, and (ii) being at rest involves being motionless at a particular point at a particular time.

This theory asks us to distinguish two things:

a) being in motion in or during an instant.
b) being in motion at an instant.

The at-at theory accepts that the arrow cannot move during an instant, but claims that the arrow can still move at an instant. It does so by occupying different locations before and after that instant.

If this is correct, the difference between rest and motion has to do with what is happening at nearby moments and has nothing to do with what is happening during a moment. The arrow counts as moving at an instant because it occupies different locations before and after that instant. The arrow counts as being at rest at an instant because it is not located at different locations before and after that instant.

Calculus

The issue is acceleration!!!

The instant must be part of a period in which the arrow is continuously in motion. instantaneous motion from instantaneous rest.

The Arrow Paradox seems especially strong to someone who would say that motion is an intrinsic property of an instant, being some propensity or disposition to be elsewhere.

Calculus: speed of an object at an instant (instantaneous velocity) is the time derivative of the object's position. This means the object's speed is the limit of its speeds during arbitrarily small intervals of time containing the instant.

The object's speed is the limit of its speed over an interval as the length of the interval tends to zero.

The derivative of position x with respect to time t, namely dx/dt, is the arrow's speed, and it has non-zero values at specific places at specific instants during the flight. The speed during an instant or in an instant, which is what Zeno is calling for, would be 0/0 and so be undefined.

Using these modern concepts, Zeno cannot successfully argue that at each moment the arrow is at rest or that the speed of the arrow is zero at every instant. Therefore, advocates of the Standard Solution conclude that Zeno's Arrow Paradox has a false, but crucial, assumption and so is unsound.

The Future?

- The only way to rescue motion requires that we assumes that facts about an objects' location in the future are already fixed.
- Suppose God were to wipe the arrow out of existence completely an instant after it moved. This would mean that it was not moving. If the arrow, though, is moving this instant, then that means it is not destroyed in the next instant. It is true now that the arrow is not being destroyed in the future!