Zeno 2

Scott O'Connor

Motion does not exist

1. Space is infinitely divisible or not infinitely divisible.
2. If space is infinitely divisible, motion is impossible.
3. If space is not infinitely divisible, motion is impossible.
4. Motion is impossible (From 1-3).

Premise 1 - the divisibility of space

- If x is infinitely divisible, x can be divided into ever smaller parts *ad infinitum*. In other words, x contains no indivisible parts, i.e. parts that cannot further be divided.
  - For example, suppose that a line, L, is infinitely divisible. Lines are divided into line segments. So every line segment of L can be divided into further smaller line segments - there is no smallest line segment.
  - Think of this process of dividing something out as merely conceptual. Don't worry whether or not we could literally do something to x to divide it in this way.
- If x is not infinitely divisible, x can be divided into a finite number of *smallest* parts, i.e. parts that cannot be divided into any smaller parts.
  - For example, suppose that a line, L, is not infinitely divisible. Then L contains a finite number of smallest line segments, i.e., line segments with some smallest extent that cannot be divided into any further line segments.

Premise 3

This handout will proceed by discussing Premise 3. See Handout 1 for discussion of Premise 2. Zeno offers two distinct arguments for Premise 3 that again come in the form of paradoxes. The strategy for each is similar. We will fist assume that space is not infinitely divisible, then prove that certain absurdities follow. If an assumption leads to an absurdity, we know the assumption is false.

Stadium Paradox

> The fourth argument is that concerning equal bodies which move alongside equal bodies in the stadium from opposite directions—the ones from the end of the stadium, the others from the middle—at equal speeds, in which he thinks it follows that half the time is equal to its double.... (Aristotle Physics, 239b33)

Suppose these rows of blocks represent some chariots in a stadium. The B's are stationary. The A's are moving from left to right. The last block in that row is called D. The Cs are moving towards from right to left. The middle block in that row is called E.

	T1	
	DAA	–>
	BBB	
<–	CEC	

	T3	
–>	DA	A
	BBB	
C	EC	<–

Compare Times 1 and 3. Suppose they are separated by a one minute interval. In this interval, D has passed one B block and two C blocks. Zeno thinks this is paradoxical. It's unclear why. For our purposes, let us assume the following:

1. There is smallest possible length, S
2. The length of each block is S.
3. There are no gaps between the blocks.
4. The blocks move with constant velocity.

It took 1 minute for D to pass two C blocks. It should take 30 seconds to pass one C block and become level with E. Suppose D passes one C block after 30 seconds. How many B blocks has it passed? Try filling out the diagram below to answer that question.

	T2	
–>	DAA	
	?	
C	EC	<–

T3 describes the moment that D and E are level. How does D relate to the B's at this moment?

We are stuck! Suppose that someone claims that D has passed *half of one B block.* Let this half be called *H.* What is H's length? You cannot, on pain of contradiction, claim that H has a length less than S. We have assumed that S is the smallest possible length, so H cannot be shorter than S.

This way of stating the paradox assumes that the length of time between T1 and T3 can be divided in two, i.e., 1 minute is divided into two 30 second intervals. Suppose that time is also atomic, that there is a smallest interval of time, a single quantum of time. Suppose also that the motion between T1 and T3 takes a single quantum of time. If this is correct, there is no T2 (which was half the interval between T1 and T3.)

Paradox still threatens. During a single quantum of time, D and E will have passed each other (as is seen in T3), but there is no moment at which they are level as is described in T2: since T1 & T3 are separated by the smallest possible time, there can be no instant between them—it would be a time smaller than the smallest time from the two moments we considered. Conversely, if one insisted that if they pass then there must be a moment when they are level, then it shows that cannot be a shortest finite interval.