

## Zeno 3

### The Arrow Paradox

The third is . . . that the flying arrow is at rest, which result follows from the assumption that time is composed of moments . . . . he says that if everything when it occupies an equal space is at rest, and if that which is in locomotion is always in a now, the flying arrow is therefore motionless. (Aristotle Physics, 239b.30)

Zeno abolishes motion, saying “What is in motion moves neither in the place it is nor in one in which it is not”. (Diogenes Laertius Lives of Famous Philosophers, ix.72)

### Outline of the Paradox

Assume the following claims:

1. Space is finitely divisible.
2. Time is composed of moment.
  - P1. An arrow must occupy a space equal to itself at each moment that it exists.
  - P2. If an arrow moves for, say, 1 minute, the arrow will occupy a space equal to itself at each moment that is it moving.
  - P3. An arrow that occupies a space equal to itself at a specific moment is not moving in that moment in that space.
  - P4. If an arrow is not moving

Consider an arrow, apparently in motion, at any instant. First, Zeno assumes that it travels no distance during that moment—‘it occupies an equal space’ for the whole instant. But the entire period of its motion contains only instants, all of which contain an arrow at rest, and so, Zeno concludes, the arrow cannot be moving.

” a moving arrow must occupy a space equal to itself during any moment. That is, during any moment it is at the place where it is. But places do not move.

So, if in each moment, the arrow is occupying a space equal to itself, then the arrow is not moving in that moment because it has no time in which to move; it is simply there at the place. The same holds for any other moment during the so-called “flight” of the arrow. So, the arrow is never moving. Similarly, nothing else moves. The source for Zeno’s argument is Aristotle (Physics, Book VI, chapter 5, 239b5-32).

### The Standard Solution

The standard solution to the paradox uses the “at-at” theory of motion:

- (i) being in motion involve being at different places at different times, and (ii) being at rest involves being motionless at a particular point at a particular time.

This theory asks us to distinguish two things:

- a) being in motion in or during an instant.
- b) being in motion at an instant.

The at-at theory accepts that the arrow cannot move during an instant, but claims that the arrow can still move at an instant. It does so by occupying different locations before and after that instant.

If this is correct, the difference between rest and motion has to do with what is happening at nearby moments and has nothing to do with what is happening during a moment. The arrow counts as moving at an instant because it occupies different locations before and after that instant. The arrow counts as being at rest at an instant because it is not located at different locations before and after that instant.

The instant must be part of a period in which the arrow is continuously in motion.

instantaneous motion from instantaneous rest.

The Arrow Paradox seems especially strong to someone who would say that motion is an intrinsic property of an instant, being some propensity or disposition to be elsewhere.

Calculus: speed of an object at an instant (instantaneous velocity) is the time derivative of the object’s position.

This means the object’s speed is the limit of its speeds during arbitrarily small intervals of time containing the instant.

The object’s speed is the limit of its speed over an interval as the length of the interval tends to zero.

The derivative of position  $x$  with respect to time  $t$ , namely  $dx/dt$ , is the arrow's speed, and it has non-zero values at specific places at specific instants during the flight.

The speed during an instant or in an instant, which is what Zeno is calling for, would be  $0/0$  and so be undefined.

Using these modern concepts, Zeno cannot successfully argue that at each moment the arrow is at rest or that the speed of the arrow is zero at every instant. Therefore, advocates of the Standard Solution conclude that Zeno's Arrow Paradox has a false, but crucial, assumption and so is unsound.

### **Response**

Stuff before and a moment determines if its moving. That's odd.

Suppose God were to wipe the arrow out of existence completely an instant after it moved. This would mean that it was not moving. But notice here that we are assuming that the future is fine in the future.