16

THE PARADOXES OF MOTION AND THE POSSIBILITY OF CHANGE

Reality and the appearance of change

The concepts of space, time, and motion are mutually dependent, although the precise nature of their relationship is contentious. One of the purposes of the present chapter is to examine that relationship by studying certain logical and metaphysical problems besetting the concept of motion. When an object moves, it undergoes a change of its spatial location during an interval of time. Movement, thus, is a species of change—and the notion of change, as we saw in Chapter 13, is far from straightforward, admitting as it does of many important distinctions. How any kind of change at all is possible is something of a mystery, rooted partly in the mysterious nature of time itself. It is little wonder, then, that in all ages there have been philosophers who have denied the reality of change, maintaining that it belongs only to the realm of appearances. But even this claim harbours a paradox, for even if only the appearances change, *something* changes and so change is real. To deny the reality of change altogether, it seems, one must deny even the appearance of change—unless, as I very much doubt, one can make clear sense of a distinction between appearance of change and change of appearance. And yet to deny even the appearance of change is to repudiate the very phenomena which have led philosophers to suppose that there is a problem about change in the first place. There cannot even appear to be a problem, it seems, unless change in some sense is real. For this reason, we are likely to find any argument which purports to show that change is impossible especially perplexing.

Motion presents a crucial challenge for the thesis that change is not illusory, for no change seems more real than movement and if movement can be shown, for any reason, to be impossible, then it will be difficult to

resist the perplexing conclusion that change in general is impossible. This helps to explain the importance and abiding interest of the famous paradoxes of motion that we owe to Zeno of Elea, which are the central concern of the present chapter. The paradoxes are four in number and go by various names, but, following venerable precedent, I shall call them the Racecourse, the Achilles, the Arrow, and the Moving Blocks. I shall not be at all concerned with the history of the paradoxes or the ancient textual evidence for their provenance, their original form, and the intended purpose of the philosopher who allegedly first thought of them.[1] The first two paradoxes, as we shall see, are really just variants of a single paradox, while the other two are genuinely distinct both from the first two and from each other. It is generally considered that the first two paradoxes present a greater challenge than do either the third or the fourth, which are regarded by some philosophers—wrongly, I think—as trifling or obviously confused. However, because the first two paradoxes have received considerably more attention than the others, I shall spend rather more time discussing them.

The Paradox of the Racecourse

The Paradox of the Racecourse may be set out as follows. A runner, A, has to run a finite distance—say, of 400 metres—in a finite time, by running from a point X to a point Y. He sets off at a certain time, t_0, aiming to arrive at the later time t_1. Clearly, however, before A can arrive at his ultimate destination, Y, he must first run half of the distance between X and Y, reaching the midpoint, Z, at time $t_{1/2}$, which is intermediate between t_0 and t_1. But, on arriving at Z, A still has the second half of his total distance yet to run. Before A can reach Y, then, he must first run half of that remaining distance, reaching the midpoint between Z and Y—call it Z'—at time $t_{3/4}$, which is intermediate between $t_{1/2}$ and t_1. But even on arriving at Z', A still has a quarter (half of one half) of his total distance yet to run, the first half of which (one-eighth of the total) he must run before he can reach Y, as is depicted in Fig. 16.1. Now, in our description of the runner's task, we have obviously set out upon an infinite regress. For, however close to Y the runner gets, some distance between himself and Y will still remain and he

[1] For more on the historical context of Zeno's paradoxes, see Richard Sorabji, *Time, Creation and the Continuum: Theories in Antiquity and the Early Middle Ages* (London: Duckworth, 1983), ch. 21. See also Gregory Vlastos, 'Zeno of Elea', in Paul Edwards (ed.), *The Encyclopedia of Philosophy* (New York: Macmillan, 1972), vol. viii, pp. 369–79.

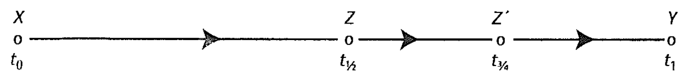

Fig. 16.1

will have to complete the first half of that distance before he completes all of it: but, having completed the first half of it, he is then just faced once more with a task of completing some distance between himself and *Y*. Thus, he cannot, it seems, complete his original task of reaching *Y*, because his completion of that task requires his prior completion of an infinite series of similar tasks.

Here it may be objected that there is no reason why the runner should not complete his original task of reaching *Y* within a finite period of time, because each remaining half-distance is shorter than the previous one and so will take him less time to complete. If the time he takes to run the first half-distance—the distance from *X* to *Z*—is, let us say, half a minute, then the time he will take to run the next half-distance, from *Z* to *Z′*, will be just a quarter of a minute (assuming that he runs at a constant speed). The next half-distance will take him one-eighth of a minute and the half-distance after that only one-sixteenth of a minute—and so on. Thus the sum of the times he takes to run the ever-decreasing half-distances may be expressed as the sum of a converging infinite series, $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots$, and it can be proved mathematically that this infinite series of ever-decreasing quantities has a sum equal to 1. In short, it can be proved that the runner will take exactly one minute to arrive at his ultimate destination and so will certainly arrive there after a finite period of time.

However, this response arguably does not go to the heart of the problem. For the problem is not to explain how the runner can arrive at his ultimate destination in a finite period of time, given that he can indeed reach that destination, but rather to explain how he can ever arrive there *at all*. For, in order to arrive there, it seems that he must complete an infinite series of tasks—running the first half-distance (from *X* to *Z*), then running the second half-distance (from *Z* to *Z′*), then running the third half-distance (from *Z′* to the midpoint between *Z′* and *Y*), then running the fourth half-distance, and so on ad infinitum. But it is impossible, surely, to complete an infinite series of tasks, since such a series has, by definition, no last member. I shall return to this claim in due course, for despite its intuitive appeal, it may rest on a confusion. But before doing so,

I want to consider some other possible responses to the Paradox of the Racecourse.

Is the paradox self-defeating?

One response which is quite tempting is to say that the paradox is self-defeating, for the following reason. In describing the task confronting the runner, we began by pointing out that before *A* can arrive at his ultimate destination, *Y*, he must first run half of the distance between *X* and *Y*. However, it may now be objected that the paradox only works on the assumption that *A* can at least run that first half-distance, from *X* to *Z*, since only if *A* can reach *Z* can there be a time at which half of the total distance still remains for him to run. And the same applies to all the other half-distances in the series. But if *A* can indeed run from *X* to *Z*, then he can complete a distance between two points, which is all that his original task amounts to. Hence, it seems, the way in which the paradox is set up concedes the very thesis that it is supposed to undermine, namely, that movement by an object from one point of space to another is possible.

However, this response, too, is questionable. For, very arguably, we should see the propounder of the paradox as presenting a *reductio ad absurdum* argument against the possibility of movement. That is to say, we should see him as presenting an argument which shows how, if we assume that movement is possible, we run into a contradiction, thus demonstrating that movement is not in fact possible. The propounder of the paradox need not be seen as conceding that it really *is* possible for *A* to run from *X* to the first midpoint, *Z*, but merely as assuming that this is possible for the sake of argument, in order to show that movement between one point of space and another is in fact *im*possible, with the implication that *A* cannot, after all, really run from *X* to *Z*.

An inverted version of the paradox

If this objection to the proposed response is not found convincing, it may be urged that there is, in any case, another way of propounding the Paradox of the Racecourse which entirely escapes the alleged difficulty. In this version, what the paradox is designed to show is not that *A* cannot ever reach his ultimate destination, *Y*, but rather that *A* cannot even *begin* to move—that he cannot leave his starting point, *X*. This alternative version of the paradox may be described as follows. As in the first version, it is pointed out that before *A* can arrive at his ultimate destination, *Y*, he must first run half of the distance between *X* and *Y*, reaching the midpoint, *Z*, at a time $t_{1/2}$ which is intermediate between t_0 and t_1. But now it may be remarked that, likewise, before *A* can reach *Z*, he must first reach the midpoint between *X* and *Z*—call it *Z′*—at a time $t_{1/4}$, which is intermediate between t_0 and $t_{1/2}$, as depicted in Fig. 16.2.

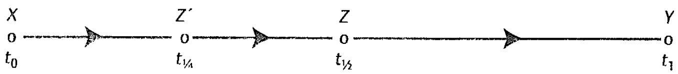

Fig.16.2

In our new description of the runner's task, we again seem to be faced with an infinite regress: before *A* can arrive at his ultimate destination, *Y*, he must first complete half of the distance from *X* to *Y*; but before he can do that, he must complete half of that half-distance; and before he can do that, he must complete half of that half-half-distance—and so on ad infinitum. But how, then, can *A* even begin to move? For in order to move any distance at all, *A* must already have moved a lesser distance: so he now seems to face an infinite series of tasks with no *first* member rather than, as in the original version of the paradox, an infinite series of tasks with no *last* member.

A common-sense objection to the paradox

Given the availability of this inverted version of the paradox, it might seem that we lose nothing by concentrating on the original version, which has the added advantage of a certain intuitive appeal. (I shall, though, return to the inverted version of the paradox later, since we shall discover reasons for doubting whether the two versions can be resolved in the same way.) Now, concerning the original version of the paradox, we must be on guard against those unphilosophically minded individuals who may be inclined to dismiss it in the style of Samuel Johnson. Boswell describes a conversation between himself and Johnson on the topic of Berkeley's view that perceptible objects are merely collections of ideas, and reports Johnson as giving a large stone an enormous kick, expostulating 'I refute it thus!'[2] The

[2] See James Boswell, *The Life of Samuel Johnson* (London: Dent, 1906), vol. i, p. 292.

implication of Johnson's remark is that everyday experience and common sense sufficiently demonstrate that stones are not merely collections of ideas; and, no doubt, he would urge with equal vehemence that they demonstrate that movement is possible. After all, it may be said, we can quite plainly *see* that people can run from one place to another, so where is the difficulty? If our runner, *A*, has a race of 400 metres to run and has a stride of two metres, then there will come a moment in the race at which he needs just one more stride to complete it—and he can surely take that last stride. What was represented as constituting an infinite series of tasks seems, to common sense, to amount in fact to a merely finite series of tasks consisting of 200 successive strides, which is easily completable by taking the final stride. However, matters are unfortunately not quite as simple as common sense suggests—and even if common sense is right in insisting that motion is possible, that in itself does nothing to remove the paradox. To take even a single stride, *A* must move his leg from one point of space to another and this is a task which is fundamentally no different in nature from the task of moving himself from *X* to *Y*. The fact that we can divide his run into 200 successive strides makes the underlying problem no easier to solve. And, in any case, exactly the same problem would arise if, instead of having *A* run from *X* to *Y*, we had him freewheeling a bicycle from *X* to *Y*, in which case we couldn't divide the continuous forward movement of the bicycle's frame into a finite series of sub-movements analogous to the successive strides of a runner.

Infinite series and supertasks

It seems, then, that we must take the Paradox of the Racecourse seriously. But perhaps it still admits of a solution which does not require us to deny the possibility of continuous movement. Recall that I suggested earlier that the root of the difficulty presented by the paradox is that it is apparently impossible to complete an infinite series of tasks, since such a series has, by definition, no last member. And we cannot deny, it seems, that the runner has to complete an infinite series of tasks—unless, of course, we deny that space is infinitely divisible. Certainly, if space is *not* infinitely divisible, then there will be a point in the race so close to *Y* that the distance between that point and *Y* is not divisible into two half-distances. And perhaps *A* will be able to move from that point to *Y* discontinuously and instantaneously. However, as we shall see when we come to discuss the

Paradox of the Moving Blocks, the notion of such discontinuous and instantaneous motion raises new problems, so it would be better not to assume that this is possible if we can resolve the Racecourse Paradox without doing so.

Let us then focus attention instead upon the claim that, because an infinite series of tasks has, by definition, no last member, it is therefore impossible to complete such a series of tasks. Much depends here on what exactly is meant by 'completing an infinite series of tasks'. If by this is meant 'completing the last task in the series after completing all of its predecessors', then, of course, we must say that it is impossible to complete an infinite series of tasks, simply because such a series has no last member. But if, as seems altogether more sensible, what we should mean by 'completing an infinite series of tasks' is just 'completing all of the tasks in the series in the order in which they occur in the series', then it is far from obvious that this is impossible. Indeed, we have already noted that we can calculate how long it will take to complete an infinite series of tasks, each of which takes half as long as the preceding one. Provided that space and time are both continuous and thus infinitely divisible, there is no problem in supposing that such an infinite series of tasks may exist, and that one does indeed exist in the case of the movement of a point-particle from one point of space to another. Our imaginary runner is no point-particle, of course, which is partly why the Paradox of the Racecourse seems so contrary to common sense and everyday experience. And, indeed, modern physics leads us to believe that point-particles do not actually exist. But these merely contingent facts have no bearing on the question of whether the paradox has any logical force. What I am now suggesting is that it does not, after all, have any logical force, but only appears to do so owing to a confusion over what should be understood by 'completing an infinite series of tasks'.

Against this suggestion, it may be urged that the notion of completing an infinite series of tasks, even if understood in the way recommended earlier, is still problematic. Consider, for instance, the well-known *Thomson's lamp* paradox, which involves such an infinite series of tasks and is thus often described as being concerned with a so-called 'supertask'.[3] The lamp possesses a button which, if pushed, will turn the lamp on if it is already off and turn it off if it is already on. Initially, the lamp is off and the

[3] See James F. Thomson, 'Tasks and Super-tasks', *Analysis* 15 (1954), 1–13, reprinted in Richard M. Gale (ed.), *The Philosophy of Time: A Collection of Essays* (Garden City, NY: Anchor Doubleday & Co., Inc., 1967).

'supertask' is to push the button an infinite number of times during the course of one minute, this being accomplished by pushing it for the first time at the beginning of the minute, for the second time after half a minute has elapsed, for the third time after three-quarters of a minute has elapsed, for the fourth time after seven-eighths of a minute has elapsed—and so on ad infinitum. We may add that no button-pushing is to occur that does not belong to this sequence. After each odd-numbered push of the button, the lamp will be on and after each even-numbered push of the button, it will be off. The problem is to say whether the lamp will be on or off when exactly one minute has elapsed. Of course, a physically real lamp would either burn out before the minute had elapsed or simply go on shining continuously after a certain point—and, in any case, there is a physical limit to how rapidly a material object like a lamp's button can move. But we are supposed to be considering the problem from a purely logical point of view, abstracting away from the constraints of physical law, which are merely contingent. This understood, it seems that we have to say that the lamp will be either on or off when exactly one minute has elapsed—there is no other state in which it can be—and yet it also seems that its being in one of those states at that time cannot be the result of any of the button-pushings in the series, because for any such button-pushing which results in the lamp's being on there is a subsequent button-pushing which results in its being off, and vice versa.

This is, indeed, a most peculiar state of affairs but not, it seems, a logically impossible one. We simply have to conclude, I think, that a world in which the 'supertask' is completable is a world in which causal determinism does not reign universally. At the end of the minute, the lamp will be either on or off, but not as a result of any button-pushing in the series. Since, as we have described the supertask, all of the button-pushings that occur do so in the course of the series, the state of the lamp at the end of the minute will not be determined by any button-pushing at all. If we also assume that only a button-pushing can causally determine the state of the lamp at any time, then it follows that the state of the lamp at the end of the minute is causally undetermined. It is important to appreciate here that every button-pushing in the series occurs *before* one minute has elapsed: there is no button-pushing in the series which occurs when exactly one minute has elapsed and so, given that the only button-pushings that occur do so in the course of the series, there is no button-pushing at all that occurs when exactly one minute has elapsed. The reason, of course, why no button-pushing in the series occurs when exactly one minute has elapsed is that the series has no last member. For, since every button-pushing in the series must occur during the course of the minute, no button-pushing in the series can occur *after* the minute has elapsed, whence it follows that if there were a button-pushing in the series which occurred when exactly one minute had elapsed, that button-pushing would have to be the last in the series—and there is no such last member.

Why does the paradox seem so compelling?

My tentative verdict concerning the Racecourse Paradox is, then, that it fails to demonstrate the impossibility of continuous motion because it is erroneously assumed that it is logically impossible to complete an infinite series of tasks, when in fact it is logically possible to complete an infinite series of tasks even within a finite period of time. As we have seen, one reason why the paradox may seem compelling is that one may have a confused understanding of what should be meant by 'completing an infinite series of tasks', taking this to mean 'completing the last task in the series after completing all of its predecessors'. But why should anyone make this mistake—why should anyone suppose that the infinite series of tasks to be completed by the runner *has* a 'last member'? For the following reason, perhaps. Clearly, in order to complete the task of running from X to Y, A must actually arrive at Y: arriving at any other point between X and Y, no matter how close it is to Y, will not suffice. But *every one* of the infinite series of tasks that A must complete in order to arrive at Y involves his arrival at a point between X and Y which is distinct from Y. It may seem, thus, that, even having completed all of those infinitely many tasks, there is still something remaining for A to do, namely, arrive at Y itself: and this, it may seem, is his 'last' task, which succeeds all his infinitely many previous tasks. But, in fact, it would seem that the proper thing to say is that, although there is no *single* task in the infinite series of tasks by completing which A arrives at Y, none the less, by completing *all* of these infinitely many tasks, A arrives at Y—there is nothing further that A need do in order to arrive there than to complete every task in the series. Thus, the *whole* task of running from X to Y, the performance of which results in A's arrival at Y, is just the sum of the infinitely many subtasks in the series, even though no one of those subtasks is such that, by performing it, A arrives at Y. This fact—if we accept that it is a fact—is sufficiently surprising, however, to make entirely understandable one's temptation to suppose, erroneously, that there must be a 'last' task for the runner to complete in order to arrive at Y. Hence, it is perfectly understandable why we should seem to be faced with a paradox, drawn as we are both to saying that the runner must have a 'last' task to complete and to saying that, because the series of tasks which he has to complete is infinite, there can be no such 'last' task for him to complete.

More on inverted versions of the paradox

It may be wondered whether this resolution of the Paradox of the Racecourse also works for the inverted version of the paradox, which is intended to demonstrate that the runner cannot even *begin* to move, because he has an infinite series of tasks to complete which has no *first* member. One reason why the inverted version of the paradox may seem to be more difficult to resolve in the foregoing fashion is this. In the original version of the paradox, the propounder of the paradox grants us—if only for the sake of argument—that the runner can complete any single subtask in the infinite series of subtasks confronting him. But then it is open to us to maintain, as we did above, that the runner's *whole* task of running from X to Y is just the sum of all those subtasks and is hence completable by him simply in virtue of his completion of each subtask in the series. But in the inverted version of the paradox, the propounder of the paradox does *not* grant us, even for the sake of argument, that the runner can complete *any* of the subtasks in the infinite series of subtasks confronting him: rather, he lets the burden of proof for this lie with us. Consequently, he will not simply allow us to assume that each of these subtasks is completable and then argue that we can, without contradiction, identify the whole task with the sum of all the subtasks. Moreover, and even more importantly, there is a significant disanalogy between the two versions of the paradox in the following respect. In the original version of the paradox, in which the question is how the runner can ever arrive at his ultimate destination, Y, the problem seems to arise because *none* of the subtasks which the runner is taken to be able to complete involves him arriving at Y. But in the inverted version of the paradox, in which the question is how the runner can ever leave his point of departure, X, *all* of the subtasks which he is required to be able to complete involve him leaving X. Hence, since the very question at issue is how the runner can ever leave X, it doesn't help us answer this question to say that his whole task of running from X to Y is the sum of infinitely many subtasks each of which involves him leaving X.

What we *could* say, though, by analogy with what we said regarding the original version of the paradox, is that the runner's whole task of running from X to Y is the sum of *another* infinite series of subtasks *none* of which involves him leaving X. Each of these subtasks has a starting point, but none of them has the same starting-point as the starting point, X, of the whole task which is the sum of all these subtasks. The starting point of the 'first' subtask in the infinite series that I now have in mind is the midpoint between X and Y, point Z; the starting point of the 'second' subtask in the series is the midpoint between X and Z, point Z' of Fig. 16.2; and so on. And the reason why I put the words 'first' and 'second' in quotation marks here is that the 'first' subtask (that of completing the distance from Z to the ultimate destination, Y) is performed *later* than the 'second' subtask (that of completing the distance from Z' to Z), and so on. To divide up the runner's whole task in this way is, indeed, just to reverse the way in which it is divided up in the original version of the paradox. In effect, it is to take the first of our two earlier diagrams, Fig. 16.1, exchange X for Y, and read time as running from right to left in the diagram, as in Fig. 16.3.

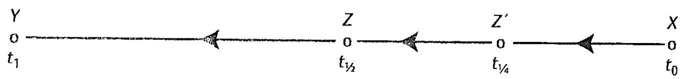

Fig. 16.3

Now, it may be noticed that this third diagram looks as though it is simply the mirror image, reflected from left to right, of our second diagram, which was this:

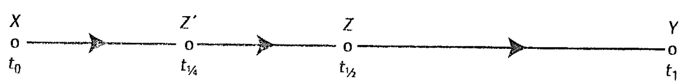

Fig. 16.2

But in this case appearances are misleading. For, as we are interpreting these two diagrams, Fig. 16.3 represents the following two subtasks of the runner: to run from Z to Y and, prior to that, to run from Z' to Z. In contrast, the subtasks that Fig. 16.2 represents are: to run from X to Z and, prior to that, to run from X to Z'. Thus, the supertask of which Fig. 16.2 is a partial representation is one in which all of the runner's subtasks are nested within the interval between X and Z, whereas the supertask of which Fig. 16.3 is a partial representation is one in which all of the runner's subtasks are strung out in a non-overlapping fashion between Y and X. Consequently, we can resolve the version of the Racecourse Paradox represented by Fig. 16.3 in a way which is modelled on our resolution of the original version of the paradox, as represented in Fig. 16.1 (of which Fig. 16.3 is merely a relabelled version). But we cannot construct a solution to the version of the paradox represented by Fig. 16.2 in exactly the same way.

So what *can* we do to resolve the version of the paradox represented by Fig. 16.2? One problem posed by this version of the paradox is that, if the runner is to move any distance at all, he must *already* have moved some (lesser) distance: if space and time are continuous, then there is no least distance for him to move and so no distance which he moves before he moves any other distance. However, our solution to the version of the paradox represented by Fig. 16.3 shows us how it can be possible for the runner to move in such a way that, for any distance that he has moved, he has already moved a lesser distance. So this problem cannot be what is distinctive of the version of the paradox represented by Fig. 16.2. In fact, I think that all that is truly distinctive of this version of the paradox is that it raises the problem of how something can *begin* to move—that is (as we might be tempted to put it), how it can be the case that at a certain instant of time an object is moving, even though it is not moving at any prior instant of time. And at the root of this problem, I think, is the more general problem of how something can be moving *at an instant of time* at all. But this, as we shall see, is the problem raised by the third of Zeno's paradoxes, the Arrow, which we shall examine shortly.

The Achilles Paradox

Before examining the Arrow Paradox, I should briefly mention the second of Zeno's four paradoxes of motion, the Achilles. This, as I mentioned earlier, is really just a variant of the Racecourse Paradox and may be set out as follows. Achilles, A, runs faster than the Tortoise, T, and accordingly A gives T a head start in a race between the two of them. Suppose that T starts at time t_0 and A starts at the later time t_1. Let P_1 be the point in the race at which T arrives when A starts to run, that is, at t_1. By the time, t_2, that A has arrived at P_1, T will have moved on to a point, P_2, some way ahead of A. Similarly, by the time, t_3, that A has arrived at P_2, T will have moved on to another point, P_3, which is still some way ahead of A. And so on ad infinitum (if space and time are continuous and thus infinitely divisible). At each successive time in this infinite series of times, T is ahead of A by a smaller distance—but T is always ahead of A by *some* distance, however small. How, then, can A ever catch up with T? Of course, it can be pointed out that the intervals of time between successive times in the series are progressively smaller, so that if A can catch up with T, he can do so in a finite period of time, since the sum of an infinite converging series of finite quantities can, as we noted earlier, be finite. But, as in the Paradox of the Racecourse, the prior question to be answered is how A can catch up with T *at all*. In effect, indeed, the Achilles Paradox is just a version of the Racecourse Paradox in which the runner, Achilles, has a moving finishing-post instead of a fixed one. That being so, however, the paradox can be resolved in the same way as we resolved the original version of the Racecourse Paradox, and for that reason I shall give it no further attention here.

The Paradox of the Arrow and instantaneous velocity

I turn next, then, to the Paradox of the Arrow. Suppose an arrow to be moving through the air. At any instant of time during its passage from one place to another, the arrow will occupy a part of space which fits it exactly. As it is sometimes put, the arrow will always take up, at any instant of time, a place exactly equal to itself. But how, then, can the arrow ever be *changing* its place—since at any instant of time it is wholly in just *one and the same* place? It is true that the arrow is supposedly in different places at different instants of time, for it must be if it is to move at all. But the problem is that we can apparently never catch it in the act of *changing* its place at any instant of time—and if it never changes its place at any instant of time, how can it be anything other than a purely stationary arrow?

One response that is often made to this supposed paradox is to say that it rests upon a misunderstanding of the nature of motion and, more especially, a misconception of what it is for an object to have a velocity *at an instant of time*. Thus, it may be said, because velocity simply *is* rate of change of distance with respect to change of time, we can only define velocity at an instant of time in terms of distance moved during periods of time surrounding that instant. More precisely, it is proposed that we should define an object's velocity at an instant of time, t, as having the limiting value towards which an infinite series of ratios converges, each member of the series being the ratio of a distance moved by the object to the amount of time taken by it to move that distance, the periods of time in quesion being smaller and smaller intervals surrounding the instant t. We can think of it in something like this way: as a rough approximation of the object's velocity at t, we can say that the value of its velocity at t, in metres per second, is equal to the distance in metres moved by the object during a period of one second which has t as its midpoint, divided by one second. A somewhat less rough approximation would be provided by reducing the period to half a second. A still less rough approximation would be provided by reducing the period to a quarter of a second. And so on ad infinitum. If we now take this infinite series of ratios of distance to time, we shall find that the successive values of the ratios converge upon a limiting value, which may be defined as the object's instantaneous velocity at t.

How is this supposed to dissolve the Paradox of the Arrow? In the following way. If instantaneous velocity should be conceived in the foregoing fashion, then it cannot make sense to ascribe a velocity—and hence motion—to an object *at an instant of time* unless the object undergoes a change of position during a period of time including that instant. No object could acquire an instantaneous velocity of, let us say, 10 metres per second, at an instant of time t, while having zero velocity at all times surrounding t and thus failing to move any distance. Clearly, an object with an instantaneous velocity of 10 metres per second at t does not move any distance at all *at t*. But it must, according to the foregoing conception of instantaneous velocity, move some distance over a period of time which *includes t*. Now, the problem supposedly raised by the Paradox of the Arrow was this. The arrow never appears to be, at any instant of time, *changing* its place—that is, moving at that very instant. But if it never moves at any instant of time, how can it be anything other than a purely stationary arrow? However, according to the conception of instantaneous velocity that has just been advanced, it is simply a mistake to suppose that what distinguishes a moving arrow from a stationary arrow at an instant of time is something that only concerns how the arrows are at that instant. So we should not be at all surprised by the fact that there is no *apparent* difference between a moving and a stationary arrow at an instant of time—and certainly should not infer that, because there is no apparent difference between them, there can be no real difference between them. For, if the foregoing proposal is correct, the real difference between the arrows at an instant of time, *t*, arises from real differences between them at *other* instants of time before and after *t*—in particular, from the fact that the moving arrow is, but the stationary arrow is not, in different places before and after *t*.

However, the foregoing conception of instantaneous velocity is not invulnerable to challenge (as we began to see in Chapter 13). One problem with it is this. Because, on this conception, the velocity of an object at an instant is *defined* in terms of distances moved by the object during periods of time surrounding the instant, it seems that we cannot explain *why* an object changes its position over a period of time by reference to the velocity which it possesses during that period—for any such explanation would be circular. We would just be saying, in effect, that the object changes its position over a certain period of time because it moves a certain distance during that period, which is just to say that it moves a certain distance because it moves a certain distance. But, very plausibly, we *can and should* explain why an object changes its position over a period of time by reference to the velocity which it possesses during that period. It may be true that we can only *measure* the object's velocity at any time by measuring the distance that it moves during a period which includes that time: but this doesn't imply that we should think of the fact that the object has that velocity as simply *consisting* in the fact that the ratio of the distance moved by the object to the length of time it takes to move that distance has a certain value.

Does this mean that the Paradox of the Arrow remains unresolved? Not necessarily. For, drawing on a proposal first sketched in Chapter 13, we can perhaps say that an instantaneous velocity of an object at a time *t* is a *directional tendency* which it possesses at *t*, in virtue of which, if it continues to possess it, it will undergo a subsequent change of spatial position. On this conception, the object's change of position is a consequence of its velocity, not something in terms of which its velocity is defined. But we can still say that there is a real difference, at an instant of time, between a moving and a stationary arrow—a difference which, moreover, does *not* concern how those objects are at other times. For we can say that the arrows differ in respect of their directional tendencies. This is not a difference between them that one could hope to *observe* at *t*, because dispositions—of which tendencies are a variety—are not straightforwardly observable. What we can observe are their *manifestations*. We can observe, for instance, the *dissolving* of the sugar, but not its *solubility*. But its solubility is quite as real a feature of the sugar as is its dissolving. And its solubility explains *why* it dissolves, when it is immersed in an appropriate solvent. In a similar fashion, we may maintain, an object's instantaneous velocity explains why it undergoes a change of spatial position. But it is because an instantaneous velocity is a species of disposition and hence not straightforwardly observable that a moving and a stationary arrow may not *appear* to differ at any instant of time—a fact which the propounder of the Arrow Paradox relies upon in order to perplex us. An instantaneous 'snapshot' of the two arrows at an instant of time would reveal no difference between them: but that, according to the view now being recommended, is because the difference between them, real though it is, is simply not the sort of difference that could be revealed by a snapshot.

How can something begin to move?

We can draw on this alternative conception of an instantaneous velocity to say something about the inverted version of the Paradox of the Racecourse, represented in Fig. 16.2. The problem posed by this version of the paradox is how the runner can even *begin* to move, that is (as we put it), how it can be the case that at a certain instant of time an object is moving, even though it is not moving at any prior instant of time. If an instantaneous velocity is a 'directional tendency', then there seems to be no reason why an object should not acquire this at an instant of time and thus *before* the object has moved any distance at all. By contrast, if an instantaneous velocity can only be ascribed to an object at an instant of time, *t*, in virtue of distances moved by the object during periods of time surrounding *t*, then it might appear to make no sense to ascribe an instantaneous velocity to an object at a time, *t*, before the object has moved any distance at all. However, it may be urged on behalf of the latter conception of instantaneous velocity that, at the instant at which the runner starts the race, his instantaneous velocity should be *zero*—and that this is, in fact, the value of the velocity assigned to him by the method of computation associated with this conception. Indeed, according to this conception of instantaneous velocity, the solution to the inverted version of the Racecourse Paradox is to say that the runner does *not* in fact 'begin to move', in the sense suggested earlier: that is to say, it is not the case, according to this conception, that an object can acquire, at an instant of time, *t*, a non-zero velocity even though it has zero velocity at all instants of time prior to t. Rather, it will be said, this is how we should understand what it is for an object to 'begin to move' at an instant of time, t: it is for the object to have zero velocity at t but a non-zero velocity at all instants of time succeeding t (until the object stops moving). However, it must still be acknowledged that this account of what it is for an object to 'begin to move' has the counterintuitive consequence that an object cannot acquire a *non-zero* velocity until it has already moved some distance, which prompts the question: how did the object manage to move that distance? That we should find this puzzling provides further evidence, I think, that we do intuitively think of an object's velocity as being something which *explains* why it undergoes a change of position—a notion which, as we have seen, appears to be incompatible with the conception of instantaneous velocity now under scrutiny. But the question of which of the two conceptions is ultimately superior is not one that I shall attempt to settle definitively here.

The Paradox of the Moving Blocks and discrete space-time

The fourth and last of Zeno's four paradoxes of motion is the Paradox of the Moving Blocks, the exact purport of which is somewhat obscure. However, I shall present what seems to be one quite plausible and interesting reconstruction of it. Suppose we have three rows of moving blocks (all of the same size), row A, row B, and row C, with row A moving from east to west at a certain speed and row C moving from west to east at the same speed, while row B, which lies between the other two, is stationary. And suppose we consider a moment of time, t_1, at which the faces of all the blocks are exactly lined up with each other, as is depicted in Fig. 16.4. Now consider a later moment of time, t_2, at which the trailing face of block A_3 is exactly lined up with the leading face of block C_1, as in Fig. 16.5.

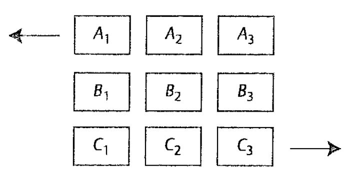

Fig. 16.4

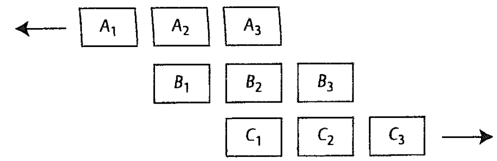

Fig. 16.5

The puzzle is supposed to be this: how can it be the case that, during the interval between t_1 and t_2, block A_3 has passed by *two* complete C blocks while, in the same amount of time, it has passed by only *one* B block—despite the fact that all of the blocks are of exactly the same size?

One's initial thought may be that there is nothing very puzzling about this. Because the A blocks and the C blocks are travelling at the same speed but in opposite directions, their velocity relative to each other is twice the velocity of an A block relative to a B block, since the B blocks are stationary. Perhaps the original propounder of the paradox did not fully comprehend the fact that one and the same object may have two different velocities relative to two different objects which are themselves moving relative to each other. Be that as it may, the example of the moving blocks does appear to create a problem if one supposes that space and time are *discrete* rather than continuous—as one might be tempted to suppose in order to overcome Zeno's other paradoxes of motion (assuming that one is not satisfied with the solutions of these that were proposed earlier). If space and time are discrete, then there is a least possible distance and a least possible length of time, neither of which is divisible. And then the problem posed by the moving blocks is this. Suppose that the moments of time t_1 and t_2 are separated by the least possible length of time, so that block A_3 passes two C blocks, C_1 and C_2, in the least possible length of time. At what time, then, did block A_3 pass block C_1? It must, surely, have passed it at some time *between* t_1 and t_2, when the blocks were spatially related to one another in the way depicted in Fig. 16.6.

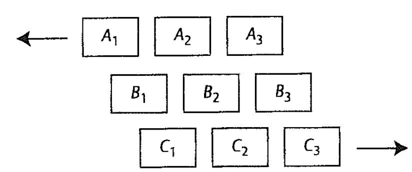

Fig. 16.6

But if t_1 and t_2 are separated by the least possible length of time, then there is, of necessity, *no* moment of time that lies between t_1 and t_2 and so no time at which block A_3 can have passed block C_3. Furthermore, we see that the situation depicted in Fig. 16.6 is one in which, at the time at which block A_3 has just passed block C_3, it has passed only *half* of block B_3, so that, whatever distance the blocks measure from front to back, it should be possible for half of that distance to exist. But what, then, if the blocks measure the least possible distance from front to back? The implication of all this seems to be that there cannot, after all, be either a least possible distance or a least possible length of time, so that space and time must be continuous rather than discrete. But how good the reasoning is for this conclusion I leave to the reader to judge for him or herself.[4]

[4] For further discussion of Zeno's paradoxes see, in addition to items already referred to in this chapter, Max Black, *Problems of Analysis: Philosophical Essays* (London: Routledge and Kegan Paul, 1954), Part 2; Adolf Grünbaum, *Modern Science and Zeno's Paradoxes* (London: George Allen and Unwin, 1968); and R. M. Sainsbury, *Paradoxes*, 2nd edn. (Cambridge: Cambridge University Press, 1995), ch. 1.

17

TENSE AND THE REALITY OF TIME

The A series and the B series

The paradoxes of motion that we discussed in the preceding chapter were designed to persuade philosophers that there is something suspect about our conception of change and our apparent experience of it. Motion is perhaps the most obvious and fundamental species of change and so, if the concept of motion can be convicted of harbouring a contradiction, making real motion impossible, we may have to conclude that reality itself is unchanging. This in turn would have a severe impact upon our conception of time as a pervasive feature of reality, especially if we take the view that time without change is impossible (a view that we examined in some detail in Chapter 13). If time without change is impossible and change is impossible, it follows that time itself is impossible—that is to say, it follows that temporality cannot be a genuine feature of reality, in the sense that elements of reality cannot genuinely stand in temporal relations to one another (relations such as *being earlier than* or *being simultaneous with*).

Of course, nothing that we concluded in the preceding chapter gives support to the foregoing line of argument against the reality of time, for we were not persuaded by the alleged paradoxes of motion that even motion—let alone change in general—is impossible. In the present chapter, however, I shall examine another and perhaps more compelling way in which this line of argument against the reality of time may be pursued—one which we owe to the Cambridge philosopher J. M. E. McTaggart, who wrote extensively on this topic in the early years of the twentieth century.[1] (Of course, if McTaggart's argument is correct, he really did no such thing, since there are in reality no 'years' and 'centuries'—but let us pass over the

[1] See J. M. E. McTaggart, *The Nature of Existence*, vol. 2 (Cambridge: Cambridge University Press, 1927), ch. 33.