39
SPACE, ABSOLUTE AND RELATIONAL

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The problem

It is unavoidable in fundamental metaphysical disputes that the framing of the issue under discussion is likely to beg the question at hand. So I might begin this essay by saying that the topic is the nature of physical space, but one set of partisans will immediately object that there is no such entity as physical space, and *a fortiori* no nature of it. We must proceed with caution.

Let us therefore begin with the thesis that *the physical world has a spatial aspect*. That is, it is correct to say of some physical objects that they have particular geometrical shapes, and stand in various spatial relations to one another. If we accept this thesis, then two questions immediately arise: first, what *is* (in exact mathematical detail) this spatial structure and, second, what *has* this spatial structure. Various possible answers to the first question are easy to articulate: the spatial structure might be Euclidean, or it might be some non-Euclidean geometry of constant positive or negative curvature, or perhaps a Riemannian geometry of variable curvature. It could be continuous or discrete at a fundamental level. The mathematical elucidation of these various possibilities is a straightforward matter, and falls to the mathematicians rather than to philosophers.

But having the mathematical structures in hand leaves several foundational issues completely open. One question is what it means, exactly, to assert that the physical world has one of these spatial structures rather than another. This sort of question was taken up most extensively in the early twentieth century, with Hans Reichenbach's *Philosophy of Space and Time* (Reichenbach 1958) as *locus classicus*. Reichenbach argued that the attribution of geometrical structure amounts to claims about the observable behavior of certain physical bodies, denominated *rigid bodies*. According to Reichenbach, claims about, for example, the relative sizes of objects are given empirical content (which is to say, for him, given any content at all) only by reference to methods for measuring, and those methods in turn require the identification of rigid bodies. But since the notion of a rigid body is exactly that of a body that does not appreciably change its geometrical shape in normal circumstances, an evident circularity ensues. Reichenbach concluded that the geometrical structure of the world could only be settled together with an identification of rigid bodies (and, with this, propositions about the existence of "universal forces" that equally deform all bodies) as a package deal, it being possible for different packages to yield the same empirical consequences. Choice among these packages would then, according to empiricist criteria of meaning, be something like a matter of convention. That is, according to Reichenbach the physical world *per se* does not have any particular geometrical structure, rather the geometrical structure is just one element of a collection of propositions/definitions/conventions that face the tribunal of experience as a corporate body. So one strand of the logical empiricism of the early twentieth century held that the geometrical structure of the world is partly a matter of convention rather than an "objective" fact.

This line of thought seems to have largely evaporated nowadays, together with the empiricist criterion of meaning. The physical systems called "rigid bodies" (or, in the case of temporal structure, "clocks") do not really play any privileged role in modern physics, nor is it any longer thought that the meaning of all physical terms needs to be reduced to claims about the immediately observable properties of macroscopic objects. No physical system is *stipulated* to be rigid, or to be a good clock. In fact, no physical system is thought to be perfectly rigid or a perfectly good clock. Rather, all physical systems are treated as subject to the same laws of physics, and measuring sticks and clocks ought to be subject to the same physical analysis in terms of their microscopic physical components as all other composite objects. But the laws governing the microscopic parts – the fundamental laws of physics – are themselves specified in spatiotemporal terms. So contemporary approaches to the nature of space have turned from the analysis of clocks and rods to the analysis of fundamental laws: what sort of spatial structure is implied by the fundamental physical laws, and what is the status of this structure?

Already in the last paragraph, though, we can see an embarrassment for our topic. For modern physics – most obviously the theory of relativity but, as we will see, even Newtonian mechanics – does not postulate pure *spatial* structure so much as *spatiotemporal* structure to the physical world. In order that physics be brought to bear, we need to consider both time and space, not space alone. Indeed, in a straightforward sense the theory of relativity denies that there is any such thing as "the spatial aspect of the physical world." And given the way we initially exposited our topic, that would mean that, strictly speaking, according to relativity there is no subject matter for the analysis of the nature of space to address! (Even in the old-style Reichenbachian approach, it is essential to consider what happens when one *moves a measuring rod from one place to another*, or *sets a measuring rod in motion*, and these questions involve time as much as space.)

So a metaphysical treatment of space without time is really not possible, and anyone interested in "the nature of space" should begin by studying the theory of Relativity. But since this article in meant to be about space alone, and since Relativity will be treated elsewhere in this volume (Hawley, this volume), we will continue as best we can, making the appropriate *caveats*.

Newton and Leibniz

The traditional absolute-vs.-relational debate about the nature of space derives from the correspondence between Leibniz and Samuel Clarke (Alexander 1984). In the course of the correspondence, the issue arises whether Newton's philosophy respects Leibniz's principle of sufficient reason. Clarke asserts that it does, but that the relevant sufficient reason may sometimes be the mere will of God. As an example, Clarke notes that according to Newton's physics God would have had to make a choice about exactly how to locate the material universe in Absolute Space, and that such a decision could not be based on the intrinsic moral superiority of one way of locating it over another. That is, according to Newton an empty, infinite Euclidean space existed before the existence of any matter, and when God decided to create the material universe he made an essentially arbitrary choice among the infinitude of ways he could situate and orient the matter in this infinite space. The example of two possible material universes that differ only with respect to the location of the matter in the space (all relations among corresponding material elements being identical) has come to be known as a *Leibniz shift*, although it was Clarke who raised the example as a criticism of Leibniz's understanding of sufficient reason.

Leibniz, in a classic argumentative reversal, turns Clarke's own example against Newton. God, according to Leibniz, is so constrained by the principle of sufficient reason that he would be incapable of making an arbitrary choice between morally equivalent possibilities. Since the mere location and orientation of matter in Absolute Space would make no *moral* difference (neither of these possibilities is *better* than the other), God could not have created the world if creating it required such a choice. Hence, according to Leibniz, the existence of the material world refutes Newton's postulation of an Absolute Space that can exist independently of matter.

The question at issue here is not what the spatial structure of the world *is* but rather what that spatial structure *inheres in*. Newton and Leibniz agree that the relevant spatial structure is given by Euclidean geometry, but they disagree about what *has* that spatial structure. For Newton, it is the structure of Absolute Space, an entity distinct from material bodies. Material bodies stand in spatial relations to one another, but only in virtue of being located in regions of Absolute Space that bear exactly those spatial relations. Due to the symmetries of Euclidean geometry, different dispositions of matter in Absolute Space can exhibit identical spatial relations among corresponding pieces of matter. It is among these various distinct possibilities that God must choose.

Since Leibniz denies God the ability to make such a choice, he must deny that there are really distinct possibilities here. This is accomplished by a particular sort of relational analysis of spatial structure: the spatial aspect of the world is *nothing but* the set of spatial relations among material bodies. Since Leibniz-shifted worlds agree about all the spatial relations *among the bodies* (they disagree only about where the bodies are situated in Absolute Space), given Leibniz's account they do not really disagree about spatial structure at all. The supposed Leibniz-shifted distribution of matter in Absolute Space is not a distinct possibility from the unshifted situation, so God need not choose between them.

In addition to the appeal to the principle of sufficient reason, Leibniz makes an independent appeal to the principle of identity of indiscernibles. If we grant Absolute Space, then in some sense the shifted and unshifted distributions of matter would "look the same": the *qualitative appearance* of these worlds to their inhabitants would be identical. Leibniz concludes that we could therefore not have distinct ideas of these two possibilities, apparently on the thesis that the content of an idea must be specified in some qualitative way. It is not merely that a distinct, Leibniz-shifted distribution of matter is not *possible* (as would follow from his account of spatial structure) but that it is not *thinkable*.

How do these arguments stand up to scrutiny? One should first note that the whole discussion takes place under the assumption that the spatial structure is Euclidean: Euclidean space, being homogenous and isotropic, would admit of Leibniz shifts that move all the matter (by rotation or translation) while keeping the relative distances between objects the same. Were Newton or Leibniz to postulate a spatial structure that lacked the appropriate symmetries, it is unclear that the arguments could be formulated in the first place. So these arguments about what *has* the spatial structure are not entirely independent of questions about what the spatial structure *is*.

Second, neither the principle of sufficient reason nor the principle of identity of indiscernibles is known to be true. The apparently random behavior of, for example, atomic decay casts doubt on the former, whose use here requires dubious theological hypotheses in any case. The principle of identity of indiscernibles also lacks justification. And in this particular case, it seems clearly inapplicable. Supposing Newtonian Absolute Space to exist, and supposing it to have been *possible* for matter to have been distributed differently from the way it actually is in Absolute Space (by, say, everything being shifted 3 meters to the north), still these possibilities are not indiscernible: indeed, we would know which of the two obtains! So in whatever sense these possibilities are qualitatively identical, it is not a sense that would create some unknowable physical fact.

Since neither of Leibniz's arguments against Absolute Space establishes the untenability of Newton's theory, we still have on the table two radically different alternative accounts of the ontology that supports the spatial structure of the universe. According to Leibniz, the spatial aspect of the world is nothing but a set of relations among material bodies, so if there were no material bodies there would be no spatial facts at all. According to Newton, spatial structure is, in the first instance, the structure of Absolute Space, and hence exists independently of material bodies.

Newton's Scholium

Newton's arguments in favor of Absolute Space are found in the Scholium to Definition VII in the *Principia* (Newton 1962: 6–12). Newton there distinguishes true, absolute space, time and motion from their merely relative cousins. There are, of course, relative spatial relations among material bodies, relations between events and material clocks, and relative motions of things, but for Newton all of these are derivative matters. Just as there is Absolute Space, whose nonmaterial parts stand in geometrical relations to one another, so there is Absolute Time, which passes independently of the motions of material clocks. Newton's Absolute Space itself persists through time, so that Absolute Motion can be defined in terms of a body's changing location in Absolute Space.

Once again, we have found ourselves embroiled in questions not just of spatial structure but of spatiotemporal structure: motion, for Newton, involves both space and time. And so again we have drifted away from our announced topic – space – into the wider arena of space and time together. This widening of scope is unavoidable, since Newton's argument is directly an argument in favor of Absolute Motion and only derivatively (via the definition of Absolute Motion) for Absolute Space. The argument makes use of a bucket filled with water.

Newton points out that there are observable effects of certain kinds of motion: when the water in a bucket *spins* it climbs the sides of the bucket, forming a concave surface, and when it does not spin the surface is flat. So there is a real, verifiable physical distinction between these two states. But the relevant distinction does not coincide with the relative motion of the water and the bucket: sometimes the surface is flat when the water and bucket are at relative rest and sometime it is concave. Newton's explanation is the natural one: in the former case both the water and the bucket are not spinning, and in the latter case both are. But spinning *relative to what*? Not, evidently, to each other, or to any material object in their immediate environment. Newton suggests that the relevant spinning – the spinning that has observable effects – must be Absolute Spinning, a change of location in Absolute Space.

There have been two famous objections to Newton's inference. One, associated with Mach, observes that the phenomena still might be explained by appeal only to the relative motion of bodies: not the relative motion of the water to the bucket, or the water to the room, or even the water to the Earth (the bulging of the Earth at the equator and the Coriolis effect show that the Earth is, in some physically significant sense, spinning) but perhaps the relative motion of the water to the *fixed stars*. For whenever the surface of the water is concave it is spinning relative to the fixed stars: who's to say that it is not that relative motion which accounts for the effect?

The short answer to this objection is that it is merely the logical outline of a possible explanation of the effect, not an actual explanation. Newton's theory really does account for the phenomenon, while Mach points out only that all alternative theories have not been ruled out. But Mach does not offer such a theory, nor have any subsequent physical theories supported his conjecture. Even in general relativity there is a distinction between rotating and non-rotating bodies that makes no mention of the fixed stars, or of any material bodies relative to which the motion is defined.

The second objection is much more serious. It notes that the phenomenon Newton discusses does not depend on the Absolute Motion of the bucket (as Newton has defined it) but only on its Absolute Acceleration. Newton's law of motion is $F = mA$, and what the bucket demonstrates is that given this law the evident presence of forces indicates the (not immediately evident) presence of acceleration. But this gets us to Absolute Space only via a series of further inferences: acceleration is change in velocity and velocity is change of place, so Absolute Acceleration implies Absolute Velocity implies Absolute Place. This chain is a bit embarrassing for Newton because although his theory implies observable effects of acceleration, it also implies *no* observable effects of

Absolute Velocity: no mechanical experiment could indicate the Absolute Velocity of an inertially moving laboratory.

This tension has been resolved in the modern approach to space-time structure. If we think of the universe throughout its whole history as a four-dimensional object, then persisting bodies trace out trajectories (world lines) in this four-dimensional manifold. From this perspective, the acceleration of a body corresponds to a bending of its world line, and Newton's law of inertia just says that a body subject to no forces traces out a straight trajectory in space-time. The First and Second Laws together get rewritten, schematically, as $F = m$BEND, where "BEND" signifies a mathematical quantity that measures the curving of the body's trajectory. The definition of this quantity, as it turns out, does not require the identification of anything like Absolute Velocity or Absolute Place: it only requires that the spatiotemporal structure be rich enough to distinguish straight from curved trajectories, and to quantify the curvature. Newton postulated more spatiotemporal structure than he needed to make sense of his dynamical laws.

Details of various possible spatiotemporal structures and the physics they support can be found in Earman (1989: 27–40) and Sklar (1976: 194–210). Without entering into details, we can see that the question has again drifted from what has the spatiotemporal structure to what is the spatiotemporal structure. Although Newton's rotating bucket experiment is purported to refute a position like Leibniz's, according to which spatiotemporal structure is just a matter of relations among bodies, it does not in fact make direct contact with that issue at all. If Leibniz is unable to account for the phenomena, it is not because he locates the spatiotemporal structure in material bodies, but because the structure he postulates is not strong enough to do the physical work. Newton succeeds not because his Absolute Space is independent of matter, but rather because the individual points of his Absolute Space are postulated to persist through time, thereby defining a spatiotemporal structure rich enough to define his laws of motion. But, as we have seen, this structure is even richer than he needs: the laws of motion can be defined with less structure, so the persistence of points of space through time is physically otiose.

Are there any arguments that address the relational/absolute controversy directly, and do not devolve into questions about the mathematical particulars of the spatiotemporal structure?

The plenum

Even if a Relationist and an Absolutist settle on the same sort of spatiotemporal structure, they may end up with different explanatory resources. If the material contents of the world intuitively form a plenum, with no "vacuum," then it becomes hard to see how *physical* considerations could become relevant. The Absolutist may postulate a certain spatiotemporal structure of space–time itself, while the Relationist postulates the very same spatiotemporal relations among parts of matter, but if every supposed location in the Absolutist space–time contains some matter, then the difference in locutions will be hard to make out. Or rather, the Relationist and Absolutist may disagree about the class of distinct physical *possibilities* (the Absolutist will maintain that there is a distinct Leibniz-shifted physical possibility, while the Relationist denies this), it is not clear that this dispute about possibilities will have any bearing on the physical explanation of the actual world.

If the material world is not a plenum, though, the situation is quite different. Both the Relationist and the Absolutist must give truth conditions to the claim that there is a vacuum in some location: for the Absolutist, it means that there is a part of absolute space that is empty of all matter, while for the Relationist it means that there are certain pieces of matter such that *there is nothing at all* that bears a particular set of spatial relation to them. But the Relationist's lack of anything at all to quantify over "in the vacuum" restricts the sorts of analyses of spatial notions he or she has available.

Until now, we have not even asked the most fundamental question of both the Absolutist and the Relationist: what are the most basic spatial (or spatiotemporal) structures? It is often presumed that the fundamental spatial structure (whether between bodies or parts of absolute space) is a distance relation: any two elements have some distance between them, and the whole geometry is nothing but the set of these distances. But geometrical structure is not, in fact, specified in such a way. The mathematical object called a "metric" does not directly specify the distance between points: it rather allows one to integrate along a continuous curve to get a *path length*. And from these path lengths, distances can be defined: the spatial distance between two points is the minimal length of a continuous path that connects them.

The definition of distance in terms of path length has nontrivial consequences. Consider, for example, the triangle inequality: given any three points, the sum of the distances between two pairs of points is greater than or equal to the distance between the last pair. This inequality is an analytical consequence of the definition of distance given above: since conjoining a path from *A* to *B* with a path from *B* to *C* yields a path from *A* to *C*, and since the length of such a conjoined path is just the sum of the lengths of the paths conjoined, the length of the minimal path from *A* to *C* cannot be longer than the sum of the length of the minimal path from *A* to *B* with the length of the minimal path from *B* to *C*. If one were to start with *distance* as the primitive notion rather than *path length*, no such derivation of the Triangle Inequality would be forthcoming. Indeed, the physicist Julian Barbour, who has pursued relational formulations of physics with the greatest vigor, is reduced to postulating the Triangle Inequality (and an infinitude of other, more complex inequalities) as unexplained constraints on the distances among objects (Barbour 1999: 42; see also Maudlin 1993).

Once again the advantage that accrues to the Absolutist here does not derive from absolutism *per se*. Rather, it is because the Absolutist automatically has a plenum. Even if matter is only scattered in space, for the Absolutist all of the various continuous paths that connect a pair of points *exist* and have a determinate length. The Relationist, by contrast, can give truth conditions for the claim "there is a vacuum," but has no points in the vacuum or continuous paths running through the vacuum to quantify over. This is why a Relationist who admits the possibility of a vacuum must take the distance relation as primitive rather than derived.

But the notion of a vacuum and the corresponding notion of a plenum are only as clear as the notion of matter itself. For the Absolutist, there is a vacuum if there is space devoid of all matter, for the Relationist if there fails to be anything at all that bears certain geometrical relations to existent material bodies. But if we are unsure what constitutes a material body in the first place, then we will be unsure whether any vacuum exists. We have no problems in a Democritean physics of the Full and the Empty, but if a gravitational field or an electromagnetic field or a quantum field counts as matter, then a vacuum may not be even physically possible. In this case, the Relationist finds himself on even ground with the Absolutist again, able to replicate the Absolutist definitions of distance in terms of path length. And we are inclined to wonder, at this point, whether there is any contentful dispute between the two sides.

The hole argument

The most recent wrinkle in the absolute/relational debate is an argument inspired by some remarks of Einstein and formalized by John Earman and John Norton that goes by the name "the hole argument" (Earman and Norton 1987). The argument aims to show that an Absolutist (or, in alternative terminology a Substantivalist) about space–time will necessarily be committed to radical indeterminism in nature, at least if the laws of physics take a common mathematical form.

The hole argument applies only to space–time structure: there is no obvious form that concerns spatial structure alone. The "hole" is a delimited region of space–time, and the argument is intended to show that the Substantivalist must accept that for any physically possible state of the universe there exists an alternative, distinct state of affairs that perfectly agrees with the first outside of the hole, but disagrees with it inside. If true, then indeterminism follows: specifying the complete physical state of the world outside the hole, and the laws of nature, does not suffice to determine the exact state inside the hole.

The indeterminism at issue does not involve any *observable* property: if one accepts that different states inside the hole are possible, no observation or experiment would reveal which one was realized. Rather, the indeterminism involves only in which particular part of space–time various observable events occur. Since the particular parts are not *per se* observable, the different possibilities will be qualitatively identical. So this is not the indeterminism of quantum theory or dice tossing, where one can describe beforehand the various possible outcomes of an experiment, and can verify afterwards which actually occurred. Indeed, no one outside the hole could have the linguistic resources to specify various distinct possibilities inside the hole, and even if they could, people inside the hole could not determine which of the possibilities obtains. So this would be indeterminism of a particularly ghostly sort.

The hole argument depends on conceptualizing the world as a set of fields on a manifold called "space–time." The fields would include electromagnetic fields, matter fields, etc., and also *the metric field that specifies the spatiotemporal structure itself*. It will help to visualize a particular solution of the fundamental equations of physics as these fields painted on to an elastic sheet. The laws of physics themselves specify only *how the various fields are related to one another at each point (or infinitesimally small region) in the manifold.*

Take the elastic sheet and choose a closed region (the "hole"). Tack the edges of the hole down so that part of the sheet cannot move. Next, stretch the sheet (with the painted fields) inside the hole in some way. Looked at from above, the painted fields will now look different inside, and just the same outside. Finally, release the elastic sheet so it returns to its original shape *and paint on to this original shape the fields as they looked when the sheet was stretched*. The region inside the hole now has two images on it: the original and the distorted copy. These two images will evidently disagree about what the value of the "fields" are at particular points on the sheet inside the hole, although they agree completely outside.

Now if the laws of nature constrain only the relations of the fields at each point, then if one of these images satisfies the laws then the other will, since every point in one image has a corresponding point in the other where the field relations are identical. So there appear to be two distinct solutions to the laws of nature that agree outside the hole but disagree inside: radical indeterminism. Earman suggests that the way out of this indeterminism is to *deny the physical reality of the elastic sheet*, which is supposed to correspond to denying substantivalism about space–time. It is not evident what the resulting positive doctrine about spatiotemporal structure would be, and in particular whether it would be a recognizable form of relationism.

The exact form of the hole argument is rather complex, and there are several points at which it can be attacked. It turns on the claim that a particular ontological account of spatiotemporal structure commits one to specific claims about a mathematical construct (the "distorted" image): that it represents a situation that is both metaphysically possible and non-actual. Jeremy Butterfield has argued, using counterpart theory, that both mathematical models represent the same physical situation, not alternatives (Butterfield 1989), and I have argued that if the spatiotemporal structure of space–time is metaphysically essential to its parts, then one of the mathematical models does not represent a metaphysical possibility (Maudlin 1990). Other analyses of the argument can be found in Brighouse (1994), Hoefer (1996), Leeds (1995) and Rynasciewicz (1994). Suffice it to say that since issues of representation, ontology, modality and determinism all intersect in the hole argument, there is considerable fodder for philosophical analysis.

There is, at present, no agreed resolution of the absolute/relational debate. The inevitable extension of the argument to space–time, the obscurity of the physical nature of a "vacuum," and the unclarity of the significance of certain symmetries in the mathematical formalism all conspire to make it hard to even formulate clearly an Absolutist or Relationist version of contemporary physical theory. Given that it is no longer clear exactly what the argument is about, it is hardly surprising that it remains unsettled.

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Further reading

J. Barbour, *The End of Time* (Oxford: Oxford University Press, 1999) is the most sustained recent attempt by a physicist to develop a completely relational account of space–time structure. J. Earman, *World Enough and Space–Time* (Cambridge, MA: MIT Press, 1989) is a technically sophisticated examination of the absolute/relational controversy, ending with a presentation of the hole argument; the work focuses on space–time rather than just space. M. Friedman, *Foundations of Space–Time Theories* (Princeton NJ: Princeton University Press, 1983) provides a general discussion of approaches to space–time ontology, with special attention to coordinate-free presentations of the physics (a more mathematically demanding work than Sklar, below). N. Huggett, *Space from Zeno to Einstein* (Cambridge, MA: MIT Press, 1999) contains a collection of classical texts on the nature of space, including Zeno, Newton, Leibniz and Clarke, Kant, Mach and Einstein, with useful commentary by Huggett. M. Jammer, *Concepts of Space* (Cambridge, MA: Harvard University Press, 1954) gives an account of the history of accounts of space from antiquity. A sustained argument against relationism with particular attention to the contingent features space can have that are independent of the material contents of space is found in G. Nerlich, *The Shape of Space* (Cambridge: Cambridge University Press, 1976). L. Sklar, *Space, Time, and Spacetime* (Berkeley: University of California Press, 1976) is the classic overview of both philosophical and mathematical issues surrounding accounts of the nature of space and space–time.