Problem 1

Theorem 1 (Problem 1). *If $x = \log_9(12)$, then x is irrational.*

Proof (long form).

1. By the definition of logs, we can say $x = \log_9(4 * 3) = \log_9(3) + \log_9(4) = \frac{1}{2} + \log_9(4)$

Lemma 1. *Any rational plus an irrational will be an irrational number.*

Proof of lemma.

a. Assume for contradiction rational + irrational = rational

a.a. We can write the first rational as a/b and the second as c/d for some $a, b, c, d \in \mathbb{Z}$ so $a/b+$ irrational $= c/d$.

a.b. Subtracting a/b from each side yields irrational $= c/d - a/b$

a.c. $c/d - a/b$ can be represented as the fraction $\frac{ad-bc}{bd}$, which is rational, contradicting a.

b. Therefore, a rational + an irrational = an irrational.

□

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2. For x to be rational, $y = \log_9(4)$ must be rational by Lemma 1.
3. $y = \log_9(4)$ can be rewritten as $9^y = 4$.
4. Taking the square root of each side yields $3^y = 2$.
5. Assume for contradiction y is rational, $y = p/q$ for some $p, q \in \mathbb{Z}$

5.1. we can rewrite $9^y = 4$ as $3^{p/q} = 2$ or $3^p = 2^q$.

5.2. 3^p will always be odd, while 2^p will always be even. Therefore, there can't exist any $p, q \in \mathbb{Z}$ to satisfy this equation.

5.3. By step 3.5, p and q cannot be integers. Therefore y is not rational by contradiction.

6. By block **??**, and Lemma 1, x cannot be rational since it is made up of the sum of an irrational number, y.

□