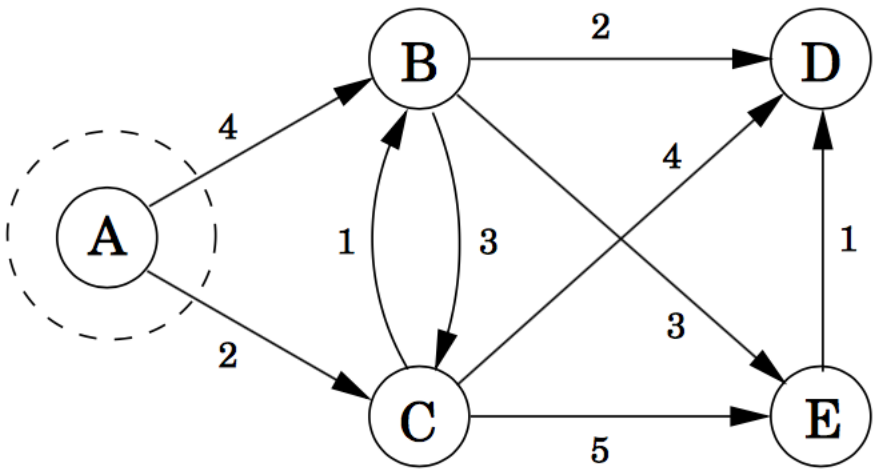
The problems for this assignment will use the following graph, G



Problem 1

Here is an optimal solution vector, X for the primal LP for shortest paths from A to E in G:
$(x_{AB}, x_{AC}, x_{BC}, x_{CB}, x_{BD}, x_{BE}, x_{CD}, x_{CE}, x_{ED}) = (0, 1, 0, 1, 0, 1, 0, 0, 0)$ Yielding an optimal value of 6.

Problem 2

Here is an optimal solution vector, π, for the dual LP for shortest paths from A to E in G:
$(\pi_A, \pi_B, \pi_C, \pi_D, \pi_E) = (0, 3, 2, 5, 6)$ Yielding an optimal value of 6.

Problem 3

Lemma 1. *The dual solution π above is optimal.*

Proof (long form).

1. By problem 1, we know that 6 is a feasible solution to the primal LP.
2. Therefore, by weak duality, every feasible solution to the dual has value at most 6.
3. Since we've found a feasible solution, π to the dual equal to the upper bound, π must be optimal.

$\square$