Problem 1

Theorem 1 (Checkerboard)**.** *John has an 8x8 checkerboard and thirty-one 1x2 dominoes. John places the dominoes one by one on the checkerboard. He places each domino to cover exactly two adjacent, previously uncovered squares (two of the 64). When he is done, he has covered all but two of the squares. Is it possible that the two squares remaining uncovered are the two white corner squares?*

Proof (long form).

1. Consider any state of the board after some number of domino placements.

1.1. We will show that John maintains the following invariant at all times:
The number of black squares covered is the same as the number of white squared covered.

1.2. The invariant is initially true, when no squares are covered.

1.3. Consider any way that John can place a single domino.

1.3.1. Assume the invariant holds just before John places the domino.

1.3.1.1. That is, just before the action, the number of uncovered white squares is the same as uncovered black squares.

1.3.1.2. Each possible action covers two adjacent squares–one black square and one white square.

1.3.1.3. Hence, the number of uncovered white and black squares always changes by (-1, -1) respectively.

1.3.1.4. Hence, their values remain equivalent and the invariant holds, after the action.

1.3.2. By block 1.3.1, if the invariant holds before the action, it holds after.

1.4. By block 1.3, for each action, if the invariant holds before the action, it holds after.

1.5. Since the invariant is initially true, and each of John's actions preserves it, the invariant holds.

1.6. The invariant also implies that the last two uncovered squares can never both be the same color.

1.7. It follows that the last two uncovered squares cannot be the two corner white squares.

2. By block 1, after any sequence of actions that John might have taken, it is impossible for him to end with only the two white corners uncovered.

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