Problem 1

Theorem 1 (All pairs have different sums)**.** *Prove or disprove: It is possible to label each square of an 8 x 8 checkerboard with an integer, so that no two squares are labeled with the same integer and no two adjacent squares (up, down, left, right) have labels that differ by 8 or more.*

Proof (long form).

1. Assume you have filled some chessboard with integers to satisfy the theorem.

1.1. Construct a set S of chessboard squares by doing the following...

1.2. Add the square with the smallest integer not in S into S, stopping when the squares in S are touching at least 8 squares not in S.

1.3. Call T the set of chessboard squares adjacent to those in S but not in S.

1.4. Call the largest numbered square in S, s, where the number in s is $num(s)$.

1.5. Since there are 8 squares in T and the numbers in the squares in T must be unique integers $> num(s)$, the lower bound on the largest numbered square in T will be $num(s) + 8$.

1.6. However, by the definition of the theorem, there is no square in S that the largest square in T can be adjacent to.

1.7. Contradiction.

2. By block 1 you cannot fill a chessboard with integers to satisfy the theorem.

3. That is, the theorem is false.

□