Problem 1

Theorem 1 (from the proof-guide handout)**.** *25 boys and 25 girls sit around a table. It is always possible to find a person that sits next to two girls (one on each side of the person).*

Note: I'm assuming here that the seats have been numbered from $1 - 50$ where consecutive integers are seats that are next to each other and seat 50 is next to seat 1

Proof (long form).

1. Assume for contradiction we have some ordering where there is no person who has a girl on both sides of them.

1.1. Call the odd numbered chairs S_o and the evens S_e.

1.2. In each of these sets there cannot be two consecutive girls.

Lemma 1. *In both S_o and S_e there are not two consecutive boys.*

Proof of lemma.

a. Assume for contradiction there are two consecutive boys in one of the even or odd sets.

a.1. Since there are the same number of girls and boys, to fill all of the seats there must be two consecutive girls in one of the sets.

a.2. Contradiction by 1.2.

b. By block a there are no two consecutive boys in the odd or even sets.

□

1.3. By Lemma 1 there are no two boys next to each other

1.4. By 1.2 and 1.3, S_o and S_e must both be ordered in the form $\{..., boy, girl, boy, girl, ...\}$

1.5. Each set has 25 people in it, so one set has 12 girls and 13 boys while the other has 13 girls and 12 boys.

1.6. (case 1.6) *Suppose S_o has 13 girls.*

1.6.1. Because of 1.4, the set must begin and end with a girl.

1.6.2. This means that since the table is circular, someone is sitting next to two girls.

1.6.3. Contradiction.

1.7. (case 1.7) *Suppose S_e has 13 girls.*

1.7.1. Because of 1.4, the set must begin and end with a girl.

1.7.2. This means that since the table is circular, someone is sitting next to two girls.

1.7.3. Contradiction.

1.8. Case 1.6 or 1.7 must hold, therefore we have a contradiction.

2. By block 1, every arrangement of boys and girls will result in at least one person sitting next to two girls.

□