Download this LaTeX file (along with assignment-macros-0.tex) to use as a template for turn-in 5.

Theorem 1. *A robot sits on an infinite 2-dimensional integer grid. The location of the robot at any time is given by a pair of integers* (i, j)*. The location is initially* $(21, 15)$*. At each time step, the robot moves as follows. If its current location is* (i, j)*, it moves to one of the two locations* $(j, i - j)$ *or* $(j, i \cdot j)$*. The robot can never reach a location* (i, j) *where* $i = 10$*.*

Proof (long form).

1. Consider any sequence of moves, M, made by the Robot.

1.1. We will show the Robot maintains the following invariant at all times:

Each coordinate i *and* j *of the Robot's location* (i, j) *is odd.*

1.2. The invariant is initially true when the Robot is located at $(21, 15)$.

1.3. Consider some move in the sequence M. Call this move m.

1.3.1. Assume the invariant holds just before m.

1.3.1.1. Assume m takes the robot to $(j, i - j)$.

1.3.1.1.1. By 1.3.1 we know the invariant holds before m is made, so j and i are odd.

1.3.1.1.2. Since i and j are odd, j and $i - j$ will be odd. *(need lemma for* $i - j$ *is odd?)*

1.3.1.1.3. That is, the coordinates of the new location of the Robot will be odd.

1.3.1.2. assume m takes the robot to $(j, i \cdot j)$.

1.3.1.2.1. By 1.3.1 we know the invariant holds before m is made, so j and i are odd.

1.3.1.2.2. Since i and j are odd, j and $i \cdot j$ will be odd. *(need lemma for* $i \cdot j$ *is odd?)*

1.3.1.2.3. That is, the coordinates of the new location of the Robot will be odd.

1.3.1.3. By block 1.3.1.1 and block 1.3.1.2, regardless of which way m takes the Robot, the new coordinates will be odd.

1.3.1.4. Hence, the invariant holds after the move m.

1.3.2. By block 1.3.1, if the invariant holds before m, it holds after.

1.4. By block 1.3, for each move in the sequence M, if the invariant holds before the move, it holds after.

1.5. Since the invariant is initially true, and each move in the sequence preserves it, the invariant holds.

1.6. The invariant also implies i can never equal 10 since 10 is not odd.

2. By block 1, after any sequence of moves the Robot might make, it is impossible for it to reach some coordinates (i, j) where $i = 10$.

□