MATH120 Homework 1

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Problem 1: Find $det(A)$ given the following: $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

Taking the determinent in respect to the bottom row will yield:

$$(0)det(\begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix}) - (0)det(\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}) + (7)det(\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}) =$$

$$(0) - (0) + (7)(4) = 28$$

Problem 2: Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

(a) Find A^T and B^T.

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ -1 & 2 \end{bmatrix}, \quad B^T = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$$

(b) Find AB and BA (if the product is not defined, write *NOT DEFINED*)

$$AB = \begin{bmatrix} (1,2,-1)\cdot(1,1,-1) \\ (3,4,2)\cdot(1,1,-1) \end{bmatrix} = \begin{bmatrix} (1)(1)+(2)(1)+(-1)(-1) \\ (3)(1)+(4)(1)+(2)(-1) \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

BA is not defined as you are attempting to multiply a 3x1 matrix by a 2x3. The columns of B (1) are not equal to the rows of A (2).

Problem 3: Let $L : \mathbb{R}^2 \mapsto \mathbb{R}^3$ be a linear transormation subject to:

$$L(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \qquad L(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

(a) Find the matrix representation of L with respect to the standard bases of $\mathbb{R}^2$ and $\mathbb{R}^3$.

We know what the standard basis of $\mathbb{R}^2$ maps to under L, so finding the matrix representation for the tranformation becomes trivial: $L(\vec{x}) = A\vec{x}$ where $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 1 & 0 \end{bmatrix}$

(b) Find the matrix representation of L with respect to the basis $\{\vec{e}_1, \vec{e}_2\}$ of $\mathbb{R}^2$ and the standard basis of $\mathbb{R}^3$, where $\vec{e}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$.

We can say $\vec{x} = C[\vec{x}]_B$ where B is the basis $\{\vec{e}_1, \vec{e}_2\}$ and C is the matrix formed by using the basis vectors as column vectors, $C = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$.

The transformation we're looking for here maps a vector with respect to the basis B to a vector with respect to the standard basis of $\mathbb{R}^3$. We can call this transformation D.
Since both D and L map to the same vectors, we know for any $\vec{x}$, $D([\vec{x}]_B) = L(\vec{x}) = A\vec{x} = AC[\vec{x}]_B$.

$$AC = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} (1,-1)\cdot(1,1) & (1,-1)\cdot(0,-1) \\ (2,0)\cdot(1,1) & (2,0)\cdot(0,-1) \\ (1,0)\cdot(1,1) & (1,0)\cdot(0,-1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \\ 1 & 0 \end{bmatrix}$$