Math 120 Optimization: Homework 3

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Problem 1: Convert the following linear programming problem into *standard form*:

$$\begin{aligned} \text{maximize } & 2x_1 + x_2 \\ \text{subject to } & 0 \leq x_1 \leq 2 \\ & x_1 + x_2 \leq 3 \\ & x_1 + 2x_2 \leq 5 \\ & x_2 \geq 0. \end{aligned}$$

We can add some variables to turn the inequalities into equalities so the problem becomes:

$$\begin{aligned} \text{maximize } & 2x_1 + x_2 \\ \text{subject to } & x_1 + s_1 = 2 \\ & x_1 + x_2 + s_2 = 3 \\ & x_1 + 2x_2 + s_3 = 5 \\ & x_1, x_2, s_1, s_2, s_3 \geq 0. \end{aligned}$$

Problem 2: Suppose that a computer supplier has two warehouses, one located in city A and another in city B. The supplier receives orders from two customers, one in city C and another in city D. The customer in city C orders 50 units, and the customer in city D orders 60 units. The number of units at the warehouse in city A is 70 and the number of units at the warehouse in city B is 80. The cost of shipping each unit from A to C is 1, from A to D is 2, from B to C is 3, from B to D is 4.

(a) Formulate the problem of deciding how many units from each warehouse should be shipped to each customer to minimize the total shipping cost.

(Not sure what "Formulate the problem" means, but hopefully this will work?)

Let x_y be the number of items shipped from warehouse x to city y.

$$\begin{aligned} \text{minimize } & a_c + 2a_d + 3b_c + 4b_d \\ \text{subject to } & a_c + a_d \leq 70 \\ & b_c + b_d \leq 80 \\ & a_c + b_c = 50 \\ & a_d + b_d = 60. \end{aligned}$$

(b) Express the problem as an equivalent standard form linear programming problem.

We can add some variables to turn the inequalities into equalities so the problem becomes:

$$\begin{aligned} \text{minimize } & a_c + 2a_d + 3b_c + 4b_d \\ \text{subject to } & a_c + a_d + s_1 = 70 \\ & b_c + b_d + s_2 = 80 \\ & a_c + b_c = 50 \\ & a_d + b_d = 60 \\ & a_c, a_d, b_c, b_d, s_1, s_2 \geq 0. \end{aligned}$$

Problem 3: A cereal manufacturer wishes to produce 1000 pounds of a cereal that contains exactly 10% fiber, 2% fat, and 5% sugar (by weight). The cereal is to be produced by combining four items of raw food material in appropriate proportions. These four items are named a, b, c, d and they have certain combinations of fiber, fat, and sugar content, and are available at various prices per pound:

Item	a	b	c	d
% fiber	3	8	16	4
% fat	6	46	9	9
% sugar	20	5	4	0
Price/lb	2	4	1	2

The manufacturer wishes to find the amounts of each item to be used to produce the cereal in the least expensive way. Formulate the problem as a linear programming problem and convert it to the standard form.

The problem here becomes:

$$
\begin{aligned}
\text{minimize } & 2a + 4b + c + 2d \\
\text{subject to } & a + b + c + d = 1000 \\
& 3a + 8b + 16c + 4d = 10 \cdot 1000 \\
& 6a + 46b + 9c + 9d = 2 \cdot 1000 \\
& 20a + 5b + 4c = 5 \cdot 1000 \\
& a, b, c, d \geq 0
\end{aligned}
$$

Problem 4: Let the equation

$$A\vec{x} = \vec{b},$$

where $A = \begin{bmatrix} 1 & 1 & -1 & 0 & 4 \\ 0 & 1 & 1 & -1 & 0 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix}^T$, and $\vec{b} = \begin{bmatrix} 1 & 2 \end{bmatrix}^T$.

(a) Find a 2×2 matrix B whose columns are from the columns of A and are linearly independent.

We can choose the first and fourth columns of A to form the vector

$$B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

(b) For the basis B you choose from (a), find the solution of the equation

$$B\vec{x}_B = \vec{b},$$

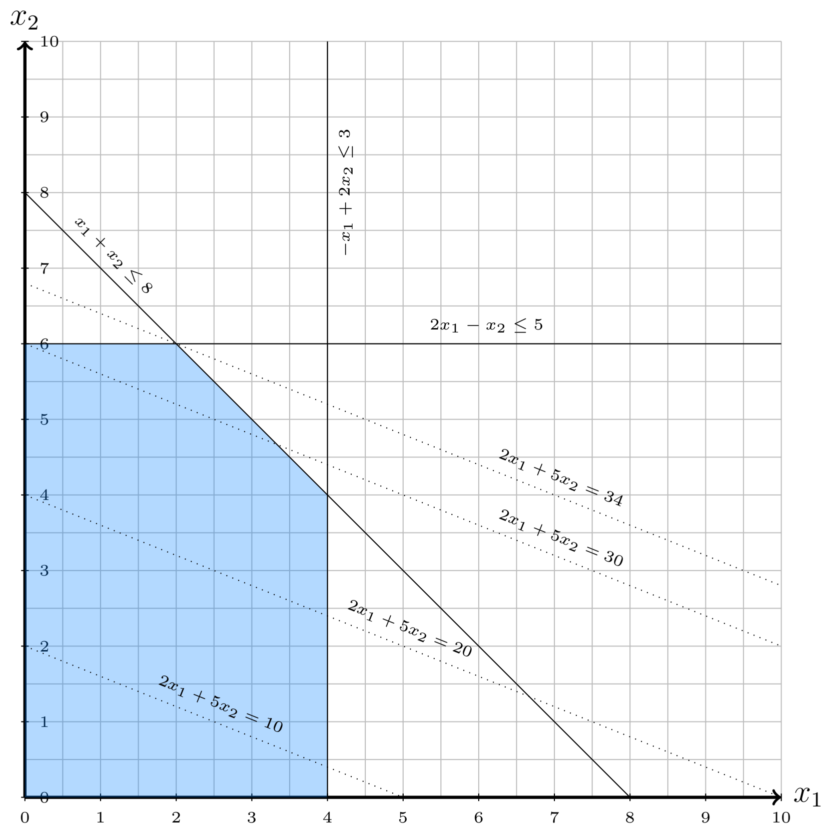
where $\vec{x}_B$ is a 2×1 matrix.

From this equation, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Longrightarrow \vec{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Problem 5: Solve the following 2 dimensional linear programming problem graphically:

$$
\begin{aligned}
\text{maximize } & 2x_1 + 5x_2 \\
\text{subject to } & 0 \leq x_1 \leq 4 \\
& 0 \leq x_2 \leq 6 \\
& x_1 + x_2 \leq 8.
\end{aligned}
$$



We can see from the dotted lines I've drawn in for different values of c in $2x_1 + 5x_2 = c$ that the maximum c we will have is the top dotted line when $c = 34, x_1 = 2, x_2 = 6$.