Math 120 Optimization: Homework 5

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Solve the following linear programming problems using the matrix form (i.e. the tableau) of simplex method.

Problem 1:

$$\begin{aligned}\text{maximize } & 2x_1 + x_2 \\ \text{subject to } & x_1 \leq 2 \\ & x_1 + x_2 \leq 3 \\ & x_1 + 2x_2 \leq 5 \\ & x_1, x_2 \geq 0.\end{aligned}$$

Putting this into standard form will make it look like:

$$\begin{aligned}\text{minimize } & -2x_1 - x_2 \\ \text{subject to } & x_1 + s_1 = 2 \\ & x_1 + x_2 + s_2 = 3 \\ & x_1 + 2x_2 + s_3 = 5 \\ & x_1, x_2, s_1, s_2, s_3 \geq 0.\end{aligned}$$

Given this LP, the initial tableau will look like:

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 0 & 2 \\ 1 & 1 & 0 & 1 & 0 & 3 \\ 1 & 2 & 0 & 0 & 1 & 5 \\ -2 & -1 & 0 & 0 & 0 & 0 \end{array}\right]$$

We'll column 1 look like column 3 to get:

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & 1 & 0 & 1 \\ 0 & 2 & -1 & 0 & 1 & 3 \\ 0 & -1 & 2 & 0 & 0 & 4 \end{array}\right]$$

Next we'll make column 2 look like column 4 to get:

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 5 \end{array}\right]$$

So we get an optimal value of 5 when $(x_1, x_2) = (2, 1)$.

Problem 2:

$$\begin{aligned} \text{maximize } & 6x_1 + 4x_2 + 7x_3 + 5x_4 \\ \text{subject to } & x_1 + 2x_2 + x_3 + 2x_4 \leq 20 \\ & 6x_1 + 5x_2 + 3x_3 + 2x_4 \leq 100 \\ & 3x_1 + 4x_2 + 9x_3 + 12x_4 \leq 75 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

Puttin this into standard form will make it look like:

$$\begin{aligned} \text{minimize } & -6x_1 - 4x_2 - 7x_3 - 5x_4 \\ \text{subject to } & x_1 + 2x_2 + x_3 + 2x_4 + s_1 = 20 \\ & 6x_1 + 5x_2 + 3x_3 + 2x_4 + s_2 = 100 \\ & 3x_1 + 4x_2 + 9x_3 + 12x_4 + s_3 = 75 \\ & x_1, x_2, x_3, x_4, s_1, s_2, s_3 \geq 0. \end{aligned}$$

Given this LP, the inital tableau will look like:

$$\left[\begin{array}{ccccccc|c} 1 & 2 & 1 & 2 & 1 & 0 & 0 & 20 \\ 6 & 5 & 3 & 2 & 0 & 1 & 0 & 100 \\ 3 & 4 & 9 & 12 & 0 & 0 & 1 & 75 \\ -6 & -4 & -7 & -5 & 0 & 0 & 0 & 0 \end{array}\right]$$

First we'll make column 3 look like column 7:

$$\left[\begin{array}{ccccccc|c} 2/3 & 14/9 & 0 & 2/3 & 1 & 0 & -1/9 & 35/3 \\ 5 & 11/3 & 0 & -2 & 0 & 1 & -1/3 & 75 \\ 1/3 & 4/9 & 1 & 4/3 & 0 & 0 & 1/9 & 25/3 \\ -11/3 & -8/9 & 0 & 13/3 & 0 & 0 & 7/9 & 175/3 \end{array}\right]$$

Next we'll make column 1 look like column 6:

$$\left[\begin{array}{ccccccc|c} 0 & 16/15 & 0 & 14/15 & 1 & -2/15 & -1/15 & 5/3 \\ 1 & 11/15 & 0 & -2/5 & 0 & 1/5 & -1/15 & 25 \\ 0 & 1/5 & 1 & 22/15 & 0 & -1/15 & 2/15 & 10/3 \\ 0 & 9/5 & 0 & 11/5 & 0 & 11/15 & 8/15 & 340/3 \end{array}\right]$$

So we get an optimal value of 340/3 when $(x_1, x_2, x_3, x_4) = (15, 0, 10/3, 0)$.

Problem 3:

$$\begin{aligned} \text{minimize } & x_1 + x_2 - x_3 \\ \text{subject to } & 2x_1 + x_2 + x_3 \leq 14 \\ & 4x_1 + 2x_2 + 3x_3 \leq 28 \\ & 2x_1 + 5x_2 + 5x_3 \leq 30 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

Puttin this into standard form will make it look like:

$$\begin{aligned} \text{minimize } & x_1 + x_2 - x_3 \\ \text{subject to } & 2x_1 + x_2 + x_3 + s_1 = 14 \\ & 4x_1 + 2x_2 + 3x_3 + s_2 = 28 \\ & 2x_1 + 5x_2 + 5x_3 + s_3 = 30 \\ & x_1, x_2, x_3, s_1, s_2, s_3 \geq 0. \end{aligned}$$

Given this LP, the inital tableau will look like:

$$\left[\begin{array}{cccccc|c} 2 & 1 & 1 & 1 & 0 & 0 & 14 \\ 4 & 2 & 3 & 0 & 1 & 0 & 28 \\ 2 & 5 & 5 & 0 & 0 & 1 & 30 \\ 1 & 1 & -1 & 0 & 0 & 0 & 0 \end{array}\right]$$

First we'll make column 3 look like column 6:

$$\left[\begin{array}{cccccc|c} 8/5 & 0 & 0 & 1 & 0 & -1/5 & 8 \\ 14/5 & -1 & 0 & 0 & 1 & -3/5 & 4 \\ 2/5 & 1 & 1 & 0 & 0 & 1/5 & 6 \\ 7/5 & 2 & 0 & 0 & 0 & 1/5 & 6 \end{array}\right]$$

So we get an optimal value of -6 when $(x_1, x_2, x_3) = (0, 0, 6)$.

Problem 4:

$$\begin{aligned} \text{maximize } & 2x_1 + 4x_2 + 3x_3 + x_4 \\ \text{subject to } & 3x_1 + x_2 + x_3 + 4x_4 \leq 12 \\ & x_1 - 3x_2 + 2x_3 + 3x_4 \leq 7 \\ & 2x_1 + x_2 + 3x_3 - x_4 \leq 10 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

Puttin this into standard form will make it look like:

$$
\begin{aligned}
\text{minimize } & -2x_1 - 4x_2 - 3x_3 - x_4 \\
\text{subject to } & 3x_1 + x_2 + x_3 + 4x_4 + s_1 = 12 \\
& x_1 - 3x_2 + 2x_3 + 3x_4 + s_2 = 7 \\
& 2x_1 + x_2 + 3x_3 - x_4 + s_3 = 10 \\
& x_1, x_2, x_3, x_4, s_1, s_2, s_3 \geq 0.
\end{aligned}
$$

Given this LP, the inital tableau will look like:

$$
\left[\begin{array}{ccccccc|c}
3 & 1 & 1 & 4 & 1 & 0 & 0 & 12 \\
1 & -3 & 2 & 3 & 0 & 1 & 0 & 7 \\
2 & 1 & 3 & -1 & 0 & 0 & 1 & 10 \\
-2 & -4 & -3 & -1 & 0 & 0 & 0 & 0
\end{array}\right]
$$

First we'll make column 2 look like column 7:

$$
\left[\begin{array}{ccccccc|c}
1 & 0 & -2 & 5 & 1 & 0 & -1 & 2 \\
7 & 0 & 11 & 0 & 0 & 1 & 3 & 37 \\
2 & 1 & 3 & -1 & 0 & 0 & 1 & 10 \\
6 & 0 & 9 & -5 & 0 & 0 & 4 & 40
\end{array}\right]
$$

Then we'll make column 4 look like column 5:

$$
\left[\begin{array}{ccccccc|c}
1/5 & 0 & -2/5 & 1 & 1/5 & 0 & -1/5 & 2/5 \\
7 & 0 & 11 & 0 & 0 & 1 & 3 & 37 \\
11/5 & 1 & 13/5 & 0 & 1/5 & 0 & 4/5 & 52/5 \\
7 & 0 & 7 & 0 & 1 & 0 & 3 & 42
\end{array}\right]
$$

So we get an optimal value of 42 when $(x_1, x_2, x_3, x_4) = (0, 52/5, 0, 2/5)$.