Math 120 Optimization: Homework 6

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Problem 1: Consider the following linear programming problem

$$\begin{aligned} \text{minimize } & -x_1 + 2x_2 - x_3 \\ \text{subject to } & x_1 + 3x_2 + x_4 = 4 \\ & 2x_1 + 6x_2 + x_3 + x_4 = 5 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

(a) Form the associated artificial problem and carry out the Phase I in the Two-Phase Simplex Method.

(b) From the final tableau for Phase I, find the initial canonical tableau for phase II (you don't need to solve the original problem).

Problem 2: Consider the following linear programming problem

$$\begin{aligned} \text{minimize } & -2x - 3y - 4z \\ \text{subject to } & 3x + 2y + z = 10 \\ & 2x + 5y + 3z = 15 \\ & x, y, z \geq 0. \end{aligned}$$

(a) Form the associated artificial problem and carry out the Phase I in the Two-Phase Simplex Method.

(b) From the final tableau for Phase I, find the initial canonical tableau for phase II (you don't need to solve the original problem).

Problem 3: Consider a standard form linear programming problem with

$$A = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \end{bmatrix}, \vec{b} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}, \vec{c} = \begin{bmatrix} 6 \\ c_2 \\ 4 \\ 5 \end{bmatrix}.$$

Suppose that we are told that the reduced cost coefficient vector corresponding to some basis is $\vec{r}^T = [0, 1, 0, 0]$.

(a) Find an optimal feasible solution to the problem;

(b) Find c_2.

Problem 4: Consider the linear program

$$\begin{aligned} \text{minimize } & 4x_1 + 3x_2 \\ \text{subject to } & 5x_1 + x_2 \geq 11 \\ & 2x_1 + x_2 \geq 8 \\ & x_1 + 2x_2 \geq 7 \\ & x_1, x_2 \geq 0. \end{aligned}$$

Write down the corresponding dual problem.