Math 120 Optimization: Homework 6

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Problem 1: Consider the following linear programming problem

$$\begin{aligned} \text{minimize } & -x_1 + 2x_2 - x_3 \\ \text{subject to } & x_1 + 3x_2 + x_4 = 4 \\ & 2x_1 + 6x_2 + x_3 + x_4 = 5 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

(a) Form the associated artificial problem and carry out the Phase I in the Two-Phase Simplex Method.

The artificial problem will look like:

$$\begin{aligned} \text{minimize } & y_1 + y_2 \\ \text{subject to } & x_1 + 3x_2 + x_4 + y_1 = 4 \\ & 2x_1 + 6x_2 + x_3 + x_4 + y_2 = 5 \\ & x_1, x_2, x_3, x_4, y_1, y_2 \geq 0. \end{aligned}$$

Meaning the tableau will look like:

$$\left[\begin{array}{cccccc|c} 1 & 3 & 0 & 1 & 1 & 0 & 4 \\ 2 & 6 & 1 & 1 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

First we'll remove the positive 1's in the bottom row to get:

$$\left[\begin{array}{cccccc|c} 1 & 3 & 0 & 1 & 1 & 0 & 4 \\ 2 & 6 & 1 & 1 & 0 & 1 & 5 \\ -3 & -9 & -1 & -2 & 0 & 0 & -9 \end{array}\right]$$

(b) From the final tableau for Phase I, find the initial canonical tableau for phase II (you don't need to solve the original problem).

After running Simplex, we'll get the tableau:

$$\left[\begin{array}{cccccc|c} 0 & 0 & -1 & 1 & 2 & -1 & 3 \\ 1/3 & 1 & 1/3 & 0 & -1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

To transform this back into the original problem, we can reconstruct the tableau using this solution:

$$\left[\begin{array}{cccc|c} 0 & 0 & -1 & 1 & 3 \\ 1/3 & 1 & 1/3 & 0 & 1/3 \\ -1 & 2 & -1 & 0 & 0 \end{array}\right]$$

Problem 2: Consider the following linear programming problem

$$\begin{aligned} \text{minimize } & -2x - 3y - 4z \\ \text{subject to } & 3x + 2y + z = 10 \\ & 2x + 5y + 3z = 15 \\ & x, y, z \geq 0. \end{aligned}$$

(a) Form the associated artificial problem and carry out the Phase I in the Two-Phase Simplex Method.

The artificial problem will look like:

$$\begin{aligned} \text{minimize } & y_1 + y_2 \\ \text{subject to } & 3x + 2y + z + y_1 = 10 \\ & 2x + 5y + 3z + y_2 = 15 \\ & x, y, z, y_1, y_2 \geq 0. \end{aligned}$$

Meaning the tableau will look like:

$$\left[\begin{array}{ccccc|c} 3 & 2 & 1 & 1 & 0 & 10 \\ 2 & 5 & 3 & 0 & 1 & 15 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

(b) From the final tableau for Phase I, find the initial canonical tableau for phase II (you don't need to solve the original problem).

Runnin Simplex on this will net us a matrix looking like:

$$\left[\begin{array}{ccccc|c} 1 & 0 & -1/11 & 1 & -2/11 & 20/11 \\ 0 & 1 & 7/11 & -2/5 & 3/11 & 25/11 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

Going back to the original problem for phase 2 gives us:

$$\left[\begin{array}{ccc|c} 1 & 0 & -1/11 & 20/11 \\ 0 & 1 & 7/11 & 25/11 \\ -2 & -3 & -4 & 0 \end{array}\right]$$

Problem 3: Consider a standard form linear programming problem with

$$A = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \end{bmatrix}, \vec{b} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}, \vec{c} = \begin{bmatrix} 6 \\ c_2 \\ 4 \\ 5 \end{bmatrix}.$$

Suppose that we are told that the reduced cost coefficient vector corresponding to some basis is $\vec{r}^T = [0, 1, 0, 0]$.

(a) Find an optimal feasible solution to the problem;

$(x_1, x_2, x_3, x_4) = (8, 0, 9, 7)$ with a value of $48 + 36 + 35 = 119$

(b) Find c_2.

$c_2 = 6 + 12 + 10 + 1 = 29$

Problem 4: Consider the linear program

$$\begin{aligned} \text{minimize } & 4x_1 + 3x_2 \\ \text{subject to } & 5x_1 + x_2 \geq 11 \\ & 2x_1 + x_2 \geq 8 \\ & x_1 + 2x_2 \geq 7 \\ & x_1, x_2 \geq 0. \end{aligned}$$

Write down the corresponding dual problem.

For this problem,

$$A = \begin{bmatrix} 5 & 1 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}, \vec{b} = \begin{bmatrix} 11 \\ 8 \\ 7 \end{bmatrix}, \vec{c} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}.$$

So the dual will be:

$$\begin{aligned} \max\ & \vec{b} \cdot \vec{y} \\ \text{subject to } & A^T \vec{y} \leq \vec{c} \\ & \vec{y} \geq 0. \end{aligned}$$

Where

$$A^T = \begin{bmatrix} 5 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}, \vec{c} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \vec{b} = \begin{bmatrix} 11 \\ 8 \\ 7 \end{bmatrix}.$$