# Multi-Objective Optimization of a Composite Longboard Deck

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Introduction:

In the world of longboarding, many factors come into play when choosing a board. Wheel diameter, wheel hardness, length, and shape all determine what kind of ride the longboard provides. The largest factor, however, is what material the board is made of. Almost all longboards are made with plies of wood that are glued together perpendicular to the grain to create a cross-grain pattern. Most longboards have 7 – 11 of these plies which helps keeps the board thin and light, without compromising the wood's strength or integrity (*The Longboard Glossary*, 2020) . The most common material longboard decks are made of is maple, which provides a sturdy ride with minimal flex. Over the past couple of years, Bamboo has grown in popularity as a material for longboard decks due to its light weight, flexibility, and environmental sustainability. This has given rise to boards with levels of Flex. Flex is a mostly arbitrary ranking system used by manufacturers to classify how much weight the boards can take. This Flex system commonly goes from 1-5 with 1 being the stiffest and 5 being the most flexible.

| FLEX RATING | WEIGHT (LBS) | WEIGHT (KG) |
|---|---|---|
| FLEX 1 | 175 - 230+ | 80 - 105+ |
| FLEX 2 | 150 - 210+ | 68 - 95+ |
| FLEX 3 | 150 - 200+ | 68 - 90 |
| FLEX 4 | 120 - 170+ | 55 - 77+ |
| FLEX 5 | 80 - 140+ | 35 - 65+ |

*Figure 1 - Flex Rating Table (ref 5)*

The most common Flex value is 1-2 due to the increasing weight restrictions with higher Flex. Flex 2 is the preferred choice for most cruiser boards, boards meant to focus on relaxed riding and small to medium hills. The flex is a double edged sword for this type of ride; it helps alleviate fatigue on the rider's legs by giving a pseudo suspension system over rough surfaces, but also makes the rider more susceptible to 'speed wobbles' at higher speeds due to decreased stability. While going down larger hills, these 'speed wobbles' can spell out certain injury for a long boarder if they are not careful managing their speed. Another problem that plagues long boarders is board weight. Due to the wider and longer boards, they are often heavier than standard skateboards and require more energy to accelerate the board while pushing and become uncomfortable to carry for long periods of time. For my optimization problem, I will be minimizing both the mass of a longboard as well as its deflection to provide a comfortable, light, and sturdy ride.

Design:

For my design, I will be optimizing a standard 42-inch (1.0668 m) Flex 2 longboard and using a point load to represent an 80 kg rider.

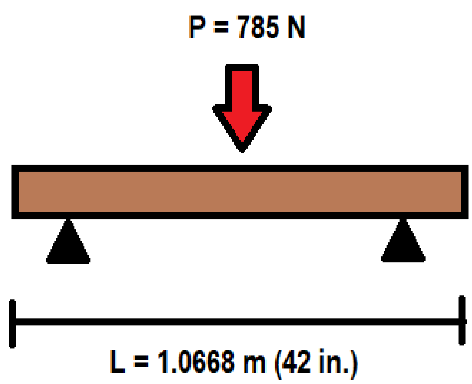


*Figure 2 - Simplified Force Diagram of Long Board*

As discussed in the introduction, my two objective functions will be the mass and deflection of the longboard. I will be optimizing these two functions by changing the width and thickness of the longboard as my design variables. For my constraint functions my first constraint is that the rupture stress must be less than the stress allowed for the given material. My next 4 constraint functions are the bounds I set on the longboard, which are that the thickness cannot be less than .5 cm and no greater than 1.5 cm to prevent 'wheel bite' (rubbing of the bottom of the board against the top of the wheels). For width, I set up the bounds as being from .203 meters (8 inches) to .254 meters (10 inches) which is the standard range of a longboard's width.

Objective Functions: $f_1(x) = L * w * t * \rho \quad (kg)$

$$f_2(x) = \frac{L^3 P}{48 I_{zz} E} \quad (m)$$

Design variables: width (w) & thickness (t)

Constraint functions:

$$g_1(x) = \frac{3PL}{2wt^2\sigma_a} - 1 \leq 0 \qquad g_4(x) = \frac{.005}{t} - 1 \leq 0$$

$$g_2(x) = \frac{.2032}{w} - 1 \leq 0 \qquad g_5(x) = 1 - \frac{.015}{t} \leq 0$$

$$g_3(x) = 1 - \frac{.254}{w} \leq 0$$

*Figure 3 - Formal Problem Statement*

In order to optimize both mass and deflection, I will be using a modified weighted sum approach that creates a composite board of different percentages of bamboo and maple. Instead of using the 'a' values of the weighted sum equation to signify importance, I will be giving both deflection and mass equal importance (.5) and instead be using 'a' as my values for percentage of maple and bamboo, respectively. Since the percentages of maple and bamboo are changing with each iteration, the mechanical properties of the long board deck will also change. To account for this, I made a second set of objective functions and constraints ($g_1$) that utilizes the 'rule of mixtures' to adjust the density, Young's Modulus, and allowable stress, based off the percent volume of each material present in the skateboard (*Composites, n.d)*.

| Maple: | Bamboo: |
|---|---|
| $\rho_m = 675\ kg/m^3$ | $\rho_b = 345\ kg/m^3$ |
| $E_m = 12.6$ GPa | $E_b = 10.3$ GPa |
| $\sigma_m = 38$ MPa | $\sigma_b = 35.92$ MPa |

Objective Functions: $f_1'(x) = L * w * t * (a_1 * \rho_m + a_2 * \rho_b) \quad (kg)$

$$f_2'(x) = \frac{L^3 P}{48 I_{zz}(a_1 * E_m + a_2 * E_b)} \quad (m)$$

$$\Phi(x) = .5 * \left[\frac{f_1'(x) - f_1^{min}}{|f_1^{min}|}\right] + .5 * \left[\frac{f_2'(x) - f_2^{min}}{|f_2^{min}|}\right]$$

Design variables: width (w) & thickness (t)

Constraint functions: $g_1(x) = \frac{3PL}{2wt^2(a_1 * \sigma_m + a_2 * \sigma_b)} - 1 \le 0$

$g_2(x) = \frac{.2032}{w} - 1 \le 0$ $\qquad$ $g_3(x) = 1 - \frac{.254}{w} \le 0$

$g_4(x) = \frac{.005}{t} - 1 \le 0$ $\qquad$ $g_5(x) = 1 - \frac{.015}{t} \le 0$

*Figure 4 - Problem Statement for Weighted Sum Application (Mechanical Properties gathered from ref 2, 3 and 4)*

The Analysis tool I will be using to evaluate this problem will be MATLAB. I will be using a Sequential Quadratic Programming (SQP) approach using the Matlab program 'fmincon'. I chose this approach because SQP excels in constrained, non-linear minimization. It also handles infeasible $x^0$ which was helpful when switching starting conditions during optimization. SQP also only handles first order information so analytical gradients were not needed.

Results:

After running through the 9 optimizations with 3 different $x^0$ values I arrived at the solution in Figure 6 below.

| % of Maple ($a_1$) | % of Bamboo ($a_2$) | $x^0$ (m) | | $g_1(x^*)$ (MPa) | $g_2(x^*)$ (m) | $g_3(x^*)$ (m) | $g_4(x^*)$ (m) | $g_5(x^*)$ (m) | $f_1(x^*)$ (kg) | $f_2(x^*)$ (m) | $x^*$ (m) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0.1 | 0.005 | 1.68271E-09 | -3.078E-05 | -0.2499615 | -0.6188113 | -0.448517 | 0.98100022 | 0.03339804 | 0.203206 | 0.01312 |
| 0.15 | 0.85 | 0.1 | 0.005 | 1.59902E-08 | -1.11E-16 | -0.25 | -0.6171652 | -0.454772 | 1.11689498 | 0.03253967 | 0.2032 | 0.01306 |
| 0.25 | 0.75 | 0.1 | 0.005 | 1.43924E-08 | -1.11E-16 | -0.25 | -0.6160679 | -0.458942 | 1.20686424 | 0.03199922 | 0.2032 | 0.01302 |
| 0.4 | 0.6 | 0.2 | 0.01 | -5.54001E-14 | 0 | -0.25 | -0.6144278 | -0.465174 | 1.34087835 | 0.03123213 | 0.2032 | 0.01297 |
| 0.5 | 0.5 | 0.2 | 0.01 | -4.56302E-14 | 0 | -0.25 | -0.6133383 | -0.469315 | 1.42960377 | 0.03074765 | 0.2032 | 0.01293 |
| 0.6 | 0.4 | 0.2 | 0.01 | -3.93019E-14 | 0 | -0.25 | -0.6122518 | -0.473443 | 1.51784255 | 0.03028315 | 0.2032 | 0.01289 |
| 0.75 | 0.25 | 0.25 | 0.015 | -8.53095E-13 | 0 | -0.25 | -0.6106277 | -0.479615 | 1.64930149 | 0.02962123 | 0.2032 | 0.01284 |
| 0.85 | 0.15 | 0.25 | 0.015 | -9.56346E-13 | 0 | -0.25 | -0.6095488 | -0.483715 | 1.73634994 | 0.02920159 | 0.2032 | 0.01281 |
| 1 | 0 | 0.25 | 0.015 | -1.02673E-12 | 2.2204E-16 | -0.25 | -0.6079359 | -0.489843 | 1.8660508 | 0.02860209 | 0.2032 | 0.01275 |

*Figure 6 - Optimized Solution for the 9 different boards*

Looking at Figure 6, we can see that all x* values are feasible and that the width of the longboard stays relatively unchanged throughout each iteration, hovering around the lowest value .2032 meters. However, as the board becomes less bamboo and more maple, the thickness decreases to keep the mass minimized but also because maple is sturdier and does not require the same thickness as bamboo. As maple percentage increases and bamboo percentage decreases, we see an increase in the minimal weight of the board but a decrease in the deflection of the board. To find the optimal design, we need to look at the Pareto Frontier in Figure 7.

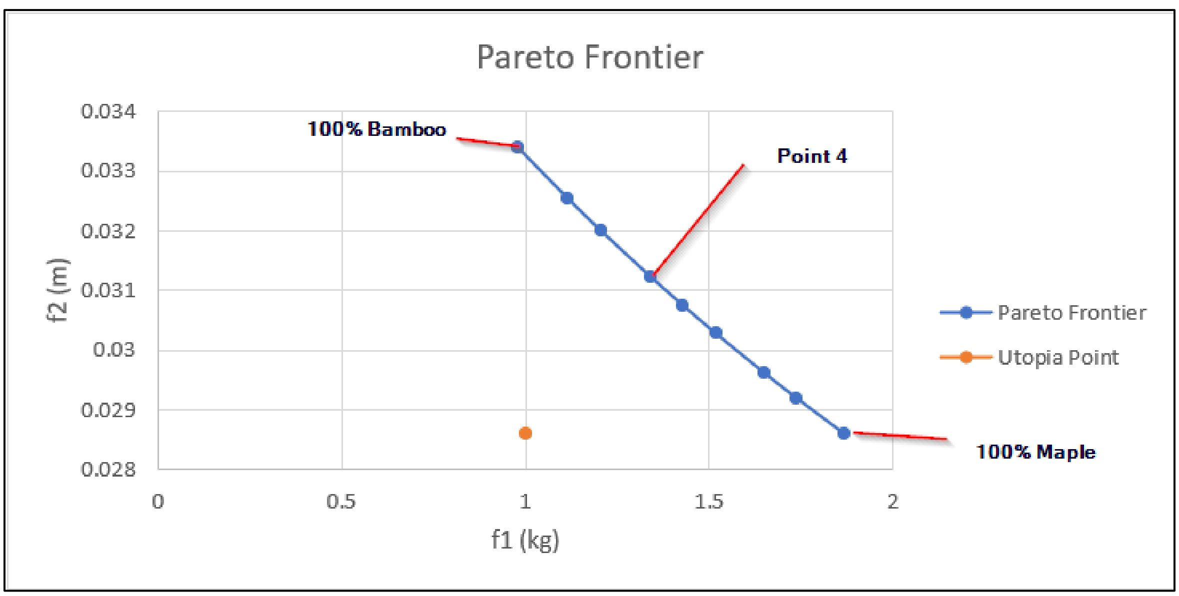


*Figure 7 - Pareto Frontier of Composite Board*

Looking at the Pareto Frontier, it is clear the optimal point along the frontier that is closest to the Utopia point is point 4. This brings us to our optimal solution of a board weighing 1.34 kg with a max deflection of 3.12 cm. This was achieved using a composite board of 60% bamboo and 40% maple, with a width of .2032 m and a thickness of 1.29 cm.

Conclusion:

Our answer does make sense in the context of the real world, as most cruiser boards on the market have a similar composition of bamboo and maple, with similar dimensions. All the constraints for the problem were satisfied with one of them being active. The problem may be missing a constraint that would have helped the width of the board change more, I believe if I were to redo the problem I would add a penalty to the minimum width of the board as this decreases stability as well. The answer is a local minimum in the sense that it is not at the true global minimum, but only the global minimum for the 'a' bounds. If I were to redo the problem with a much smaller step size between material percentages, I would be able to reach the true global minimum of the problem. This solution does satisfy the Kuhn-Tucker Conditions as the x* is feasible and in the bounds we set, all $g_j(x^*)$ are less than 0 except $g_2$ and solving the third Kuhn-Tucker condition we get:

$$\{\frac{6.6}{103.4}\} + \{\frac{-.003}{-.3e\text{-}5}\} + \lambda_2\{\frac{-4.92}{-77.09}\} = \{\frac{0}{0}\} \rightarrow \lambda_2 = 1.34 \geq 0 \quad \text{Kuhn-Tucker satisfied}$$

The problem did have some significant set up time as each equation had to be tweaked with every iteration to adjust the mechanical properties present in the constrain and objective functions, however the computational cost was relatively low as 'fmincon' was used which does not have a massive draw on the computers processing. As mentioned above, if I were to do this problem again, I would make it more accurate by lowering the percentage step size of the materials and add more constraints to make width more competitive.

References:

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Lee, A. W. C., Bai, X., & Peralta, P. N. (1994). Selected physical and mechanical properties of giant timber bamboo grown in south carolina. Forest Products Journal, 44(9), 40. Retrieved from https://search.proquest.com/scholarly-journals/selected-physical-mechanical-properties-giant/docview/214634490/se-2?accountid=13360

Loaded Longboards Flex Chart. (n.d.). Retrieved December 03, 2020, from https://www.evo.com/loaded-longboards-flex-chart

The Longboard Glossary. (2020). Retrieved December 03, 2020, from https://www.thelongboardstore.com/the-longboard-glossar

## Appendix A: Code used

```
function [f,grad_f] = mass(x)
L = 1.0668; %m
rho = 345; %kg/m^3
f = L * x(1) * x(2) * rho; %kg
if nargout > 1
    grad_f = [];

end
```

```
function [f,grad_f] = deflection(x)
P = 80 * 9.81; %`785 N
E = 12.6e9; %Pa
Izz = x(1) * x(2)^2 /12;
L = 1.0668; %m
%f = P / (24 * E * Izz) * ((L/2)^4 - 4*L^3 * L /2 + 3*L^4);
f = L^3 * P*50 / ( 48* Izz* E);
if nargout > 1
    grad_f = [];


end
```

```
function [f,grad_f] = deflection2(x)
global a1 a2 a3 a4 a5 a6 a7 a8 a9
a = a9;
P = 80 * 9.81; %`785 N
E = a(1) * 12.6e9 + a(2) * 10.2e9; %Pa
Izz = x(1) * x(2)^2 /12;
L = 1.0668; %m
%f = P / (24 * E * Izz) * ((L/2)^4 - 4*L^3 * L /2 + 3*L^4);
f = L^3 * P*50 / ( 48* Izz* E);
if nargout > 1
    grad_f = [];


end
```

```
function [g,h,grad_g,grad_h] = boardcon(x)
P = 80 * 9.81; %`785 N
L = 1.0668; %m
g(1) = 3*P*L / (2*x(1)*x(2)^2*35.92e6) - 1;
g(2) = .2032 / x(1) -1;
g(3) = 1 - .254 / x(1);
g(4) = .005 / x(2) - 1;
g(5) = 1 - .019/x(2);
h = [];

if nargout > 2
    grad_g = [-x(2);-x(1)];
    grad_h = [];
end
end
```

```
function [g,h,grad_g,grad_h] = boardcon2(x)
global a1 a2 a3 a4 a5 a6 a7 a8 a9
a = a9;

P = 80 * 9.81; %`785 N
L = 1.0668; %m
s = a(1) * 38e6 + a(2) * 35.92e6;
g(1) = 3*P*L / (2*x(1)*x(2)^2*s) - 1;
g(2) = .2032 / x(1) -1;
g(3) = 1 - .254 / x(1);
g(4) = .005 / x(2) - 1;
g(5) = 1 - .019/x(2);
h = [];

if nargout > 2
    grad_g = [-x(2);-x(1)];
    grad_h = [];
end
end
```

```
function phi = weightsum1(x)
global a1 a2 a3 a4 a5 a6 a7 a8 a9
a = a9;

%Constant Variables

L = 1.0668; %m
P = 80 * 9.81; %`785 N
Izz = x(1) * x(2)^2 /12;

% Composite Variables
E = a(1) * 11.31e9 + a(2) * 15.03e9; %Pa
rho = a(1) * 675 + a(2) * 345; %kg/m^3

phi1 =  .5 * ((L * x(1) * x(2) * rho -.9810) / .9810)^2;
phi2 =  .5 * ((P / (24 * E * Izz) *...
        ((L/2)^4 - 4*L^3 * L /2 + 3*L^4) - .0103) / .0103)^2;
phi = phi1 + phi2;
end
```

```
function [f,grad_f] = mass2(x)
global a1 a2 a3 a4 a5 a6 a7 a8 a9
a = a9;

L = 1.0668; %m
rho = a(1) * 675 + a(2) * 345; %kg/m^3
f = L * x(1) * x(2) * rho; %kg
if nargout > 1
    grad_f = [];

end
```

```
clc
clear all

A = [];
b = [];
x0 = [.25;.015];
options = optimoptions(@fmincon,'Algorithm','sqp');
Aeq = [];
beq = [];
lb = [];
ub = [];
xstar = fmincon(@mass,x0,A,b,Aeq,beq,lb,ub,@boardcon,options);
[fx1,fgrad] = mass(xstar);
xstar2 = fmincon(@deflection,x0,A,b,Aeq,beq,lb,ub,@boardcon,options);
[fx2,fgrad2] = deflection(xstar2);
lowestf = [fx1;fx2];
global a1 a2 a3 a4 a5 a6 a7 a8 a9
a1 = [0;1];
a2 = [.15;.85];
a3 = [.25;.75];
a4 = [.4;.6];
a5 = [.5;.5];
a6 = [.6;.4];
a7 = [.75;.25];
a8 = [.85;.15];
a9 = [1;0];
xstarphi = fmincon(@weightsum1,x0,A,b,Aeq,beq,lb,ub,@boardcon2,options);
```

```
fx11 = mass2(xstarphi);
fx21 = deflection2(xstarphi);
finalf1 = [fx11;fx21]
[g,h,grad_g,grad_h] = boardcon2(xstarphi);
T = table(g);
T1 = table(transpose(x0));
T2 = table(transpose(finalf1));
T3 = table(transpose(xstarphi));
filename = 'Final Table.xlsx';
writetable(T,filename,'Sheet',2,'Range','E10','WriteVariableNames',false);
writetable(T1,filename,'Sheet',2,'Range','C10','WriteVariableNames',false);
writetable(T2,filename,'Sheet',2,'Range','J10','WriteVariableNames',false);
writetable(T3,filename,'Sheet',2,'Range','L10','WriteVariableNames',false);
```

## Appendix B: Excel workbook

| % of Maple ($a_1$) | % of Bamboo ($a_2$) | $x^0$ (m) | | $g_1(x^*)$ (MPa) | $g_2(x^*)$ (m) | $g_3(x^*)$ (m) | $g_4(x^*)$ (m) | $g_5(x^*)$ (m) | **$f_1(x^*)$ (kg)** | **$f_2(x^*)$ (m)** | **$x^*$ (m)** | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0.1 | 0.005 | 1.68271E-09 | -3.078E-05 | -0.2499615 | -0.6188113 | -0.448517 | 0.98100022 | 0.03339804 | 0.203206 | 0.01312 |
| 0.15 | 0.85 | 0.1 | 0.005 | 1.59902E-08 | -1.11E-16 | -0.25 | -0.6171652 | -0.454772 | 1.11689498 | 0.03253967 | 0.2032 | 0.01306 |
| 0.25 | 0.75 | 0.1 | 0.005 | 1.43924E-08 | -1.11E-16 | -0.25 | -0.6160679 | -0.458942 | 1.20686424 | 0.03199922 | 0.2032 | 0.01302 |
| 0.4 | 0.6 | 0.2 | 0.01 | -5.54001E-14 | 0 | -0.25 | -0.6144278 | -0.465174 | 1.34087835 | 0.03123213 | 0.2032 | 0.01297 |
| 0.5 | 0.5 | 0.2 | 0.01 | -4.56302E-14 | 0 | -0.25 | -0.6133383 | -0.469315 | 1.42960377 | 0.03074765 | 0.2032 | 0.01293 |
| 0.6 | 0.4 | 0.2 | 0.01 | -3.93019E-14 | 0 | -0.25 | -0.6122518 | -0.473443 | 1.51784255 | 0.03028315 | 0.2032 | 0.01289 |
| 0.75 | 0.25 | 0.25 | 0.015 | -8.53095E-13 | 0 | -0.25 | -0.6106277 | -0.479615 | 1.64930149 | 0.02962123 | 0.2032 | 0.01284 |
| 0.85 | 0.15 | 0.25 | 0.015 | -9.56346E-13 | 0 | -0.25 | -0.6095488 | -0.483715 | 1.73634994 | 0.02920159 | 0.2032 | 0.01281 |
| 1 | 0 | 0.25 | 0.015 | -1.02673E-12 | 2.2204E-16 | -0.25 | -0.6079359 | -0.489843 | 1.8660508 | 0.02860209 | 0.2032 | 0.01275 |