# Foundations for statistical inference - Sampling distributions

Daniel Moscoe

In this lab, you will investigate the ways in which the statistics from a random sample of data can serve as point estimates for population parameters. We're interested in formulating a *sampling distribution* of our estimate in order to learn about the properties of the estimate, such as its distribution.

**Setting a seed:** We will take some random samples and build sampling distributions in this lab, which means you should set a seed at the start of your lab. If this concept is new to you, review the lab on probability.

## Getting Started

### Load packages

In this lab, we will explore and visualize the data using the **tidyverse** suite of packages. We will also use the **infer** package for resampling.

Let's load the packages.

```
library(tidyverse)
library(openintro)
library(infer)
```

### The data

A 2019 Gallup report states the following:

> The premise that scientific progress benefits people has been embodied in discoveries throughout the ages – from the development of vaccinations to the explosion of technology in the past few decades, resulting in billions of supercomputers now resting in the hands and pockets of people worldwide. Still, not everyone around the world feels science benefits them personally.
>
> **Source:** World Science Day: Is Knowledge Power?

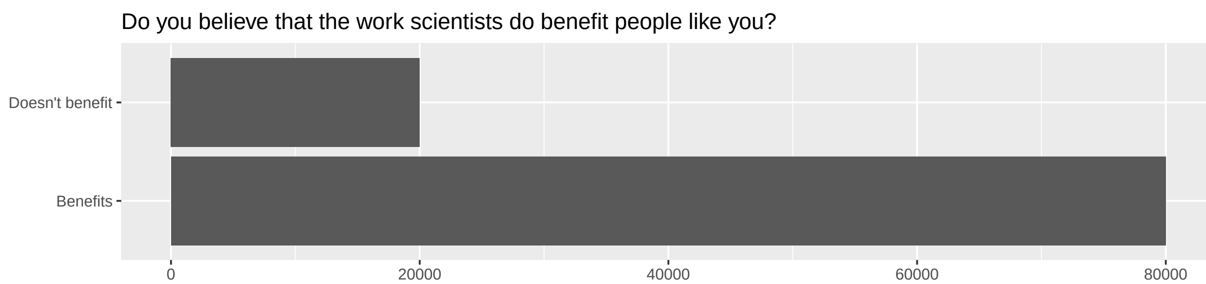
The Wellcome Global Monitor finds that 20% of people globally do not believe that the work scientists do benefits people like them. In this lab, you will assume this 20% is a true population proportion and learn about how sample proportions can vary from sample to sample by taking smaller samples from the population. We will first create our population assuming a population size of 100,000. This means 20,000 (20%) of the population think the work scientists do does not benefit them personally and the remaining 80,000 think it does.

```
global_monitor <- tibble(
  scientist_work = c(rep("Benefits", 80000), rep("Doesn't benefit", 20000))
)
```

The name of the data frame is `global_monitor` and the name of the variable that contains responses to the question *"Do you believe that the work scientists do benefit people like you?"* is `scientist_work`.

We can quickly visualize the distribution of these responses using a bar plot.

```
ggplot(global_monitor, aes(x = scientist_work)) +
  geom_bar() +
  labs(
    x = "", y = "",
    title = "Do you believe that the work scientists do benefit people like you?"
  ) +
  coord_flip()
```




We can also obtain summary statistics to confirm we constructed the data frame correctly.

```
global_monitor %>%
  count(scientist_work) %>%
  mutate(p = n /sum(n))
```

```
## # A tibble: 2 x 3
##   scientist_work      n     p
## * <chr>           <int> <dbl>
## 1 Benefits        80000   0.8
## 2 Doesn't benefit 20000   0.2
```

## The unknown sampling distribution

In this lab, you have access to the entire population, but this is rarely the case in real life. Gathering information on an entire population is often extremely costly or impossible. Because of this, we often take a sample of the population and use that to understand the properties of the population.

If you are interested in estimating the proportion of people who don't think the work scientists do benefits them, you can use the `sample_n` command to survey the population.

```
samp1 <- global_monitor %>%
  sample_n(50)
```

This command collects a simple random sample of size 50 from the `global_monitor` dataset, and assigns the result to `samp1`. This is similar to randomly drawing names from a hat that contains the names of all in the population. Working with these 50 names is considerably simpler than working with all 100,000 people in the population.

1. Describe the distribution of responses in this sample. How does it compare to the distribution of responses in the population. **Hint:** Although the `sample_n` function takes a random sample of observations (i.e. rows) from the dataset, you can still refer to the variables in the dataset with the same names. Code you presented earlier for visualizing and summarizing the population data will still be useful for the sample, however be careful to not label your proportion `p` since you're now calculating a sample statistic, not a population parameters. You can customize the label of the statistics to indicate that it comes from the sample.

```
samp1 %>%
  count(scientist_work) %>%
  mutate(p_hat = n / sum(n))
```

```
## # A tibble: 2 x 3
##   scientist_work      n p_hat
## * <chr>           <int> <dbl>
## 1 Benefits           40   0.8
## 2 Doesn't benefit    10   0.2
```

In this sample, 40 respondents, or 80%, answered that the work scientists do benefits them. Ten respondents, or 20%, answered that it does not. The sample proportion in this case is equal to the population proportion. In general, this will not be the case, even though the sample proportion is an unbiased estimator of the population proportion.

If you're interested in estimating the proportion of all people who do not believe that the work scientists do benefits them, but you do not have access to the population data, your best single guess is the sample proportion.

```
samp1 %>%
  count(scientist_work) %>%
  mutate(p_hat = n /sum(n))
```

```
## # A tibble: 2 x 3
##   scientist_work      n p_hat
## * <chr>           <int> <dbl>
## 1 Benefits           40   0.8
## 2 Doesn't benefit    10   0.2
```

Depending on which 50 people you selected, your estimate could be a bit above or a bit below the true population proportion of 0.2. In general, though, the sample proportion turns out to be a pretty good estimate of the true population proportion, and you were able to get it by sampling less than 1% of the population.

2. Would you expect the sample proportion to match the sample proportion of another student's sample? Why, or why not? If the answer is no, would you expect the proportions to be somewhat different or very different? Ask a student team to confirm your answer.

I would not expect the sample proportion above to match exactly the sample proportion of another student's sample. That's because which members of the population are selected for a sample vary each time a sample is selected. While the sample proportions found by different students may vary, I would expect them to be close, and for their sample proportion to be near the true population proportion of 80%.

3. Take a second sample, also of size 50, and call it `samp2`. How does the sample proportion of `samp2` compare with that of `samp1`? Suppose we took two more samples, one of size 100 and one of size 1000. Which would you think would provide a more accurate estimate of the population proportion?

```r
samp2 <- global_monitor %>%
  sample_n(50)

samp2 %>%
  count(scientist_work) %>%
  mutate(p_hat = n / sum(n))
```

```
## # A tibble: 2 x 3
##   scientist_work      n p_hat
## * <chr>           <int> <dbl>
## 1 Benefits           43  0.86
## 2 Doesn't benefit     7  0.14
```

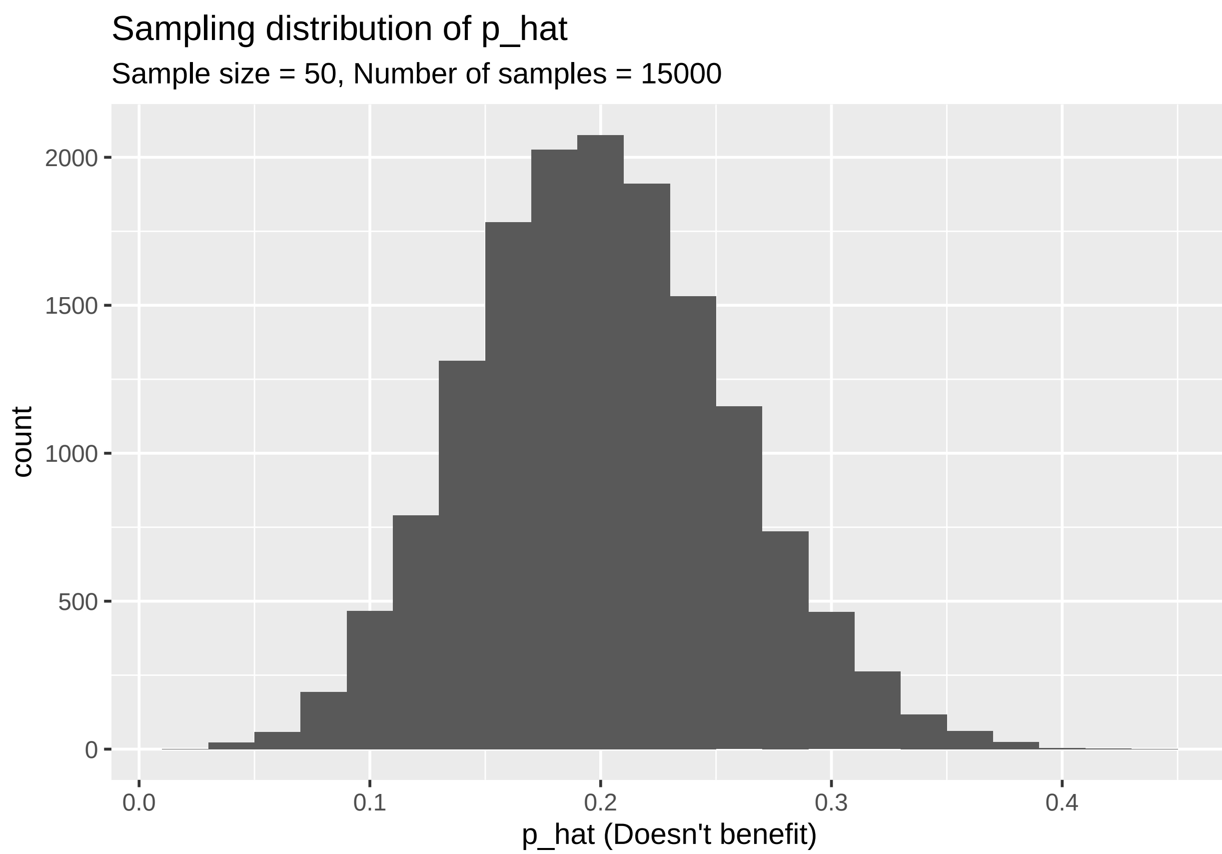
The sample proportion in samp2 is slightly greater (0.86) than the sample proportion in samp1 (0.80). If we took two more samples of size 100 and size 1000, I would expect the larger samples to provide more accurate estimates of the population proportion.

Not surprisingly, every time you take another random sample, you might get a different sample proportion. It's useful to get a sense of just how much variability you should expect when estimating the population proportion this way. The distribution of sample proportions, called the *sampling distribution (of the proportion)*, can help you understand this variability. In this lab, because you have access to the population, you can build up the sampling distribution for the sample proportion by repeating the above steps many times. Here, we use R to take 15,000 different samples of size 50 from the population, calculate the proportion of responses in each sample, filter for only the *Doesn't benefit* responses, and store each result in a vector called `sample_props50`. Note that we specify that `replace = TRUE` since sampling distributions are constructed by sampling with replacement.

```r
sample_props50 <- global_monitor %>%
                    rep_sample_n(size = 50, reps = 15000, replace = TRUE) %>%
                    count(scientist_work) %>%
                    mutate(p_hat = n /sum(n)) %>%
                    filter(scientist_work == "Doesn't benefit")
```

And we can visualize the distribution of these proportions with a histogram.

```r
ggplot(data = sample_props50, aes(x = p_hat)) +
  geom_histogram(binwidth = 0.02) +
  labs(
    x = "p_hat (Doesn't benefit)",
    title = "Sampling distribution of p_hat",
    subtitle = "Sample size = 50, Number of samples = 15000"
  )
```




Next, you will review how this set of code works.

4. How many elements are there in `sample_props50`? Describe the sampling distribution, and be sure to specifically note its center. Make sure to include a plot of the distribution in your answer.

There are 15000 rows in sample_props50. The sampling distribution is nearly normal, with center very near 0.2, the true population proportion for those who believe science doesn't benefit them. The plot of the distribution is above, titled "Sampling distribution of p_hat."

## Interlude: Sampling distributions

The idea behind the `rep_sample_n` function is *repetition*. Earlier, you took a single sample of size `n` (50) from the population of all people in the population. With this new function, you can repeat this sampling procedure `rep` times in order to build a distribution of a series of sample statistics, which is called the **sampling distribution**.

Note that in practice one rarely gets to build true sampling distributions, because one rarely has access to data from the entire population.

Without the `rep_sample_n` function, this would be painful. We would have to manually run the following code 15,000 times

```
global_monitor %>%
  sample_n(size = 50, replace = TRUE) %>%
  count(scientist_work) %>%
  mutate(p_hat = n /sum(n)) %>%
  filter(scientist_work == "Doesn't benefit")
```

```
## # A tibble: 1 x 3
##   scientist_work      n p_hat
##   <chr>           <int> <dbl>
## 1 Doesn't benefit     7  0.14
```

as well as store the resulting sample proportions each time in a separate vector.

Note that for each of the 15,000 times we computed a proportion, we did so from a **different** sample!

5. To make sure you understand how sampling distributions are built, and exactly what the `rep_sample_n` function does, try modifying the code to create a sampling distribution of **25 sample proportions** from **samples of size 10**, and put them in a data frame named `sample_props_small`. Print the output. How many observations are there in this object called `sample_props_small`? What does each observation represent?

```
sample_props_small <- global_monitor %>%
  rep_sample_n(size = 10, reps = 25, replace = TRUE) %>%
  count(scientist_work) %>%
  mutate(p_hat = n / sum(n)) %>%
  filter(scientist_work == "Doesn't benefit")

head(sample_props_small)
```

```
## # A tibble: 6 x 4
## # Groups:   replicate [6]
##   replicate scientist_work      n p_hat
##       <int> <chr>           <int> <dbl>
## 1         1 Doesn't benefit     2   0.2
## 2         2 Doesn't benefit     4   0.4
## 3         3 Doesn't benefit     3   0.3
## 4         5 Doesn't benefit     1   0.1
## 5         6 Doesn't benefit     1   0.1
## 6         7 Doesn't benefit     3   0.3
```

There are 25 observations in sample_props_small. Each observation represents a sample proportion from a sample of size $n = 10$ drawn from the population of 100,000 respondents. For each sample, p-hat is calculated, which is the fraction of respondents in the sample responding that scientific work does not benefit them. These p-hat are estimators for the population proportion.

## Sample size and the sampling distribution

Mechanics aside, let's return to the reason we used the `rep_sample_n` function: to compute a sampling distribution, specifically, the sampling distribution of the proportions from samples of 50 people.

```
ggplot(data = sample_props50, aes(x = p_hat)) +
  geom_histogram(binwidth = 0.02)
```

The sampling distribution that you computed tells you much about estimating the true proportion of people who think that the work scientists do doesn't benefit them. Because the sample proportion is an unbiased estimator, the sampling distribution is centered at the true population proportion, and the spread of the distribution indicates how much variability is incurred by sampling only 50 people at a time from the population.

In the remainder of this section, you will work on getting a sense of the effect that sample size has on your sampling distribution.

6. Use the app below to create sampling distributions of proportions of *Doesn't benefit* from samples of size 10, 50, and 100. Use 5,000 simulations. What does each observation in the sampling distribution represent? How does the mean, standard error, and shape of the sampling distribution change as the sample size increases? How (if at all) do these values change if you increase the number of simulations? (You do not need to include plots in your answer.)

Each observation in one of the sampling distributions represents a sample proportion of respondents reporting that science doesn't benefit them. In the sampling distribution, each observation is a sample. Within a sample, each observation is the report of one respondent. For all these sampling distributions, the mean is very near the true population proportion, 0.2. As sample size increases, the standard error of the sampling distribution decreases, and the shape of the distribution becomes more strongly peaked at the true population proportion.

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## More Practice

So far, you have only focused on estimating the proportion of those who think the work scientists doesn't benefit them. Now, you'll try to estimate the proportion of those who think it does.

Note that while you might be able to answer some of these questions using the app, you are expected to write the required code and produce the necessary plots and summary statistics. You are welcome to use the app for exploration.

7. Take a sample of size 15 from the population and calculate the proportion of people in this sample who think the work scientists do enhances their lives.

Using this sample, what is your best point estimate of the population proportion of people who think the work scientists do enhances their lives?

```
sample_props_single <- global_monitor %>%
rep_sample_n(size = 15, reps = 1, replace = TRUE) %>%
  count(scientist_work) %>%
  mutate(p_hat = n / sum(n)) %>%
  filter(scientist_work == "Benefits")
sample_props_single
```

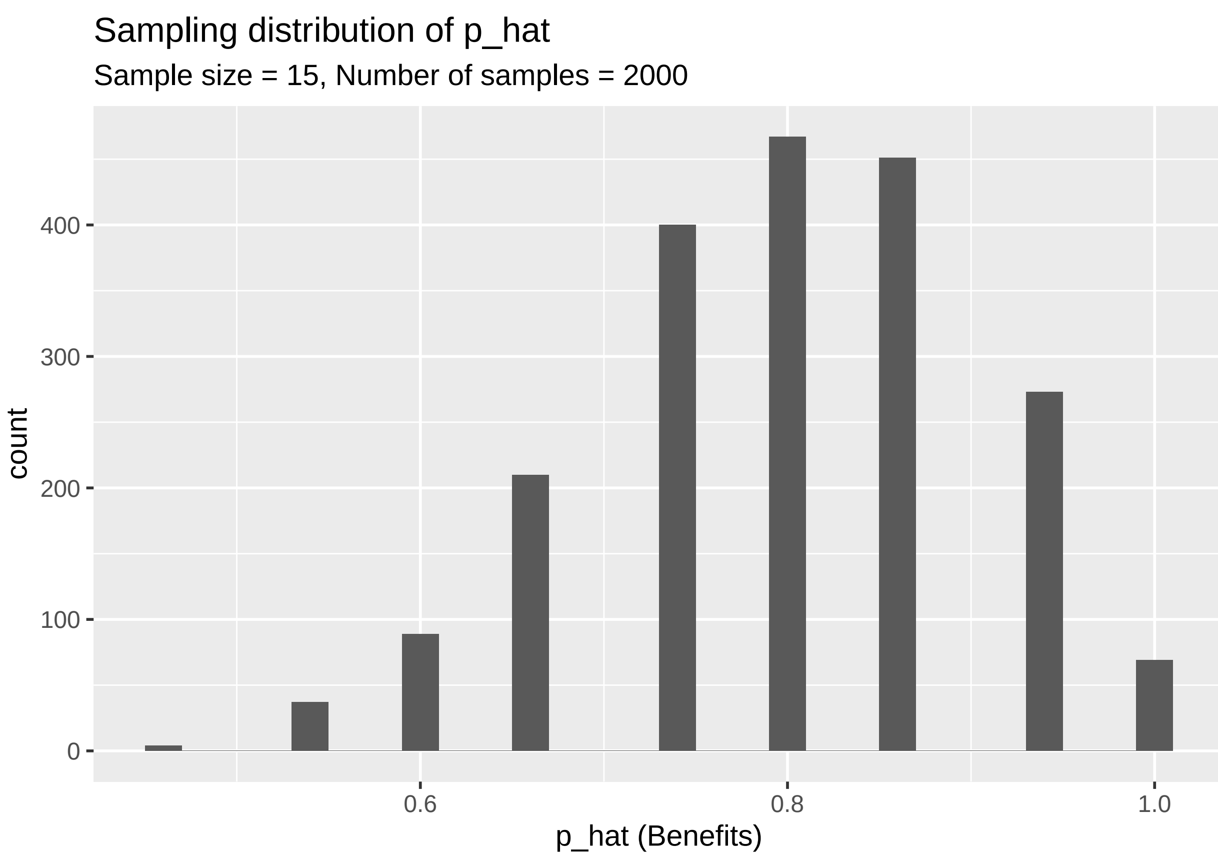
```
## # A tibble: 1 x 4
## # Groups:   replicate [1]
##   replicate scientist_work     n p_hat
##       <int> <chr>          <int> <dbl>
## 1         1 Benefits          14 0.933
```

In this particular sample, 14 out of 15 respondents report that the work scientists do does benefit their lives. This is an unusual result, giving a sample proportion of 0.933, which is far from the population proportion of 0.80.

8. Since you have access to the population, simulate the sampling distribution of proportion of those who think the work scientists do enhances their lives for samples of size 15 by taking 2000 samples from the population of size 15 and computing 2000 sample proportions. Store these proportions as `sample_props15`. Plot the data, then describe the shape of this sampling distribution. Based on this sampling distribution, what would you guess the true proportion of those who think the work scientists do enhances their lives to be? Finally, calculate and report the population proportion.

```
sample_props15 <- global_monitor %>%
  rep_sample_n(size = 15, reps = 2000, replace = TRUE) %>%
  count(scientist_work) %>%
  mutate(p_hat = n / sum(n)) %>%
  filter(scientist_work == "Benefits")

ggplot(data = sample_props15, aes(x = p_hat)) +
  geom_histogram(binwidth = 0.02) +
  labs(
    x = "p_hat (Benefits)",
    title = "Sampling distribution of p_hat",
    subtitle = "Sample size = 15, Number of samples = 2000"
  )
```




The shape of the distribution is unimodal at p__hat = 0.8, but the data is skewed left. Based on this sampling distribution, my best guess for the true proportion of those who think the work scientists do enhances their lives is about 0.8. The mean of the samples collected here is 0.7983, very close to the population proportion of 0.80. The standard error of the sampling distribution is 0.1043.

```r
mean(sample_props15$p_hat)
```

```
## [1] 0.7983
```

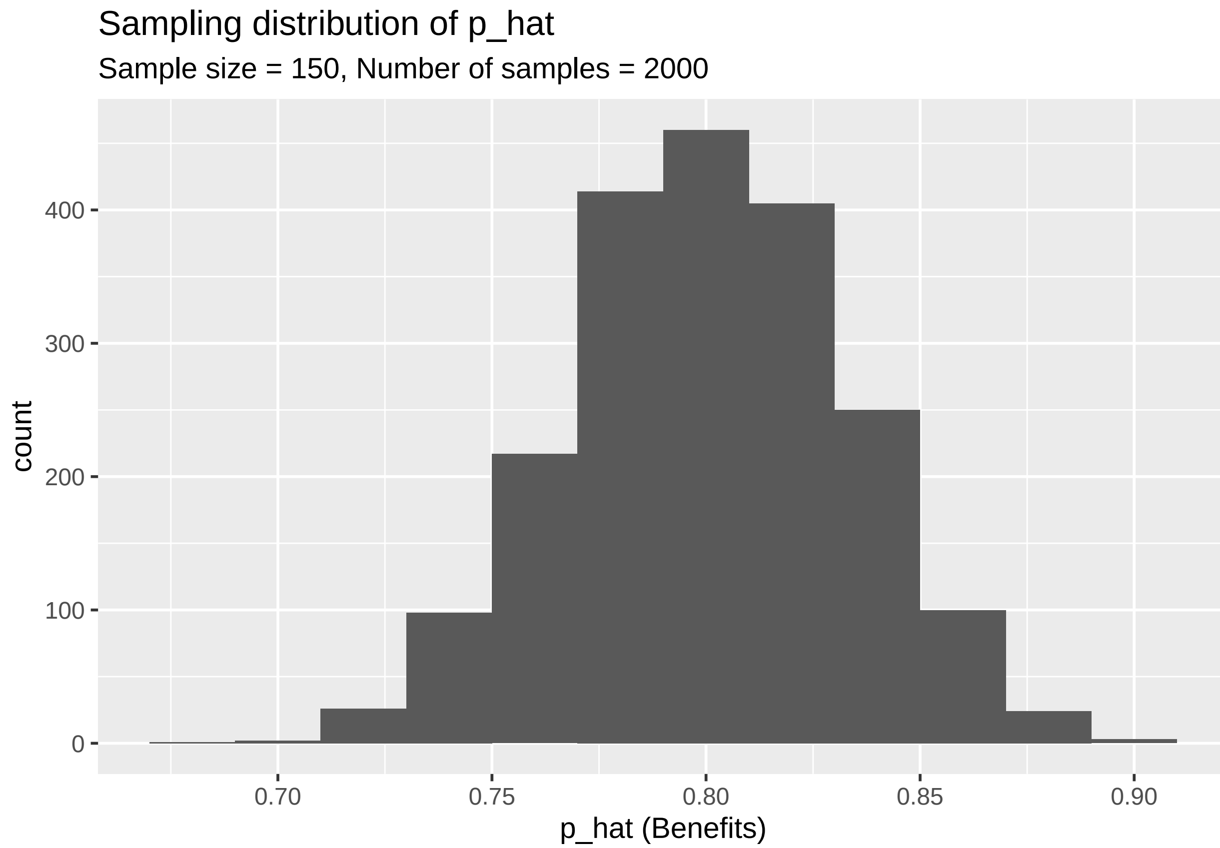
```r
sd(sample_props15$p_hat)
```

```
## [1] 0.1043301
```

9. Change your sample size from 15 to 150, then compute the sampling distribution using the same method as above, and store these proportions in a new object called `sample_props150`. Describe the shape of this sampling distribution and compare it to the sampling distribution for a sample size of 15. Based on this sampling distribution, what would you guess to be the true proportion of those who think the work scientists do enhances their lives?

```r
sample_props150 <- global_monitor %>%
  rep_sample_n(size = 150, reps = 2000, replace = TRUE) %>%
  count(scientist_work) %>%
  mutate(p_hat = n / sum(n)) %>%
  filter(scientist_work == "Benefits")

ggplot(data = sample_props150, aes(x = p_hat)) +
  geom_histogram(binwidth = 0.02) +
  labs(
    x = "p_hat (Benefits)",
    title = "Sampling distribution of p_hat",
    subtitle = "Sample size = 150, Number of samples = 2000"
  )
```




The shape of the distribution is unimodal and symmetric. It appears very close to the normal distribution. Compared to the sampling distribution for samples of size 15, this distribution is more symmetric. Based on this distribution, I would estimate the true proportion to be 0.80. The mean of the samples collected here is 0.80, and the standard error of the sampling distribution is 0.0325.

```
mean(sample_props150$p_hat)
```

```
## [1] 0.80057
```

```
sd(sample_props150$p_hat)
```

```
## [1] 0.03246471
```

10. Of the sampling distributions from 2 and 3, which has a smaller spread? If you're concerned with making estimates that are more often close to the true value, would you prefer a sampling distribution with a large or small spread?

Of the sampling distributions from exercises 8 and 9, the distribution from 9, with larger samples, had smaller spread. A smaller spread means that estimates drawn from this sampling distribution (i.e., samples of larger size) will more often be close to the true value than samples of smaller size.

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