

APPENDIX 1: HUMIDITY CONVERSION EQUATIONS

(Revised 7/96)

Computer-efficient algorithms for converting among several humidity units, as used in HCON, are given here. They utilize vapor pressure formulations developed by A. Buck (1981).

DP = dew or frost point in deg C
 e = vapor pressure in millibars
 es = saturation vapor pressure in millibars
 P = pressure in millibars
 r = mixing ratio by weight in ppm
 RH = relative humidity in percent
 rho = absolute humidity in g/m³
 rhos = absolute humidity at saturation
 T = temperature in deg C
 Tk = absolute temperature in K

Saturation vapor pressure (es) = f1(T) = e/RH

Dew/frost point (DP) = f2(e) (e)
 = f2[r x P/(622 x 10³ + r)] (r)
 = f2(RH x f1(T)/100) (RH)
 = f2(rho x Tk/216.7) (rho)

Vapor pressure (e) = f1(DP) (DP)
 = r x P/(622 x 10³ + r) (r)
 = RH x f1(T)/100 (RH)
 = rho x Tk/216.7 (rho)

Mixing ratio (r), ppmw = (18.02/ M.W. of gas) x 10⁶ x e/(P-e) (e)
 = (18.02/ M.W. of gas) x 10⁶ x f1(DP)/[P - f1(DP)] (DP)
 = (18.02/ M.W. of gas) x 10⁶ x RH x es/(100 x P - RH x es) (RH)
 = (18.02/ M.W. of gas) x 10⁶ x rho x Tk/(216.7 x P - rho x Tk) (rho)

Relative humidity (RH) = 100 x f1(DP)/f1(T) (DP)
 = 100 x e/es (e)
 = 100 x r x P/[(622x10³ + r) x es] (r)
 = 100 x rho x Tk/(216.7 x es) (rho)

Absolute humidity (rho)= 216.7 x f1(DP)/Tk (DP)
 = 216.7 x e/Tk (e)
 = 0.2167 x r x P/[(622 + .001 x r) x Tk] (r)
 = 216.7 x RH x es/(100 x Tk) (RH)

mixing ratio by volume (ppmv) = mixing ratio by weight (ppmw) x (M.W. of gas)/ 18.02

grains/lb = r x 0.007

Precipitable cm per km = rho/10

NOTE 1: $f_1(DP)$ and $f_2(e)$ are variations on vapor pressure formulations found in Buck, A: J Appl Met 20, pp 1527-1532 (1981). They are given by:

e vs. DP or es vs. T:

$$f_1(DP) = EF \times a_w \times \exp [(b_w - DP/dw) \times DP/(DP + cw)] \quad (\text{over water})$$

$$= EF \times a_i \times \exp [(b_i - DP/di) \times DP/(DP + ci)] \quad (\text{over ice})$$

DP vs. e or T vs. es:

$$f_2(e) = dw/2 \times [b_w - s - ((b_w - s)^2 - 4 cw \times s/dw)^{1/2}] \quad (\text{over water})$$

$$= di/2 \times [b_i - s - ((b_i - s)^2 - 4 ci \times s/di)^{1/2}] \quad (\text{over ice})$$

where:

$$a_w = 6.1121$$

$$b_w = 18.678$$

$$c_w = 257.14$$

$$d_w = 234.5$$

$$a_i = 6.1115$$

$$b_i = 23.036$$

$$c_i = 279.82$$

$$d_i = 333.7$$

$$s = \ln (e/EF) - \ln (a_w \text{ or } a_i)$$

$$EF_w = 1 + 10^{-4} [7.2 + P (0.0320 + 5.9 \times 10^{-6} T^2)],$$

$$EF_i = 1 + 10^{-4} [2.2 + P (0.0383 + 6.4 \times 10^{-6} T^2)],$$

where P is in millibars and T is in °C.

NOTE 2: RH is defined here using es with respect to ice below freezing. However, RH is also frequently defined using es with respect to water, even below freezing.

NOTE 3: These conversions are intended for use with moist air rather than pure water vapor. They therefore include EF, the enhancement factor, which corrects for the slight departure of the behavior of water in air from that of a pure gas.

NOTE 4: The definitions f_1 and f_2 for ice agree with an extrapolation of NBS values down to -120 deg C, within 0.5%.