Question 2

Find the cylindrical coordinate expression for $F(x, y, z)$.

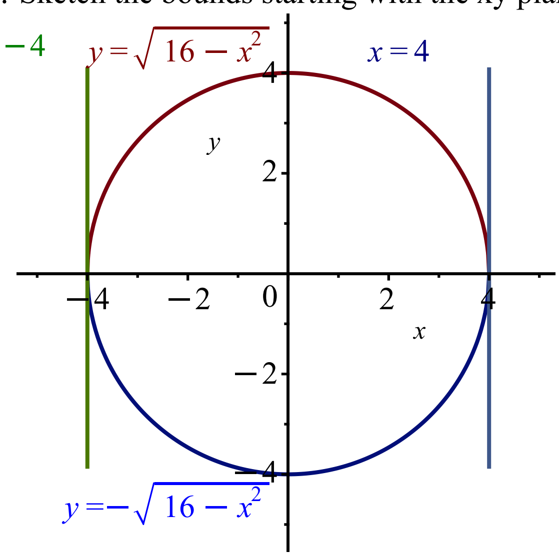
$$F(x, y, z) = 7ze^{x^2 + y^2 + z^2}$$

$x^2 + y^2 = r^2 \Rightarrow F(r, \theta, z) = 7\, z \cdot e^{r^2 + z^2}$

Question 3

$$\int_{-2}^{2} \int_{-4}^{4} \int_{-\sqrt{16 - x^2}}^{\sqrt{16 - x^2}} \left(x^2 + y^2\right)^{\frac{3}{2}}\, dy\, dx\, dz = \frac{8192\,\pi}{5}$$

1. Sketch the bounds starting with the xy plane

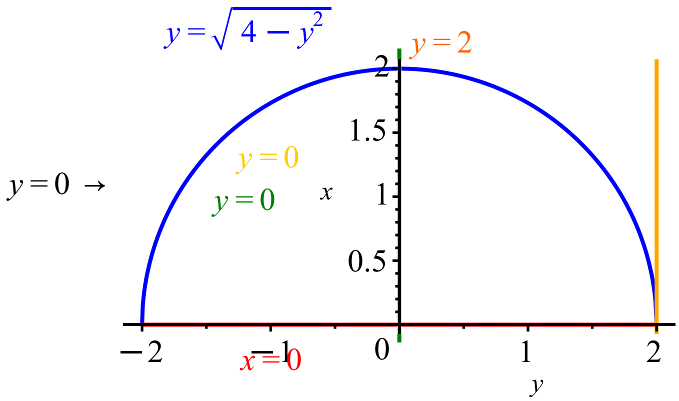


2. Convert to polar: the circle can just be $0 \le r \le 4$ and $0 \le \theta \le 2\pi$. z will stay -2 to 2
3. $\int_{-2}^{2} \int_{0}^{2\pi} \int_{0}^{4} r^3 \cdot r\, dr\, d\theta\, dz = \frac{8192\,\pi}{5}$

Question 4

$$\int_{0}^{2} \int_{0}^{\sqrt{4 - y^2}} \int_{0}^{4 - x^2 - y^2} 1\, dz\, dx\, dy = 2\,\pi$$

1. $0 \le z \le 4 - r^2$
2. Sketch yx plane



3. $\int_0^{\frac{\pi}{2}} \int_0^2 \int_0^{4-r^2} 1 \, dz \, r dr \, d\theta = 2\pi$

Question 5

$$\int_{-1}^{0} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^1 x \cdot y^2 \, dz \, dy \, dx = -\frac{2}{15}$$

Question 6

$$\int_0^2 \int_0^{2\pi} \int_0^2 z \cdot e^{-r^2} r \, dr \, d\theta \, dz = 4\left(\frac{1}{2} - \frac{e^{-4}}{2}\right)\pi$$

$$\frac{\int_0^2 \int_0^{2\pi} \int_0^2 z^2 \cdot e^{-r^2} r \, dr \, d\theta \, dz}{\int_0^2 \int_0^{2\pi} \int_0^2 z \cdot e^{-r^2} r \, dr \, d\theta \, dz} = \frac{4}{3}$$

Question 7

1. Bounds for z are given and can be converted to cylindrical

$r^2 \leq z \leq \sqrt{12 - r^2}$

2. set equal to eachother to find intersection

$r^2 = \sqrt{12 - r^2} \Rightarrow r = \sqrt{3}$

$$\frac{\int_0^{2\pi}\int_0^{\sqrt{3}}\int_{r^2}^{\sqrt{12-r^2}} z\cdot r \,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta}{\int_0^{2\pi}\int_0^{\sqrt{3}}\int_{r^2}^{\sqrt{12-r^2}} r \,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta} = \frac{15\sqrt{3}}{8\left(8 - \frac{15\sqrt{3}}{4}\right)}$$

Question 8

1. Bounds for z are given and can be converted to cylindrical
$r \leq z \leq 12 - r^2$
2. Set $r = 12 - r^2 \Rightarrow r = 3$ is where they intesect so now $0 \leq r \leq 3$
and because it circles all the way around $0 \leq \theta \leq 2\pi$

$$\int_0^{2\pi}\int_0^{3}\int_{r}^{12-r^2} 1 \,\mathrm{d}z\, r\mathrm{d}r\,\mathrm{d}\theta = \frac{99\pi}{2}$$