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with(plots) :
# Define the system of differential equations
sys := {diff(x1(t), t) = -1/10·x1(t) + 1/50·x2(t) + 0.8, diff(x2(t), t) = 2/25·x1(t) - 1/10
· x2(t) + 0.1};
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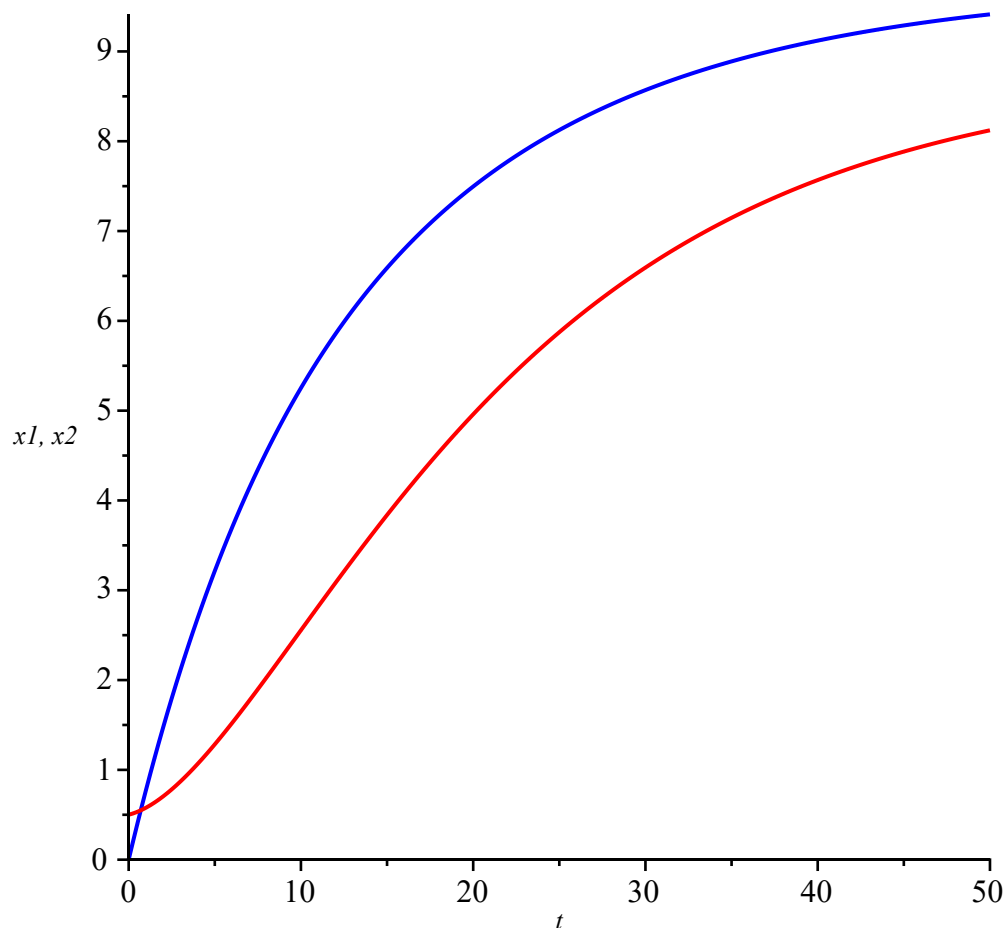
$$\text{sys} := \left\{ \frac{d}{dt} x_1(t) = -\frac{x_1(t)}{10} + \frac{x_2(t)}{50} + 0.8, \frac{d}{dt} x_2(t) = \frac{2x_1(t)}{25} - \frac{x_2(t)}{10} + 0.1 \right\} \quad (1)$$

```
# Define initial conditions
ics := {x1(0) = 0, x2(0) = 0.5};
```

$$\text{ics} := \{x_1(0) = 0, x_2(0) = 0.5\} \quad (2)$$

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# Solve the system
sol := dsolve(sys union ics, {x1(t), x2(t)}, numeric);
odeplot(sol, [[t, x1(t)], [t, x2(t)]], t = 0 .. 50, color = [blue, red], legend = ["x1(t)", "x2(t)"]);
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```
sol := proc(x_rkf45) ... end proc
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— x1(t) — x2(t)

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sol := dsolve(sys union ics, {x1(t), x2(t)}, explicit);
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$$sol := \left\{ x1(t) = -\frac{167 e^{-\frac{3t}{50}}}{24} - \frac{157 e^{-\frac{7t}{50}}}{56} + \frac{205}{21}, x2(t) = -\frac{167 e^{-\frac{3t}{50}}}{12} + \frac{157 e^{-\frac{7t}{50}}}{28} + \frac{185}{21} \right\} \quad (3)$$

# Load necessary package

with(LinearAlgebra) :

# Define the coefficient matrix A

$$A := \langle \langle -1/10, 1/50 \rangle | \langle 2/25, -1/10 \rangle \rangle;$$

# Define the constant vector B

$$B := \langle 0.8, 0.1 \rangle;$$

# Define the variable vector X

$$X := \langle x1(t), x2(t) \rangle;$$

$$\begin{aligned} A &:= \begin{bmatrix} -\frac{1}{10} & \frac{2}{25} \\ \frac{1}{50} & -\frac{1}{10} \end{bmatrix} \\ B &:= \begin{bmatrix} 0.8 \\ 0.1 \end{bmatrix} \\ X &:= \begin{bmatrix} x1(t) \\ x2(t) \end{bmatrix} \end{aligned} \quad (4)$$

# Compute eigenvalues and eigenvectors

$$(\lambda, V) := \text{Eigenvectors}(A);$$

$$\lambda, V := \begin{bmatrix} -\frac{3}{50} \\ -\frac{7}{50} \end{bmatrix}, \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 1 & 1 \end{bmatrix} \quad (5)$$

$$Xp := -\text{MatrixInverse}(A) \cdot B;$$

$$Xp := \begin{bmatrix} 9.76190476190476 \\ 8.80952380952381 \end{bmatrix} \quad (6)$$

# Define general solution

$$\begin{aligned} X\_general &:= C1 \cdot \exp(\lambda[1] \cdot t) \cdot V[., 1] + \\ &\quad C2 \cdot \exp(\lambda[2] \cdot t) \cdot V[., 2] + Xp; \end{aligned}$$

# Display the full general solution

$$X\_general;$$

$$X\_general := \begin{bmatrix} \frac{C1 e^{-\frac{3t}{50}}}{2} - \frac{C2 e^{-\frac{7t}{50}}}{2} + 9.76190476190476 \\ C1 e^{-\frac{3t}{50}} + C2 e^{-\frac{7t}{50}} + 8.80952380952381 \end{bmatrix} \quad (7)$$

# Define initial conditions

ics := {x1(0)=0, x2(0)=0.5};

# Solve for C1 and C2

constants := solve({C1 \* V[1, 1] + C2 \* V[1, 2] + Xp[1]=0,  
C1 \* V[2, 1] + C2 \* V[2, 2] + Xp[2]=0}, {C1, C2});

# Substitute back into the general solution

X\_final := subs(constants, X\_general);

# Display the final explicit solution

X\_final;

$$\begin{aligned} ics &:= \{x1(0)=0, x2(0)=0.5\} \\ constants &:= \{C1 = -14.16666667, C2 = 5.357142857\} \\ X\_final &:= \begin{bmatrix} -7.083333335 e^{-\frac{3t}{50}} - 2.678571428 e^{-\frac{7t}{50}} + 9.76190476190476 \\ -14.16666667 e^{-\frac{3t}{50}} + 5.357142857 e^{-\frac{7t}{50}} + 8.80952380952381 \end{bmatrix} \end{aligned} \quad (8)$$

$$Xp := \begin{bmatrix} 10.4761904761905 \\ 3.09523809523810 \end{bmatrix} \quad (9)$$