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Exercise 12

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Homework Problem 12.1.

Consider the operator $T \colon L^2(0,1) \mapsto L^2(0,1)$ defined by

$$Tu(x) \coloneqq \int_0^x u(t) \, \mathrm{d}t \text{ for } x \in [0,1].$$

Find the adjoint of T with respect to the standard inner product.

Homework Problem 12.2. (homework problem 10.1 revisited)

Derive first order optimality conditions for the boundary-controlled modification of the floor-heating problem

$$\text{Minimize} \quad \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega_{\mathrm{obs}})} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.\,t.} \quad \begin{cases} -\operatorname{div}(\kappa \, \nabla y) = 0 & \text{in } \Omega \\ \kappa \, \dfrac{\partial}{\partial n} y = \alpha \, (u - y) & \text{on } \Gamma \end{cases}$$

$$\text{and} \quad u \in L^2(\Gamma).$$

You are not expected to turn in your solutions.