IM 60002/Advanced Decision Modeling
HW Assignment#1
Due Date: Feb 12, 2025 (in-class)

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(1) Which of the following functions is convex, concave or neither? Why?

a. $f(x_1, x_2) = 2x_1^2 - 4x_1x_2 - 8x_1 + 3x_2$

b. $f(x_1, x_2) = x_1 e^{-(x_1+3x_2)}$

c. $f(x_1, x_2) = -x_1^2 + 10x_1 - 3x_2^2 + 4x_1x_2 - 10x_2$

d. $f(x_1, x_2) = x_1x_2$

e. $f(x) = e^x - 1$

f. $Q = K^\alpha L^\beta$ with $\alpha, \beta > 0$ and $\alpha + \beta < 1$.

(2) Let $f_1, f_2, \dots, f_k: R^n \to R$ be the convex functions. Consider the function defined by $f(x) = max\{f_1(x), f_2(x), \dots, f_k(x)\}$, show that *f* is convex.

(3) Let $h: E_n \to E_1$ be a quasi-convex function and let $g: E_1 \to E_1$ be a non-decreasing function, then show that the composite function $f: E_n \to E_1$ defined as $f(x) = g[h(x)]$ is a quasi-convex.

(4) We want to design a cone-shaped paper drinking cup of volume *V* that requires a minimum of paper to make. That in we want to minimize its surface area $\pi r\sqrt{r^2 + h^2}$ where *r* is the radius of the open end and *h* is height. Find the radius *r* at which surface area is minimized (in terms of *V* and *r*).

(5) Minimize $2x_1^2 + x_2^2$

S.t $\quad x_1 + x_2 = 1$

If we change the RHS from 1 to 1.05, compute the value of the objective function.

(6) Suppose we have a refinery that must ship finished goods to some strange tanks. Suppose further that there are two pipelines, A & B, to do the shipping. The cost of shipping *x* units on A in ax^2; and on B in by^2, $a > 0, b > 0$. How can we ship Q units that minimizes the cost? What happens to the cost if Q increases by *r%*.

(7) Solve the following constrained optimization problem using the method of Lagrange multiplier.

Max $\quad \ln x + 2\ln y + 3\ln z$

s.t $\quad x + y + z = 60$

Estimate the change in the optimal objective function value if the RHS increases from 60 to 65.

(8) Minimize $\quad f(x) = x_1^2 + x_2^2 + x_3^2$

s.t $\quad 2x_1 + x_2 - 5 \le 0$; $x_1 + x_3 - 2 \le 0$; $1 - x_1 \le 0$; $2 - x_2 \le 0$; $-x_3 \le 0$

Write the necessary KKT conditions and find the optimal solution.

(9) Write the necessary condition for optimality for the following problems

a. Maximize $f(x) = x_1^3 - x_2^2 + x_1x_3^2$

s.t $\quad x_1 + x_2^2 + x_3 = 5$; $5x_1^2 - x_2^2 - x_3 \ge 2$; $x_1, x_2, x_3 \ge 0$

b. Minimize $f(x) = x_1^4 + x_2^2 + 5x_1x_2x_3$

s.t $\quad x_1^2 - x_2^2 + x_3^3 \le 10$; $x_1^3 + x_2^2 + 4x_3^2 \ge 10$

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