NLP Algorithms

Dichotomous Search

Initialization Step Choose the distinguishability constant, $2\varepsilon > 0$, and the allowable final length of uncertainty, $l > 0$. Let $[a_1, b_1]$ be the initial interval of uncertainty, let $k = 1$, and go to the Main Step.

Main step

1. If $b_k - a_k < l$, stop; the minimum point lies in the interval $[a_k, b_k]$.

   Otherwise, consider λ_k and μ_k defined below, and go to step 2.

   $$\lambda_k = \frac{a_k + b_k}{2} - \varepsilon, \qquad \mu_k = \frac{a_k + b_k}{2} + \varepsilon$$

2. If $\theta(\lambda_k) < \theta(\mu_k)$, let $a_{k+1} = a_k$ and $b_{k+1} = \mu_k$. otherwise, let $a_{k+1} = \lambda_k$ and $b_{k+1} = b_k$.

   Replace k by $k + 1$ and go to step 1.

Note that the length of uncertainty at the beginning of iteration $k + 1$ is given by

$$b_{k+1} - a_{k+1} = \frac{1}{2}(b_1 - a_1) + 2\varepsilon\left(1 - \frac{1}{2^k}\right)$$

Golden Section Method

Initialization Step Choose an allowable final length of uncertainty, $l > 0$. Let $[a_1, b_1]$ be the initial interval of uncertainty, let $\lambda_1 = a_1 + (1-\alpha)(b_1 - a_1)$ and $\mu_1 = a_1 + \alpha(b_1 - a_1)$, where $\alpha = 0.618$. Evaluate $\theta(\lambda_k)$ and $\theta(\mu_k)$, let $k = 1$, and go to the Main Step.

Main step

1. If $b_k - a_k < l$, stop; the minimum point lies in the interval $[a_k, b_k]$.

   Otherwise, consider if $\theta(\lambda_k) > \theta(\mu_k)$, go to step 2; and if $\theta(\lambda_k) \le \theta(\mu_k)$, go to step 3.

2. Let $a_{k+1} = \lambda_k$ and $b_{k+1} = b_k$. furthermore, let $\lambda_{k+1} = \mu_k$ and let $\mu_{k+1} = a_{k+1} + \alpha(b_{k+1} - a_{k+1})$.

   Evaluate $\theta(\mu_{k+1})$ and go to Step 4.

3. Let $a_{k+1} = a_k$ and $b_{k+1} = \mu_k$. furthermore, let $\mu_{k+1} = \lambda_k$ and let

   $\lambda_{k+1} = a_{k+1} + (1-\alpha)(b_{k+1} - a_{k+1})$. Evaluate $\theta(\lambda_{k+1})$ and go to Step 4.

4. Replace k by $k + 1$ and go to step 1.

The Fibonacci search

Initialization Step Choose an allowable final length of uncertainty, $l > 0$ and distinguishability constant, $\varepsilon > 0$. Let $[a_1, b_1]$ be the initial interval of uncertainty, and choose the number of observation n to be taken such that $F_n > \frac{(b_1 - a_1)}{l}$. Let $\lambda_1 = a_1 + \left(\frac{F_{n-2}}{F_n}\right)(b_1 - a_1)$ and $\mu_1 = a_1 + \left(\frac{F_{n-1}}{F_n}\right)(b_1 - a_1)$. Evaluate $\theta(\lambda_I)$ and $\theta(\mu_I)$, let $k = 1$, and go to the Main Step.

Main step

1. If $\theta(\lambda_k) > \theta(\mu_k)$, go to step 2; and if $\theta(\lambda_k) \le \theta(\mu_k)$, go to step 3.
2. Let $a_{k+1} = \lambda_k$ and $b_{k+1} = b_k$. Furthermore, let $\lambda_{k+1} = \mu_k$ and let $\mu_{k+1} = a_{k+1} + \left(\frac{F_{n-k-1}}{F_{n-k}}\right)(b_{k+1} - a_{k+1})$. If $k = n - 2$ go to step 5; otherwise evaluate $\theta(\mu_{k+1})$ and go to step 4.
3. Let $a_{k+1} = a_k$ and $b_{k+1} = \mu_k$. Furthermore, let $\mu_{k+1} = \lambda_k$ and let $\lambda_{k+1} = a_{k+1} + \left(\frac{F_{n-k-2}}{F_{n-k}}\right)(b_{k+1} - a_{k+1})$. If $k = n - 2$ go to step 5; otherwise, evaluate $\theta(\lambda_{k+1})$ and go to Step 4.
4. Replace k by $k + 1$ and go to step 1.
5. Let $\lambda_n = \lambda_{n-1}$ and $\mu_n = \lambda_{n-1} + \varepsilon$. If $\theta(\lambda_n) > \theta(\mu_n)$, let $a_n = \lambda_n$ and $b_n = b_{n-1}$. Otherwise, if $\theta(\lambda_n) \le \theta(\mu_n)$, $a_n = a_{n-1}$ and $b_n = \lambda_n$. Stop; the optimal solution lies in the interval $[a_n, b_n]$.

Bisection search Method

Initialization Step Let $[a_1, b_1]$ be the initial interval of uncertainty, and let ℓ be the allowable final interval of uncertainty. Let n be the smallest positive integer such that $(1/2)^n \le \ell/(b_1 - a_1)$. Let $k = 1$ and go to the Main Step.

Main Step

1. Let $\lambda_k = (1/2)(a_k + b_k)$ and evaluate $\theta'(\lambda_k)$. If $\theta'(\lambda_k) = 0$, stop; λ_k is an optimal solution. Otherwise, go to Step 2 if $\theta'(\lambda_k) > 0$, and go to Step 3 if $\theta'(\lambda_k) < 0$.
2. Let $a_{k+1} = a_k$ and $b_{k+1} = \lambda_k$. Go to Step 4.
3. Let $a_{k+1} = \lambda_k$ and $b_{k+1} = b_k$. Go to Step 4.
4. If $k = n$, stop; the minimum lies in the interval $[a_{n+1}, b_{n+1}]$. Otherwise, replace k by $k + 1$ and repeat Step 1.

Cyclic Coordinate Method

Initialization Step Choose a scalar $\varepsilon > 0$ to be used for terminating the algorithm, and let $\mathbf{d}_1, \ldots, \mathbf{d}_n$ be the coordinate directions. Choose an initial point $\mathbf{x}_1$, let $\mathbf{y}_1 = \mathbf{x}_1$, let $k = j = 1$, and go to the Main Step.

Main Step

1. Let λ_j be an optimal solution to the problem to minimize $f(\mathbf{y}_j + \lambda \mathbf{d}_j)$ subject to $\lambda \in R$, and let $\mathbf{y}_{j+1} = \mathbf{y}_j + \lambda_j \mathbf{d}_j$. If $j < n$, replace j by j + 1, and repeat Step 1. Otherwise, if $j = n$, go to Step 2.
2. Let $\mathbf{x}_{k+1} = \mathbf{y}_{n+1}$. If $\|\mathbf{x}_{k+1} - \mathbf{x}_k\| < \varepsilon$, then stop. Otherwise, let $\mathbf{y}_1 = \mathbf{x}_{k+1}$, let $j = 1$, replace k by $k + 1$, and go to Step 1.

Hooke and Jeeves

Initialization Step Choose a scalar $\varepsilon > 0$ to be used in terminating the algorithm. Choose a starting point $\mathbf{x}_1$, let $\mathbf{y}_1 = \mathbf{x}_1$, let $k = j = 1$, and go to the Main Step.

Main Step

1. Let λ_j be an optimal solution to the problem to minimize $f(\mathbf{y}_j + \lambda \mathbf{d}_j)$ subject to $\lambda \in R$, and let $\mathbf{y}_{j+1} = \mathbf{y}_j + \lambda_j \mathbf{d}_j$. If $j < n$, replace j by j + 1, and repeat Step 1. Otherwise, if $j = n$, let $\mathbf{x}_{k+1} = \mathbf{y}_{n+1}$. If $\|\mathbf{x}_{k+1} - \mathbf{x}_k\| < \varepsilon$, stop; otherwise, go to Step 2.

2. Let $\mathbf{d} = \mathbf{x}_{k+1} - \mathbf{x}_k$, and let $\hat{\lambda}$ be an optimal solution to the problem to minimize $f(\mathbf{x}_{k+1} + \lambda \mathbf{d})$ subject to $\lambda \in R$. Let $\mathbf{y}_1 = \mathbf{x}_{k+1} + \hat{\lambda}\mathbf{d}$, let $j = 1$, replace k by $k + 1$, and go to Step 1.

Steepest Descent

Initialization Step Let $\varepsilon > 0$ be the termination scalar. Choose a starting point $\mathbf{x}_1$, let $k = 1$, and go to the Main Step.

Main Step

If $\|\nabla f(\mathbf{x}_k)\| < \varepsilon$, stop; otherwise, let $\mathbf{d}_k = -\nabla f(\mathbf{x}_k)$, and let λ_k be an optimal solution to the problem to minimize $f(\mathbf{x}_k + \lambda \mathbf{d}_k)$ subject to $\lambda \geq 0$. Let $\mathbf{x}_{k+1} = \mathbf{x}_k + \lambda_k \mathbf{d}_k$, replace k by $k + 1$, and repeat the Main Step.