SCS 43XX

Quantum Mechanics in Computing

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Labsheet 02

Multiple Systems in Qiskit

In this section, we'll explore the behavior of multiple systems.

1. Import necessary classes: Start by importing `Statevector` and `Operator`, as well as the square root function from NumPy

Below is the importation:

```
from qiskit.quantum_info import Statevector, Operator
from numpy import sqrt
```

Figure 1: Imports

2. Tensor Products:
Create two state vectors representing $|0\rangle$ and $|1\rangle$, and use the tensor method to create a new vector, $|0\rangle \otimes |1\rangle$.

Answer: Below is the tensor product in Qiskit:

```
zero = Statevector.from_label("0")
one = Statevector.from_label("1")
psi = zero.tensor(one)
display(psi.draw("latex"))
```

Output:

$$|01\rangle$$

Figure 2: Basis states tensor products 2-system

3. Another Tensor Product

Create state vectors representing the $|+\rangle$ and $\frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle)$ states, and combine them to create a new state vector. We'll assign this new vector to the variable `psi`.

Answer: Below is the tensor product in Qiskit:

```
plus = Statevector.from_label("+")
i_state = Statevector([1 / sqrt(2), 1j / sqrt(2)])
psi = plus.tensor(i_state)

psi.draw("latex")
```

Output:

$$\frac{1}{2}|00\rangle + \frac{i}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{i}{2}|11\rangle$$

Figure 3: Advanced tensor products 2-system

4. More Tensor Products

Create state vectors representing the

$$|+i\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$

$$|-i\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$

states, and combine them to create a new state vector. Assign this new vector to the variable `phi`.

Answer:

```python
plus = Statevector.from_label("+")
minus_i = Statevector.from_label("l")
phi = plus.tensor(minus_i)
display(phi.draw("latex"))
```

Output:

$$\frac{1}{2}|00\rangle - \frac{i}{2}|01\rangle + \frac{1}{2}|10\rangle - \frac{i}{2}|11\rangle$$

Figure 4: Advanced tensor products 2-system

Obtain the tensor using the operator.

Answer:

```python
display((plus ^ minus_i).draw("latex"))
```

Output:

$$\frac{1}{2}|00\rangle - \frac{i}{2}|01\rangle + \frac{1}{2}|10\rangle - \frac{i}{2}|11\rangle$$

Figure 5: Advanced tensor products 2-system

5. Tensor products of operators

The `Operator` class also has a `tensor` method. Create the X and I gates and display their tensor product.

Answer:

```python
X = Operator([[0, 1], [1, 0]])
I = Operator([[1, 0], [0, 1]])

X.tensor(I)
```

Output:

```
Operator([[0.+0.j, 0.+0.j, 1.+0.j, 0.+0.j],
          [0.+0.j, 0.+0.j, 0.+0.j, 1.+0.j],
          [1.+0.j, 0.+0.j, 0.+0.j, 0.+0.j],
          [0.+0.j, 1.+0.j, 0.+0.j, 0.+0.j]],
         input_dims=(2, 2), output_dims=(2, 2))
```

Figure 6: X and I tensor product

Obtain $H \otimes I \otimes X$.

Answer:

```python
H = Operator.from_label("H")
I = Operator.from_label("I")
X = Operator.from_label("X")
display(H.tensor(I).draw("latex"))
display(H.tensor(I).tensor(X).draw("latex"))
```

Output:

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \end{bmatrix}$$

$$\begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 \\ \frac{\sqrt{2}}{2} & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & \frac{\sqrt{2}}{2} \\ 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & -\frac{\sqrt{2}}{2} & 0 & 0 \\ \frac{\sqrt{2}}{2} & 0 & 0 & 0 & -\frac{\sqrt{2}}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & -\frac{\sqrt{2}}{2} \\ 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & -\frac{\sqrt{2}}{2} & 0 \end{bmatrix}$$

Figure 7: H and I and X tensor product

It is possible to use the operator

```
display((H ^ I ^ X).draw("latex"))
```

Figure 8: H and I and X tensor product

6.Use Compound Operations on Compound States
Compound states can be evolved using compound operations.

$$(H \otimes I)|\phi\rangle \quad \text{for} \quad |\phi\rangle = |+\rangle \otimes |-i\rangle$$

Perform the about transformation on the state $|\phi\rangle$ (phi).

Answer: :

```
display(phi.evolve(H ^ I).draw("latex"))
```

Output:

$$\frac{\sqrt{2}}{2}|00\rangle - \frac{\sqrt{2}i}{2}|01\rangle$$

Figure 9: Compound Operator Operation

7. Controlled Operation
Calculate $CX|\psi\rangle$ for

$$|\psi\rangle = |+\rangle \otimes |0\rangle.$$

To be clear, this is a CX operation where the left-hand qubit is the control and the right-hand qubit is the target. The result is the Bell state:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

Answer:

```
CX = Operator(
    [[1, 0, 0, 0],
     [0, 1, 0, 0],
     [0, 0, 0, 1],
     [0, 0, 1, 0]])
psi = plus.tensor(zero)
display(psi.evolve(CX).draw("latex"))
```

Output:

$$\frac{\sqrt{2}}{2}|00\rangle + \frac{\sqrt{2}}{2}|11\rangle$$

Figure 10: Controlled NOT

8.Partial Measurement

The `measure` method is usedto simulate a measurement of a quantum state vector. This method returns two items: the simulated measurement result and the new state vector after the measurement.

By default, the `measure` method measures all qubits in the state vector. Alternatively, it's possible to provide a list of integers as an argument, which causes only those qubit indices to be measured.

To demonstrate this, the following quantum state is created:

$$|w\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$$

Then, qubit number 0 (the rightmost qubit) is measured.

Answer:

```
w = Statevector([0, 1, 1, 0, 1, 0, 0, 0] / sqrt(3))
display(w.draw("latex"))

result, state = w.measure([0])
print(f"Measured: {result}\nState after measurement:")
display(state.draw("latex"))

result, state = w.measure([0,1])
print(f"Measured: {result}\nState after measurement:")
display(state.draw("latex"))
```

Output:

$$\frac{\sqrt{3}}{3}|001\rangle + \frac{\sqrt{3}}{3}|010\rangle + \frac{\sqrt{3}}{3}|100\rangle$$

```
Measured: 0
State after measurement:
```

$$\frac{\sqrt{2}}{2}|010\rangle + \frac{\sqrt{2}}{2}|100\rangle$$

```
Measured: 01
State after measurement:
```

$$|001\rangle$$

Figure 11: Partial Measurement