SCS 43XX

Quantum Mechanics in Computing

University of Colombo, School of Computing

Tutorial 6

Qubit state manipulation with Single Qubit Gates

The Pauli gates (X, Y, Z) are the three Pauli matrices $(\sigma_x, \sigma_y, \sigma_z)$ and act on a single qubit. Respectively, they perform a rotation around the x, y and z axes of the Bloch sphere by π radians.

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

1. Apply an X-gate to the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ in Dirac Notation (by looking at the columns of the X-gate matrix).
2. Apply a Y-gate to the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ in Dirac Notation (by looking at the columns of the Y-gate matrix).
3. Apply a Z-gate to the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ in Dirac Notation (by looking at the columns of the Z-gate matrix).

Hadamard gate maps the basis states $|0\rangle \mapsto \frac{|0\rangle+|1\rangle}{\sqrt{2}}$ and $|1\rangle \mapsto \frac{|0\rangle-|1\rangle}{\sqrt{2}}$ (it creates an equal superposition state if given a computational basis state). The two states $(|0\rangle + |1\rangle)/\sqrt{2}$ and $(|0\rangle - |1\rangle)/\sqrt{2}$ are sometimes written $|+\rangle$ and $|-\rangle$ respectively. The Hadamard gate performs a rotation of π about the axis $(\hat{x} + \hat{z})/\sqrt{2}$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

(a) Represent the state i in exponential form $(e^{i\theta})$

(b) The $|i\rangle$ state is defined by $|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$. Substitute in i in exponential form, how many radians has the qubit been rotated around the Bloch Sphere and does it line up with the $|i\rangle$ state on the Bloch Sphere?

(c) Represent -1 in exponential form $(e^{i\theta})$

(d) The $|-\rangle$ state is defined by $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. Substitute in -1 in exponential form, how many radians has the qubit been rotated around the Bloch Sphere and does it line up with the $|-\rangle$ state on the Bloch Sphere?

(e) Represent $-i$ in exponential form $(e^{i\theta})$

(f) The $|-i\rangle$ state is defined by $|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$. Substitute in $-i$ in exponential form, how many radians has the qubit been rotated around the Bloch Sphere and does it line up with the $|-i\rangle$ state on the Bloch Sphere?

2. Find

   (a) $H|0\rangle$

   (b) $H|1\rangle$

   (c) $H|+\rangle$

   (d) $H|-\rangle$