SCS 43XX

Quantum Mechanics in Computing

University of Colombo, School of Computing

Tutorial 4

Quantum States, Matrix Form, Dirac Notation, Block Sphere

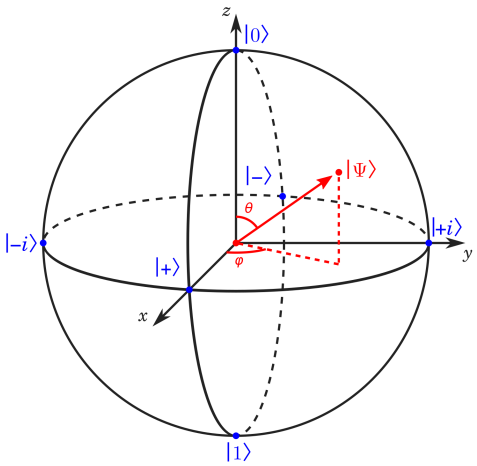
1. An arbitrary qubit state can be represented as $|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$. The probability of measuring the $|0\rangle$ state is $|\alpha|^2$ and the probability of measuring the $|1\rangle$ state is $|\beta|^2$.

- (a) What is the probability of measuring $|0\rangle$ with a qubit in the state $|\psi\rangle = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$?
- (b) What is the probability of measuring $|1\rangle$ with a qubit in the state $|\psi\rangle = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$?
- (c) What is the probability of measuring $|0\rangle$ when a qubit is in the state $|\psi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$?
- (d) What is the probability of measuring $|1\rangle$ when a qubit is in the state $|\psi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$?

2. For an arbitrary qubit state $|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$, the probability of measuring $|0\rangle$ plus the probability of measuring $|1\rangle$ must equal 1 since when we measure a qubit, we will get one of those outcomes. This gives us the equation $|\alpha|^2 + |\beta|^2 = 1$. Are these qubit states valid qubit states? Why?

- (a) $|\psi\rangle = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$
- (b) $|\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- (c) $|\psi\rangle = \begin{pmatrix} \frac{\sqrt{2}}{5} \\ \frac{3}{5} \end{pmatrix}$

The Bloch sphere is a geometric representation of a qubit state $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$, where θ and ϕ are spherical coordinates. It visualizes pure states as points on the surface of the unit sphere in three dimensions.



1. Draw Out a Bloch Sphere and plot the following qubit states
   - $|0\rangle$
   - $|1\rangle$
   - $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
2. What does the vertical position (higher or lower) of a qubit on the Bloch Sphere represent in terms of its state and the probability of measuring $|0\rangle$ or $|1\rangle$?
3. What does it mean when the state's position falls right on the X and Y axis respectively ?
4. Briefly describe the difference between Pure and Mixed states. How are the mixed states represented in the Bloch Sphere?

6. Convert the following qubit states from matrix to Dirac Notation:

- (a) $|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$
- (b) $|\psi\rangle = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$
- (c) $|\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

7. If we measure a qubit in the state $|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ as $|0\rangle$, what would we measure if we were to measure the qubit again? Why?