SCS 43XX

Quantum Mechanics in Computing

University of Colombo, School of Computing

Tutorial 5

Introduction to Phase and Phase Shift Gates

1. If we have a qubit in superposition that has a relative phase of $e^{5\pi i/4}$, on the Bloch Sphere how many radians has the qubit 'spun' around the z-axis
2. Simplify the qubit state $|\phi\rangle = \alpha e^{i\phi}|0\rangle + \beta e^{i\psi}|1\rangle = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$ by omitting the global phase
3. Simplify the following qubit states by first factoring out the phase from the 0 state, creating a global and relative phase, then omitting the global phase. (HINT for (c): represent -1 as a complex number in exponential form, -1 = $e^{i\pi}$)
   - $e^{i\theta}\alpha|0\rangle + e^{i\phi}\beta|1\rangle$
   - $e^{\pi i/2}\alpha|0\rangle + e^{3\pi i/4}\beta|1\rangle$
   - $e^{3\pi i/2}\alpha|0\rangle - \beta|1\rangle$

The phase shift is a family of single-qubit gates that map the basis states $|0\rangle \to |0\rangle$ and $|1\rangle \to e^{i\phi}|1\rangle$. The probability of measuring a $|0\rangle$ or a $|1\rangle$ is unchanged after applying this gate, however it modifies the phase of the quantum state.

$$P(\varphi) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{bmatrix}$$

1. Apply an S-gate to a qubit in the state $|\psi\rangle = \alpha|0\rangle + e^{i\theta}\beta|1\rangle$. What happens to the relative phase of the qubit?
2. Apply an $S^\dagger$-gate to a qubit in the state $|\psi\rangle = \alpha|0\rangle + e^{i\theta}\beta|1\rangle$. What happens to the relative phase of the qubit?
3. Apply a T-gate to a qubit in the state $|\psi\rangle = \alpha|0\rangle + e^{i\theta}\beta|1\rangle$. What happens to the relative phase of the qubit?
4. Apply a $T^\dagger$-gate to a qubit in the state $|\psi\rangle = \alpha|0\rangle + e^{i\theta}\beta|1\rangle$. What happens to the relative phase of the qubit?

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = P(\pi)$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = P\left(\frac{\pi}{2}\right) = \sqrt{Z}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{bmatrix} = P\left(\frac{\pi}{4}\right) = \sqrt{S} = \sqrt[4]{Z}$$