SCS 43XX

Quantum Mechanics in Computing

University of Colombo, School of Computing

Tutorial 3

Unitary and Hermitian Matrices. Eigenvectors

Unitary matrices, denoted as U, are used in quantum computing to represent quantum gates, ensuring that

$$UU^\dagger = I$$

where $U^\dagger$ is the conjugate transpose of U.

Hermitian matrices, denoted as H, represent observable quantities in quantum mechanics, satisfying:

$$H = H^\dagger$$

Conjugate: For a matrix A, the conjugate A^* is obtained by taking the complex conjugate of each element in A.

Conjugate Transpose: The conjugate transpose (or Hermitian adjoint) of a matrix A, denoted as $A^\dagger$, is obtained by taking the transpose of A and then taking the complex conjugate of each element.

1. Let $A = \begin{bmatrix} 1 & 0 & 4 \\ 17 & 10 & 18 \end{bmatrix}$, find A^T.
2. Let $A = \begin{bmatrix} 3+2i & 2e^{\pi i/3} & 4 \\ 1-7i & 10 & -3i \end{bmatrix}$, find A^* (Hint: $z = re^{i\theta}$, $z^* = re^{-i\theta}$).
3. Let $A = \begin{bmatrix} 3+2i & 2e^{\pi i/3} & 4 \\ 1-7i & 10 & -3i \end{bmatrix}$, find $A^\dagger$.
4. Is the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$, Unitary? Prove.
5. Is the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$, Hermitian? Prove.

Eigenvectors of a matrix A are non-zero vectors v that satisfy:

$$Av = \lambda v,$$

where λ is the eigenvalue.

Eigenvalues are scalars that indicate how the eigenvector v is scaled during the transformation by A.

1. When we apply a matrix to one of it's eigenvectors, what happens to the eigenvector geometrically?
2. Find the eigenvalue when the matrix $\begin{bmatrix} 1 & 9 \\ 4 & 1 \end{bmatrix}$ applied to its eigenvector $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$
3. Geometrically what happens to the vector when the eigenvalue is greater than 1 and less that 1 respectively
4. Find the eigenvalues and eigenvectors of the matrices $\begin{bmatrix} -2 & 1 & 1 \\ 6 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 2 \\ -1 & -1 & -1 \end{bmatrix}$
5. Show that the Hermitian matrix $\begin{bmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ has only two real eigenvalues and the three eigenvectors are orthonormal.