# HW 8 - MATH403

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## Problem 1 (Chapter 24, Exercise 10)

Let $H$ be a proper subgroup of a finite group $G$. Show that $G$ is not the union of all conjugates of $H$.

*Proof.* $\square$

## Problem 2 (Chapter 24, Exercise 16)

Find all the Sylow 3-subgroups of $S_4$.

*Proof.* $\square$

## Problem 3 (Chapter 24, Exercise 22)

Show that every group of order 56 has a proper nontrivial normal subgroup.

*Proof.* $\square$

## Problem 4 (Chapter 24, Exercise 26)

How many Sylow 5-subgroups of $S_5$ are there? Exhibit two.

*Proof.* $\square$

## Problem 5 (Chapter 24, Exercise 40)

Suppose that $G$ is a group of order 60 and $G$ has a normal subgroup $N$ of order 2. Show that

a. $G$ has normal subgroups of orders 6, 10, and 30.

b. $G$ has subgroups of orders 12 and 20.

c. $G$ has a cyclic subgroup of order 30.

*Proof.* $\square$

## Problem 6 (Chapter 24, Exercise 60)

Determine the groups of order 45.

*Proof.* □

## Problem 7 (Chapter 12, Exercise 26)

Determine $U(\mathbb{R}[x])$.

*Proof.* □

## Problem 8 (Chapter 12, Exercise 40)

Let $M_2(\mathbb{Z})$ be the ring of all $2 \times 2$ matrices over the integers and let $R = \left\{ \begin{bmatrix} a & a+b \\ a+b & b \end{bmatrix} \middle| \, a, b \in \mathbb{Z} \right\}$. Prove or disprove that $R$ is a subring of $M_2(\mathbb{Z})$.

*Proof.* □

## Problem 9 (Chapter 12, Exercise 54)

Show that $4x^2 + 6x + 3$ is a unit in $\mathbb{Z}_8[x]$.

*Proof.* □

## Problem 10

Calculate the number of different conjugacy classes in $S_5$ and write down a representative element for each conjugacy class.

*Proof.* □

## Problem 11

Prove that the 3-cycles in $A_5$ do form a single conjugacy class but that the 5-cycles do not.

*Proof.* □