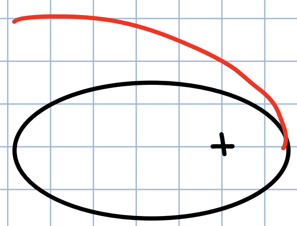
ORBITAL MECHANICS

See results in the recording

(A) $r_A' > r_A$ ⟹ A is the periapsis of orbits (1) Transfer (3) is the more efficient regardless the final orbit



A = periapsis

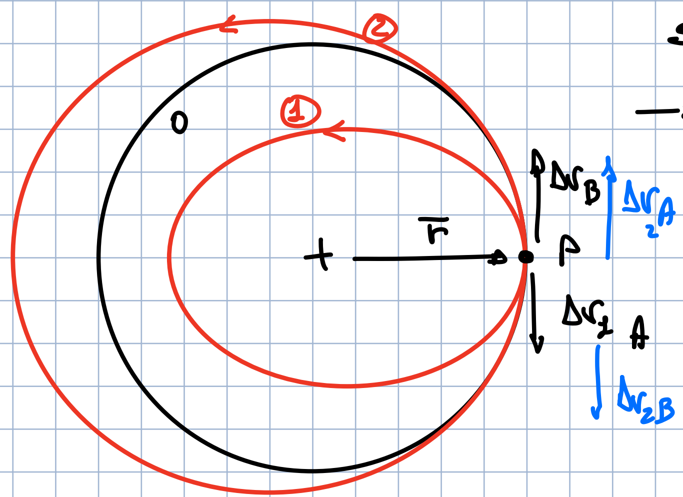
(C) $r_A > r_A'$ ⟹ A' is the periapsis of orbits (1) Transfer (3') is the more efficient regardless the final orbit.

(B) inner circular orbit

- Transfer (3') is better if $r_B' > r_B$ ⟹ Transfer should terminate at an apoapsis of the outher orbit.
- Transfer (3) is better if $r_B > r_D'$ ⟹ transfer should terminate at an apoapsis of the outher orbit where speed is the lowest.

PHASING MANOEUVRES

- create a time difference
- change the position of s/c on the same orbit (eg. rendez-vous)
- move on a new location over equator (communicate with a different ground station eg. telecom s/c GEO, weather forecast).



s/c ⓪, s/c₁ ① $-\Delta v_{1A}$
→ come back at P earlier than s/c ⓪ and then $+\Delta v_{1B}$ to have again orbit ⓪

s/c ⓪, s/c₂ ② $+\Delta v_{2A}$ → come back @ P later than s/c ⓪ $-\Delta v_{2B}$

How to design these manoeuvres?
Eq (1.58) to compute a

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$\boxed{a_{PH} = \left(\frac{T\sqrt{\mu}}{2\pi}\right)^{2/3}} \quad (3.14)$$

↳ of phasing orbit

from $a_{PH} \to r_a, r_p$

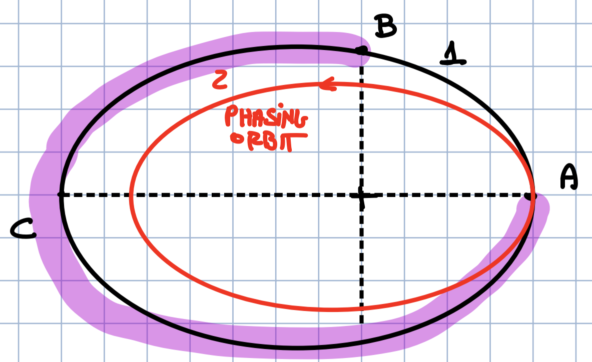
② $r_p = \bar{r}$ $\quad a = \frac{r_a + r_p}{2} \Rightarrow r_a = 2a_{PH} - r_p$

① $r_a = \bar{r}$ $\quad a = \frac{r_a + r_p}{2} \Rightarrow r_p = 2a_{PH} - r_a$

example

We have two satellite on the same orbit.

2 s/c at A, B of orbit $\boxed{1}$



DATA

$r_A = 6800$ Km

$r_C = 13600$ Km

$\mu_E = 398600$ Km^3/s^2

A chosen s/c performs a phasing manoeuvre to catch s/c B when it comes back at point A after 1 chosen revolution on orbit 2.

$$T_2 = T_1 - \Delta t_{AB}$$

$$a_1 = \frac{r_A + r_C}{2} = 10200 \text{ km}$$

$$T_1 = 2\pi \sqrt{\frac{a_1^3}{\mu}} = 10252 \text{ s}$$

From eq (2.28) $\Delta t_{AB} = \sqrt{\frac{a^3}{\mu}} \left(E_B - e_1 \sin E_B \right)$

$$e_1 = \frac{r_C - r_A}{r_C + r_A} = 0.3333$$

$$E_B = 2 \operatorname{tg}^{-1} \left(\sqrt{\frac{1-e_1}{1+e_1}} \operatorname{tg} \frac{\theta_B}{2} \right)$$

but $\theta_B = \frac{\pi}{2}$ $\quad E_B = 1.2310$ rad $\rightarrow \Delta t_{AB} = 1458.7$ s

$$T_2 = T_1 - \Delta t_{AB} = 8756.3 \text{ s}$$

So we can compute a_2

$$a_2 = \left(\frac{T_2 \sqrt{\mu}}{2\pi} \right)^{2/3} = 3182.1 \text{ km}$$

② r_A

$$r_D = 2a_2 - r_A = 11564 \text{ km}$$

Eq (3.10) and (3.11)

$$h_1 = \sqrt{2\mu \frac{r_A r_C}{r_A + r_C}} = 60116 \frac{\text{km}^2}{s}$$

$$h_2 = \sqrt{2\mu \frac{r_A r_D}{r_A + r_D}} = 58426 \frac{\text{km}^2}{s}$$

$$v_{A_1} = \frac{h_1}{r_A} = 8.8406 \text{ km/s}$$

$v_{A_2} < v_{A_1} \rightarrow$ OK it is correct

$$v_{A_2} = \frac{h_2}{r_A} = 8.5921 \text{ km/s}$$

$$\Delta v_{APH1\, ① \rightarrow ②} = v_{A_2} - v_{A_1} = -0.24851 \text{ km/s}$$ s/c breaks

at the end of phasing orbit ②

$$\Delta v_{APH_2\, ② \rightarrow ①} = +0.24851 \text{ km/s}$$ s/c accelerates

$$\Delta v_{TOT} = |\Delta v_{APH_1}| + |\Delta v_{APH_2}| = 2|\Delta v_{APH1}| = 0.4970 \text{ km/s}$$

Δv_{PH} can be reduced by decreasing the difference in a_1 and a_2.

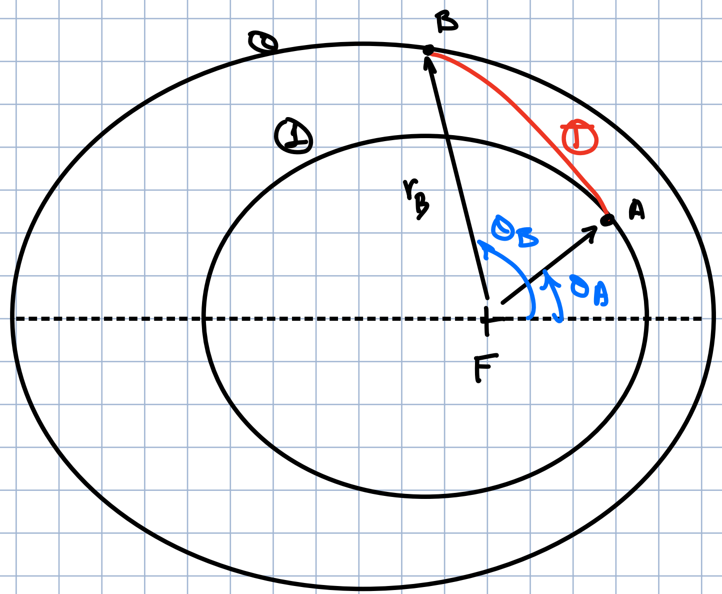
$$nT_2 = KT_1 - \Delta t_{AB}$$ $\rightarrow$ see Exe Lab

n, K integer number

NOTE check if r_P of phasing orbit is larger than R_{PLANET}

after u, k orbit of s/c 1 and 2 we will meet again.

NON-HOHMANN TRANSFER WITH COMMON APSE LINE



TRANSFER: CO-AXIAL
CO-FOCAL
SECANT to 1 and 2

shorter transfer time

NOTE if they are not co-axial the transfer can not be calculated in a close form.

$r_A, r_B \in 1, 2$
$\in T$

$r_{A1} = r_{AT}$

$r_{B2} = r_{BT}$

Transfer orbit (e_T, h_T) by using eq (1.26)

$$\begin{cases} r_A = \dfrac{h_T^2}{\mu} \dfrac{1}{1+e_T\cos\theta_A} \\ r_B = \dfrac{h_T^2}{\mu} \dfrac{1}{1+e_T\cos\theta_B} \end{cases} \qquad (3.15)$$

Solve h_T, e_T

$$\begin{cases} \dfrac{h_T^2}{\mu} = r_A(1+e_T\cos\theta_A) \\ \dfrac{h_T^2}{\mu} = r_B(1+e_T\cos\theta_B) \end{cases}$$

$$r_A\left(1+e_T\cos\theta_A\right)=r_B\left(1+e_T\cos\theta_B\right)$$

$$r_A+r_A e_T\cos\theta_A=r_B+r_B e_T\cos\theta_B$$

$$r_A-r_B=e_T\left(\cos\theta_B\, r_B-\cos\theta_A\, r_B\right)$$

$$\boxed{e_T=\frac{r_A-r_B}{r_B\cos\theta_B-r_A\cos\theta_A}}$$

(3.16)

$$h_T^2=\mu r_A\left(1+\frac{r_A-r_B}{r_B\cos\theta_B-r_A\cos\theta_A}\cos\theta_A\right)$$

$$h_T^2=\mu r_A\left(\frac{r_B\cos\theta_B-r_A\cancel{\cos\theta_A}+\cancel{r_A}\cos\theta_A-r_B\cos\theta_A}{r_B\cos\theta_B-r_A\cos\theta_A}\right)$$

$$=\mu r_A r_B\,\frac{\cos\theta_B-\cos\theta_A}{r_B\cos\theta_B-r_A\cos\theta_A}$$

$$\boxed{h_T=\sqrt{\mu r_A r_B\,\frac{\cos\theta_B-\cos\theta_A}{r_B\cos\theta_B-r_A\cos\theta_A}}}\qquad(3.17)$$

e_T, h_T $\Rightarrow$ can calculate $\underline{v}_A$, $\underline{v}_B$

NOTE if set $\theta_A=0$, $\theta_B=\pi$ $\Rightarrow$ Hohmann transfer

$$e_T=\frac{r_A-r_B}{-r_B-r_A}=\frac{r_B-r_A}{r_B+r_A}\qquad(\text{see eq } 3.10)$$

$$h_T=\sqrt{\mu r_A r_B\,\frac{-1-1}{-r_B-r_A}}=\sqrt{2\mu\frac{r_A r_B}{r_B+r_A}}$$

same eq obtained before (3.7) $\Rightarrow$ the eq 3.17 is just more general.

Q & A session

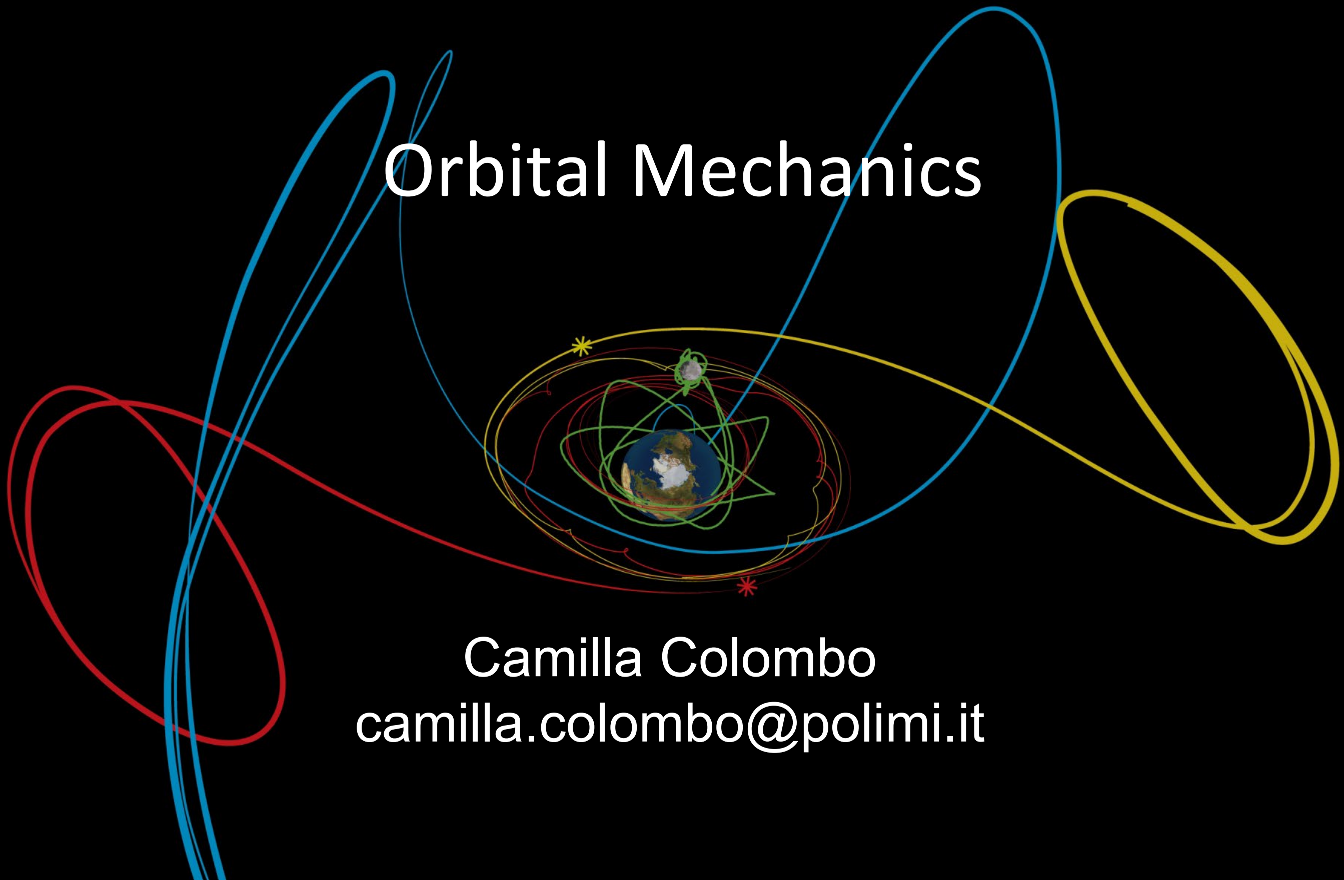
$\epsilon = LAT - LMT$

$LMT = GMT + \lambda$

$LAT = GAT + \lambda$

$\lambda = LAT - GAT = LMT - GMT$

Orbital Mechanics



Camilla Colombo
camilla.colombo@polimi.it

3. ORBIT MANOEUVRES

Propellant mass consumption

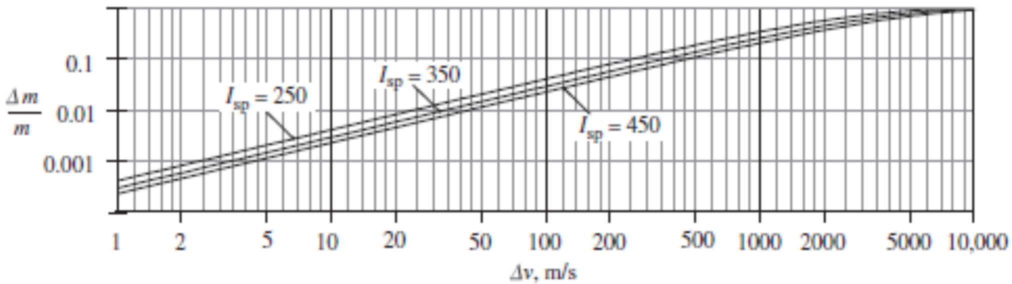


FIGURE 6.1

Propellant mass fraction versus Δv for typical specific impulses.

Hohmann transfer efficiency

$r_{A'} > r_A$

A = periapsis of the first orbit

A' = apoapsis of the first orbit

$r_{A'} = r_A$

$r_{A'} < r_A$

$\frac{\Delta r_3'}{\Delta r_3}$

all larger than one

So (3) is the most-efficient therefore,

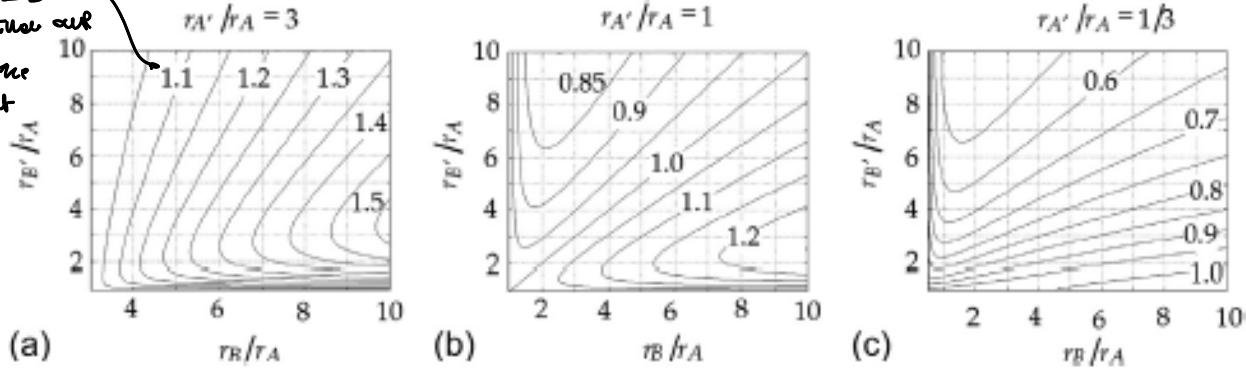
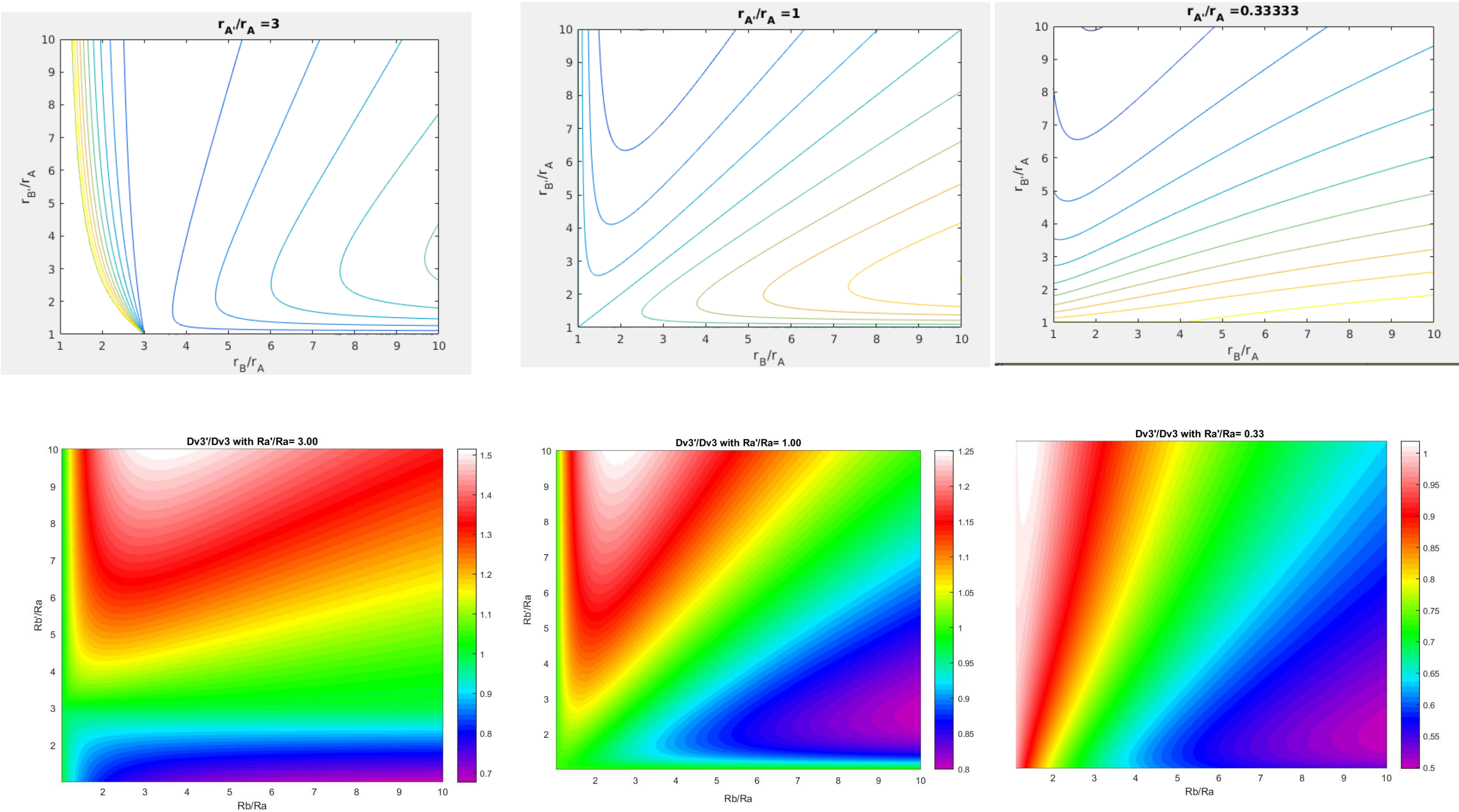


FIGURE 6.5

Contour plots of $\Delta v_{total})_{3'}/\Delta v_{total})_3$ for different relative sizes of the ellipses in Figure 6.4. Note that $r_B > r_{A'}$ and $r_{B'} > r_A$.

Hohmann transfer efficiency examples



Bi-elliptic transfer

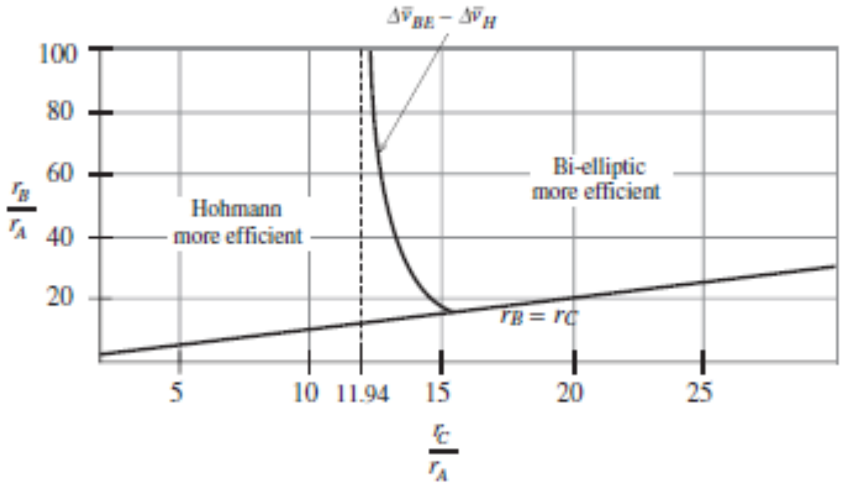
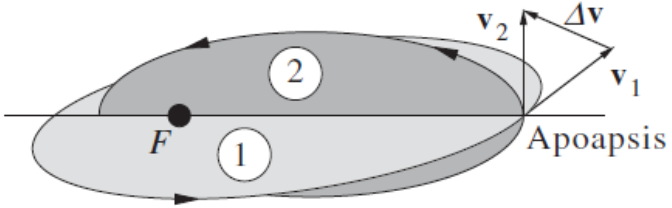
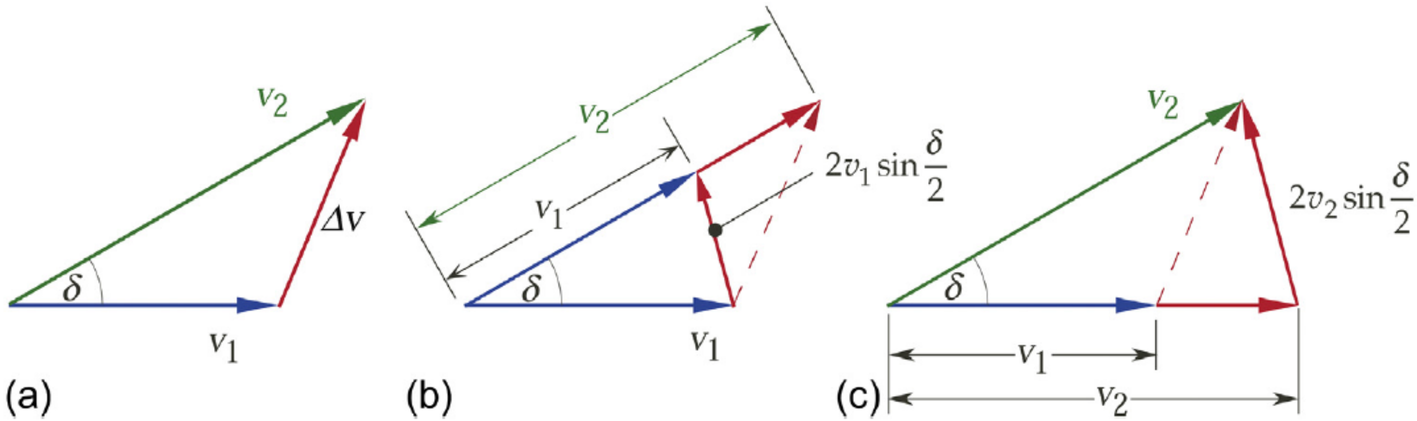


FIGURE 6.8

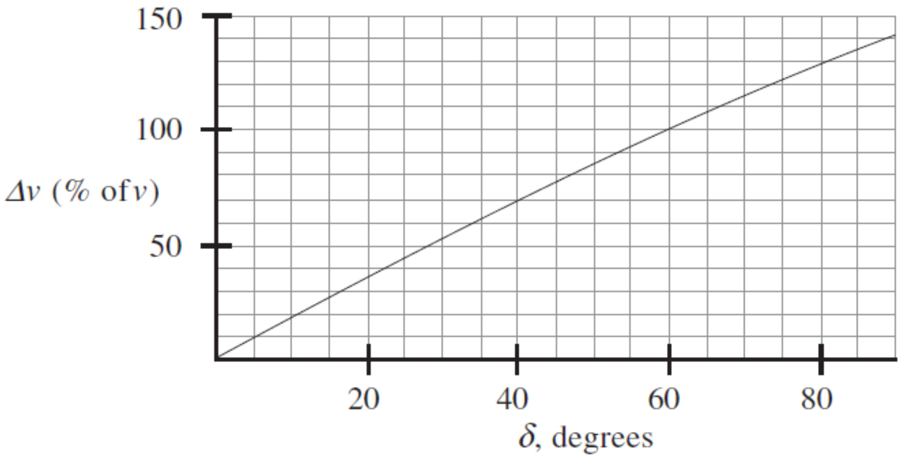
Orbits for which the bi-elliptic transfer is either less efficient or more efficient than the Hohmann transfer.

Plane change manoeuvre

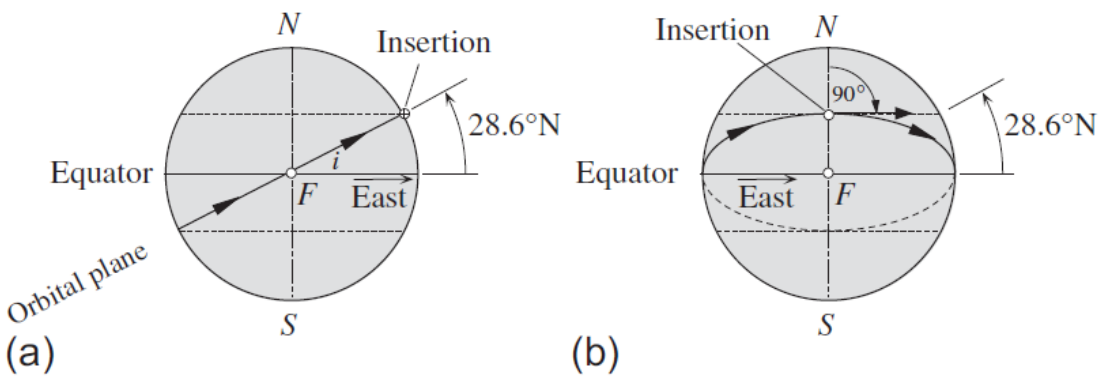




Plane change manoeuvre

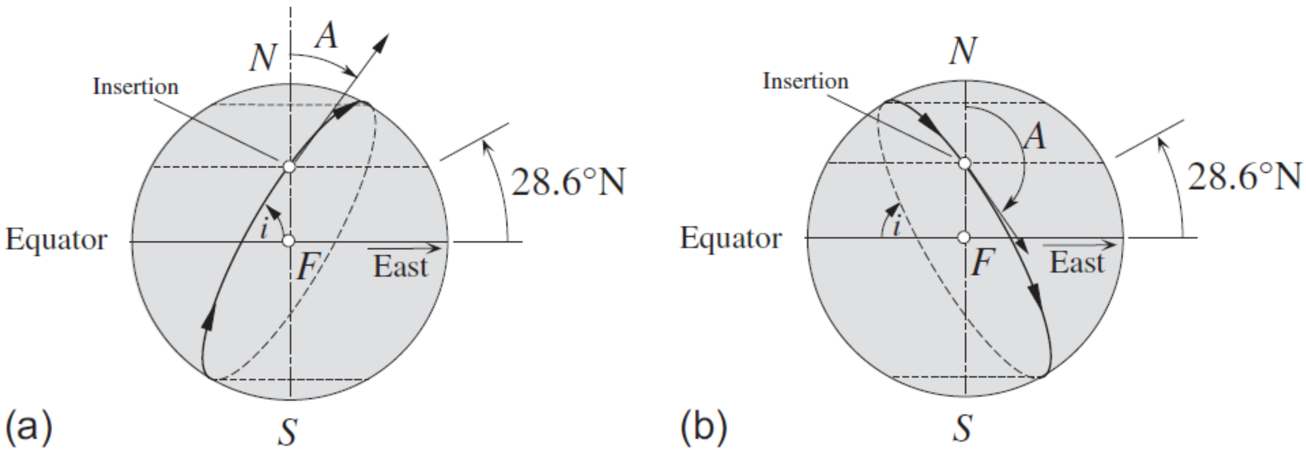


Launch geometry

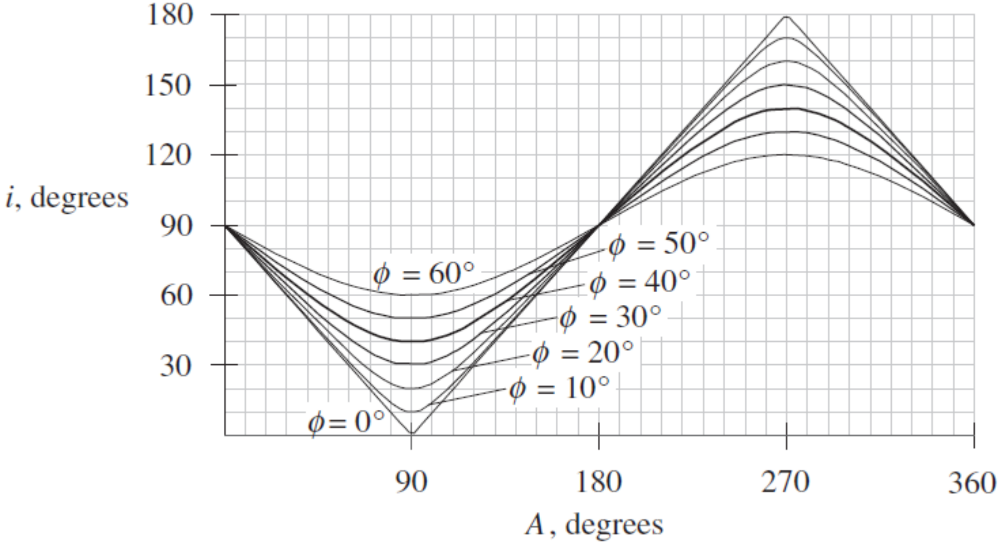


POLITECNICO
MILANO 1863

Launch geometry



Launch geometry



Lambert's problem (1)

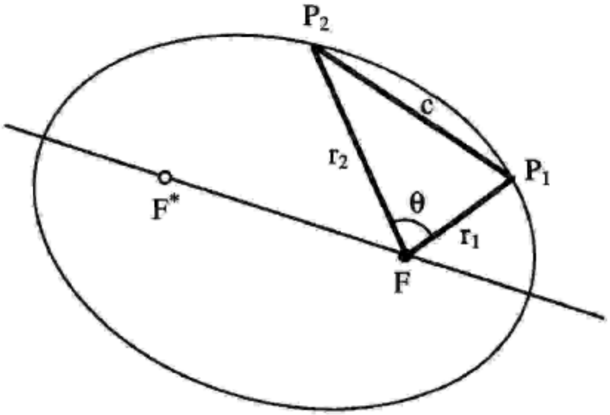


Fig. 4.1 Transfer Orbit Geometry

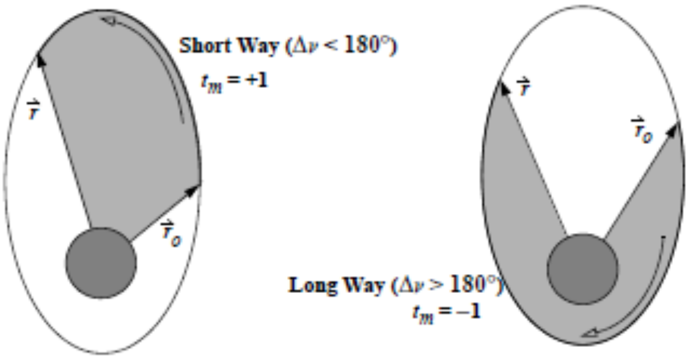


Figure 7-8. **Transfer Methods, t_m, for the Lambert Problem.** Traveling between the two specified points can take the long way or the short way. For the long way, the change in true anomaly exceeds 180°.

POLITECNICO
MILANO 1863

Lambert's problem (2)

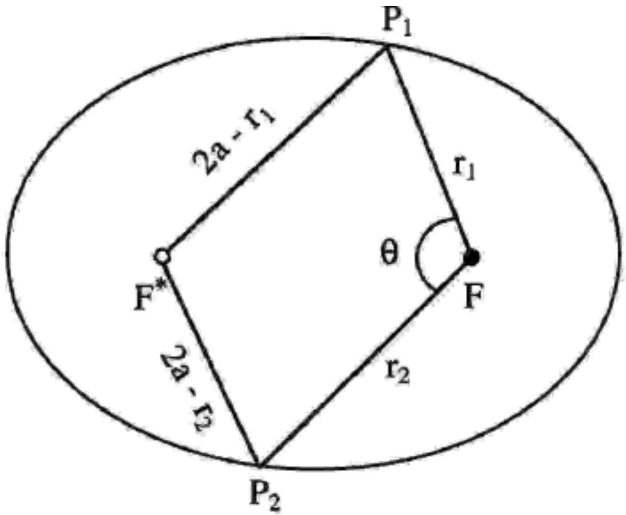


Fig. 4.2 A Geometric Property of Ellipses

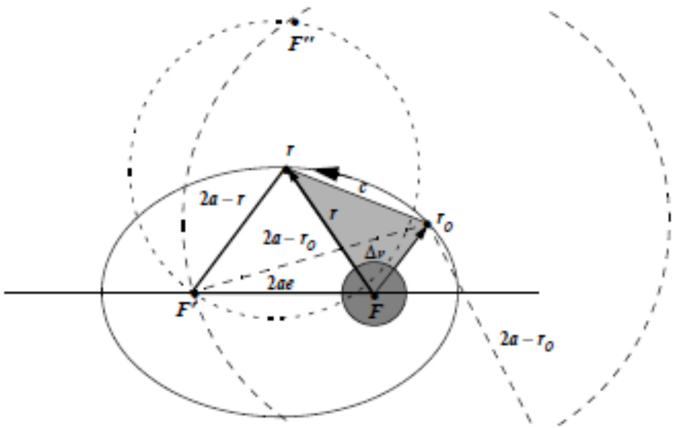


Figure 7-10. Geometry for the Lambert Problem (I). This figure shows how we locate the secondary focus—the intersection of the dashed circles. The chord length, c, is the shortest distance between the two position vectors. The sum of the distances from the foci to any point, r or r_0, is equal to twice the semimajor axis.

POLITECNICO
MILANO 1863

Lambert's problem (3)

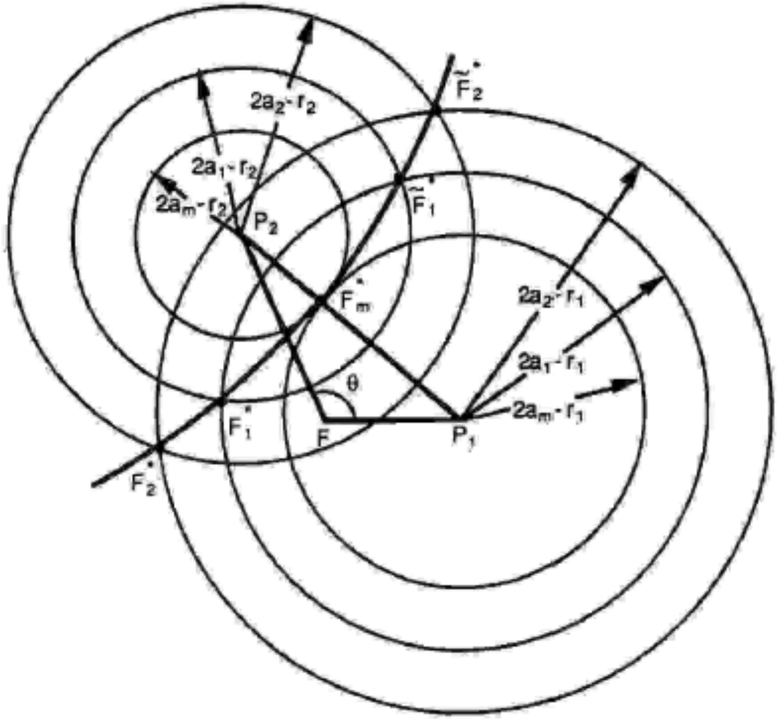


Fig. 4.3 Vacant Focus Locations

Lambert’s problem (4)

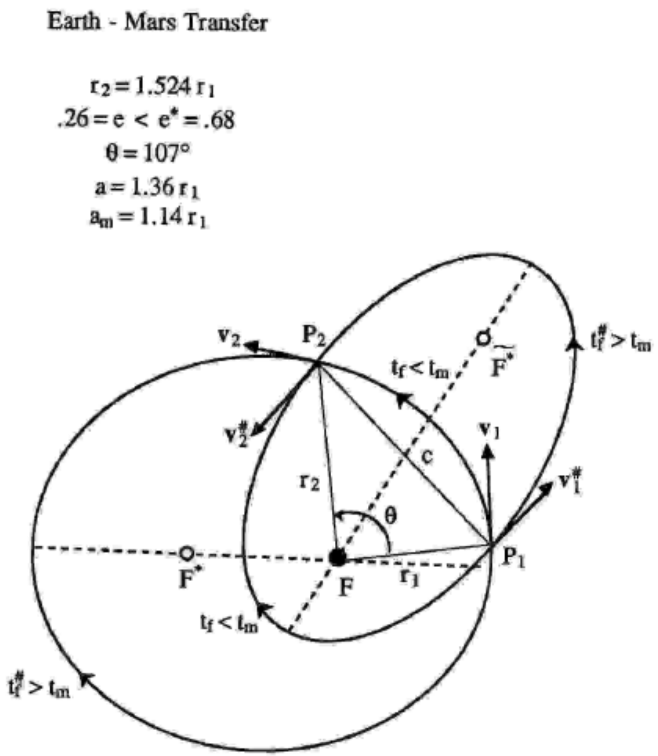


Fig. 4.4 Two Elliptic Transfer Orbits with the Same Value of a

Lambert’s problem (5)

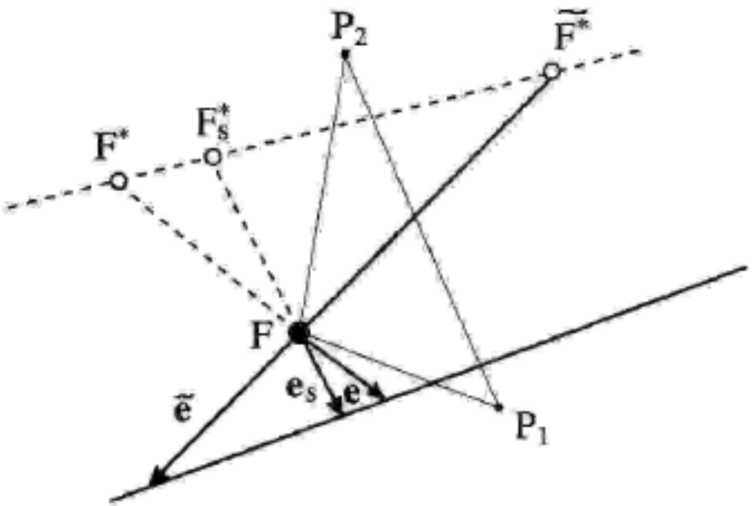


Fig. 4.5 Locus of Eccentricity Vectors

Lambert’s problem (6)

The time required to traverse an elliptic arc between two specified endpoints ”depends only on the semi-major axis of the ellipse, and on two geometric properties of the space triangle, namely the chord and the sum of the radii from the focus to point P_1 and P_2”

Lambert, 1761

Lambert’s problem (7)

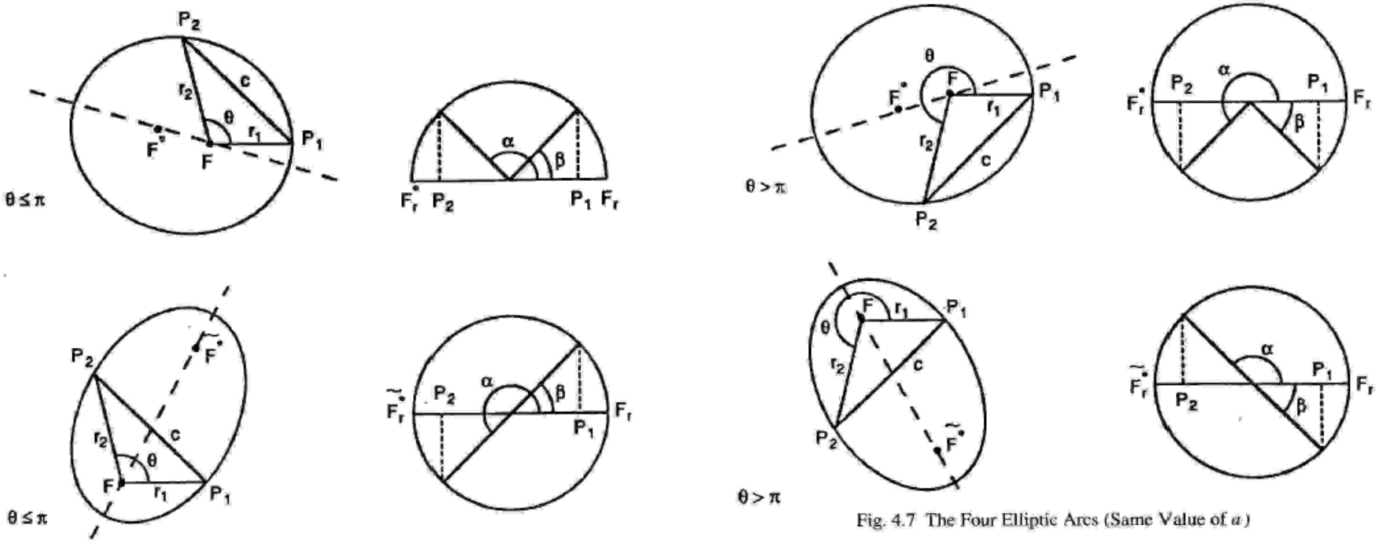


Fig. 4.7 The Four Elliptic Arcs (Same Value of a)

Lambert's problem (8)

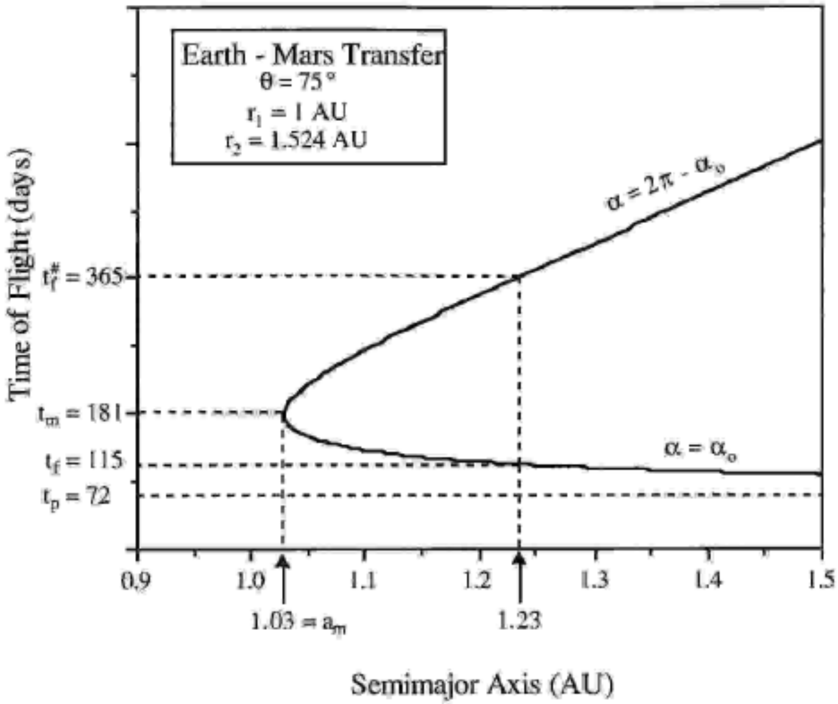
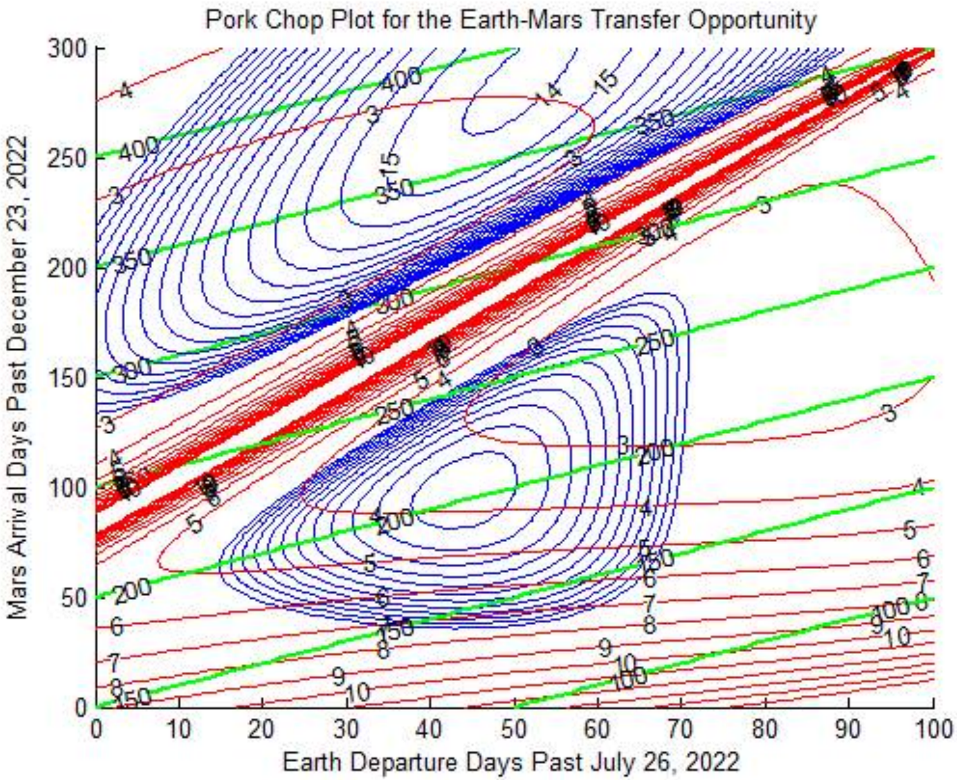


Fig. 4.8 Transfer Time vs. Semimajor Axis

Transfer design strategy – pork-chop graph (1)



Transfer design strategy – pork-chop graph (2)

Table 7.2: **Keplerian elements and their rates, with respect to the mean ecliptic and equinox of J2000, valid for the time–interval 1800 AD - 2050 AD.**

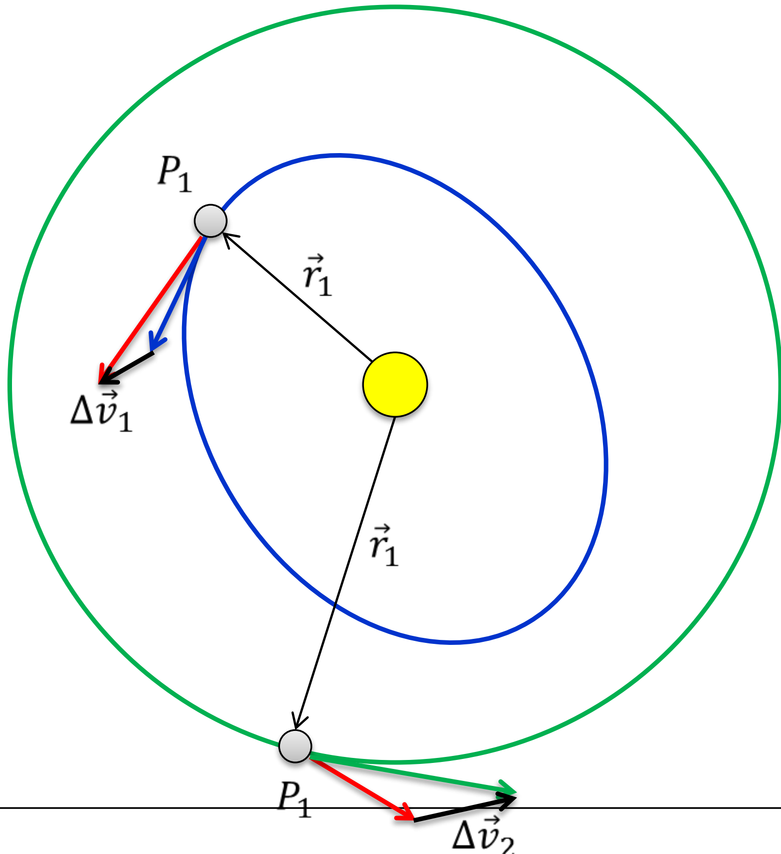
Planet	semimaj.axis a [AU] ($\dot{a}$) [AU/Cy]	eccentricity e ($\dot{e}$) [1/Cy]	inclination i [deg] ($\dot{i}$) [deg/Cy]	true anomaly ν [deg] ($\dot{\nu}$) [avg.deg/y]	long.perihelion Π [deg] ($\dot{\Pi}$) [deg/Cy]	long.asc.node Ω [deg] ($\dot{\Omega}$) [deg/Cy]
Mercury	**0.38709927** (0.00000037)	**0.20563593** (0.00001906)	**7.00497902** (-0.00594749)	**252.25032350** (1494.72674111)	**77.45779628** (0.16047689)	**48.33076593** (-0.12534081)
Venus	**0.72333566** (0.00000390)	**0.00677672** (-0.00004107)	**3.39467605** (-0.00078890)	**181.97909950** (585.17815387)	**131.60246718** (0.00268329	**76.67984255** (-0.27769418)
Earth	**1.00000261** (0.00000562)	**0.01671123** (-0.00004392)	**-0.00001531** (-0.01294668)	**100.46457166** (359.99372450)	**102.93768193** (0.32327364	**0.0** (0.0)
Mars	**1.52371034** (0.00001847)	**0.09339410** (0.00007882)	**1.84969142** (-0.00813131)	**-4.55343205** (191.40302685)	**-23.94362959** (0.44441088)	**49.55953891** (-0.29257343)
Jupiter	**5.20288700** (-0.00011607)	**0.04838624** (-0.00013253)	**1.30439695** (-0.00183714)	**34.39644051** (30.34746128)	**14.72847983** (0.21252668)	**100.47390909** (0.20469106)
Saturn	**(9.53667594** (-0.00125060)	**0.05386179** (-0.00050991)	**2.48599187** (0.00193609)	**49.95424423** (12.22493622)	**92.59887831** (-0.41897216)	**113.66242448** (-0.28867794)
Uranus	**19.18916464** (-0.00196176)	**0.04725744** (-0.00004397)	**0.77263783** (-0.00242939)	**313.23810451** (4.28482028)	**170.95427630** (0.40805281)	**74.01692503** (0.04240589)
Neptune	**30.06992276** (0.00026291)	**0.00859048** (0.00005105)	**1.77004347** (0.00035372)	**-55.12002969** (2.18459453)	**44.96476227** (-0.32241464)	**131.78422574** (-0.00508664)
Pluto	**39.48211675** (-0.00031596)	**0.24882730** (0.00005170)	**17.14001206** (0.00004818)	**238.92903833** (1.45207805)	**224.06891629** (-0.04062942)	**110.30393684** (-0.01183482)

Table 7.3: **Physical properties of Sun and planets.**

Celestial Body	Diameter [km]	(Earth = 1)	Rotational period*	Oblateness	Axial tilt [deg]	Mass Earth = 1	Mass par. [km^3s^{-2}]
Sun	1,392,000	(109)	≈ 25.4 days	$\approx 10^{-5}$	7.25	333,432	$1.327\ 10^{11}$
Mercury	4,879	(0.38)	58.65 days	0.0	2.0	0.055	$2.232\ 10^{4}$
Venus	12,104	(0.95)	-243.02 days	0.0	177.4	0.815	$3.257\ 10^{5}$
Earth	12,742	(1.0)	23 hrs 56 min	0.0034	23.45	1.000	$3.986\ 10^{5}$
Mars	6,780	(0.53)	24 hrs 37 min	0.005	25.19	0.107	$4.305\ 10^{4}$
Jupiter	139,822	(10.97)	9 hrs 55 min	0.065	3.12	317.830	$1.268\ 10^{8}$
Saturn	116,464	(9.14)	10 hrs 40 min	0.108	26.73	95.159	$3.795\ 10^{7}$
Uranus	50,724	(3.98)	-17.24 days	0.03	97.86	14.536	$5.820\ 10^{6}$
Neptune	49,248	(3.87)	16 hours 7 min	0.02	29.56	17.147	$6.896\ 10^{6}$
Pluto	2,390	(0.19)	-6.38 days	0.0	119.6	0.002	$0.797\ 10^{3}$

* Negative numbers indicate retrograde rotation.

Transfer design strategy – pork-chop graph (3)



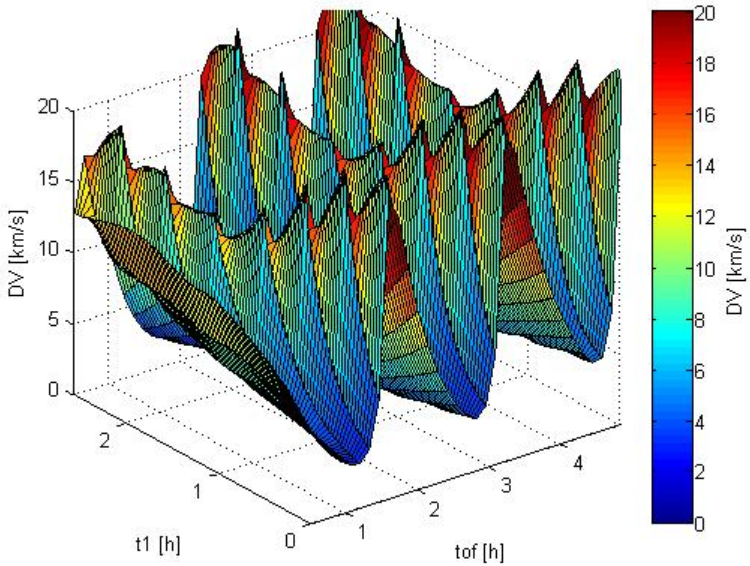
Initial orbit
Final orbit
Transfer

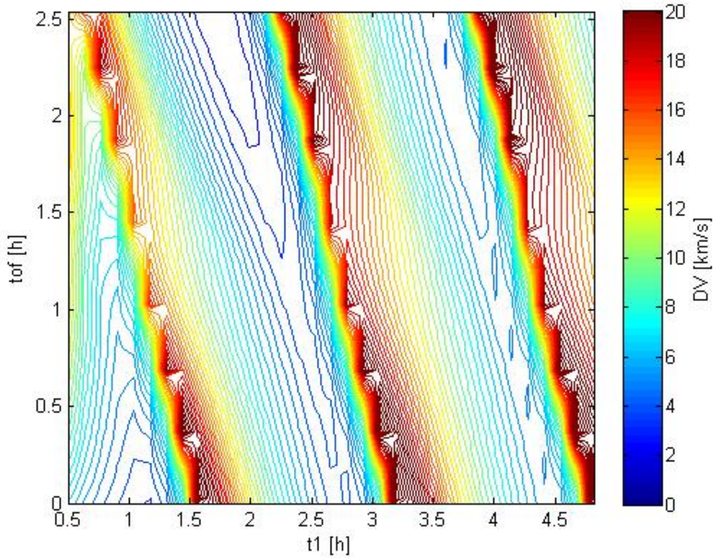
Lambert's Problem

For any possible P1 and P2

Transfer design strategy – pork-chop graph (4)

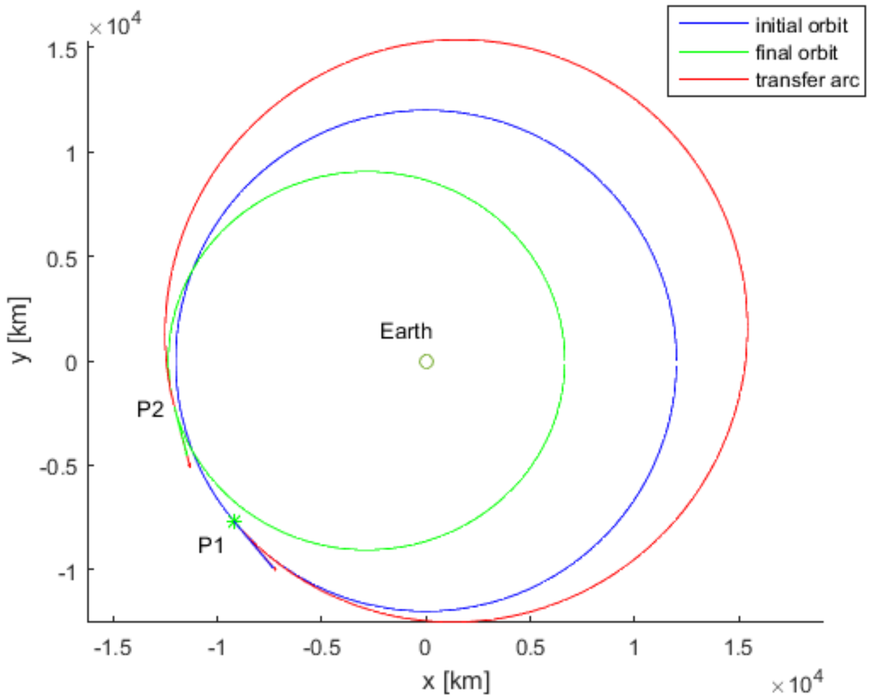
Plot the total cost considering any possible initial position (on the initial orbit) and final position (on the final orbit)





Transfer design strategy – pork-chop graph (5)

Chosen solution (minimum in the Pork-Chop graph)



Transfer design strategy – pork-chop graph (6)

Data

- $\{a, e, i, \Omega, \omega, \vartheta\}_1$ (first orbit)
- $\{a, e, i, \Omega, \omega, \vartheta\}_2$ (second orbit)
- tof_{max}

1. Compute initial and final orbits

2. Compute transfer arc using Lambert solver for any possible initial position (on the initial orbit, at time t_1) and final position (on the final orbit, at time $t_1 + tof$)

3. Compute the total cost of the transfer for any computed arc

4. Plot the total cost as function of initial time (starting position on the initial orbit) and time of flight (which depends on the arrival position on the final orbit)

5. Choose the minimum and plot the selected transfer arc