ORBITAL MECHANICS

suggestion => follow theory, exercise and labs step by step
il will help with the understanding of the material

Fondamentals of Astrodynamics and Application -> Vallado
↳ for perturbation

-> Curtis problem -> Altro libro -- Vedere registrazione

Everything will be on Beep.

GENERAL AND RESTRICTED TWO-BODY PROBLEM

obj : describe motion of planets or satellite (S/C) around Sun / Planet.
(e.g Moon @ Earth , Ganimede @ Jupiter , S/C @ Earth,
planet @ Sun , S/C @ Sun)

NOTE

Understanding the motion of planets / Sun and Star is a need
much older than the first space flight (Sputnik 1957).
Such understanding was needed for:
- measuring time
- navigation using celestial body.

Nowadays -> predicting S/C for:
- scientific mission (remote sensing mission)
- exploration mission
- technological demonstration
- Space is providing different services
↳ Private interest in Space.

Mission founded by private sources to obtain a return on investment

→ Space situational awareness (threat to space activities)

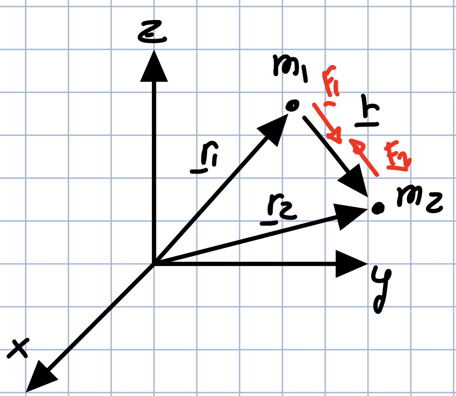
- space wheather
- Near Earth objects   bodies @ Sun (asteroids comets)
  - ↳ for avoiding collision
  - for mining
- space debries
  - ~ 4000 s/c active
  - ~ 300'000 trackable object (non active)
  - ~ millions smaller than 1 cm.

NEWTON'S GRAVITATION LAW

$$\boxed{\underline{F} = G\,\frac{m_1 \cdot m_2}{r^2}\,\hat{r}} \quad (1.1)$$

$\hat{r} \Rightarrow |\hat{r}| = 1$ $\qquad$ $\hat{r} = \frac{\underline{r}}{r}$ $\qquad$ $r = \|\underline{r}\|$

$G = 6{,}67 \cdot 10^{-20} \frac{km^3}{kg\, s^2}$ universal constant.



$$\boxed{\underline{r} = \underline{r}_2 - \underline{r}_1} \quad (1.2)$$

e.g.

m_1	m_2
Sun	planet/ s/c
Earth	satellite/Moon
planet	Moon / s/c

Newton's Second Law

$$\left.\begin{aligned} m_1 \ddot{\underline{r}}_1 &= G\,\frac{m_1 m_2}{r^2}\,\hat{r} \\ m_2 \ddot{\underline{r}}_2 &= -G\,\frac{m_1 m_2}{r^2}\,\hat{r} \end{aligned}\right\} (1.3)$$

$\ddot{\underline{r}} = \frac{d^2\underline{r}}{dt^2}$ $\qquad$ $\underline{F}_1 = \underline{F}$, $\underline{F}_2 = -\underline{F}$

we want to find the relative motion between m_1 and m_2

but before → (1.2) + (1.3)

$$m_1 \ddot{\underline{r}}_1 + m_2 \ddot{\underline{r}}_2 = 0$$

Position of the baricenter G   velocity of the baricenter G

$$\underline{r}_G = \frac{m_1 \underline{r}_1 + m_2 \underline{r}_2}{m_1 + m_2} \qquad \underline{v}_G = \dot{\underline{r}}_G = \frac{m_1 \dot{\underline{r}}_1 + m_2 \dot{\underline{r}}_2}{m_1 + m_2}$$

acceleration of G

$$\underline{a}_G = \dot{\underline{v}}_G = \boxed{\ddot{\underline{r}}_G = \frac{m_1 \ddot{\underline{r}}_1 + m_2 \ddot{\underline{r}}_2}{m_1 + m_2} = \emptyset}$$

No external forces ⟹ G moves on a straight line with constant velocity.

$$\boxed{\underline{r}_G(t) = \underline{r}_{G0} + \underline{v}_{G0}\, t}$$

Looking back at equations (1.2) (1.3)

$$\ddot{\underline{r}}_1 = G m_2 \frac{\hat{r}}{r^2} = \begin{cases} \ddot{x}_1 = G m_2 \dfrac{x_2 - x_1}{r^3} \\ \ddot{y}_1 = G m_2 \dfrac{y_2 - y_1}{r^3} \\ \ddot{z}_1 = G m_2 \dfrac{z_2 - z_1}{r^3} \end{cases}$$

it is a vectorial equation.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\ddot{\underline{r}}_2 = G m_1 \frac{\hat{r}}{r^2} = \begin{cases} \ddot{x}_2 = G m_1 \dfrac{x_1 - x_2}{r^3} \\ \ddot{y}_2 = G m_1 \dfrac{y_1 - y_2}{r^3} \\ \ddot{z}_2 = G m_1 \dfrac{z_1 - z_2}{r^3} \end{cases}$$

Describe motion of m_2 with respect m_1

$$m_1 \ddot{\underline{r}}_1 = \underline{F} \longrightarrow \ddot{\underline{r}}_1 = \frac{\underline{F}}{m_1}$$

$$m_2 \ddot{\underline{r}}_2 = -\underline{F} \longrightarrow \ddot{\underline{r}}_2 = -\frac{\underline{F}}{m_2}$$

$$\ddot{\underline{r}}_2 - \ddot{\underline{r}}_1 = \ddot{\underline{r}} = -\frac{\underline{F}}{m_2} - \frac{\underline{F}}{m_1} = -\frac{G m_1}{r^2}\hat{r} - \frac{G m_2}{r^2}\hat{r} = -\frac{G(m_1+m_2)}{r^2}\hat{r}$$

$$\boxed{\ddot{\underline{r}} = -G\frac{(m_1+m_2)}{r^2}\hat{r} \quad \text{or} \quad \ddot{\underline{r}} = -G\frac{(m_1+m_2)}{r^3}\underline{r}} \quad (1.4)$$

Equation of two Body Problem.

$$\boxed{\mu = G(m_1+m_2)}$$ 2BP constant

RESTRICTED TWO BODY PROBLEM

Hp: $m_1 \gg m_2$ m_2 does not affect motion of m_1

m_1 @ center of reference frame.

(1.4) becomes

$$\ddot{\underline{r}} = \frac{-G(m_1+\cancel{m_2})}{r^3}\underline{r} \Rightarrow \boxed{\ddot{\underline{r}} = -\frac{G m_1}{r^3}\underline{r}} \quad (1.5)$$

and now μ becomes

$$\boxed{\mu = G m_1}$$ Planetary constant

No longer depends on the smaller body.

$$\boxed{\ddot{\underline{r}} = -\frac{\mu}{r^3}\underline{r}} \quad (1.6)$$

Knowing (r_0, v_0) Numerically integrate $(1.6) \Rightarrow \underline{r}(t), \underline{v}(t)$

Motion of m_2 in the gravitational field of m_1

NOTE m_1 assumed to be spherically simmetric

$\Rightarrow$ mass is all concentrated @ origin of RF (point mass)

Gravitational force in (1.6) can be expressed in terms of a gradient of a scalar function, the gravity potential V of m_1

$$\boxed{\underline{F} = \nabla V \quad \text{where} \quad V = \frac{\mu}{r}} \quad (1.7)$$

For a spherical simmetric body $V = \frac{\mu}{r}$

For Non-Simmetric body add additional terms