POLITECNICO
MILANO 1863

Spacecraft Attitude Dynamics

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Nonlinear attitude control

Nonlinear stability

Lyapunov's Second Stability Theorem

Consider an autonomous nonlinear dynamic system described by:

$$\dot{\underline{x}} = f(\underline{x}), f(\underline{x}^*) = 0$$

Where $\underline{x}^*$ is an isolated equilibrium point. If there exists in some finite neighbourhood D , of the equilibrium point $\underline{x}^*$ a scalar function $V(\underline{x})$ with continuous first partial derivative with respect to $\underline{x}$ such that the following conditions hold

(i) $V(\underline{x}) > 0$ for all $\underline{x} \neq \underline{x}^*$ in D *and* $V(\underline{x}^*) = 0$

(ii) $\dot{V}(\underline{x}) < 0$ for all $\underline{x} \neq \underline{x}^*$ in D except for $\dot{V}(\underline{x}^*) = 0$

Then the system is said to be **asymptotically stable.** If D includes all possible states then the system is said to be **globally asymptotically stable.**

DCM Control- slew motion - Lyapunov Function

$$I\frac{d\underline{\omega}}{dt} = I\underline{\omega} \times \underline{\omega} + \underline{u}$$

$$\dot{A}_{B/N} = -[\omega \wedge]A_{B/N}$$

Equations of motion

$$A_e = AA_d^T$$

Attitude Error function

$$A_d = I_{3\times3}, \underline{\omega}_d = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Commanded state

Candidate Lyapunov functions

$$V = \frac{1}{2}\underline{\omega}^T J\underline{\omega} + k_2 tr(I - A)$$

Quaternion Control- Lyapunov Function for slew motions

$$I\frac{d\underline{\omega}}{dt} = I\underline{\omega} \times \underline{\omega} + \underline{u}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2}\begin{pmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{pmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Equations of motion

$$\begin{bmatrix} q_{1e} \\ q_{2e} \\ q_{3e} \\ q_{4e} \end{bmatrix} = \begin{bmatrix} q_{4c} & q_{3c} & -q_{2c} & -q_{1c} \\ -q_{3c} & q_{4c} & q_{1c} & -q_{2c} \\ q_{2c} & -q_{1c} & q_{4c} & -q_{3c} \\ q_{1c} & q_{2c} & q_{3c} & q_{4c} \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Error function

$$q_c \quad \underline{\omega}_c = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Commanded state

Candidate Lyapunov functions

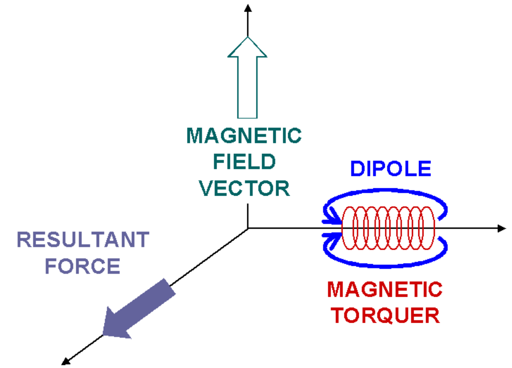
$$V = \frac{1}{2}\underline{\omega}^T J\underline{\omega} + k_2(1 - q_{4e})$$

$$V = \frac{1}{2}\underline{\omega}^T J\underline{\omega} + k_2(1 - {q_{4e}}^2)$$

$$V = \frac{1}{2}\underline{\omega}^T J\underline{\omega} + 2k_2 H(q_{4e})$$

Spin rate damping: B-dot proportional control

$\underline{m}$ Magnetic moment $\underline{m} = -k_b \underline{\dot{B}_m}$



$$\underline{M_c} = \underline{m} \Lambda \underline{B} = -k_b \underline{\dot{B}_m} \Lambda \underline{B}.$$

rate of change of the kinetic energy of the satellite $\dot{E}_k = \omega \underline{I} \dot{\omega}$

$$\dot{E}_k \approx \underline{\omega}^T \underline{M_c} = \underline{\omega}^T \left(\underline{m} \Lambda \underline{B} \right) = k_b \underline{\dot{B}}^T \left(\underline{\omega}^T \Lambda \underline{B} \right) = -k_b \underline{\dot{B}}^T \underline{B}$$

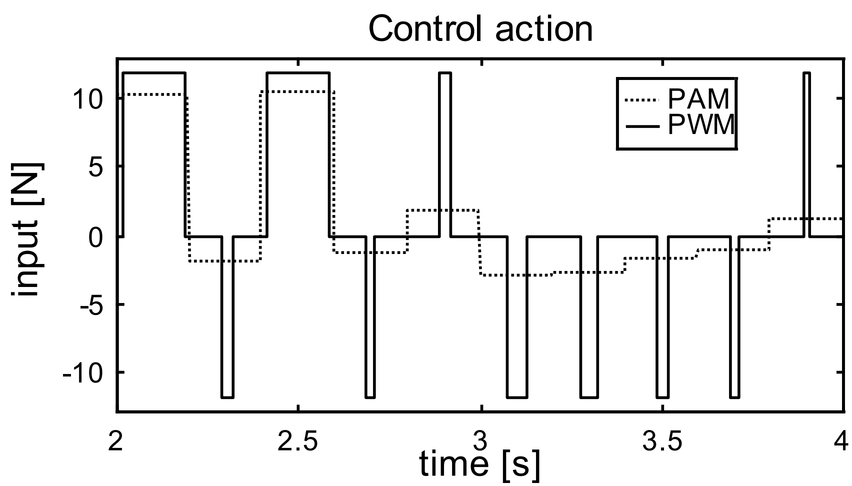
Simplified logic $\underline{m} = -m_0 \, \mathrm{sgn}(\underline{\dot{B}_m})$

General on-off thruster control

On-off controller

Sign-based logic $\vec{u}_{real} = T\,\mathrm{sgn}(\vec{u}_{ideal})$

Sampled integral-based logic



Schmidt-Trigger Logic

Nonlinear controller decoupled for each axis, phase plane analysis.

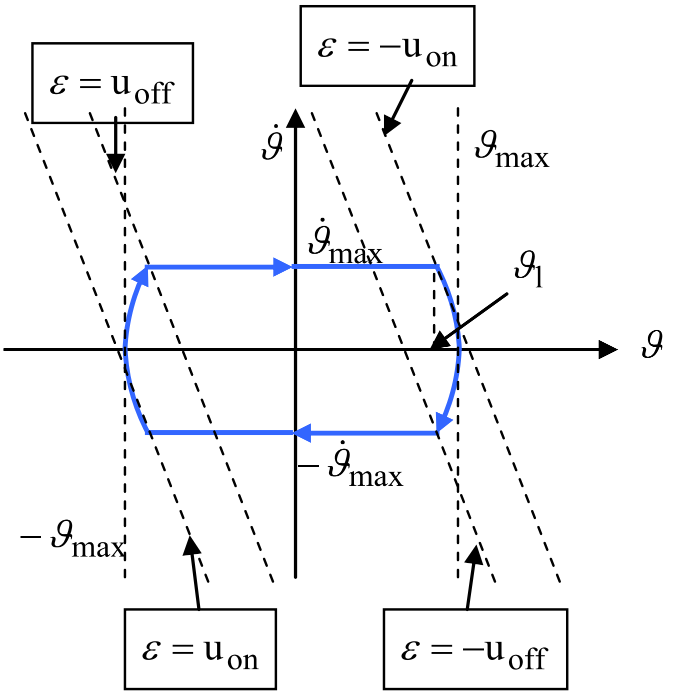
Nonlinear switch called “Schmitt trigger”, based on $\varepsilon = \theta + \tau\dot{\theta}$

u_{on}, u_{off} determined considering ϑ_{max}, $d\vartheta_{max}$ τ.

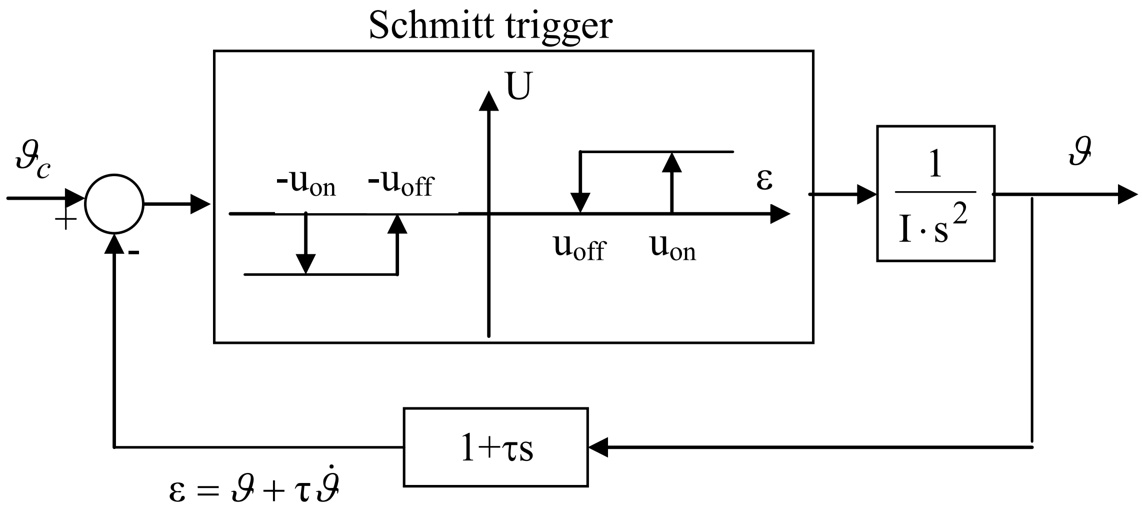
$$\vartheta_1 = \vartheta_{\max} - \frac{d\vartheta_{max}^2}{2u_c}$$

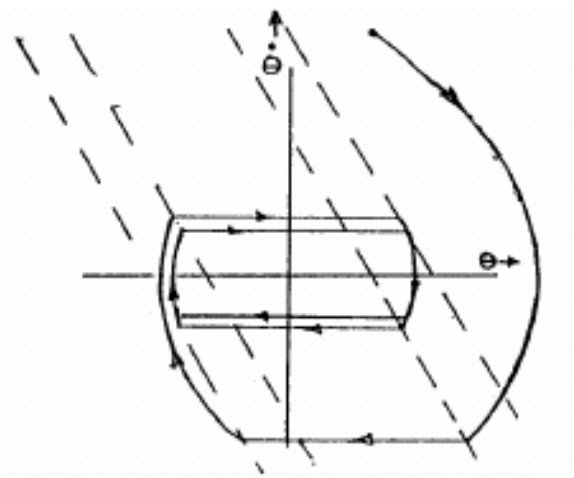
$u_c = M/I$ is the control command

$$u_{on} = \tau d\vartheta + \vartheta_1$$
$$u_{off} = -\tau d\vartheta + \vartheta_1$$



Schmidt-Trigger Logic





Pulse-Width-Pulse-Frequency Modulator

Varies both the width of the control pulse and the frequency of the switchings.