ORBITAL MECHANICS

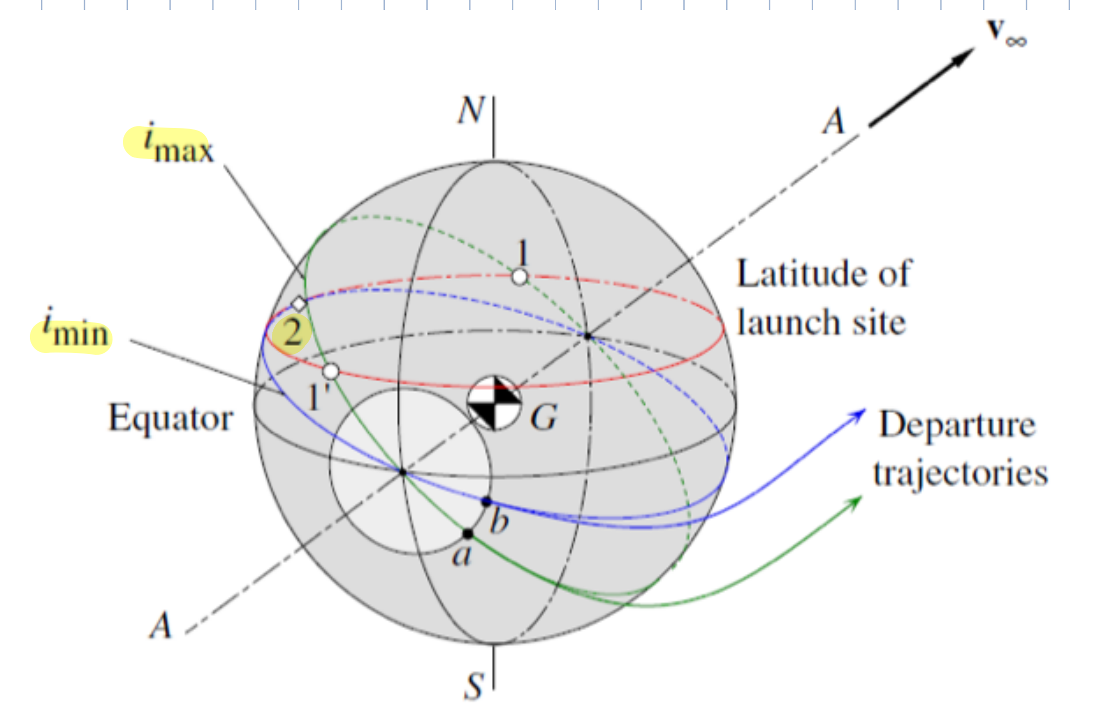
NOTE THERE IS NO CONSTRAINT ON THE TOTAL ΔV ONLY ON ΔV_1.

The injection circle comes from the fact that the asymptote of the hyperbola must be parallel to $\underline{v_1}$ (Hohmann transfer) and between the asymptote and the apse line we have the β angle.

Set of possible perigee → Injection circle.

Asymptote => direction of ∞ regardless of the type of transfer.

All of these hyperbolae, even if for the planet they have different plane, for the point of view of the sun they are all the same.



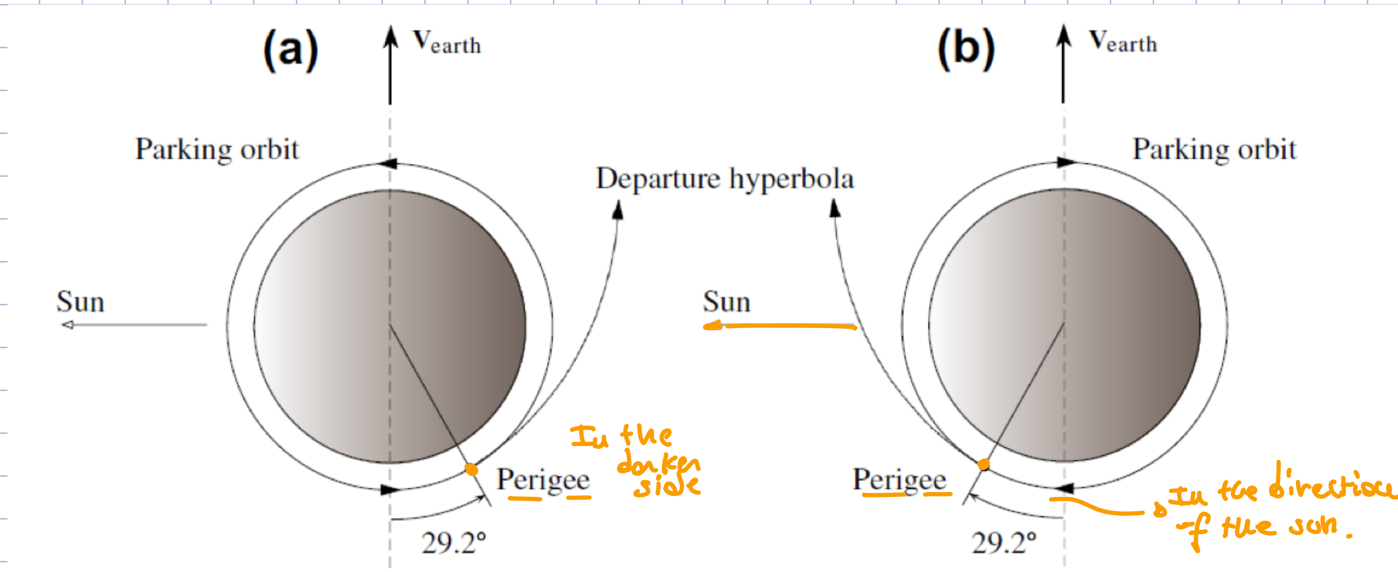
other constraint exist due to launcher site

$$i_{MIN} < i_{HYPERBOLA} < i_{MAX} \qquad (4.25)$$

↳ In theory can be up to $\frac{\pi}{2}$ but usually it is limited due to launcher constraint.

correspond to launcher site latitude or launcher injection orbit must contain focus of orbit (centre of planet)

As the Earth rotate beneath it, there are 2 possible launch opportunities per day because i_{HYP} can be reached twice per day.



They are the same hyperbola at ∞. This is the same drawing we did last time.

- check out user manual of Ariane 6.

We could solve this problem using the spherical geometry → where the perigee point is the unknown we want to determine.

① i_{HYP} does not matter for helio leg as long as $\underline{v_\infty}$ (asymptote) is in the direction I need $\underline{r}_{S/C} \stackrel{\Delta}{=} \underline{v_\infty} + \underline{V_1}$

② $i_{MIN} < i_{HYP} < i_{MAX}$ → launcher injection selection
↳ site of launch

③ Due to Earth's rotation launch site (λ_{LAUNCH}, ϕ_{LAUNCH}) intersect the parking orbit twice per day.

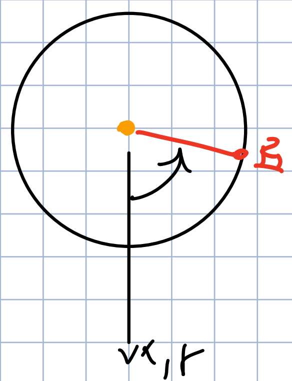
At helio leg

$\underline{R}_1(t_0)$ Earth @ Sun at t_0

$\underline{V}_D^{s/c} = \underline{V}_1 + \underline{V}_\infty$

$\underline{R}_1(t_0)$, $\underline{V}_1$ → i_{EARTH} wrt SUN

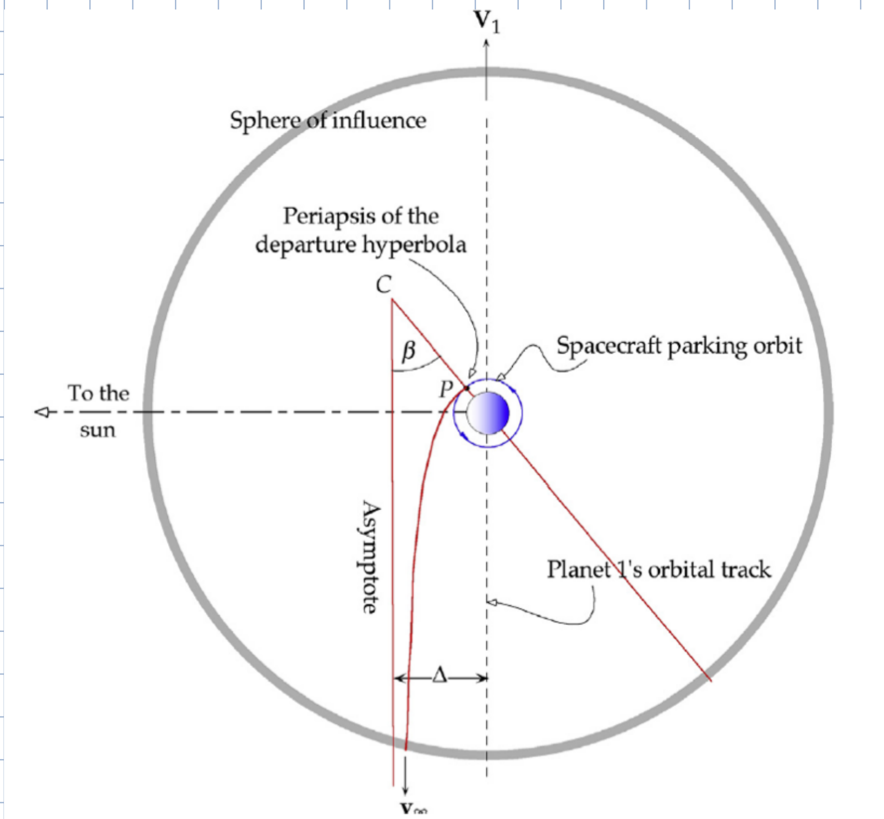
$\underline{R}_1(t_0)$, $\underline{V}_D^{s/c}$ → $i_{HELIO} \neq i_{EARTH}$



$\underline{V}_\infty$ // asymptote in general case →

$\underline{V}_D^{s/c} = \underline{V}_1 + \underline{V}_\infty$ → asymptote

↳ given by Lambert



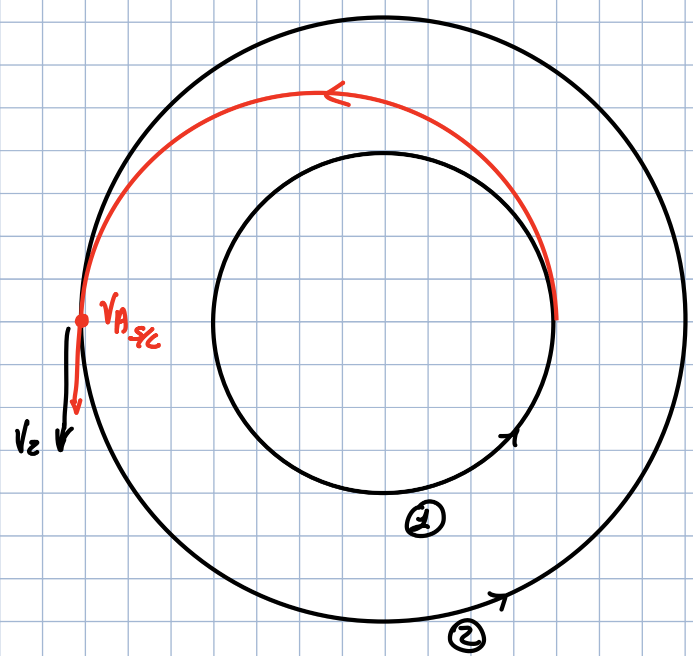
If mission to OUTER to inner $\underline{V}_{s/c} < \underline{V}_1$ → hyperbola asymptote is opposite direction than $\underline{V}_1$.

$\underline{V}_\infty$ // $\underline{V}_1$ but has on opposite direction

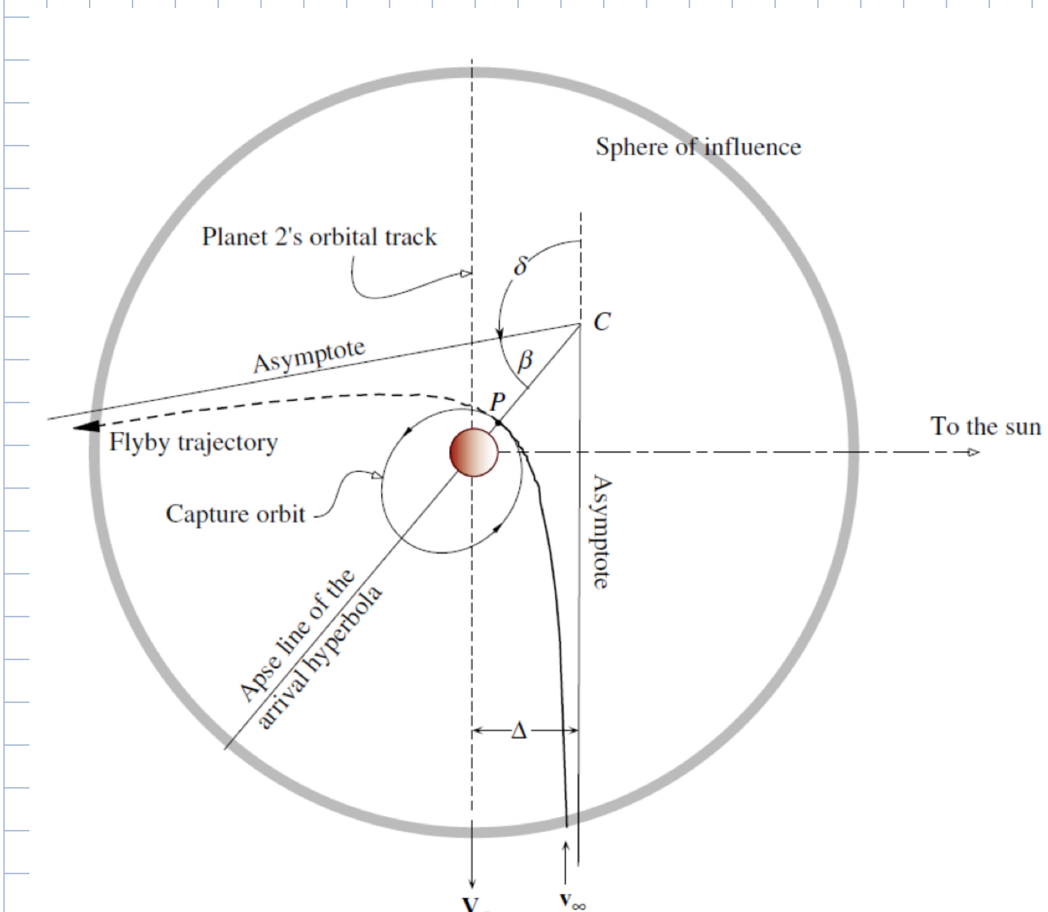
PLANET RENDEZVOUS

S/C arrives at SOI of planet 2 from helio-centric trajectory and enters with $\underline{v}_\infty$.

MISSION FROM INNER PLANET TO OUTER.



$\Delta V_A = V_2 - V_A^{s/c} > 0$



using the relative absolute we will never have problem

$v_A = v_2 + v_T$

(4.26)

$\underline{v_A}_{s/c} = \underline{v_\infty} + \underline{v_2}$

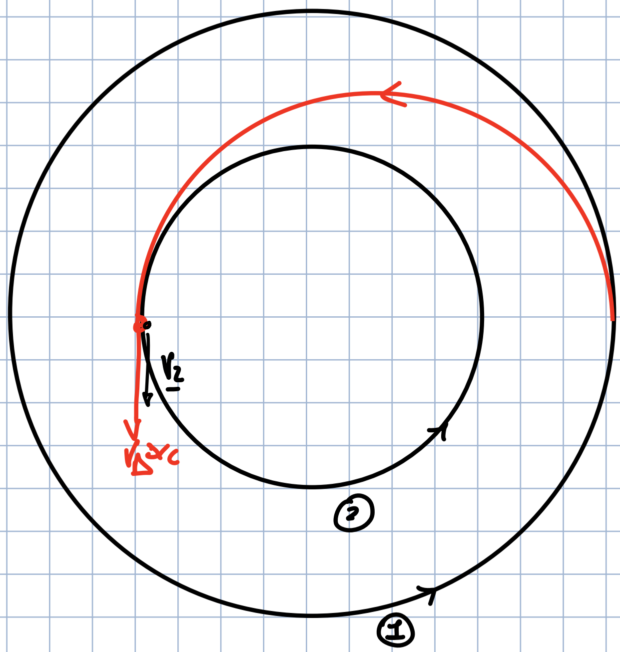
Relative velocity wrt to the planet.

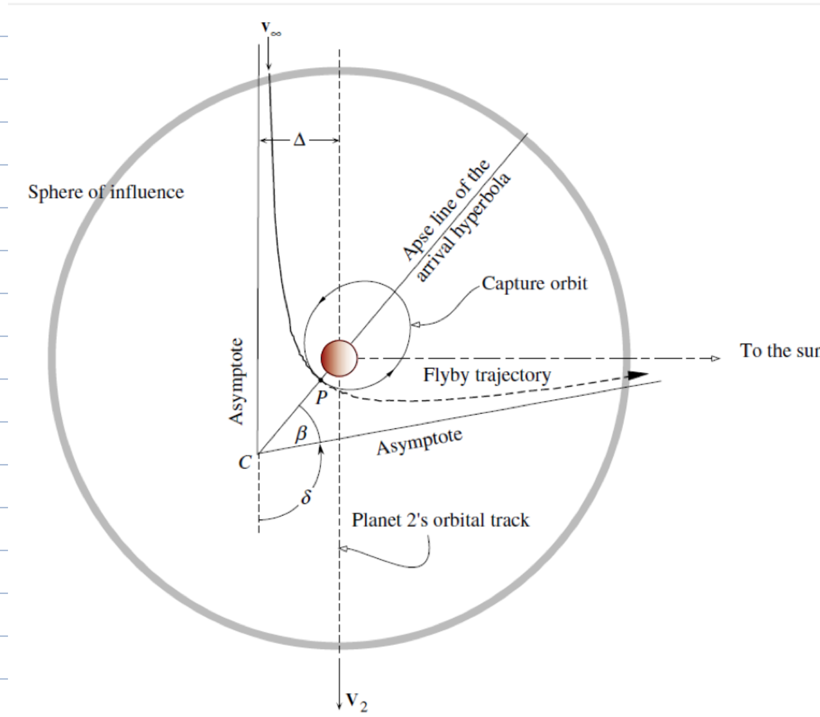
$\underline{V_2} > \underline{V_A}\,s/c$

$a_2 > a_T$

→ planet which is moving faster sees s/c arriving the other way around.

MISSION from OUTHER TO INNER PLANET





$$\Delta V_A^{s/c} = V_2 - V_A^{s/c} < 0$$

$$V_2 < V_A^{s/c}$$

The planet sees you arriving with same direction

$$|\Delta V_A^{s/c}| = v_\infty$$

- After the s/c reaches the sphere of influence of the planet 2 things can happen

1) Rendezvous mission ⇒ s/c stays in orbit ② planet 2
2) Fly-by mission ⇒ the s/c doesn't stop at the perigee of the hyperbola → The s/c continues on it's heliocentric leg after fly by of planet 2.

PLANET fly-by

FLY-BY : S/C enters in the SOI of planet 2 along hyperbolic trajectory (v_∞, e, h) but it doesn't stop at perigee of hyp (to move on a closed orbit) but continues along hyperbola to get again in heliocentric leg [leg number 2] to target another planet.

	PLANET ABSOLUTE HELIO VELOCITY	S/C ABSOLUTE HELIO VELOCITY	S/C VELOCITY REL. TO THE PLANET
ENTRY HYPERBOLA	$\underline{v}_{pl}$	$\underline{v}_{s/c}^-$	$\underline{v}_\infty^-$
EXIT HYPERBOLA	$\underline{v}_{pl}$	$\underline{v}_{s/c}^+$	$\underline{v}_\infty^+$

$$\underline{v}_{s/c}^- = \underline{v}_{pl} + \underline{v}_\infty^- \qquad (4.27)$$
$$\underline{v}_{s/c}^+ = \underline{v}_{pl} + \underline{v}_\infty^+$$

$\downarrow$ wrt SUN $\quad \downarrow$ wrt SUN $\quad \downarrow$ wrt planet

FLY-BY PRODUCES AN HELIOCENTRIC CHANGE IN VELOCITY

$$\Delta \underline{v}_{flyby} = \underbrace{\underline{v}_{s/c}^+}_{\text{exit SOI wrt sun}} - \underbrace{\underline{v}_{s/c}^-}_{\text{enter SOI wrt sun}} = \underline{v}_{pl} + \underline{v}_\infty^+ - (\underline{v}_{pl} + \underline{v}_\infty^-)$$

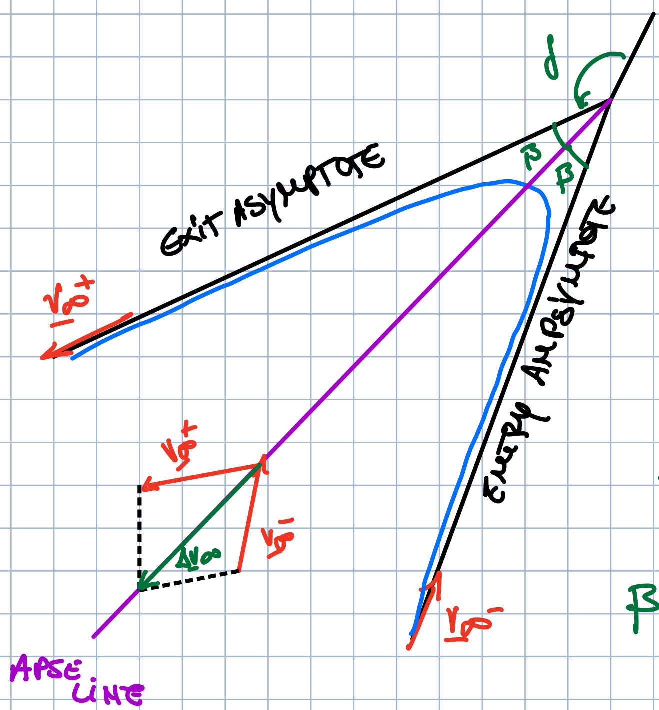
$$\Delta \underline{v}_{FLY\,BY} = \cancel{\underline{v}_{pl}} + \underline{v}_\infty^+ - \cancel{\underline{v}_{pl}} - \underline{v}_\infty^- = \underline{v}_\infty^+ - \underline{v}_\infty^- = \Delta \underline{v}_\infty$$

assumption $\underline{v}_{pl} =$ constant

→ Remark the same in magnitude and direction FLY/BY (hyperbole in the SOI much faster) time planet moves around sun.

$$\Delta \underline{v}_{FLY\text{-}BY} = \underline{v}_\infty^+ - \underline{v}_\infty^- = \Delta \underline{v}_\infty \qquad (4.28)$$

$\underline{v}_\infty^+$, $\underline{v}_\infty^-$ lie on the same hyperbole at SOI boundaries at entry and exit of SOI. Parallel to asymptotes of hyperbole



δ = TURN ANGLE BETWEEN THE ENTRY AND EXIT ASYMPTOTE

$v_\infty^- = v_\infty^+$ have the same magnitude

because $\mathcal{E}_{HYP}$ = CONSTANT (4.29)

$$\Delta \underline{v}_\infty = \underline{v}_\infty^+ - \underline{v}_\infty^- = \Delta \underline{v}_{FLYBY}$$

β = angle between asymptote and hyperbole apse line

$\Delta \underline{v}_{FLYBY}$ because $\underline{v}_\infty$ is rotated $\underline{v}_\infty^+ \neq \underline{v}_\infty^-$ (4.30)

I can play with hyperbola design to define $\Delta \underline{v}_{FLYBY} = \Delta \underline{v}_\infty$ fly-by as a natural $\Delta \underline{v}$ manoeuvre in the heliocentric frame.

Target planet → enter SOI → exit SOI → triangle of velocities

$$\underline{v}_{S/C}^+ = \underline{v}_{S/C}^-$$