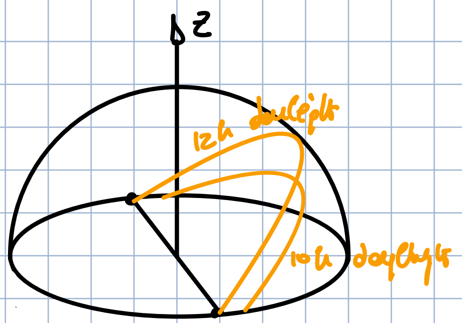
ORBITAL MECHANICS



The sun rise and set at different point than East and West because the Earth is inclined ⇒ The Sun will be visible for more or less than 12 h during a year ⇒ So the sun will follow an arc that connect two point that are exactly 180° apart only if a daylight last for exactly 12 h.

COORDINATE TRANSFORMATIONS

By spherical trigonometry we will see how coordinates of a celestial object can be transformed into a different reference frame.
Another way to solve this problem, as illustrated in Curtis, is to use rotation matrix ⇒ Usefull to write a computer code. But by using rotation matrix make the comprension of what is happening very difficult.

Spherical trigonometry → give a greater insight of what is happening.

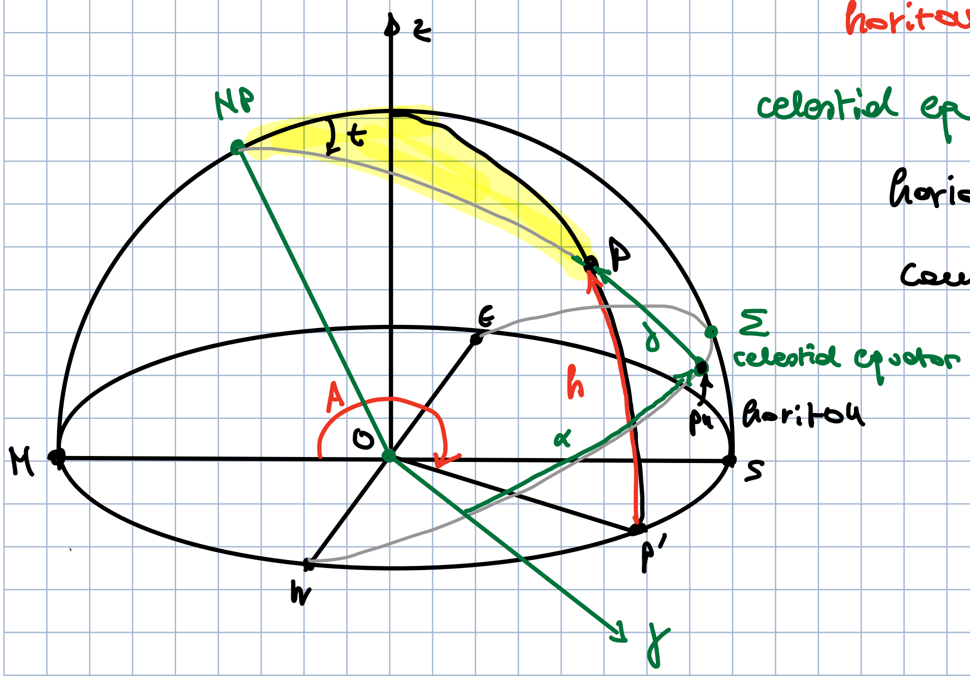
example

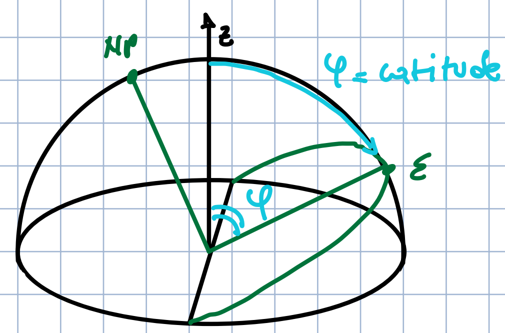
The altitude h and the azimuth A of a star are to be determinated given its right ascension, declination and place of observation.

horizon RF : A, h

celestial equator RF : γ, α, δ

horizon and equator are connected through the latitude.





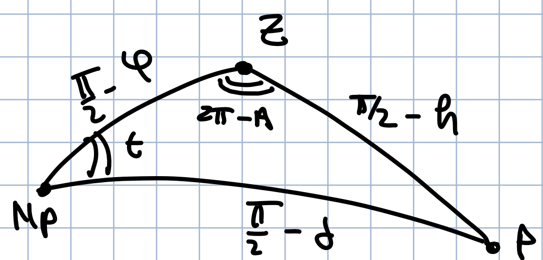
We can identify a particular triangle

LATITUDE : intersection of the azimuth to the arc of a great circle $\widehat{NP\,\Sigma}$

↳ copy here

$\varphi = \widehat{z\Sigma}$

↳ This is wrong.



ASTRONOMICAL TRIANGLE

ARCS

$\widehat{NP\,z}$ $\qquad \left.\begin{aligned} \widehat{NP\,\Sigma} &= \frac{\pi}{2} \\ \widehat{z\,\Sigma} &= \varphi \end{aligned}\right\} \Rightarrow \widehat{NP\,z} = \frac{\pi}{2} - \varphi$

$\widehat{zP}$ $\qquad \left.\begin{aligned} \widehat{z\,P'} &= \frac{\pi}{2} \\ \widehat{P\,P'} &= h \end{aligned}\right\} \Rightarrow \widehat{z\,P} = \frac{\pi}{2} - h$

$\widehat{NP\,P}$ $\qquad \left.\begin{aligned} \widehat{NP\,P''} &= \frac{\pi}{2} \\ \widehat{P\,P''} &= \delta \end{aligned}\right\} \Rightarrow \widehat{NP\,P} = \frac{\pi}{2} - \delta$

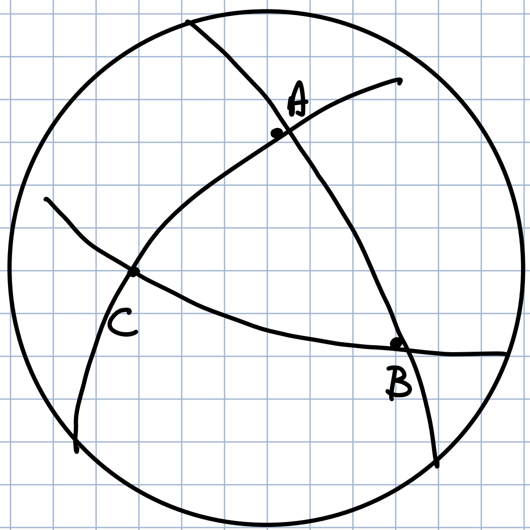
DIEDRAL ANGLE

$Z\widehat{NP}P = t$ local hour angle

$NP\hat{Z}P = 2\pi - A$

$Z\hat{P}NP$ unknown $\Rightarrow$ it doesn't have any particular name.

SPHERICAL GEOMETRY



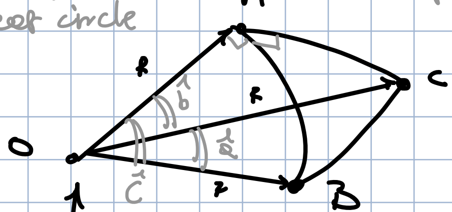
Three point on a sphere.

PLANAR TRIANGLE	SPHERICAL TRIANGLES
- sides	- arcs of a circumpherence with radius R (= radius of a sphere)
- segments: shortest ways to connect two points	- geodetic lines (normal to the geodetic line is $\perp$ to the surface)

$\Downarrow$

meridian = arcs of a great circle are the geodetic lines of a sphere

$\llcorner$ perpendicular to the radii only if they are meridian or arcs of a great circle



center of the sphere

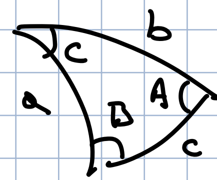
We are considering only the angular distance $\Rightarrow$ We are not interested to the radius R=1 eg.

NOTE

in a spherical triangle the sum of the diedral angle is NOT π (but is larger than π and it can vary)

Also in the spherical triangle there are theorems (cosine, sine tanget etc)

diedral angle



COSINE LAW

$$\boxed{\begin{aligned} \cos b &= \cos c \cos a + \sin c \sin a \cos \hat{B} \\ \cos a &= \cos b \cos c + \sin b \sin c \cos \hat{A} \\ \cos c &= \cos a \cos b + \sin a \sin b \cos \hat{C} \end{aligned}} \qquad (2.59)$$

SINE LAW

$$\boxed{\frac{\sin b}{\sin \hat{B}} = \frac{\sin c}{\sin \hat{C}} = \frac{\sin a}{\sin \hat{A}}} \qquad (2.60)$$

COTANGENT LAW

$$\boxed{\begin{aligned} \cot g\, c \sin a &= \cos a \cos \hat{B} + \sin \hat{B} \cot g\, \hat{C} \\ \cot g\, c \sin b &= \cos b \cos \hat{A} + \sin \hat{A} \cot g\, \hat{C} \\ \cot g\, a \sin c &= \cos c \cos \hat{B} + \sin \hat{B} \cot g\, \hat{A} \\ \cot g\, a \sin b &= \cos b \cos \hat{C} + \sin \hat{C} \cot g\, \hat{A} \\ \cot g\, b \cdot \sin c &= \cos c \cos \hat{A} + \sin \hat{A} \cot g\, \hat{B} \\ \cot g\, b \sin a &= \cos a \cos \hat{C} + \sin \hat{C} \cot g\, \hat{B} \end{aligned}} \qquad (2.61)$$

CONTINUING WITH THE EXAMPLE

COSINE LAW $\longrightarrow$

$$\cos\left(\frac{\pi}{2}-h\right)=\cos\left(\frac{\pi}{2}-\varphi\right)\cos\left(\frac{\pi}{2}-\delta\right)+\sin\left(\frac{\pi}{2}-\varphi\right)\sin\left(\frac{\pi}{2}-\delta\right)\cos t$$

SINE LAW $\longrightarrow$ $$\frac{\sin(2\pi-A)}{\sin(2\pi-\delta)}=\frac{\sin t}{\sin(2\pi-h)}$$

unknowns are h, A given α, δ, φ, time of observation

solution LATER time $\Rightarrow t$

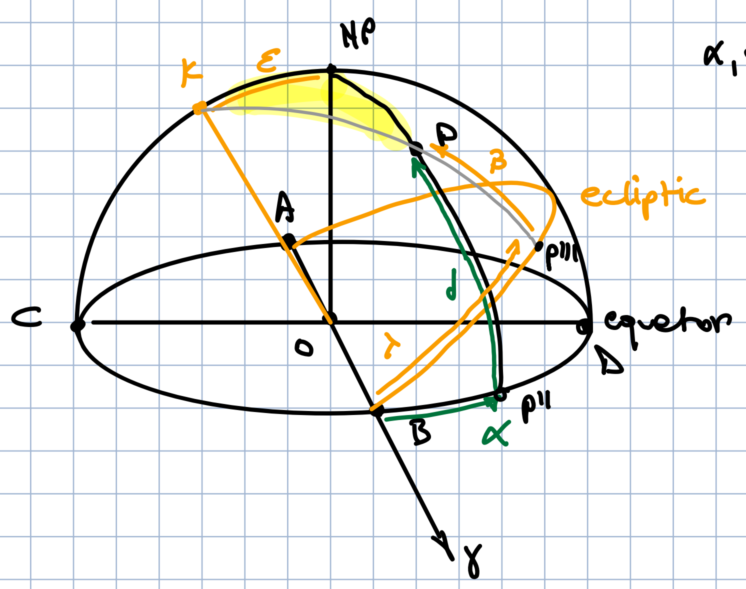
$$\sin h=\sin\varphi\sin\delta+\cos\varphi\cos\delta\cos t \longrightarrow h$$

$$\frac{-\sin A}{\cos\delta}=\frac{\sin t}{\cos h} \longrightarrow \sin A=\frac{-\sin t}{\cos h}\cos\delta$$

example

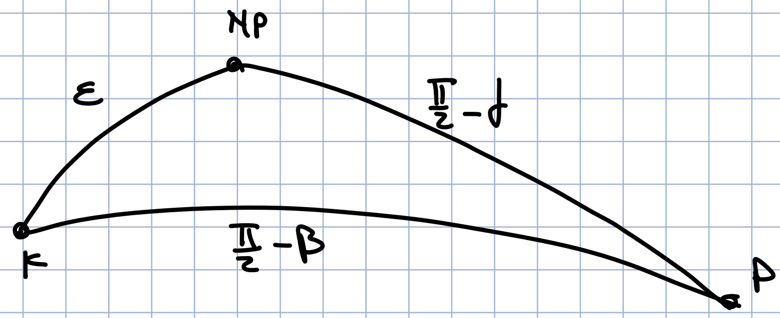
Knowing α and δ of a celestial object find its celestial longitude and latitude (λ, β)

$\varepsilon = \widehat{KNP}$



α, δ of planets/stars/moons

tabular data ephemerides



ARCS

$\widehat{KNP} = \varepsilon$

$\widehat{NP\,P}$

$$\left.\begin{aligned} \widehat{NP\,P''} &= \frac{\pi}{2} \\ \widehat{P\,P''} &= \delta \end{aligned}\right\} \quad \widehat{NP\,P} = \frac{\pi}{2} - \delta$$

$\widehat{KP}$

$$\left.\begin{aligned} \widehat{K\,P'''} &= \frac{\pi}{2} \\ \widehat{P\,P'''} &= \beta \end{aligned}\right\} \quad \widehat{KP} = \frac{\pi}{2} - \beta$$

DIEDRAL ANGLES

$NP\,\hat{K}\,P = \frac{\pi}{2} - \lambda$

$K\,\hat{NP}\,P = \frac{\pi}{2} + \alpha$

$K\,\hat{P}\,NP =$ unknown

COSINE LAW

$$\cos\left(\frac{\pi}{2} - \beta\right) = \cos\left(\frac{\pi}{2} - \delta\right)\cos\varepsilon + \sin\left(\frac{\pi}{2} - \delta\right)\sin\varepsilon\cos\left(\frac{\pi}{2} + \alpha\right)$$

SINE LAW

$$\frac{\sin\left(\frac{\pi}{2} + \alpha\right)}{\sin\left(\frac{\pi}{2} - \beta\right)} = \frac{\sin\left(\frac{\pi}{2} - \lambda\right)}{\sin\left(\frac{\pi}{2} - \delta\right)}$$

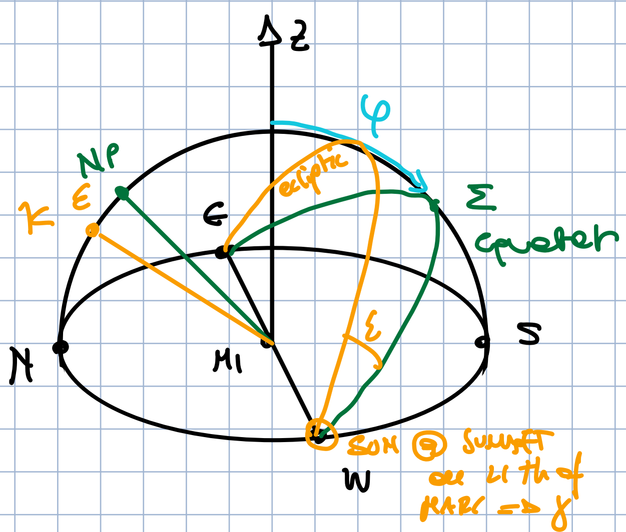
Solution

$$\sin\beta = \cos\varepsilon\sin\delta - \cos\delta\sin\varepsilon\sin\alpha \longrightarrow \beta$$

$$\frac{\cos\alpha}{\cos\beta} = \frac{\cos\lambda}{\cos\delta} \longrightarrow \cos\lambda = \frac{\cos\alpha}{\cos\beta}\cos\delta$$

example

Situation in Milan on 21st March at Sunset



On 21st March the sun is on the Equator but Sun ∈ ecliptic (always)

As the sun is setting => it is at W point of the local horizon.

We see the ecliptic, the equator, horizon the observer can be obtain only in a specific time instant because ? rotates around NP.

MEASUREMENT OF TIME

SIDEREAL TIME

it is measured by rotation of Earth wrt fixed stars

sidereal day

- time it takes for a distant star to return to its same position above same meridian
- time between two successive passages of the vernal equinox across a given observer meridian

SIDEREAL TIME = T = hour angle of the vernal equinox.

APPARENT SOLAR TIME

apparent solar day

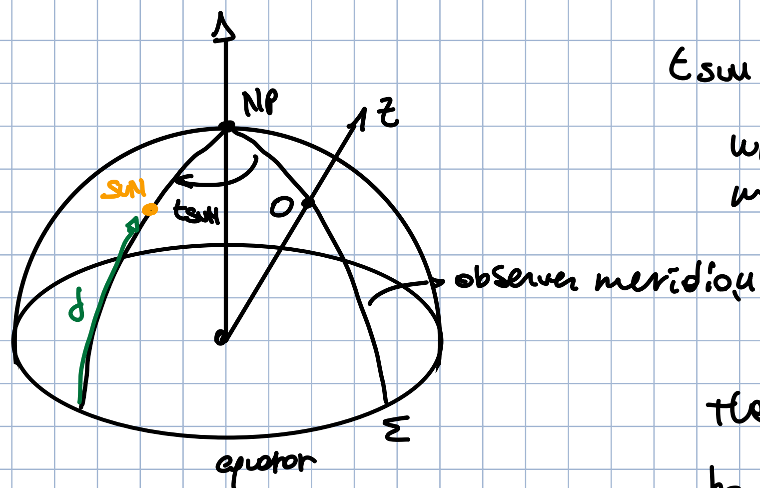
- interval of time between 2 successive transit of the sun over the observer meridian (from the lower meridian = starts at midnight)

apparent solar time $= t + \pi$ (2.62)
(apparent local time)
LAT

↳ Because it is not measured from midday but it is measured from midnight ⇒ so i need to sum pi.

LOCAL = it varies with the observer, changing with Z point

L.A.T changes from observer to observer



t_{sun} = local hour angle of the sun

when the sun is above the observer meridian it is midday

the sun at midday does not need to be at zenith, depends on latitude.

At midnight the sun is at lower meridian (at the dir side)

$$t_{sun}(\text{midnight}) = \pi$$

$$t_{sun}(\text{midday}) = \emptyset$$

Those time are measured in radiants or in degrees but it is possible to use hours.

$LAT = t + 12h$ [h]

$LAT = t + \pi$ [rad]

$LAT = t + 180°$ [deg]

to get h ⇒ $\left[\frac{1}{15}\right] h/°$

LAT is not constant because the sun "moves" around the Earth of 1° per day approx (360° in 365.25 days)

- elliptical Earth's orbit around sun
- Sun on ecliptic not equatorial (obliquity)

MEAN SOLAR TIME

mean solar angle: angle between the observer meridian and "medium" Sun (angular mean velocity)

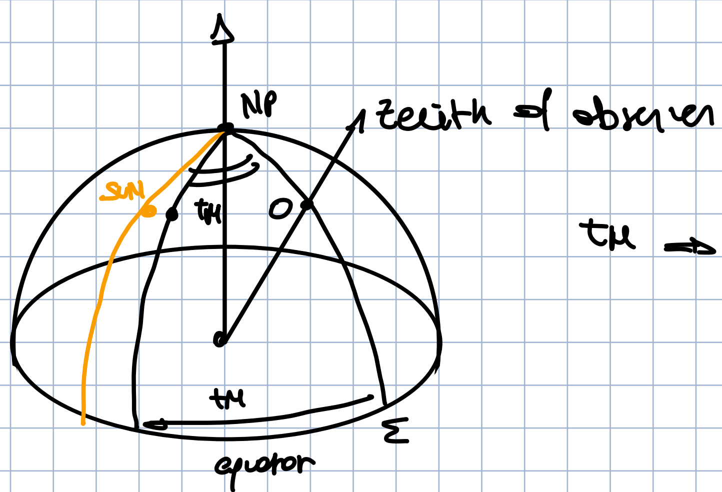
mean solar time: hour angle of the mean sun + π

$\left[\times \frac{1}{15}\ h/° \text{ to get } h\right]$

$$\text{MEAN SOLAR TIME} \triangleq \text{LMT} \triangleq t_M + 12h \qquad (2.63)$$

CIVIL TIME
LOCAL TIME

$$\text{LMT}(°) = t_M + 180°$$



tM ⇒ considering the mean motion of the Sun ⇒ we are not considering apogee and perigee and obliquity of Earth's spin axis.