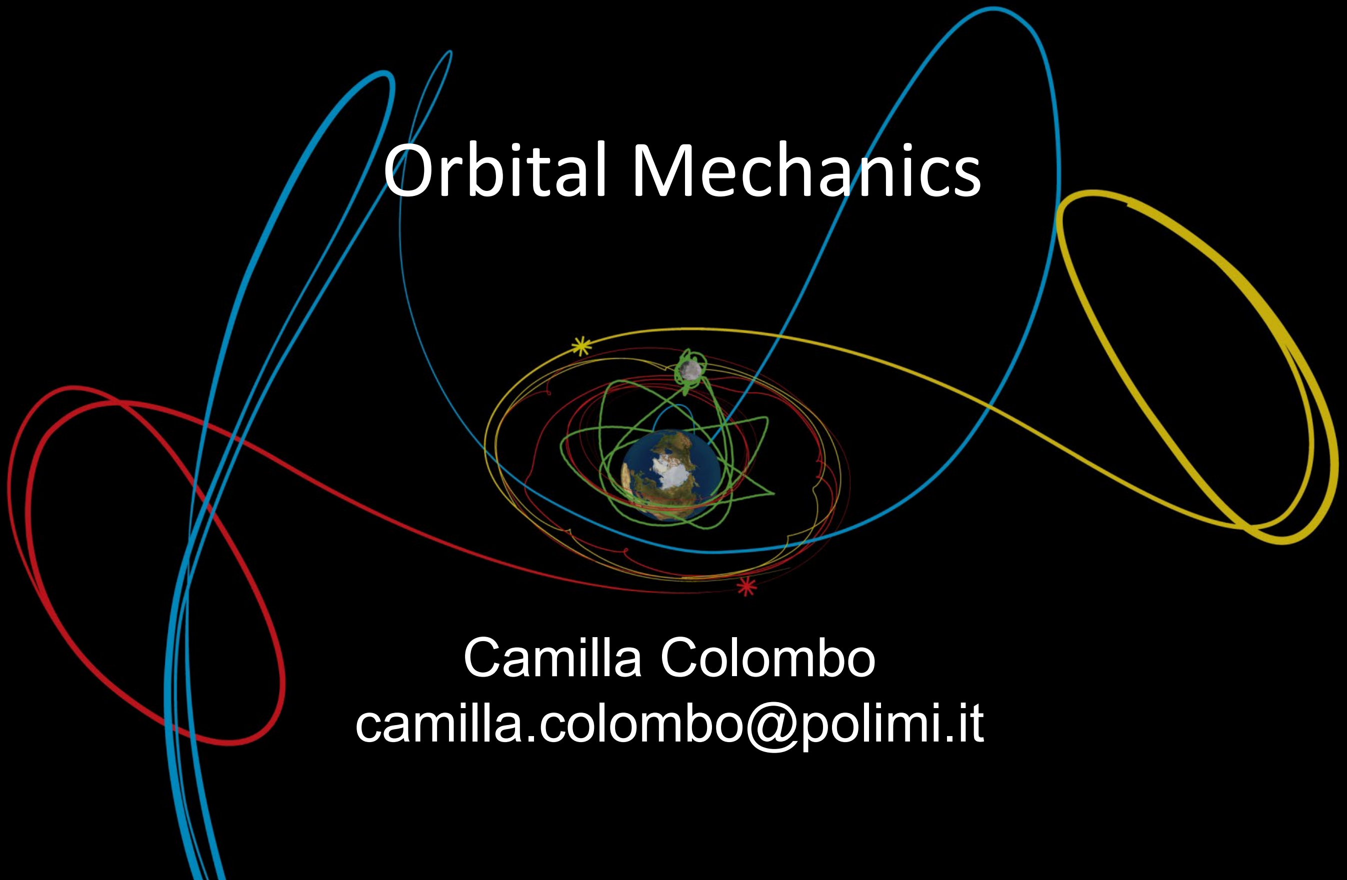
Orbital Mechanics



Camilla Colombo
camilla.colombo@polimi.it

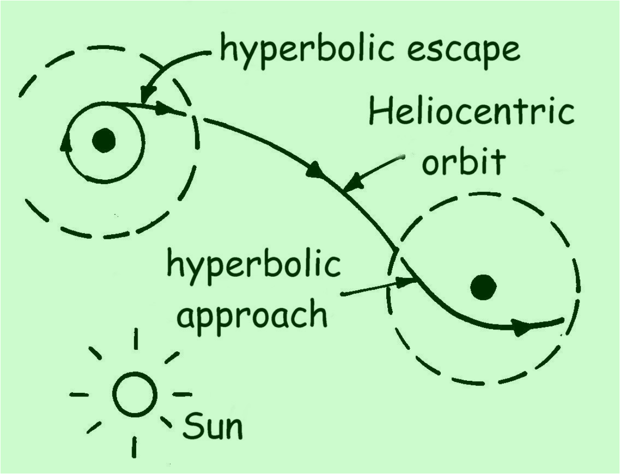
4. INTERPLANETARY TRAJECTORIES

Interplanetary trajectories

Patched conics method

Three phases (generally):

1. Escape from Earth orbit
2. Transfer in solar gravity field (e.g., Hohmann transfer)
3. Capture/flyby at target planet



In each phase consider only **one** gravity field

$\Downarrow$

Use three conic sections 'patched' at boundaries to define overall mission

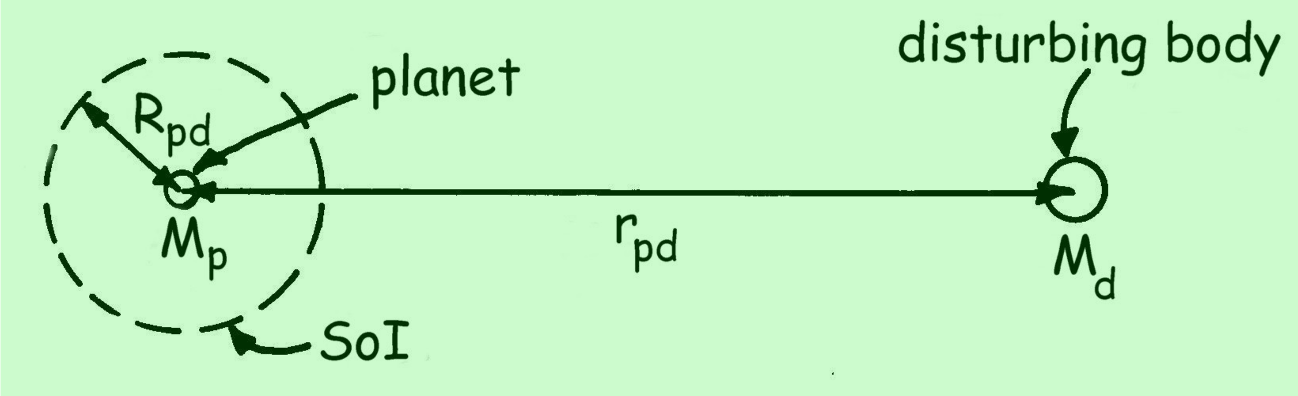
Notes:

- Energy considerations tell us that a S/C approaching a planet with finite speed will arrive and depart on a hyperbolic trajectory.
- Patched conic method is approximate $\Rightarrow$ useful for preliminary mission planning (Δv, transfer times computation)

How do we define appropriate ‘boundaries’? $\Rightarrow$ Spheres of influence

From Cornelisse et al, Section 15.3, p.355

$$\boxed{R_{pd} = r_{pd}\left(\frac{M_p}{M_d}\right)^{\frac{2}{5}}}$$



For p = Earth, d = Sun: $\boxed{R_{ES} = 0.92 \times 10^6 \text{ km}}$

(Mean Earth-Moon distance 0.38×10^6 km)

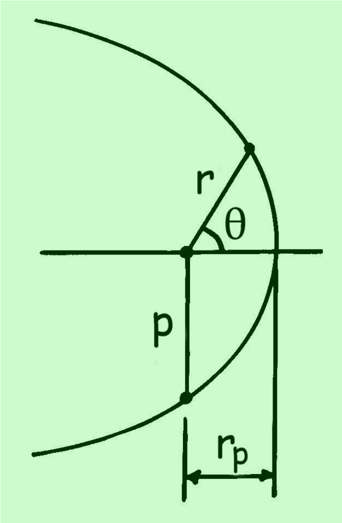
For p = Jupiter, d = Sun: $\boxed{R_{JS} = 48.20 \times 10^6 \text{ km}}$

Parabola. (e = 1)

Definition: when θ = 90° (or 270°) (for a general conic):

Semi-latus rectum

$$r = \frac{a\left(1-e^2\right)}{1+e\cos 90^\circ} \quad \Rightarrow \quad \boxed{r = a\left(1-e^2\right) \equiv p}$$



For a parabola, $a = p/(1-e^2) \rightarrow \infty$

Also $r_p = \frac{p}{\left(1+\cos 0^\circ\right)} = \frac{p}{2} \quad \Rightarrow \quad \boxed{r = \frac{2r_p}{\left(1+\cos\theta\right)}}$

Note that $r \rightarrow \infty$ as $\theta \rightarrow 180^\circ$ an open "orbit"

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Orbit energy is: $\varepsilon = -\dfrac{\mu}{2a} = 0$

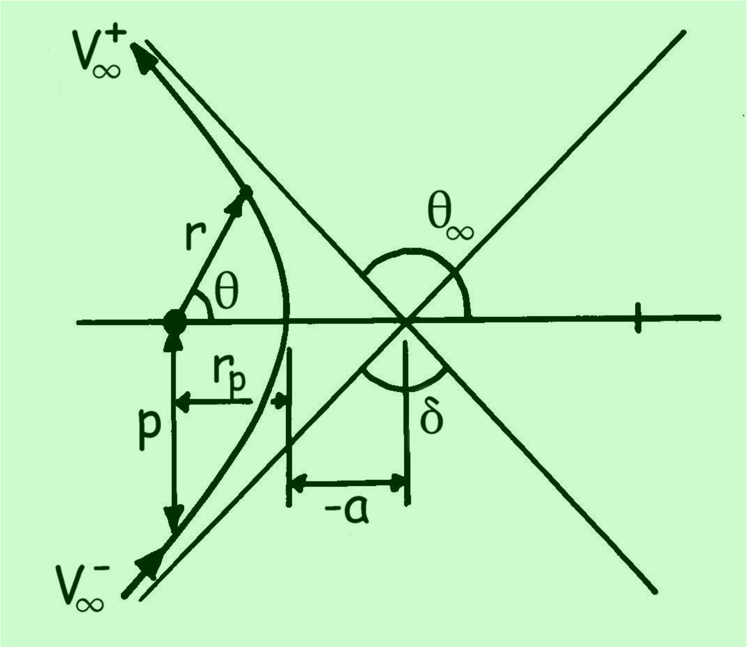
so that along trajectory $\dfrac{1}{2}V^2 - \dfrac{\mu}{r} = 0$

$\Longrightarrow$ as $r \rightarrow \infty$, $V_\infty = 0$

$\Longrightarrow$ Minimum energy escape trajectory $\boxed{V_{esc,min} = \sqrt{\dfrac{2\mu}{r}}}$

Hyperbola. ($e > 1$)

- $$a = p/(1-e^2) < 0$$



- $$r_p = \frac{p}{\left(1 + e\cos 0^\circ\right)} = \frac{p}{1+e}$$ Hence $$r = \frac{r_p\left(1+e\right)}{1+e\cos\theta}$$

- Note that $r \to \infty$ as $1 + e\cos\theta \to 0$ ...

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... $\Rightarrow$ an "open trajectory"

Therefore direction at great distance is given by

$$\boxed{\theta_\infty = \cos^{-1}\left(-\frac{1}{e}\right)}$$

- Energy is positive, so excess velocity is positive

$$\varepsilon = -\frac{\mu}{2a} > 0 \quad \Rightarrow \quad V_\infty > 0$$

- Velocity at great distance:

  Energy equation –

$$\frac{1}{2}V^2 - \frac{\mu}{r} = -\frac{\mu}{2a} \quad \Rightarrow \quad \text{As } r \to \infty,\ V_\infty^2 = -\frac{\mu}{a}$$

$$\Rightarrow \quad \boxed{V_\infty = \sqrt{\frac{-\mu}{a}}\ , \quad a < 0}$$

- Eccentricity:

  At periapsis, $\theta = 0^o$

$$\Rightarrow \quad r_p = \frac{\left(-\frac{\mu}{V_\infty^2}\right)\left(1 - e^2\right)}{\left(1 + e\right)}$$

$$\Rightarrow \quad \boxed{e = 1 + \frac{r_p V_\infty^2}{\mu}}$$

- Deflection angle δ:

  From diagram,

$$\theta_\infty = 90^\text{o} + \frac{\delta}{2} \Rightarrow \cos\theta_\infty = \cos\left(90^\text{o} + \frac{\delta}{2}\right) = -\frac{1}{e}$$

$$\therefore \sin\left(\frac{\delta}{2}\right) = \frac{1}{e} \Rightarrow \boxed{\delta = 2\sin^{-1}\left(\frac{1}{e}\right)}$$

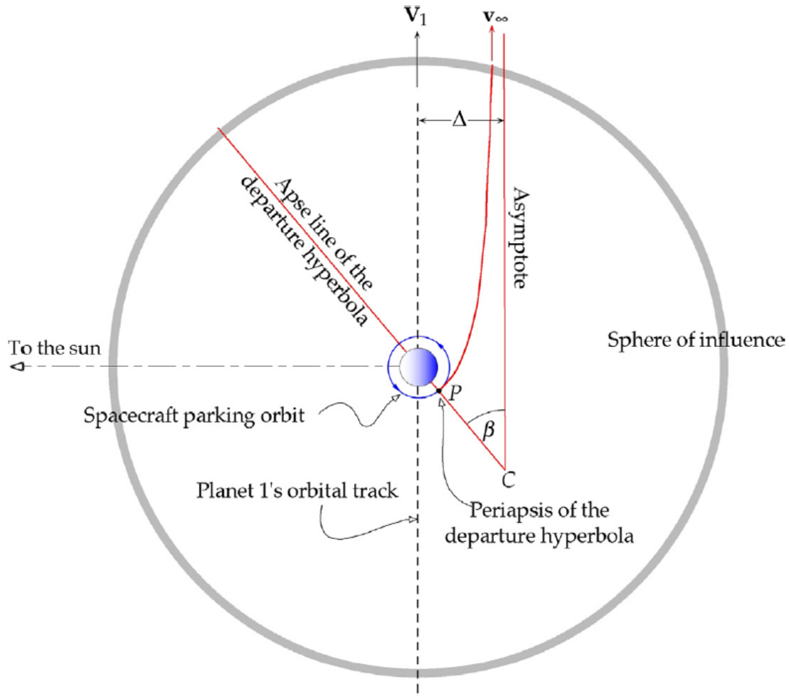


FIGURE 8.8

Departure of a spacecraft on a mission from an inner planet to an outer planet.

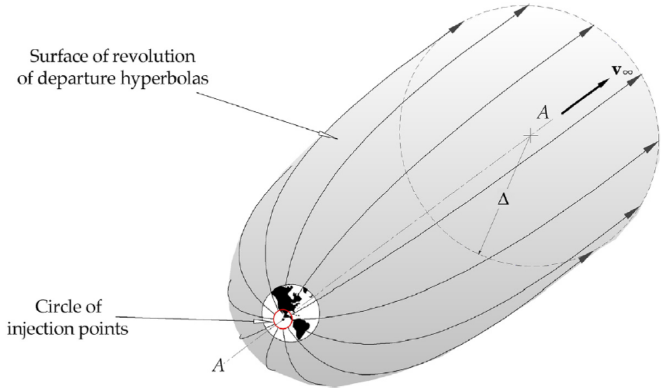


FIGURE 8.9

Locus of possible departure trajectories for a given v_∞ and r_p.

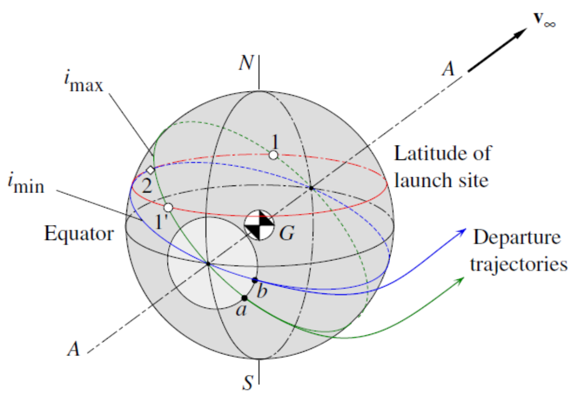


FIGURE 8.10

Parking orbits and departure trajectories for a launch site at a given latitude.

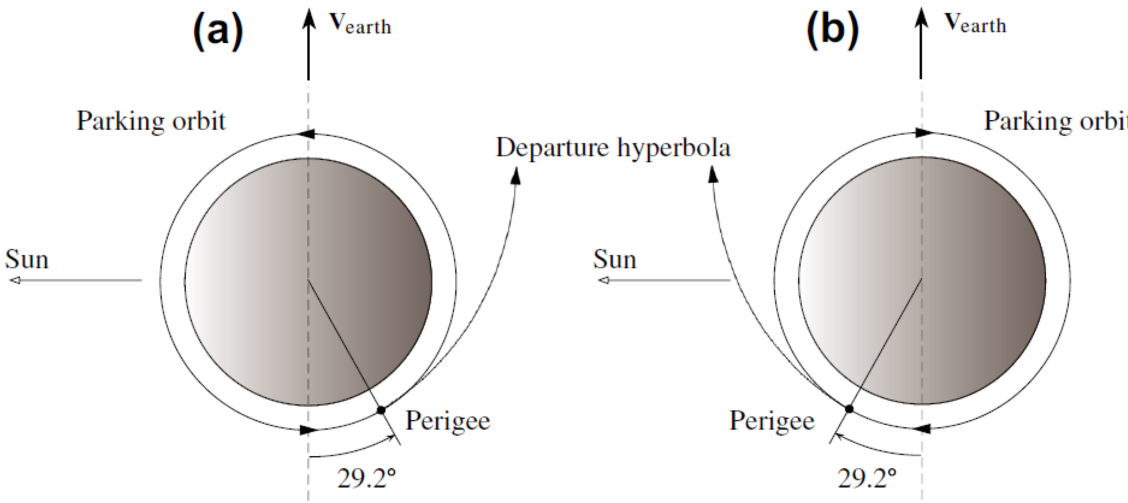


FIGURE 8.12

Departure trajectory to mars initiated from (a) the dark side and (b) the sunlit side of the earth.

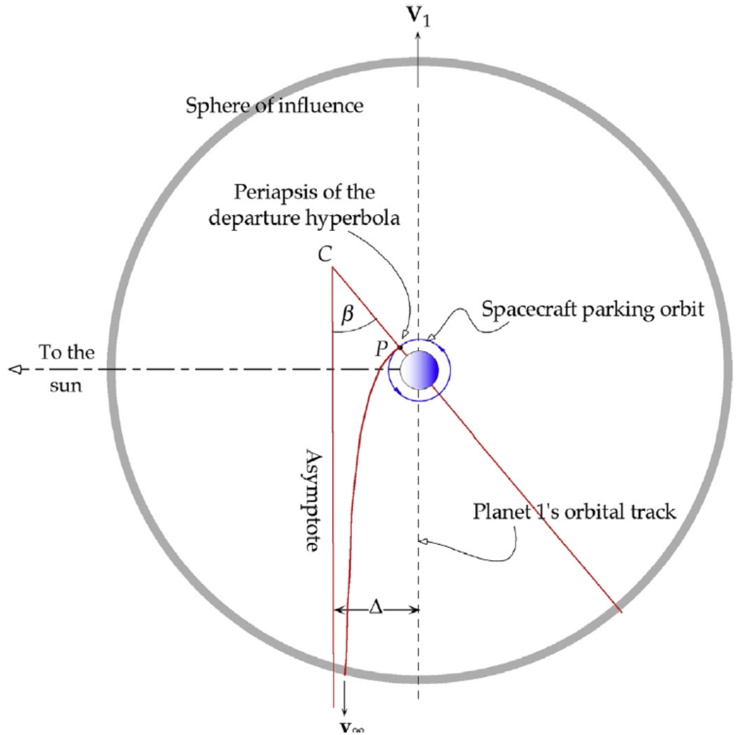
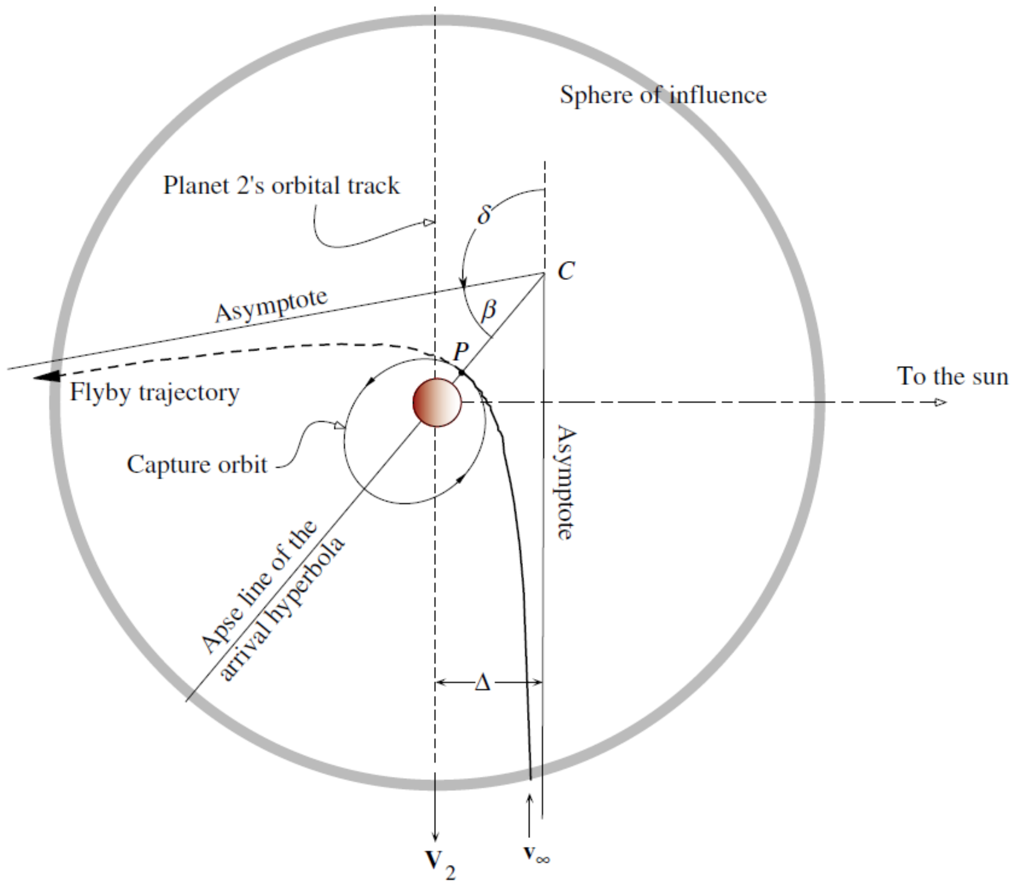


FIGURE 8.11

Departure of a spacecraft on a trajectory from an outer planet to an inner planet.



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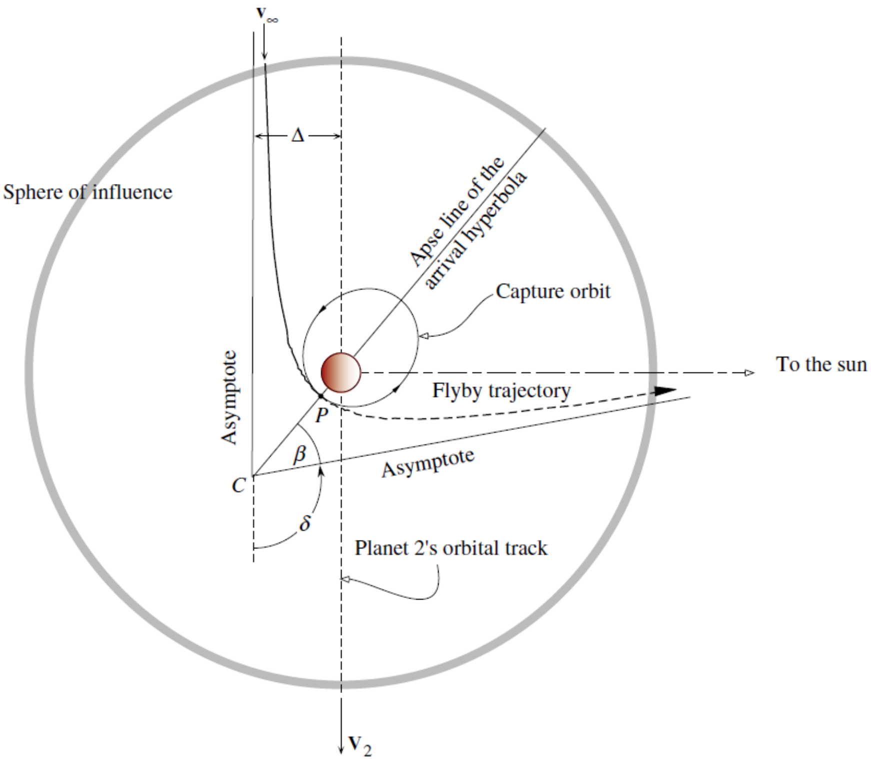


$\mathbf{v}_\infty$
Δ
Sphere of influence
Apse line of the arrival hyperbola
Capture orbit
To the sun
Flyby trajectory
Asymptote
P
β
C
Asymptote
δ
Planet 2's orbital track
$\mathbf{V}_2$

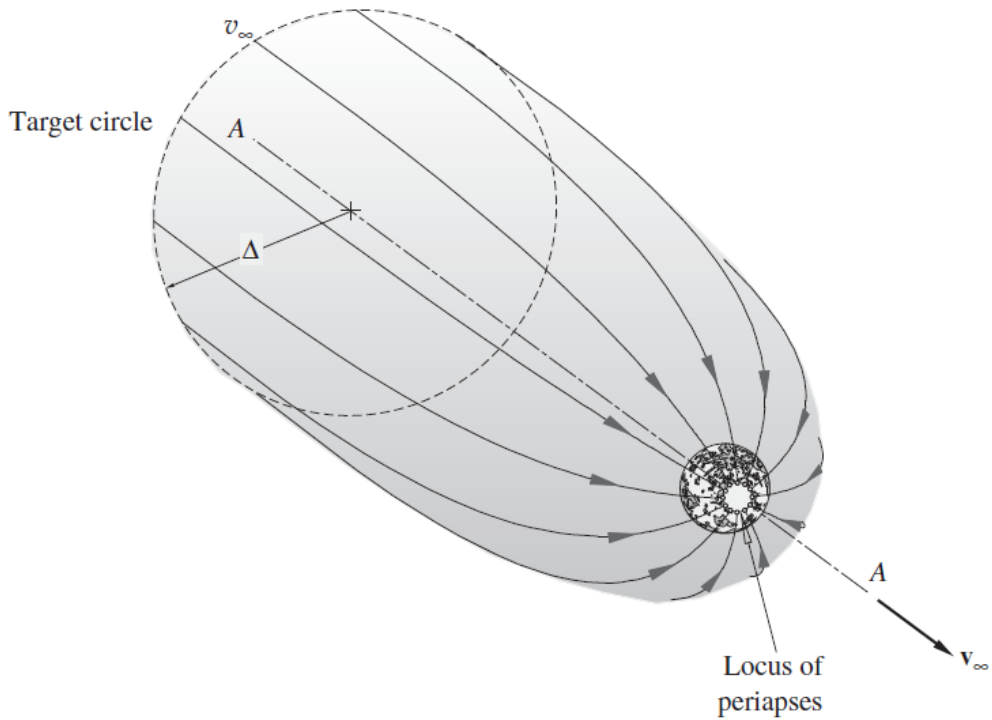


FIGURE 8.15

Locus of approach hyperbolas to the target planet.

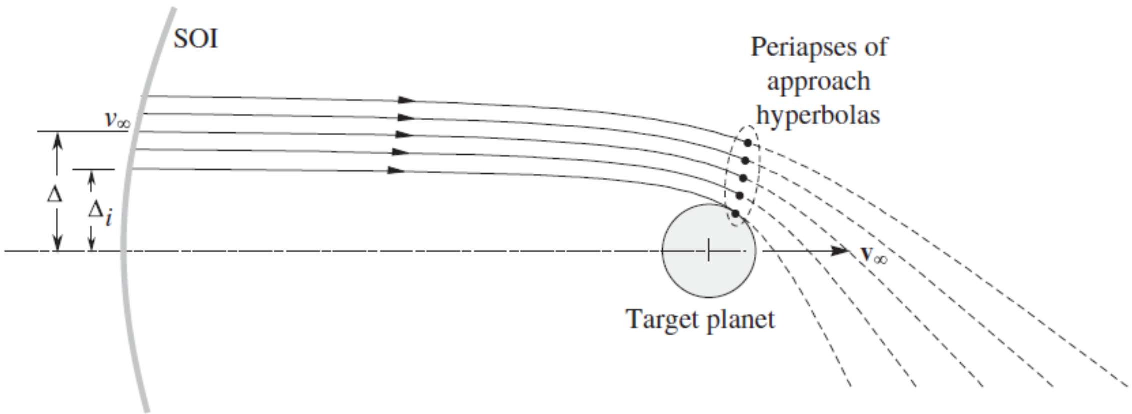


FIGURE 8.16

Family of approach hyperbolas having the same v_∞ but different Δ.

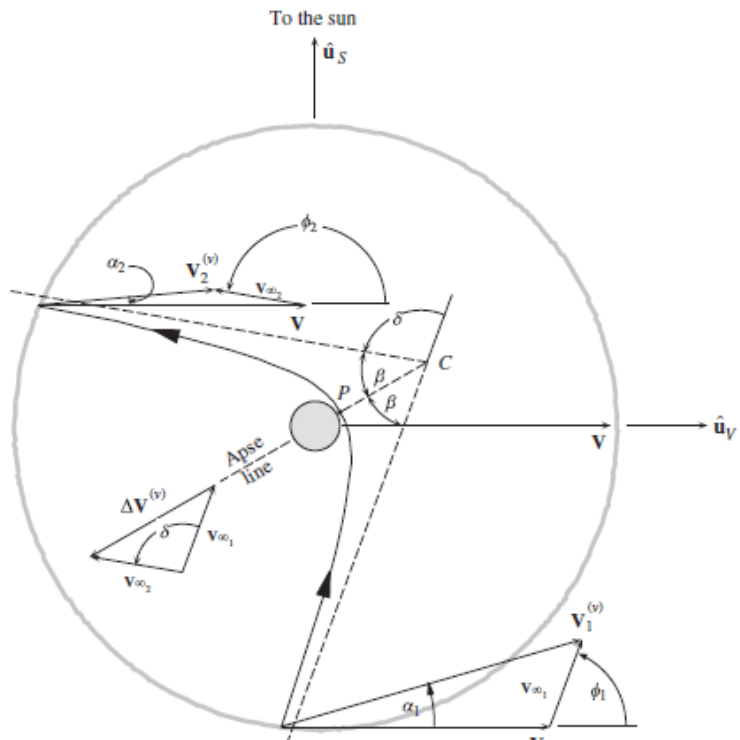


FIGURE 8.18

Leading-side planetary flyby.

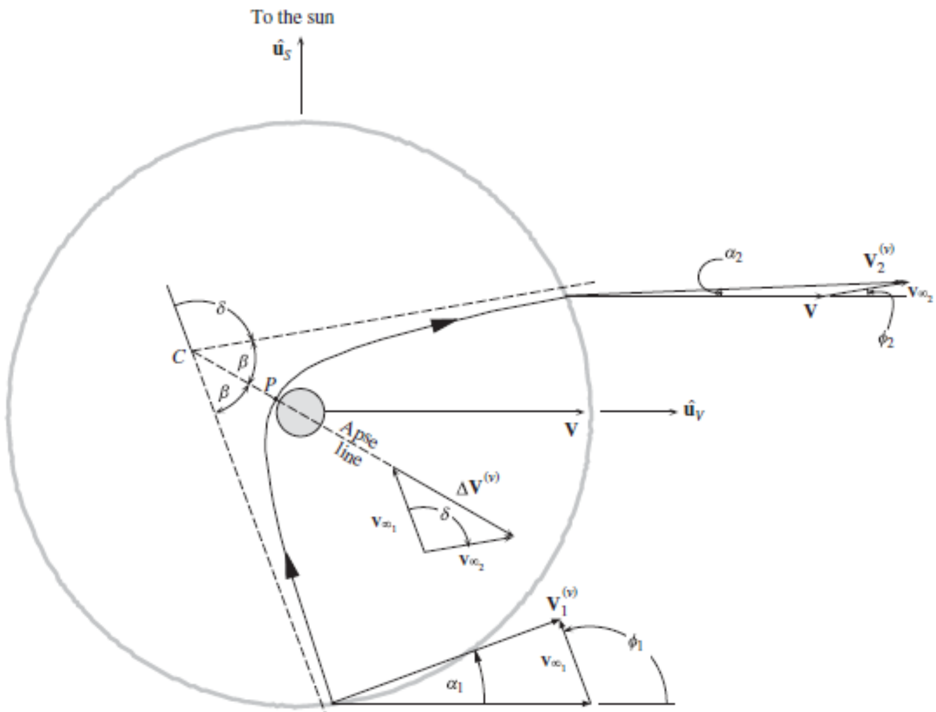


FIGURE 8.19

Trailing-side planetary flyby.

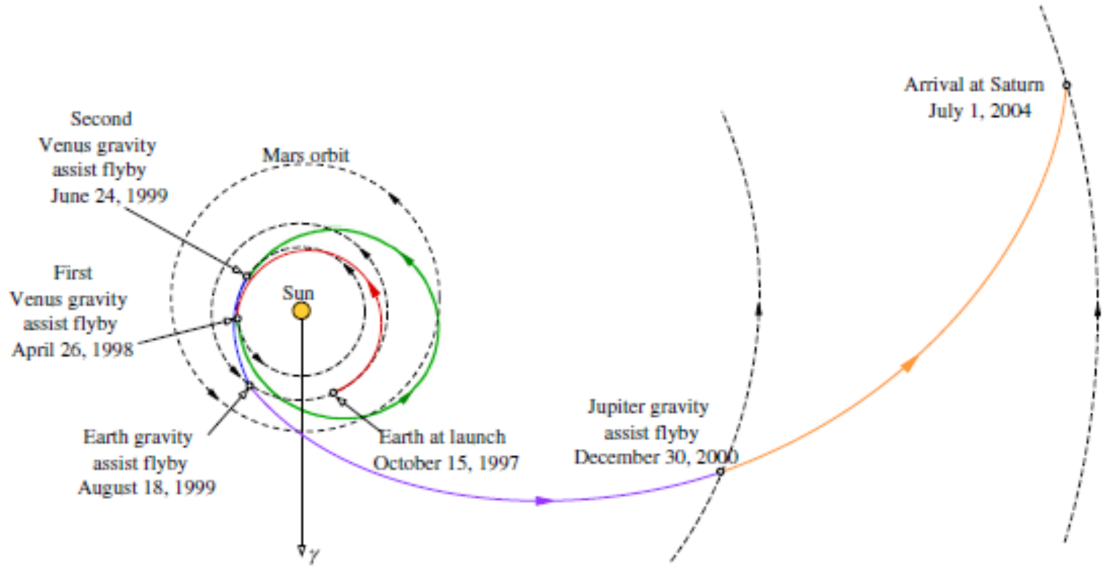


FIGURE 8.24

Cassini's 7-year mission to saturn.

ROLE	Mercury observation & exploration
LAUNCH DATE	27 Jan 2017 (planned) | Arrival Jan 2024
LAUNCHER/LOCATION	Ariane 5/Kourou
LAUNCH MASS	4100 kg
ORBIT	Cruise: Heliocentric transfer orbit At Mercury: MPO: polar orbit, 400×1508 km, 2.3-hr period MMO: polar orbit, 400×11 824 km, 9.3-hr period
NOMINAL MISSION	1 year
+ ESA's first mission to Mercury will provide clues on how planets form and interact with the Sun +	

Key mission dates (for a 2017 launch)

Date	Mission event
2017	Launch
16 July 2018	Earth flyby
22 September 2019	First Venus flyby
4 May 2020	Second Venus flyby
23 July 2020	First Mercury flyby
14 April 2021	Second Mercury flyby
6 July 2022	Third Mercury flyby
29 December 2022	Fourth Mercury flyby
4 February 2023	Fifth Mercury flyby
1 January 2024	Arrival at Mercury
1 April 2025	End of nominal mission
1 April 2026	End of extended mission

Table 8.1 Planetary Orbital Elements and Their Centennial Rates

	a(AU) $\dot{a}$(AU/Cy)	e $\dot{e}$(1/Cy)	i(°) $\dot{i}$(°/Cy)	Ω(°) $\dot{\Omega}$(°/Cy)	ϖ(°) $\dot{\varpi}$(°/Cy)	L(°) $\dot{L}$(°/Cy)
Mercury	0.38709927	0.20563593	7.00497902	48.33076593	77.45779628	252.25032350
	0.00000037	0.00001906	−0.00594749	−0.12534081	0.16047689	149,472.67411175
Venus	0.72333566	0.00677672	3.39467605	76.67984255	131.60246718	181.97909950
	0.00000390	−0.00004107	−0.00078890	−0.27769418	0.00268329	58,517.81538729
Earth	1.00000261	0.01671123	−0.00001531	0.0	102.93768193	100.46457166
	0.00000562	−0.00004392	−0.01294668	0.0	0.32327364	35,999.37244981
Mars	1.52371034	0.09339410	1.84969142	49.55953891	23.94362959	−4.55343205
	0.0001847	0.00007882	−0.00813131	−0.29257343	0.44441088	19,140.30268499
Jupiter	5.20288700	0.04838624	1.30439695	100.47390909	14.72847983	34.39644501
	−0.00011607	0.00013253	−0.00183714	0.20469106	0.21252668	3034.74612775
Saturn	9.53667594	0.05386179	2.48599187	113.66242448	92.59887831	49.95424423
	−0.00125060	−0.00050991	0.00193609	−0.28867794	−0.41897216	1222.49362201
Uranus	19.18916464	0.04725744	0.77263783	74.01692503	170.95427630	313.23810451
	−0.00196176	−0.00004397	−0.00242939	0.04240589	0.40805281	428.48202785
Neptune	30.06992276	0.00859048	1.77004347	131.78422574	44.96476227	−55.12002969
	0.00026291	0.00005105	0.00035372	−0.00508664	−0.32241464	218.45945325
(Pluto)	39.48211675	0.24882730	17.14001206	110.30393684	224.06891629	238.92903833
	−0.00031596	0.00005170	0.00004818	−0.01183482	−0.04062942	145.20780515

Source: From Standish et al. (2013). Used with permission.

Example: Venus orbiter

Estimate the ΔV required for an Earth-Venus transfer. Assume:

- Coplanar circular orbits for Earth/Venus
- Hohmann transfer
- Initial Earth parking orbit: 200 km, circular
- Final Venusian orbit: 200 km, circular, arbitrary inclination

<u>Data</u>

Earth:

- Gravity constant, $\mu_E = 3.986 \times 10^5$ km^3/s^2
- Mean distance from the Sun $r_E = 1.496 \times 10^8$ km

Venus:

- Gravity constant, $\mu_V = 3.249 \times 10^5$ km^3/s^2
- Mean distance from the Sun $r_V = 1.081 \times 10^8$ km

Sun: gravity constant $\mu_S = 1.327 \times 10^{11}$ km^3/s^2

Earth departure

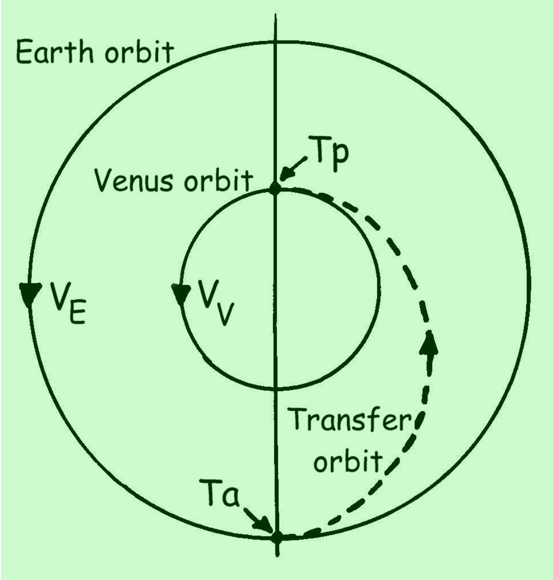
A.Hohmann Transfer (Sun's sphere of influence)

$$\frac{1}{2}V_{Ta}^2 = \frac{\mu_S}{r_E} - \frac{\mu_S}{2a_T} \qquad \text{where} \qquad a_T = \frac{1}{2}(r_E + r_V)$$

$$V_{Ta} = 27.28 \text{ km/s}$$

$$V_E = \sqrt{\frac{\mu_S}{r_E}} = 29.78 \text{ km/s}$$

Both relative to the Sun



Therefore relative to the Earth:

$$V_{dep} = V_E - V_{Ta} = 2.50 \text{ km/s}$$

(Need to slow down by this much)

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B. Earth escape (Earth's sphere of influence)

On the hyperbola $\varepsilon_B = \varepsilon_A \Rightarrow \frac{1}{2}V_\infty^2 = \frac{1}{2}V_p^2 - \frac{\mu_E}{r_p}$

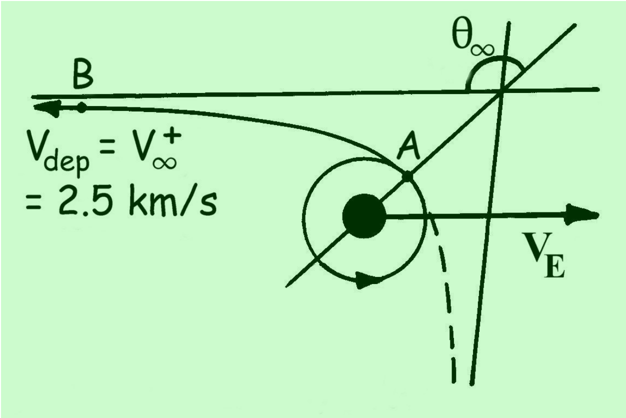
where $r_p = R_E + 200$ km = 6578 km

$$\therefore \; V_p^2 = 2\left(\frac{1}{2}V_\infty^2 + \frac{\mu_E}{r_p}\right) \Rightarrow V_p = 11.29 \text{ km/s}$$

$$V_{circ} = \sqrt{\frac{\mu_E}{r_p}} = 7.78 \text{ km/s} \Rightarrow \therefore \; \Delta V_{esc} = V_p - V_{circ} = 3.51 \text{ km/s}$$

Also, from Eq. $e = 1.1031$

from Eq. $\theta_\infty = 155.03°$ (for timing)



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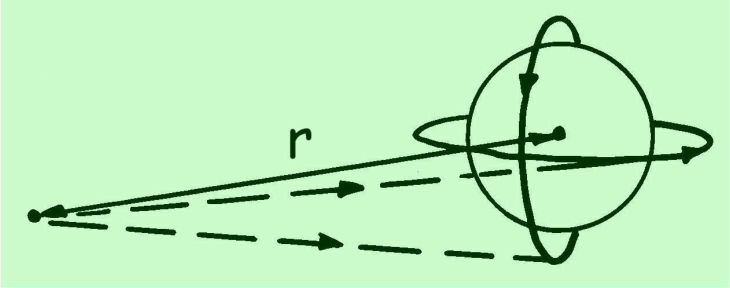
Venus approach

A. Arbitrary inclination of Venus' orbit (Sun's sphere of influence)

$$\text{As } r \to \infty,\ \Delta V \to 0$$



B. Hohmann transfer (Sun's sphere of influence)

$$V_{Tp} r_V = V_{Ta} r_E \quad \Rightarrow \quad V_{Tp} = 37.75 \text{ km/s}$$

$$V_V = \sqrt{\frac{\mu_S}{r_V}} = 35.04 \text{ km/s}$$

Both relative to the Sun

Therefore relative to Venus: $V_{ap} = V_{Tp} - V_V = 2.71 \text{ km/s}$

C. Venus capture (Venus sphere of influence ~ 0.62 × 10^6 km)

On the hyperbola

$$\varepsilon_B = \varepsilon_A \qquad \Rightarrow \qquad \frac{1}{2}\left(V_\infty^-\right)^2 = \frac{1}{2}V_p^2 - \frac{\mu_V}{r_p}$$

where $r_P = R_V + 200$ km = 6052 + 200 km = 6252 km

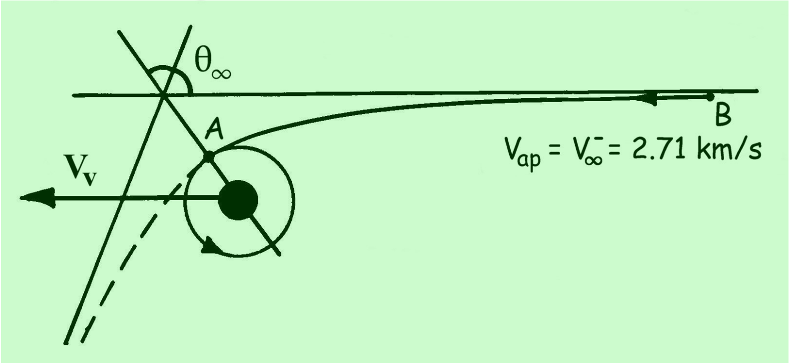
hence $V_p = 10.55$ km/s, $V_{circ} = \sqrt{\dfrac{\mu_V}{r_p}} = 7.21$ km/s

$\Rightarrow \Delta V_{cap} = V_p - V_{circ} = 3.34$ km/s

Also, e = 1.1413

ϑ_∞ =151.18°

(aerobraking)



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Total ΔV for the entire mission

$$\text{Total } \Delta V = \Delta V_{esc} + \Delta V_{cap} + \Delta = \left(6.85 + \Delta\right) \text{km/s}$$

Δ is small: - Mid-course manoeuvre corrections

- Arbitrary inclination

"Swing-by" missions

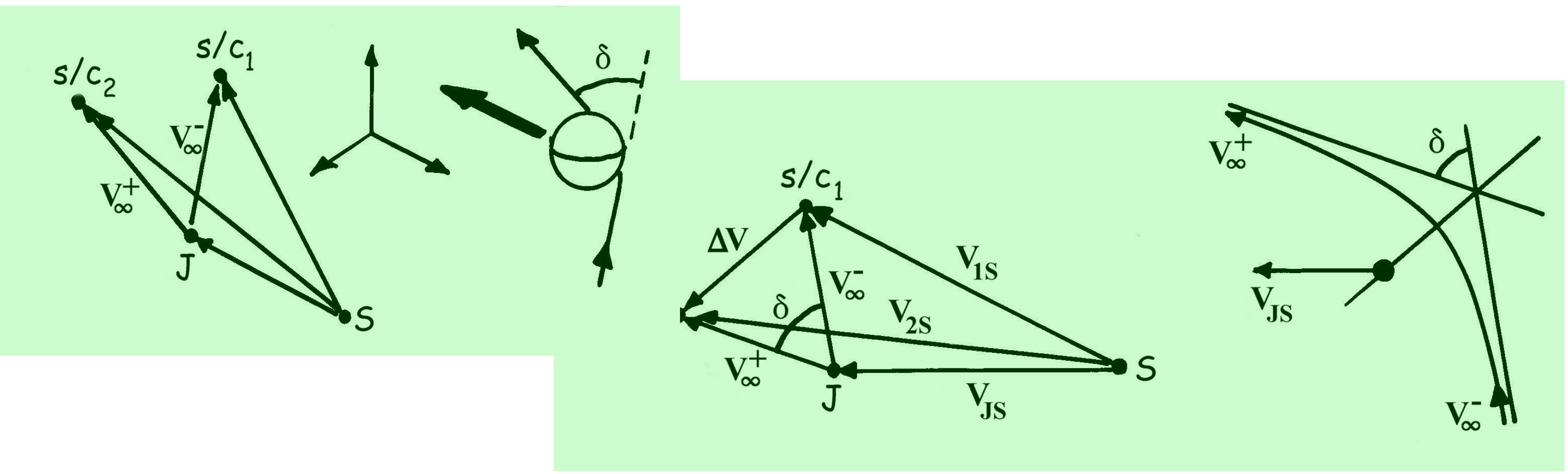
Consider Jupiter swing-bys

Recall Jovian sphere of influence : For p = Jupiter, d = Sun: $R_{JS} = 48.20 \times 10^6$ km

A. Hyperbolic passage 'behind' Jupiter (e.g., Voyager-type)

$\Downarrow$

heliocentric velocity is increased



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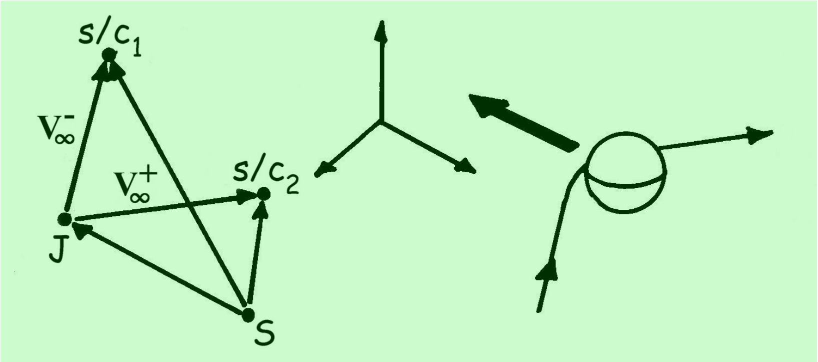
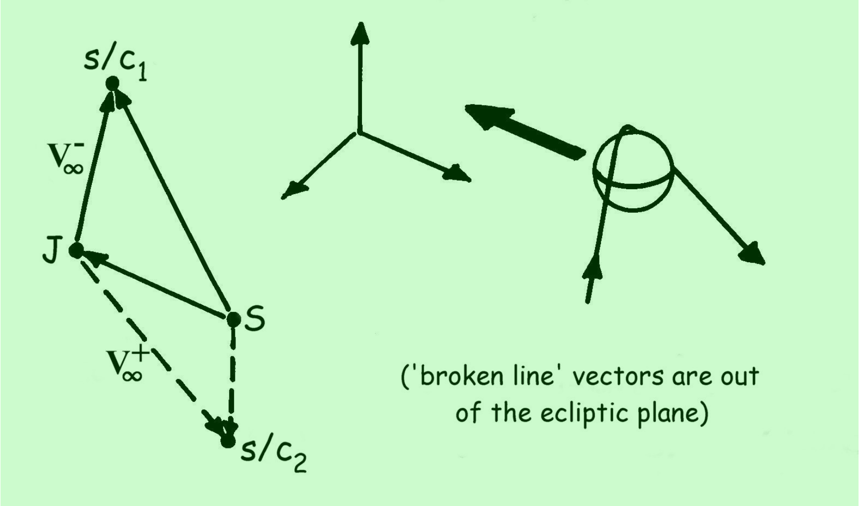
B. Hyperbolic passage 'in front' of Jupiter

$\Downarrow$

Heliocentric velocity decreased



C. Out of ecliptic plane manoeuvres (e.g., Ulysses-type)



Example

A S/C is launched to perform a Pluto fly-by mission, via a Jupiter swing-by manoeuvre. After Earth escape, the Earth-Jupiter transfer orbit has a perihelion distance of 1 AU, and a Sun-relative perihelion speed of 41 km/s. If the closest approach to Jupiter's centre is 6 Jupiter radii, estimate:

a) the change in Sun-relative speed due to swing-by

b) a and e of the heliocentric orbit after the swing-by

<u>Data</u>

Earth:

- Mean distance from the Sun $r_E = 1\ \text{AU} = 1.5 \times 10^8$ km

Jupiter:

- Gravity constant, $\mu_J = 1.3 \times 10^8\ \text{km}^3/\text{s}^2$
- Mean distance from the Sun $r_J = 7.8 \times 10^8$ km
- Planet radius $R_J = 7.2 \times 10^4$ km

Sun: gravity constant $\mu_S = 1.3 \times 10^{11}\ \text{km}^3/\text{s}^2$

A. Earth-Jupiter transfer (Sun's sphere of influence)

$$\frac{1}{2}V_p^2 - \frac{\mu_S}{r_E} = -\frac{\mu_S}{2a_T} \quad \Rightarrow \quad a_T = 24.8408\times 10^8 \text{ km}$$

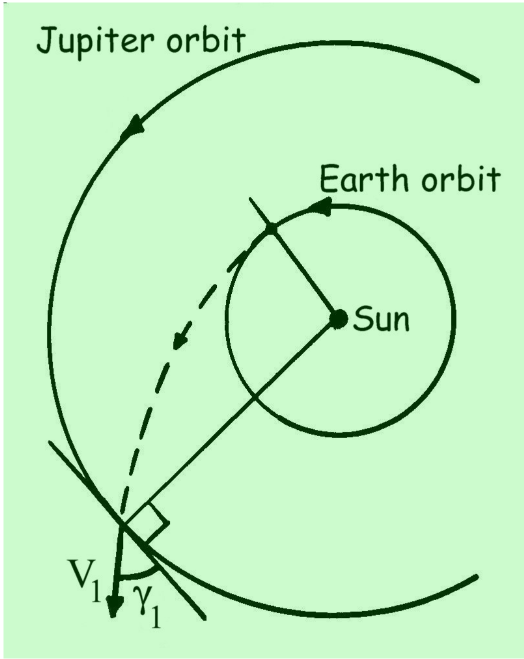
$$r_E = r_p = a_T\left(1-e_T\right) \quad \Rightarrow \quad e_T = 0.939615$$

B. Jupiter arrival (Sun's sphere of influence)

$$\frac{1}{2}V_1^2 - \frac{\mu_S}{r_J} = -\frac{\mu_S}{2a_T} \quad \Rightarrow \quad V_1 = 16.7631 \text{ km/s}$$

Relative to the Sun

$$\cos^2\gamma_1 = \frac{\mu_S a_T\left(1-e_T^2\right)}{r_J^2 V_1^2} \quad \Rightarrow \quad \gamma_1 = 61.9425^o$$



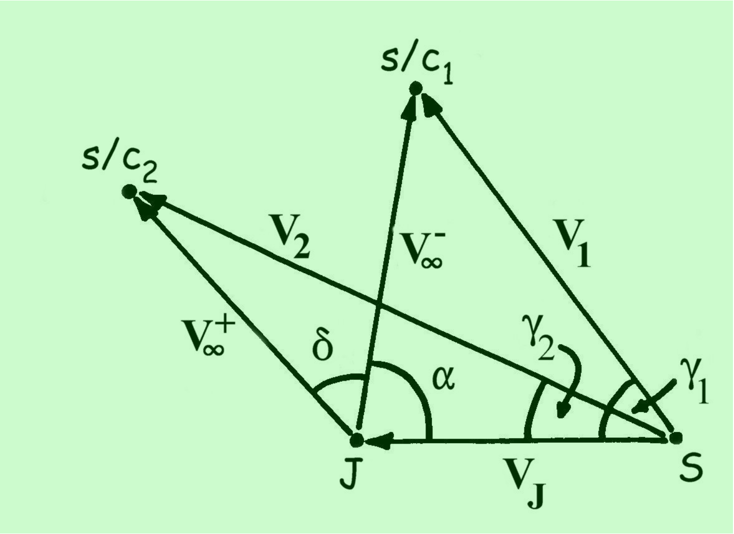
C. Jupiter swing-by (Jupiter's sphere of influence)

$$V_J = \sqrt{\frac{\mu_S}{r_J}} = 12.9099 \text{ km/s}$$ Relative to the Sun

- Swing-by relative velocity diagram
  Solve geometry: from V_1, V_J and γ_1

$$V_\infty^- = V_\infty^+ = 15.6233 \text{ km/s}$$

$$\alpha = 71.2370^\circ$$



- Swing-by hyperbola

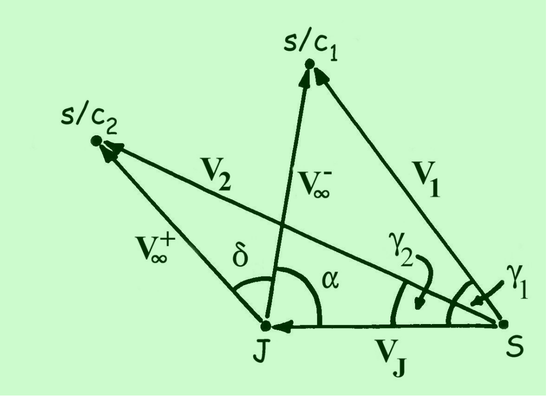
$$e_S = 1 + \frac{r_p V_\infty^2}{\mu_J}, \text{where } r_p = 6R_J$$

$$\Downarrow$$

$$e_S = 1.811121$$

Turn angle

$$\delta_S = 2\sin^{-1}\left(\frac{1}{e_S}\right) \quad \Rightarrow \quad \delta_S = 67.0285^o$$



Solve geometry: from V_J, V_∞^+, $\alpha+\gamma_s$

$V_2 = 26.6792$ km/s and $\gamma_2 = 22.9434^o$ Relative to the Sun

Therefore (answers)

a) Sun-relative speed change

$$\boxed{= V_2 - V_1 = 9.9161 \text{ km/s}}$$

b) *a* and *e* post encounter trajectory (Sun's sphere of influence)

$$\frac{1}{2}V_2^2 - \frac{\mu_S}{r_J} = -\frac{\mu_S}{2a} \quad \Rightarrow \quad \boxed{a = -3.4351\times10^8 \text{ km}}$$

From Eq. $\cos^2\gamma = \dfrac{\mu\, a\,(1-e^2)}{r^2V^2} \Rightarrow e^2 = 1-\left(\dfrac{r_J^2V_2^2\cos^2\gamma_2}{\mu_S a}\right) \Rightarrow \boxed{e = 3.037056}$

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