Spacecraft Attitude Dynamics

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Actuators

Inertia, Reaction wheels, CMGs – momentum exchange devices

These actuators are able to change momentum more than torque → Torque is the effect of the exchange of angular momentum.

Divided into three main devices

A **reaction wheel**

- can oscillate around zero for precise pointing
- actively controlled through feedback

– controlled by a motor through a feedback loop → motor should manage to get the reaction wheel rotating with a certain speed. The sc is gaining momentum if reaction wheel loose momentum and vice versa.

An **inertia wheel**

- Inertia wheels spin at a constant speed for stability
- Once spun up they are left to passively spin.

CONTROL MOMENT GYRO

A **CMG** spins at a constant speed but can be tilted around an axis orthogonal to the spin axis.

-These are used for performing large slew maneuvers.

- More torque at less power than RW

It does not change it's rotational speed → has two axis ⊥ to rotation axis where we can rotate to produce a reaction → they can typically produce more torque than the other two systems.

The fact that these actuators are exchanging momentum the spacecraft is called also platform

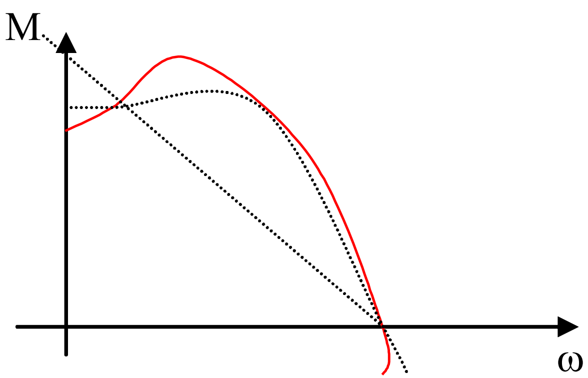
platform = spacecraft – actuators

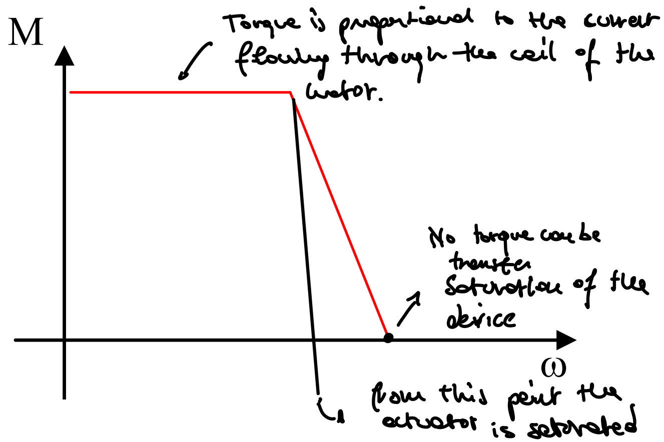
Inertia, Reaction wheels, CMGs – momentum exchange devices

All these actuators can only exchange angular momentum with the satellite but not produce a net torque

$$I_r \dot{\omega}_r = M_r$$
$$I_z \dot{\omega}_z = -I_r \dot{\omega}_r$$

The torque M_r is in general provided by an electric motor that has its typical operational curve





Inertia, Reaction wheels, CMGs – momentum exchange devices

contribution of the exchange angular momentum device $A \rightarrow$ orientation of the angular momentum device

general problem of control with RW, IW or CMG

$\underline{h}$ = angular momentum in the principal axis frame

total angular momentum of the s/c

$\rightarrow$ angular momentum of the actuators

$$\underline{h} = I\underline{\omega} + A\underline{h}_r$$

$\hookrightarrow$ angular momentum of the platform

$I\underline{\omega}$ = momentum of the platform

we are interested in the relative angular velocity because rotation depends on the relative angular velocity.

$\underline{h}_r \rightarrow [\;\;] \rightarrow$ n any number of actuators

$$I\underline{\dot{\omega}} + \underline{\omega} \wedge I\underline{\omega} + \dot{I}\underline{\omega} + \dot{A}\underline{h}_r + A\underline{\dot{h}}_r + \underline{\omega} \wedge A\underline{h}_r = \underline{T}$$

System dynamics

Effect of the control action

All this terms can be compacted in one terms that we called control Torque T_c $\rightarrow$ that it is possible to carry it in the right hand side

Additional equations for the relative dynamics of the rotors

Torque produced by the motor that drives the actuators

Inner behaviour of the actuator $\leftarrow$

$$\dot{A}\underline{h}_r + A\underline{\dot{h}}_r = \underline{M}_r$$

There is one equation for each exchange moment devices

$\dot{I}\underline{\omega}$ exists only for CMG, and can be neglected if these are small compared to the satellite

$\dot{A}\underline{h}_r$ $\rightarrow$ consider control — exists only for CMG, and indicates the relative attitude rotation in the spacecraft body frame

Assume CMG $\underline{\dot{h}}_r = 0$ $\rightarrow$ this exists only for Reaction Wheel and Inertia wheel

$A\underline{\dot{h}}_r$ $\rightarrow$ can exist for any devices $\rightarrow$ for Inertial wheel and Reaction wheel. — in general it exists for RW and IW, not for CMG since these have often constant spin velocity — CMG can change also the spin rates

$\underline{\omega} \wedge A\underline{h}_r$ exists for any actuator, could be neglected for RW since they are characterized by nominal $\underline{h}_r = 0$

It is always present $\rightarrow$ for any devices

Equations of motion for reaction wheels

$$\begin{cases} I\underline{\dot{\omega}} + \underline{\omega} \wedge I\underline{\omega} + \underline{\omega} \wedge A\underline{h}_r + A\underline{\dot{h}}_r = \underline{T} \\ \underline{M}_c = -\underline{\omega} \wedge A\underline{h}_r - A\underline{\dot{h}}_r \\ I_r\underline{\dot{\omega}}_r = \underline{M}_r \end{cases}$$

↳ control action

Disturbance Torque
external disturbance
⇒ It is not the control action

$T = T_{\text{Airdrag}} + T_{\text{gravity gradient}} + T_{\text{mag field}} + T_{\text{solar pressure}}$

Number of Rows of A = 3, Number of columns = number of actuators

Equation for the evaluation of the control law $I\underline{\dot{\omega}} + \underline{\omega} \wedge I\underline{\omega} = \underline{T} + \underline{M}_c$

↳ Assume that we have this Dynamics know to design the control law
⇒ we know how to design a control law of a linearized system ⇒ so first of all we need to linearized the system dynamics, then neglecting the disturbances and then write the state space representation.

Any control design technique can be used to calculate $\underline{M}_c$

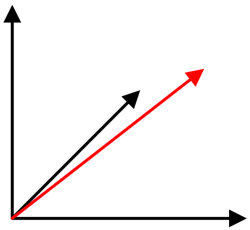
Then $\underline{M}_r$ can be evaluated, i.e., $\underline{\dot{h}}_r = I_r\underline{\dot{\omega}}_r$

$$A\underline{\dot{h}}_r = -\underline{M}_c - \underline{\omega} \wedge A\underline{h}_r$$

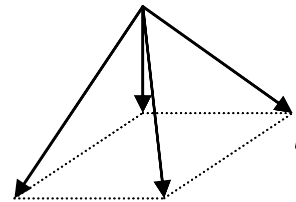
$$\underline{\dot{h}}_r = -A^*\left(\underline{M}_c + \underline{\omega} \wedge A\underline{h}_r\right)$$

The input from the actuators is $\underline{M}_c$ so we want to calculate it from the $\underline{M}_c$ that we want to obtain.
Easiest configuration 3 reaction wheel oriented to the principal axis of inertia
↳ configuration is not fail safe one fail and we will lose control along one inertia axis

Typical configurations for RW



Three axes and diagonal



Another very common configuration

pyramid

4 reaction wheel

In this way I can also generate the same change of momentum without reaching saturation → we have different way to achieve the same effect without running into saturation problem.

$$A = \begin{bmatrix} 1 & 0 & 0 & 1/\sqrt{3} \\ 0 & 1 & 0 & 1/\sqrt{3} \\ 0 & 0 & 1 & 1/\sqrt{3} \end{bmatrix}$$

Actuators

$$A = \begin{bmatrix} -a & a & a & -a \\ -a & -a & a & a \\ a & a & a & a \end{bmatrix}$$

Action in principal axis

$$a = \frac{1}{\sqrt{3}}$$

If I need to produce a reaction along the x axis we will need to use all 4 actuators

$$A^* = \begin{bmatrix} 5/6 & -1/6 & -1/6 \\ -1/6 & 5/6 & -1/6 \\ -1/6 & -1/6 & 5/6 \\ 1/2\sqrt{3} & 1/2\sqrt{3} & 1/2\sqrt{3} \end{bmatrix}$$

If this logic is used this matrix tend to distribute the load to all the other actuators.

$$A^* = \begin{bmatrix} -b & -b & b \\ b & -b & b \\ b & b & b \\ -b & b & b \end{bmatrix}$$

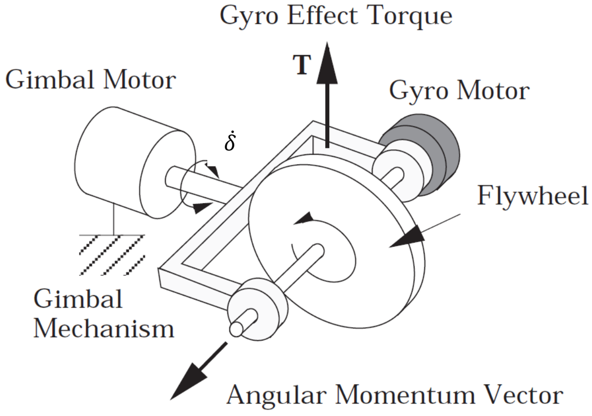
$$b = \frac{\sqrt{3}}{4}$$

Fail safe approach.

If we know that there is an axis that required higher reaction to generate I can change the shape of the pyramid in order to have higher coefficient where they are required.

So this is a fail safe system even if I lose one actuator I need to remove one column of the A matrix but A remain non singular so I can calculate the inverse and determine the control law.

CMG → control moment gyros



(a) Single gimbal CMG

$$A = \begin{bmatrix} -a & a & a & -a \\ -b & -b & b & b \\ c & c & c & c \end{bmatrix}$$

$$a^2 + b^2 + c^2 = 1$$

$$h_{CMG} = A\underline{h}_r = h_r \begin{bmatrix} -\sin\delta \\ \cos\delta \\ 0 \end{bmatrix}$$

$$\dot{h}_{CMG} = h_r \begin{bmatrix} -\cos\delta \\ -\sin\delta \\ 0 \end{bmatrix} \dot{\delta}$$

CMG

General control problem with CMG actuators

$$I\underline{\dot{\omega}} + \underline{\omega} \wedge I\underline{\omega} + \dot{A}\underline{h}_r + \underline{\omega} \wedge A\underline{h}_r = \underline{T}$$

$$\begin{cases} I\underline{\dot{\omega}} + \underline{\omega} \wedge I\underline{\omega} + \dot{A}\underline{h}_r + \underline{\omega} \wedge A\underline{h}_r = \underline{T} \\ M_c = -\underline{\omega} \wedge A\underline{h}_r - \dot{A}\underline{h}_r \end{cases}$$

$$I\underline{\dot{\omega}} + \underline{\omega} \wedge I\underline{\omega} = \underline{T} + \underline{M}_c$$

$$\dot{A}\underline{h}_r = -\underline{M}_c - \underline{\omega} \wedge A\underline{h}_r$$

$\dot{A} \rightarrow$ real action that the actuator used to do: changing orientation of CMG around their dof produces the effect wanted

The problem is solved when the matrix $\dot{A}$ is evaluated

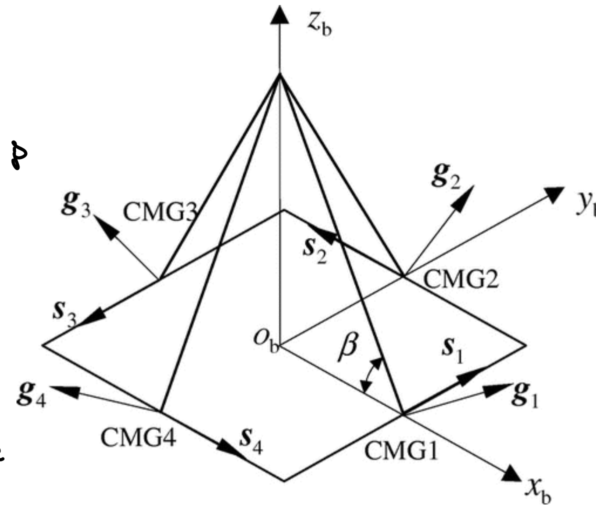
CMG configuration modelling

$$\mathrm{h_{CMG}} = \mathrm{A}\underline{\mathrm{h}}_{\mathrm{r}}$$

A matrix depends on the configuration angle β and on what the actuators are doing $\delta_{1,2,3,4}$.

$$= \begin{bmatrix} -\cos\beta\sin\delta_1 & -\cos\delta_2 & \cos\beta\sin\delta_3 & \cos\delta_4 \\ \cos\delta_1 & -\cos\beta\sin\delta_2 & -\cos\delta_3 & \cos\beta\sin\delta_4 \\ \sin\beta\sin\delta_1 & \sin\beta\sin\delta_2 & \sin\beta\sin\delta_3 & \sin\beta\sin\delta_4 \end{bmatrix} \begin{Bmatrix} \mathrm{h_r} \\ \mathrm{h_r} \\ \mathrm{h_r} \\ \mathrm{h_r} \end{Bmatrix}$$

Owning same spin rate



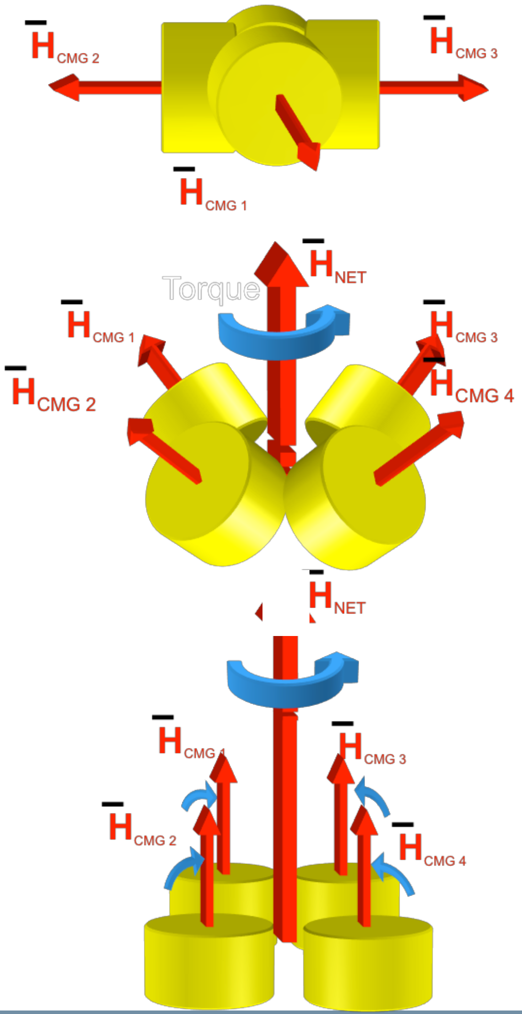
$$\dot{\mathrm{h}}_{\mathrm{CMG}} = \dot{\mathrm{A}}\underline{\mathrm{h}}_{\mathrm{r}} = \mathrm{B}\dot{\delta} = \mathrm{h_r} \underbrace{\begin{bmatrix} -\cos\beta\cos\delta_1 & \sin\delta_2 & \cos\beta\cos\delta_3 & -\sin\delta_4 \\ -\sin\delta_1 & -\cos\beta\sin\delta_2 & \sin\delta_3 & \cos\beta\cos\delta_4 \\ \sin\beta\cos\delta_1 & \sin\beta\sin\delta_2 & \sin\beta\cos\delta_3 & \sin\beta\sin\delta_4 \end{bmatrix}}_{B} \underbrace{\begin{Bmatrix} \dot{\delta}_1 \\ \dot{\delta}_2 \\ \dot{\delta}_3 \\ \dot{\delta}_4 \end{Bmatrix}}_{\dot{\delta}}$$

There is a possibility the δ angle produce a B matrix no longer invertible ⇒ limiting capacity to control torque around one axis look at the following page.

The control equation is

$$\dot{A}\underline{h}_r = -\underline{M}_c - \underline{\omega} \wedge A\underline{h}_r = B\underline{\dot{\delta}} \quad \rightarrow \quad \underline{\dot{\delta}} = -B^*\left[\underline{M}_c + \underline{\omega} \wedge A\underline{h}_r\right]$$

Saturation / singularity in CMGs

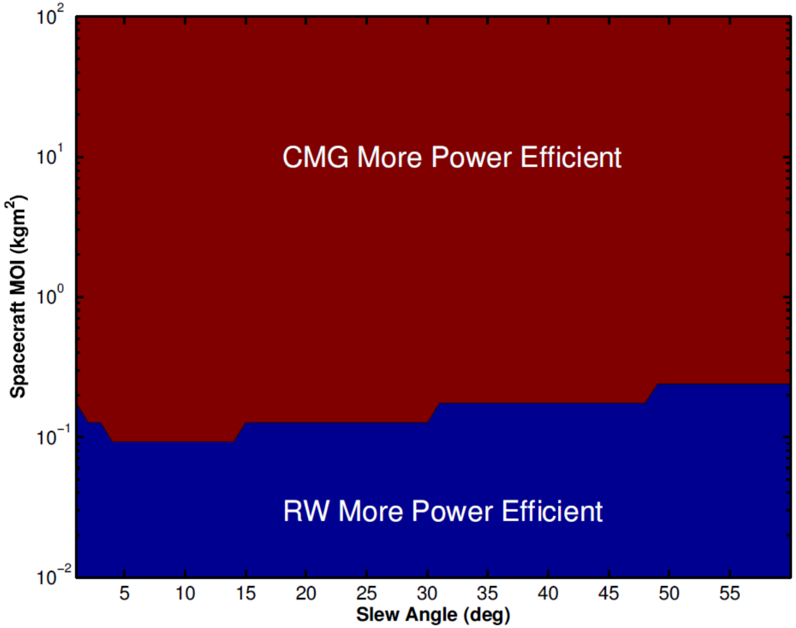


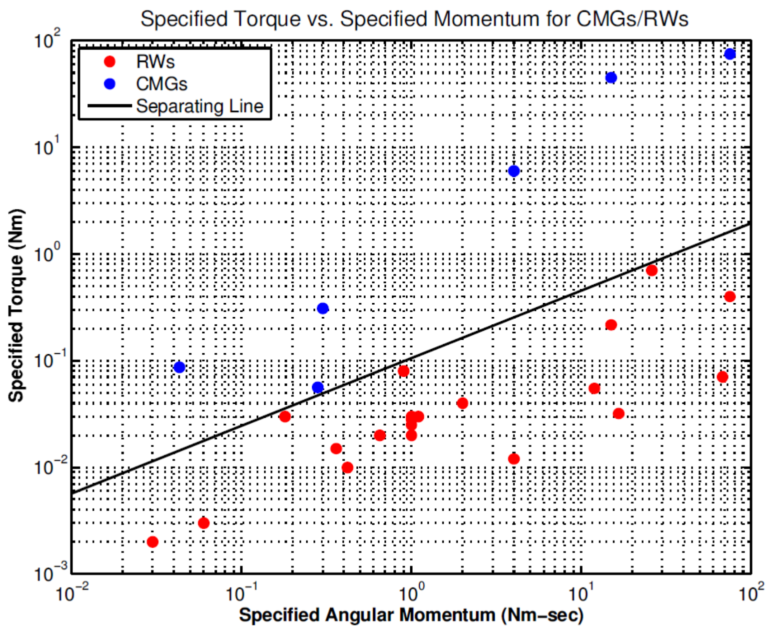
Can be avoided by adding the null motion command

$$B\underline{\dot{\delta}_n} = 0$$

$$\underline{\dot{\delta}} = -B^*\left[\underline{M}_c + \underline{\omega} \wedge A\underline{h}_r\right] + k\underline{\dot{\delta}_n}$$

How to decide if it is better to use CMG or RW





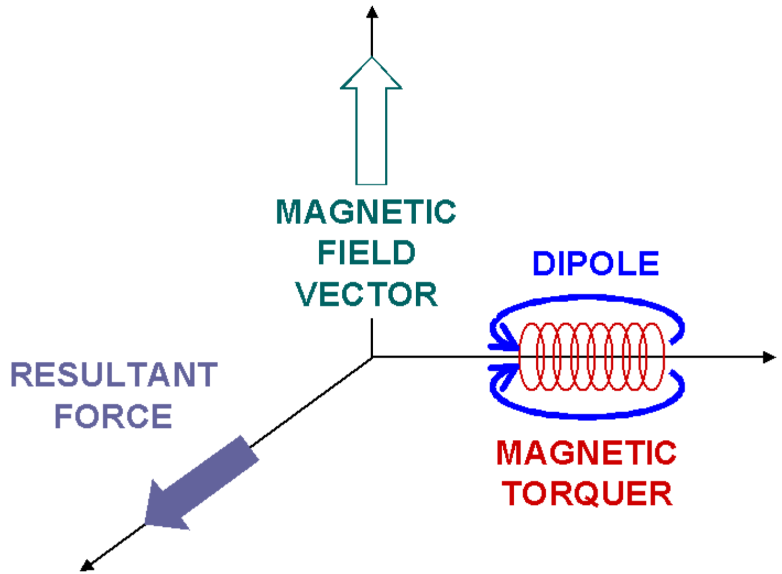
Magnetic Torque Rods

$$\underline{M} = \underline{D} \wedge \underline{B}$$

$$\underline{D} = \mu n S \underline{I}$$

μ	Magnetic permeability
n	Number of coil windings
S	Area of the coil
$\underline{I}$	Current intensity

Typically produce torques of $10^{-2} - 10^{-6}$ N-M



Control with magnetic torquers

It is never possible to generate three independent components of the control torque

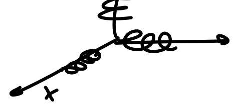
$$\underline{M} = \underline{D} \wedge \underline{B} = -\underline{B} \wedge \underline{D}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & -B_x & 0 \end{bmatrix} \begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix}$$

$\underline{D} = [-B\,\wedge]^{-1}\underline{M}$ $\longrightarrow$ $[-B\,\wedge]$ is a singular matrix, $\underline{D}$ cannot be evaluated

It is not
possible because $\underline{B}$ is singular

Control with magnetic torquers



→ I can generate an arbitrary $\underline{D}$ vector

Torque due to 2 magnetic torquers and third actuator (reaction wheel)

$$D_z = 0$$ → not have the coil around the z axis.

$$\begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & -B_x & 0 \end{bmatrix} \begin{Bmatrix} D_x \\ D_y \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -\dot{h}_z \end{Bmatrix}$$

Control torque

→ No more singular.

$$\begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} 0 & B_z & 0 \\ -B_z & 0 & 0 \\ B_y & -B_x & 1 \end{bmatrix} \begin{Bmatrix} D_x \\ D_y \\ -\dot{h}_z \end{Bmatrix}$$

→ Actuators

In this case we can find an inverse law to derive the control

$$\begin{Bmatrix} D_x \\ D_y \\ -\dot{h}_z \end{Bmatrix} = \frac{1}{B_z} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ B_x & B_y & B_z \end{bmatrix} \begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix}$$

Control with magnetic torquers

Approximate solution

$$\underline{M} = \underline{D} \wedge \underline{B}$$
$$\underline{B} \wedge \underline{M} = \underline{B} \wedge \left(\underline{D} \wedge \underline{B}\right)$$
$$\underline{B} \wedge \underline{M} = \left(\underline{B} \cdot \underline{B}\right)\underline{D} - \left(\underline{D} \cdot \underline{B}\right)\underline{B}$$

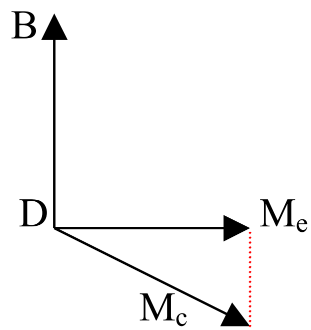
We can assume that $\underline{D}$ is generated in such a way to be always perpendicular to $\underline{B}$

average along orbit
⇒ for inclined orbit

$$\underline{D} \cdot \underline{B} = 0 \quad \rightarrow \quad \underline{B} \wedge \underline{M} = B^2 \underline{D} \quad \rightarrow \quad \underline{D} = \frac{\underline{B \wedge M_c}}{B^2} \quad \rightarrow \quad \underline{M_{\text{eff}}} = \frac{1}{B^2}\left(\underline{B} \wedge \underline{M_c}\right) \wedge \underline{B}$$

The effective control is equal to the desired control only if the desired control $\underline{M_c}$ is orthogonal to $\underline{B}$

We will not be able to control well the spacecraft. Because $\underline{B}$ is not always orthogonal.



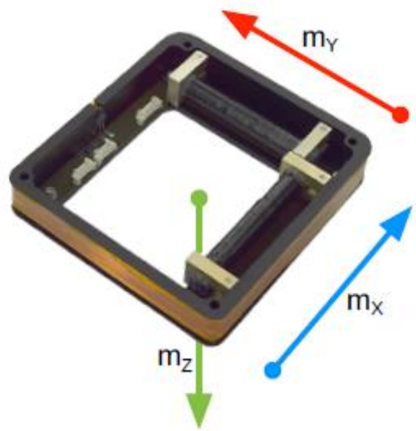
For equatorial orbit this approx should not be used
⇒ I can use the solution on the previous case.

Magnetic torquers

GOMSPACE

Description	Value	Unit
Mass	156	g
Size	90.5 x 96.9 x 17.2	mm

Condition	Min	Typ.	Max	Unit
PWM voltage	0	3.3	4	V
Dipole moment @3.3V (Z-axis)		0.355		Am^2
Dipole moment @3.3V (X- and Y-axis)		0.300		Am^2



Magnetic torquers

Advantages

- Lightweight
- Energy Efficient
- Reliable
- No fuel

Disadvantages

- Small torques
- Need to be in LEO → Rely on external magnetic field is stronger at low altitude
- Cannot produce torque in all three axis

Thrusters for attitude control

→ Rather low specific impulse

↗ But we have low requirements
Extremely simple system

Cold-gas thrusters use a non-reactive gas, stored at high pressure (around 30 MPa), commonly use Nitrogen~Isp 70s or Helium~Isp 175s. Helium saves mass but is more prone to leakage and more expensive.

(Tank/valve/Nozzle) → very simple system
the complication could arise from the type of valve we want → impulse response or variable thrust

Primary Thruster → Isp + level of thrust both equally important
Attitude Thruster → Isp most important parameter because the action on the s/c are relatively small

Monopropellant uses a propellant that is decomposed catalytically. Hydrazine~Isp 242s is by far the most common. They do not require high pressure but are highly toxic.

→ Propellant exhaust must not come in contact with any part of the spacecraft because it could damage it.

One more step wrt the cold gas thruster → the gas must be decompose before being ejected so we could have some delay which we must taken into account.

Electric Thrusters use electric fields to accelerate the expelled ionized mass at high velocity. High Isp 500-3000s.

$$I_{SP} \equiv \frac{F_{thrust}}{g_0 \dot{m}}$$

→ The other two methods required very little power → This is not the case for the electric thruster → Gas must be kept ionized and we must generate the magnetic field to accelerate the gas molecule.
We need a lot more energy so we will need a much higher electric thruster.
very little level of thrust → actuators because we
they are modelled by a linear
... level of thrust.

Thrusters

whereas the other actuators are modelled as impulsive actuators broadly.

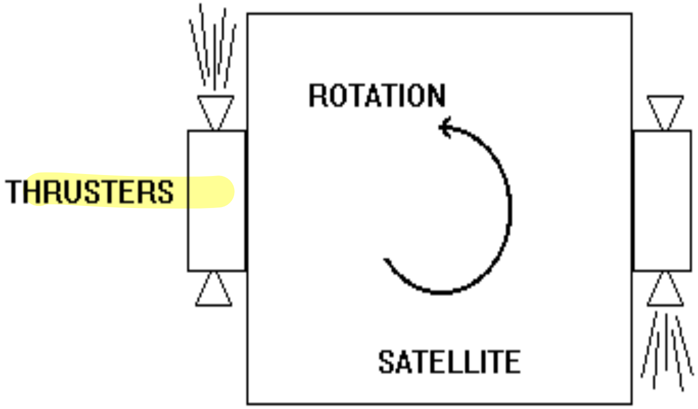
Advantages:

Do not depend on environmental disturbances for their use i.e. gravitational field or magnetic field.

Disadvantages:

They use propellant, not useful for precision control

$$\underline{F} = -\dot{m}\underline{v}_{rel}$$
$$T = \underline{r} \times \underline{F}$$



Example data sheet – Airbus mono-propellant thrusters

1N Monopropellant Thruster Key Technical Characteristics	
Characteristics	
Thrust Nominal	1 N
Thrust Range	0.320 … 1.1 N
Specific Impulse, Nominal	220 s
Pulse, Range	200 … 223 s
Mass Flow, Nominal	0.44 g/s
Mass Flow, Range	0.142 … 0.447 g/s
Inlet Pressure Range	5.5 … 22 bar
Minimum Impulse Bit	0.01 … 0.043 Ns
Nozzle Expansion Ratio	80
Mass, Thruster with valves	290 g
Propellant	Hydrazine (N_2H_4), High-Purity Grade

$MIB = \Delta t_{\min}$ → like an integral

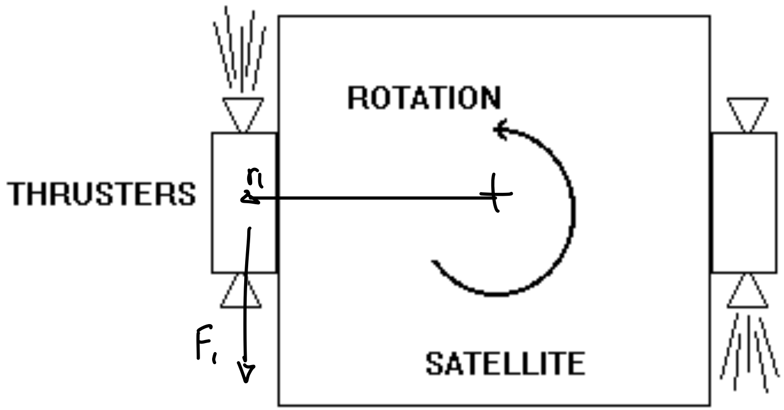
↳ minimum input bit

Thrusters for attitude control

$$\underline{F}_1 = \underbrace{\begin{Bmatrix} f_x \\ f_y \\ f_z \end{Bmatrix}}_{\underline{f}_1} F$$

$$T_{couple} = \underline{r}_1 \wedge \underline{F} + \underline{r}_2 \wedge -\underline{F} = (\underline{r}_1 - \underline{r}_2) \wedge \underline{F}$$

$$\hat{\underline{R}} = [\underline{r}_1 \wedge \underline{f}_1]$$

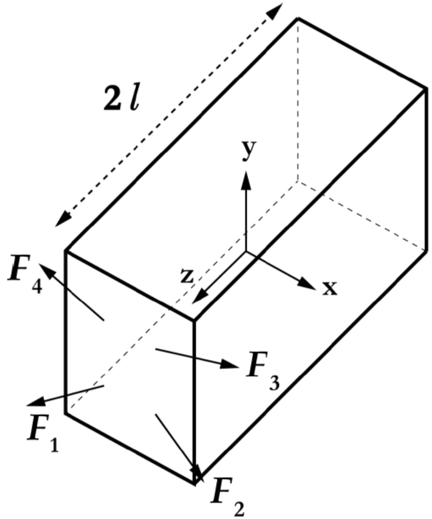


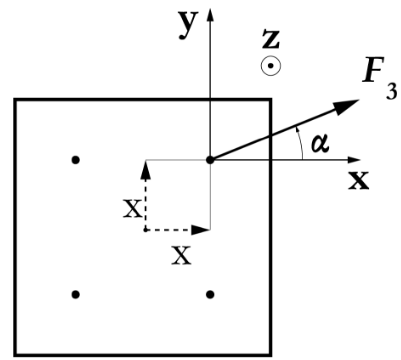
We need to have 12 thrusters to have 6 moment without any residual thrust.

For a system of thrusters $\underline{T}_{tot} = [\hat{R}]F$

↳ combination of the thrust value

Thruster configuration matrix

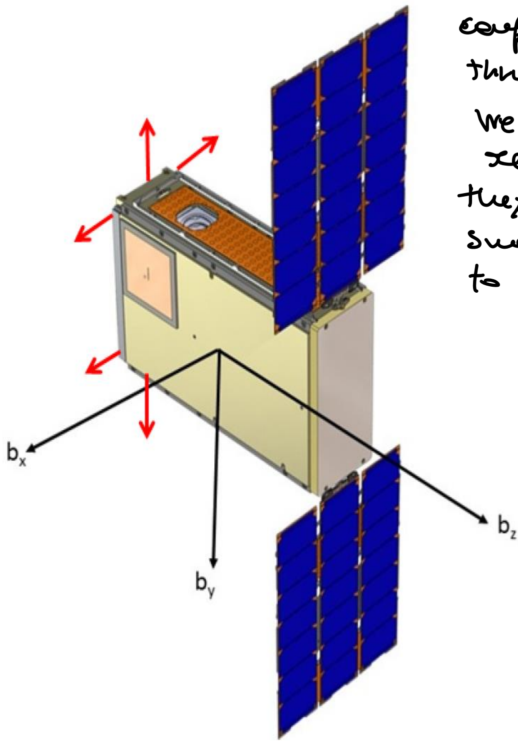




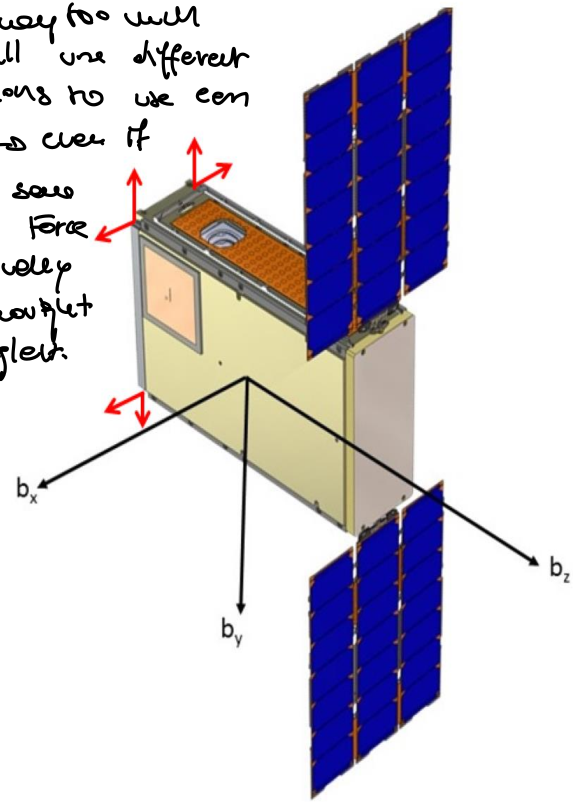
$$[\hat{R}] = \begin{bmatrix} l\,sin\,\alpha & 0 & 0 \\ 0 & l\,cos\,\alpha & 0 \\ 0 & 0 & x\,sin\,\alpha - x\,cos\,\alpha \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

NASA Biosentinal 6U Cubesat thruster configuration

12 thruster very too much
so we will use different
configurations so we use less
thruster → check if
we have some
residual force
they are really
small enough
to be neglect.



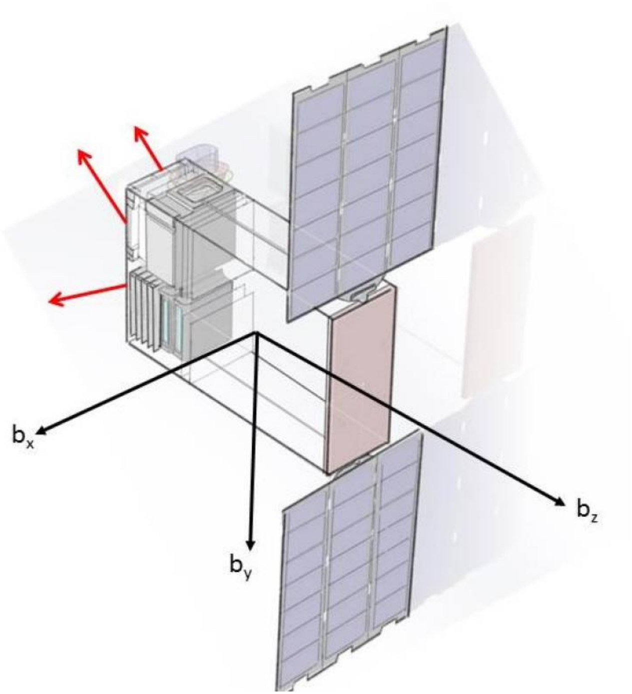
6 thrusters



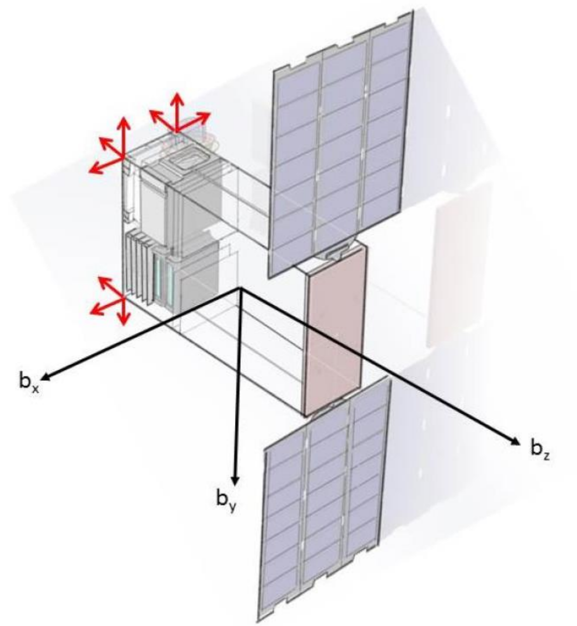
8 thrusters

NASA Biosentinal 6U Cubesat thruster configuration

I can use less thrusters if the nozzle can be tillted.



Cold gas thruster configuration



Electric propulsion configuration

Example of control allocation for thrusters

The controller is computing the desired input $\underline{u}_{ideal}$

Map the desired input $\underline{u}_{ideal} = \hat{R}\underline{u}_c$

The Moore-Penrose pseudo inverse cannot be used since $\underline{u}_c \geq 0$

However, we can define a modified inverse $\underline{u}_c = \hat{R}^*\underline{u}_{ideal} + \gamma w$

If $\underline{u}_{ideal}$ give some value of negative thrust.

-> we need to use γw

Where $w > 0$ is the null space vector of $\hat{R}$ -> $\hat{R}w = 0$ -> combination of the thrusters that generate zero torque.

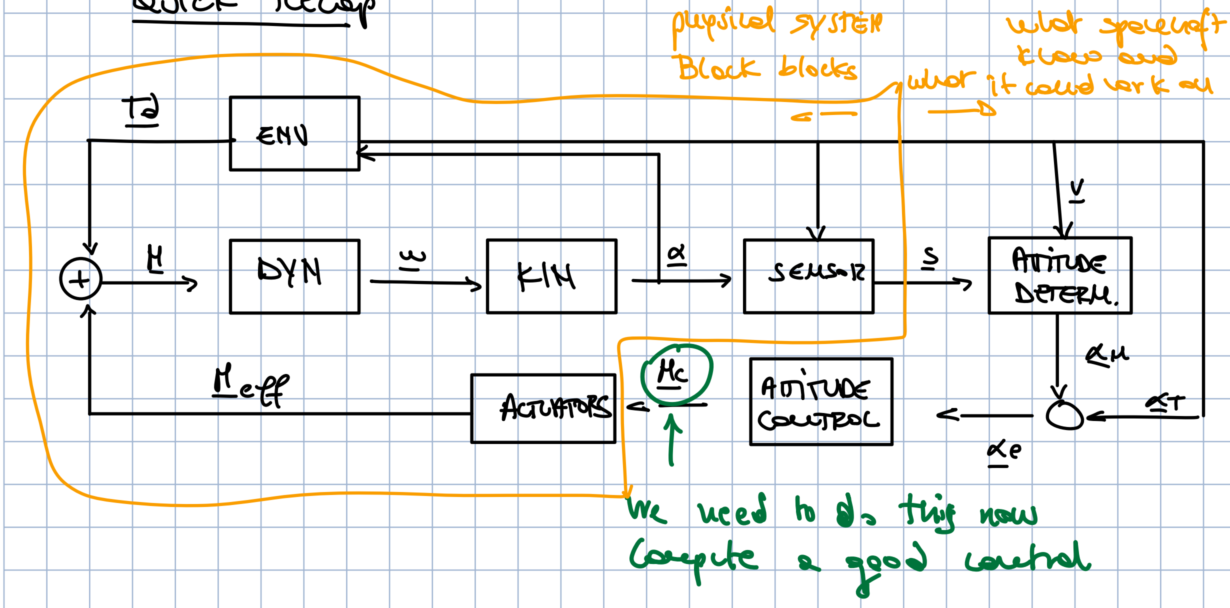
$$\gamma = \max_{i=1,..,N}(\hat{R}^*\underline{u}_{ideal})_i / w_i$$

Example:

$$[\hat{R}] = \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \qquad w = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

listen here

Quick recap



$\underline{\alpha}$ = where the s/c is looking wrt a ref frame.

$\underline{\omega}$ angular velocity in an inertial frame.

$\underline{T_d}$ = disturbance torque

$\underline{S}$ sensor measurement

$\underline{V}$ some vector wrt another reference system

Comparing $\underline{V}$ and $\underline{S}$ we get the attitude determination $\underline{\alpha}_M \simeq \underline{\alpha}$ but not exactly

We should check if $\underline{\alpha}_M \simeq \underline{\alpha}$ if they are very different we are using some wrong information to model the attitude so also the control action will be flawed.

In our simulation we need to check if $\underline{\alpha}$ is close to $\underline{\alpha}_M$

Which logic we need to implement inside the ATTITUDE CONTROL BLOCK?

Generally it is necessary to understand how far from a target orientation in space our spacecraft is.

Then after we have determined the distance of the s/c to the reference we can compute $\underline{M_c}$. Then we need to project the $\underline{M_c}$ onto the actuators to determine $\underline{M_{eff}}$.

I can assume $\underline{\alpha}$ as the state of my system and use my knowledge to design a correct control system capable to obtain the stability for the pointing requirement.

We need to talk about :

- What is a Target ? → Target requirement are not only a direction but there are some more thing to do.
- Lissen lemon 8/12/20