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Spacecraft Attitude Dynamics

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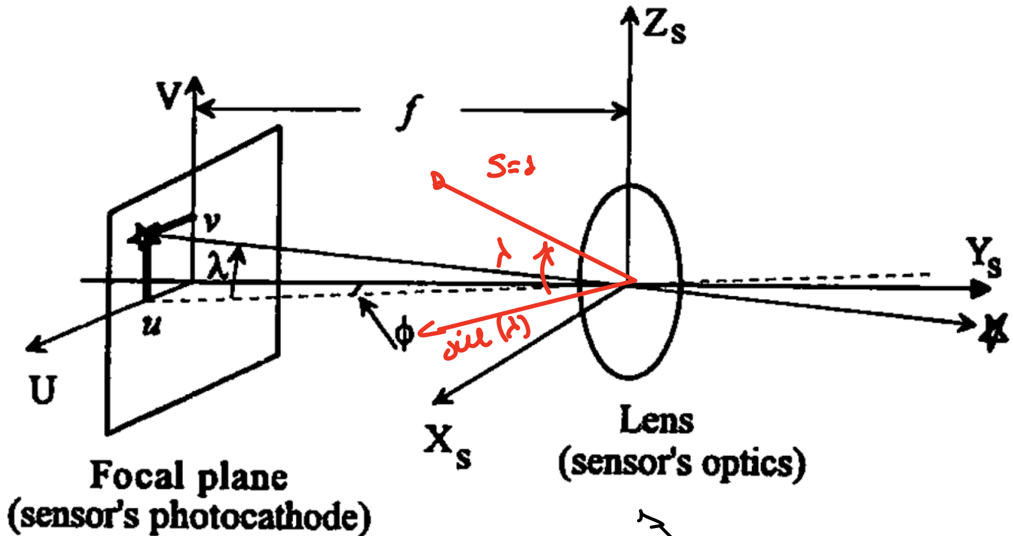
Attitude sensors

Star sensors



cosλ sin ϕ
– cosλ cosϕ
sin λ

the opposite

$$\mathbf{S} = \begin{bmatrix} -\sin(\phi)\cos(\lambda) \\ \cos(\phi)\cos(\lambda) \\ -\sin(\lambda) \end{bmatrix}.$$

$\tan\phi = u/f$ and $\tan\lambda = (v/f)\cos(\phi)$,

Star sensors

- Most accurate attitude reference sensors - arcseconds $\frac{1}{3600}$ of a degree
- Expensive
- radiation from the Sun can deteriorate performance
- Low up-date rates <5 Hz
- Can only perform attitude acquisition below 1-2 degrees per second
- 5 -20 degree FOV

- Star tracker – tracks single star and keeps it in the field of view
- Star mapper – star sensor is fixed and maps updated of the observed sky

- Star sensors have to perform the following operations:
- Phase 1: data acquisition
- Phase 2: correct positioning in space of the acquired data
- Phase 3: interpretation of the data acquired (star identification) that requires analysis of a star catalogue → Might be quite complicated
- Phase 4: attitude determination

It is important to know the intensity of light because can help the identification of a star Magnitude = integer $\log\left(\frac{I_{ref}}{I_{star}}\right)$ → less it is higher the luminosity greater lower Sun = −23 very bright.

Star sensors

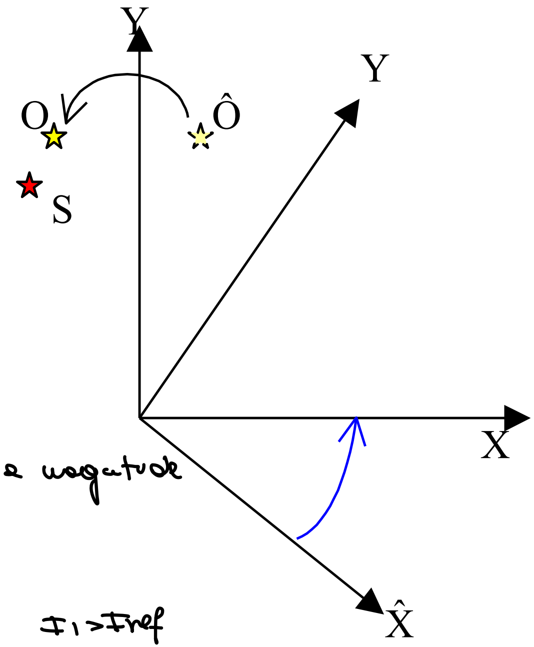
Attitude determination requires first the identification of the stars observed.

hat = observation

$$\hat{O}_1 \xrightarrow{A} O_1 \rightarrow S_1$$
$$\hat{O}_2 \xrightarrow{A} O_2 \rightarrow S_2$$
$$\hat{O}_3 \xrightarrow{A} O_3 \rightarrow S_3$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$\hat{O}_n \xrightarrow{A} O_n \rightarrow S_n$$

$\hat{O}_i$ local reference

O_i inertial reference



In the field of view of the sensor could see n stars that have a magnitude high enough to be detected by the satellite and not too bright

Relative magnitude $\underbrace{m_1 - m_{ref}}_{\text{Apparent magnitude}} = -2.5 \log\left(\frac{I_1}{I_{ref}}\right)$

$I_1 > I_{ref}$

$-2.5 \log\left(\frac{I_1}{I_{ref}}\right)$

$-2.5 + 1$

Star sensors

direction in space
unit vector

$$\underline{\hat{O}}_i = A_{B/N}\underline{S}_i$$

↳ rotation to Body frame from inertial

Observed star vectors in the body frame

All unit vectors identified in the body frame

Star catalogue

$$3\left\{\overbrace{[\underline{\hat{O}}_1 \quad \underline{\hat{O}}_2 \quad \cdots \quad \underline{\hat{O}}_n]}^{n}\right. = A_{B/N}\, 3\left\{\underbrace{\overbrace{[\underline{S}_1 \quad \underline{S}_2 \quad \cdots \quad \underline{S}_n]}^{n}}_{\text{Made of independent columns}}\right.$$

We need to compute $A_{B/N}$

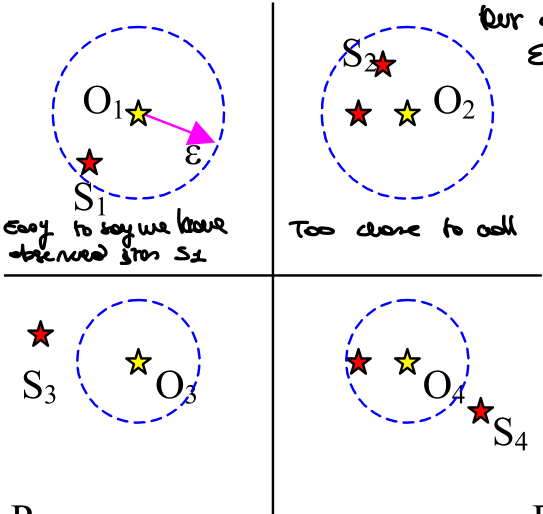
$$A_{B/N} = [\underline{\hat{O}}_1 \quad \underline{\hat{O}}_2 \quad \cdots \quad \underline{\hat{O}}_n][\underline{S}_1 \quad \underline{S}_2 \quad \cdots \quad \underline{S}_n]^*$$

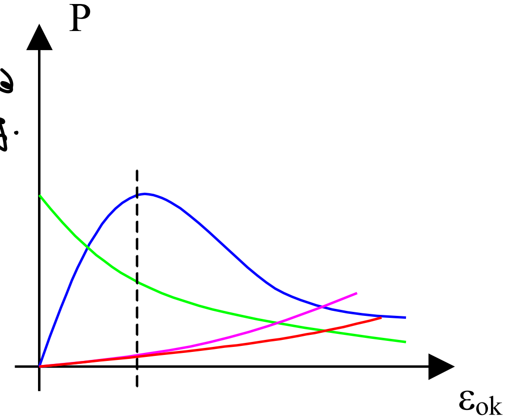
3xn for $\underline{\hat{O}}$ and $\underline{S}$ we cannot find the inverse
so we need to do the pseudo inverse.

* Denotes the pseudo inverse $\quad B^* = B^T(BB^T)^{-1}$

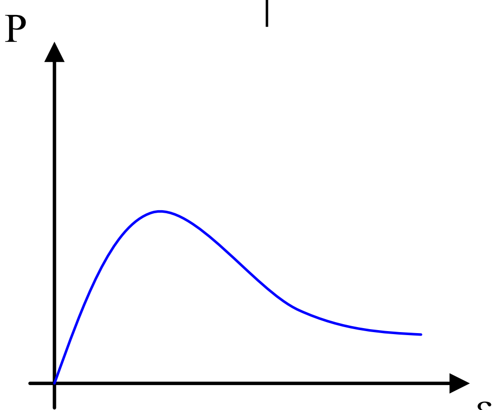
Star sensors

→ the observation will be 100% spot on → so we need to search in the region around our observation in the catalog.
ε → small

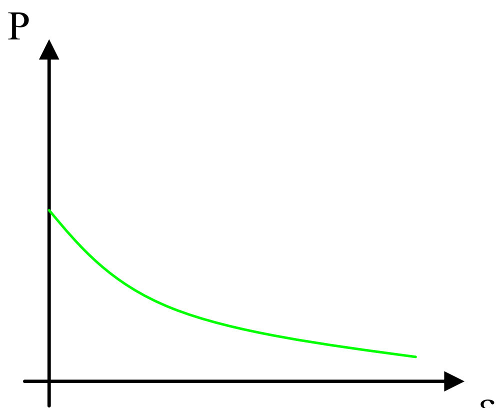




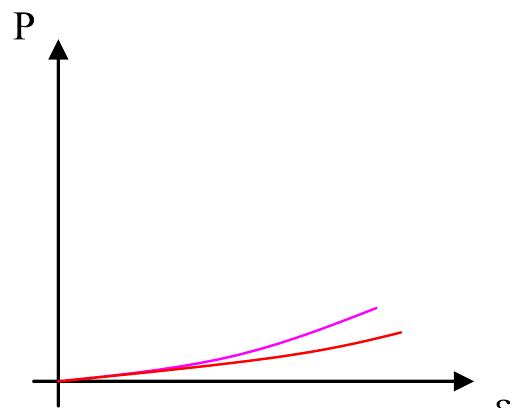
The optimal search radius ε maximizes the probability to have, for each observed star, a unique and correct association with the catalogue



Correct

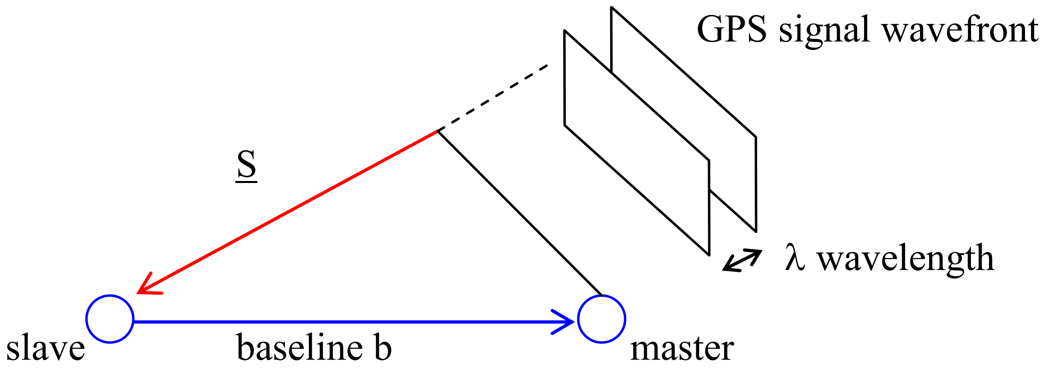


No association



Uncertain and wrong

Use of GPS sensors for attitude determination



path difference of the signal received by the two antennas $\underline{S}^T b$

transforming the vector $\underline{S}$ from geocentric to body frame $\underline{S} = AS$

$$\Delta r = S^T A^T b$$

the unknown is the rotation matrix A

Use of GPS sensors for attitude determination

$\Delta r_{11} = S_1^T A^T b_1$ baseline 1 GPS satellite 1

$\Delta r_{21} = S_2^T A^T b_1$ baseline 1 GPS satellite2

$\Delta r_{12} = S_1^T A^T b_2$ baseline 2 GPS satellite 1

$\Delta r_{22} = S_2^T A^T b_2$ baseline 2 GPS satellite2

Two baselines are required; having three GPS satellites in view of one single baseline would not allow to determine the rotation around the baseline

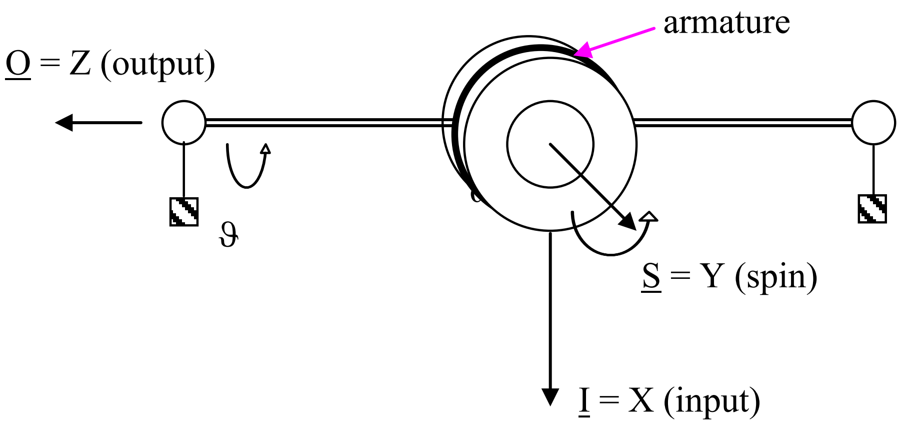
To determine the attitude, the following function can be minimized

$$J = \sum_{i=1}^{N.S} \sum_{j=1}^{N.b} \left(\Delta r_{ij} - S_i^T A^T b_J \right)$$

To get the best estimate of matrix attitude A.

Not yet used as standard ⇒ In a future with an bigger number of satellite to get reference from will be much more precise.
obviously the satellite should be big enough for precise measurement b should be sufficient high.

Inertial sensors – Mechanical Gyros

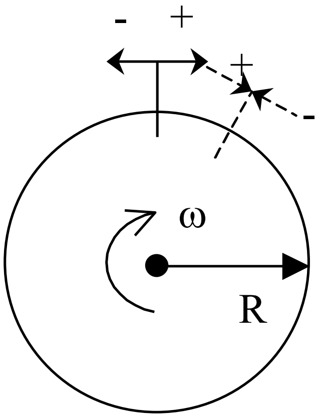


RATE GYRO (RG) $\bar{\vartheta} = -\frac{I_R \omega_R \omega_x}{k}$ $\omega_x = -\frac{k\bar{\vartheta}}{I_r \omega_r}$

RATE INTEGRATING GYRO (RIG) $\bar{\dot{\vartheta}} = -\frac{I_R \omega_R \omega_x}{c}$ $\omega_x = -\frac{c\bar{\dot{\vartheta}}}{I_r \omega_r}$

Constrain the gyros only with damping

Ring laser Gyro – Fibre optic gyro



For simplicity, the description will assume a circular optical path

$$ct^{-} = (2\pi - \omega t^{-})R \qquad t^{-} = \frac{2\pi R}{c + \omega R}$$

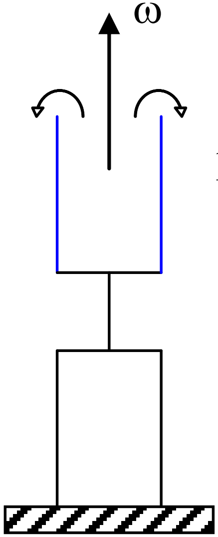
$$ct^{+} = (2\pi + \omega t^{+})R \qquad t^{+} = \frac{2\pi R}{c - \omega R}$$

$$\Delta t = (t^{+} - t^{-}) = \frac{2\pi R(c + \omega R) - 2\pi R(c - \omega R)}{(c^{2} - \omega^{2}R^{2})} = \frac{4\pi R^{2}\omega}{(c^{2} - \omega^{2}R^{2})} \cong \frac{4\pi R^{2}\omega}{c^{2}} = \frac{4A\omega}{c^{2}}$$

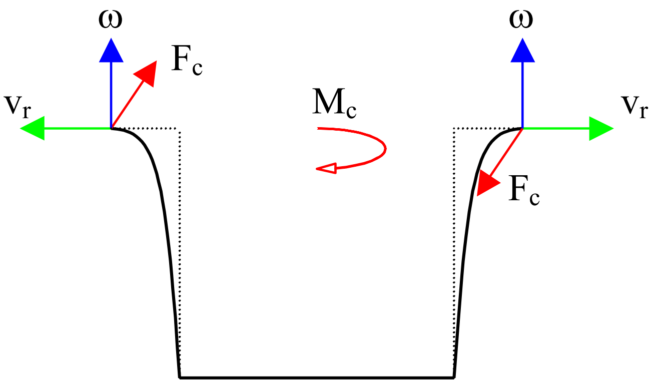
$$\omega = \frac{\Delta t c^{2}}{4A}$$

RING LASER GYRO (RLG) → optical path is a cavity
FIBER OPTIC GYRO (FOG) → optical path is made of a fiber optic

Solid sate gyro



$F_c = -2\underline{\omega} \wedge \underline{v}_r$



Sensor update rates

Star sensor – 1-5 Hz

Sun Sensor – 1-5 Hz

Earth Horizon sensor – 1 Hz

Magnetometer – 5 Hz

Gyro – 40-1000 Hz

Approximate sensor accuracy

Reference Object	Potential accuracy
Stars	1 arc second
Sun	8 arc minute
Earth (horizon)	10 arc minutes
Magnetometer	300 arc minutes

1 arcminute is 1/60th of a degree
1 arcsecond is 1/360th of a degree