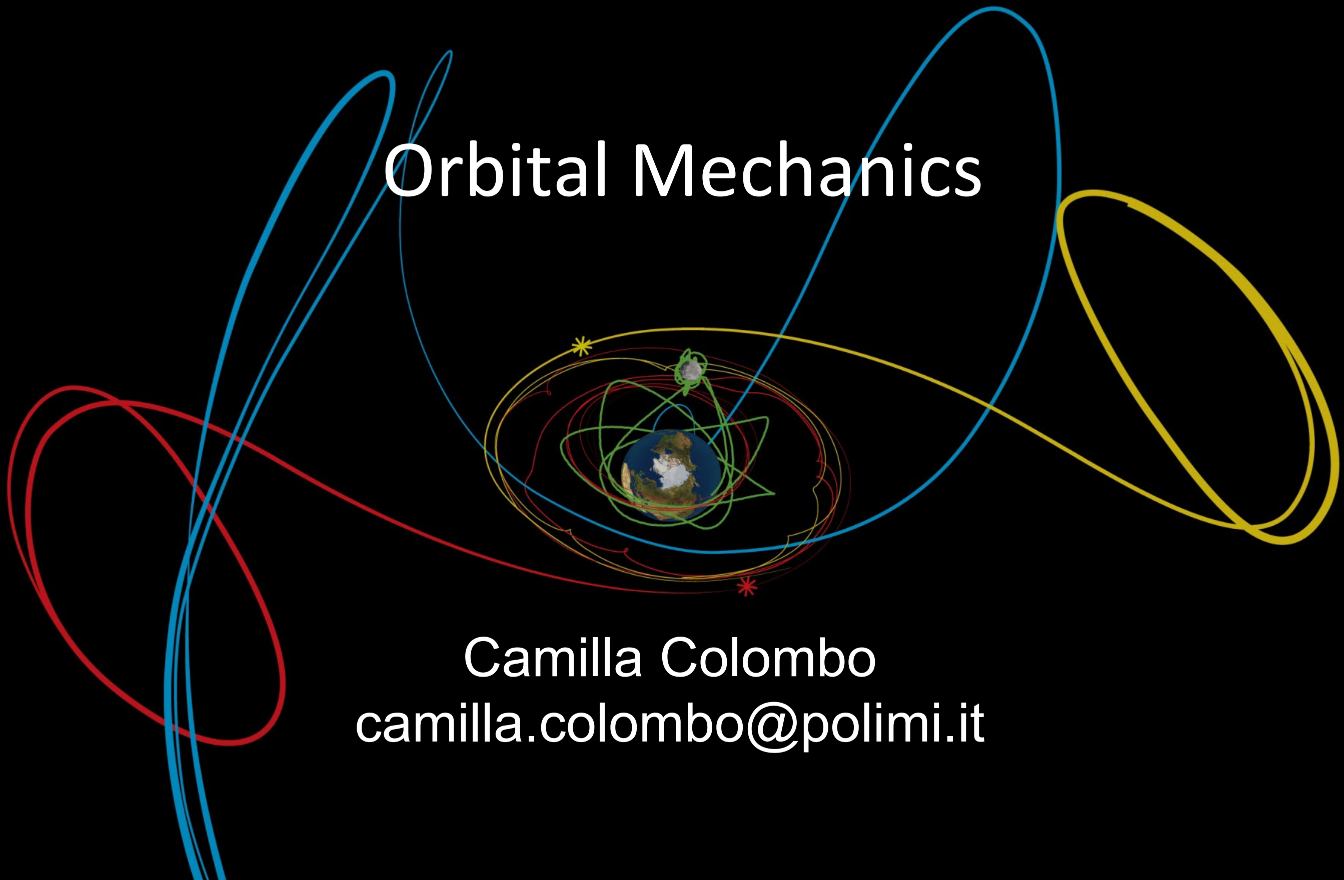
Orbital Mechanics



Camilla Colombo
camilla.colombo@polimi.it

1. TWO BODY PROBLEM

1. The two body problem

General and restricted two-body problem

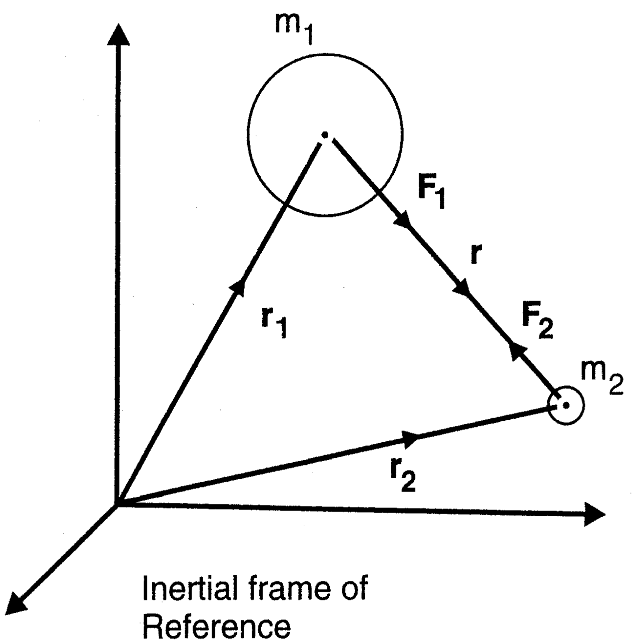
- Newton's Laws
  - Gravity $\mathbf{F}_1 = \dfrac{Gm_1m_2}{r^2}\dfrac{\mathbf{r}}{r}$
  - Motion

    second law: $m_1\ddot{\mathbf{r}}_1 = \mathbf{F}_1$

    $m_2\ddot{\mathbf{r}}_2 = \mathbf{F}_2$

    third law: $\mathbf{F}_1 = -\mathbf{F}_2$

    hence $\ddot{\mathbf{r}}_2 - \ddot{\mathbf{r}}_1 = -G(m_1 + m_2)\dfrac{\mathbf{r}}{r^3}$



$G = 6.67\times10^{-20}\,\mathrm{km}^3/\mathrm{s}^2\mathrm{kg}$

(notation $\ddot{\mathbf{r}} \equiv \dfrac{d^2\mathbf{r}}{dt^2}$

POLITECNICO MILANO 1863

If we let $m_1 >> m_2$, and $\mu = G \cdot m_1$, then

$$\ddot{\mathbf{r}} + \frac{\mu}{r^3}\mathbf{r} = 0 \qquad \text{(where } \mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1\text{)}$$

- Moment of momentum (orbital angular momentum):

$$\boxed{m\mathbf{h} = \mathbf{r} \times m\mathbf{V}\ , \quad \frac{d\mathbf{h}}{dt} = \mathbf{0}} \quad (1.12) \quad (1.13)$$

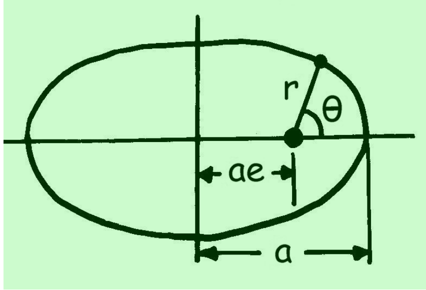
- Orbital energy

$$\boxed{\frac{1}{2}V^2 - \frac{\mu}{r} = \varepsilon = -\frac{\mu}{2a}} \quad (1.10) \quad \Longrightarrow \quad \boxed{V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

POLITECNICO MILANO 1863

The solution of equations of motion is

$$\boxed{r = \frac{a(1-e^2)}{(1+e\cos\theta)}} \quad (1.42) \qquad \boxed{r = \frac{h^2/\mu}{(1+e\cos\theta)}} \quad (1.26)$$



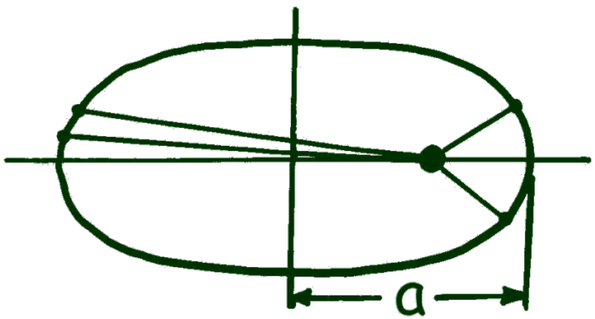
which is the polar equation of a conic section

This is the ‘equation of unperturbed motion’ of m_2 with respect to m_1, assuming mass of m_2 to be negligible.

POLITECNICO MILANO 1863

- Kepler’s Laws of Planetary Motion:
  - Laws 1 and 2:
  - Law 3: Orbit Period

$$\boxed{\tau = 2\pi\sqrt{\frac{a^3}{\mu}}, \quad \mu = Gm_1} \qquad (1.59)$$



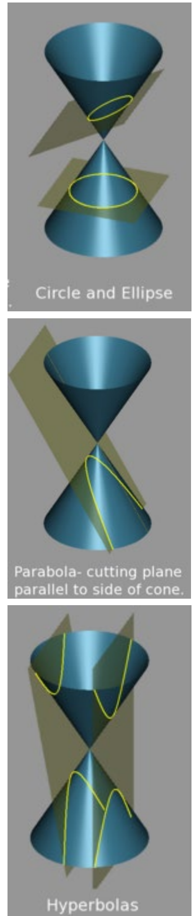
POLITECNICO MILANO 1863

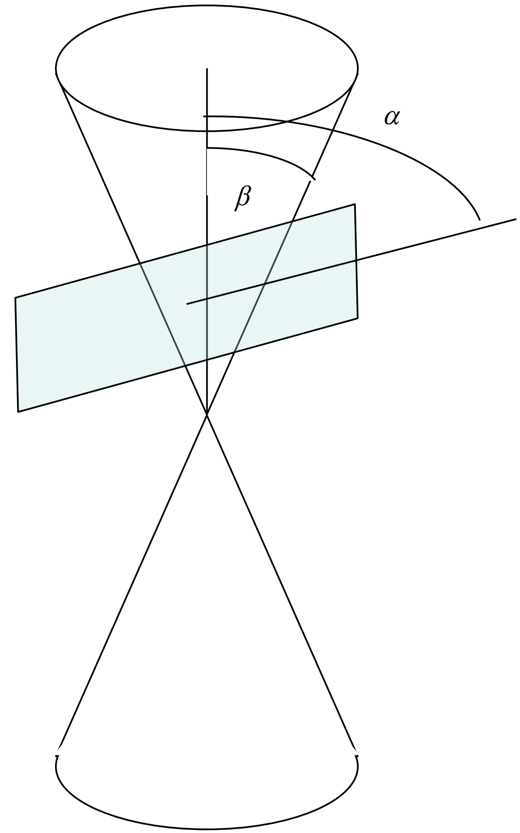
Properties of conic sections

$$\boxed{r = \frac{a(1-e^2)}{(1+e\cos\theta)}}$$

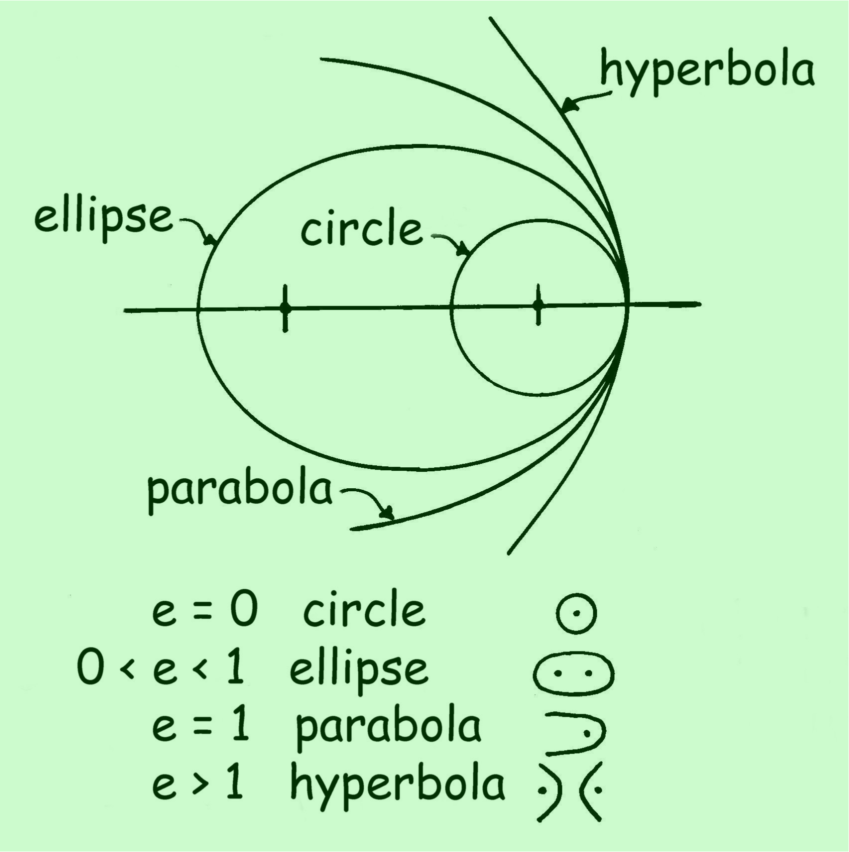
Trajectory is:

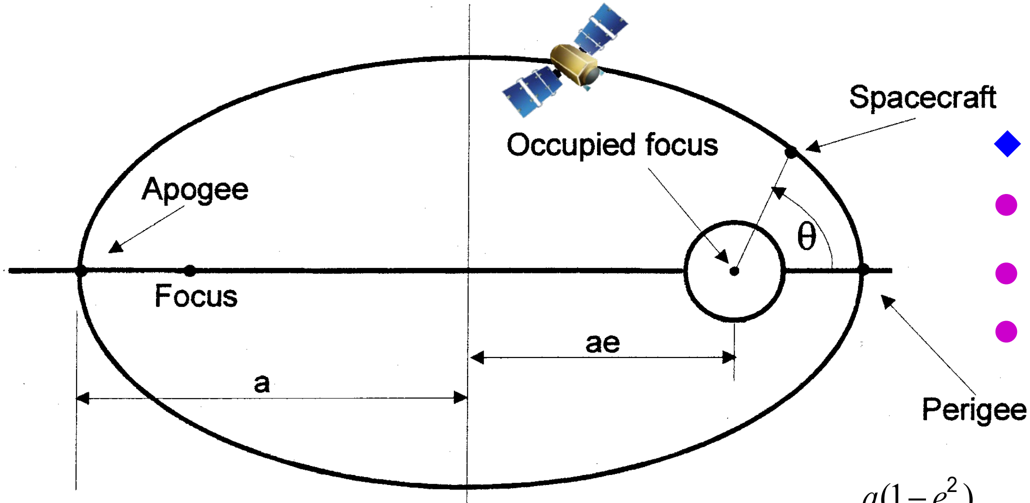
- circular for $e = 0$
- elliptic for $0 < e < 1$
- parabolic for $e = 1$
- hyperbolic for $e > 1$





Types of conics





$$r = \frac{a(1-e^2)}{(1+e\cos\theta)}$$

◆ **In-plane orbit elements**

- size : semi-major axis, a
- shape : eccentricity, e
- orbital position : true anomaly, θ

◆ **Orbit period:**

$$\tau = 2\pi\sqrt{\frac{a^3}{\mu}}$$

where μ = Gm

◆ **Orbit shape:**

- perigee $r_p = a(1-e)$
- apogee $r_a = a(1+e)$

$$r_a + r_p = 2a$$

$$\frac{r_a - r_p}{r_a + r_p} = \frac{a(1+e) - a(1-e)}{2a} = \frac{2ae}{2a} = e$$

POLITECNICO MILANO 1863

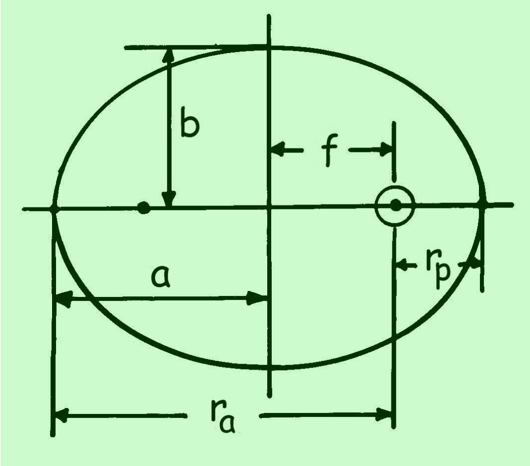
Ellipse. ($0 < e < 1$)

From Eq. (2): $$r_p = \frac{a\left(1-e^2\right)}{1+e\cos 0^{\circ}} = \frac{a\left(1-e^2\right)}{1+e} = a(1-e) \qquad (1.45)$$

$$r_a = \frac{a\left(1-e^2\right)}{1+e\cos 180^{\circ}} = \frac{a\left(1-e^2\right)}{1-e} = a(1+e) \qquad (1.46)$$

$$f = a - r_p = a - a(1-e) = ae \qquad (1.43)$$

(f is the distance between the geometric centre of the ellipse and its focus)



From Eq. (1.10): $\varepsilon < 0$

Semi-minor axis $b = a\left(1-e^2\right)^{\frac{1}{2}}$

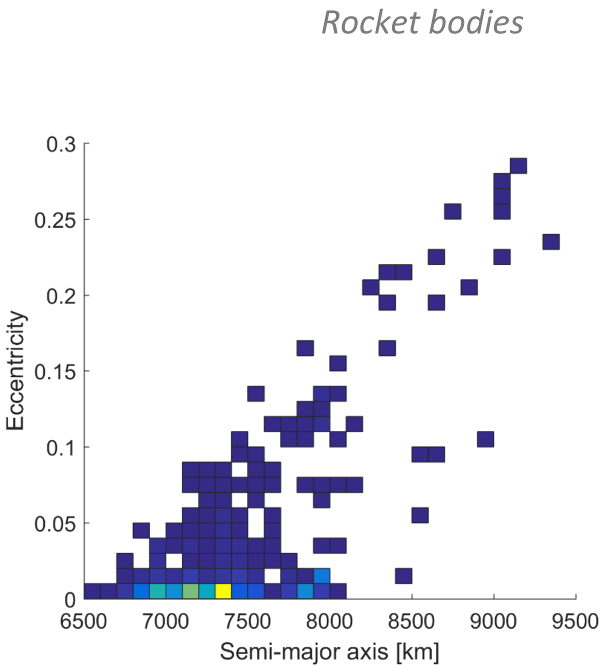
Circle. ($e = 0$)

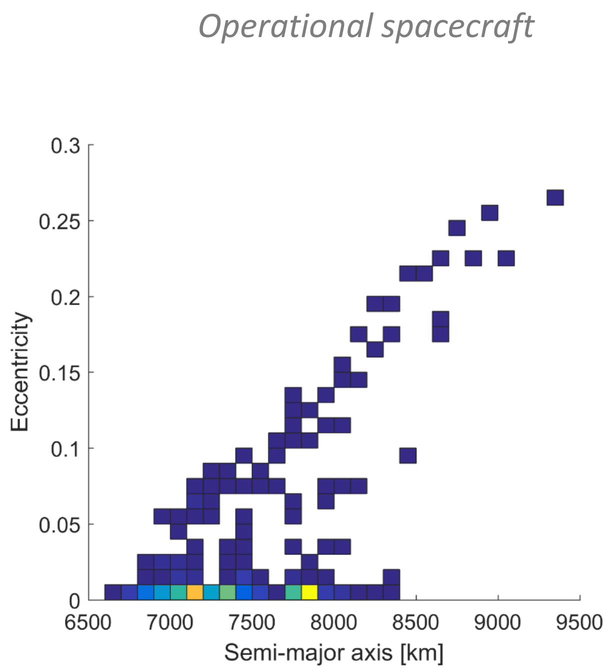
From Eq. (1.42): $r = a$

from Eq. (1.10): $V_{circ}^2 = \mu\left(\frac{2}{r} - \frac{1}{r}\right) = \frac{\mu}{r}$ $\Rightarrow$ $\boxed{V_{circ} = \sqrt{\frac{\mu}{r}}}$ (1.62)

From Eq. (1.10): $\boxed{\varepsilon = -\frac{\mu}{2a} < 0}$

Low Earth Orbit

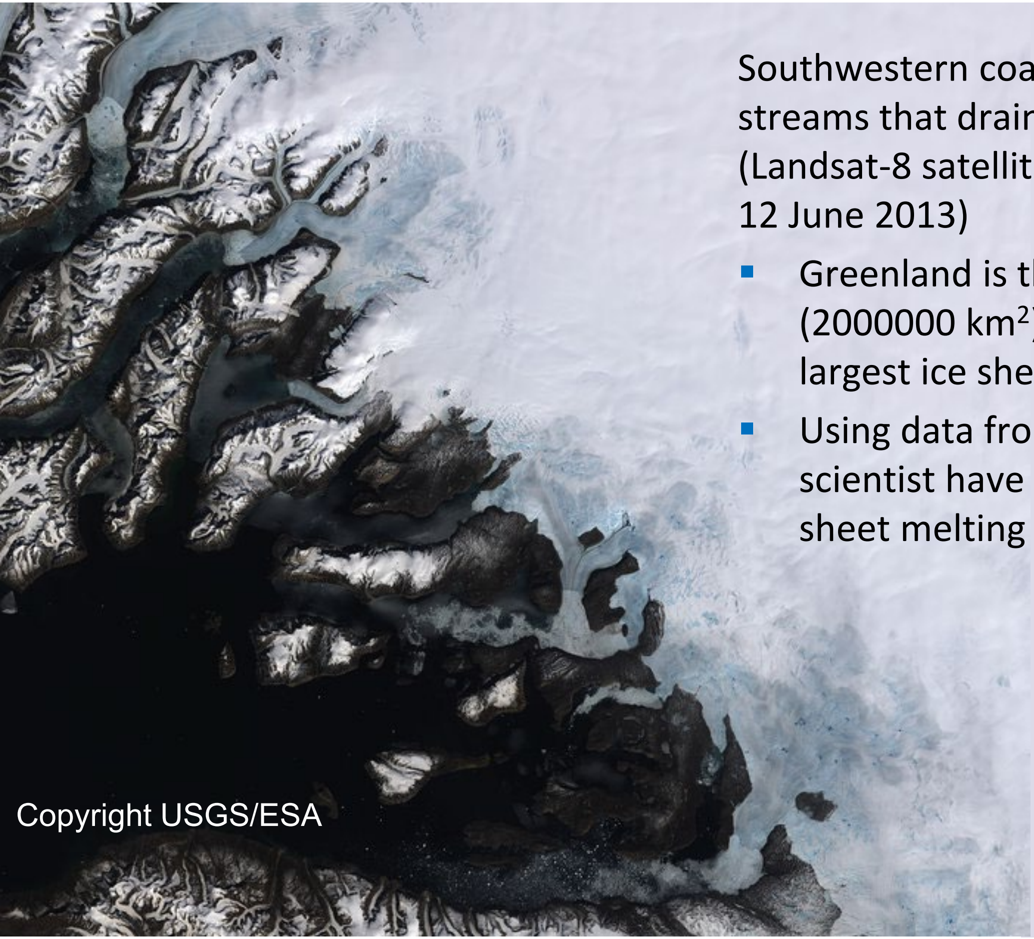




Number of objects in each semi-major axis and eccentricity bin. Bin number = [29, 29].

- Data from 2013 space debris population kindly provided by Technical University of Braunschweig

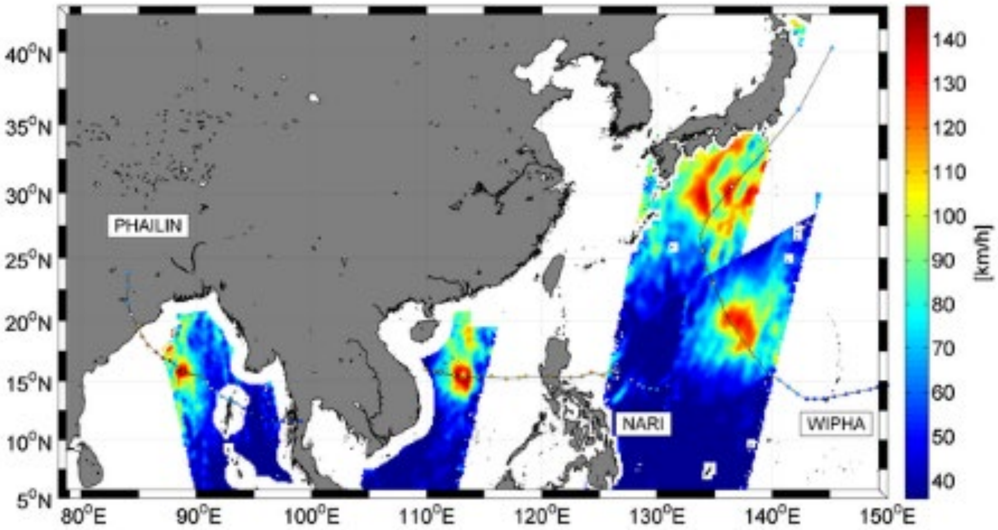
LEO missions: climate change



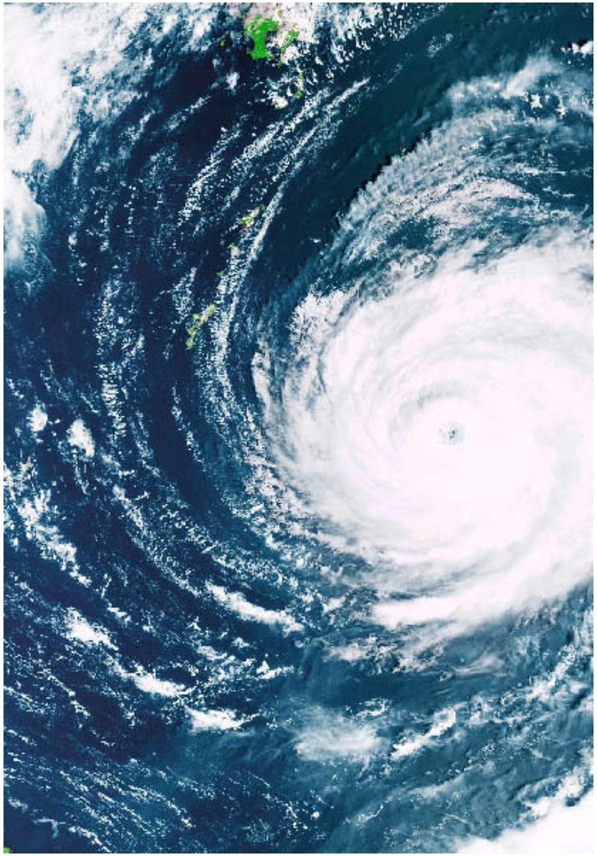
Southwestern coast of Greenland, multiple ice streams that drain the Greenland ice sheet (Landsat-8 satellite's Operational Land Imager on 12 June 2013)

- Greenland is the world's largest island (2000000 km^2) and home to the second largest ice sheet after Antarctica.
- Using data from Earth-observing satellites scientist have discovered that the rate of ice sheet melting is increasing.

LEO missions: weather forecast



SMOS's microwave radiometer captured wind speed readings from three different typhoons during 10–15 October 2013.

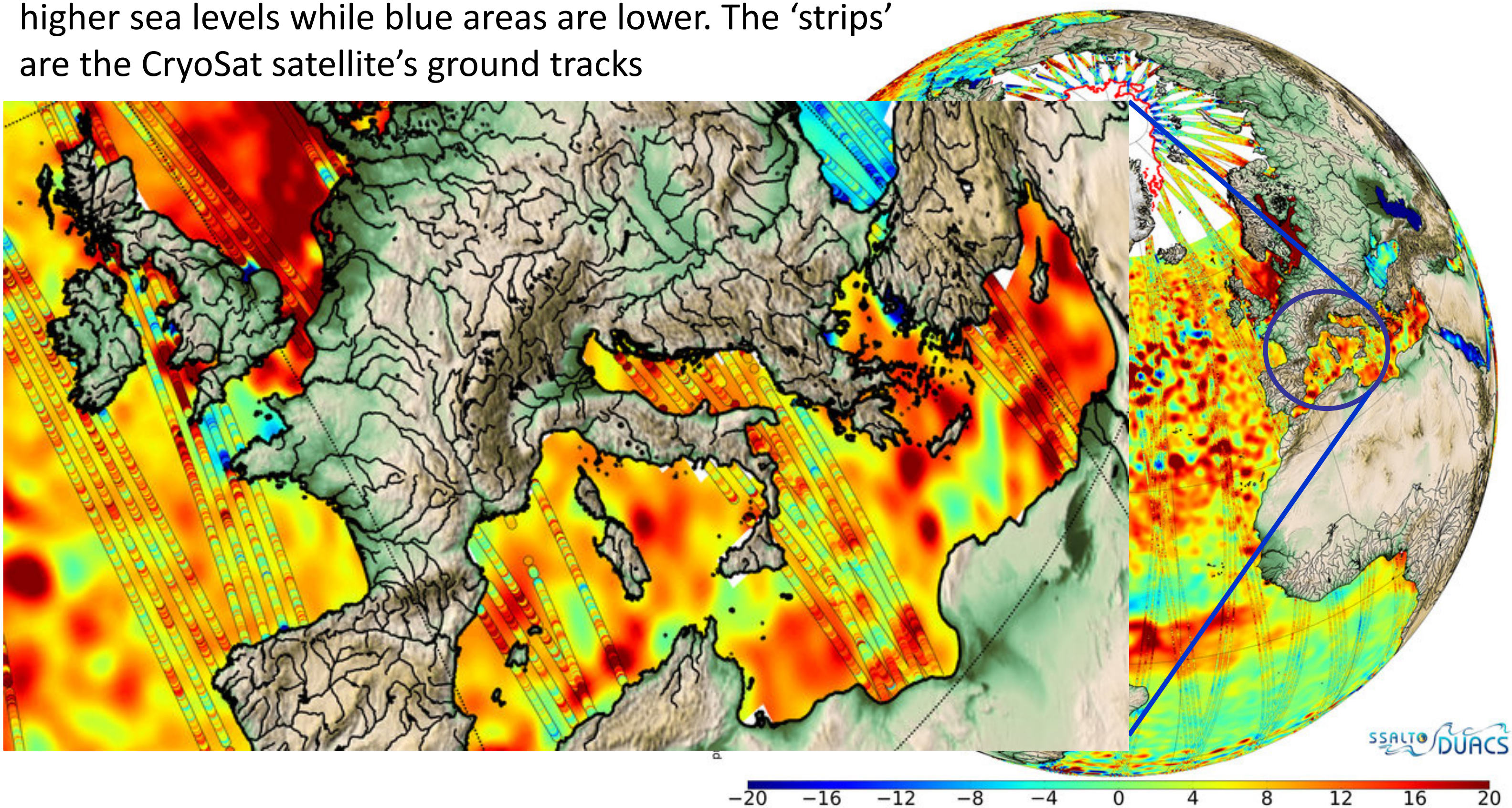


4 September 2004 MERIS image of Typhoon Songda as it approached the Japanese homeland.

- http://www.esa.int/Our_Activities/Observing_the_Earth/The_Living_Planet_Programme/Earth_Explorers/SMOS

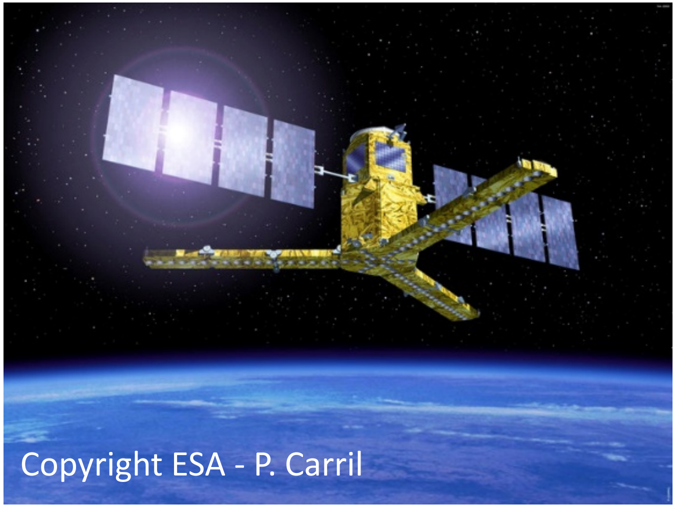
LEO missions: sea-surface topography

Detail of sea-surface topography (2013): red represents higher sea levels while blue areas are lower. The 'strips' are the CryoSat satellite's ground tracks

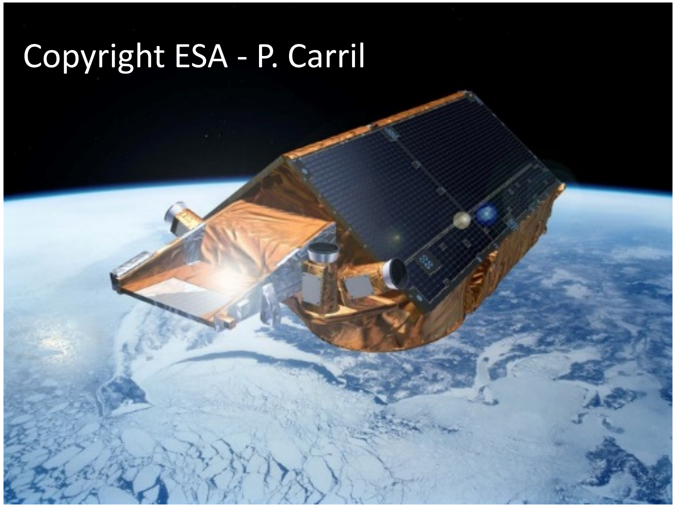


LEO missions

- The Soil Moisture and Ocean Salinity (SMOS) Earth Explorer satellite. Data from SMOS will result in a better understandiong of Earth's water cycle.



- The information from the CryoSat will lead to a better understanding of how the volume of ice on Earth is changing and, in turn, a better appreciation of how ice and climate are linked



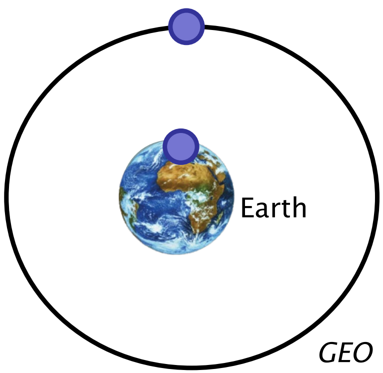
Geostationary Earth Orbit

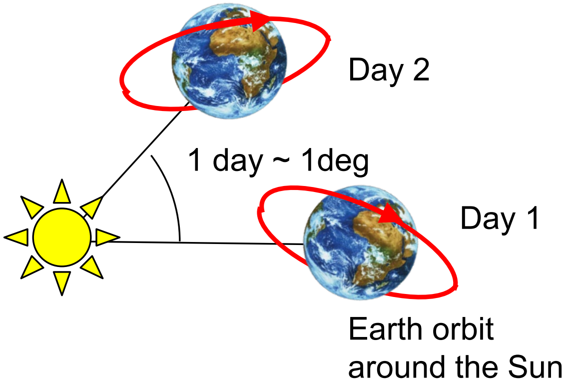
Geostationary Earth orbit:

- Circular, equatorial orbit
- Spacecraft remains 'stationary' above a fixed point on the equator

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

→ period T = 23 hours 56 min (one sidereal day)
radius r = 42164 km (height h = 35786 km)

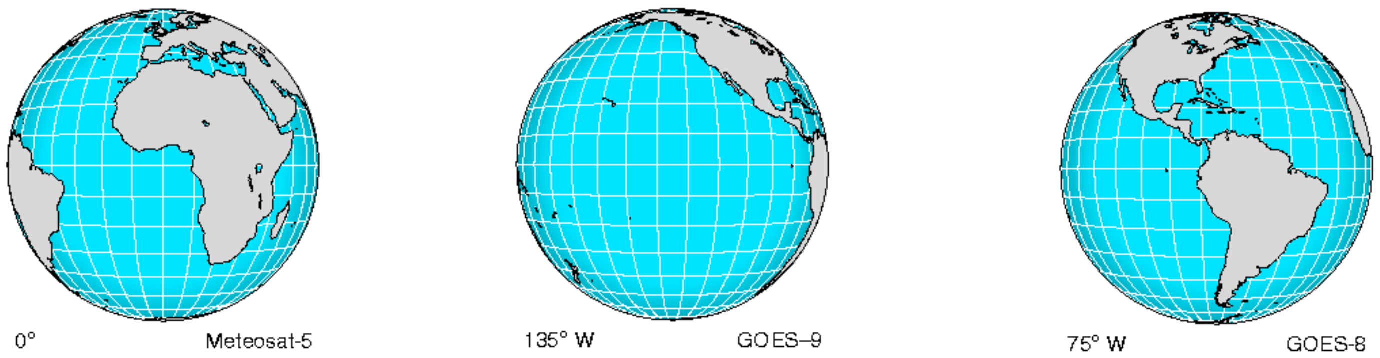




Geostationary Earth Orbit

- GEO provides a real-time, hemispherical view (but poor spatial resolution)
- Minimum of 3 spacecraft for full global coverage (international cooperation)
- Good view of low latitudes, but poor view of the poles (shortened view): Coverage is good up to 70°latitude (North and South), beyond which the spacecraft elevation falls below 10°

GEO view at different longitudes



POLITECNICO
MILANO 1863

Parabola. (e = 1)

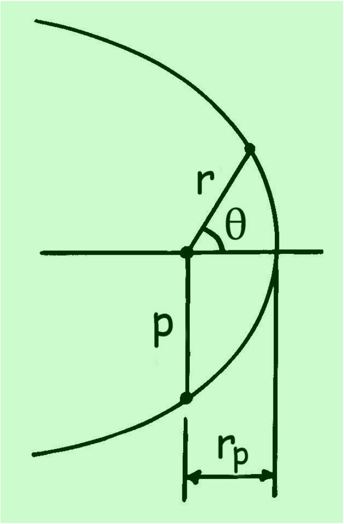
Definition: when θ = 90° (or 270°) (for a general conic):

Semi-latus rectum

$$r = \frac{a\left(1-e^2\right)}{1+e\cos 90^\circ} \quad \Rightarrow \quad \boxed{r\Big|_{\theta=90^\circ} = a\left(1-e^2\right) \equiv p} \qquad (1.70)$$

For a parabola, $a = p\big/(1-e^2) \rightarrow \infty$

Also $r_p = \frac{p}{\left(1+\cos 0^\circ\right)} = \frac{p}{2} \quad \Rightarrow \quad \boxed{r = \frac{2r_p}{\left(1+\cos\theta\right)}} \qquad (1.73)$



Note that $r \rightarrow \infty$ as $\theta \rightarrow 180^\circ$ an open "orbit"

POLITECNICO
MILANO 1863

Orbit energy is: $\varepsilon = -\frac{\mu}{2a} = 0$

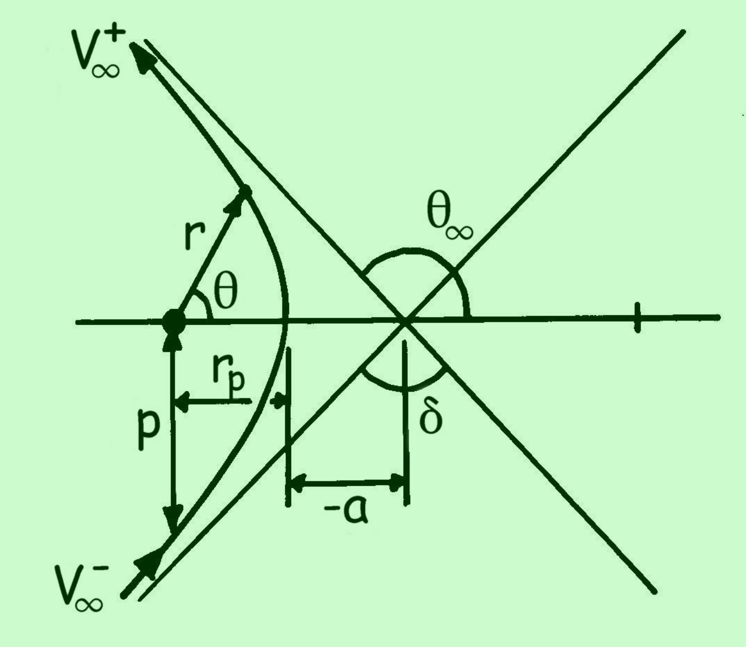
so that along trajectory $\frac{1}{2}V^2 - \frac{\mu}{r} = 0$

$\Rightarrow$ as $r \to \infty$, $V_\infty = 0$

$\Rightarrow$ Minimum energy escape trajectory $\boxed{V_{esc,min} = \sqrt{\frac{2\mu}{r}}}$ (1.67)

Hyperbola. ($e > 1$)

- $a = p/(1 - e^2) < 0$



- $r_p = \dfrac{p}{(1 + e\cos 0^\circ)} = \dfrac{p}{1+e}$ Hence $r = \dfrac{r_p(1+e)}{1 + e\cos\theta}$

- Note that $r \to \infty$ as $1 + e\cos\theta \to 0$ ...

POLITECNICO MILANO 1863

... $\Rightarrow$ an "open trajectory"

Therefore direction at great distance is given by

$$\boxed{\theta_\infty = \cos^{-1}\left(-\frac{1}{e}\right)} \qquad (1.76)$$

- Energy is positive, so excess velocity is positive

$$\underline{\varepsilon = -\frac{\mu}{2a} > 0} \quad \Rightarrow \quad \underline{V_\infty > 0}$$

- Velocity at great distance:

  Energy equation –

$$\frac{1}{2}V^2 - \frac{\mu}{r} = -\frac{\mu}{2a} \quad \Rightarrow \quad \text{As } r \to \infty,\ V_\infty^2 = -\frac{\mu}{a}$$

$$\Rightarrow \quad \boxed{V_\infty = \sqrt{\frac{-\mu}{a}}\ , \quad a<0} \qquad (1.85)$$

- Eccentricity:

  At periapsis, θ = 0°

$$\Rightarrow \quad r_p = \frac{\left(-\frac{\mu}{V_\infty^2}\right)\left(1-e^2\right)}{\left(1+e\right)}$$

$$\Rightarrow \quad \boxed{e = 1 + \frac{r_p V_\infty^2}{\mu}} \qquad (1.89)$$

- Deflection angle δ:

  From diagram,

$$\theta_\infty = 90^o + \frac{\delta}{2} \Rightarrow \cos\theta_\infty = \cos\left(90^o + \frac{\delta}{2}\right) = -\frac{1}{e}$$

$$\therefore \sin\left(\frac{\delta}{2}\right) = \frac{1}{e} \Rightarrow \boxed{\delta = 2\sin^{-1}\left(\frac{1}{e}\right)} \quad (1.80)$$

Let's look to another parameter of an elliptical orbit.

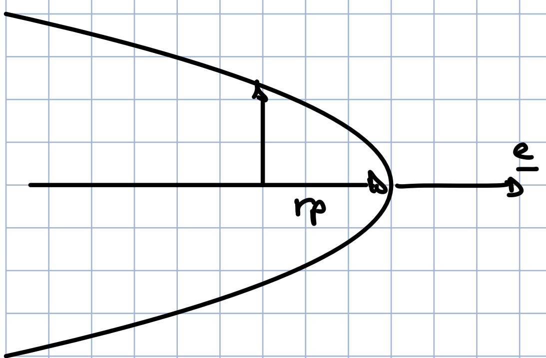
$$p = r\left(\theta = \frac{\pi}{2}\right) = a(1-e^2) \qquad (1.70)$$

→ valid also for a parabola

$$\boxed{a_p = \frac{p}{1-e^2} \longrightarrow \infty} \qquad (1.71)$$

$$r_p = r(\theta = 0) = \frac{p}{2} \Rightarrow \boxed{r_p = \frac{p}{2}} \qquad (1.72)$$

for parabolic orbit.



$$\boxed{r = \frac{2r_p}{1+\cos\theta}} \qquad (1.73)$$

Alternative formula for a parabolic orbit.

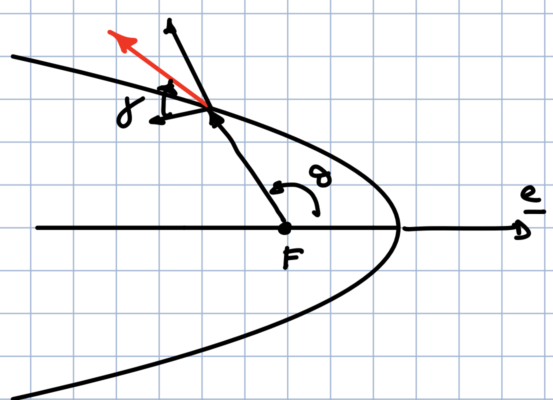
$$\text{Eq}(1.37) \rightarrow \tan\gamma = \frac{e\sin\theta}{1+e\cos\theta} \longrightarrow \tan\gamma = \frac{\sin\theta}{1+\cos\theta}$$

RECALL ⇒ $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ $\qquad 1+\cos\theta = 2\cos^2\frac{\theta}{2}$

$$\tan\gamma = \frac{\cancel{2}\sin\frac{\theta}{2}\,\cancel{\cos\theta/2}}{\cancel{2\cos^2}\frac{\theta}{2}} = \frac{\sin\theta/2}{\cos\frac{\theta}{2}} \Longrightarrow \tan\gamma = \tan\frac{\theta}{2}$$

$$\Rightarrow \boxed{\gamma = \frac{\theta}{2}} \qquad (1.74)$$

For a parabolic orbit → direct relation between the flight path angle γ and the true anomaly θ.



HYPERBOLIC TRAJECTORY.

$e>1$ (eq 1.26 $r = \frac{h^2}{\mu} \frac{1}{1+e\cos\theta}$)

$e>1 \Rightarrow r \to \infty$

$1 + e\cos\theta \to 0 \qquad \theta_\infty = \cos^{-1}\left(-\frac{1}{e}\right)$ (1.75

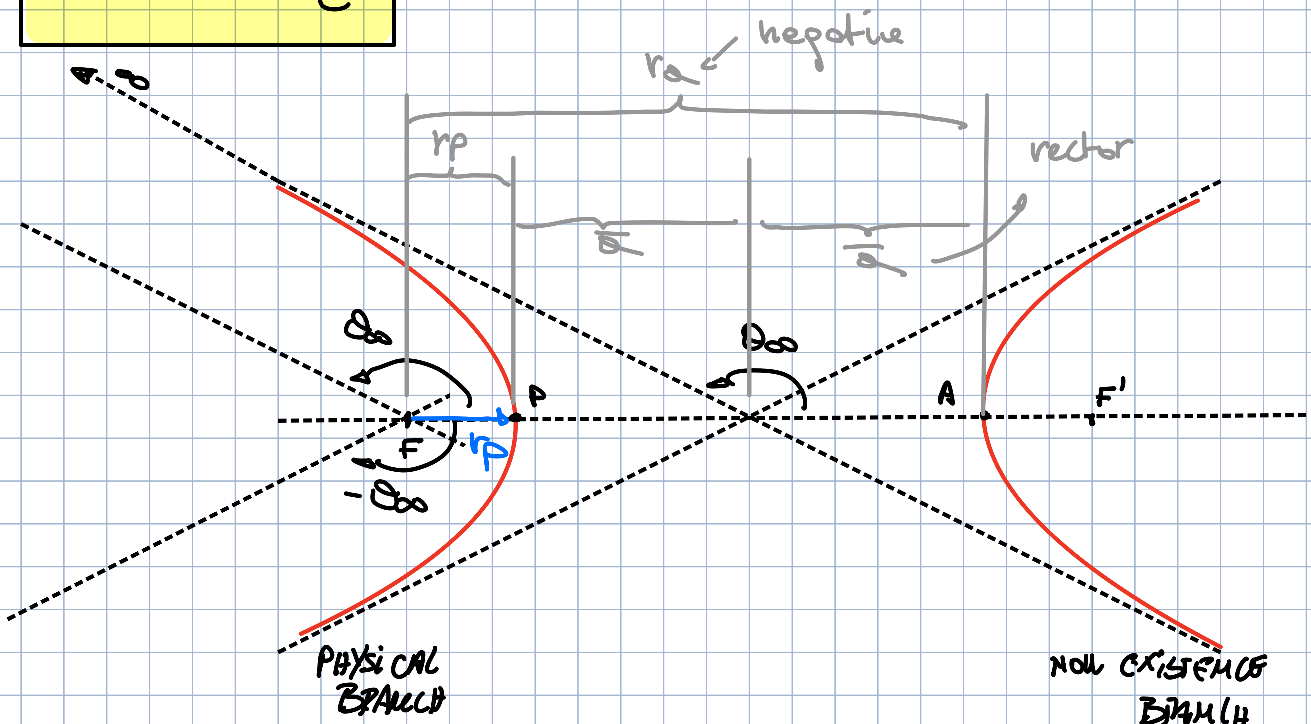
$r \to \infty$ for θ_∞ = TRUE ANOMALY ASYMPTOTE

$\frac{\pi}{2} < \theta_\infty < \pi$

After p — Before Apocenter

$\cos\theta_\infty = -\frac{1}{e} \to \sin\theta_\infty = \sqrt{1-\cos^2\theta_\infty} = \sqrt{1-\frac{1}{e^2}}$

$$\boxed{\sin\theta_\infty = \frac{\sqrt{e^2-1}}{e}} \qquad (1.76)$$



At θ_∞ we will never intersect the trajectory.

$\theta \in [-\theta_\infty ; +\theta_\infty]$ REAL HYPERBOLIC CURVE

$\theta \in [\theta_\infty ; 2\pi - \theta_\infty]$ VACANT FOCUS HYPERBOLA

↳ PHYSICALLY IMPOSSIBLE → requires repulsive gravitational force.

from : eq (1.35) $r_p = \frac{h^2}{\mu} \frac{1}{1+e}$ $r_p > 0$

eq (1.39) $r_a = \frac{h^2}{\mu} \frac{1}{1-e}$ $r_a < 0$ as $e > 1$

↳ is negative because it belongs to the vacant focus part of the hyperbola

from : eq (1.42) p still defined → (exist for every conic)

$$\boxed{p = r\left(\theta = \frac{\pi}{2}\right) = a(1-e^2)} \quad (1.77)$$

$$\boxed{a_{Hy} = \frac{p}{1-e^2} < 0} \quad (1.78)$$

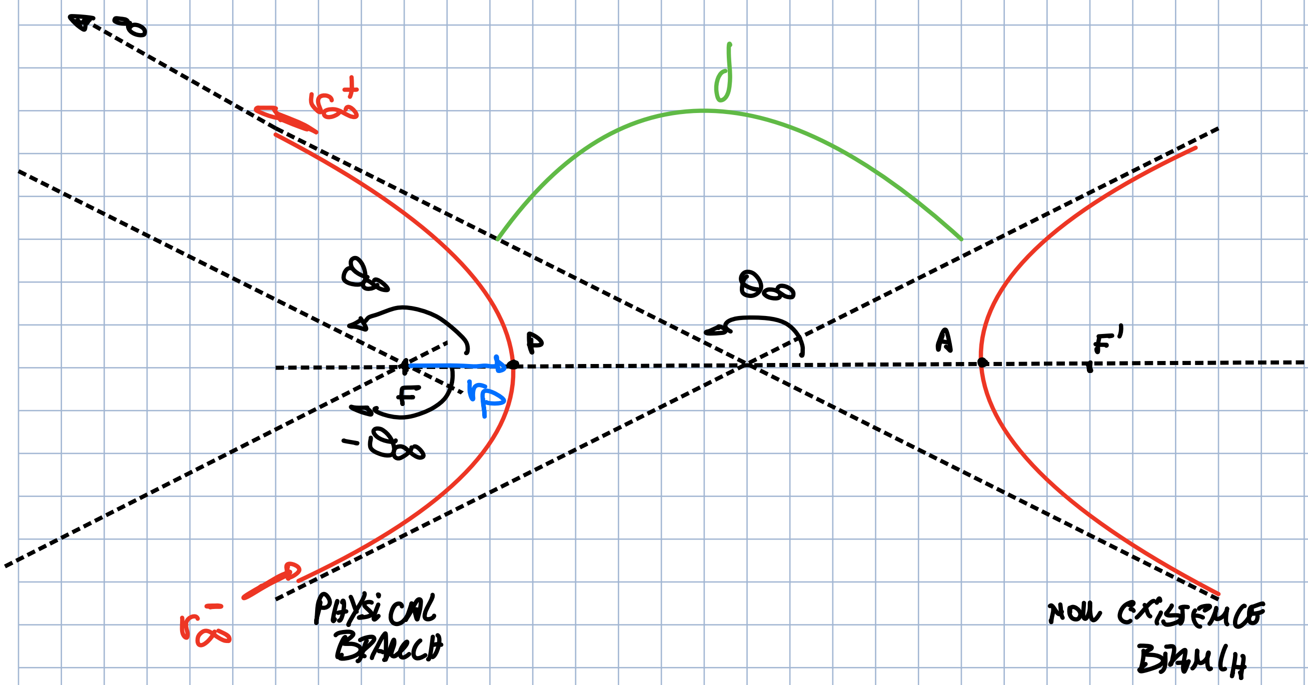
↳ negative because it is measure outside the real part of the hyperbola.

$\overline{PA} = |r_a| - r_p = -r_a - r_p$

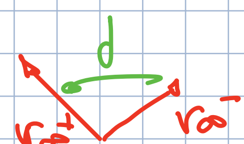
$$r_p = a(1-e) = -\bar{a}(1-e) = \underset{\substack{\uparrow \\ \text{module of } a}}{\bar{a}}(e-1)$$

$$\boxed{r_p = \bar{a}(e-1)}$$

$$r_a = a(1+e) = -\bar{a}(1+e) \Rightarrow \boxed{r_a = -\bar{a}(1+e)}$$



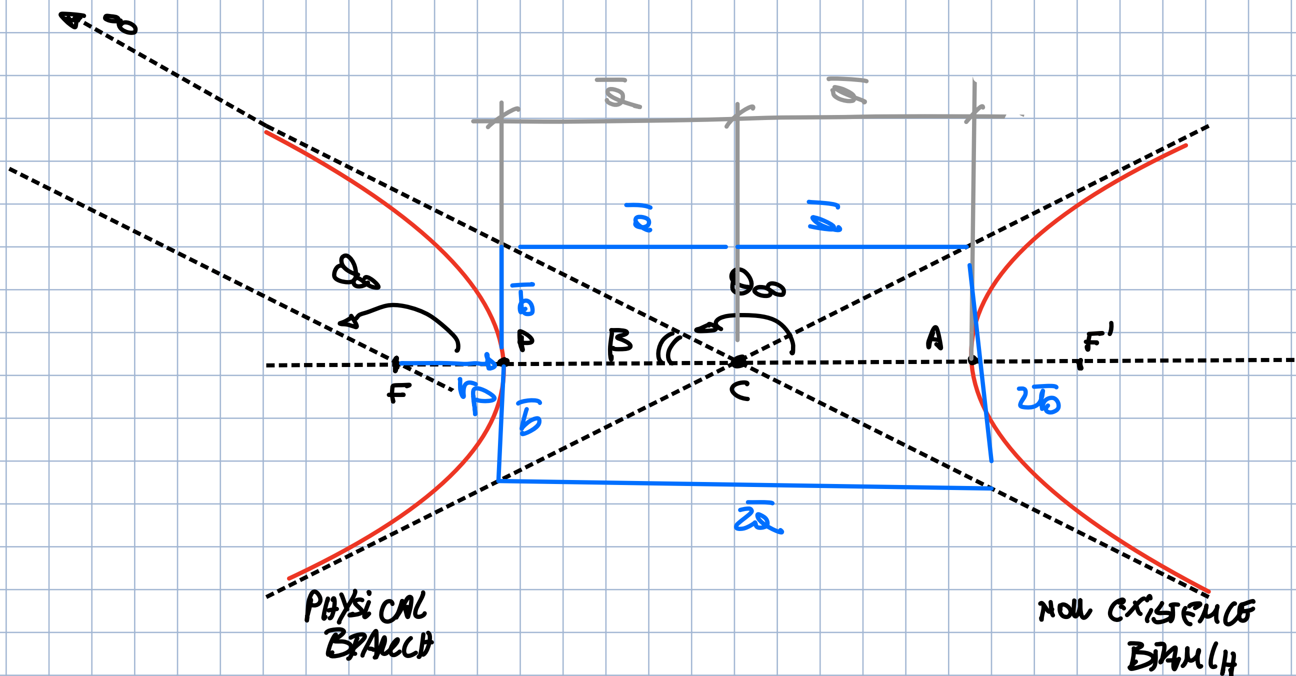
δ = DEFLECTION ANGLE – TURN ORBIT angle through which the velocity is rotated orbiting m_1 @ F.



$$\theta_\infty = \frac{\pi}{2} + \frac{\delta}{2} \rightarrow \cos\theta_\infty = \cos\left(\frac{\pi}{2} + \frac{\delta}{2}\right) \rightarrow \cos\theta_\infty = -\sin\frac{\delta}{2}$$

therefore from eq (1.75)

$$\sin\frac{\delta}{2} = \frac{1}{e} \qquad \boxed{\delta = 2\sin^{-1}\left(\frac{1}{e}\right)} \quad (1.80)$$



$\bar{b}$ = semi minor axis.

$$\bar{b} = \bar{a}\,tg\,\beta = \bar{a}\,tg(\pi - \theta_\infty) = \bar{a}\,tg(-\theta_\infty) = -\bar{a}\,tg\,\theta_\infty$$

$$= -\bar{a}\,\frac{\sin\theta_\infty}{\cos\theta_\infty} = +\bar{a}\,\frac{\sqrt{e^2-1}}{e}\,e = \bar{a}\sqrt{e^2-1}$$

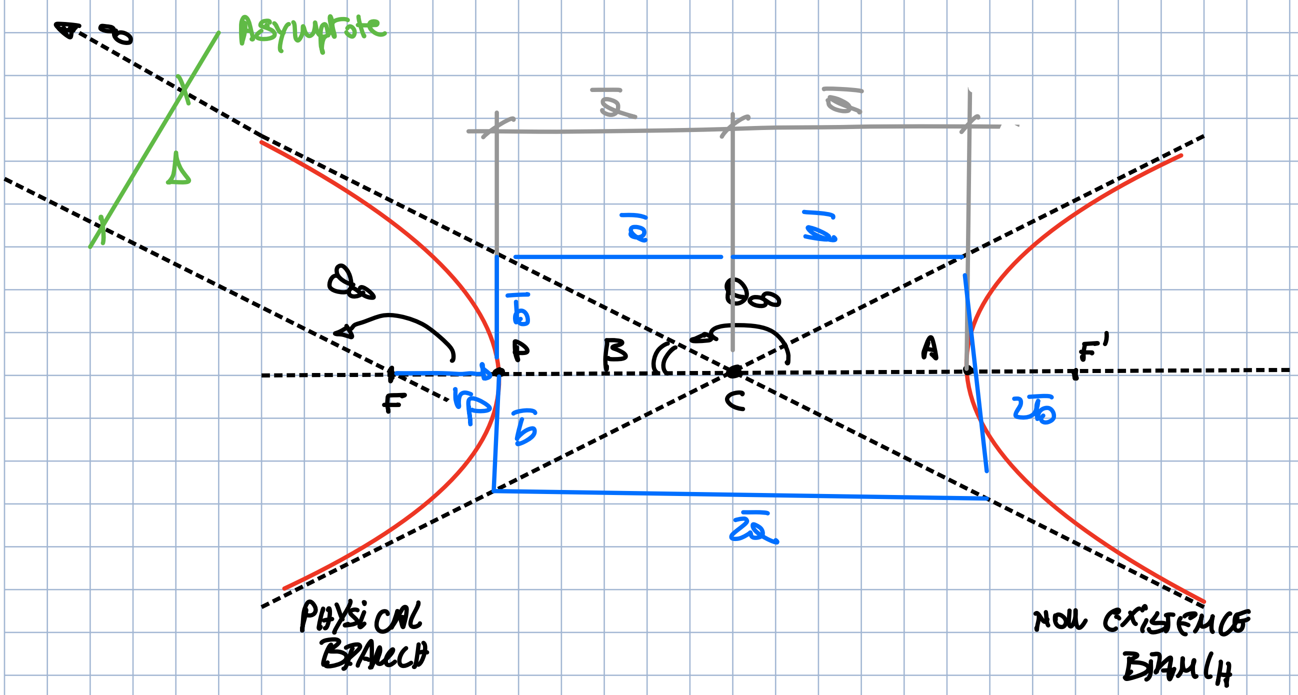
$$\boxed{\bar{b} = \bar{a}\sqrt{e^2-1}} \qquad (1.81)$$

NOTE

$$b = a\sqrt{1-e^2} = -\bar{a}\sqrt{1-e^2} = -\bar{a}\sqrt{-(e^2-1)} = -\bar{a}\,i\sqrt{(e^2-1)}$$

$$\Rightarrow \quad \boxed{b = -\underset{\text{complex unit}}{i}\,\bar{b}}$$

b is complex because it can not be defined inside the hyperbola.



$$\Delta = (r_p + \bar{a}) \sin\beta = (r_p + \bar{a}) \sin(\pi - \theta_\infty) = (r_p + \bar{a}) \sin\theta_\infty$$

↑ asymptote

$$= (\underbrace{\bar{a}(e-1)}_{\text{eq } (1.79)} + \bar{a}) \sin\theta_\infty = \bar{a} e \frac{\sqrt{e^2-1}}{e} = \bar{a}\sqrt{e^2-1}$$

$$\Delta = \bar{b} = \bar{a}\sqrt{e^2-1} \qquad (1.82)$$

Δ = distance between asymptote and focus = IMPACT PARAMETER AIMING RADIUS

As $r_p = \frac{p}{1+e} = \bar{a}(e-1)$

From eq (1.42) $\quad r = \frac{p}{1+e\cos\theta} = \frac{r_p(1+e)}{1+e\cos\theta}$

$$\boxed{r = \frac{\bar{a}(e^2-1)}{1+e\cos\theta}} \qquad (1.83)$$

From eq 1.55

$$\varepsilon = -\frac{\mu}{2a} = \frac{\mu}{2\bar{a}} \qquad \boxed{\varepsilon = \frac{\mu}{2\bar{a}}} \qquad (1.84)$$

As $a_{Hyp} < 0 \Rightarrow \varepsilon_{hyp} > 0 \Rightarrow$ the excess velocity at $\infty > 0$

From (1.56)

$$\frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \qquad \text{At } \infty \qquad \frac{v_\infty^2}{2} - \cancelto{0}{\frac{\mu}{r_\infty}} = -\frac{\mu}{2a}$$

$$v_\infty = \sqrt{-\frac{\mu}{a}} \qquad \boxed{v_\infty = \sqrt{\frac{\mu}{\bar{a}}}} \qquad (1.85)$$

v_∞ is the speed at which m_2 arrives at ∞

HYPERBOLIC EXCESS SPEED OR THIRD COSMIC SPEED

NOTE

Curtis book we a as $\bar{a}$.

$$\boxed{\frac{v^2}{2} - \frac{\mu}{r} = \frac{v_\infty^2}{2}} \qquad (1.86)$$

Multiply (1.86) by 2

$$v^2 - \frac{2\mu}{r} = v_\infty^2$$

$$\Downarrow$$

$$v^2 - v_{esc}^2 = v_\infty^2$$

$$\boxed{v = v_{esc}^2 + v_\infty^2} \qquad (1.87)$$

v_∞^2 represents the excess kinetic energy over that energy which is required to escape the center of attraction.

$$C_3 = v_\infty^2 = \text{CHARACTERISTIC ENERGY} \qquad (1.88)$$

C_3 = measure of the required energy for interplanetory mission.

C_3 launcher $\geq$ C_3 mission $\rightarrow$ design requirements

$$[v_\infty^2] = [C_3] = \frac{km^2}{s^2}$$

From eq 1.85 and eq 1.75

$$v_\infty^2 = -\frac{\mu}{a} \qquad a = -\frac{\mu}{v_\infty^2}$$

$$r_p = a(1-e) \rightarrow r_p = -\frac{\mu}{v_\infty^2}(1-e)$$

$$-\frac{r_p v_\infty^2}{\mu} = 1-e \qquad \boxed{e = 1 + \frac{r_p v_\infty^2}{\mu}} \qquad (1.89)$$

e as a function of r_p and v_∞ $\rightarrow$ important parameters for the hyperbola

ORBIT RAPPRESENTATION

TIME OF FLIGHT ON ELLIPTICAL ORBIT, KEPLER'S TIME LAW

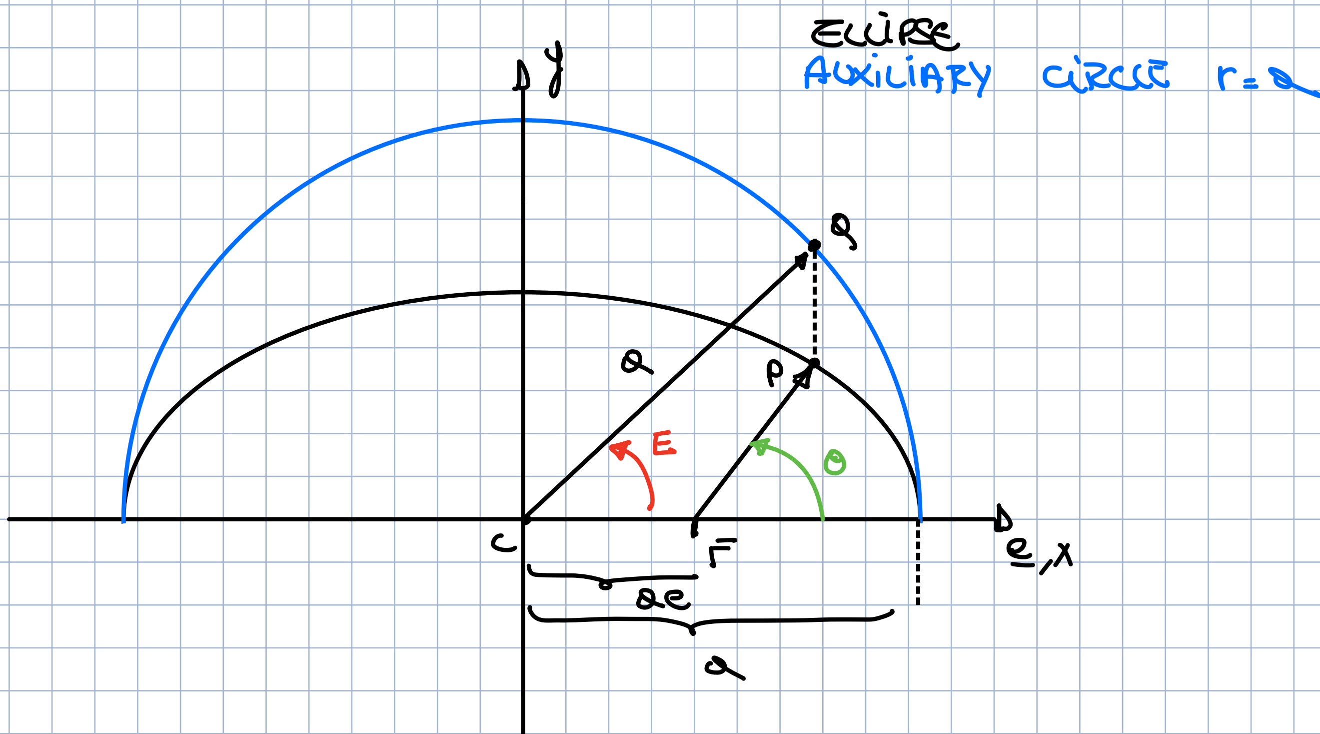
$r(\theta)$ we want to relate $r(t) \Rightarrow r(\theta(t))$

Kepler introduced another time variable to ease the process of determining $r(\theta(t))$.

E = eccentric anomaly

① Relation E and t : $E(t)$
② Relation E and θ : $\theta(E)$

$\left.\begin{matrix} E(t) \\ \theta(E) \end{matrix}\right\} \theta(E(t))$



Point are represented wrt the centre

$x_P = x_Q \qquad y_P / y_Q = \frac{b}{a}$

$$P: \begin{cases} x = a\cos E \\ y = b\sin E \end{cases} \qquad \begin{pmatrix} x_P = x_Q \\ y_P = \frac{b}{a} y_Q \quad y_Q = a\sin E \end{pmatrix} \qquad (2.1)$$

using θ instead

$$P: \begin{cases} x = ae + r(\theta)\cos\theta \\ y = r(\theta)\sin\theta \end{cases} \qquad (2.2)$$

① RELATION BETWEEN E and $t \Rightarrow E(t)$

Let's focus on x $\qquad (2.1a) = (2.2a)$

drop the dependency to ease the notation

$$a\cos E = ae + r(\theta)\cos\theta$$

$$r\cos\theta = a\cos E - ae$$

Using eq (1.42) $\qquad r(1+e\cos\theta) = a(1-e^2)$

$$r + re\cos\theta = a(1-e^2)$$

substituting $r\cos\theta$ from eq (2.3)

$$r + e\underbrace{(a\cos E - ae)}_{eq(2.3)} = a(1-e^2)$$

$$r + ea\cos E - \cancel{a}e^2 = a - \cancel{a}e^2$$

$$\boxed{r = a(1 - e\cos E)} \qquad (2.4)$$

defined link between position of the satellite and the eccentric anomaly $\Rightarrow r(E)$.

$r(E)$ ellipse $a > 0$ $\cos E < 1$ and $e < 1$ $\Rightarrow r > 0$

Compare (2.4) with (1.42). In eq (2.4) there is a linear relation between r and $\cos E$

To get time dependency we derive eq (2.4)

$$\dot{r} = \frac{dr}{dt}$$

$$\dot{r} = a e \sin E \, \dot{E} \qquad (2.5)$$

From eq 2.56

$$\frac{1}{2} v^2 - \frac{\mu}{r} = -\frac{\mu}{2a}$$

v^2 in terms of v_r and v_θ

$$\left.\begin{aligned} v_r &= \dot{r} \\ v_\theta &= r\dot{\theta} \end{aligned}\right\} \text{From orbit kinematics} \quad \text{eq } (1.23)$$

$$v^2 = \dot{r}^2 + \left(r \frac{h}{r^2}\right)^2 = \dot{r}^2 + \frac{h^2}{r^2}$$

$\hookrightarrow$ eq (1.14) $\qquad \dot{\theta} = \frac{h}{r^2}$

$$\frac{1}{2}\left(\dot{r}^2 + \frac{h^2}{r^2}\right) - \frac{\mu}{r} = -\frac{\mu}{2a}$$

$$\dot{r}^2 = \frac{2\mu}{r} - \frac{\mu}{a} - \frac{h^2}{r^2} \qquad \text{but } h^2 = p\mu \quad \text{eq } 1.38$$

$$= \frac{\mu(2ra - r^2 - ap)}{ar^2} \qquad \text{but } p = a(1-e^2)$$

$$\dot{r}^2 = \frac{\mu(2ra - r^2 - a^2 + a^2e^2)}{ar^2}$$

$$\frac{\dot{r}^2 a r^2}{\mu} = -(a-r)^2 + a^2e^2 \qquad (2.6)$$

From eq (2.4)

$$a - r = ae\cos\epsilon$$

$(a-r)$ into eq (2.6)

$$\frac{\dot{r}^2 a r^2}{\mu} = a^2e^2 - a^2e^2\cos^2\epsilon$$

$$\frac{\dot{r}^2 a r^2}{\mu} = a^2e^2\underbrace{(1-\cos^2\epsilon)}_{\sin^2\epsilon}$$

$\dot{r}$ From eq 2.5

$$\frac{\cancel{a^2}\cancel{e^2}\cancel{\sin^2\epsilon}\,\dot{\epsilon}^2 a r^2}{\mu} = \cancel{a^2}\cancel{e^2}\cancel{\sin^2\epsilon}$$

$$\dot{\epsilon}^2 = \frac{\mu}{a r^2} \longrightarrow \boxed{\dot{\epsilon} = \frac{1}{r}\sqrt{\frac{\mu}{a}}} \qquad (2.7)$$

$\dot{\epsilon} = \frac{d\epsilon}{dt}$ time derivative of the Eccentric anomaly.

taking $r(\epsilon)$ from (2.4) separate variables

$$\frac{d\epsilon}{dt} = \sqrt{\frac{\mu}{a}}\,\frac{1}{a(1-e\cos\epsilon)}$$

$$d\epsilon\,(1-e\cos\epsilon) = \sqrt{\frac{\mu}{a^3}}\,dt$$ → solve diff eq.

constant to add in order to integrate the equation.

$$\boxed{\epsilon - e\sin\epsilon = \sqrt{\frac{\mu}{a^3}}\,(t - t_p)} \qquad (2.8)$$

↳ solve by seeing $\epsilon = 0$ at a particular position.

$\mathcal{E} = 0$ @ t_p $\Rightarrow$ time at perigee passage.

$\mathcal{E}_0 = 0$ if $t = t_0 = t_p$ time of passage from perigee.

$t_p =$ 6th constant of integration ($\underline{h}$, $\underline{e}$, $\underline{h} \cdot \underline{e} = 0$)