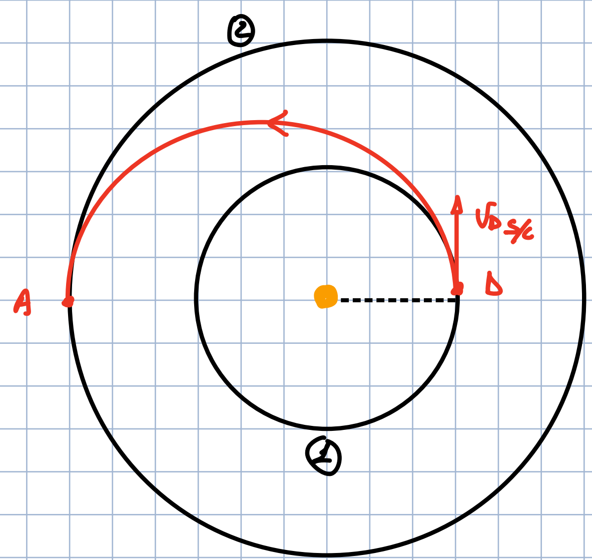
INTERPLANETARY HOHMANN TRANSFER

We use it because we know all the planet lies more or less on the ecliptic and the planet's orbit have a small eccentricity (with except for mercury and pluto)

⇒ hp : planets' orbit are coplanar and cofocal



R_1 = orbit ①

$$v_1 = \sqrt{\frac{\mu_{SUN}}{R_1}}$$

$$v_{D\,s/c} = \frac{h_T}{R_1} = \frac{1}{R_1}\sqrt{2\mu_{SUN}\frac{R_1 R_2}{R_1+R_2}} = \sqrt{2\mu_{SUN}\frac{R_2}{R_1(R_1+R_2)}}$$

$$\Delta v_D = \underbrace{v_{D\,s/c} - v_1}_{\text{excess velocity to leave Planet ①}} = \sqrt{\frac{\mu_{SUN}}{R_1}}\left(\sqrt{\frac{2R_2}{R_1+R_2}} - 1\right) \qquad (4.1)$$

Actually when a s/c is orbiting a planet for the point of view of the sun is it has the same velocity and position of the planet.

$$\Delta v_A = \underbrace{v_2 - v_{A\,s/c}}_{\text{excess velocity}} = \sqrt{\frac{\mu_{SUN}}{R_2}}\left(1 - \sqrt{\frac{2R_1}{R_1+R_2}}\right) \qquad (4.2)$$

↓ Same for Hohmann Transfer

↳ will be used to match arrival conditions

In the general case Δv_i and Δv_1 come from LAMBERT SOLVER

TRANSfer from inner to outer planet $\rightarrow \Delta v_B > 0, \Delta v_A > 0$

transfer from outer to inner planet $\rightarrow \Delta v_B < 0, \Delta v_A < 0$

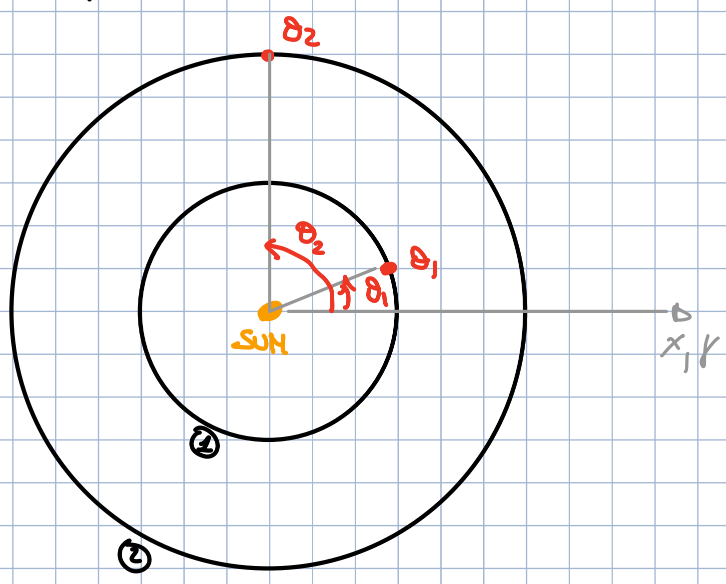
This is not enough for a Hohmann transfer or for a Lambert's transfer, we must check the phases because the planet has to be there on the arrival. We will use the ephemerides.

PLANET PHASING

We want to rendez-vous with planet (planet has to be there!)

In general this is done by PORK-CHOP PLOT.

In this case we need to compute the desired planet phasing for this to happen.



We want the planet to be there because a phasing manoeuvre will take approximately the same time as an orbit period (years!)

At departure from planet ①, planet ② should be in a location such that we can match planet ② at arrival.

We take a common reference frame.

$$\theta_1 = \theta_{10} + n_1 t \qquad (4.3)$$

$$\theta_2 = \theta_{20} + n_2 t \qquad (4.4)$$

n_1, n_2 angular velocities of planets around the sun

θ_{10} and θ_{20} mean / time anomalies of planet 1 and 2 at time t_0.

Phasing:

$$\phi = \theta_2 - \theta_1 = (\theta_{20} - \theta_{10}) + (n_2 - n_1)t$$

$$= \phi_0 + \underbrace{(n_2 - n_1)}_{\text{get longer with time}} t \qquad \boxed{\phi = \phi_0 + (n_2 - n_1)t} \quad (4.5)$$

$n_2 - n_1$ orbital angular velocity of planet 2 wrt planet 1

$T_2 > T_1 \Rightarrow n_1 > n_2 \Rightarrow n_2 - n_1 < 0 \Rightarrow$ 2 moves clockwise wrt 1

otherway around for transfer from outer to inner planet

$$\Delta\phi = (n_{\substack{\text{FINAL}\\\text{PLANET}}} - n_{\substack{\text{INITIAL}\\\text{PLANET}}})t$$

Question: if phasing at t_0 is ϕ_0 when phasing will become ϕ_0 again?

From eq (4.5)

synodic period

$$\phi_0 - 2\pi = \phi_0 + (n_2 - n_1) T_{SYN}$$

$$T_{SYN} = \frac{2\pi}{n_1 - n_2} \qquad (n_1 > n_2 \Rightarrow T_{SYN} > 0)$$

For a transfer from outer to inner planet

$$T_{syn} = \frac{2\pi}{n_2 - n_1} \qquad (n_2 > n_1 \Rightarrow T_{syn} > 0)$$

So the general formula is :

$$\boxed{T_{syn} = \frac{2\pi}{|n_1 - n_2|}} \qquad (4.6)$$

as $n_1 = \frac{2\pi}{T_1}$ and $n_2 = \frac{2\pi}{T_2}$

$$\boxed{T_{syn} = \frac{T_1 T_2}{|T_1 - T_2|}} \qquad (4.7)$$

Note : in the pork-chop plot the repetition pattern is related to the synodic period T_{syn}.

It could be useful to do an interplanetary retrograde transfer when we want to maximize the relative velocity e.g to deflect an asteroid.

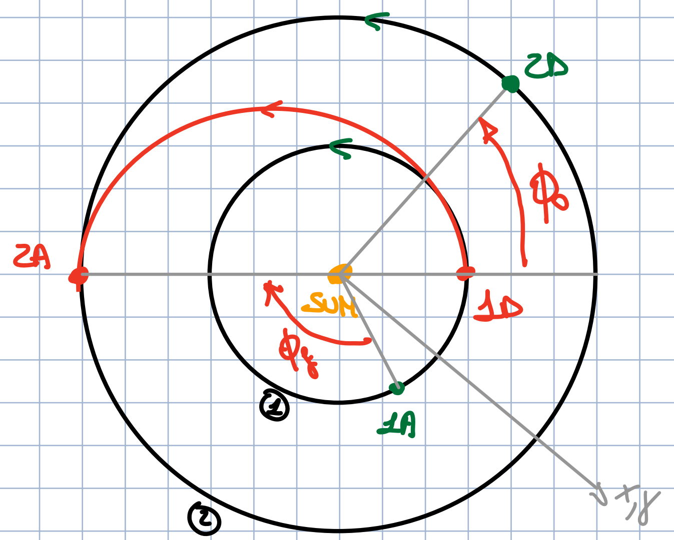
example

Mission from inner (1) to outer (2)

Transfer time

$$\underbrace{\Delta t_{12} = T_{of}}_{\text{time of flight}} = \frac{\pi}{\sqrt{\mu_{SUN}}} \sqrt{\left(\frac{R_1 + R_2}{2}\right)^3}$$

ϕ_0 = phasing at departure.



During Δt_{12} planet 2 travels the distance $n_2 \Delta t_{12}$

$\uparrow$ mean motion of planets 2

So phasing at departure

$$\phi_0 = \pi - n_2 \Delta t_{12} > 0$$

When s/c arrives at planet 2 at 2A, planet 1 has moved from its departure position $n_1 \Delta t_{12}$ so phasing arrival

$$\phi_f = \pi - n_1 \Delta t_{12} < 0$$

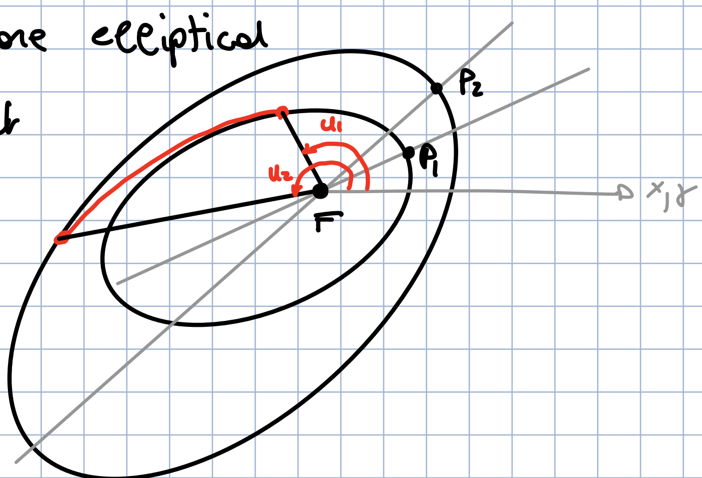
The planet 1 when s/c arrives at planet 2 has an angular position greater than 2 $\rightarrow$ Planet 1 overtake planet 2 because it is faster.

↳ REAL MISSION: USE EPHEMERIDES

exercise 4.2

compute return trip.

When orbit are elliptical with different apse-line



$u_1 = \omega_1 + \theta_1 = \theta_1^*$

$u_2 = \omega_2 + \theta_2$

If they are not coplanar $i_1 \neq i_2$

$$\lambda_1 = \underline{\Omega_1} + \underbrace{\omega_1 + \theta_1}_{\text{planet 1 project on ecliptic}}$$

$$\lambda_2 = \underbrace{\Omega_2}_{\text{on ecliptic}} + \underbrace{\omega_2 + \theta_2}_{\text{planet 2 project on ecliptic}}$$

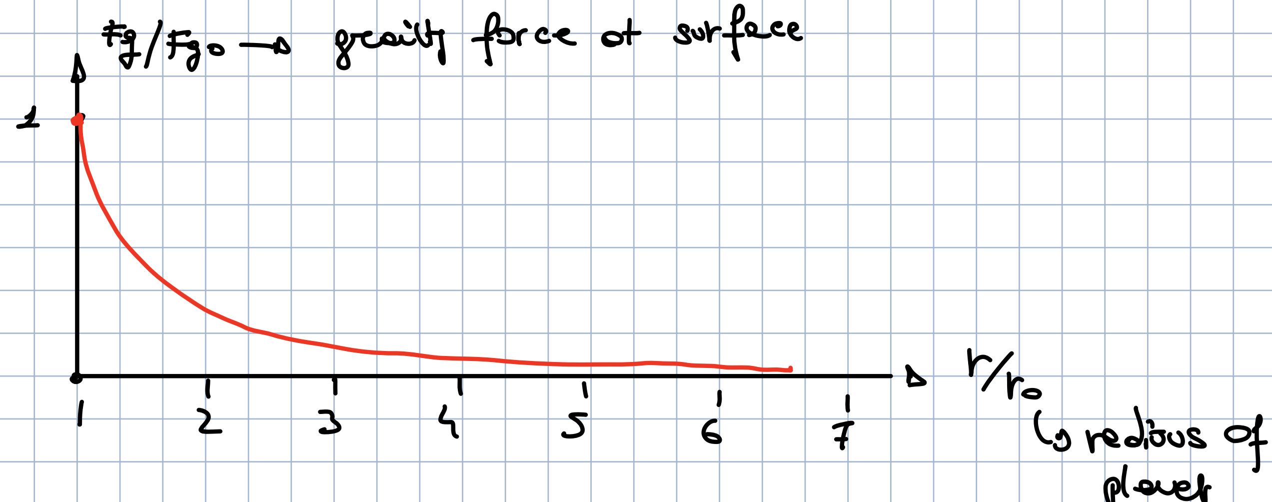
We used this kind of preliminary design for multi transfer orbit where it is computationally impossible to use a pork chop plot because it tell us already all the combination.

Listen 1:00 }

SPHERE OF INFLUENCE

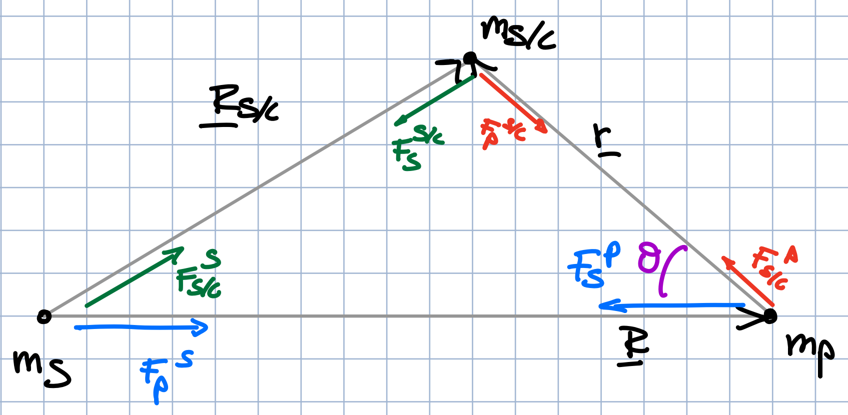
The sun gravity is much stronger than planet gravity. If close to planet → planet's gravity is larger than sun's gravity.

DEFINING SPHERE OF INFLUENCE (SOI) to define region of interest of mission



at $r/r_0 = 10$ $F_g/F_{g0} \simeq 0.01$ (1 %)

Let's consider the three body problem



along the gray line we will have all the gravitational forces.

$\underline{F}_p^{s/c}$ force due to the planet acting on the space craft.

$\underline{R}$, $\underline{R}_{s/c}$ absolute position wrt sun

$\underline{r}$ relative position wrt planet

$$\begin{cases} \underline{F}_p^{s/c} = -G \dfrac{m_{s/c}\, m_p}{r^3}\, \underline{r} \\ \underline{F}_s^{s/c} = -G \dfrac{m_{s/c}\, m_s}{R_{s/c}^3}\, \underline{R}_{s/c} \\ \underline{F}_s^{p} = -G \dfrac{m_p\, m_s}{R^3}\, \underline{R} \end{cases} \quad (4.8)$$

set of forces

$$\underline{R}_{s/c} = \underline{R} + \underline{r} \quad (4.9)$$

Apply cosine law

$$R_{s/c} = \left(R^2 + r^2 - 2Rr\cos\theta\right)^{1/2} = R\left(1 + \left(\frac{r}{R}\right)^2 - 2\frac{r}{R}\cos\theta\right)^{1/2}$$

we can say that within the planet sphere of influence

$\frac{r}{R} \ll 1$ $\quad R_{s/c} \simeq R \quad (4.10)$

The sun sees s/c and planet at the same position.

1) EQUATION OF MOTION OF THE S/C IN THE INERTIAL REFERENCE FRAME

$$m_{s/c}\,\underline{\ddot{R}}_{s/c} = \underline{F_S}^{s/c} + \underline{F_p}^{s/c}$$

$$\underline{\ddot{R}}_{s/c} = \frac{1}{\cancel{m_{s/c}}}\left(-G\frac{\cancel{m_{s/c}}\,m_s}{R_{s/c}^3}\,\underline{R}_{s/c}\right) + \frac{1}{\cancel{m_{s/c}}}\left(-G\frac{\cancel{m_{s/c}}\,m_p}{r^3}\,\underline{r}\right)$$

$$\boxed{\underline{\ddot{R}}_{s/c} = -\frac{G m_s}{R_{s/c}^3}\,\underline{R}_{s/c} - \frac{G m_p}{r^3}\,\underline{r}} \qquad (4.11)$$