IMPULSIVE MANOEUVRES

- transfer from orbit to orbit
- orbit maintenance
- close proximity operations - rendez-vous

ISTANTANEOUS @ t^- → MANOEUVRE → t^+

$\underline{r}, \underline{v}$ → $\Delta \underline{v}$ → $\underline{r}, \underline{v} + \Delta \underline{v}$

$\underline{k}_{ep}$ → $\underline{k}_{ep} + \Delta \underline{k}_{ep}$

no need to solve the equation of motion (1.6) because for istantaneous manouvre the time in wich the engine is on is much smaller than the coasting time (time when the engine is off and the space craft wait to reach a certain position)

$$\boxed{\Delta v = I_{sp}\, g_0 \ln \frac{m_0}{m_f}} \qquad (3.1)$$

m_0 = initial mass = mass @ t^-

m_f = final mass = mass @ t^+

$m_p = \Delta m = m_0 - m_f$ mass of propellant

$$\ln \frac{m_0}{m_f} = \frac{\Delta v}{I_{sp}\, g_0}$$

I_{sp} [s] = specific impulse

g_0 = gravitational acc. at sea level

$$\frac{m_0}{m_f} = \exp\left(\frac{\Delta v}{I_{sp}\, g_0}\right) \qquad \frac{m_f}{m_0} = \exp\left(-\frac{\Delta v}{I_{sp}\, g_0}\right)$$

$$\frac{m_0 - m_p}{m_0} = \exp\left(-\frac{\Delta v}{I_{sp}\, g_0}\right) \qquad 1 - \frac{m_p}{m_0} = \exp\left(-\frac{\Delta v}{I_{sp}\, g_0}\right)$$

$$\boxed{\frac{m_p}{m_0} = 1 - \exp\left(-\frac{\Delta v}{I_{sp}\, g_0}\right)} \qquad (3.2)$$

$$I_{sp} = \frac{\text{Thrust}}{\text{sea level weight rate of fuel consumption}} \quad [s]$$

g_0 = sea level standard acceleration $\left[\frac{m}{s^2}\right]$

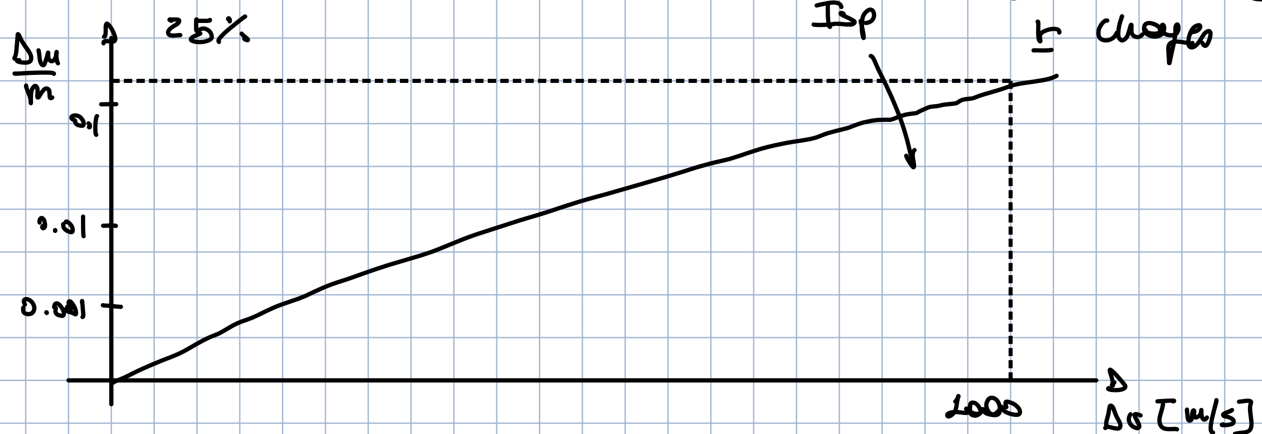
$$\frac{dm}{dt} = \frac{T}{I_{sp}\, g_0} \longrightarrow I_{sp} = \frac{T}{\underbrace{\frac{dm}{dt} g_0}_{\frac{dw}{dt}}}$$

$\frac{dw}{dt}$ @ sea level

PROPELLANT	I_{sp} [s]
Cold gas	50
Hydrazine	230
Solid propellant	290
Lox	450
Ion propulsion	>3000

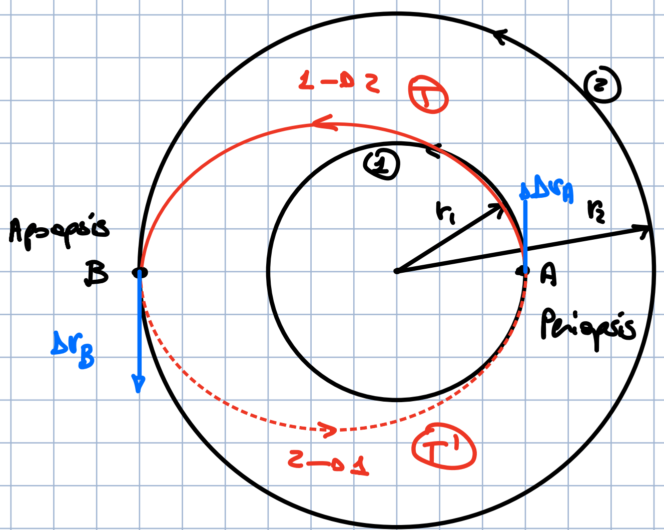
(Cold gas – Lox:) Impulsive manoeuvre high thrust and relative low I_{sp}

(Ion propulsion:) $\longrightarrow$ low thrust manoeuvres not istantaneous
t changes



HOHMANN TRANSFER

most efficient transfer between 2 coplanar, cofocal, circular orbit.



① → ② T
② → ① T'

Hohmann transfer elliptical orbit that is tangent to the two circular orbit.

$$\boxed{\Delta t = \pi \sqrt{\frac{a^3}{\mu}}}$$ ← Time required to do the transfer

$$\varepsilon = -\frac{\mu}{2a}$$ Eq (1.55)

$a \uparrow$ $\varepsilon \uparrow$

① → ② $\varepsilon \uparrow$ TRANSFER ORBIT Ⓣ

@ A Δv_A $\underline{r_A} = \underline{r_1}$ t^- ① $\underline{r_A}$ $\underline{v_A}^-$ t^+ Ⓣ $\underline{r_A}$ $\underline{v_A}^+$

ε_1 $\uparrow \varepsilon_T$

AB COASTING TIME / ORBIT

@ B Δv_B $\underline{r_B} = \underline{r_2}$

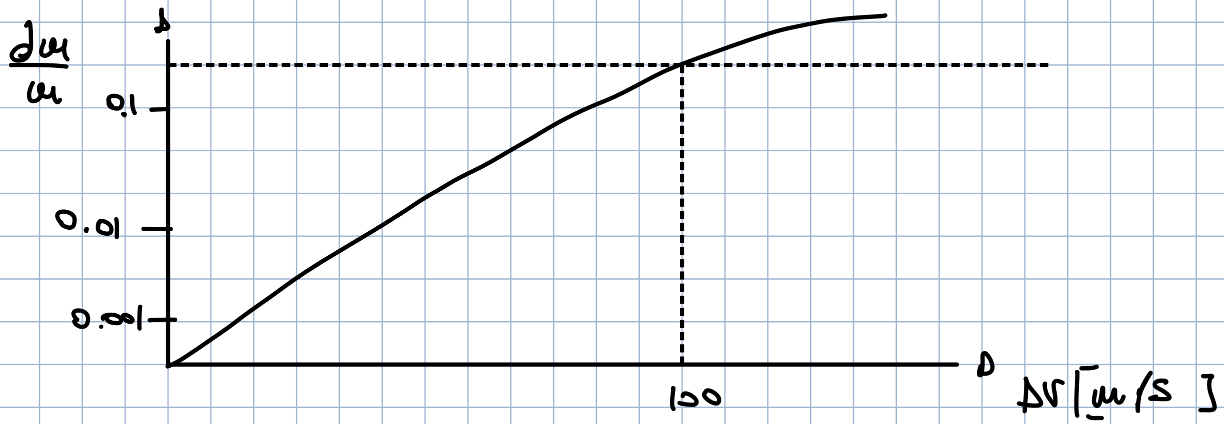
ε_T $\quad \hat{\varepsilon}_2$

ε^- $\quad$ ε^+

(T) $\quad$ (2)

$\underline{v}_B \; \underline{v}_B^-$ $\quad$ $\underline{r}_B \; \underline{v}_B^+$

$$\Delta v_{tot} = |\Delta v_A| + |\Delta v_B|$$



Δv → parameters of the orbit after impulse

v^- $\quad$ manoeuvre $\quad$ v^+

r $\longrightarrow$ r

a^- $\quad$ a^+

$$\underline{v}^+ = \underline{v}^- + \Delta\underline{v}$$

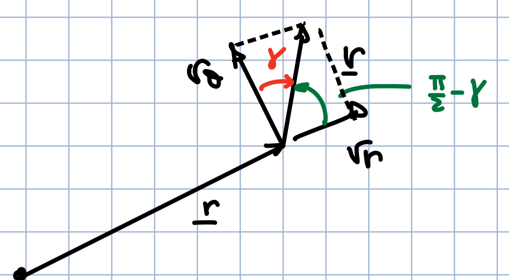
$$\boxed{a^+ = -\frac{\mu}{2}\,\frac{1}{\frac{v^{+2}}{2} - \frac{\mu}{r_A}}} \qquad (3.3)$$

inverting the eq of energy

$$\underline{h} = \underline{r} \wedge \underline{v}$$

flight path angle

$$\underline{h}^+ = \underline{r} \wedge \underline{v}^+ = r\, v^+ \sin\left(\frac{\pi}{2} - \gamma\right)$$



$\boxed{h^+ = r v^+ \cos\gamma}$ from eq $h^2 = p\mu$ $h^2 = a\mu(1-e^2)$

$$a^+(1-e^{+2})\mu = r^2 v^{+2} \cos^2\gamma$$

$$\boxed{e^+ = \sqrt{1 - \frac{r^2 v^{+2}\cos^2\gamma}{a^+\mu}}} \qquad (3.4)$$

$$\boxed{\Delta V_{TOT} = |\Delta v_A| + |\Delta v_B|} \qquad (3.5)$$

Same Δv_{TOT} would be required to go from B to A (T')

① B Δv_B opposite to motion $\varepsilon_2 \downarrow \varepsilon_T' = \varepsilon_T$

BA coasting arc

② A Δv_A opposite to motion $\varepsilon_T' \downarrow \varepsilon_1$

$\Delta v_{TOT} = |\Delta v_B| + |\Delta v_A|$ ← still the same