TIME LAW FOR HYPERBOLA

$r = a(1 - e\cos E)$ Eq (2.4)

For hyperbola $r = -\bar{a}\,(1 - e\cos(i\bar{E})) = -\bar{a}(1 - e\cosh E)$

$r = -\bar{a}(1 - e\cosh E)$ (2.29)

For ellipse $M - M_p = E - e\sin E$ Eq (2.8), (2.11)

For Hyperbola

$-i\bar{M} + i\bar{M}_p = i\bar{E} - e\sin(i\bar{E})$

$-i(\bar{M} - \bar{M}_p) = i\bar{E} + \dfrac{e\sinh\bar{E}}{i}$

$-i(\bar{M} - \bar{M}_p) = i(\bar{E} - e\sinh(\bar{E}))$

TIME LAW HYPERBOLA

$$\boxed{\bar{M} - \bar{M}_p = \bar{E} - e\sinh\bar{E}}$$

(2.30)

We can set $\bar{M}_p = \emptyset$

Relation θ, $\bar{E}$

for ellipse $\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\,\tan\frac{E}{2}$

for hyperbola $\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\;\dfrac{\sin\left(\frac{i\bar{E}}{2}\right)}{\cos\left(\frac{i\bar{E}}{2}\right)}$

$= \sqrt{\frac{1+e}{1-e}}\;\dfrac{-\frac{1}{i}\sinh\left(\frac{\bar{E}}{2}\right)}{\cosh\left(\frac{\bar{E}}{2}\right)}$

$= \sqrt{\frac{1+e}{1-e}}\;\underbrace{i}_{=\sqrt{-1}}\,\tanh\left(\frac{\bar{E}}{2}\right)$

<0

$= \sqrt{\frac{1+e}{e-1}}\,\tanh\left(\frac{\bar{E}}{2}\right)$

$$\boxed{\operatorname{tg}\frac{\theta}{2} = \sqrt{\frac{1+e}{e-1}}\ \operatorname{tgh}\left(\frac{\bar{E}}{2}\right)} \qquad (2.31)$$

$\bar{E}$ in an hyperbola evolves in an exponential way.

SUMMARY AND PARABOLIC CASE

$\underline{r}(t), \underline{v}(t)$ to $\theta(\underline{r}(0), \underline{v}(0))$ $\theta \to t$

$t \longrightarrow$ E (ellipse), D (parabola), F (hyperbola) $\longrightarrow \theta$ (The anomaly) $\longrightarrow \underline{v}, \underline{r}$

$\Rightarrow$ Relationship between t and eccentric anomaly

① Ellipse $t \to E$

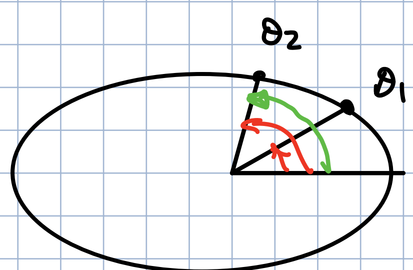
$$\sqrt{\frac{\mu}{a^3}}\,(t - t_p) = E - e\sin E \qquad (2.10)$$

$$M - M_p = E - e\sin E \qquad (2.12)$$

$M_p = 0$

$\to$ it can be used to calculate any time between two point.

$$\underbrace{(t - t_p)}_{\Delta t} = \sqrt{\frac{a^3}{\mu}}\,(E - e\sin E)$$



$$M_2 - M_1 = E_2 - e\sin E_2 - (E_1 - e\sin E_1)$$

② Hyperbola $t \to F$ $(F = \bar{E})$

$$\sqrt{\frac{\mu}{a^3}}\,(t - t_p) = e\sinh F - F \qquad (2.30)$$

$$\bar{M} - \bar{M}_p = e \sinh F - F$$
$$\shortparallel$$
$$0$$

③ Parabola $t \to D$

$$\sqrt{\frac{\mu}{p^3}}\,(t - t_p) = \frac{1}{2}\left(D + \frac{D^3}{3}\right) \quad (2.32) \quad \to \text{close form.}$$

<u>Recall</u>: $h^2 = p\mu \Rightarrow \sqrt{\frac{\mu}{p^3}} = \sqrt{\frac{\mu^4}{h^6}} = \frac{\mu^2}{h^3}$

NOTE

$t - t_p = \Delta t$ from pericenter

$E(t)$, $F(t)$ need to be solved numerically whereas $D(t)$ can be solve with a close form.

Relationship between eccentric anomaly and true anomaly

① Ellipse $E \to \theta$ $\qquad tg\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\, tg\frac{E}{2} \quad (2.17)$

② Hyperbola $F \to \theta$ $\qquad tg\frac{\theta}{2} = \sqrt{\frac{1+e}{e-1}}\, tgh\frac{F}{2} \quad (2.31)$

③ Parabola $D \to \theta$ $\qquad tg\frac{\theta}{2} = D \quad (2.33)$

GEOCENTRIC RIGHT ASCENSION AND DECLINATION FRAME

Let's describe orbits in 3D

well defined : celestial sphere

right ascention (RA)

declination (Dec)

$\Bigg\}$ location of stars, planets, celestial bodies, s/c

→ Inertial Geocentric Equatorial reference frame.

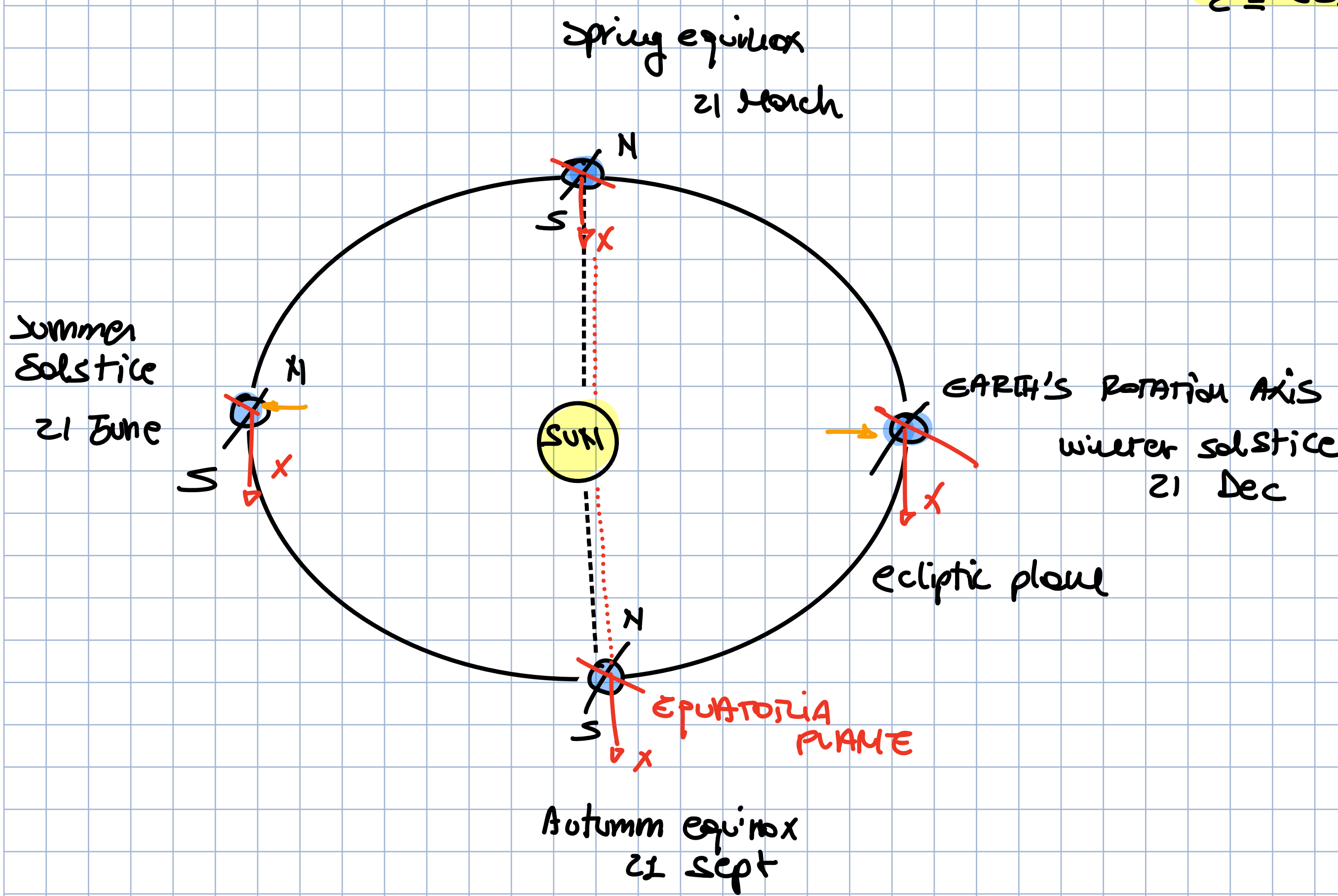
Orbits @ Earth can be defined:

- wrt equatorial plane (⊥ Earth's rotation axis)
- wrt ecliptic plane : Earth's orbit plane around the sun.

however the spin axis of the Earth is not ⊥ to the ecliptic frame.

obliquity angle

$\varepsilon = 23.4°$

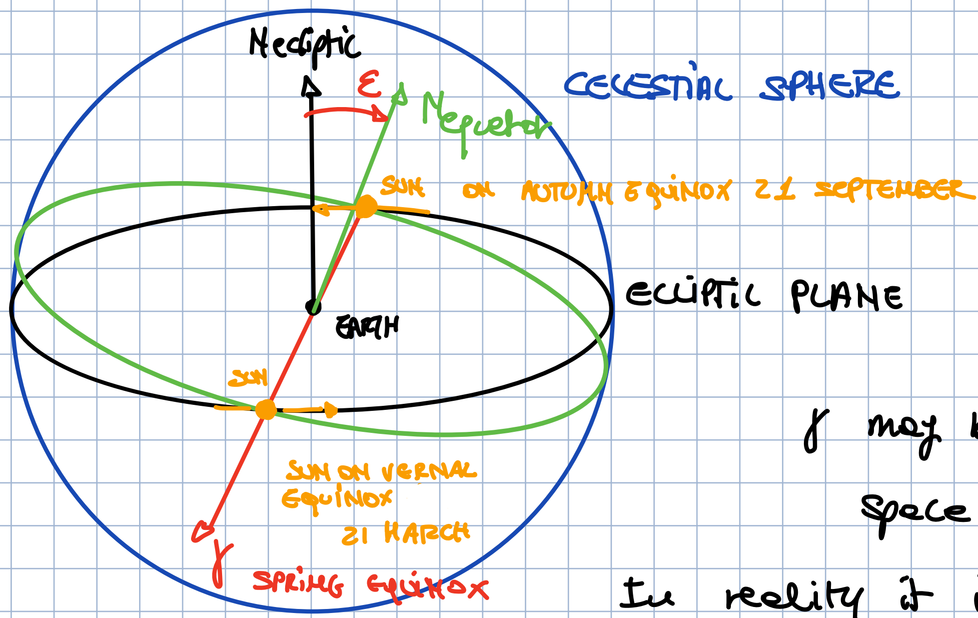


The equatorial plane intersect the ecliptic plane on the line that passes between the spring equinox and the Autumn equinox. we take that line as the x axis of our inertial reference frame,

$x = \gamma$ → greek letter for ARES CONSTELLATION
↳ vernal equinox line

now adays the vernal equinox line point at the pisces constellation.

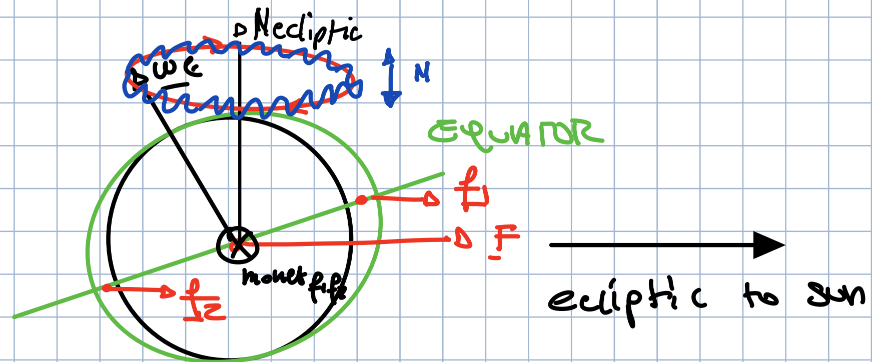
Now lets take a look from the point of view of the Earth.



γ may be consider fixed in space

In reality it is processing at 1....° per century westward → cause: gravity of sun and moon on an non spherical distribution of Earth mass.

Centrifugal forces due to rotation of Earth about its own axis ⇒ Earth bulges outwards at its equators



F = central force

f1, f2 - Sun's gravity on bulging side

① f1, f2 produce a net clock wise moment about centre i.e. of the Earth

② Moment would tend to align equator to ecliptic

③ Earth is spinning $\underline{\omega}_e$ 360° per day (counter clock wise way) the total effect is to rotate the angular momentum vector in the direction of moment (into page)

(4) spin axis precesses in counterclockwise way.

(5) MOON'S gravity is acting (exerting a torque)

(6) TOTAL EFFECT → PRECESSION of SPIN AXIS →

γ precession with a period of 26000 years

(7) MOON'S attraction superimposes a SMALL NUTATION

ε varies with amplitude of $0.0025°$ over a period of 18.6 years → Period in which the position of Sun Earth and Moon repeats.

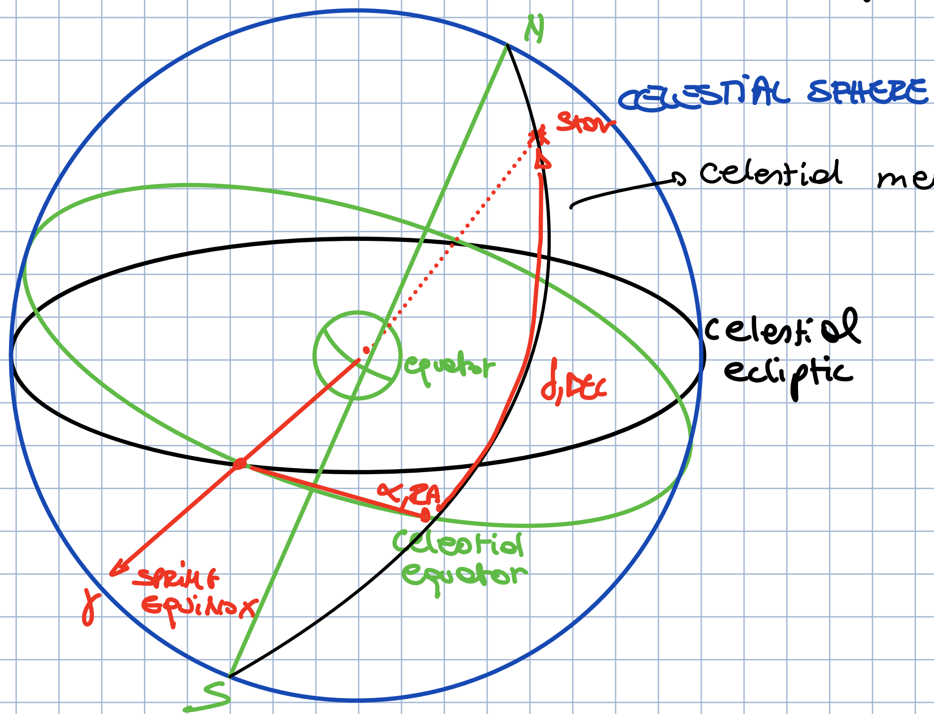
NASA SPICE SYSTEM → J2000 reference frame

↓

the direction of the x axis is taken at January 1st 2000.

↳ If we do not consider this we will make large mistake over long distances.

How to describe the position of a body in this reference system?



α, RA = RIGHT ASCENSION = longitude on celestial sphere starting from γ $\quad 0<\alpha<2\pi$

δ, DEC = Declination = latitude on the celestial sphere

Positive north

Negative south $\quad -\frac{\pi}{2}<\delta<\frac{\pi}{2}$

These are spherical coordinates.

I Earth centered equatorial frame (ECEq frame)

$\hat{z}$ in NP

$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\underline{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

$\underline{r} = r\hat{r}$

$\hat{r} = \cos\delta\cos\alpha\,\hat{i} + \cos\delta\sin\alpha\,\hat{j} + \sin\delta\,\hat{k}$

$\underline{\underline{!}}\ \ell\hat{i} + m\hat{j} + n\hat{k}$

$$\ell = \frac{x}{r}, \quad m = \frac{y}{r}, \quad n = \frac{z}{r}$$

$$\delta = \sin^{-1}(n)$$

$$\alpha = \begin{cases} \cos^{-1}\left(\dfrac{\ell}{\cos\delta}\right) & m > 0 \\ 2\pi - \cos^{-1}\left(\dfrac{\ell}{\cos\delta}\right) & m \leq 0 \end{cases}$$