POLITECNICO MILANO 1863

Spacecraft Attitude Dynamics

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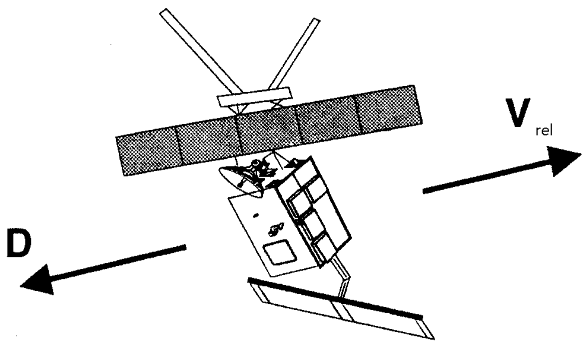
Disturbance Torques – SRP and air drag

Force due to atmospheric drag – inertial frame

The drag force on an Earth orbiting spacecraft is



where

$$\bar{F}_i = -\frac{1}{2}\rho C_D {v_{rel}}^2 \frac{\mathbf{v}_{rel}}{\|\mathbf{v}_{rel}\|} A_{cross}$$

$\rho(h,t)$ atmospheric density

$\mathbf{v}_{rel}$ relative speed

A_{cross} cross sectional area perpendicular to $\mathbf{v}_{rel}$

$C_D \approx 2.2, 1.5 < C_D < 2.6$ drag coefficient (LEO)

flat plate drag coefficient

Air density example data set

$$\bar{F}_i = -\frac{1}{2}\rho C_D {v_{rel}}^2 \frac{\mathbf{v}_{rel}}{\|\mathbf{v}_{rel}\|} A_{cross}$$

altit. (km)	10	20	30	40	50	60	70	80	90	100
density (g/cm^3)	4,02e-04	8,34e-05	1,57e-05	3,18e-06	8,37e-07	2,33e-07	5,86e-08	1,40e-08	2,99e-09	5,17e-10
altit. (km)	110	120	130	140	150	160	170	180	190	200
density (g/cm^3)	8,42e-11	1,84e-11	7,36e-12	3,78e-12	2,19e-12	1,37e-12	9,00e-13	6,15e-13	4,32e-13	3,10e-13
altit. (km)	210	220	230	240	250	260	270	280	290	300
density (g/cm^3)	2,27e-13	1,68e-13	1,26e-13	9,58e-14	7,35e-14	5,68e-14	4,43e-14	3,48e-14	2,75e-14	2,18e-14
altit. (km)	310	320	330	340	350	360	370	380	390	400
density (g/cm^3)	1,74e-14	1,40e-14	1,13e-14	9,10e-15	7,39e-15	6,02e-15	4,92e-15	4,03e-15	3,31e-15	2,72e-15
altit. (km)	410	420	430	440	450	460	470	480	490	500
density (g/cm^3)	2,25e-15	1,86e-15	1,54e-15	1,28e-15	1,07e-15	8,89e-16	7,43e-16	6,22e-16	5,22e-16	4,39e-16
altit. (km)	510	520	530	540	550	560	570	580	590	600
density (g/cm^3)	3,71e-16	3,13e-16	2,66e-16	2,26e-16	1,93e-16	1,65e-16	1,41e-16	1,22e-16	1,05e-16	9,14e-17
altit. (km)	610	620	630	640	650	660	670	680	690	700
density (g/cm^3)	7,96e-17	6,97e-17	6,12e-17	5,41e-17	4,79e-17	4,27e-17	3,82e-17	3,44e-17	3,10e-17	2,82e-17
altit. (km)	710	720	730	740	750	760	770	780	790	800
density (g/cm^3)	2,57e-17	2,35e-17	2,16e-17	1,99e-17	1,84e-17	1,71e-17	1,59e-17	1,49e-17	1,39e-17	1,31e-17
altit. (km)	810	820	830	840	850	860	870	880	890	900
density (g/cm^3)	1,23e-17	1,16e-17	1,10e-17	1,04e-17	9,86e-18	9,37e-18	8,91e-18	8,48e-18	8,09e-18	7,72e-18
altit. (km)	910	920	930	940	950	960	970	980	990	1000
density (g/cm^3)	7,37e-18	7,04e-18	6,74e-18	6,45e-18	6,18e-18	5,92e-18	5,68e-18	5,44e-18	5,22e-18	5,02e-18

Atmospheric density model

$$\rho(h,t) = \rho_0 \exp\left[-\frac{h-h_0}{H}\right]$$

ρ_0 reference density

h, h_0 actual and reference altitude

H scale height

TABLE 7-4. Exponential Atmospheric Model. Although a very simple approach, this method yields moderate results for general studies. Source: Wertz, 1978, 820, which uses the *U.S. Standard Atmosphere* (1976) for 0 km, CIRA-72 for 25–500 km, and CIRA-72 with T_∞ = 1000 K for 500–1000 km. The scale heights have been adjusted to maintain a piecewise-continuous formulation of the density.

Altitude h_{ellp} (km)	Base Altitude h_o (km)	Nominal Density ρ_o (kg/m³)	Scale Height H (km)	Altitude h_{ellp} (km)	Base Altitude h_o (km)	Nominal Density ρ_o (kg/m³)	Scale Height H (km)
0–25	0	1.225	7.249	150–180	150	2.070×10^{-9}	22.523
25–30	25	3.899×10^{-2}	6.349	180–200	180	5.464×10^{-10}	29.740
30–40	30	1.774×10^{-2}	6.682	200–250	200	2.789×10^{-10}	37.105
40–50	40	3.972×10^{-3}	7.554	250–300	250	7.248×10^{-11}	45.546
50–60	50	1.057×10^{-3}	8.382	300–350	300	2.418×10^{-11}	53.628
60–70	60	3.206×10^{-4}	7.714	350–400	350	9.158×10^{-12}	53.298
70–80	70	8.770×10^{-5}	6.549	400–450	400	3.725×10^{-12}	58.515
80–90	80	1.905×10^{-5}	5.799	450–500	450	1.585×10^{-12}	60.828
90–100	90	3.396×10^{-6}	5.382	500–600	500	6.967×10^{-13}	63.822
100–110	100	5.297×10^{-7}	5.877	600–700	600	1.454×10^{-13}	71.835
110–120	110	9.661×10^{-8}	7.263	700–800	700	3.614×10^{-14}	88.667
120–130	120	2.438×10^{-8}	9.473	800–900	800	1.170×10^{-14}	124.64
130–140	130	8.484×10^{-9}	12.636	900–1000	900	5.245×10^{-15}	181.05
140–150	140	3.845×10^{-9}	16.149	1000–	1000	3.019×10^{-15}	268.00

Eq. (7-31) requires knowledge of the actual altitude, found by subtracting the Earth's radius (6378.137 km) from the satellite's given radius (h_{ellp} = 747.2119 km). Now, if we use values from Table 7-4, Eq. (7-31) becomes

$$\rho = 3.614\times10^{-14}\,\mathrm{EXP}\left[-\frac{747.2119-700}{88.667}\right] = 2.121\,985\,4\times10^{-14}\frac{\mathrm{kg}}{\mathrm{m}^3}$$

▲ Note that the units in the exponential cancel (all are km), and the result is less than the base value at 700 km, as we would expect.

Relative orbital velocity in the inertial frame

$$\mathbf{v}_{orbit} = \frac{d\vec{R}}{dt}$$

$$\mathbf{v}_{atmosphere} = \omega_{Earth} \wedge \vec{R}$$

Dependent on the orbit that we define

$$\omega_{Earth} = [0 \quad 0 \quad \omega_{\oplus}]$$

$$\mathbf{v}_{rel} = \begin{bmatrix} \dot{x} + \omega_{\oplus} y \\ \dot{y} - \omega_{\oplus} x \\ \dot{z} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R \begin{bmatrix} \cos\theta \\ \sin\theta \cos i \\ \sin\theta \sin i \end{bmatrix}$$

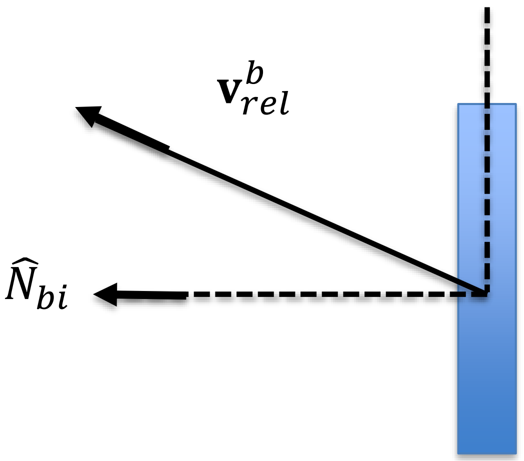
$$R = \frac{a(1 - e^2)}{1 + e\cos\theta}$$

$$\dot{\theta} = \frac{n(1 + e\cos\theta)^2}{(1 - e^2)^{3/2}}$$

$$\omega_{\oplus} = 0.000072921 rad/\sec$$

Relative velocity in body fixed coordinates

$$\mathbf{v}_{rel}^{b} = A_{B/N}\mathbf{v}_{rel} + \omega_{sat} \wedge \vec{r} \qquad \mathbf{v}_{rel} = \begin{bmatrix} \dot{x} + \omega_{\oplus} y \\ \dot{y} - \omega_{\oplus} x \\ \dot{z} \end{bmatrix}$$



$$A_{cross} = A_i(\hat{N}_{bi} \cdot \frac{\mathbf{v}_{rel}^{b}}{\|\mathbf{v}_{rel}^{b}\|})$$

for a simple cuboid spacecraft

$$\hat{n}_{s1}^{b} = [1 \quad 0 \quad 0]^T, \hat{n}_{s2}^{b} = [0 \quad 1 \quad 0]^T, \hat{n}_{s3}^{b} = [0 \quad 0 \quad 1]^T$$
$$\hat{n}_{s4}^{b} = [-1 \quad 0 \quad 0]^T, \hat{n}_{s5}^{b} = [0 \quad -1 \quad 0]^T, \hat{n}_{s6}^{b} = [0 \quad 0 \quad -1]^T$$

Torque due to atmospheric drag

The aerodynamic force acting on a flat surface is defined by:

$$\bar{F}_i = -\frac{1}{2}\rho C_D {v_{rel}}^2 \frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|}(\hat{N}_{bi}\cdot\frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|})A_i$$

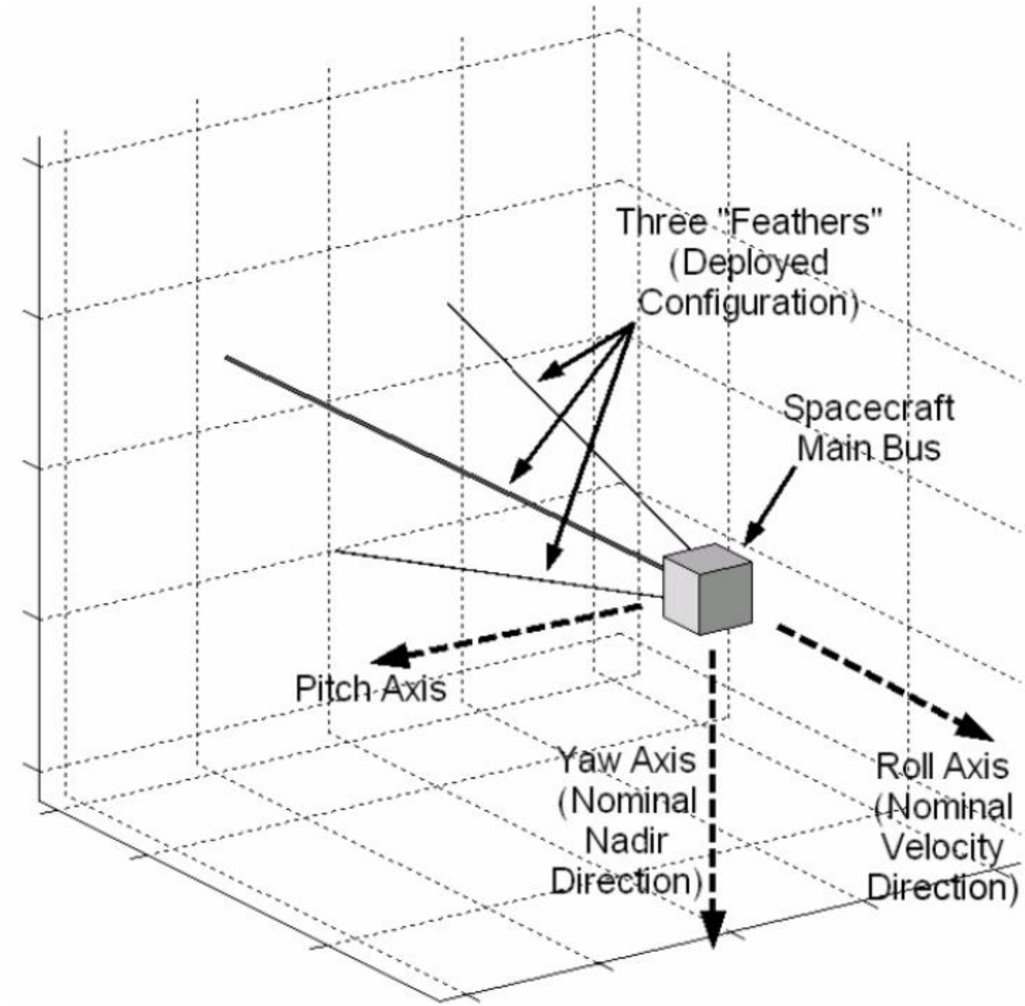
.

$$T_{aero} = -\sum_{i=1}^{n} \bar{r}_i \wedge \frac{1}{2}\rho C_D {v_{rel}}^2 \frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|}\sum_{i=1}^{n}(\hat{N}_{bi}\cdot\frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|})A_i$$

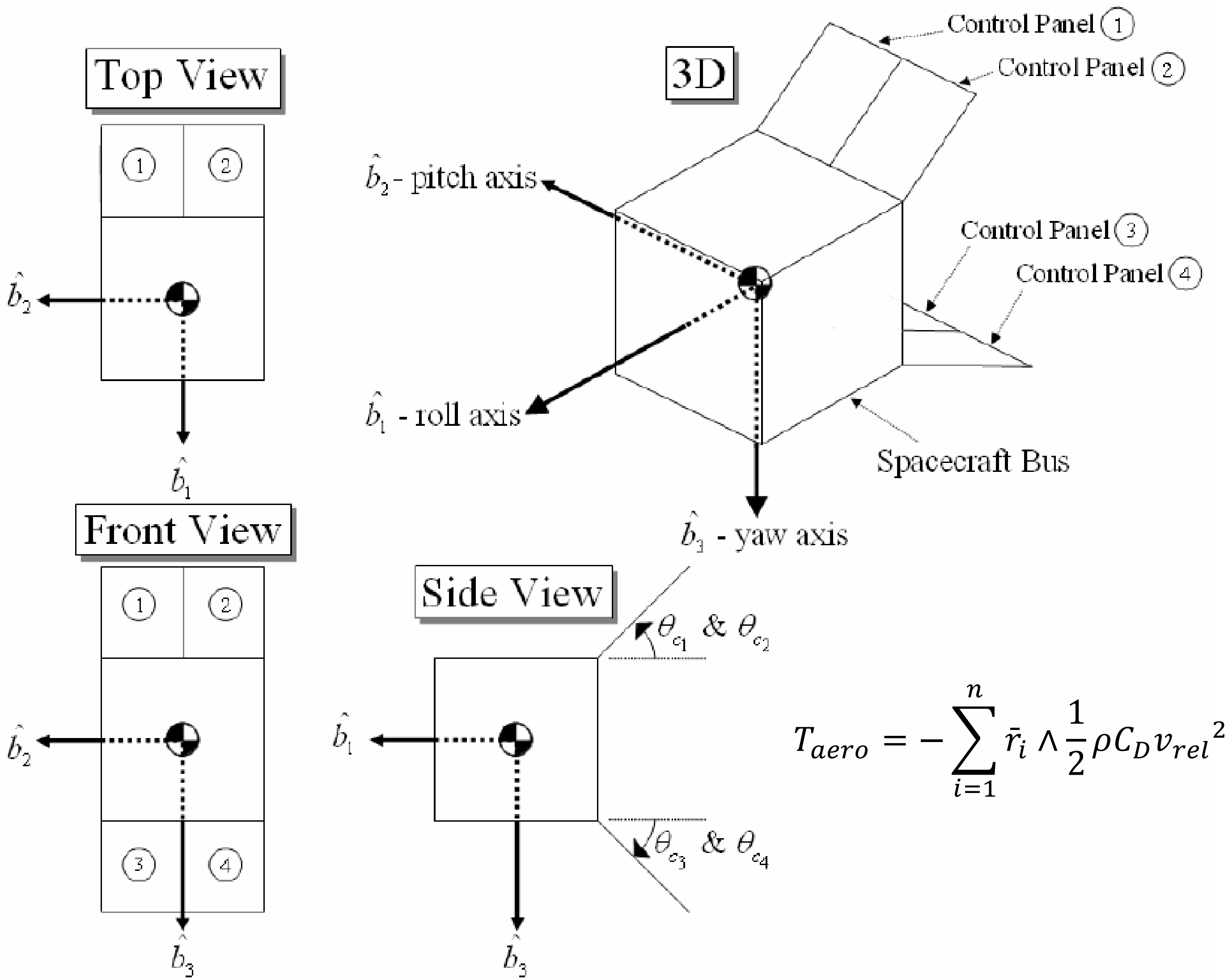
Total torque on a rigid body

$$T_{aero} = \begin{cases} -\frac{1}{2}\rho C_D {v_{rel}}^2 \sum_{i=1}^{n} \bar{r}_i \wedge \frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|}\sum_{i=1}^{n}(\hat{N}_{bi}\cdot\frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|})A_i & \hat{N}_{bi}\cdot\frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|} > 0 \\ 0 & \hat{N}_{bi}\cdot\frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|} < 0 \end{cases}$$

Shuttlecock concept



CubeSat – air drag control



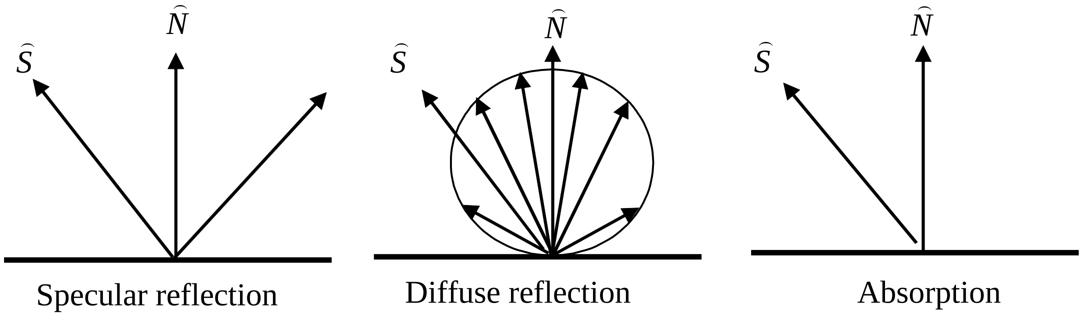
$$T_{aero} = -\sum_{i=1}^{n} \bar{r}_i \wedge \frac{1}{2}\rho C_D v_{rel}{}^2 \frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|} \sum_{i=1}^{n} (\widehat{N}_{bi} \cdot \frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|}) A_i$$

Force due to solar radiation pressure

Altitude (Km)	Direct solar radiation (W/m²)	Radiation reflected by the Earth (W/m²)	Earth radiation (W/m²)
500	1358	600	150
1000	1358	500	117
2000	1358	300	89
4000	1358	180	62
8000	1358	75	38
15000	1358	30	14
30000	1358	12	3
60000	1358	7	2

The average pressure due to radiation can be evaluated as $P = \frac{F_e}{c}$

where c is the speed of light and F_e is the power per unit surface.



$$\rho_s + \rho_d + \rho_a = 1$$

Force due to solar radiation pressure (from the Sun)

Force on a flat panel

$$\bar{F}_i = -PA\left[\rho_a(\hat{S}\cdot\widehat{N})\hat{S} + 2\rho_s(\hat{S}\cdot\widehat{N})^2\widehat{N} + \rho_d(\hat{S}\cdot\widehat{N})(\hat{S} + \frac{2}{3}\widehat{N})\right]$$

$$\bar{F}_i = -PA(\hat{S}\cdot\widehat{N})\left[(1-\rho_s)\hat{S} + (2\rho_s(\hat{S}\cdot\widehat{N}) + \frac{2}{3}\rho_d)\widehat{N}\right]$$

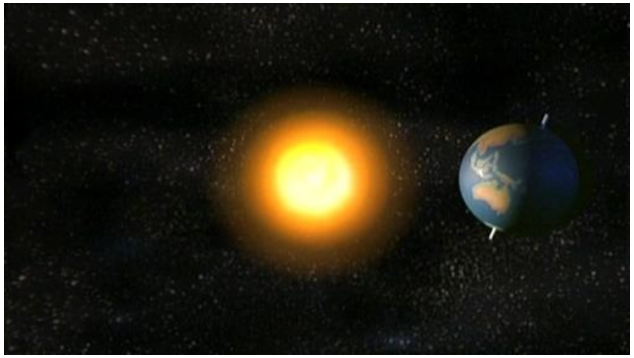
$$\bar{F}_i = -PA_i(\hat{S}_b\cdot\widehat{N}_{bi})\left[(1-\rho_s)\hat{S}_b + (2\rho_s(\hat{S}_b\cdot\widehat{N}_{bi}) + \frac{2}{3}\rho_d)\widehat{N}_{bi}\right]$$

Torque due to solar radiation pressure

$$\bar{F}_i = -PA_i(\hat{S}_b \cdot \widehat{N}_{bi})\left[(1-\rho_s)\hat{S}_b + (2\rho_s(\hat{S}_b \cdot \widehat{N}_{bi}) + \frac{2}{3}\rho_d)\widehat{N}_{bi}\right]$$

Modelling the Sun in body coordinate

$$\hat{S}_b = A_{B/N}\hat{S}_i$$



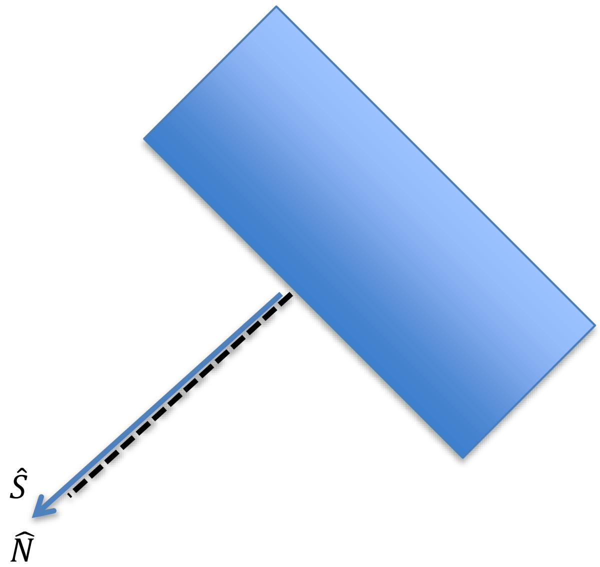
For a rigid body we have

$$T_{SRP} = \sum_{i=1}^{n} \bar{r}_i \wedge \bar{F}_i$$

$$T_{SRP} = \begin{cases} \sum_{i=1}^{n} \bar{r}_i \wedge \bar{F}_i & \hat{S}_b \cdot \widehat{N}_b > 0 \\ 0 & \hat{S}_b \cdot \widehat{N}_b < 0 \end{cases}$$

Maximum SRP Force and corresponding torque

$$\bar{F}_i = -PA(\hat{S}\cdot\widehat{N})\left[(1-\rho_s)\hat{S} + \left(2\rho_s(\hat{S}\cdot\widehat{N}) + \frac{2}{3}\rho_d\right)\widehat{N}\right]$$



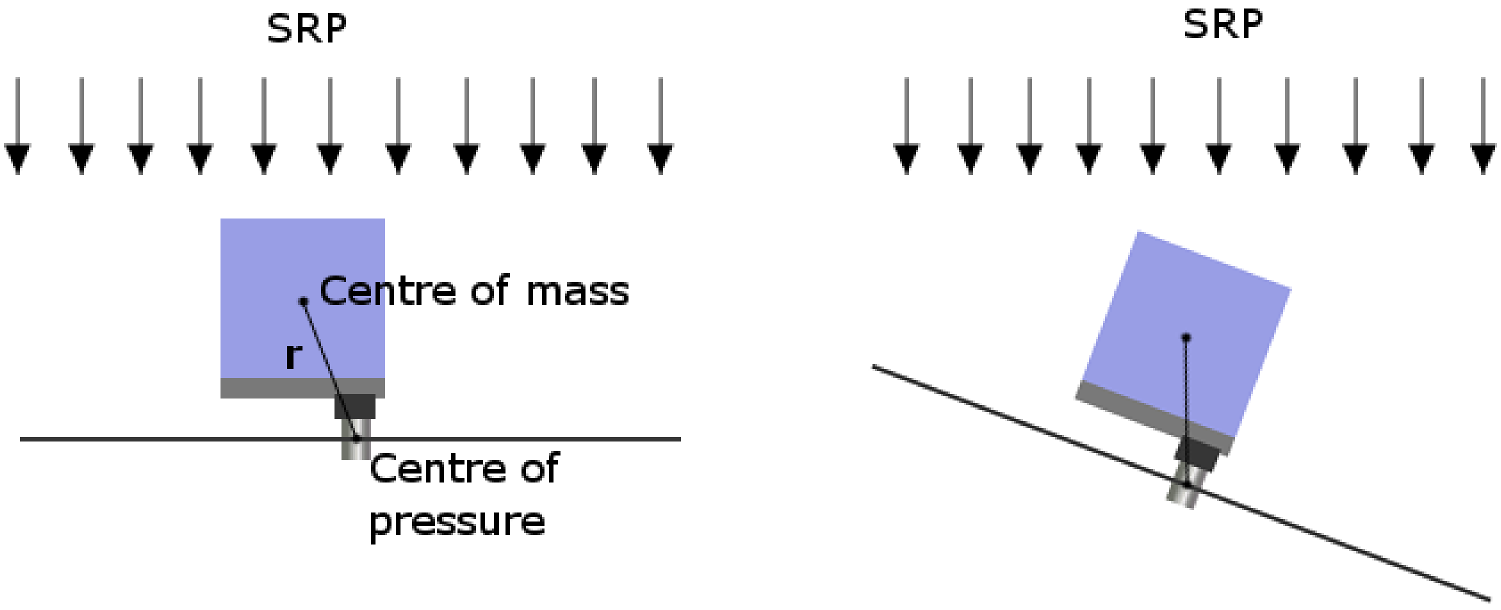
$$\hat{S} = \widehat{N}$$

then

$$\bar{F}_i = -PA\left[(1+\rho_s) + \frac{2}{3}\rho_d\right]\widehat{N}$$

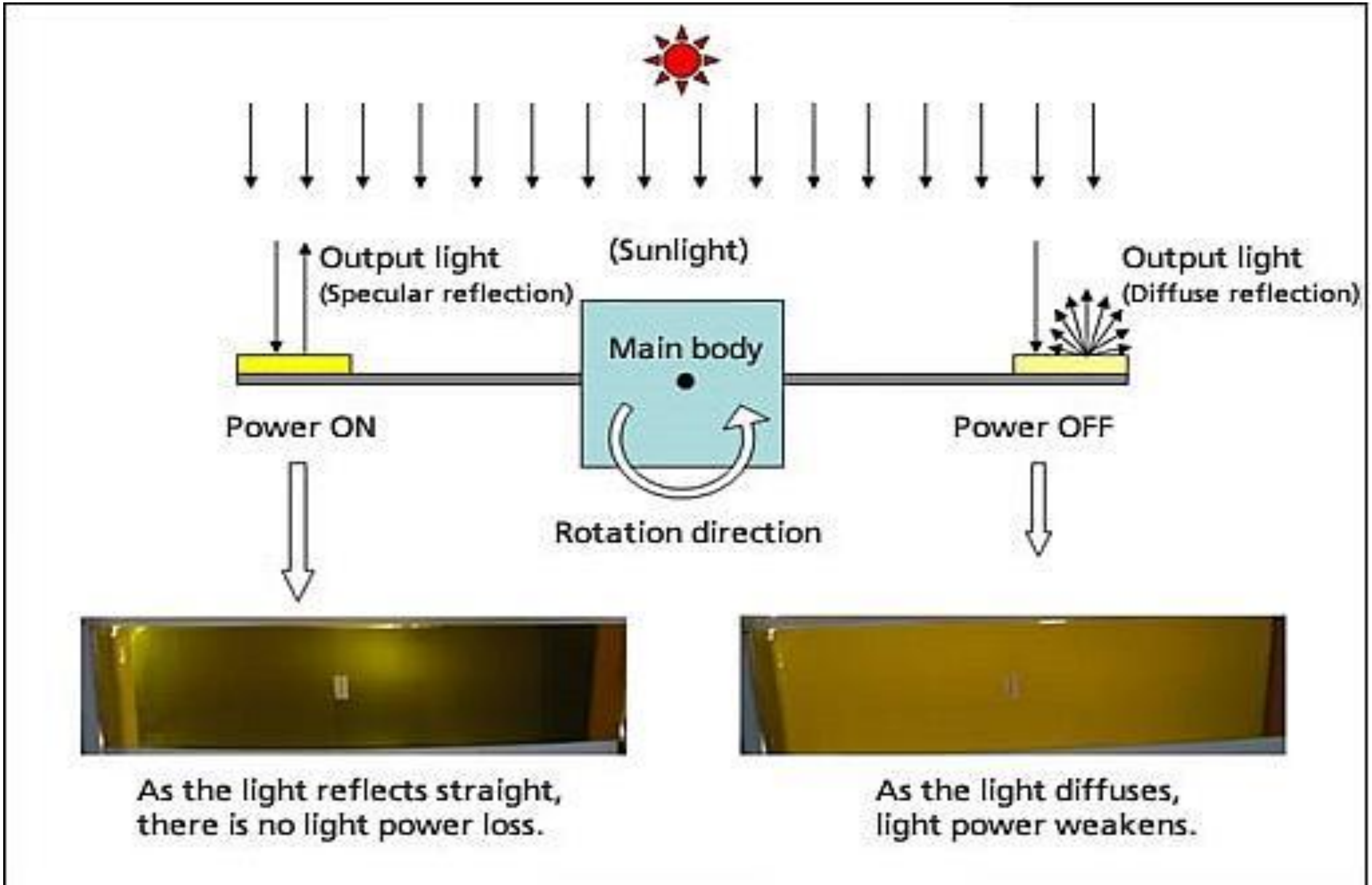
$$T_{SRP} = \sum_{i=1}^{n} \bar{r}_i \wedge \bar{F}_i$$

Exploiting SRP for attitude control - CubeSail

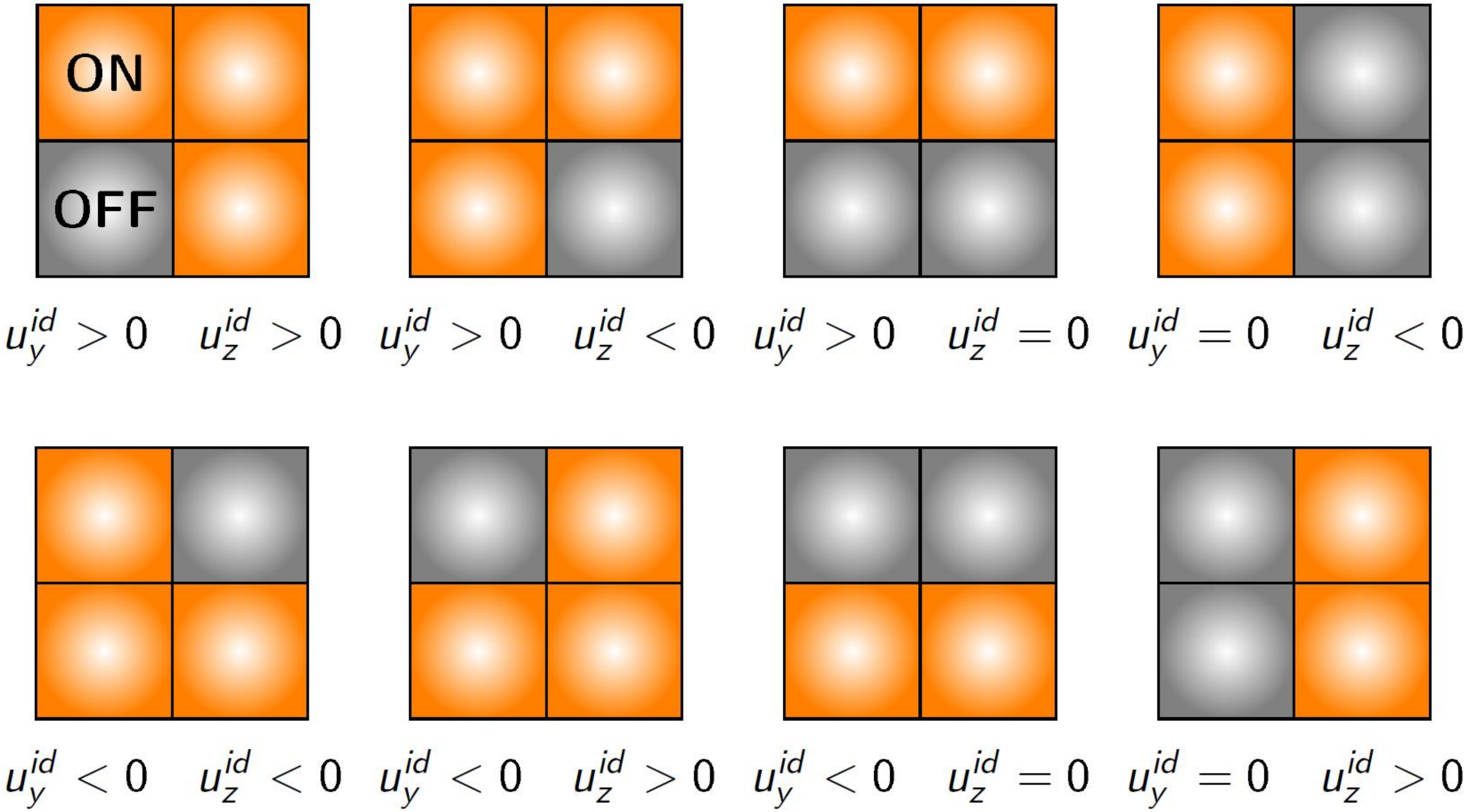


$$T_{srp} = \sum_{i=1}^{n} \bar{r}_i \wedge \bar{F}_i$$

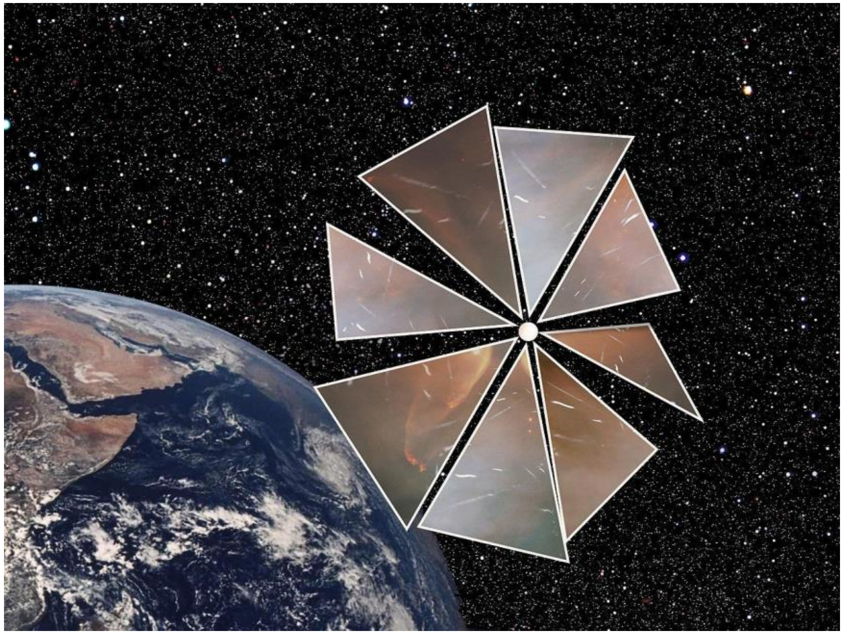
IKAROS – reflectivity control devices



Basic control logic



Future concepts for attitude control with SRP



$$\bar{F}_i = -PA_i(\hat{S}_b \cdot \widehat{N}_{bi})\left[(1-\rho_s)\hat{S}_b + \left(2\rho_s(\hat{S}_b \cdot \widehat{N}_{bi}) + \frac{2}{3}\rho_d\right)\widehat{N}_{bi}\right]$$

Summary

Preliminary estimation of relevance of torques

Disturbance	Type	Main parameters	Reference formula
Gravity gradient	Constant torque for Earth pointing, cyclic for inertial pointing	- Inertia moments - Orbit altitude	$T_{max} = \frac{3Gm_t}{2R^3}\left\|I_M - I_m\right\|$
Solar radiation	Cyclic torque for Earth pointing, constant for inertial (or Sun-oriented) pointing	- Spacecraft geometry - Panel reflectivity - Position of center of mass	$T_{max} = P_s A_s (1+q)\left(c_{ps} - c_g\right)$
Magnetic field	Cyclic torque	- Orbit altitude - Orbit inclination - Residual dipole	$T_{max} = D_s B_{max}$
Aerodynamics	Constant torque for Earth pointing, variable for inertial pointing	- Orbit altitude - Geometry and position of center of mass	$T_{max} = \frac{1}{2}\rho V^2 A_s C_D \left(c_{pa} - c_g\right)$

Passive control methods

Type	Advantages	Disadvantages
Spin-stabilised **(~1º accuracy)**	Simple, passive, long-life, provides scan motion, gyroscopic stability for large burns, inertial pointing	Poor manoeuvrability, low solar cell efficiency (cover entire drum),
Aerodynamic **(~5º accuracy)**	Simple, passive, low cost, useful for Earth pointing	Poor yaw stability, effective only in low altitude, poor accuracy, degrades orbit
Dual-spin stabilised **(~0.1º accuracy)**	Provides both fixed pointing (on de-spun platform) and scanning motion, gyroscopic stability for large burns	Require de-spin mechanism, low solar cell efficiency (cover entire drum), cost can approach 3-axis if high accuracy
Gravity-gradient **(~5º accuracy)**	Simple, low cost totally passive, long-life, provides simple passive Earth pointing mode	Low accuracy, almost no manoeuvrability, poor yaw stability, require deployment mechanism
Magnetic **(~1º accuracy)**	Simple, low cost, can be passive with use of permanent magnet or active with use of electromagnets	Poor accuracy (uncertainty in Earth's magnetic field), magnetic interference with science payload