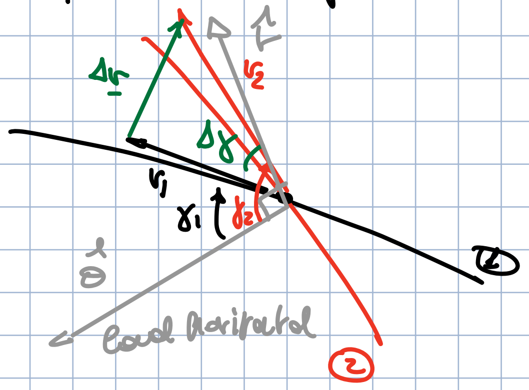
example Δv at point B



① → ② valid for every secant intersection.

$$\underline{\Delta v} = \underline{v_2} - \underline{v_1}$$

The magnitude of $\underline{\Delta v}$ is NOT $\cancel{\Delta v = |\underline{v_2} - \underline{v_1}|}$ No!!

$$\Delta v = \|\underline{\Delta v}\| = \sqrt{(\underline{v_2} - \underline{v_1}) \circ (\underline{v_2} - \underline{v_1})} = \sqrt{\underline{v_1} \circ \underline{v_1} - 2\,\underline{v_1} \circ \underline{v_2} + \underline{v_2} \circ \underline{v_2}}$$

$$\underline{v_1} \circ \underline{v_2} = v_1 v_2 \cos \Delta\gamma$$

$$\boxed{\Delta v = \sqrt{v_1^2 + v_2^2 - 2 v_1 v_2 \cos \Delta\gamma}} \qquad (3.18)$$

It is the cosine theorem for impulsive manoeuvres for coplanar orbit.

- If $\Delta\gamma = 0$ ($\underline{v_1} \parallel \underline{v_2}$) $\rightarrow$ $\Delta v = |v_2 - v_1|$ any case otherwise the triangle must be solved
- if $v_1 = v_2$ $\Rightarrow$ we are just rotating the velocity vector of an angle $\Delta\gamma$ Eq (3.18) becomes

$$v_1 = v_2 = v$$

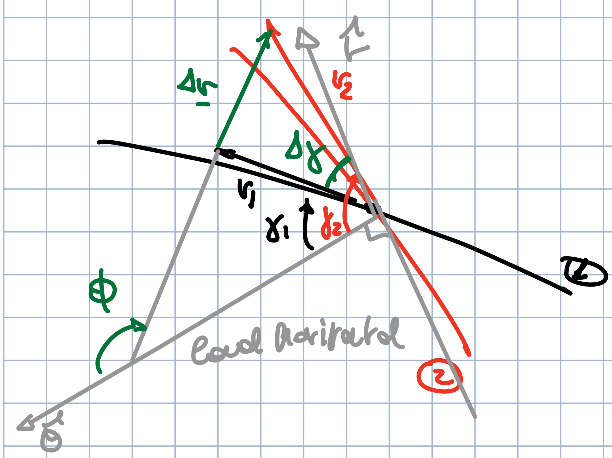
PURE ROTATION OF VELOCITY VECTOR IN ORBITAL PLANE

$$\Delta v = \sqrt{2v^2 - 2v^2 \cos \Delta\gamma} \Rightarrow \boxed{\Delta v = v\sqrt{2(1 - \cos\Delta\gamma)}} \qquad (3.19)$$

NOTE fuel expenditure is needed in both

- to change magnitude Δv
- to change the direction

the thruster need to be oriented in the direction of $\underline{\Delta v}$



Need to find the orientation of $\underline{\Delta v}$ wrt $\hat{r}, \hat{\theta}$ frame.

Need to find ϕ (flight path angle of $\underline{\Delta v}$)

DESIGN MISSION $\rightarrow$ $\underline{v_1}, \underline{v_2}$ $\rightarrow$ $\underline{\Delta v}$ $(\underline{v_1}, \underline{v_2}, \Delta\gamma)$

FLIGHT MISSION $\rightarrow$ to obtain $\underline{\Delta v}$ $\rightarrow$ Δv and ϕ $\rightarrow$ for the control of S/C.

Recall

$$tg\,\gamma = \frac{v_r}{v_\theta} \qquad (1.35)$$

$$\rightarrow \quad \boxed{tg\,\phi = \frac{\Delta v_r}{\Delta v_\theta}} \qquad (3.20)$$

Recall energy

$$\mathcal{E} = m \cdot \varepsilon \quad \rightarrow \text{where} \rightarrow \quad \varepsilon = \frac{\underline{v} \cdot \underline{v}}{2} - \frac{\mu}{r}$$

$\hookrightarrow$ specific total energy per unit mass

if an impulsive Δv is given

$$\Delta\mathcal{E} = \Delta(m\varepsilon) = m\Delta\varepsilon + \Delta m\,\varepsilon = m\left(\Delta\varepsilon + \varepsilon\frac{\Delta m}{m}\right)$$

if Δm (variation of propellant mass) is negligible

wrt m of s/c $\quad \frac{\Delta m}{m} \rightarrow 0$

$$\Rightarrow \quad \Delta \mathcal{E} = m \, \Delta \varepsilon$$

$$\Delta \varepsilon = \frac{(\underline{v} + \Delta \underline{v}) \cdot (\underline{v} + \Delta \underline{v})}{2} - \frac{\underline{v} \cdot \underline{v}}{2}$$

Recall that in an impulsive manoeuvre $\underline{r}$ does not change

$$\Delta \varepsilon = \frac{\cancel{\underline{v} \cdot \underline{v}} + \Delta \underline{v} \cdot \Delta \underline{v} + 2 \, \underline{v} \cdot \Delta \underline{v} - \cancel{\underline{v} \cdot \underline{v}}}{2}$$

$$= \frac{\Delta \underline{v} \cdot \Delta \underline{v} + 2 \, \underline{v} \cdot \Delta \underline{v}}{2} = \underline{v} \cdot \Delta \underline{v} + \frac{1}{2} \Delta v^2$$

$$\underline{v} \cdot \Delta \underline{v} = v \, \Delta v \cos(\phi - \gamma_1)$$

$$\boxed{\Delta \varepsilon = \Delta v \, v \cos(\phi - \gamma_1) + \frac{1}{2} \Delta v^2} \qquad (3.21)$$

Note there are some gravity losses during a real burn due to the rotation of the thrust.

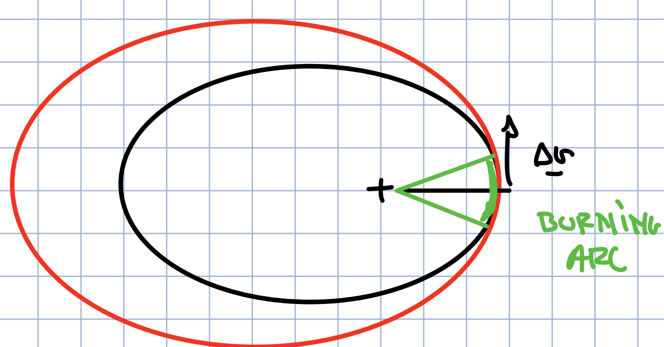
impulse $\underline{v}^- \rightarrow \underline{v}^+$

$\underline{r}^-$ $\quad \underline{v}^-$ $\quad$ ①

$\parallel$ $\quad \nparallel$

$\underline{r}^+$ $\quad \underline{v}^+$ $\quad$ ②

↳ $\Delta \underline{v} = \underline{v}^+ - \underline{v}^-$

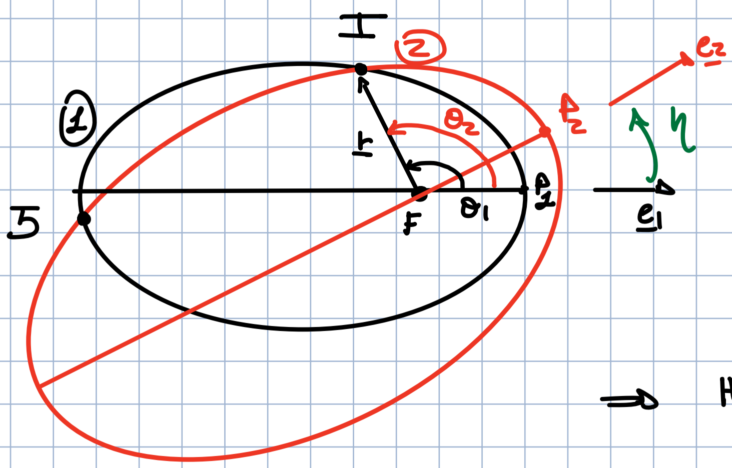


↳ gravity losses.

PHASE 0 $\quad$ PHASE A/B $\quad$ mission.

$\Delta \varepsilon$ higher for a given Δv if $v \uparrow$, if $\underline{v}$ and $\underline{\Delta v}$ are aligned

APSE LINE ROTATION



co-focal

co-planar

P_1 = periapsis ①

P_2 = periapsis ②

different apse line

$\Rightarrow$ Hohmann transfer is not possible

Transfer at one of two intersection $I, J \Rightarrow$ ROTATION OF APSE LINE (ALSO SHAPE AND SIZE)

NOTE 2 orbits can have max 2 intersection points

$\eta = \theta_1 - \theta_2$ ROTATION OF THE APSE LINE

CASE 1: GIVEN apse line rotation e_1, e_2, h_1, h_2

FIND true anomaly on ① and ② of points I, J

$I \in$ orbit ① and orbit ②

$$I \begin{cases} r_{①I} = \dfrac{h_1^2}{\mu} \dfrac{1}{1+e_1\cos\theta_1} \\ r_{②I} = \dfrac{h_2^2}{\mu} \dfrac{1}{1+e_2\cos\theta_2} \end{cases}$$

$$r_{①I} = r_{②I} \rightarrow \frac{h_1^2}{\mu}\frac{1}{1+e_1\cos\theta_1} = \frac{h_2^2}{\mu}\frac{1}{1+e_2\cos\theta_2}$$

$$h_1^2(1+e_2\cos\theta_2) = h_2^2(1+e_1\cos\theta_1) \quad (3.22)$$

set

$$\theta_2 = \theta_1 - \eta \qquad (3.23)$$

<u>Recall</u> $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\theta_1-\eta) = \cos\theta_1\cos\eta + \sin\theta_1\sin\eta$

In 3.22

$$h_1^2 - h_2^2 = h_2^2 e_1\cos\theta_1 - h_1^2 e_2\cos\theta_2$$

$$= h_2^2 e_1\cos\theta_1 - h_1^2 e_2(\cos\theta_1\cos\eta + \sin\theta_1\sin\eta)$$

$$\cos\theta_1\underbrace{(h_2^2 e_1 - h_1^2 e_2\cos\eta)}_{\text{known} = a} + \sin\theta_1\underbrace{(-h_1^2 e_2\sin\eta)}_{b = \text{known}} = \underbrace{h_1^2 - h_2^2}_{c = \text{known}}$$

$$\cos\theta_1\, a + \sin\theta_1\, b = c \qquad (3.24)$$

let's divide by a

$$\cos\theta_1 + \sin\theta_1\frac{b}{a} = \frac{c}{a}$$

$\frac{b}{a} = \mathrm{tg}\,\phi$ $\quad\phi$ auxiliary angle

$$\cos\theta_1 + \frac{\sin\phi}{\cos\phi}\sin\theta_1 = \frac{c}{a}$$

$$\cos\theta_1\cos\phi + \sin\phi\sin\theta_1 = \frac{c}{a}\cos\phi$$

recall $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos(\theta_1 - \phi) = \frac{c}{a}\cos\phi$$

$$\theta_1 - \phi = \cos^{-1}\left(\frac{c}{a}\cos\phi\right)$$

$$\boxed{\begin{array}{l} \theta_1 = \phi \pm \cos^{-1}\left(\frac{c}{a}\cos\phi\right) \\ \text{where} \quad \phi = tg^{-1}\left(\frac{b}{a}\right) \end{array}} \qquad (3.26)$$

Now known θ_1 and θ_2 $\Rightarrow$ Find Δv

$$r_{I_1} = \frac{h_1^2/\mu}{1+e_1\cos\theta_1} \qquad r_{I_2} = r_{I_1}$$

velocity components on orbit ① at point I

$$v_{\theta_1 I} = \frac{h_1}{r} = \frac{\mu}{h_1}(1+e_1\cos\theta_1)$$

$$v_{r_1 I} = \frac{\mu}{h_1} e_1 \sin\theta_1$$

$$\gamma_{1I} = tg^{-1}\left(\frac{v_{r_1 I}}{v_{\theta_1 I}}\right) \qquad v_{1I} = \sqrt{v_{\theta_1 I}^2 + v_{r_1 I}^2}$$

Same for orbit ② at point I

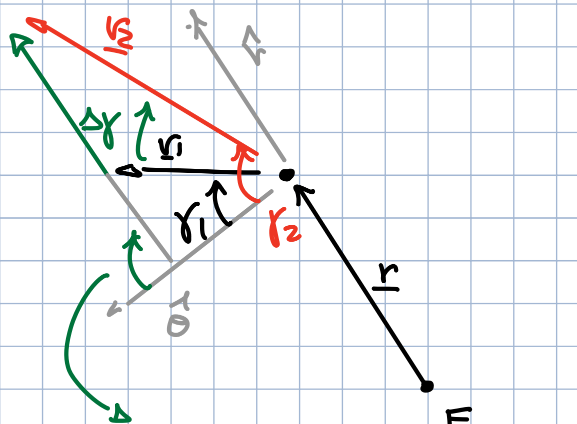
$$\theta_2 = \theta_1 - \eta$$

$$v_{\theta_2 I}, \; v_{r_2 I}, \; \gamma_{2I} \Rightarrow v_{2I} = \sqrt{v_{\theta_2 I}^2 + v_{r_2 I}^2}$$

$$\Delta v = \sqrt{v_1^2 + v_2^2 - 2 v_1 v_2 \cos(\gamma_2 - \gamma_1)}$$

we can calculate the flight path angle of the delta velocity.

$$\gamma_{\Delta v} = tg^{-1}\left(\frac{\Delta v_r}{\Delta v_\theta}\right) = tg^{-1}\left(\frac{v_{r2} - v_{r1}}{v_{\theta 2} - v_{\theta 1}}\right)$$



APPLICATION : - ORBIT MAINTENANCE

- DESIGN TRANSFER ORBIT, MISSION DESIGN

$$\underbrace{\begin{array}{ccc} \text{initial} & \longrightarrow & \text{TARGET} \\ e_1, h_1 & & e_2, h_2 \\ & & \text{apse rotation} \end{array}}_{\text{define maneuvre that makes this possible}}$$

NEXT TIME → application to flight dynamics