

ORBITAL MECHANICS

$f = \text{Earth's oblateness}$

$$f = \frac{2E - R_p}{R_E} \quad (2.57)$$

$R_p \approx R_E - 30 \text{ km}$

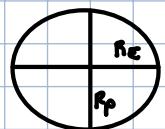
$f \approx 0.00335$

GEOPHYSICAL LATITUDE = GEOCENTRIC LATITUDE.

$$R_p = R_E \sqrt{1 - e^2_{\text{ellipse}}}$$

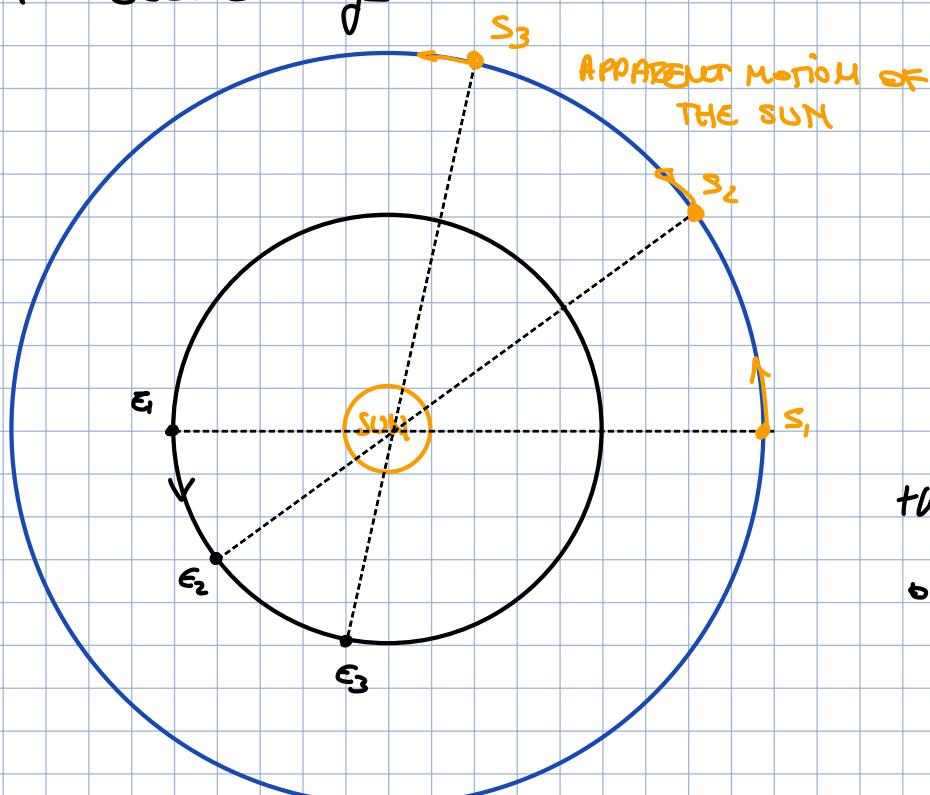
eclipse \rightarrow refers to the Earth shape.

$$f = 1 - \sqrt{1 - e^2_{\text{ellipse}}} \quad (2.58)$$

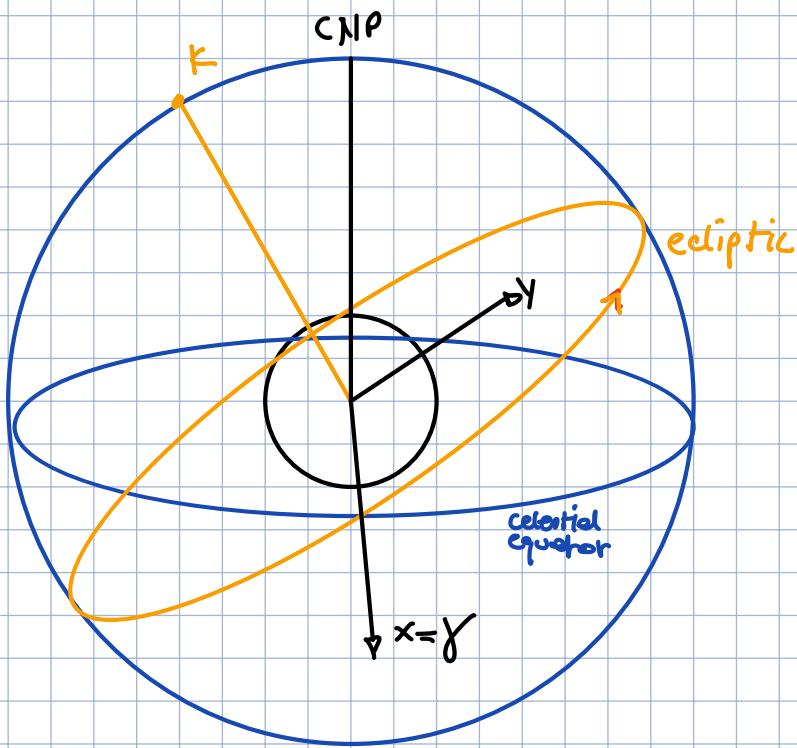


EARTH MOTIONS

$$T = 365.25 \text{ days}$$



We can project the Sun on the celestial sphere. We can observe the apparent motion of the sun due to the motion of the Earth around the sun.



APPARENT MOTION OF THE SUN

ALONG THE ECLIPТИC.

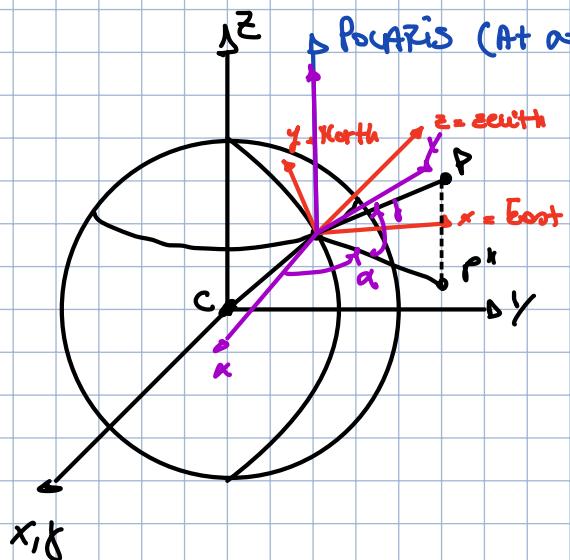
Due to precession NP produces a cone in 26000 years around K

ECLIPТИC : apparent great circle of the path of Sun on the celestial sphere during 1 years.

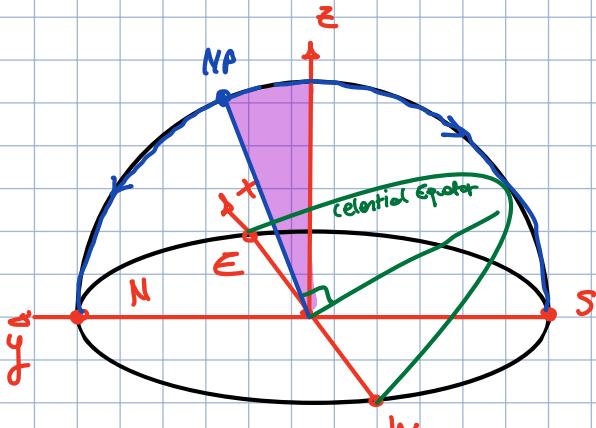
PLANE OF ECLIPТИC : plane of Earth's orbit around Sun

CELESTIAL EQUATOR : equator projected on the celestial sphere.

TOPOCENTRIC EQUATORIAL COORDINATE SYSTEM



centred at the observer



The arc of a great circle that goes

NP -> SP = observer meridian

CELESTIAL EQUATOR : plane \perp to the PN that intersect the celestial sphere and horizon by identify the points East and West.

Demonstration

Let's call s the line of intersection between the celestial equator and the horizon.

$s \in$ celestial equator plane

$s \in$ horizon plane $\sigma + \bar{\sigma}$

if s is \perp to both directions $\Rightarrow s \perp$ plane define by those two directions $\Rightarrow s \perp$ plane defined by $\sigma, \bar{\sigma}, \text{PN}$

$s \perp$ to any straight line \in to that plane.

$\Rightarrow s \perp$ to line $\bar{N}s$ and $\bar{E}s \in \sigma, \bar{\sigma}, \text{PN}$

$\Rightarrow s$ is the direction $\bar{N}\bar{E}$

\Rightarrow intersection between horizon and celestial equator identifies N and E.

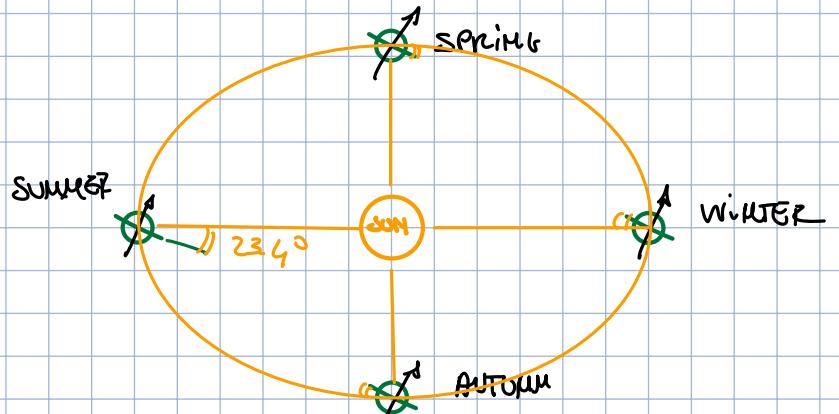
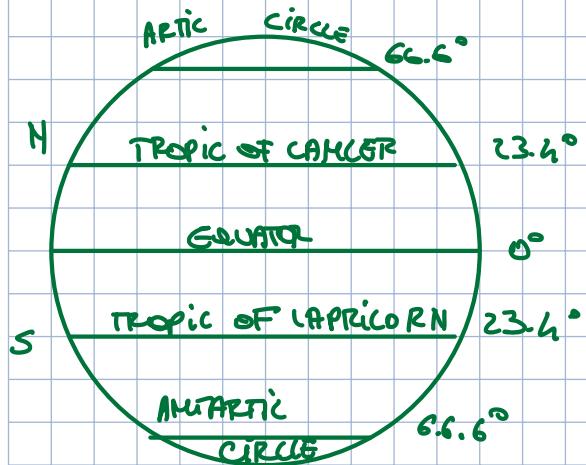
N: search polaris star project down from $\bar{\sigma}$ to horizon

S: two direction of maximum altitude of sun during the day.

$N, S \perp (N, S)$ on horizon you can see two points s to celestial equator (virtual line).

Sun rises and sets at E and W twice a year (equinoxes)

Sun rises and sets at \neq points because the ecliptic is inclined of 23.5° on the Equator.



In summer the sun is going to hit \perp the Tropic of Cancer

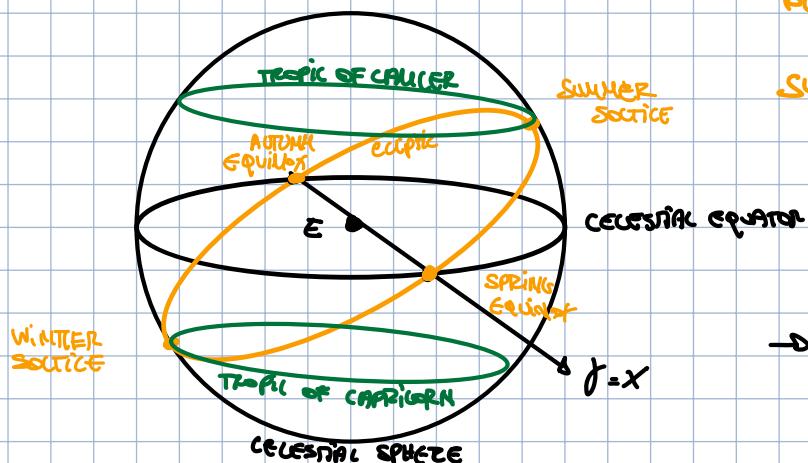
In winter the sun is going to hit \perp the Tropic of Capricorn.

$$66.6^\circ + 23.4^\circ = 90^\circ$$

Above and below 66.6° the sun is going to be above the horizon for a full day 24 h at least one time per year.

Above and below 23.4° latitude sun never rises up to 90° .

$\text{ECEF} = \text{Earth fixed earth fixed}$

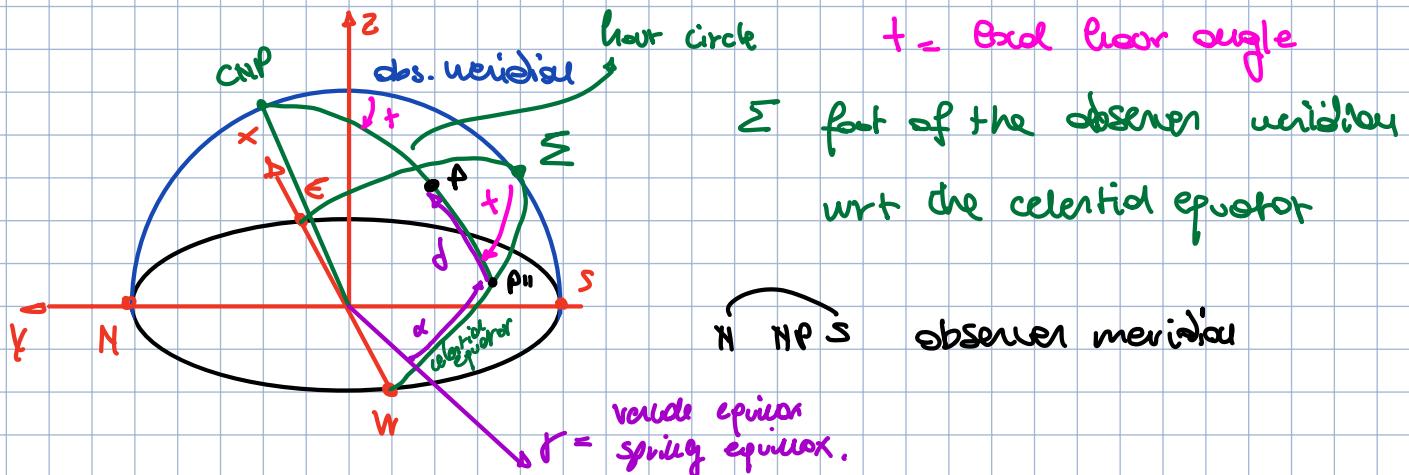


In spring and autumn the sun rises and sets at E,W

→ At sun rise and sunset ($\pm S$)
E horizon, ecliptic,
celestial equator

This is an inertial reference frame \Rightarrow we are not considering the rotation of the Earth

Let's go back to topocentric horizon coordinate system.



Whenever we want to trace the position of a star it is necessary to trace the arc of a great circle passing through C.N.P and the star p

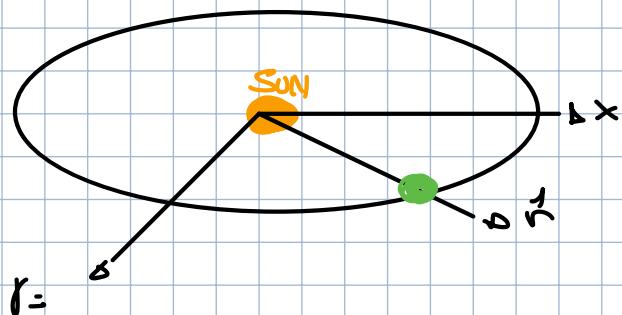
NP P P" hour circle = great circle that passes through Np and P

δ = declination distance to the celestial equator along the hour circle

α = angular distance from the vernal equinox to p^{ll}

$$-\frac{\pi}{2} \leq f \leq \frac{\pi}{2} \quad 0 \leq x < 2\pi$$

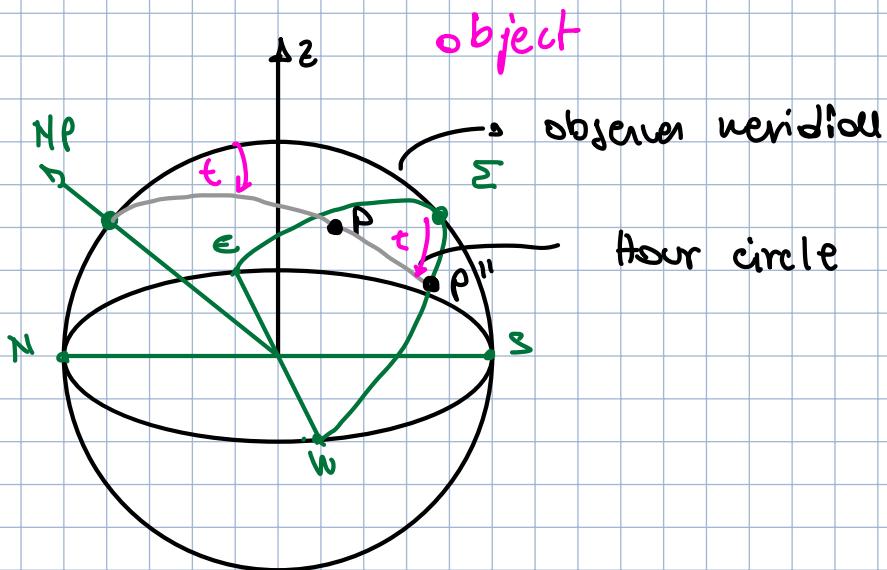
The star in the sky appears to rotate due to the Earth's rotation around the axis but α , δ are fixed while A and B are functions of time $A(t)$, $B(t)$. Also P and γ appear to rotate



pieces construction
(before f Aves)

δ, α are not influenced by the position of the observer located on the celestial sphere. δ, α change only to the variation introduced by the precession.

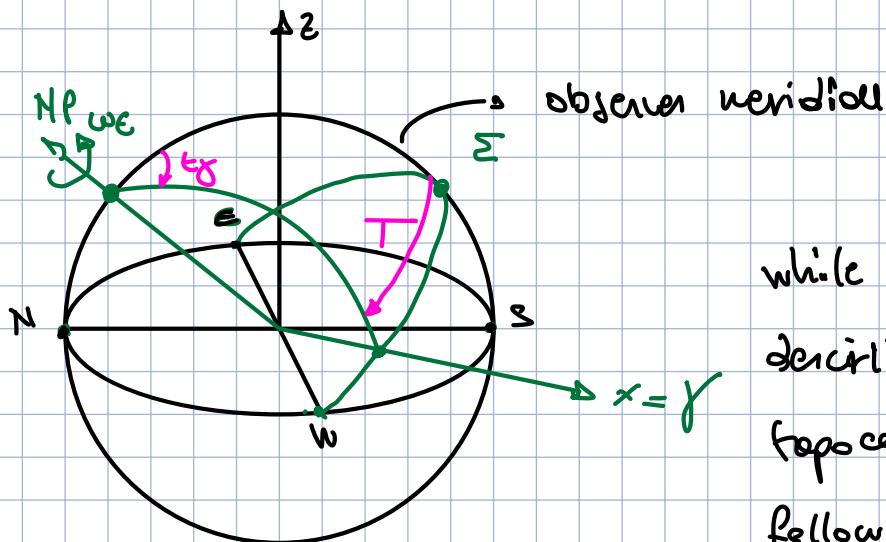
LOCAL HOUR ANGLE \Rightarrow angle measured west along the equator from the observer meridian ($NNPS$) to the hour circle ($NP'P''$) through the celestial object



As Earth rotates counter clockwise \Rightarrow P appears to rotate from E to W \Rightarrow angle Σ increases ^{locally} with the local system is using a very specific star $\rightarrow \gamma$

Because γ is fixed in space.

$$\text{For } \gamma = \begin{cases} \alpha = 0 \\ \delta = 0 \end{cases}$$

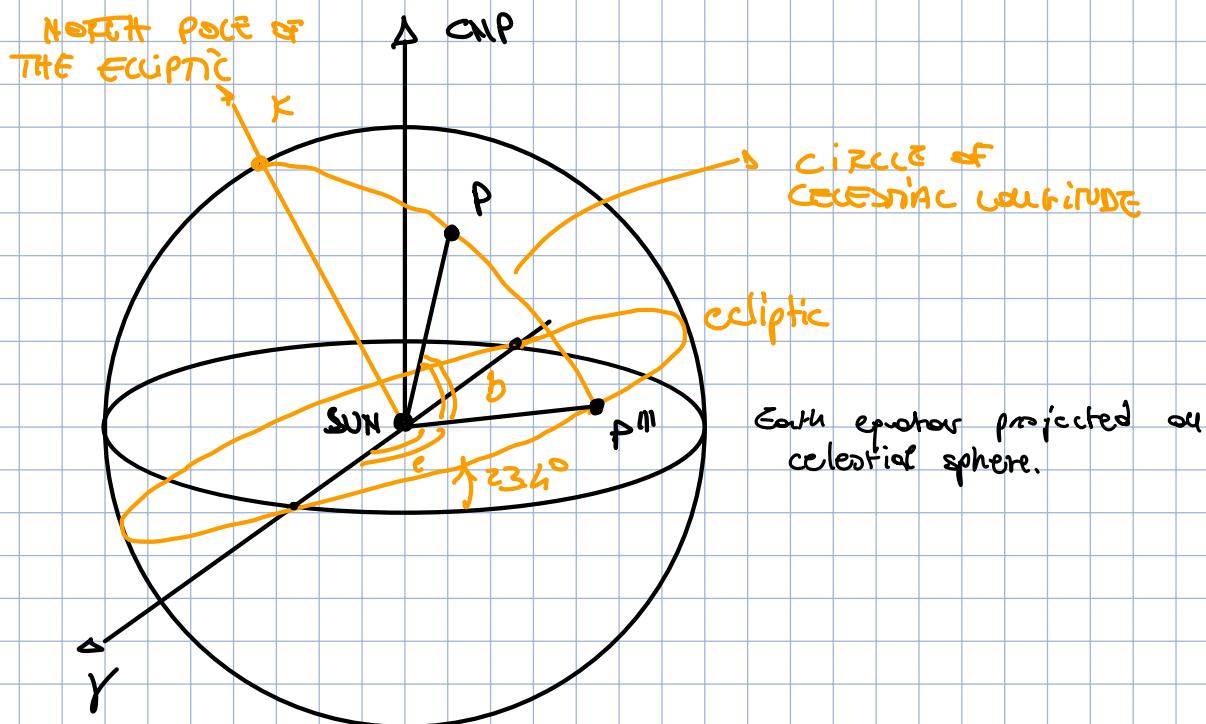


$\epsilon_f = T$ = sidereal Time

while Earth rotates around Xp, it describes a cone around Xp and has a centripetal acceleration if given follows ω , T increases from 0 to 2π

Celestial Latitude, Longitude and Heliocentric Coordinates

To describe other phenomena in the solar system, it is convenient to use the elliptic system.



ℓ = CELESTIAL LONGITUDE (from γ to ρ^{III} on the ecliptic)

$$0 \leq \theta < 2\pi$$

b = CELESTIAL LATITUDE

$$-\frac{\pi}{2} \leq b \leq \frac{\pi}{2}$$

a and b or x and y one element fixed

REFERENCE FRAME

POLE

LATITUDE-LIKE

LATITUDE-LIKE

HORIZON

$z = \text{zenith}$

$h(t) = \text{ALTITUDE}$

$A(t) = \text{Altitude}$
Clock wise from H

CELESTIAL EQUATOR

CNP = CELESTIAL
NORTH POLE

$\delta = \text{DECIMATION}$

$\lambda = \text{RIGHT ASCENSION}$

ECLIPTIC

K = NORTH POLE
ECLIPTIC

for an observer at earth

$\delta = \text{Co-latitude}$ $\phi = \text{Latitude geographic}$

$\psi = \text{geographic longitude}$.

$b = \text{CELESTIAL}$
 LATITUDE

$\ell = \text{CELESTIAL}$
 LONGITUDE