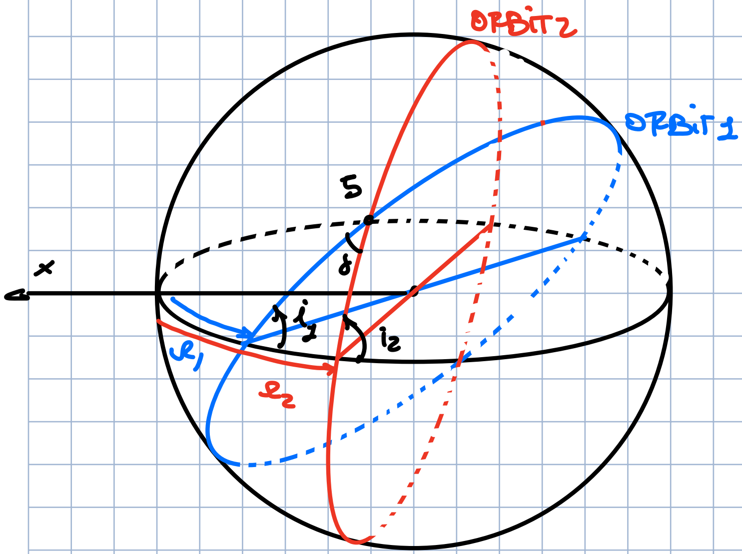
ORBITAL MECHANICS



GIVEN i_1, Ω_1

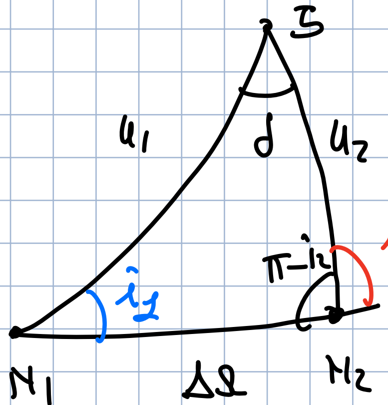
GIVEN i_2, Ω_2

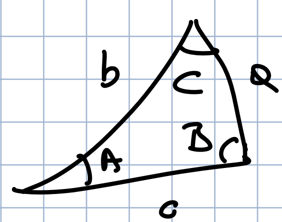
1. FIND THE intersection between two planes S r^*
2. Rotate $\underline{v}$ on the new plane

if orbit has same shape and size $\Rightarrow$ a, e do not change
$\Rightarrow$ θ do not change

CHANGE i, Ω, ω

$$u_1 = \omega_1 + \theta_1$$
$$u_2 = \omega_2 + \theta_2$$





$$\cos A = -\cos B \cos C + \sin B \cos C \cos a$$

$$\cos\delta = -\cos i_1 \cos(\pi - i_2) + \sin i_1 \sin(\pi - i_2)\cos\Delta\Omega$$

$$\cos\delta = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos\Delta\Omega \qquad (3.71)$$

Let's write it for another angle:

$$\cos i_1 = -\cos\delta \cos(\pi - i_2) + \sin\delta \sin(\pi - i_2)\cos u_2$$

$$\cos i_1 = \cos\delta \cos i_2 + \sin\delta \sin i_2 \cos u_2 \qquad (3.72)$$

for the third and final one

$$\cos(\pi - i_2) = -\cos\delta \cos i_1 + \sin\delta \sin i_1 \cos u_1$$

$$\cos i_2 = \cos\delta \cos i_1 - \sin\delta \sin i_1 \cos u_1 \qquad (3.73)$$

we could use the sine theorem

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$$\frac{\sin u_1}{\sin(\pi - i_2)} = \frac{\sin u_2}{\sin(i_1)} = \frac{\sin \Delta\Omega}{\sin \delta} \qquad (3.74)$$

from Eq (3.74) $\Rightarrow$ δ known $i_1, i_2, \Delta\Omega$

Eq (3.71) (3.72) (3.73) $\Rightarrow$ u_1, u_2

Eq (3.74) $\longrightarrow$ $\sin(u_1) = \dfrac{\sin\Delta\Omega}{\sin\delta} \sin i_2$

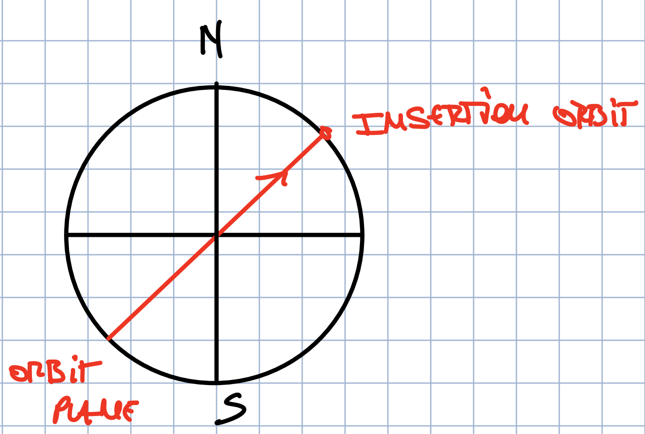
$$\sin(u_2) = \frac{\sin\Delta\Omega}{\sin\delta} \sin i_1$$

$$\left.\begin{aligned} u_1 &= \omega_1 + \theta_1 \\ u_2 &= \omega_2 + \theta_2 \\ \theta_1 &= \theta_2 \end{aligned}\right\} \longrightarrow \omega_2$$

HOW TO INJECT A SATELLITE / LAUNCHER ORBIT INJECTION

Δv change of plane is expensive $\Rightarrow$ it's done during the ascent phase. For some missions plane change must take place in orbit. e.g GEO $(i=0)$ but it is not possible to launch directly in equatorial orbit if launch site is not on the equator.

Plane of orbit must contain: - CENTER OF EARTH (FOCUS)
- POINT WHERE S/C IS INSERTED IN ORBIT



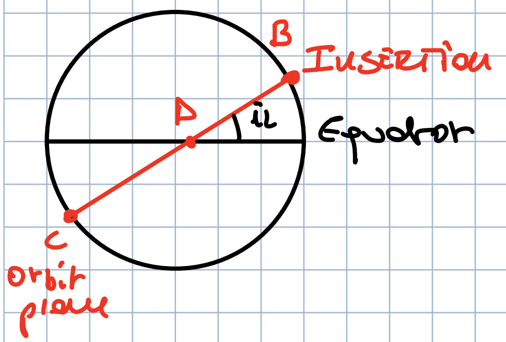
Launching S/C East take advantage of Earth's rotational velocity (0.46 km/s at the equator)

$$V_{equatorial} = 0.46 \text{ km/s}$$

$$V_{\substack{rotational \\ earth}} = V_{equatorial} \cos\phi \qquad (3.75)$$

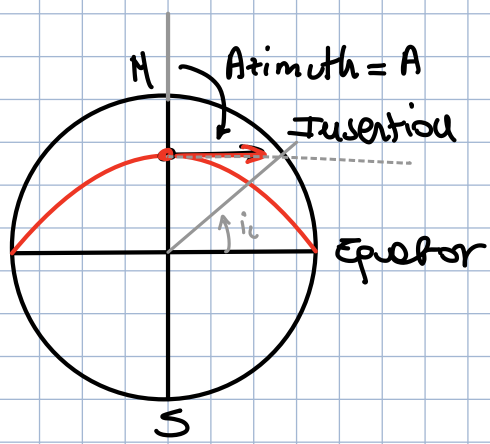
↑ diminishes at the poles

↳ ϕ = latitude



if s/c is launched at i_L (point B)

- After a quarter of the orbit the s/c will intersect the equator.
- After half of orbit it reaches the southern most point C
- After $\frac{3}{4}$ of the orbit reaches the point D (cross with the Equator)
- After 1 full orbit returns at point B to $\phi = i_L$ north.



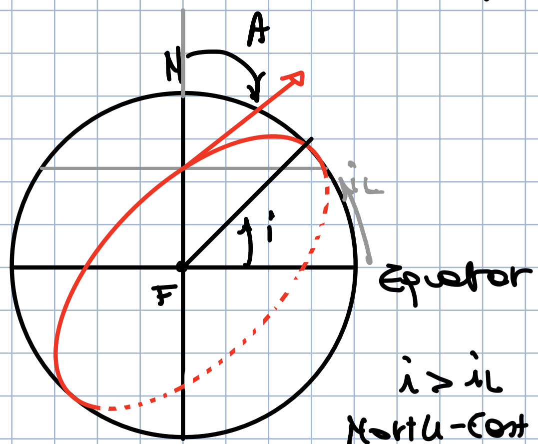
($A = \frac{\pi}{2}$ here)

Launch Azimuth is the flight direction at insertion point, measured clockwise from North on the local meridian.

What happens if the insertion direction is not exactly east?

Suppose we will launch with a north-east direction.

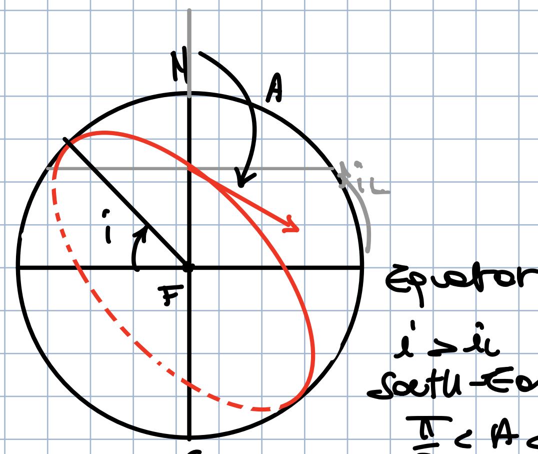
$\Rightarrow$ The inclination of insertion orbit will be higher than i_L



$i > i_L$

North-East Direction

$0 < A < \frac{\pi}{2}$



$i > i_L$

South-East direction

$\frac{\pi}{2} < A < \pi$

Suppose we will launch with a south-East direction.

It takes less advantage of Earth's rotation and $i > i_L$

Both cases orbit will have an Eastward velocity

$\Rightarrow$ PROGRADE ORBIT $\quad i_L < i < \frac{\pi}{2}$

Launch towards west produce a RETROGRADE ORBIT with $i > \frac{\pi}{2}$ $\Rightarrow$ $\frac{\pi}{2} < i < \pi$

Launch North to South produce a Polar orbit. $\quad i \geq i_L$ (3.76)

NOTE Retrograde orbit are very important because we can get sun synchronous orbit that are very important for remote sensing.

Spherical geometry can be used to obtain relation between orbit inclination i, launch platform ϕ and launch azimuth A

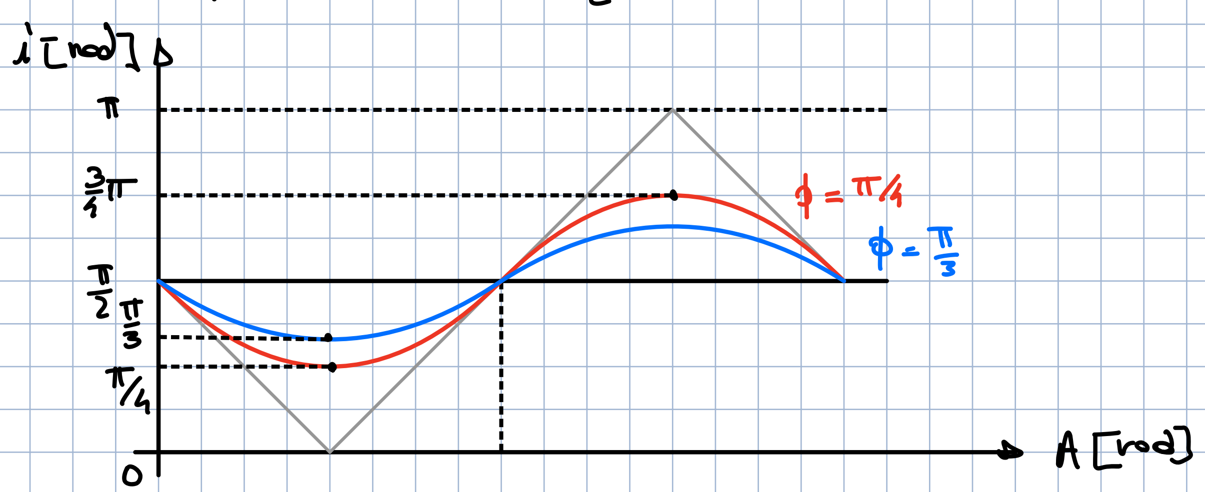
$$\cos i = \cos\phi \sin A \qquad (3.77)$$

final inclination

launch platform latitude

Azimuth

$i = \phi$ when $A = \frac{\pi}{2}$ $\Leftarrow$



- exercise

Determine the launch Azimuth for a sun-synchronous orbit if is launched on California coast ($\phi = 34.5°$ N latitude)

S/c orbit Period $T = 100$ min $(e=0)$

<u>hint</u>: Sun synchronous Orbit

$$\dot{\alpha}_{SUN} = \frac{2\pi}{365.26 \cdot 24 \cdot 3600} = 1.991 \cdot 10^{-7} \ \frac{rad}{s}$$

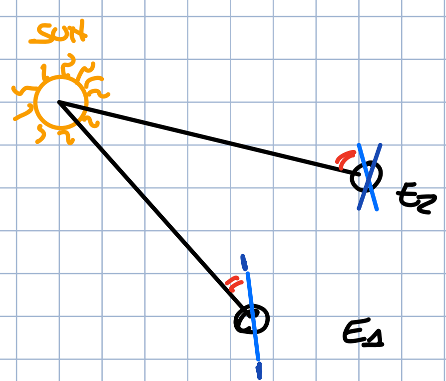
$$\dot{\Omega}_{J_2} = -\frac{3}{2} \frac{\sqrt{\mu} \, J_2 \, R_E^2}{(1-e^2)^2 a^{7/2}} \cos i \qquad J_2 = 0.00108263$$

↳ exploiting the J_2 effect

sun-synchronous $\boxed{\dot{\alpha}_{SUN} = \dot{\Omega}_{J_2}}$ => very important

(3.78)

↳ RAAN naturally drift by the same amount as apparent sun wrt Earth

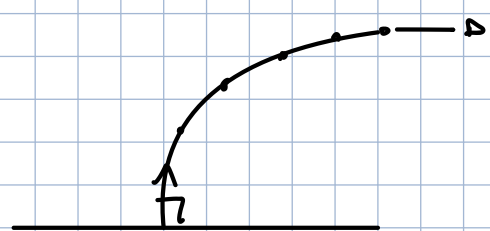


sun synchronous

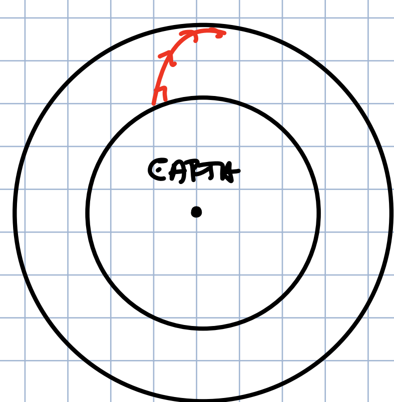
NOT SUN SYNCHRONOUS

$\phi = \alpha_{SUN} - \Omega = $ const

Definition of the insertion point?



the point we reach orbital in order to stay in orbit



INTERPLANETARY TRAJECTORY → new chapter

PATCHED CONIC METHOD

preliminary mission analysis for interplanetary mission (phase 0 / phase A mission design)

simplification : interplanetary leg (e.g. Earth Mars) is Hohmann transfer

$\underline{r}_1(t_0)$, $\underline{r}_2(t_0+\Delta t)$ such that Hohmann transfer possible.

(IN GENERAL ⇒ solve Lambert Problem)

hp : - planet's orbit lie on the same plane
- planets are synchronized so that Hohmann is possible ⇒ define right timings.

⇒ In general solve Lambert Problem (PC - AIDED)

- Let's start by considering 1 interplanetary trajectory leg:

PLANET 1 → INTERPLANETARY TRANSFER → PLANET 2

① ESCAPE ② CRUISE ③ CAPTURE / FLY-BY

① ESCAPE PLANET 1 ORBIT : hyperbolic / parabolic path

μ_1 of planet 1 - orbit such that s/c velocity at the sphere of influence of planet 1 matches the required Δv for interplanetary transfer.

② CRUISE TRANSFER IN HELIOCENTRIC SYSTEM

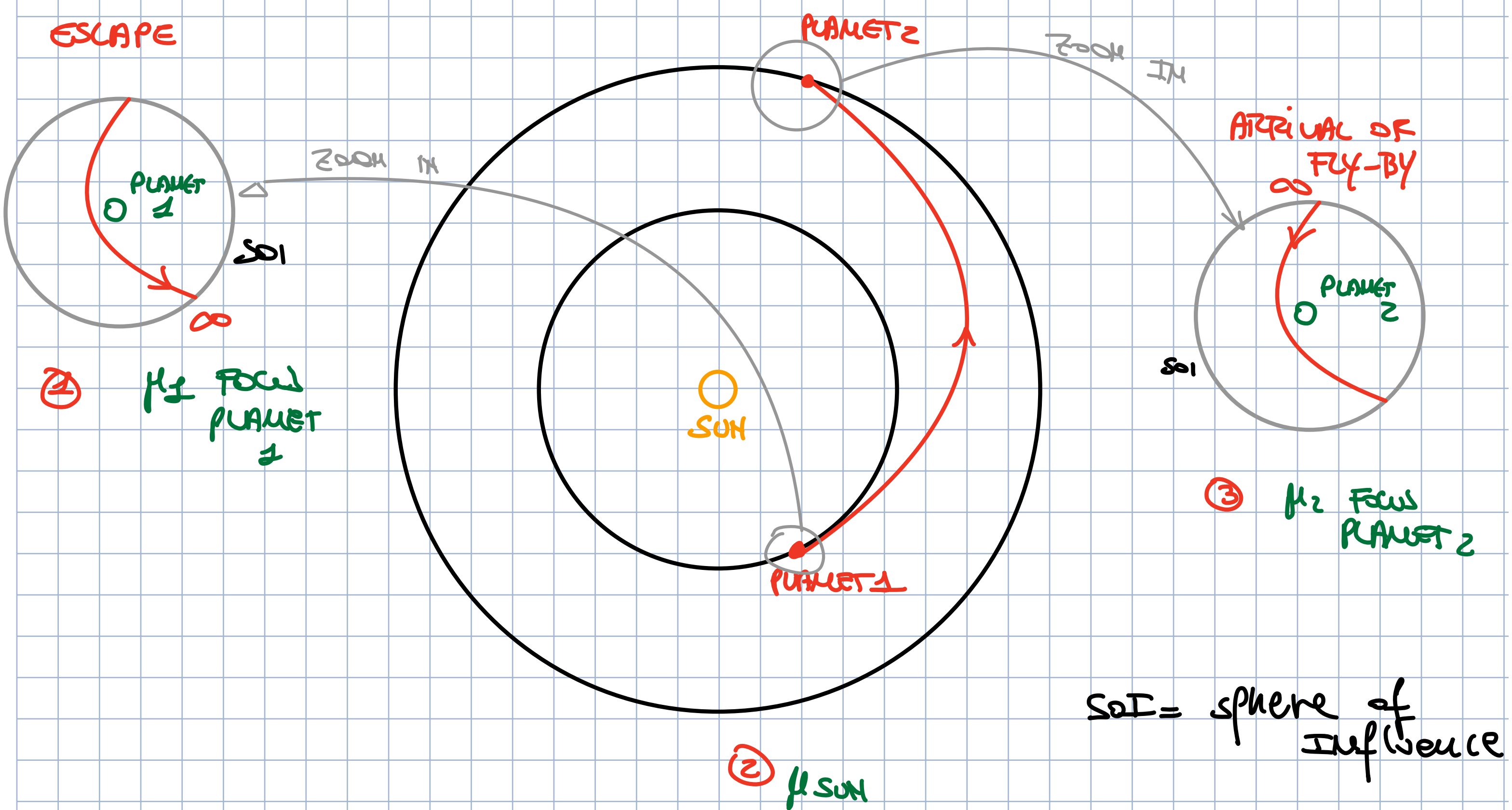
μ_{SUN} focus $\equiv$ SUN

eg Hohmann transfer if planet 1 and planet 2 are synchronised. In general we can substitute it with a Lambert arc.

③ CAPTURE / FLY-BY AT PLANET 2 : hyperbolic/parabolic path

μ_2 of planet 2 FOCUS $\equiv$ PLANET 2 orbit is such that the v_∞ velocity at the sphere of influence of planet 2 matches the required Δv for interplanetary transfer.

Specifying the perigee radius of hyperbola on phase 1 and 3 determine the Δv requirements at departure and arrival.



TIME ORDER ① $\longrightarrow$ ② $\longrightarrow$ ③

ORDER THAT WE WILL FOLLOW
FOR THE DESIGN

INTERPLANETARY 2
THEN
ESCAPE 1 AND CAPTURE/FLY BY 3