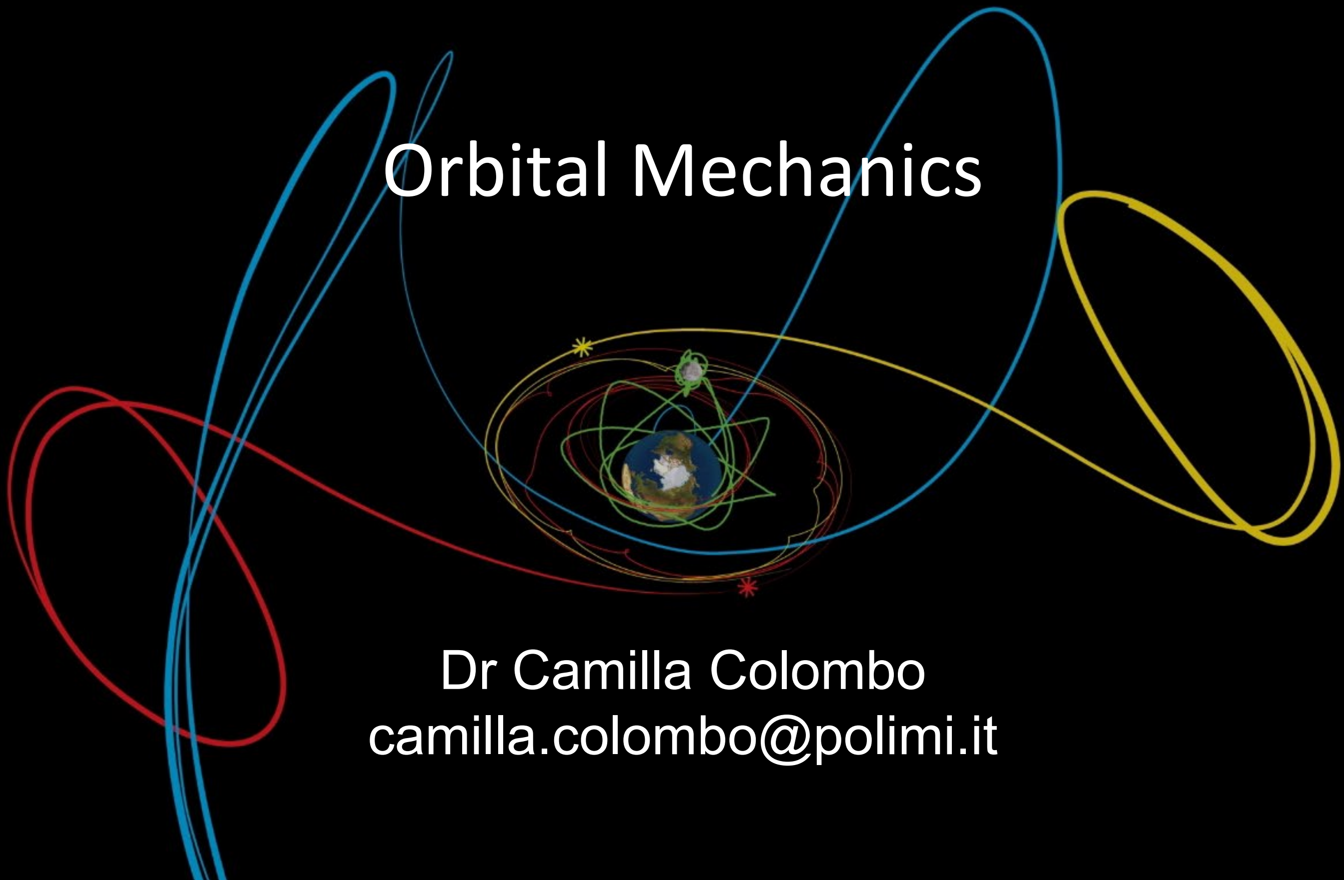

Orbital Mechanics

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5. ORBIT PERTURBATIONS

References

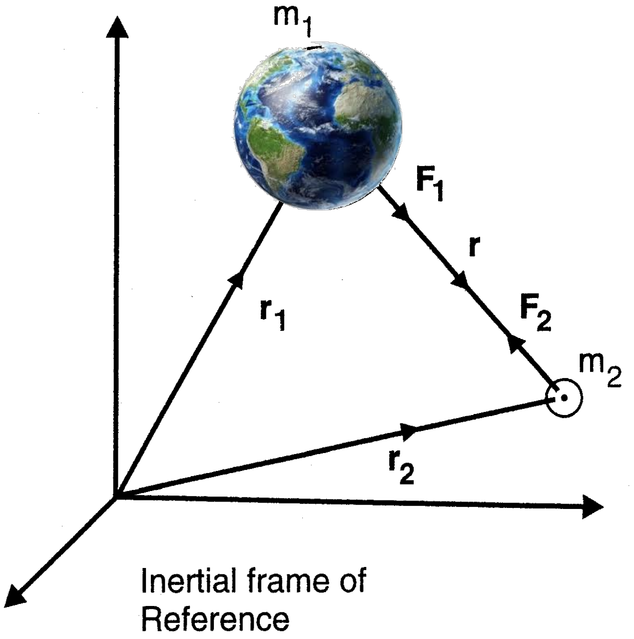
- A. Vallado, Fundamentals of Astrodynamics and Applications (Space Technology Library). 4th Edition, Springer, 2007, ISBN-13 978-0387718316. Chapter 8 and 9 (Very detailed)
- R. H. Battin, An Introduction to the Mathematics and Methods of Astrodynamics, Revised Edition, AIAA Educational Series, Reston, 1999. Chapter 10 – Gauss and Lagrange equations derivation
- H. Curtis, Orbital Mechanics for Engineering Students, Second Edition (Aerospace Engineering). 2nd Edition, Butterworth-Heinemann, 2009, ISBN-13 978-0123747785.
  Chapter 12 (Introduction to orbit perturbations)

Recall: general and restricted two-body problem

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r}$$

Keplerian orbit: closed form solution of the two-body equation of relative motion

- Only two masses in space
- Their spherical gravitational fields are the only source of interaction between them
- $m_1 >> m_2$, and $\mu = G \cdot m_1$



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5.1 The perturbed two-body problem

The introduction of a perturbing acceleration $\mathbf{a}_p$ modifies the equations of motion such that

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r} + \sum \mathbf{a}_p$$

$\mathbf{a}_p$: perturbative acceleration from each source other than spherically symmetric gravitational attraction between two bodies.

- Magnitude small compared with primary gravitational acceleration

$$\mathbf{a}_p \leq a_0, \quad \text{with } a_0 = \frac{\mu}{r^2}$$

- Exception: atmospheric drag below 100 km

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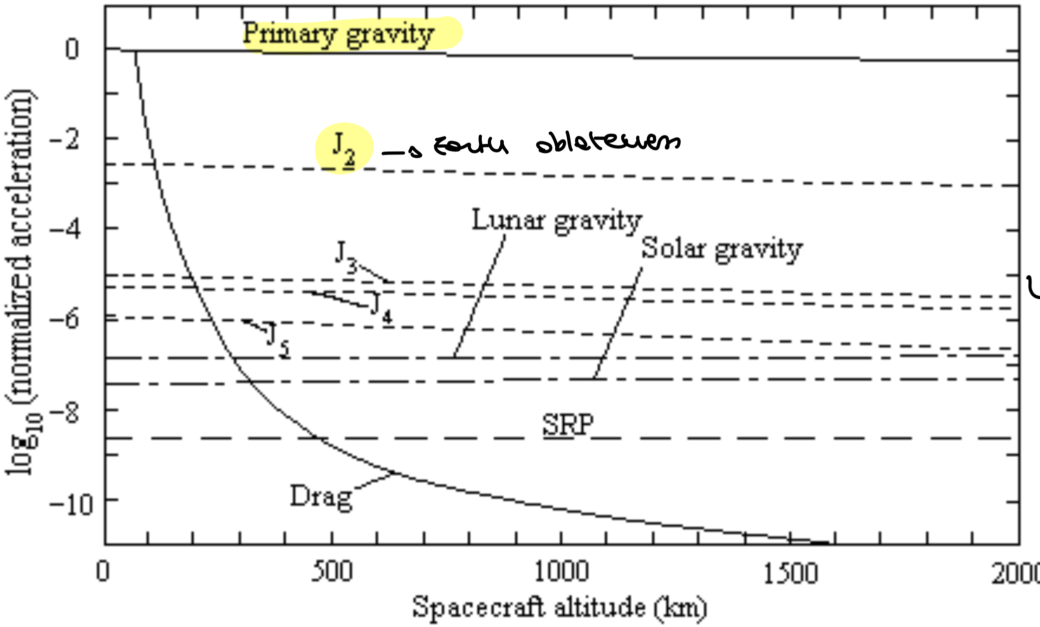
The perturbed two-body problem

from vallado



→ Earth oblateness

all these 5 due to non spherical distribution of mass.

The perturbed two-body problem

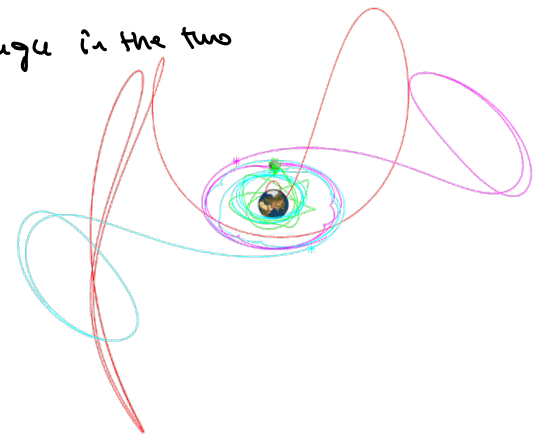
<u>Example</u>: at 1000 km

- Acceleration of Earth's oblateness $\mathbf{a}_p \approx 10^{-2} a_0$
- Acceleration due to lunar gravity $\mathbf{a}_p \approx 10^{-9} a_0$
- Acceleration due to solar radiation on conventional satellite $\mathbf{a}_p \approx 10^{-7} a_0$

The perturbed two-body problem

Trajectory design, orbit prediction and maintenance are a challenging task when the effects of orbit perturbations is relevant

- Design of planet centred orbits (i.e. planetary explorer mission)
  - Frozen orbit definition → How to design a stable orbit.
  - Orbit maintenance → How to stay on the orbit designed in the two body restriction.
  - Coverage → E.g ground track design.
- Space situation awareness → • Asteroid Defence • Space weather • Space Debries
  - Design of end-of-life disposal trajectories
    - Graveyard orbit stability
    - Prediction of spacecraft re-entry
  - Modelling of the evolution of space debris and high area-to-mass ratio objects



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5.2 Numerical methods

OPTION 1

- Starting with initial condition $(\mathbf{r}_0, \mathbf{v}_0)$
- Numerically integrate equation of motion over time period

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r} + \sum \mathbf{a}_p$$

- Find time evolution of $(\mathbf{r}(t), \mathbf{v}(t))$
- Compute orbital elements through the conversion car2kep at each time step

OPTION 2 → write dynamic of the orbital element considering the perturbations

- Described in term of variation of orbital elements
- In the unperturbed motion all Keplerian elements, except the true anomaly θ (or analogously the mean anomaly M), remain constant

Numerical methods

For conservative perturbation (e.g., Earth geopotential) which have a disturbing function R, the **Lagrange planetary equations** can be used to calculate change in elements

↳ for conservative perturbations

The derivative are no longer ∅ like two body problem

$$
\begin{cases}
\dfrac{da}{dt} = \dfrac{2}{na}\dfrac{\partial R}{\partial \lambda} \\
\dfrac{de}{dt} = -\dfrac{b}{na^3 e}\dfrac{\partial R}{\partial \omega} + \dfrac{b^2}{na^4 e}\dfrac{\partial R}{\partial \lambda} \\
\dfrac{di}{dt} = -\dfrac{1}{nab\sin i}\dfrac{\partial R}{\partial \Omega} + \dfrac{\cos i}{nab\sin i}\dfrac{\partial R}{\partial \omega} \\
\dfrac{d\Omega}{dt} = \dfrac{1}{nab\sin i}\dfrac{\partial R}{\partial i} \\
\dfrac{d\omega}{dt} = -\dfrac{\cos i}{nab\sin i}\dfrac{\partial R}{\partial i} + \dfrac{b}{na^3 e}\dfrac{\partial R}{\partial e} \\
\dfrac{d\lambda}{dt} = -\dfrac{2}{na}\dfrac{\partial R}{\partial a} - \dfrac{b^2}{na^4 e}\dfrac{\partial R}{\partial e}
\end{cases}
$$

with

- t_0: time of perigee passage
- $\lambda = -nt_0$
- $R(a, e, i, \Omega, \omega, \lambda)$ disturbing function

- System of equations solved simultaneously by numerical integration
- Equations function of orbital elements

► Battin, An Introduction to the Mathematics and Methods of Astrodynamics, Revised Edition, AIAA Educational Series, Reston, 1999. Chapter 10

Numerical methods

For non-conservative perturbation (e.g., Aerodynamic drag) the **Gauss planetary equations** can be used to calculate change in elements

↳ for non conservative effect.

component of the disturbing acceleration into the transversal, normal and out of plane contribution,

$$
\begin{cases}
\dfrac{da}{dt} = \dfrac{2a^2 v}{\mu} a_t \\[2ex]
\dfrac{de}{dt} = \dfrac{1}{v}\left(2\left(e+\cos\theta\right) a_t - \dfrac{r}{a}\sin\theta\, a_n \right) \\[2ex]
\dfrac{di}{dt} = \dfrac{r\cos\theta^*}{h} a_h \\[2ex]
\dfrac{d\Omega}{dt} = \dfrac{r\sin\theta^*}{h\sin i} a_h \\[2ex]
\dfrac{d\omega}{dt} = \dfrac{1}{ev}\left(2\sin\theta\, a_t + \left(2e + \dfrac{r}{a}\cos\theta \right) a_n \right) - \dfrac{r\sin\theta^* \cos i}{h\sin i} a_h \\[2ex]
\dfrac{d\theta}{dt} = \dfrac{h}{r^2} - \dfrac{1}{ev}\left(2\sin\theta\, a_t + \left(2e + \dfrac{r}{a}\cos\theta \right) a_n \right)
\end{cases}
$$

or equivalently

$$
\frac{dM}{dt} = n - \frac{b}{eav}\left(2\left(1 + \frac{e^2 r}{p} \right)\sin\theta\, a_t + \frac{r}{a}\cos\theta\, a_n \right)
$$

with

$$
v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} \qquad n = \sqrt{\mu/a^3}
$$

$$
h = nab
$$

$$
b = a\sqrt{1-e^2} \qquad r = \frac{p}{1+e\cos\theta}
$$

$$
p = b^2/a \qquad \theta^* = \theta + \omega
$$

► Battin, An Introduction to the Mathematics and Methods of Astrodynamics, Revised Edition, AIAA Educational Series, Reston, 1999. Chapter 10

Numerical methods

a_t, a_n, a_h are the components of the perturbing acceleration $\boldsymbol{a}_p$

They can be obtained with rotation matrix

$$\mathbf{s}_{car} = \{\mathbf{r} \quad \mathbf{v}\}^T = \{x, y, z, v_x, v_y, v_z\}^T$$

$$\begin{cases} \hat{\mathbf{t}} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|} \\ \mathbf{h} = \mathbf{r} \times \mathbf{v} \qquad \hat{\mathbf{h}} = \dfrac{\mathbf{h}}{\|\mathbf{h}\|} \\ \hat{\mathbf{n}} = \hat{\mathbf{h}} \times \hat{\mathbf{t}} \end{cases} \longrightarrow \quad \mathbf{A} = \begin{bmatrix} \hat{\mathbf{t}} & \hat{\mathbf{n}} & \hat{\mathbf{h}} \end{bmatrix}$$

$$\mathbf{a}_{\mathrm{p},tnh} = \begin{bmatrix} \hat{\mathbf{t}} & \hat{\mathbf{n}} & \hat{\mathbf{h}} \end{bmatrix}^T \mathbf{a}_{\mathrm{p,car}}$$

$$\mathbf{a}_{\mathrm{p,car}} = \begin{bmatrix} \hat{\mathbf{t}} & \hat{\mathbf{n}} & \hat{\mathbf{h}} \end{bmatrix} \mathbf{a}_{\mathrm{p},tnh}$$

Note: Gauss equations an be also used to model low-trust propulsion. In that case a_t, a_n, a_h would be the control acceleration of the low-thrust engine

Numerical methods

Solved by numerical integration: Example: code

```
% Gauss' Equations in {t,n,h} reference system (Battin pag 489)

u_tnh = feval(handleAcc,t,kep,mu,accArgs{:});
ut = u_tnh(1);
un = u_tnh(2);
uh = u_tnh(3);

v = sqrt(2*mu/r - mu/a);

Da  = 2*a^2*v/mu * ut;
De  = 1/v*(2*(e+cos(f)) * ut - r/a*sin(f) *un);
Di  = r*cos(theta)/h * uh;
DOm = r*sin(theta)/(h*sin(i)) * uh;
Dom = 1/(e*v)*(2*sin(f) * ut + (2*e+r/a*cos(f)) * un) - r*sin(theta)*cos(i)/(h*sin(i)) * uh;
Df  = h/r^2 - 1/(e*v)*(2*sin(f) * ut + (2*e+r/a*cos(f)) *un);
DM  = n - b/(e*a*v)*(2*(1+e^2*r/p)*sin(f) * ut + r/a*cos(f) * un);
```

```
b = a*sqrt(1-e^2);
p = b^2/a;
n = sqrt(mu/a^3);
h = n*a*b;

r = p/(1+e*cos(f));

theta = om+f;
```

In the unperturbed motion all Keplerian elements, except the true anomaly θ (or analogously the mean anomaly M), remain constant

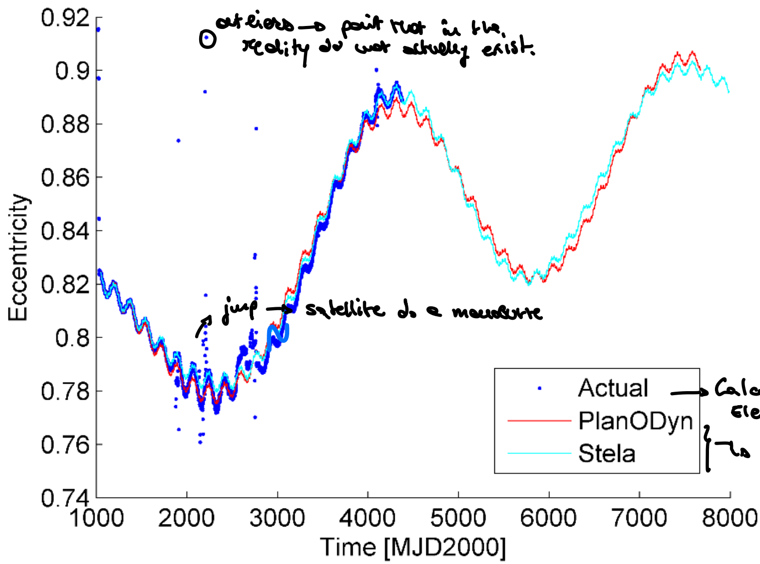
Numerical methods

first method cartesian coordinates
second method Keplerian element (Gauss or Lagrange)

If the motion is unperturbed the only Keplerian elements that are changing are θ, M, E →
Nice to use Keplerian elements method is that if I do not have perturbation I can see the only derivative that are varying are the one we expect.

In the unperturbed motion all Keplerian elements, except the true anomaly (or analogously the mean anomaly M), remain constant, but in an unperturbed problem all the orbital elements vary

If I represent motion of the orbit with all perturbations the only thing that change is θ
→ If I have perturbations we are going to integrate only the effect of these perturbation while with cartesian we have to integrate everything even if there is no perturbations.



→ Calculated with Two Line Element (TLE).
→ to Gauss and Lagrange tools from two universities

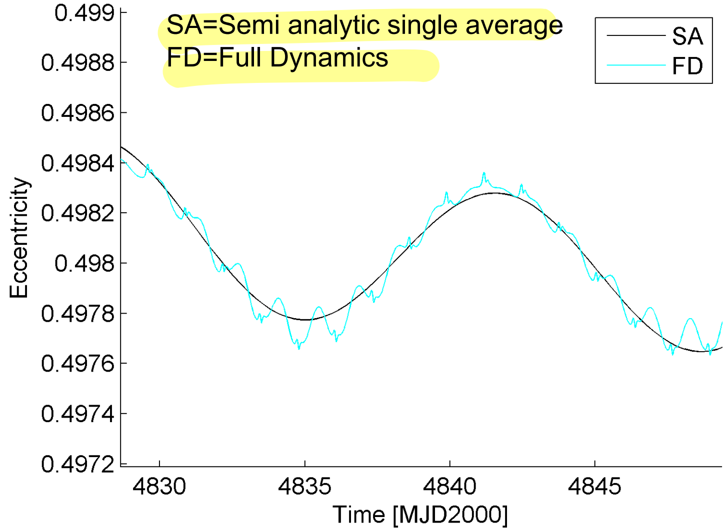
► Colombo C., Letizia F., Alessi E. M., Landgraf M., "End-of-life Earth re-entry for highly elliptical orbits: the INTEGRAL mission", The 24th AAS/AIAA Space Flight Mechanics Meeting, Jan. 26-30, 2014, Santa Fe, New Mexico

5.3 Semi-analytical methods

Start with Lagrange and Gauss planetary equation but we do not integrate them directly but I am integrating the average of variation over one orbit → So I can propagate the average variation of an orbital elements for very long period and only when I am interested in it we can add the classical integration of the Lagrange / Gauss planetary equation.

Average variation of orbital elements over one orbit revolution

- Filter high frequency oscillations
- Reduce stiffness of the problem
- Decrease computational time for long term integration



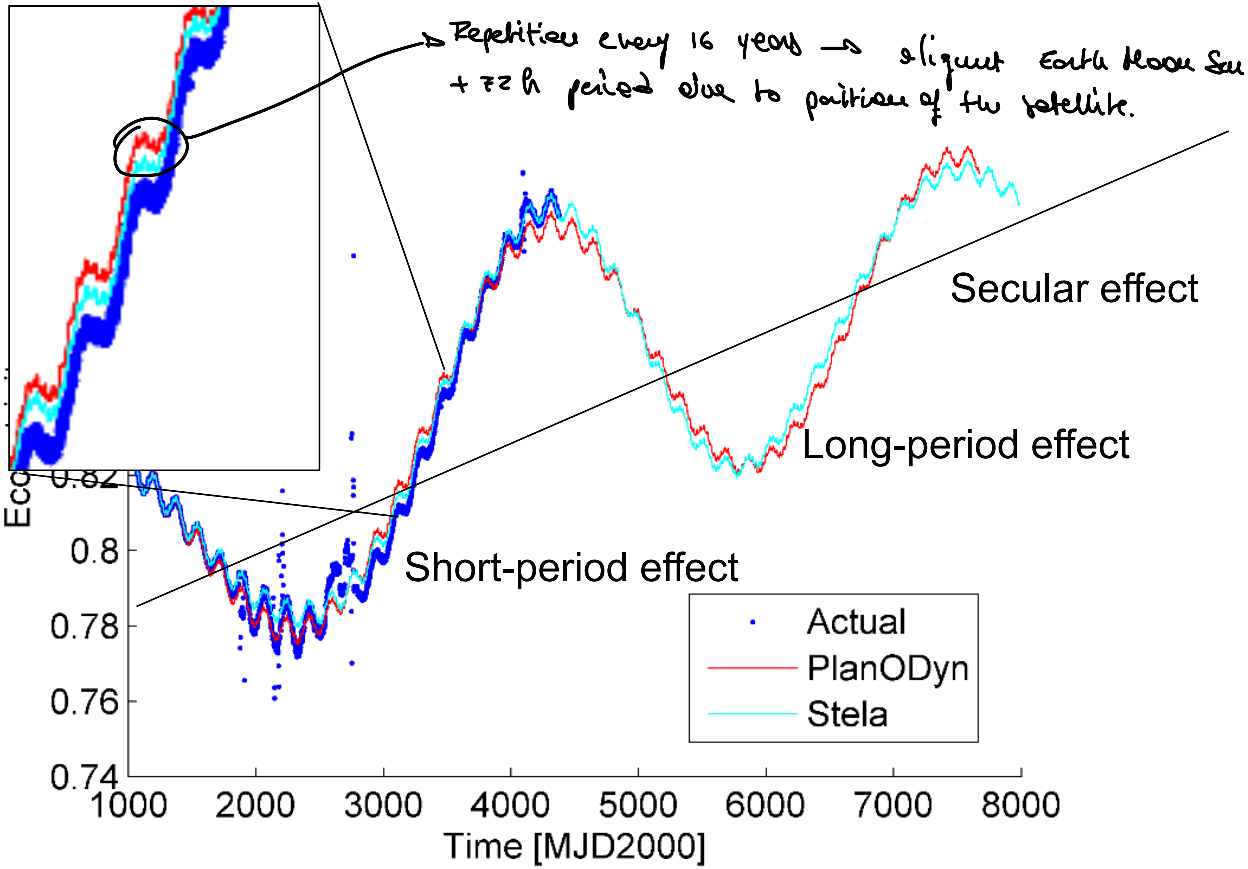
More precision over a long period of time

➢ Finkleman D., Colombo C. Cefola P., "Analysis of the suitability of analytical, semi/analytical and numerical approaches for important orbital dynamics tasks", 65th IAC, 2014, Toronto, Canada, IAC-14.C1.2.6

- There are different oscillations that can be isolated by using semi-analytical approaches:
  - Secular variations
  - Long-period variations
  - Short-period variations

Semi-analytical methods

→ Repetition every 16 years → aligment Earth Moon Sun
+ 72 h period due to position of the satellite.



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Semi-analytical methods

How to do the first one

$R = R(\underline{\alpha}) \rightarrow$ disturbing potential

$\overline{R}(a, e, i, \Omega, \omega) = \frac{1}{2\pi}\int_0^{2\pi} R(\underline{\alpha})\, dM$

no more function of the mean anomaly

For conservative orbit perturbation effects

- *R* disturbing potential function
- Planetary equations in Lagrange form $\boldsymbol{\alpha} = \begin{bmatrix} a & e & i & \Omega & \omega & M \end{bmatrix}^T$

Solar pressure

$$R = R_{\text{SRP}} + R_{\text{zonal}} + R_{3-\text{Sun}} + R_{3-\text{Moon}} \qquad \frac{d\boldsymbol{\alpha}}{dt} = f\left(\boldsymbol{\alpha}, \frac{\partial R}{\partial \boldsymbol{\alpha}}\right)$$

Average over one orbit revolution of the spacecraft around the primary planet

$$\overline{R} = \overline{R}_{\text{SRP}} + \overline{R}_{\text{zonal}} + \overline{R}_{3-\text{Sun}} + \overline{R}_{3-\text{Moon}} \qquad \frac{d\overline{\boldsymbol{\alpha}}}{dt} = f\left(\overline{\boldsymbol{\alpha}}, \frac{\partial \overline{R}}{\partial \overline{\boldsymbol{\alpha}}}\right)$$

Single average

Average over the revolution of the perturbing body around the primary planet

$$\overline{\overline{R}} = \overline{\overline{R}}_{\text{SRP}} + \overline{R}_{\text{zonal}} + \overline{\overline{R}}_{3-\text{Sun}} + \overline{\overline{R}}_{3-\text{Moon}} \qquad \frac{d\overline{\overline{\boldsymbol{\alpha}}}}{dt} = f\left(\overline{\overline{\boldsymbol{\alpha}}}, \frac{\partial \overline{\overline{R}}}{\partial \overline{\overline{\boldsymbol{\alpha}}}}\right)$$

Double average

Non-singular orbital elements

Keplerian elements → physical interpretation
other use for $i=0$ non singular for those specific cases

↳ for other application

Use some Keplerian elements combination that are non singular

Keplerian elements are undefined in some situations. Note that:

- Ω is not defined as $i \rightarrow 0$
- Ω is not defined as $e \rightarrow 0$

For example, a satellite in Geostationary Earth Orbit (GEO) requires $e \approx 0$, $i \approx 0$

A redefinition of the orbit element set is sometimes required. For example, **non-singular equinoctial orbit elements** are used:

- mean longitude, λ
- longitude drift rate, $\dot{\lambda}$
- eccentricity vector, $\mathbf{e} = e \begin{bmatrix} \cos(\Omega+\omega) \\ \sin(\Omega+\omega) \end{bmatrix}$
- inclination vector, $\mathbf{i} = I \begin{bmatrix} \sin\Omega \\ -\cos\Omega \end{bmatrix}$

The Gauss and Lagrange planetary equations are re-written accordingly

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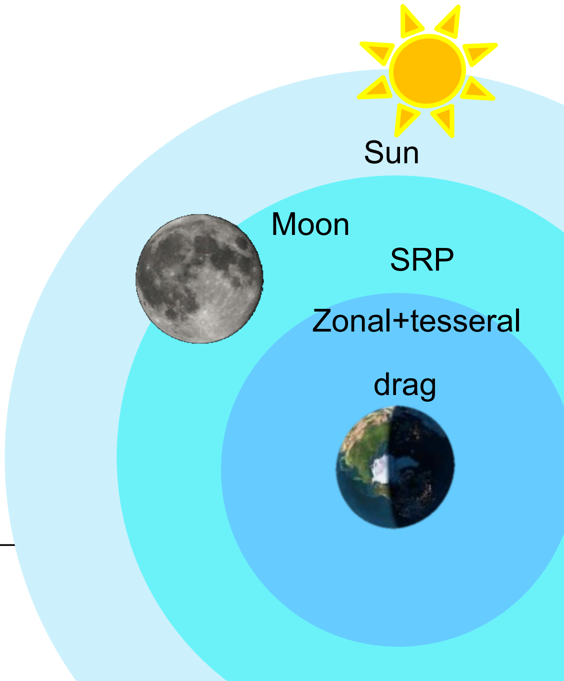
The perturbed two-body problem

Principal influence on the motion: central gravity field

Orbit perturbations

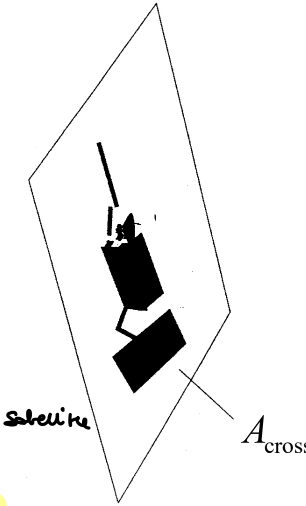
- Aerodynamic drag
- Gravity anomalies
- Third body acceleration (e.g., Moon, Sun)
- Solar radiation pressure



5.4 Aerodynamic perturbation

$\nearrow$ mean value

$\frac{A}{m} = 0.0012 \ \frac{m^2}{kg}$



The drag force on an orbit spacecraft is

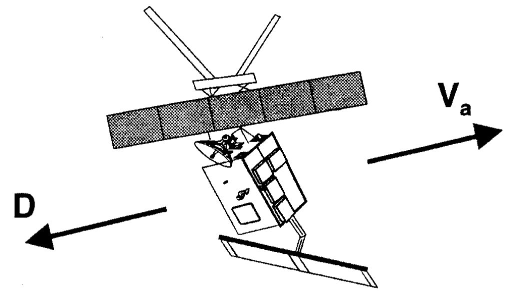
Better to work with the acceleration

$$\boxed{\mathbf{a}_{\text{Drag}} = -\frac{1}{2}\frac{A_{\text{cross}} c_D}{m}\rho(h,t) v_{rel}^{\ 2} \frac{\mathbf{v}_{rel}}{\left\|\mathbf{v}_{rel}\right\|}}$$ where

→ opposite to the direction of the relative velocity btw satellite and atmosphere velocity.

- $\rho(h,t)$ atmospheric density
- $\mathbf{v}_{rel}$ air-relative speed (atmosphere rotates) $\mathbf{v}_{rel} = \frac{d\mathbf{r}}{dt} - \boldsymbol{\omega}_{Earth} \times \mathbf{r}$
- A_{cross} reference area (i.e. cross-sectional area perpendicular to $\mathbf{v}_{rel}$)
  ↳ Depends on the attitude of the satellite
- m spacecraft mass
- c_D drag coefficient

Lift acceleration is the same with $c_D \longrightarrow c_L$



Some satellite are design to have some lift →
Instead of CD we will have CL.

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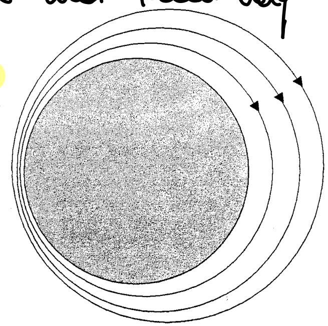
Aerodynamic perturbation

Very difficult to predict the deorbiting path of the satellite because we do not know very well the upper atmosphere → Because it depends on the solar activity and its behaving has a daily period a yearly one and every 11 years.



- Principal effects upon the orbit are to reduce its size and to circularise it (both *a* and *e* decrease)

- Uncertainties in modelling aero-perturbations:

  - Atmospheric density — Depends on the solar Activity with 1 day, 1 year, 11 years period. — *Difficult to model and predict (solar activity)*
  - Reference area A_{cross} — *Can be estimated if attitude profile and configuration of spacecraft are known*
  - Drag and lift coefficients C_D, C_L — Depends on the altitude and on the temperature — *Experimental data is sparse for the free molecular flow regime of orbital flight ($C_D \approx 2.1/2.2$)*

- Aero-force modelling is one of the major obstacles to very precise orbit determination and prediction

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Aerodynamic perturbation

Each layer has a different exponential

height

Exponential Band model

$$\rho(h,t) = \rho_0 \exp\left[-\frac{h-h_0}{H}\right]$$

ρ_0 reference density

h, h_0 actual and reference altitude

H scale height

Note: be consistent with units!

TABLE 7-4. Exponential Atmospheric Model. Although a very simple approach, this method yields moderate results for general studies. Source: Wertz, 1978, 820, which uses the *U.S. Standard Atmosphere* (1976) for 0 km, CIRA-72 for 25–500 km, and CIRA-72 with T_∞ = 1000 K for 500–1000 km. The scale heights have been adjusted to maintain a piecewise-continuous formulation of the density.

Altitude h_{ellp} (km)	Base Altitude h_o (km)	Nominal Density ρ_o (kg/m^3)	Scale Height H (km)	Altitude h_{ellp} (km)	Base Altitude h_o (km)	Nominal Density ρ_o (kg/m^3)	Scale Height H (km)
0–25	0	1.225	7.249	150–180	150	2.070×10^{-9}	22.523
25–30	25	3.899×10^{-2}	6.349	180–200	180	5.464×10^{-10}	29.740
30–40	30	1.774×10^{-2}	6.682	200–250	200	2.789×10^{-10}	37.105
40–50	40	3.972×10^{-3}	7.554	250–300	250	7.248×10^{-11}	45.546
50–60	50	1.057×10^{-3}	8.382	300–350	300	2.418×10^{-11}	53.628
60–70	60	3.206×10^{-4}	7.714	350–400	350	9.158×10^{-12}	53.298
70–80	70	8.770×10^{-5}	6.549	400–450	400	3.725×10^{-12}	58.515
80–90	80	1.905×10^{-5}	5.799	450–500	450	1.585×10^{-12}	60.828
90–100	90	3.396×10^{-6}	5.382	500–600	500	6.967×10^{-13}	63.822
100–110	100	5.297×10^{-7}	5.877	600–700	600	1.454×10^{-13}	71.835
110–120	110	9.661×10^{-8}	7.263	700–800	700	3.614×10^{-14}	88.667
120–130	120	2.438×10^{-8}	9.473	800–900	800	1.170×10^{-14}	124.64
130–140	130	8.484×10^{-9}	12.636	900–1000	900	5.245×10^{-15}	181.05
140–150	140	3.845×10^{-9}	16.149	1000–	1000	3.019×10^{-15}	268.00

Eq. (7-31) requires knowledge of the actual altitude, found by subtracting the Earth's radius (6378.137 km) from the satellite's given radius (h_{ellp} = 747.2119 km). Now, if we use values from Table 7-4, Eq. (7-31) becomes

$$\rho = 3.614\times10^{-14}\,\mathrm{EXP}\left[-\frac{747.2119-700}{88.667}\right] = 2.121\,985\,4\times10^{-14}\frac{\mathrm{kg}}{\mathrm{m}^3}$$

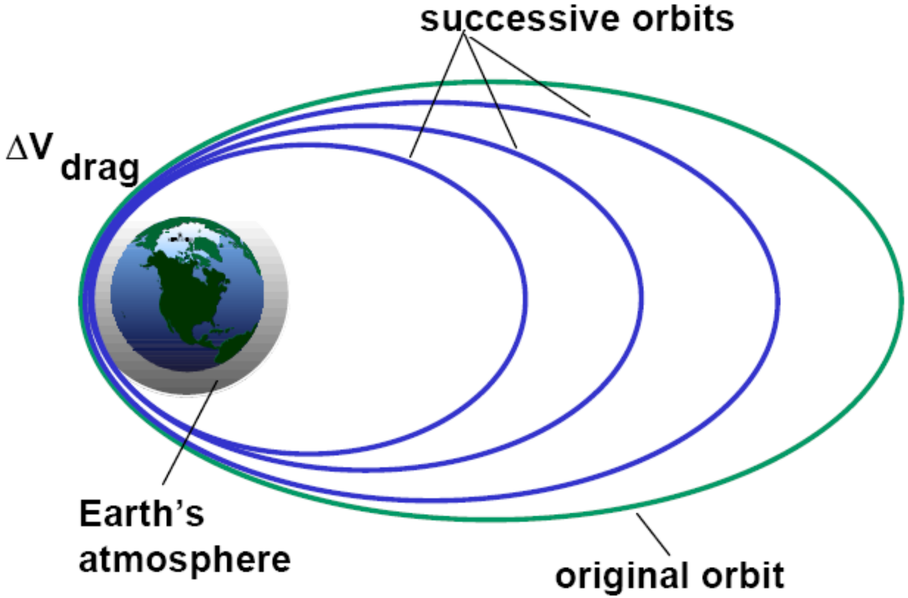
▲ Note that the units in the exponential cancel (all are km), and the result is less than the base value at 700 km, as we would expect.

Aerodynamic perturbation

Effect of drag
- Decrease the energy of the orbit
  Breking effect → reduce of a
- We can neglect the velocity of the atmosphere

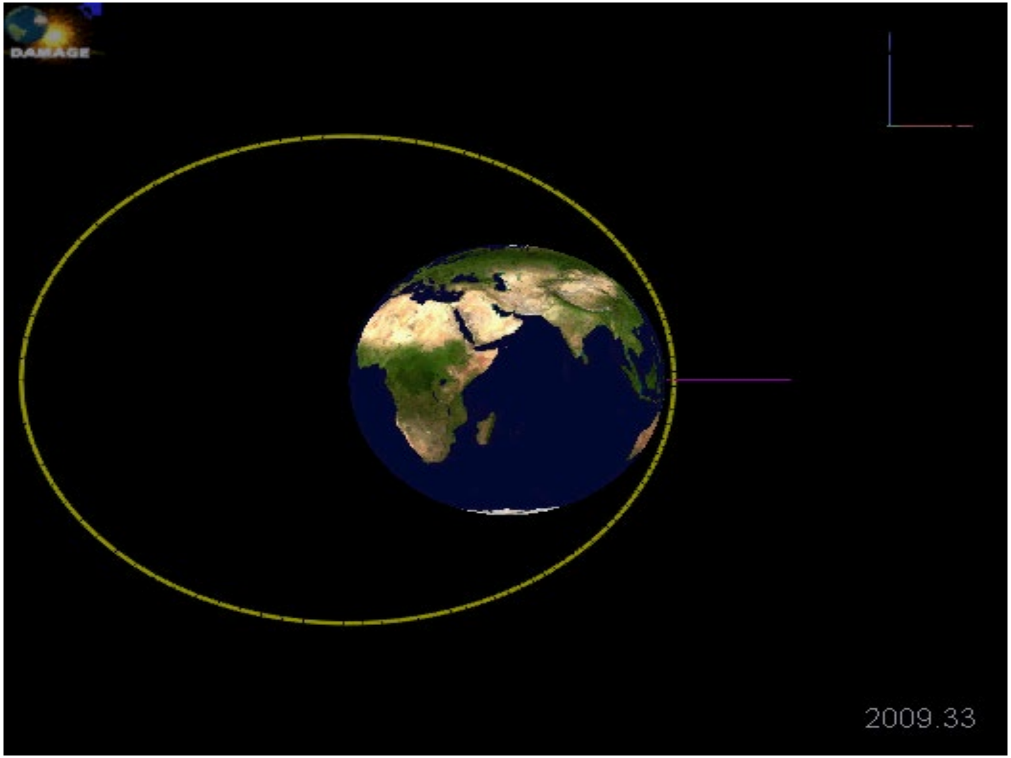
Effect of drag on an elliptical orbit → Decrease the energy of the orbit

Athmosphere revelant up to 1000 km of height



Aerodynamic perturbation

Effect of drag on an elliptical orbit

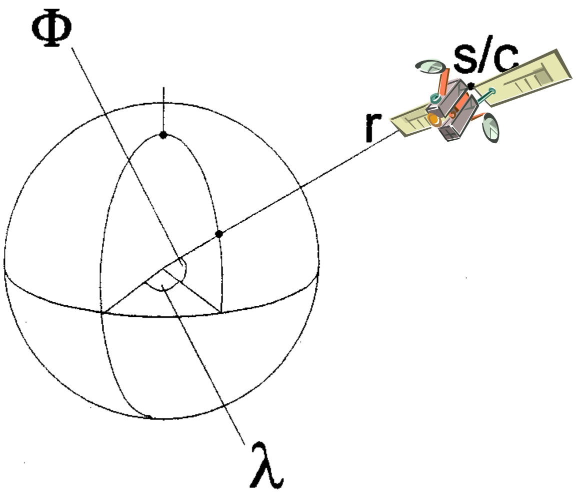


Courtesy: University of Southampton

5.5 Gravitational perturbation

Spacecraft position defined by

- geocentric distance, *r*
- latitude (i.e. angular distance from the pole), ϕ
- longitude, λ



Primary gravitational potential
(for ideal spherical Earth)

it is not only $-\frac{\mu}{r}$ → I have another two terms

$$\boxed{R(r) = -\frac{\mu}{r}} \Rightarrow \boxed{\mathbf{F} = -m\nabla R = -\frac{\mu m}{r^3}\mathbf{r}}$$

→ To be more precise we need to consider that the Earth is not symmetrical

Gravitational perturbation

The disturbing perturbing potential on top of the central gravity field can be distinguished in two contributions:

- Zonal harmonic potential: function of the geocentric distance, *r* and the latitude (i.e. angular distance from the pole)

→ Zonal (latitude)

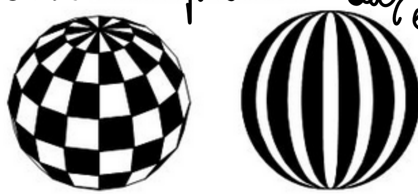
Zonal Harmonics

- Tesseral harmonic potential: function of the geocentric distance, *r* , the latitude (i.e. angular distance from the pole) and the longitude

latitude and longitude

over longitude.

(latitude and longitude)



Tesseral Harmonics Sectorial Harmonics

5.5.1 Gravitational perturbation: zonal harmonics

Zonal Harmonic Potential (distance and latitude ϕ dependent only)

$$\boxed{R(r,\phi)=\frac{\mu}{r}\left(-1+R_{ZH}\right)} \leftarrow \text{where} \quad \boxed{R_{ZH}(r,\phi)=\sum_{n=2}^{\infty} J_n \left(\frac{R_E}{r}\right)^n P_n(\cos\phi)} \quad \cos\phi=\frac{z}{\|\mathbf{r}\|}$$

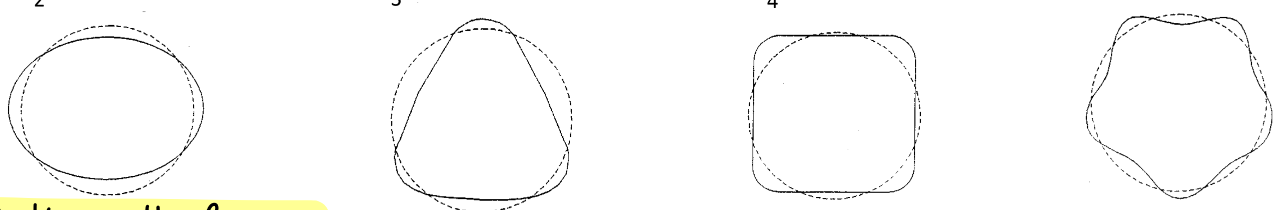
→ Legendre polynomial as function of cos φ.

- J_n are numerical values, and P_n are Legendre polynomials of degree n.

- Summation may be represented pictorially by:

$$J_2\left(\frac{R_E}{r}\right)^2 \qquad J_3\left(\frac{R_E}{r}\right)^3 \qquad J_4\left(\frac{R_E}{r}\right)^4 \qquad J_5\left(\frac{R_E}{r}\right)^5 + \ldots$$

$$J_2=1.08\cdot 10^{-3} \qquad J_3=-2.51\cdot 10^{-6} \qquad J_4=-1.60\cdot 10^{-6}$$



How far is the earth from a sphere in two dimensions → how flat are the poles

→ most important term → All planets where fluid and centrifugal force make them bulge at the equator

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Zonal harmonics: J_2 effect

$J_2 \approx 10^{-3}, J_n \leq 10^{-6}, n \geq 3$ Hence J_2 term (Earth oblateness) is dominant

$$\boxed{\text{Oblateness} = \frac{\text{Equatorial radius - Polar radius}}{\text{Equatorial radius}}}$$

$\Rightarrow$ J_2 effect is symmetric in xy axis $\Leftarrow$

- The Earth, like other planets with comparable or higher rotational rate, bulges out of the equator because of centrifugal force
- Earth equatorial radius = 21 km larger than polar radius
- Due to lack of symmetry the gravity on an orbiting body is not directed towards the centre of the Earth
- Oblateness causes variation also with latitude

Components of spacecraft position vector in an Earth-fixed (rotating) reference frame

```
a_J2_car(1) =   muP/r^3*s_sc(1)*(J2*3/2*(RE/r)^2*(5*(s_sc(3)/r)^2-1));
a_J2_car(2) =   muP/r^3*s_sc(2)*(J2*3/2*(RE/r)^2*(5*(s_sc(3)/r)^2-1));
a_J2_car(3) = - muP/r^3*s_sc(3)*(J2*3/2*(RE/r)^2*(-5*(s_sc(3)/r)^2+3));
```

Zonal harmonics: J_2 effect

We focus on the secular effect of the J_2 perturbation which acts on the right ascension of the ascending node, the argument of perigee and the true anomaly

Variation due to J_2 effect over the long term.

$$\dot{\Omega}_{SEC} = -\frac{3nR_{Earth}^2 J_2}{2p^2}\cos(i)$$

$$\dot{\omega}_{SEC} = \frac{3nR_{Earth}^2 J_2}{4p^2}(4-5\sin^2 i)$$

$$\dot{M}_{SEC} = -\frac{3nR_{Earth}^2 J_2\sqrt{1-e^2}}{4p^2}(3\sin^2 i-2)$$

$$\dot{a} = \dot{e} = \dot{i} = 0$$ → they have oscillation but the secular effect is null.

↳ oscillation over 1 orbital period.

Zonal harmonics J_2 effect: nodal regression

$\begin{cases} \text{if } 0 < i < \pi/2 \rightarrow \text{Nodal regression towards West} \\ \pi/2 < i < \pi \rightarrow \text{Retrograde orbit} \rightarrow \text{towards East} \end{cases}$

Secular effect (1): Nodal Regression (rotation positive toward East)

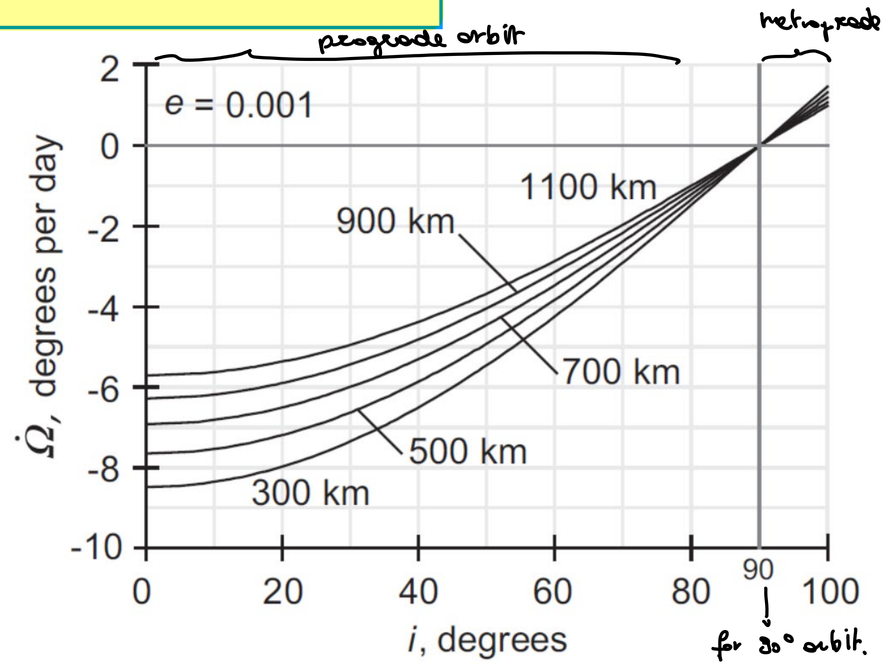
→ the further we are the less important become this effect.

$$\dot{\Omega} = -\frac{3}{2}\frac{nJ_2R_E^{\ 2}}{a^2\left(1-e^2\right)^2}\cos i \qquad n = \sqrt{\mu/a^3}$$

Per orbit

$$\Delta\Omega = \dot{\Omega}\frac{2\pi}{n}$$

$$\Delta\Omega = \frac{-3\pi J_2 R_E^{\ 2}\cos i}{a^2(1-e^2)^2} \qquad \text{(rad/rev)}$$



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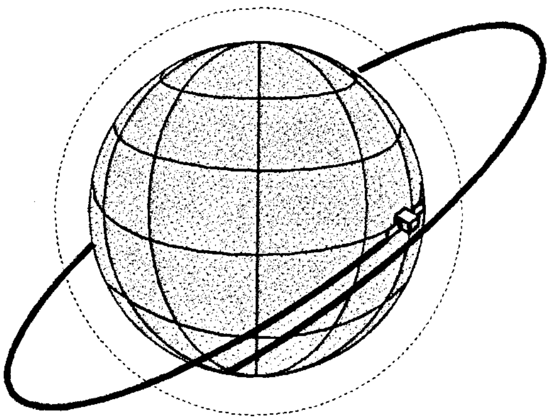
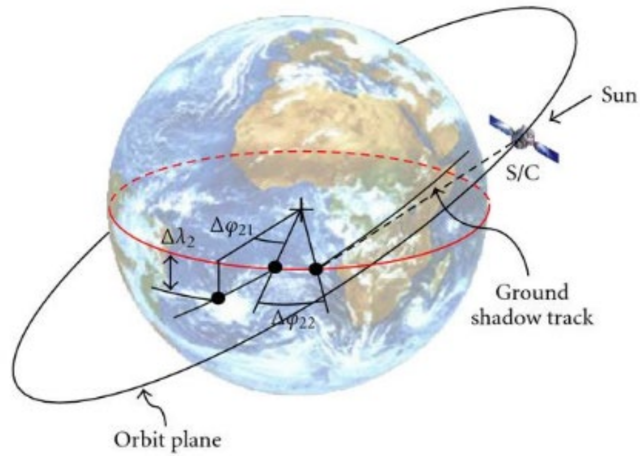
Zonal harmonics J_2 effect: nodal regression

Nodes move

orbit rotates westward

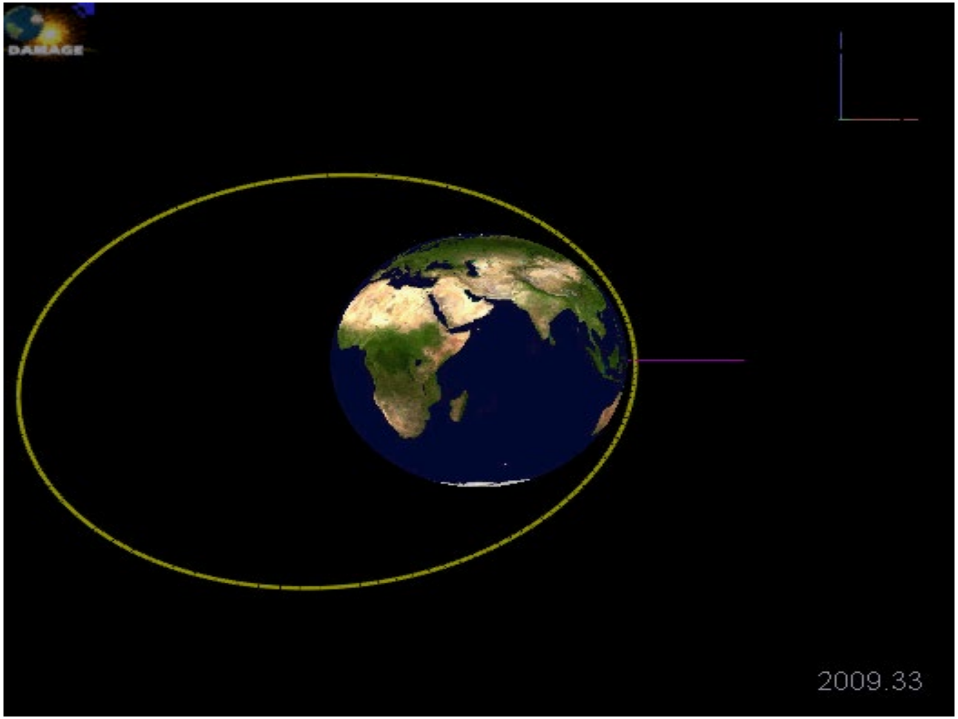
- Westward if orbit inclination is between 0° and 90° (prograde orbit)
- Eastward if orbit inclination is between 90° and 180° (retrograde orbit)





Zonal harmonics J_2 effect: nodal regression

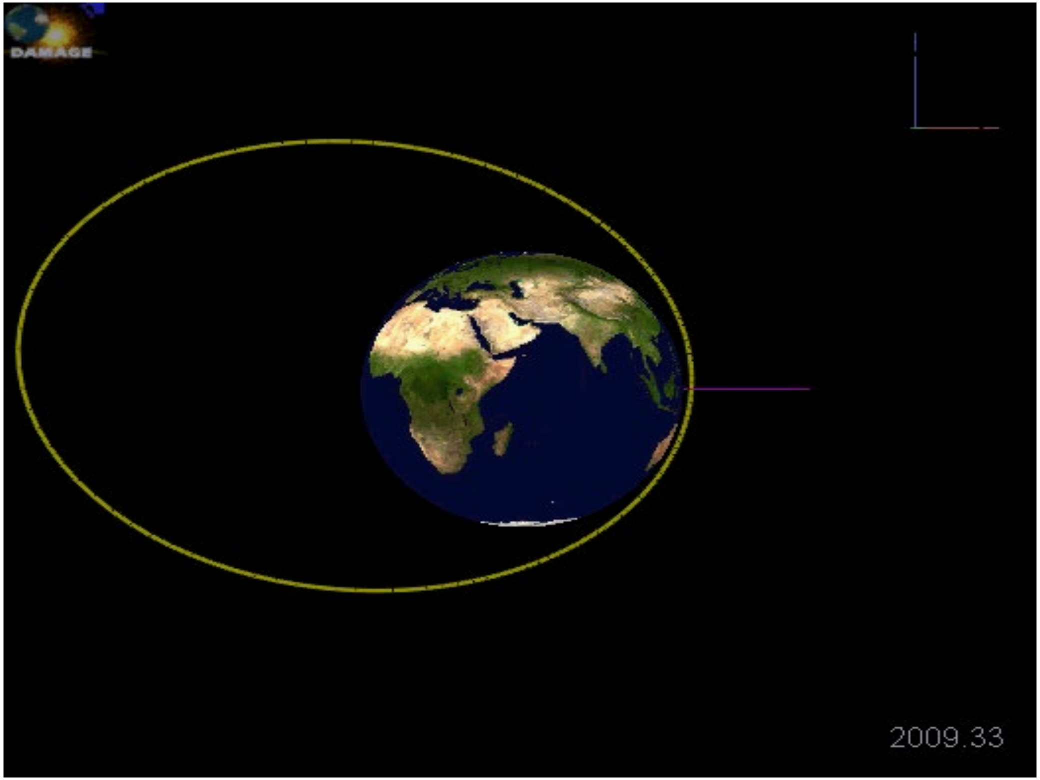
Prograde orbit



Courtesy: University of Southampton

Zonal harmonics J_2 effect: nodal regression

Retrograde orbit



Courtesy: University of Southampton

Zonal harmonics J_2 effect: perigee precession

↳ [illegible] [illegible] effect

Secular effect (2): Perigee precession (rotation positive if according to right-hand side rule)

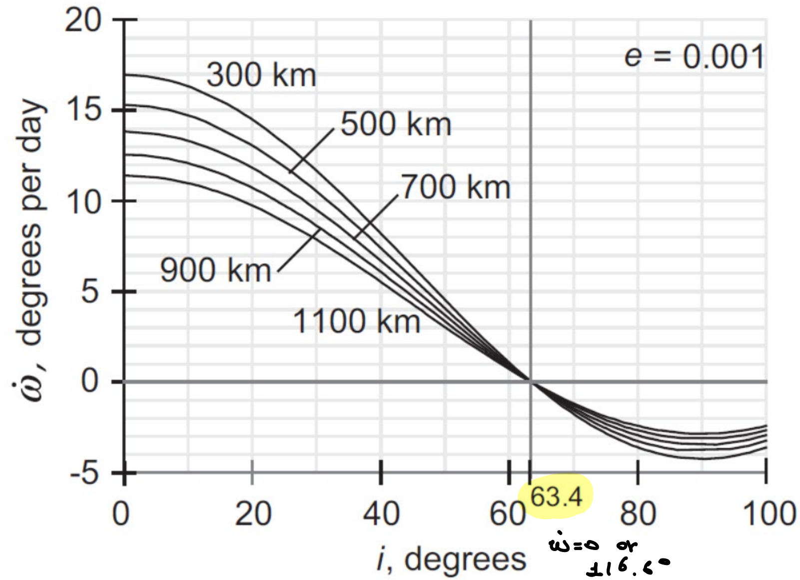
frozen orbit at $i = 63.4°$ and $i = 116.6°$ → the closer the orbit it is the easier is the effect

$$\dot{\omega} = \frac{3nJ_2R_E^2}{2a^2\left(1-e^2\right)^2}\left(2-\frac{5}{2}\sin^2 i\right) \qquad n = \sqrt{\mu/a^3}$$

Per orbit

$$\Delta\omega = \dot{\omega}\frac{2\pi}{n}$$

$$\Delta\omega = \frac{3\pi J_2 R_E^2}{a^2(1-e^2)^2}\left(2-\frac{5}{2}\sin^2 i\right) \quad \text{(rad/rev)}$$



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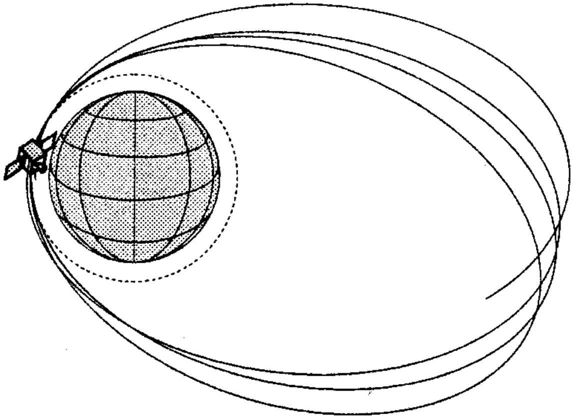
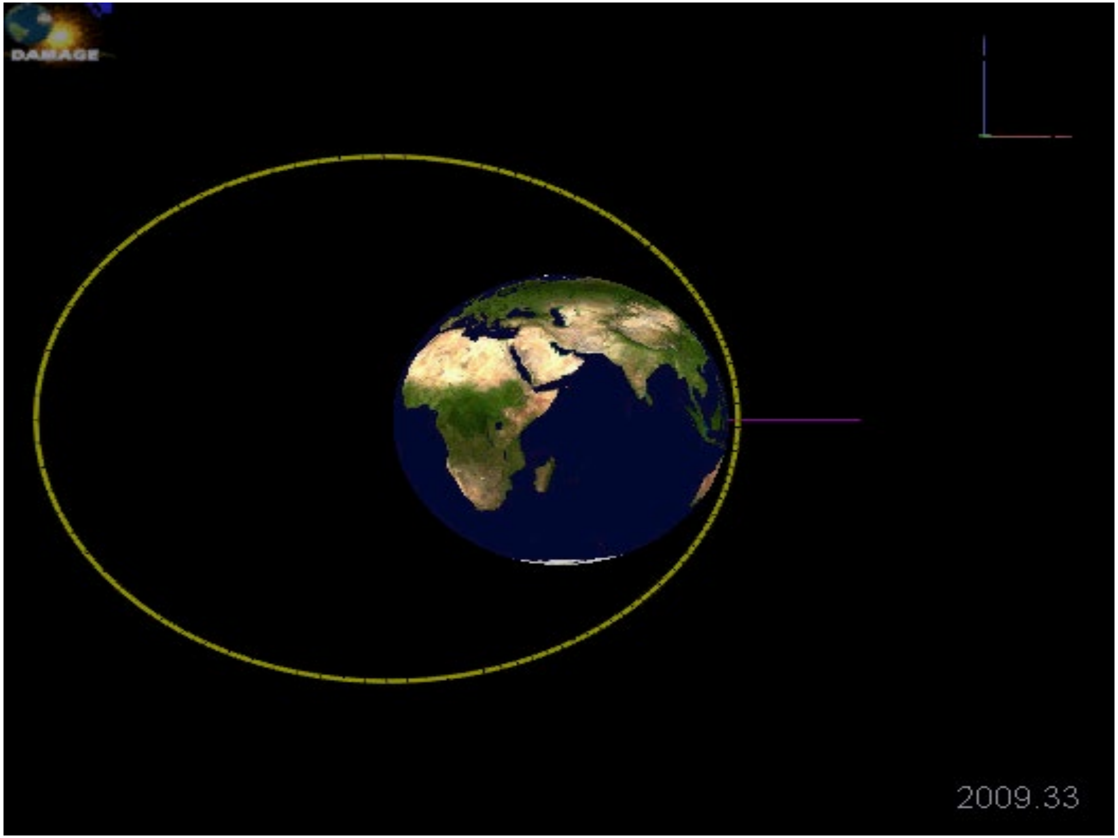
Zonal harmonics J_2 effect: perigee precession

Perigee precession (apse line rotation)

- Knowledge of the Earth oblateness (J_2) effects are vital for Earth orbit operations
- Hula-hooping effect

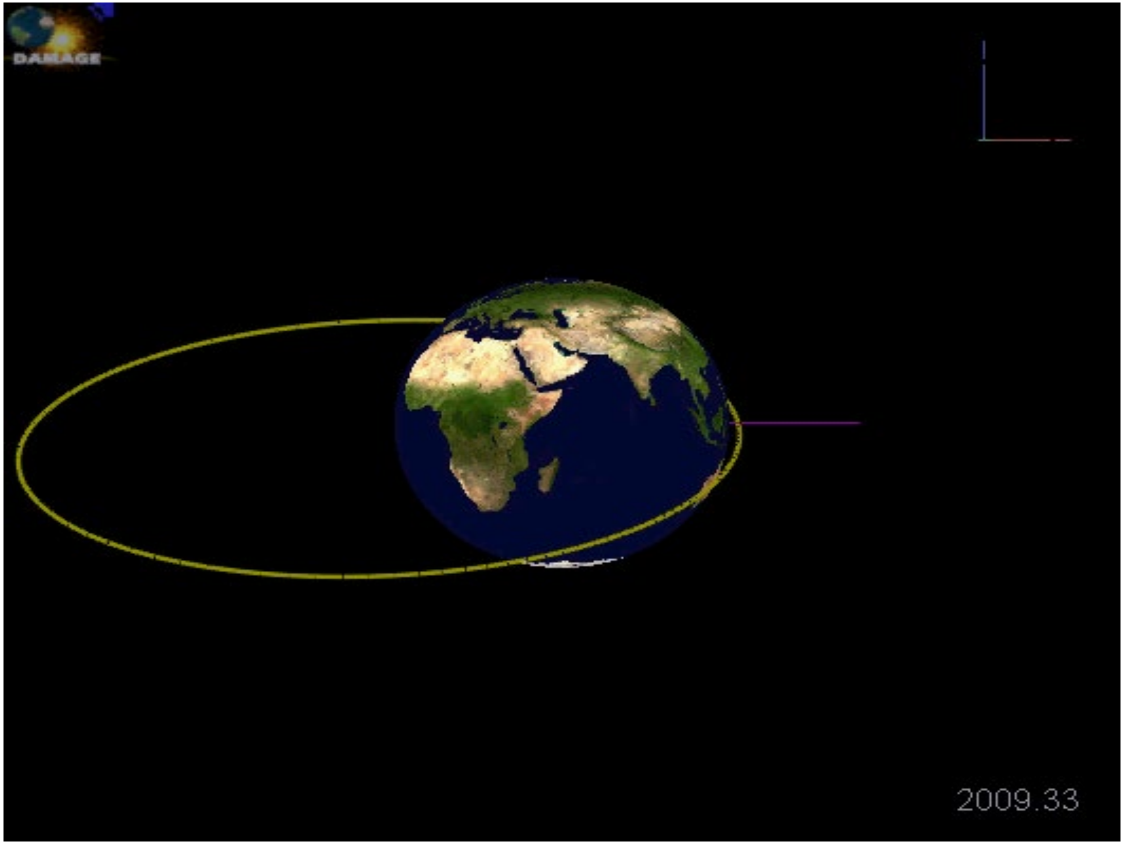


Zonal harmonics J_2 effect: perigee precession



Courtesy: University of Southampton

Zonal harmonics J_2 effect: combined effects of the Earth oblateness



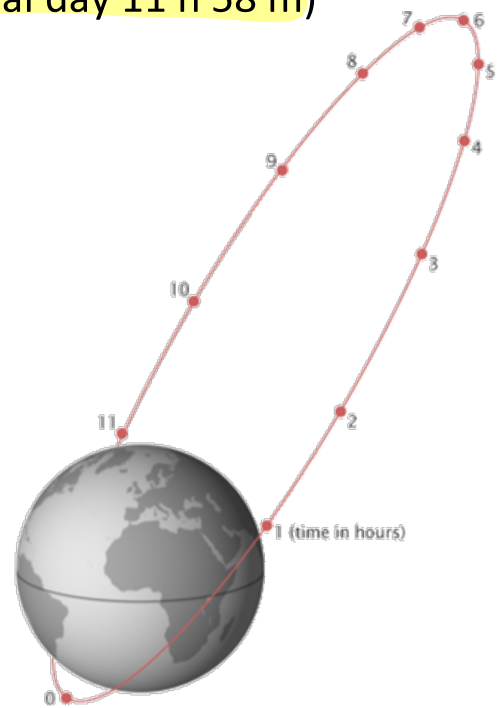
Courtesy: University of Southampton

Example: Molniya Orbit

Definition

- Highly elliptical 12 hour orbit (strictly, half a sidereal day 11 h 58 m)
- Inclined at 63.4°to the equator
- Orbital elements:
  - a = 26,560 km (orbit period)
  - e = 0.72 (h_p = 1 000 km, h_a = 39 360 km)
  - i = 63.4° ⟶ No drift of the perigee
  - ω= 270°(perigee in the Southern hemisphere)
  - Ω arbitrary

Exploitation of J_2 for defining a frozen orbit with respect to dω/dt



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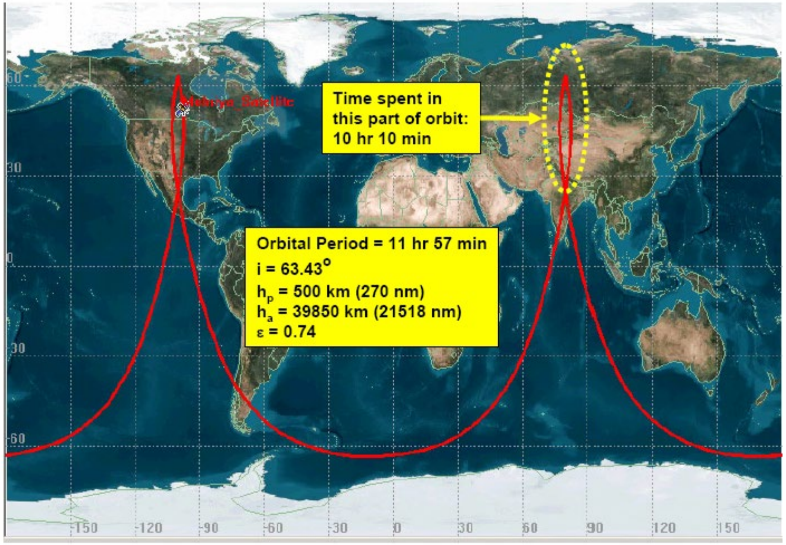
Example: Molniya Orbit

Ground track and global coverage

- Molniya orbit - used for high latitude telecoms in Russia
- 24 h regional coverage is achievable (with a high satellite elevation) at high latitudes with a minimum of 3 spacecraft in orbits with ascending nodes 120°apart (complements GEO coverage)

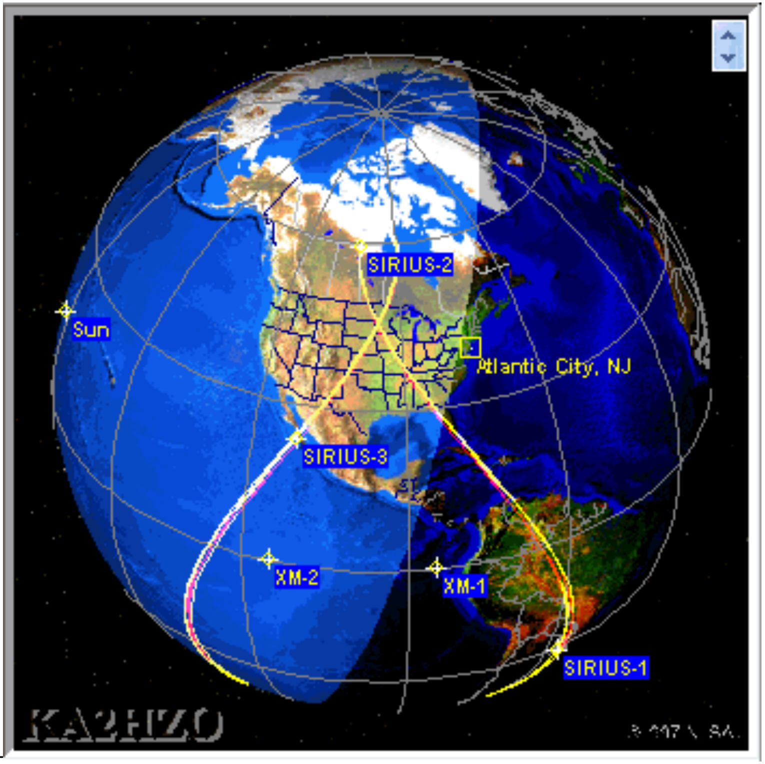


Example: Tundra orbit

- Highly elliptical 24 hour (strictly, 23 h 56 m) orbit
- Inclined at 63.4°to the equator
- Orbital elements:
  - $a = a_{GEO}$ = 42 164 km (a GEO orbit period)
  - i = 63.4°
  - Ω and mean anomaly of each satellite is offset by 120°

Exploitation of J_2 for defining a frozen orbit with respect to $d\omega/dt$

Sirius Satellite Radio



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Example: Other elliptical orbits

Orbit	Period	Inclination	Characteristics	Reference
Molniya	12 hours	63.4°	continuous coverage over Russia	King-Hele et al.
Cobra	8 hours 3 rev/d	63.4°	continuous coverage over three fixed longitudes at medium latitudes by using six satellites	Draim et al.
Magic orbits	3 hours 8 rev/d	63.4°-116.5°	smaller eccentricity, repeated groundtrack	Aerospace Corporation
Ellipso Borealis	3 hours 8 rev/d	116.5°	Sun-synchronous orbits, with a fixed line of apsides and repeating groundtrack apogees = 7,605 km, perigees = 633 km	Sabol et al.

Exploitation of J_2 for defining a frozen orbit with respect to $d\omega/dt$

Example: Principal attributes of HEO telecom orbits

Advantages

- Satellite at high elevation at high latitude ground sites – this is the over-riding benefit for comms services at high latitude
- No eclipse during comms operations
- The flexibility of its design allows targeting specific latitudes if a stable orbit is designed
- High eccentricities allows a longer time spent in the apogee region, hence offer enhanced coverage for regions at the apogee point

Example: Principal attributes of HEO telecom orbits

Disadvantages

- Ground stations must track spacecraft
- Satellite switching protocol required
- More than one spacecraft required for 24 h regional coverage
- Variation in satellite range and range-rate – this has a number of impacts upon the comms payload design:
  - Variation in time of signal propagation
  - Frequency variation due to Doppler
  - Variation in received signal power
  - Change in ground coverage pattern during each orbit
- Passage through Van Allen radiation belts each orbit – resulting in accelerated degradation of power and electronic systems
- Orbit perturbations – third body forces may disturb the perigee height, causing premature atmospheric re-entry

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Example: remote sensing mission

→ low Earth orbit → circular → close

Orbit selection for remote sensing

- Orbit inclination:

Global coverage requirement ⇒ orbit near-polar ($i \sim 90^o$)

- Orbit shape:

Invariant payload sensor range ⇒ orbit near-circular ($e \sim 0$)

- Orbit altitude:

Payload sensor resolution ⇒ low altitude

Spacecraft propellant requirement for drag compensation ⇒ high altitude

Trade-off leads to height in the region of $h \sim 700$ to 900 km ($a \sim 7080$ to 7280 km) for spacecraft with required operation for an extended period

Example: remote sensing mission

Need to define exact value of inclination, eccentricity, semi-major axis.

Payload operational requirements:

- sun-synchronous orbit
- repeat ground track
- one spacecraft axis Earth-pointing (attitude constraint)

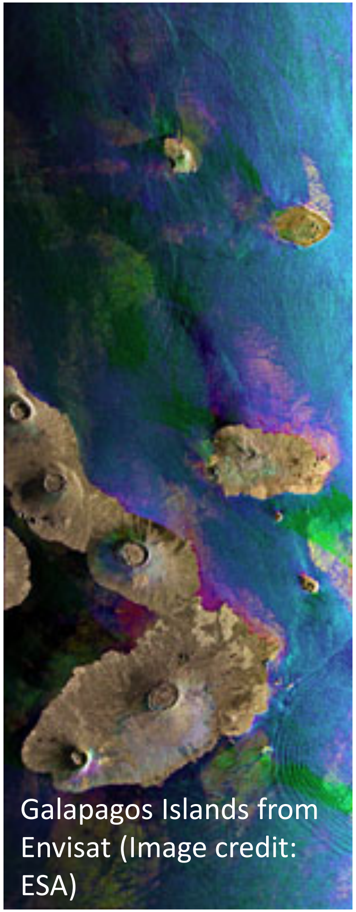
Exploitation of J_2 for defining a Sun-synchronous orbit using the J_2 effect on $d\Omega/dt$

Repetition pattern is synchronous with the sun.

$i \simeq 38°$

so they have $\dot{\Omega} = \dot{\alpha}$

Drift of the sun.



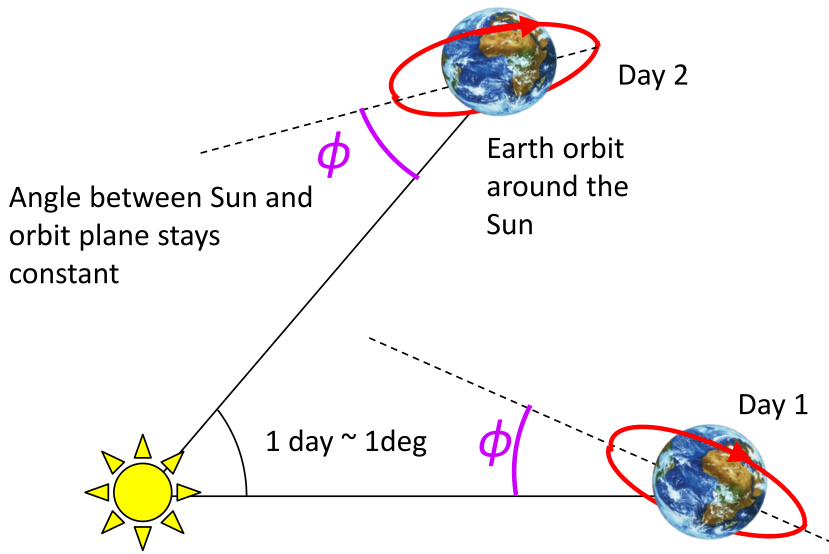
Galapagos Islands from Envisat (Image credit: ESA)

Sun-synchronous orbit

A spacecraft in a sun-synchronous orbit arrives over a particular location at the same local solar time (LST) on each pass

$\Rightarrow$ The 'solar aspect' on the ground similarly remain unchanged, apart from seasonal variations:

- Same shadow length
- Same shadow/light orientation
- Provides fixed lighting conditions for imaging (allows image comparison)



$$\dot{\phi} = \dot{\alpha}_s - \dot{\Omega} = 0$$

$$\dot{\Omega} = \dot{\alpha}_s = 360^o / 365.25 days \approx 0.986^o / day \ (East)$$

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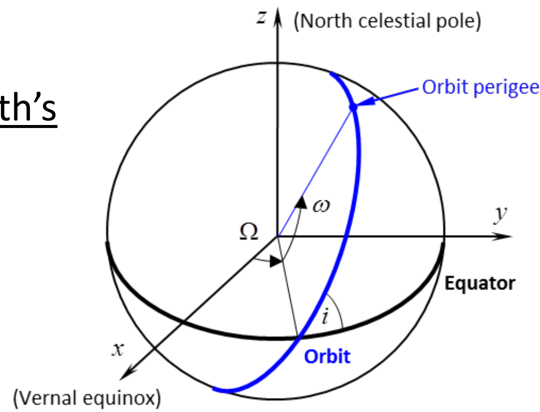
Sun-synchronous orbit

How is a Sun-synchronous orbit achieved?

Orbit node shift can be achieved using gravitational Earth's oblateness perturbation

Recall nodal regression rate due to J_2 (+ve East)

$$\dot{\Omega}_{J_2} = -\frac{3}{2} J_2 \left(\frac{R_{Earth}}{a} \right)^2 \frac{1}{\left(1-e^2\right)^2} \sqrt{\frac{\mu_{Earth}}{a^3}} \cos i$$



for e=0 $\quad \dot{\Omega}_{J_2} = -2.0647 \cdot 10^{14} a^{-7/2} \cos i$

In degrees/day, where a is in km

hence

$0^o < i < 90^o$, Node moves West

$i = 90^o$, Node is stationary

$i > 90^o$, Node moves East

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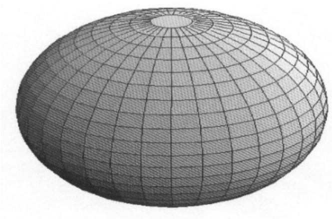
Sun-synchronous orbit

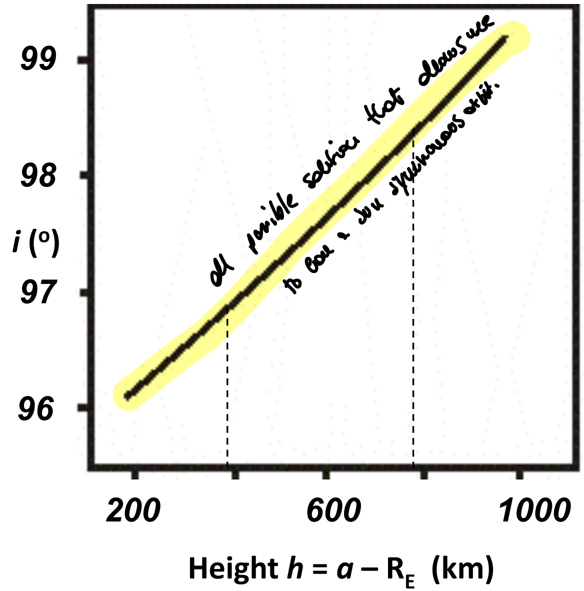
Natural orbit regression due to J_2 $\boxed{\dot{\Omega}_{J_2} = f(a,i)}$



We require to achieve $\dot{\Omega} \approx 1^{\circ}$/day Eastwards

For LEO we require $i \approx 98^{\circ}$

Sun-synchronous inclination for circular orbits, as a function of orbit altitude



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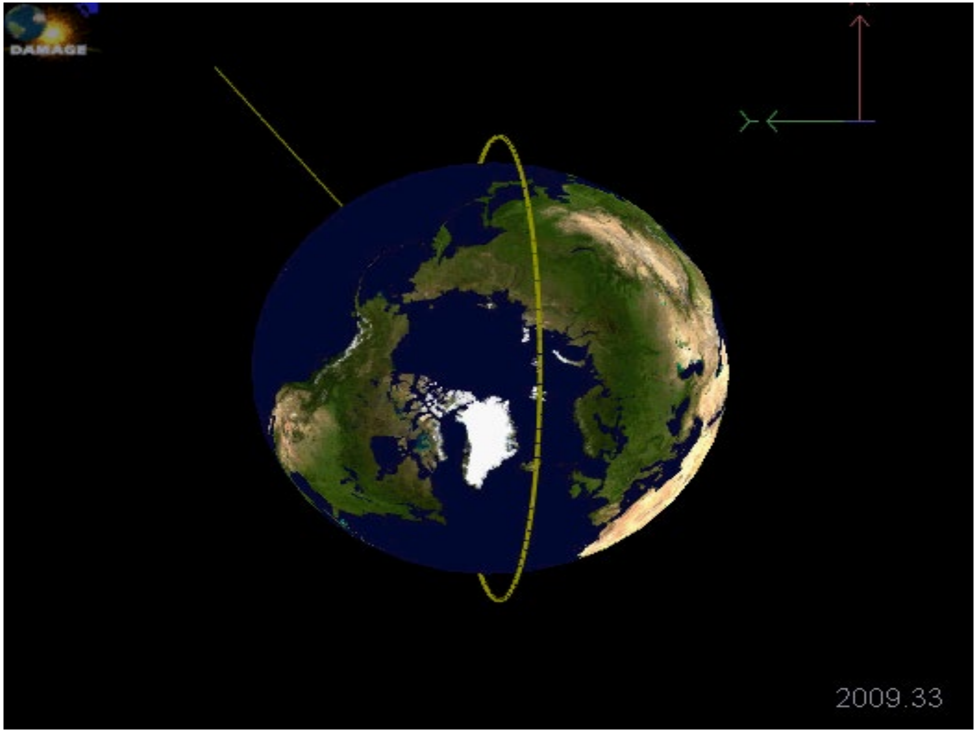
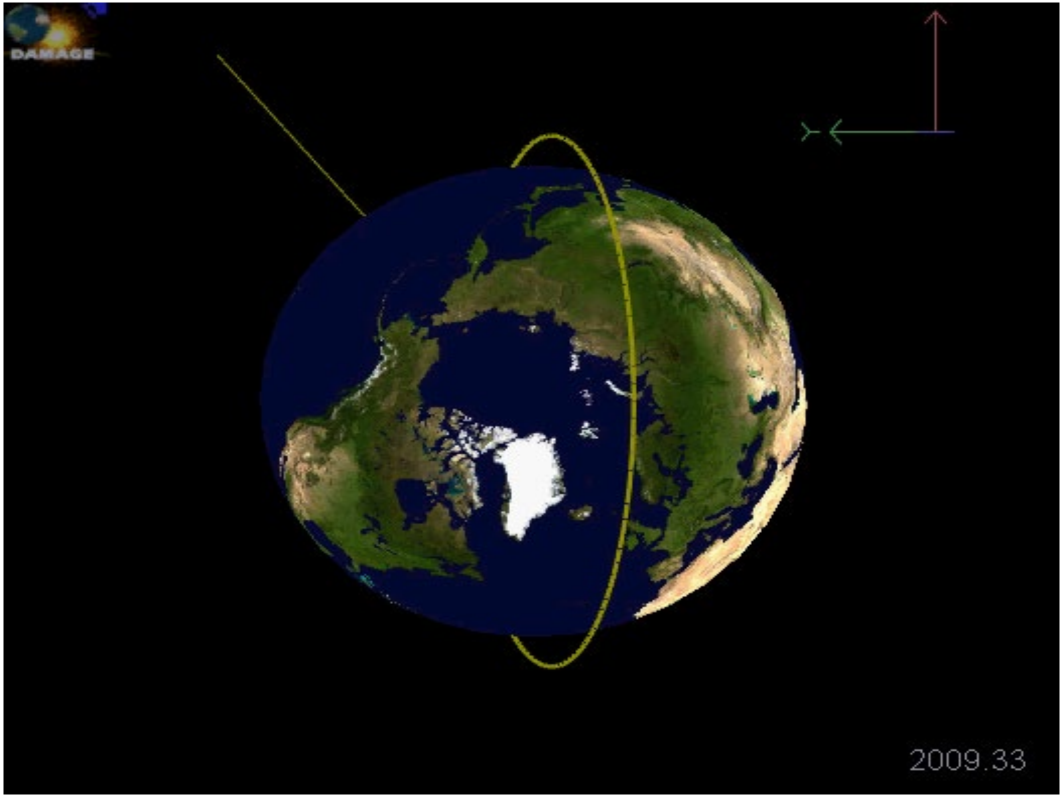
Sun-synchronous orbit



Courtesy: University of Southampton

Non Sun-synchronous orbit



Courtesy: University of Southampton

HAVE A LOOK AT SAROS YEAR

5.5.2 Gravitational perturbation: tesseral harmonics

Tesseral Harmonic Potential (distance, latitude ϕ and longitude λ dependant)

zonal harmonics

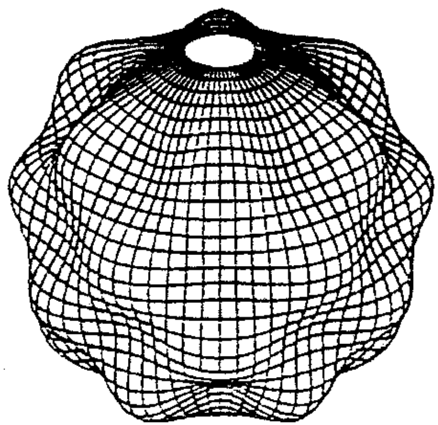
$$\boxed{R(r,\phi,\lambda)=\frac{\mu}{r}\left(-1+R_{ZH}+\sum_{n=2}^{\infty}\sum_{m=1}^{n}\left(\frac{R_E}{r}\right)^n\left(C_{nm}\cos m\lambda+S_{mn}\sin m\lambda\right)P_{nm}(\cos\phi)\right)}$$

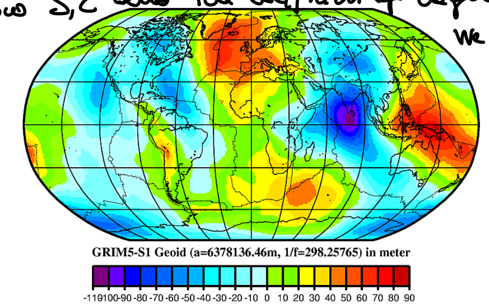
↳ Tesseral harmonics → Tiles like so need two summation.

Potential expressed as a sum of spherical harmonics

↳ function of longitude, latitude and distance

→ If we know S, C and the coefficient of Legendre polynomial we know everything



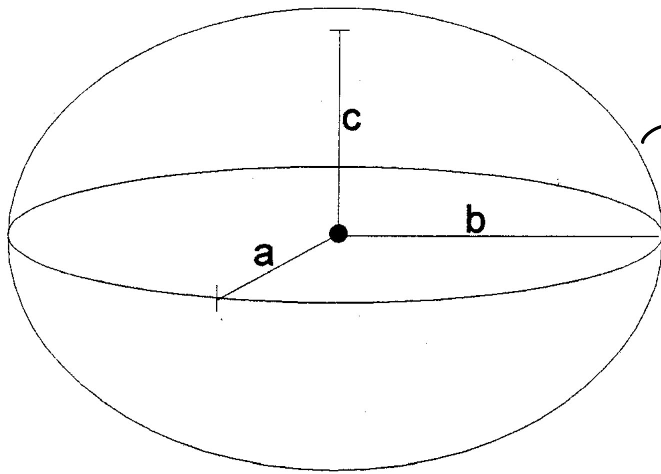


Geopotential map

Gravitational perturbation: triaxiality

- For LEO spacecraft, perturbation is small
- For GEO spacecraft the lowest order term ($n = m = 2$) has greatest effect This, in combination with Earth oblateness, gives a triaxial geometry for the Earth → some satellite feel different attraction force

↳ Earth triaxiality. Not only of an oblate planet but it is an ellipsoid



→ simplest model of an Asteroid.

$$a \neq b \neq c$$

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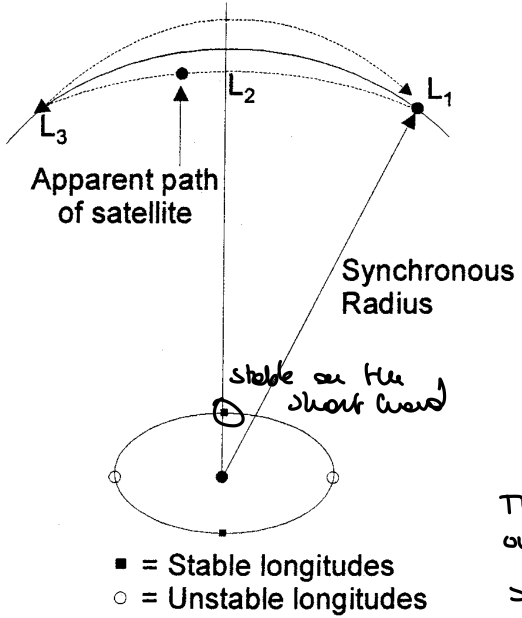
Gravitational perturbation: triaxiality

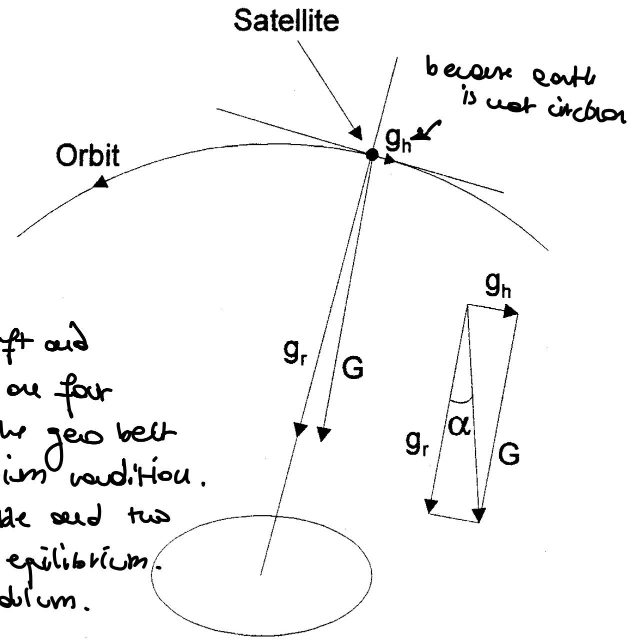
Equatorial GEO orbit on plane a,b.

The principal effect of the triaxiality perturbation on GEO spacecraft is secular changes in satellite longitude λ causing large changes in longitudinal position

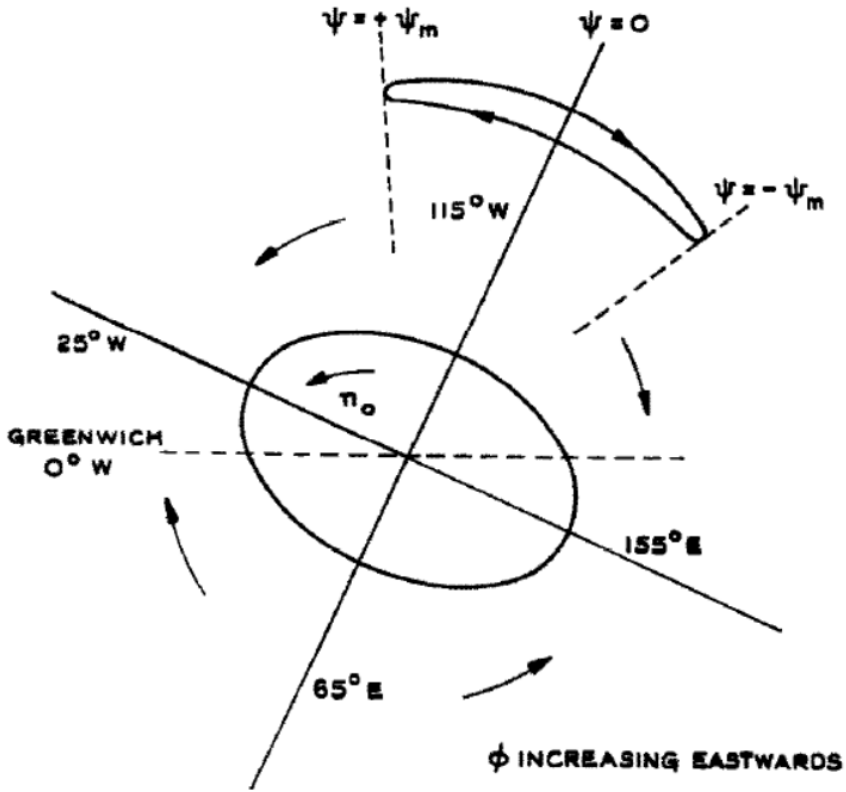
At a particular longitudes, $d^2\lambda / dt^2 = \text{constant}$



In geo we are drifted left and right → there are four location on the geo belt called equilibrium condition.
There are two stable and two unstable point of equilibrium.
Similar to a pendulum.



We can write the dynamics in terms of longitude so we can see it the difference of longitude wrt the equilibrium point. Dynamics in terms of longitude and the derivative of the longitude



Geostationary Orbital Dynamics

One of key perturbation sources at GEO:

- Earth's triaxiality

$$\boxed{R(r,\phi,\lambda)=\frac{\mu}{r}\left(-1+R_{ZH}+\sum_{n=2}^{\infty}\sum_{m=1}^{n}\left(\frac{R_E}{r}\right)^n\left(C_{nm}\cos m\lambda+S_{mn}\sin m\lambda\right)P_{nm}\left(\cos\phi\right)\right)}$$

- For the first two terms: (J_2 and $J_{2,2}$)

$$R(r,\phi,\lambda)=\frac{g_0R_0^2}{r}\left[1-J_2\frac{R_0^2}{r^2}\left(\frac{3\,sin^2\phi-1}{2}\right)+3J_{2,2}\frac{R_0^2}{r^2}\cos^2\phi\cos 2\gamma\right]$$

$\gamma=\lambda-\lambda_{22}$ $\quad$ λ_{22}stable latitude at $\frac{\pi}{2}+\tan^{-1}\frac{S_{nm}}{C_{nm}}$

$J_{2,2}=\sqrt{C_{nm}^2+S_{nm}^2}$

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Geostationary Orbital Dynamics

Key perturbation sources are:

- Differentiate gravitational potential to obtain acceleration

$$R(r,\phi,\lambda) = \frac{g_0 R_0^2}{r}\left[1 - J_2 \frac{R_0^2}{r^2}\left(\frac{3\,sin^2\phi - 1}{2}\right) + 3J_{2,2}\frac{R_0^2}{r^2}\cos^2\phi\cos 2\gamma\right]$$

$$a_r = \frac{\partial R}{\partial r} = -\frac{g_0 R_0^2}{r^2} + \frac{3}{2}J_2\frac{R_0^4}{r^4}(3\,sin^2\phi - 1) - 9J_{2,2}\frac{R_0^4}{r^4}\cos^2\phi\cos 2\gamma$$

$$a_\lambda = \frac{1}{r\cos\phi}\frac{\partial R}{\partial\lambda} = -6J_{2,2}\frac{R_0^4}{r^4}\cos\phi\cos 2\gamma$$

$$a_\phi = \frac{1}{r}\frac{\partial R}{\partial\phi} = -3J_2\frac{R_0^4}{r^4}\sin\phi\cos\phi - 6J_{2,2}\frac{R_0^4}{r^4}\cos\phi\sin\phi\cos 2\gamma$$

Earth-Fixed Rotating Reference Frame

Is usually convenient for GEO

In Earth-fixed polar coordinates (r, λ, ϕ):

$$\ddot{r} = a_r + r\left(\Omega + \dot{\lambda}\right)^2 \cos^2\phi + r\dot{\phi}^2$$

$$\ddot{\lambda} = \frac{1}{r\cos\phi}\left(a_\lambda - 2\dot{r}\left(\Omega + \dot{\lambda}\right)\cos\phi + 2r\left(\Omega + \dot{\lambda}\right)\dot{\phi}\sin\phi\right)$$

$$\ddot{\phi} = \frac{1}{r}\left(a_\phi - 2\dot{r}\dot{\phi} - r\left(\Omega + \dot{\lambda}\right)^2 \cos\phi\sin\phi\right)$$

Long-term uncontrolled dynamics

Use two equations,

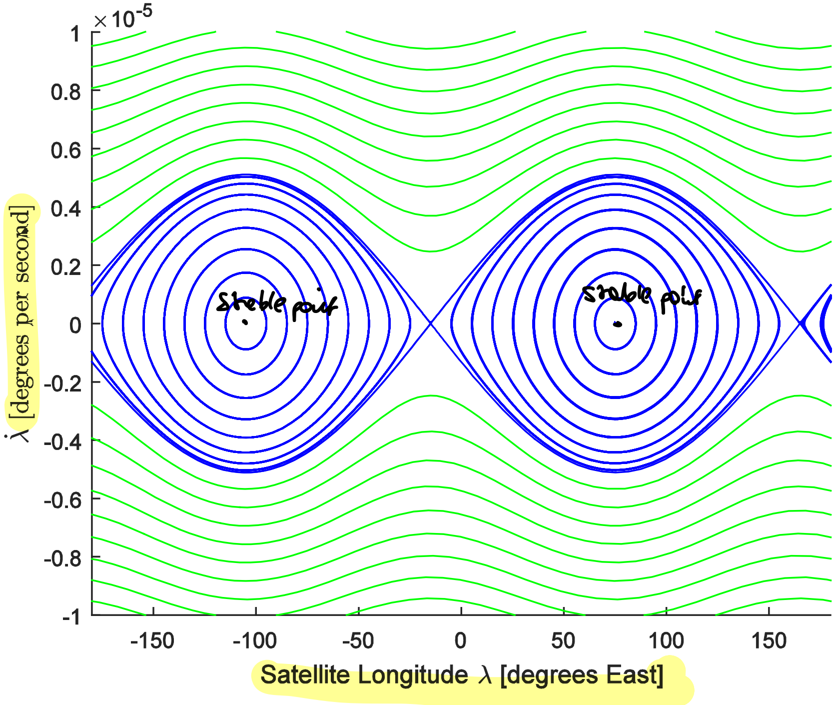
$$\ddot{\lambda} + \Omega^2 \ddot{\lambda} - 24\,\Omega^3 \mathrm{J}_{2,2} \frac{R_0^2}{r^2} \sin 2\gamma \dot{\lambda} - 18\Omega^4\, \mathrm{J}_{2,2} \frac{R_0^2}{r^2} \sin 2\gamma = 0$$

The long-term dynamics is

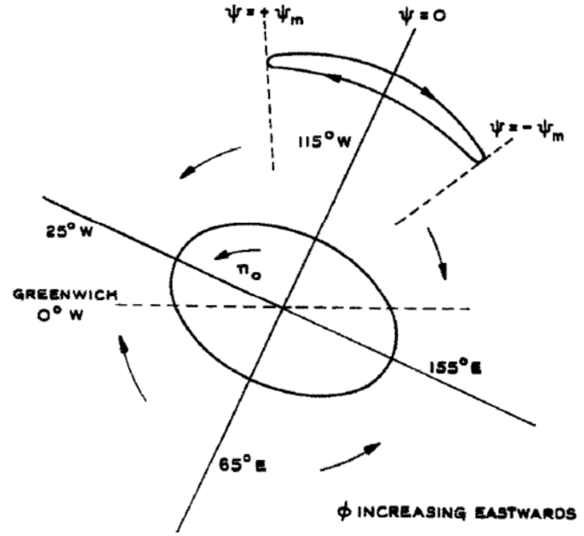
$$\ddot{\lambda} = 18\Omega^2\, \mathrm{J}_{2,2} \frac{R_0^2}{r^2} \sin 2(\lambda - \lambda_{22})$$

This is an equation of motion of a pendulum (J_{22} is negative)

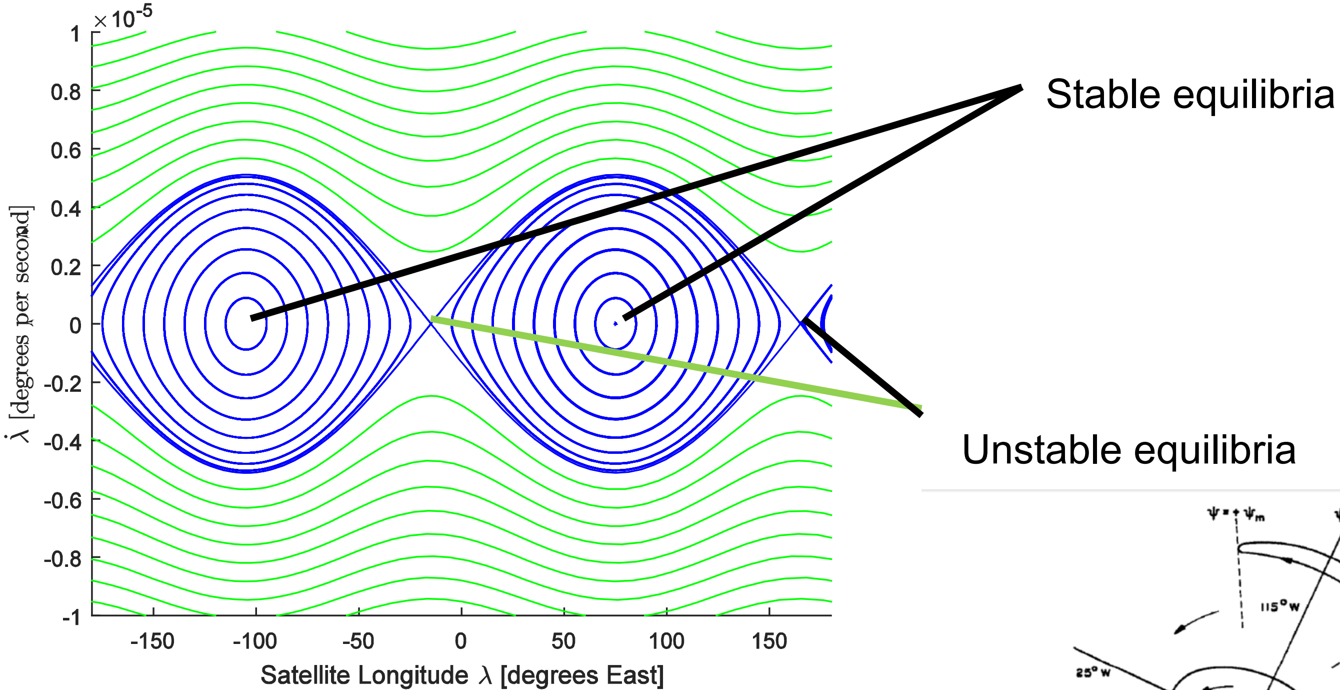
Long-term uncontrolled dynamics

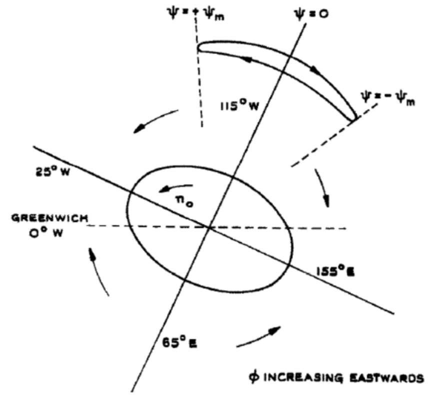


Pendulum like motion



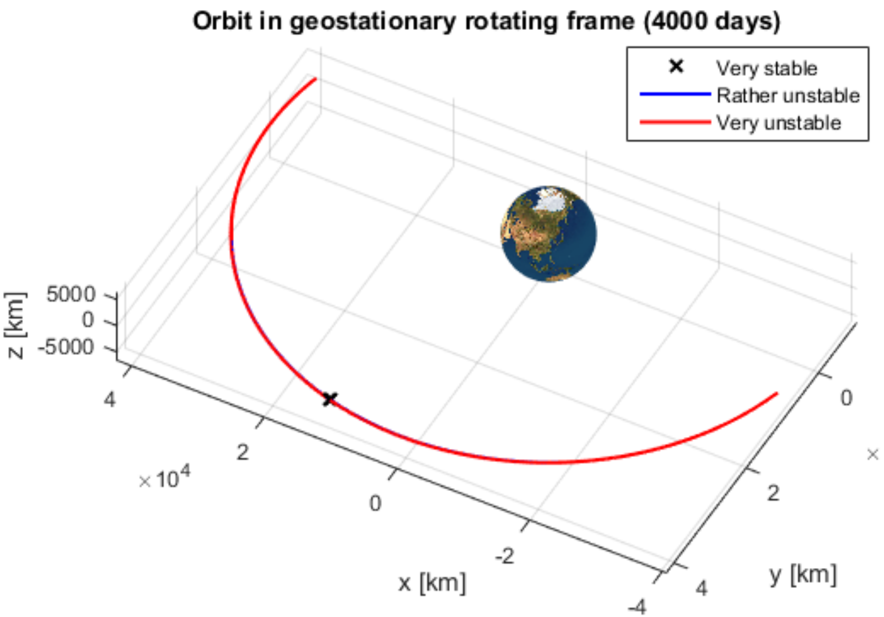
Long-term uncontrolled dynamics

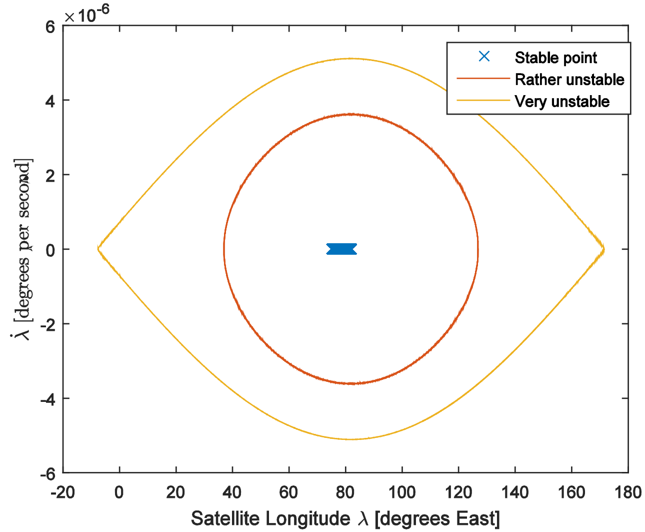




Long-term uncontrolled dynamics

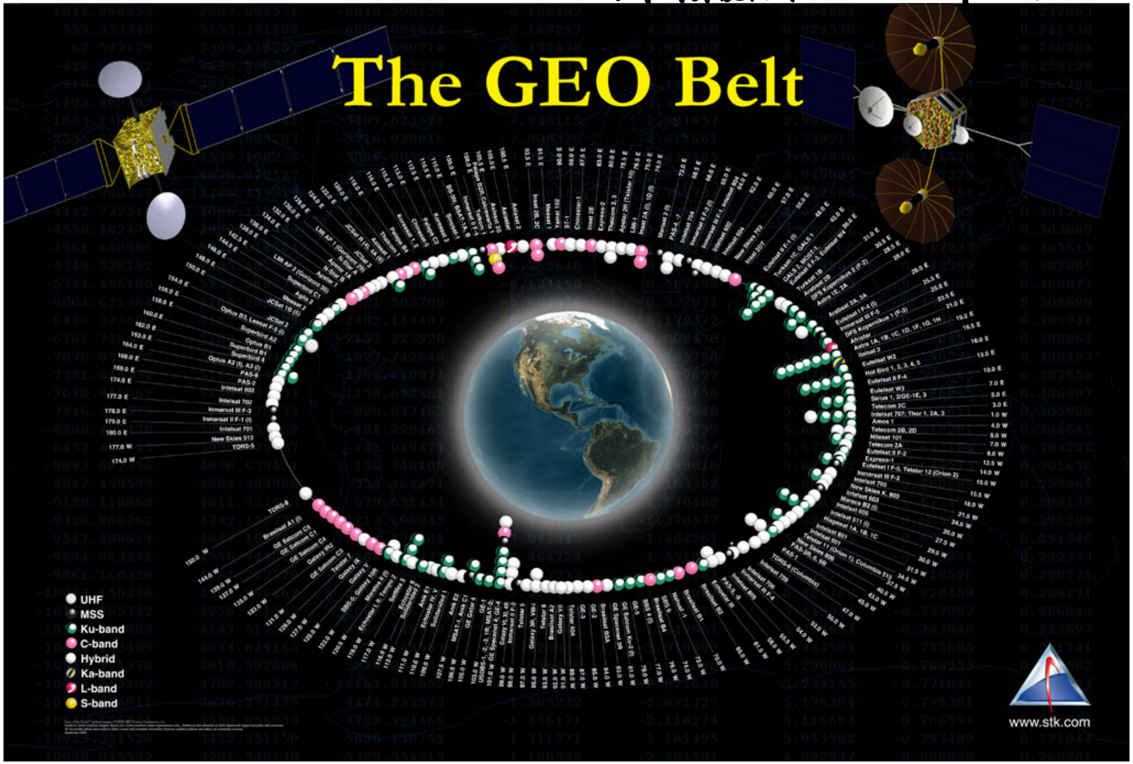
Fully numerical simulation of F=ma



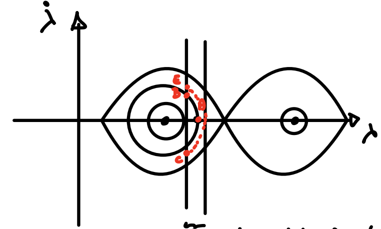


Gravitational perturbation: triaxiality
Orbit maintenance

Congestion of the GEO ring → geostationary orbit must be stable → even if we have perturbation → to keep a satellite geostationary we need a control



Station-keeping – Requirements



→ given slot

Due to Earth triaxiality we will have a drift →

The maintenance of the spacecraft within specified latitude and longitude limits using the least amount of propellant

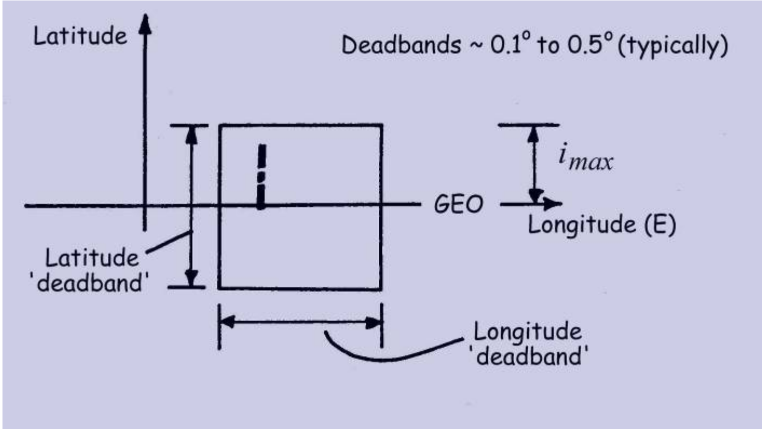
I let the satellite drift until it reaches the outer edge of the slot then I give an impulsive manoeuvre to go from B to C that they are at the same position with different velocity. then from C I let drift the satellite from C to E it is longer to the outer edges of the slot given then from E I do another impulsive manoever to go to C.

These manoeuvre are basically a phasing manoeuvre inter inertial reference frame → Every 2 month I need to do a manoeuvre.

In GEO station-keeping is performed

- to prevent communiction interference between spacecraft in adjacent GEO slots
- to minimise collision risk
- to aid management of the GEO arc

Stay within allocated East-West slot



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There are not only East and West manoeuvre, there are also North and South manoeuvre.

East West Station Keeping (EWSK) – Strategy

Within the box, perturbation forces are almost constant.

This means the dynamics can be linearised.

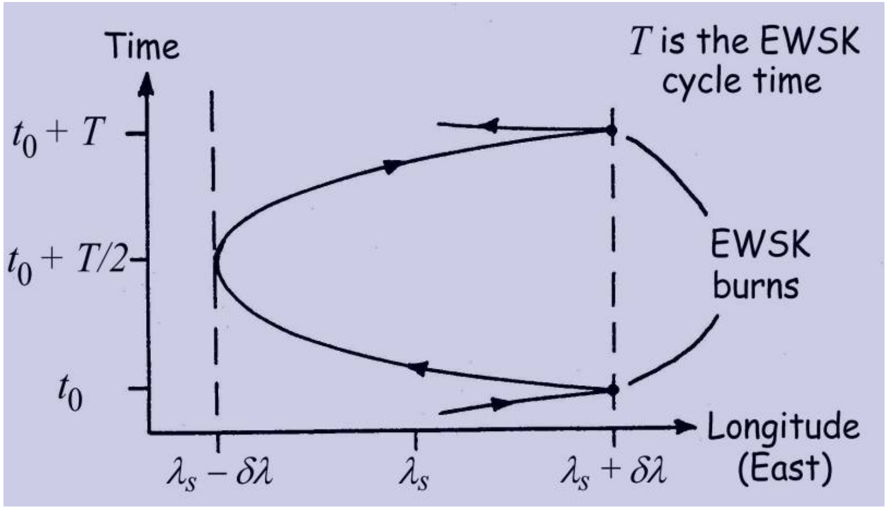
$$\frac{d\epsilon}{dt} = -6J_{2,2}\frac{R_0^4}{r^4}\cos 2\gamma \cdot r\Omega$$

$$\frac{da}{dt} = -12anJ_{2,2}\frac{R_0^2}{a^2}\sin 2\gamma$$

$$\frac{d^2\lambda}{dt^2} = -\frac{3}{2}\frac{n}{a}\frac{da}{dt} = \beta = constant$$

$$T = 2\sqrt{\frac{\delta\lambda}{\beta}}$$

$$\Delta a = T\frac{da}{dt}$$



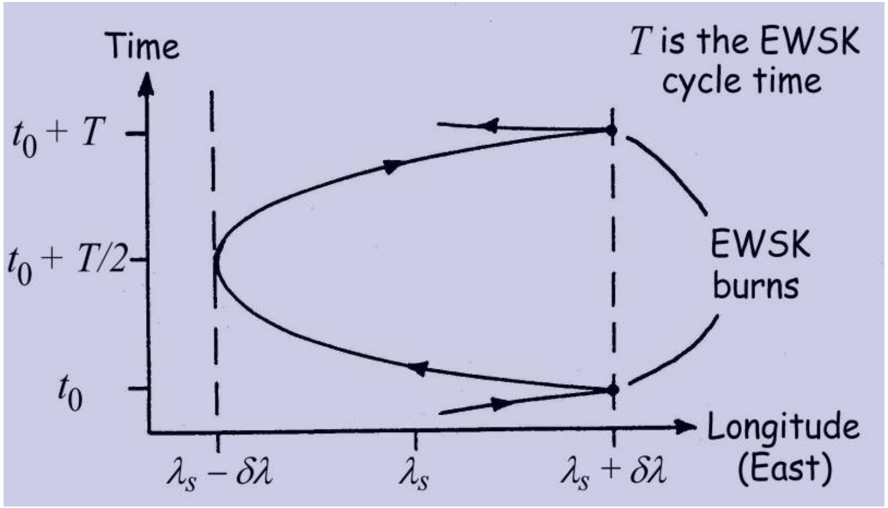
$$\Delta V_{year} = 1.75\sin(\lambda - \lambda_{22})\ \text{m/sec/year}$$

East West Station Keeping (EWSK) – Strategy

- Triaxiality changes satellite longitude (SRP changes *e*), both causing longitudinal excursions.
- EWSK operations comprise along-track secondary propulsion burns when spacecraft 'hits' longitude deadband boundary (triaxiality is the dominant effect).

EWSK for European region $d^2\lambda / dt^2 > 0$

- EWSK is ~ order of magnitude < NSSK
- EWSK cycle time ~ 1/ month, depending upon orbital position longitude and longitude deadband



5.6 Third-Body perturbation

→ Can be written in terms of Disturbing potential and then use also semi analytical technique

- Luni-Solar Perturbations
- The effect of the direct gravitational attraction of the Moon and the Sun on the spacecraft
- Principal effects are secular changes in i, e and ω

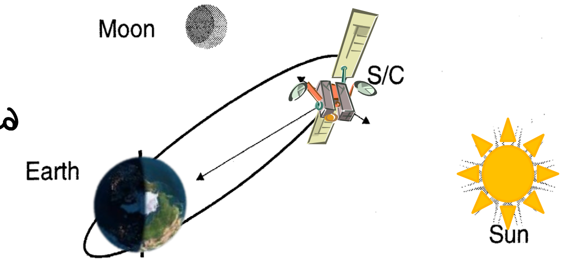
for LEO spacecraft, perturbation is small

for GEO and HEO spacecraft, effect can be significant

$$\boxed{\mathbf{a}_{3Body} = \mu_{3Body}\left(\frac{\mathbf{r}_{sc-3body}}{r_{sc-3body}^{3}} - \frac{\mathbf{r}_{Earth-3body}}{r_{Earth-3body}^{3}}\right)}$$

← formula obtained when we end e look at the 3 body motion

$$\boxed{\ddot{\mathbf{r}} = -\frac{\mu_{Earth}}{r^3}\mathbf{r} + \mathbf{a}_{3Body}}$$



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Third-Body perturbation

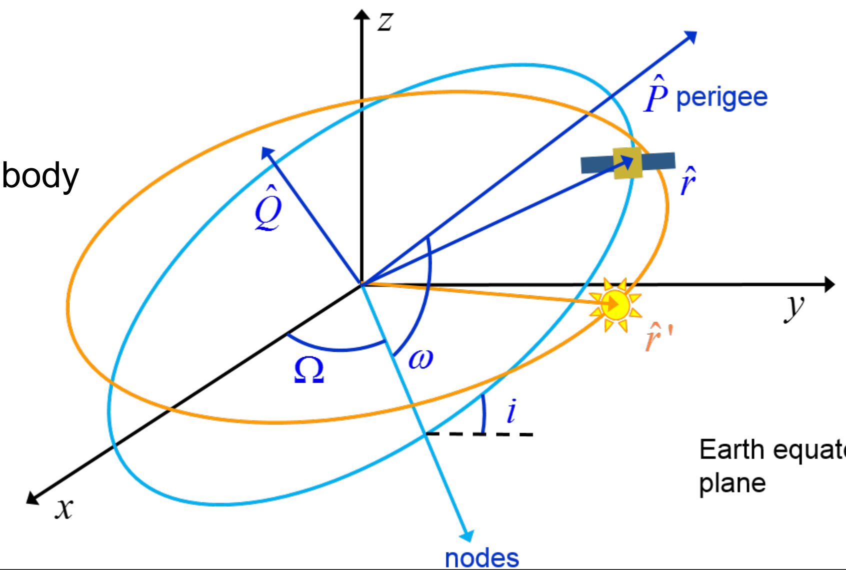
$$\boxed{R_{3B}\left(r,r'\right)=\frac{\mu'}{r'}\left(\left(1-2\frac{r}{r'}\cos\psi+\left(\frac{r}{r'}\right)^2\right)^{-1/2}-\frac{r}{r'}\cos\psi\right)}$$

μ' gravitational coefficient of the third body

$\mathbf{r}'$ position vector of third body

$\mathbf{r}$ position vector of satellite

ψ angle between satellite and third body



Earth equatorial plane

No Exam

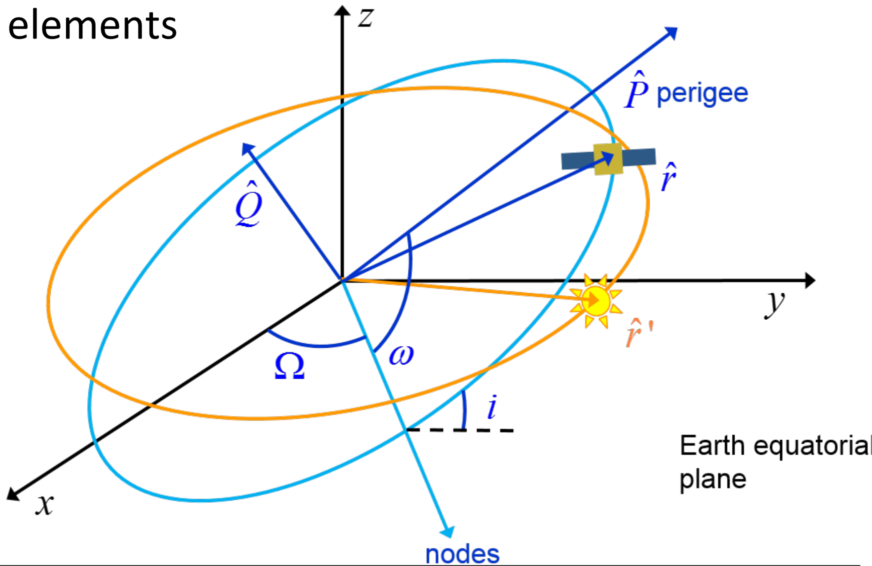
Third-Body perturbation

Derivation of averaged potential

Third body potential in terms of:

- Ratio between orbit semi-major axis and distance of the third body $\delta = \frac{a}{r'}$
- Orientation of orbit eccentricity vector with respect to third body $A = \hat{P} \cdot \hat{r}'$
- Orientation of semi-latus rectum vector with respect to third body $\mathrm{B} = \hat{\mathrm{Q}} \cdot \hat{r}'$
- Composition of rotation in orbital elements

$$\hat{P} = R_3(\Omega) R_1(i) R_3(\omega) \cdot [1 \quad 0 \quad 0]^T$$
$$\hat{Q} = R_3(\Omega) R_1(i) R_3(\omega + \pi/2) \cdot [1 \quad 0 \quad 0]^T$$
$$\hat{r}' = R_3(\Omega') R_1(i') R_3(\omega' + f') \cdot [1 \quad 0 \quad 0]^T$$



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Third-Body perturbation

Derivation of averaged potential

No Exam

Series expansion around $\delta = 0$

$$R_{3B}(r,r') = \frac{\mu'}{r'} \sum_{k=2}^{\infty} \delta^k F_k(A,B,e,E)$$

μ' gravitational coefficient of the third body

$\mathbf{r}'$ position vector of third body

E eccentric anomaly

Average over one orbit revolution

$$\bar{R}_{3B}(r,r') = \frac{\mu'}{r'} \sum_{k=2}^{\infty} \delta^k \bar{F}_k(A,B,e)$$

$$\bar{F}_k(A,B,e) = \frac{1}{2\pi} \int_{-\pi}^{\pi} F_k(A,B,e,E) \underbrace{(1 - e\cos E)dE}_{dM}$$

Partial derivatives for Lagrange equations

$A(\Omega,i,\omega,\Omega',i',u')$

$B(\Omega,i,\omega,\Omega',i',u')$

$\bar{F}_k(A,B,e)$

$$\frac{\partial \bar{F}_k}{\partial \Omega} = \frac{\partial \bar{F}_k}{\partial A}\frac{\partial A}{\partial \Omega} + \frac{\partial \bar{F}_k}{\partial B}\frac{\partial B}{\partial \Omega}$$

$$\frac{\partial \bar{F}_k}{\partial i} = \frac{\partial \bar{F}_k}{\partial A}\frac{\partial A}{\partial i} + \frac{\partial \bar{F}_k}{\partial B}\frac{\partial B}{\partial i}$$

$$\frac{\partial \bar{F}_k}{\partial \omega} = \frac{\partial \bar{F}_k}{\partial A}\frac{\partial A}{\partial \omega} + \frac{\partial \bar{F}_k}{\partial B}\frac{\partial B}{\partial \omega}$$

$$\frac{\partial \bar{F}_k}{\partial a} = \frac{k}{a}\bar{F}_k$$

$$\frac{\partial \bar{F}_k}{\partial e}$$

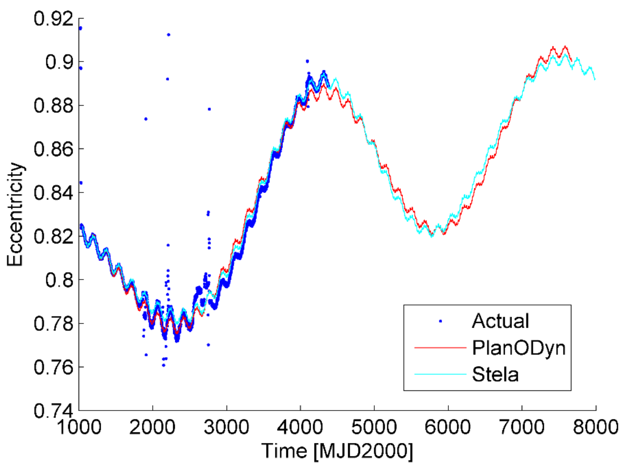
▶ *Kaufman and Dasenbrock, NASA report, 1979*

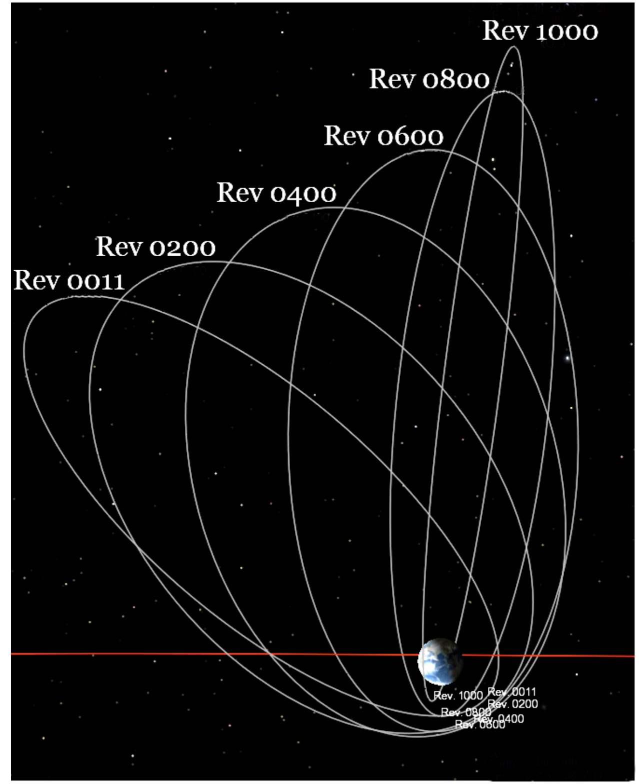
We can use 3body perturbation

Third-Body Perturbation

Effects of luni-solar gravity perturbations on Integral orbit:

- initial operational orbit: rev. 11
- initial orbit period 72 hours (3 days)
- rev. 1000 ~ 8.2 years





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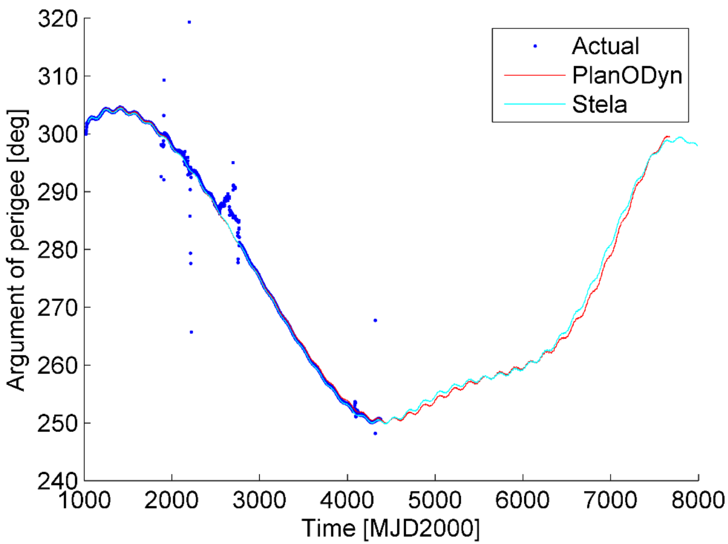
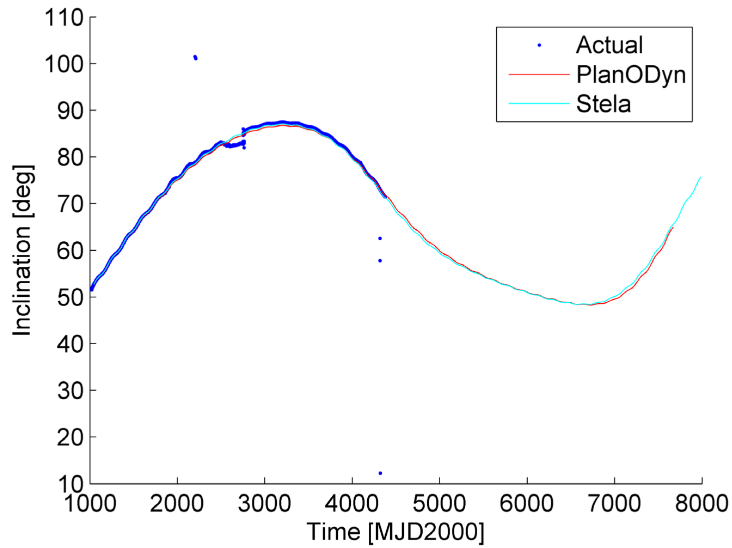
Third-Body Perturbation

ω → over the long terms remain constant.





Third-Body Perturbation

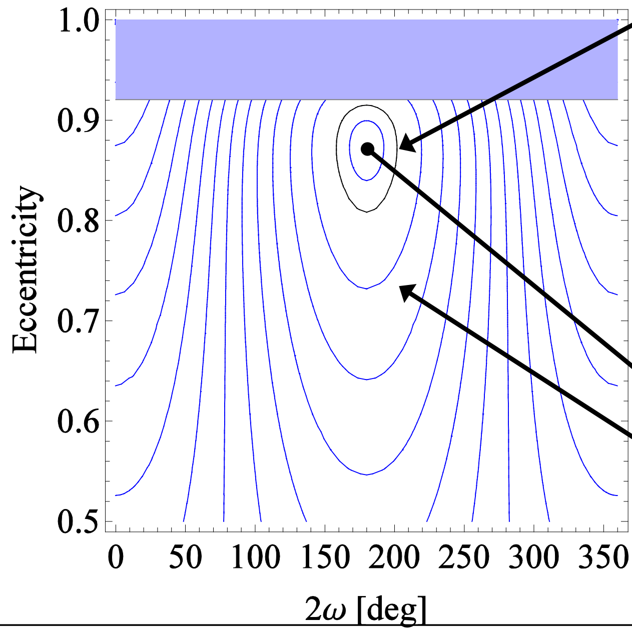
→ mean ω measured from the line where the plane of orbit intersect the plane of the third body

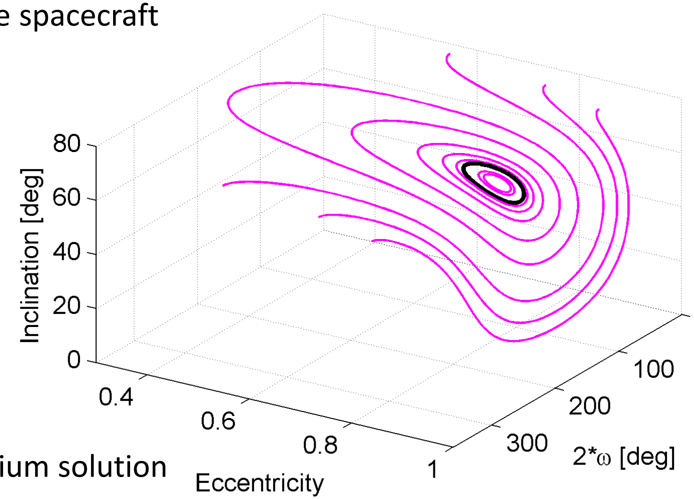
Kozai constant

$$\Theta = \left(1 - e^2\right)\cos i^2$$

Initial condition in *a*, *e*, *i*, *ω* defines a contour line in phase space

INTEGRAL-like spacecraft





Equilibrium solution

Librational solutions

argument of perigee
$\omega \rightarrow$ measured from the point of intersection between plane of orbit and the third body.

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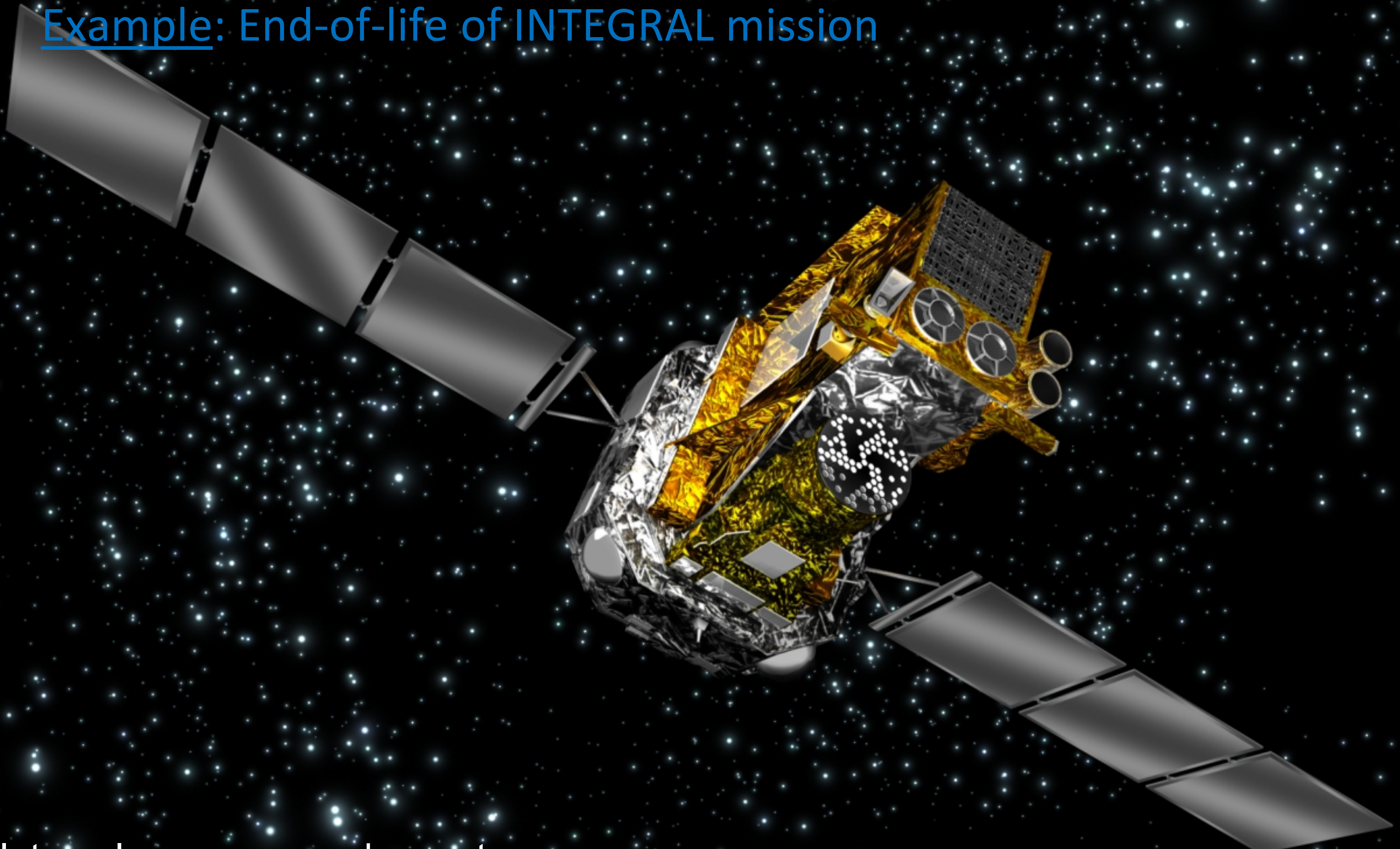
Example: End-of-life of INTEGRAL mission



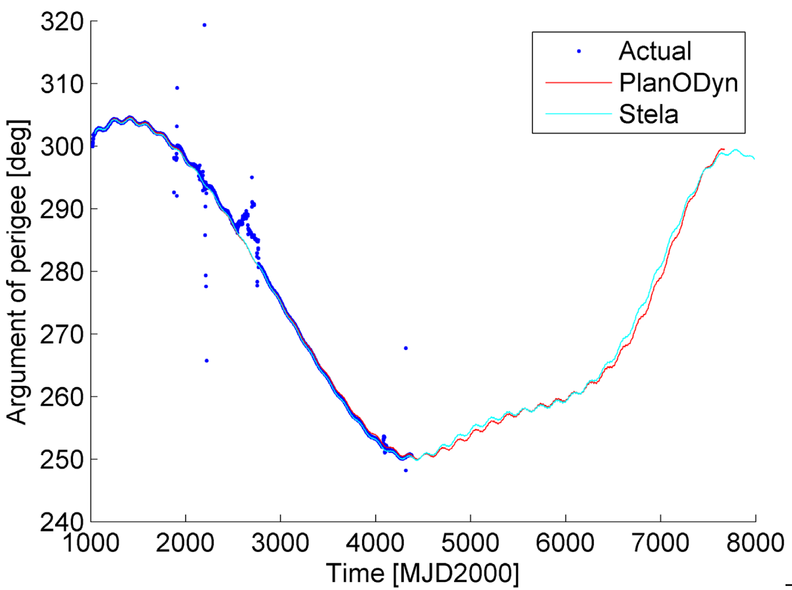
Integral: gamma-ray observatory
ESA's Integral observatory is able to detect gamma-ray bursts, the most energetic phenomena in the Universe

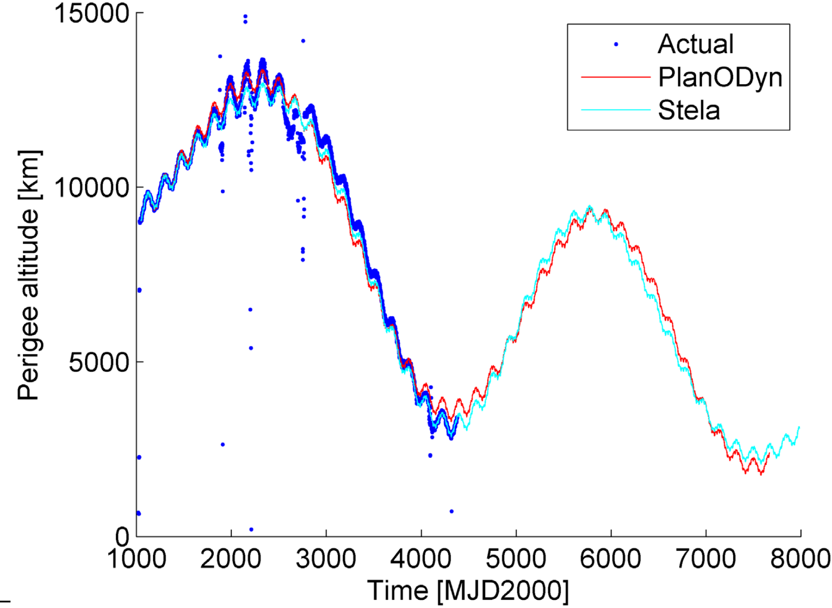
Example: End-of-life of INTEGRAL mission

Operational orbit

Mission scenario

- Propagation time: 2002/11/13 to 2021/01/01
- Initial Keplerian elements from Horizon NASA on 2002/11/13 at 00:00: a = 87736 km, e = 0.82403, i = 0.91939 rad, Ω = 1.7843 rad, ω = 5.271 rad





Example: End-of-life of INTEGRAL mission

Operational orbit

Mission scenario

- Propagation time: 2002/11/13 to 2021/01/01
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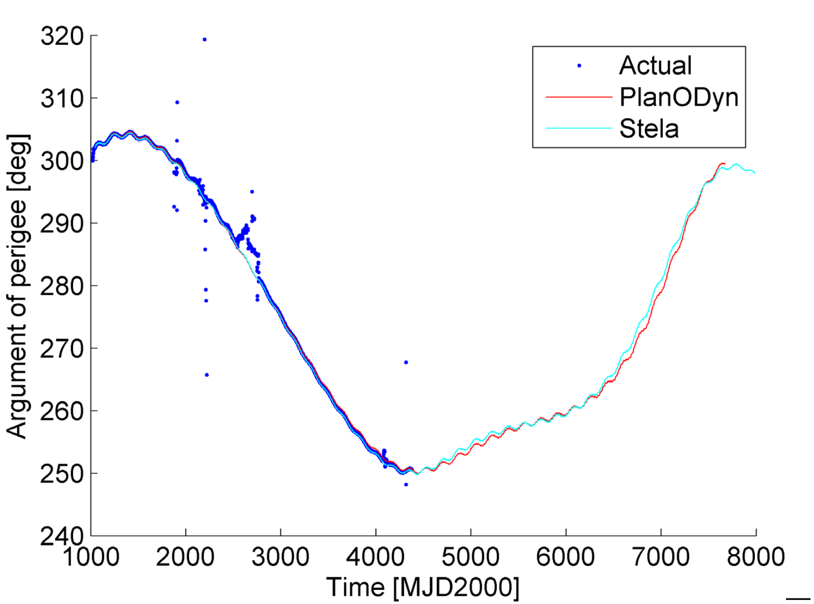
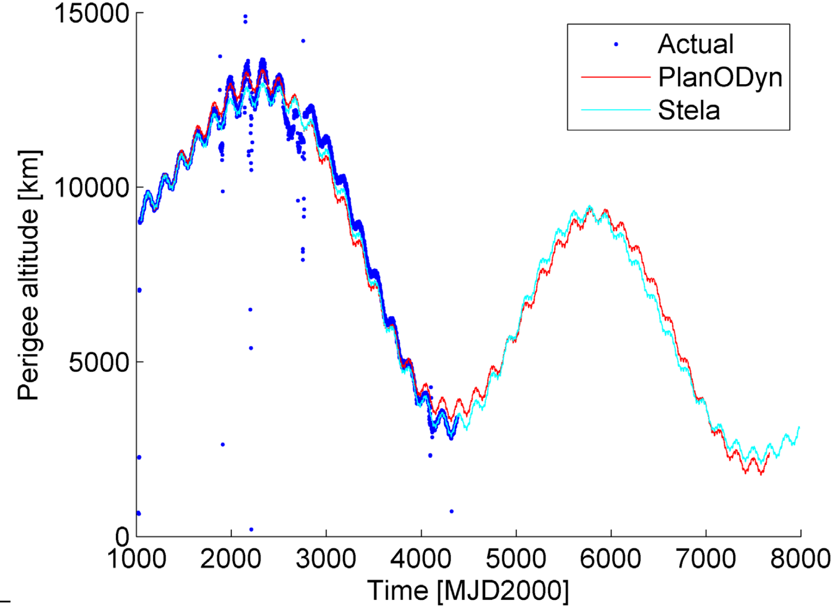
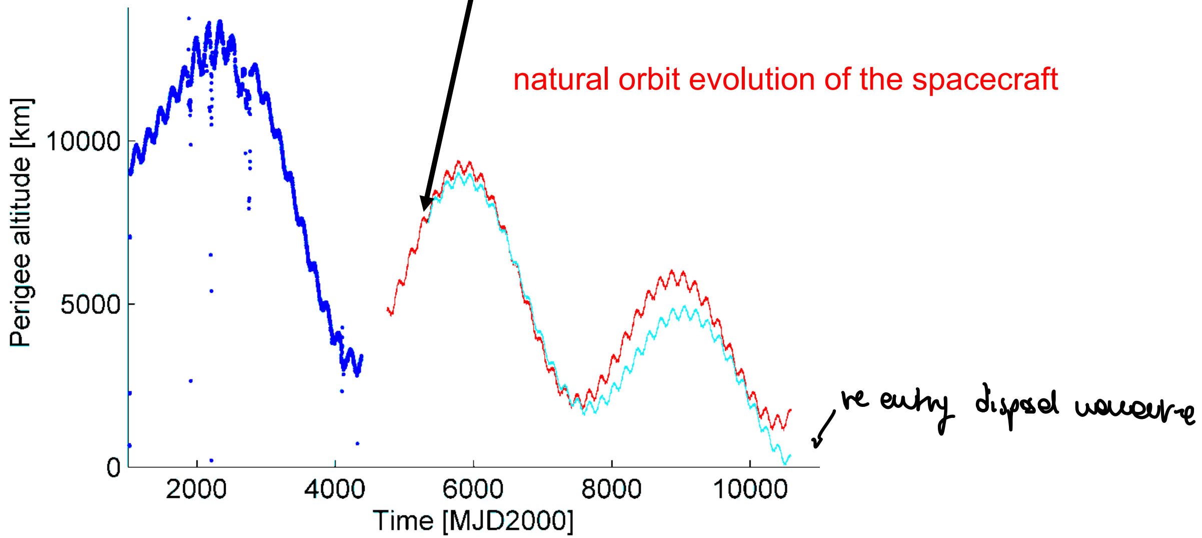




Re-entry disposal design for INTEGRAL mission

single maneuver

$$\Delta\mathbf{v} = \Delta v \begin{bmatrix} \cos\alpha\cos\beta \\ \sin\alpha\cos\beta \\ \sin\beta \end{bmatrix}$$



Re-entry disposal design for INTEGRAL mission

single maneuver

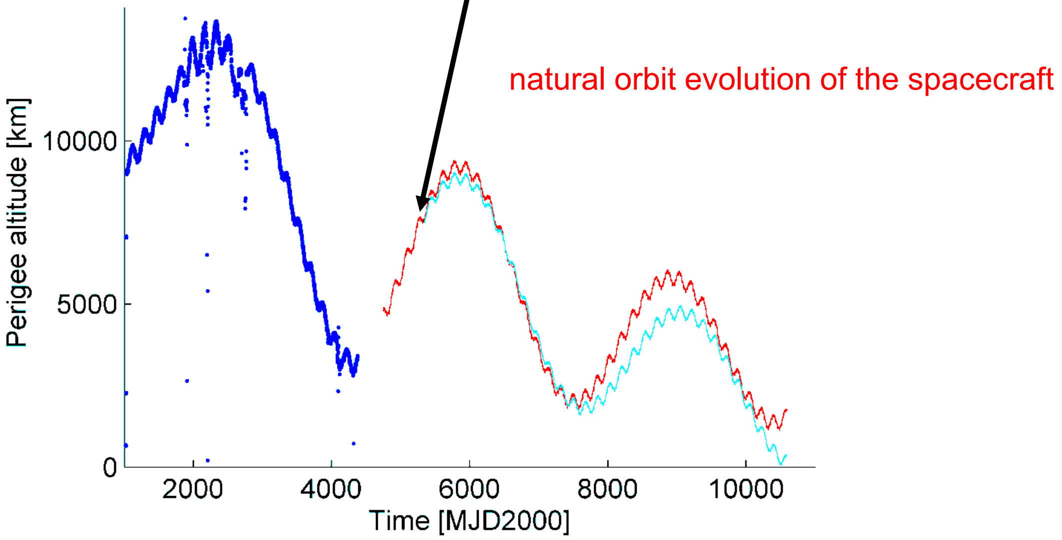
$$\Delta\mathbf{v} = \Delta v \begin{bmatrix} \cos\alpha\cos\beta \\ \sin\alpha\cos\beta \\ \sin\beta \end{bmatrix}$$

$$\Delta kep = G\left(kep\left(t_m\right), f_m, \Delta\mathbf{v}\right)$$

Gauss' planetary equations in finite-difference form

$$kep_d = kep\left(t_m\right) + \Delta kep$$

Deviated condition



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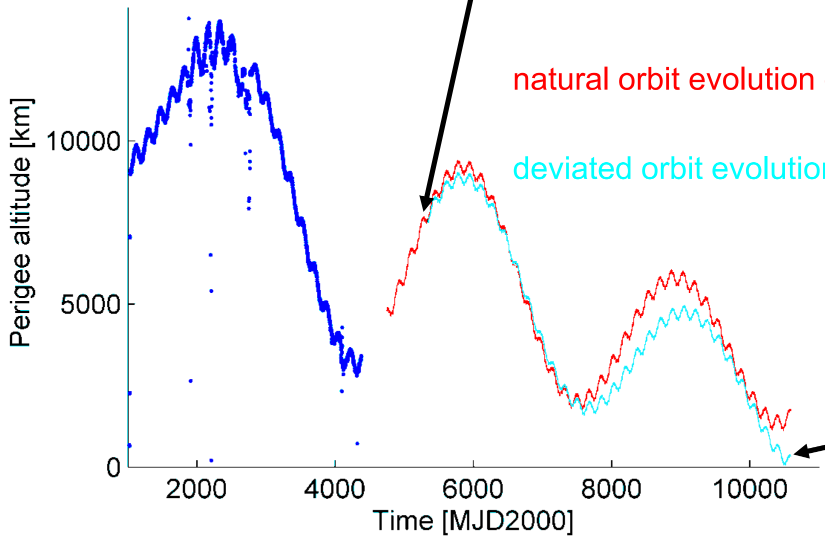
Re-entry disposal design for INTEGRAL mission

single maneuver

$$\Delta \mathbf{v} = \Delta v \begin{bmatrix} \cos\alpha\cos\beta \\ \sin\alpha\cos\beta \\ \sin\beta \end{bmatrix} \Rightarrow \Delta kep = G\left(kep\left(t_m\right), f_m, \Delta\mathbf{v}\right) \Rightarrow kep_d = kep\left(t_m\right) + \Delta kep$$

Gauss' planetary equations in finite-difference form

Deviated condition



Minimum perigee within selected time interval for disposal

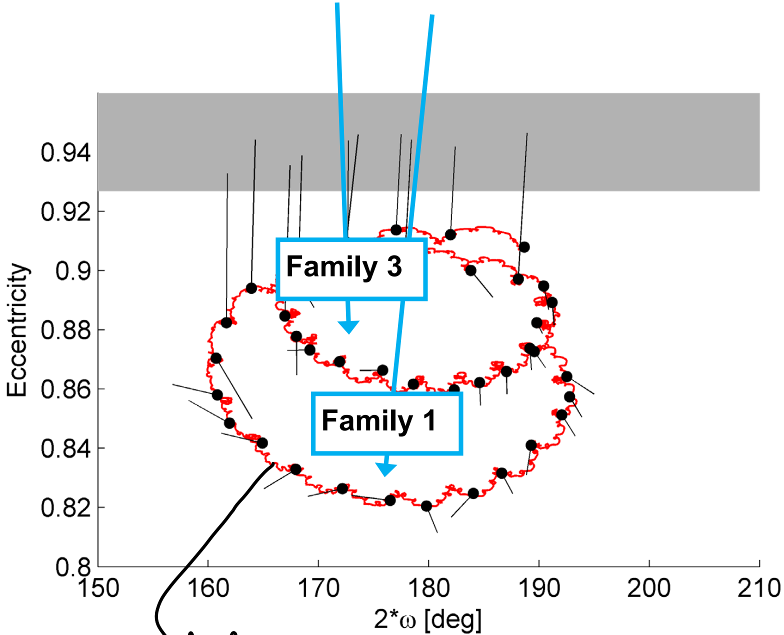
$$h_{p,\min} = \min_{t \in \Delta t_{\mathrm{disposal}}} h_p\left(t\right)$$

optimization problem → what is the minimum Δv to get a re-entry

Re-entry disposal design for INTEGRAL mission

INTEGRAL re-entry

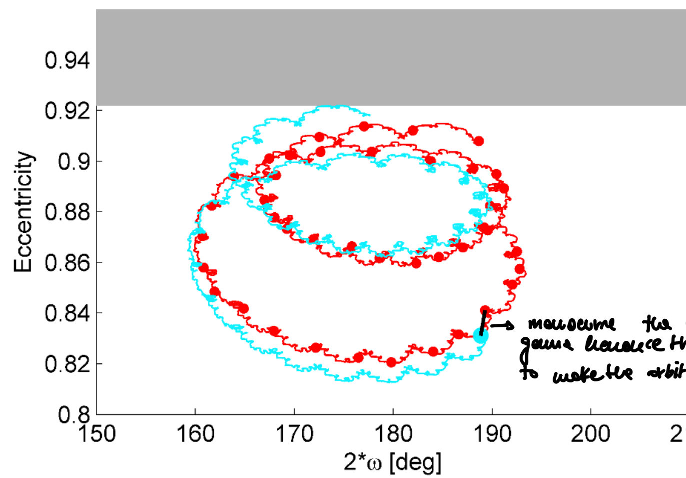
Low eccentricity conditions



All the perturbations together

The manoeuvre tends to further decrease *e*

→ the following long term propagation will reach a higher eccentricity (re-entry).
The manoeuvre is more efficient (i.e., lower Δv is required).



→ manoeuvre the moon is gone hence the oscillation to make the orbit re-enters

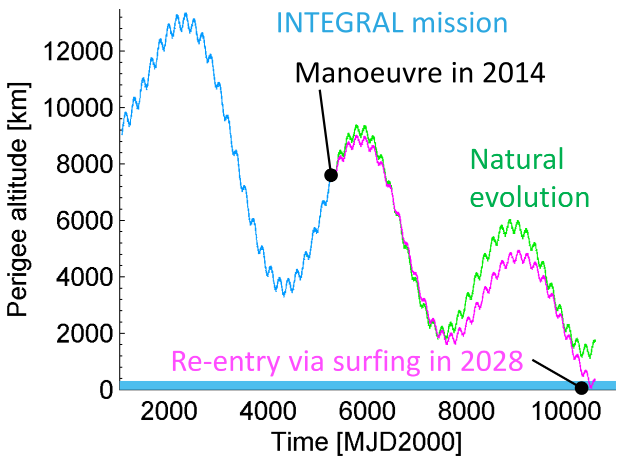
Manoeuvre performed on 08/08/2014

Re-entry in 2028

when we are already on a large elliptical orbit → try to give a single manoeuvre at the apogee in order to lower the perigee.

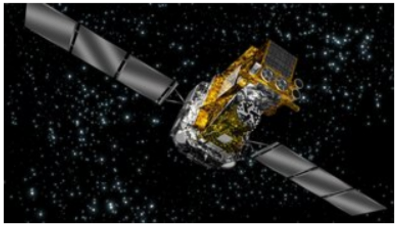
Re-entry disposal design for INTEGRAL mission

Luni-solar perturbation surfing made re-entry of INTEGRAL mission possible



INTEGRAL MANOEUVRES FOR THE FUTURE

From www.esa.int



Integral: gamma-ray observatory

23 January 2015 Since 2002, ESA's Integral spacecraft has been observing some of the most violent events in the Universe, including gamma-ray bursts and black holes. While it still has years of life ahead, its fuel will certainly run out one day.

Integral, one of ESA's longest-serving and most successful space observatories, has begun a series of four thruster burns carefully designed to balance its scientific life with a safe reentry in 2029.

That seems far off, but detailed planning and teamwork now will ensure that the satellite's eventual entry into the atmosphere will meet the Agency's guidelines for minimising space debris.

Making these disposal manoeuvres so early will also minimises fuel usage, allowing ESA to exploit the valuable satellite's lifetime to the fullest.

This is the first time that a spacecraft's orbit is being adjusted, after 12 years in space, to achieve a safe reentry 15 years in the future, while maximising valuable science return for the subsequent seven to eight years.

"Our four burns will use about half of the estimated 96 kg of fuel available," says Richard Southworth, spacecraft operations manager at ESA's Space Operations Centre, ESOC, in Darmstadt, Germany.

"This will influence how Integral's orbit evolves, so that even after we run out of propellant we will still have a safe reentry in February 2029 as a result of natural orbit decay.

"No further manoeuvres are required between now and then and Integral can continue to operate."

5.7 Solar Radiation Pressure (SRP) perturbation

How to model it?

Direct electromagnetic radiation from the sun exerts a pressure on spacecraft of the order of 10^{-6} N/m^2 at Earth orbit

Magnitude of the SRP acceleration is

cross Area wrt the sun

$$a_{SRP} = p_{SR@1AU} \frac{AU^2}{\left\|\mathbf{r}_{sc-Sun}\right\|^2} c_R \frac{A_{Sun}}{m}$$

reflectivity coefficient

$$\mathbf{a}_{SRP} = -a_{SRP} \frac{\mathbf{r}_{sc-Sun}}{\left\|\mathbf{r}_{sc-Sun}\right\|}$$

opposite from the spacecraft to the sun

where $p_{sr} = 4.5 \times 10^{-6}$ N/m^2 at $r_s = 1$ AU

c_R is a factor dependent upon the optical properties of the spacecraft surface

r_s is the spacecraft-to-Sun distance

A/m is the spacecraft area-to-mass ratio relative to the Sun

For LEO spacecraft, effect is small relative to other surface forces (i.e. drag)

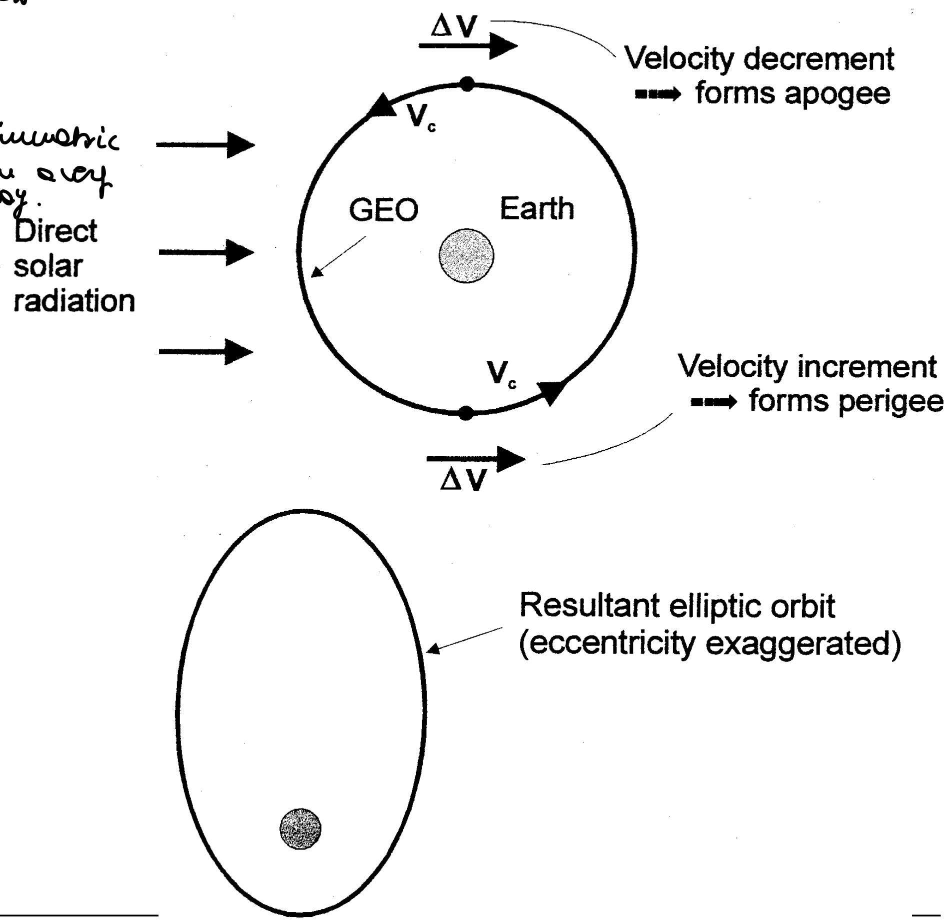
Solar Radiation Pressure (SRP) perturbation

↳ More relevant for higher orbit

It changes , Ω, ω, e, i

↳ Because it is not simmetric

↳ It change it in a very complex way.



For GEO spacecraft, effect can be considerable – particularly for spacecraft with large array surface areas

Principal effect is on GEO eccentricity

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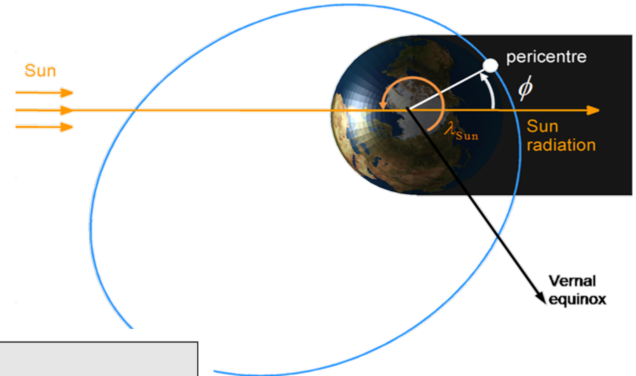
Solar radiation pressure and J_2 perturbation

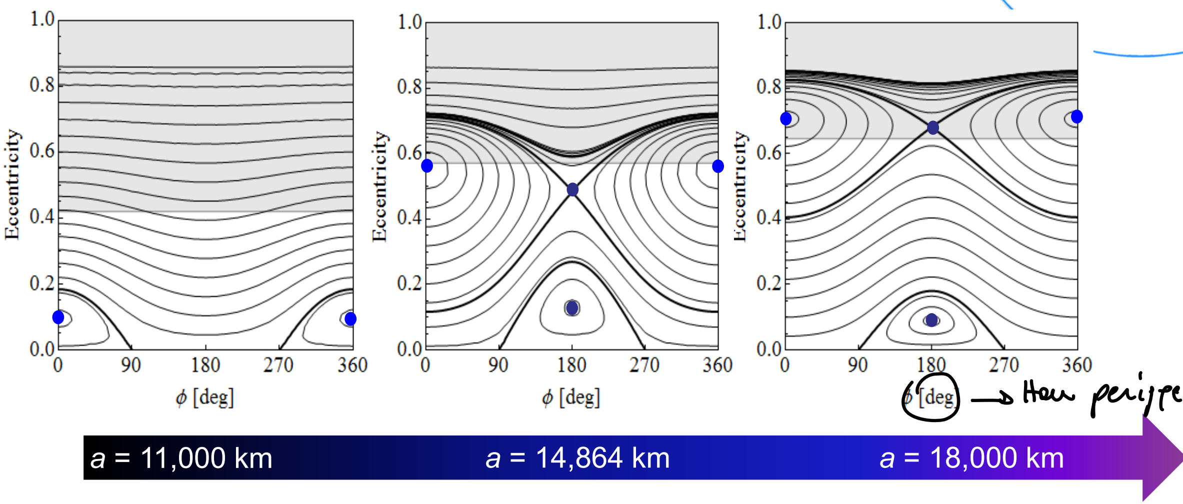
Example of a planar orbit: orbital elements:

- Semi-major axis, a
- Eccentricity, e
- Sun radiation/pericentre angle, ϕ

Evolution of the orbit in the phase space

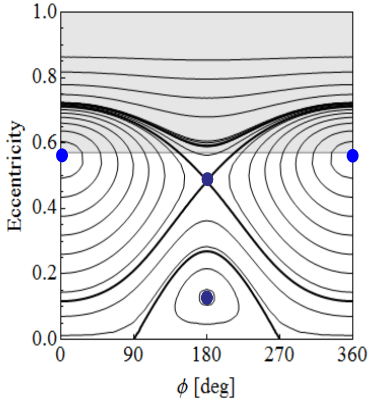


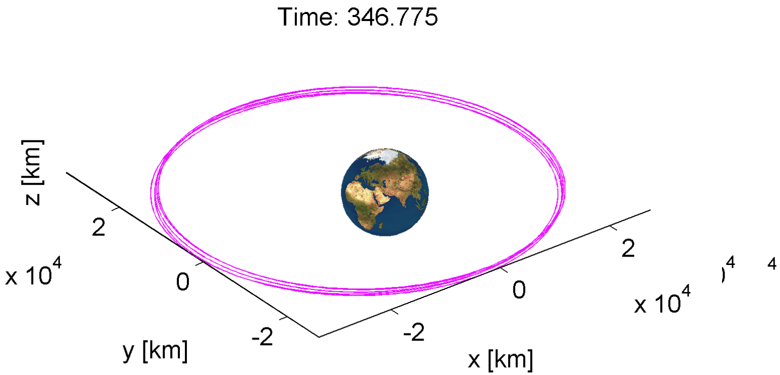
Equatorial plane ≡ Ecliptic plane

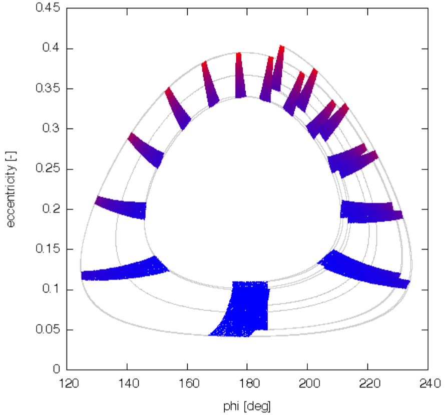


→ How perigee is oriented wrt Sun.

Solar radiation pressure and J_2 perturbation



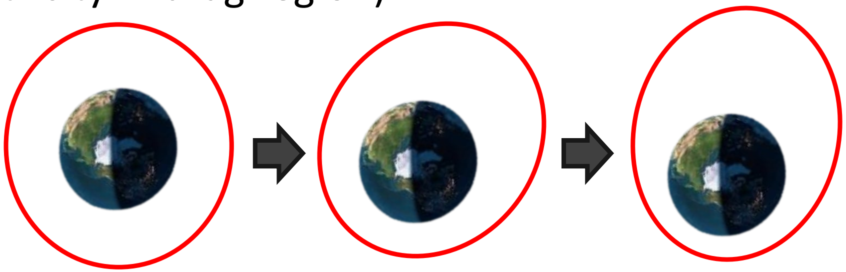




Example: Deorbiting with sail passive Sun stabilisation

Deploy area-increasing device to augment effect of solar radiation pressure

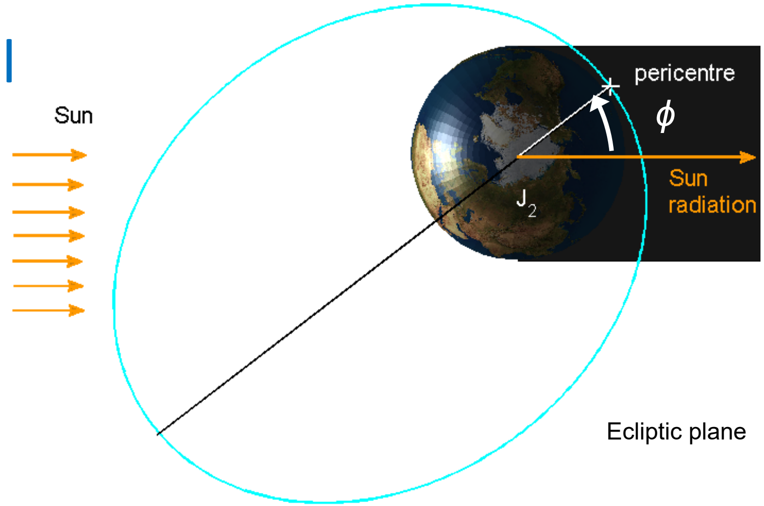
Phase 1: Passive eccentricity increase due to SRP from initial circular orbit (until reach critical eccentricity in drag region)

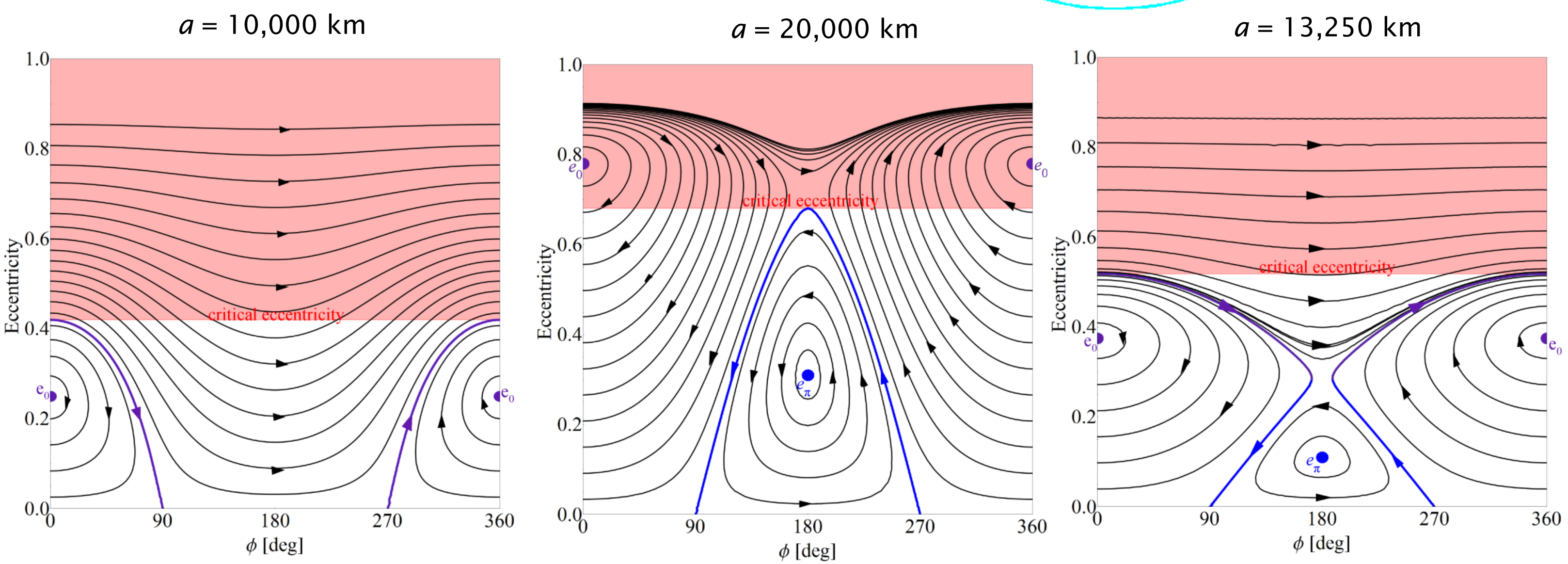


Phase 2: Deorbit augmented through drag

► Lücking, Colombo, McInnes, “A Passive Satellite Deorbiting Strategy for MEO using Solar Radiation Pressure and the J_2 Effect”, Acta Astronautica, 2012.

Example: Deorbiting with sail passive Sun stabilisation





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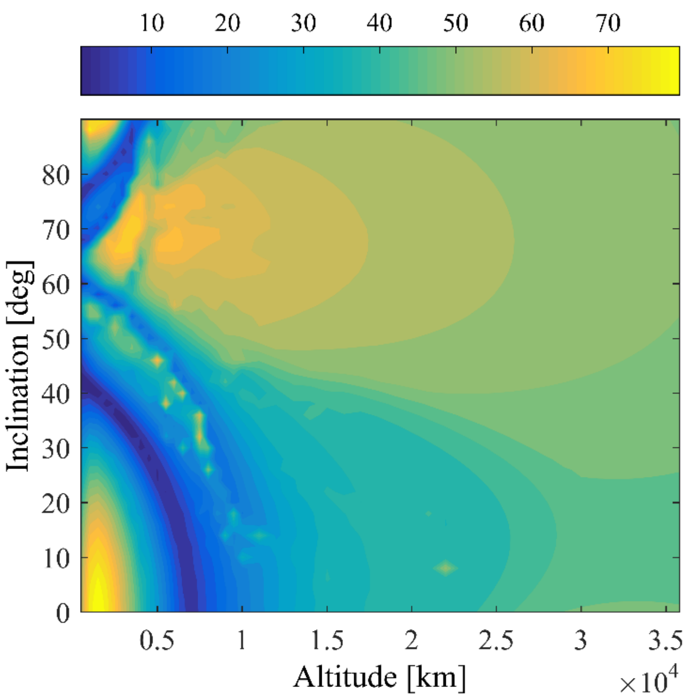
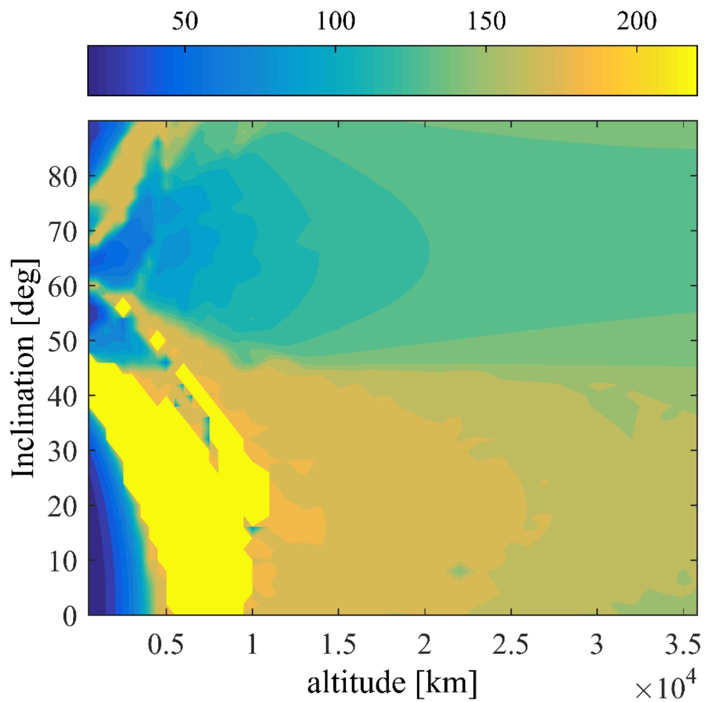
Example: Deorbiting with sail passive stabilisation

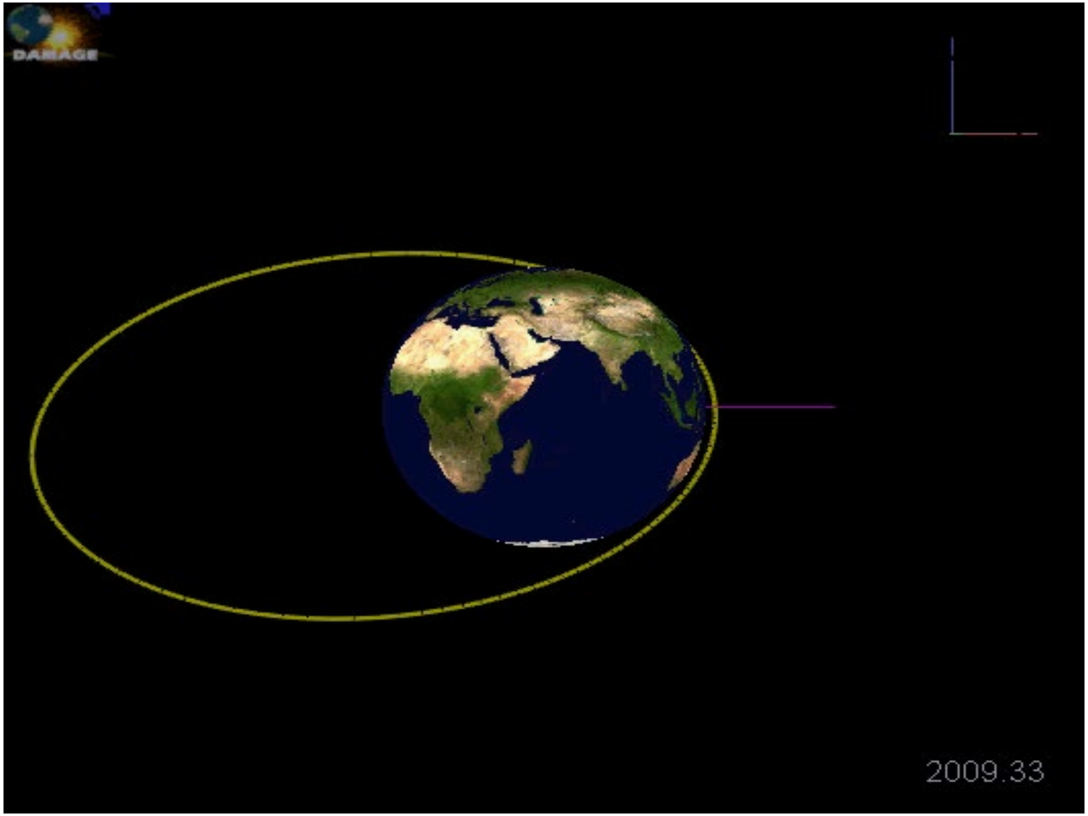
Area-to-mass times reflectivity coefficient [m^2/kg] to de-orbit from circular orbit and Ω_0 = 0 degrees



Time to de-orbit from circular orbit



All Perturbations…



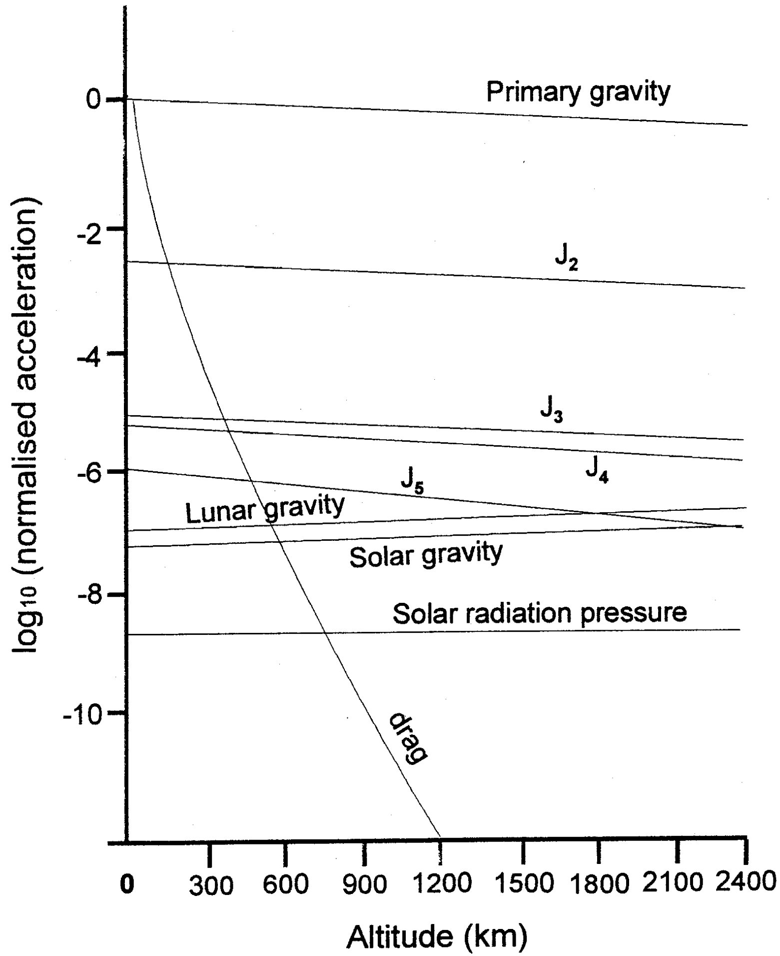
Southampton UNIVERSITY OF

Courtesy: University of Southampton

Summary

- Example to show relative (and approximate) magnitudes of perturbations for LEO (moderately high solar activity, with a drag A/m ratio of ~0.005 m^2/kg)

- Acceleration normalised to 1 g



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