ORBITAL MECHANICS

Today we will finish the theory behind the planet fly-by.

ENTRY CONDITION $\underline{V}_{s/c}^- = \underline{V}_{pl} + \underline{v}_\infty^-$

EXIT CONDITION $\underline{V}_{s/c}^+ = \underline{V}_{pl} + \underline{v}_\infty^+$

$$\Delta \underline{V}_{FLYBY} = \underline{V}_{s/c}^+ - \underline{V}_{s/c}^- = \underline{v}_\infty^+ - \underline{v}_\infty^- = \Delta \underline{v}_\infty$$

↳ planetocentric reference

! HELIO reference

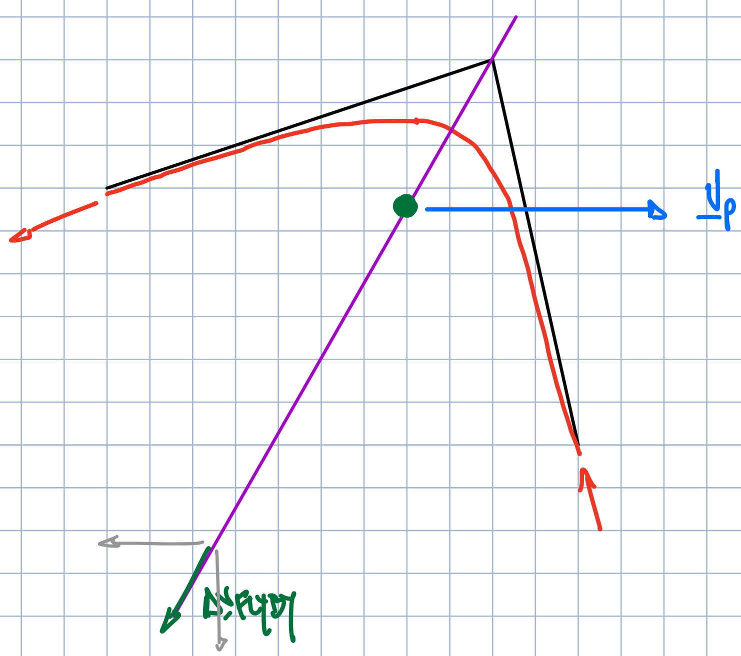
$\Delta \underline{V}_{FLYBY} \rightarrow$ Direction of the apse line of the hyperbola → that's why we need to design the hyperbola knowing the $\Delta \underline{v}$ I need to give.

$\Delta \underline{V}_{FLYBY}$ is directed along the apse line of the HYPERBOLA

ΔV_{FLYBY} depends on v_∞ and β

⇒ design the hyperbola so that I can use it as an "heliocentric free manoeuvre".

LEADING-SIDE HYPERBOLA

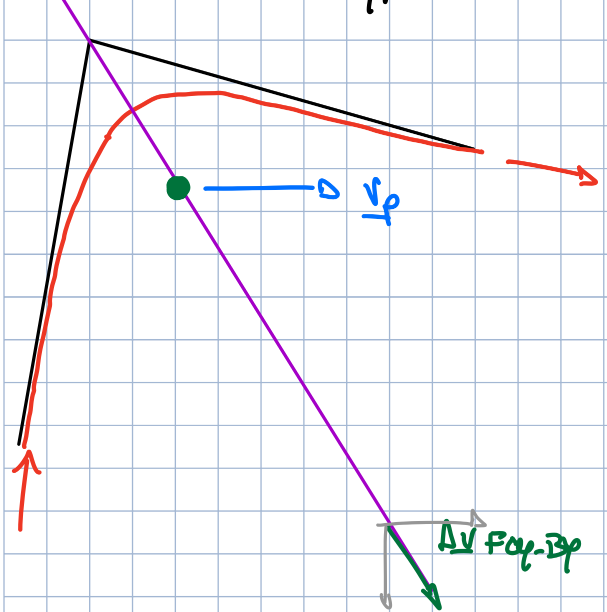


The planet is moving in such a way that the component of $\Delta \underline{V}_{fly\text{-}by}$ along the velocity of the planet will be negative.

ΔV_{FLYBY} component along the $\underline{V}_p$ direction < 0

⇒ Decrease the heliocentric velocity. $|\underline{v}_{s/c}^{+}| < |\underline{v}_{s/c}^{-}|$

TRAILING-SIDE HYPERBOLA



$\underline{\Delta v}_{FLY\text{-}BY}$ COMPONENT ALONG planet velocity $\underline{v}_p > 0$

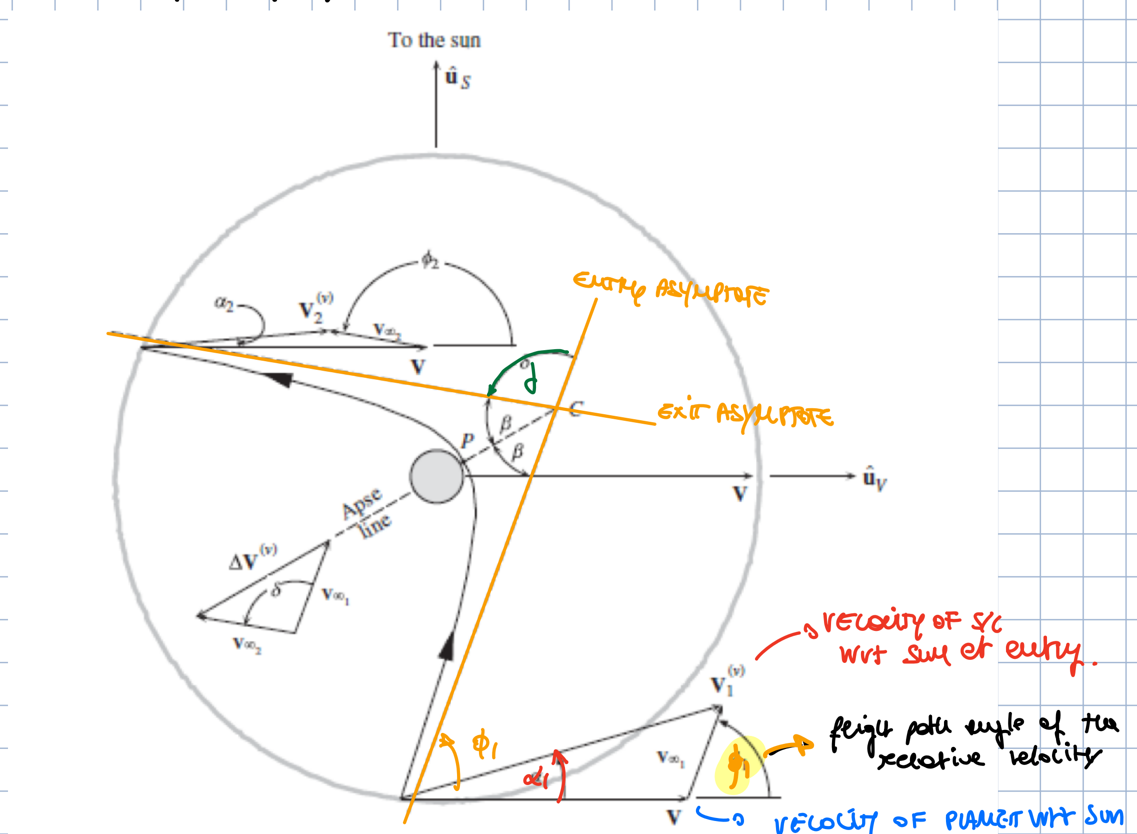
⇒ INCREASE IN THE HELIOCENTRIC

$|\underline{v}_{s/c}^{+}| > |\underline{v}_{s/c}^{-}|$

HINT: when you pass in front of a planet (leading-side) the planet sees you ⇒ planet "reduces" your velocity

when you pass behind the planet (trailing-side) the planet doesn't see you ⇒ you (s/c) "steal" heliocentric energy to the planet.

LEADING SIDE PLANETARY FLY-BY.



Let's define (not needed if we have a s/c → we have ephemerides)

$\hat{u}_V$: unit vector in the direction of planet velocity $= \hat{\theta}$ (circular orbit)

$\hat{u}_S$: unit vector in the direction to the Sun $= -\hat{r}$ (circular orbit)

$\hat{u}_S \perp \hat{u}_V$ → as we assume planet circular orbit.

ENTRY OF SOI

notation s/c
anti s/c

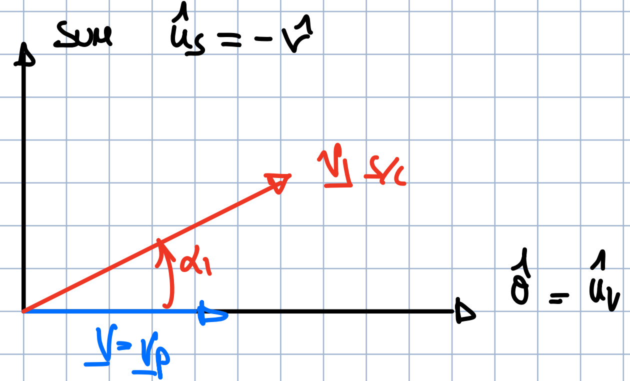
$$\underline{V}_{s/c} = \underline{V}_1 = [V_1^{s/c}]_V \, \hat{u}_V + [V_1^{s/c}]_S \, \hat{u}_S$$

$$[V_1^{s/c}]_V = V_1^{s/c} \cos\alpha_1$$
$$[V_1^{s/c}]_S = V_1^{s/c} \sin\alpha_1$$

α_1 angle between $\underline{V}_1^{s/c}$ and $\underline{V}$ (planet velocity wrt Sun)
= flight path angle of $\underline{V}_1^{s/c}$ when it enter the SOI.

$$[V_1^{s/c}]_V = V_{\theta_1}$$

$$[V_1^{s/c}]_S = -V_{r_1}$$
↳ as $\hat{u}_S = -\hat{r}$



So we can use V_r and V_θ

Recall
$$V_\theta = \frac{\mu}{h}(1 + e\cos\theta) \qquad \text{eq } (1.34)$$

$$V_r = \frac{\mu}{h}(e\sin\theta) \qquad \text{eq } (1.33)$$

$$[V_1^{s/c}]_V = V_{\theta_1} = \frac{\mu_{SUN}}{h_1}(1 + e_1\cos\theta_1)$$

$$-[V_1^{s/c}]_S = V_{r_1} = \frac{\mu_{SUN}}{h_1}(e_1\sin\theta_1)$$

We are characterizing the velocity of the space craft in the heliocentric point of view.

$\left.\begin{array}{l} e_1 \\ h_1 \\ \theta_1 \end{array}\right\}$ approach trajectory to planet heliocentric leg (μ_{sun})

$$\underline{v}_p = v_p \, \hat{u}_v$$

$$v_p = \sqrt{\frac{\mu_{sun}}{r_p}}$$ hp: planet on circular orbit

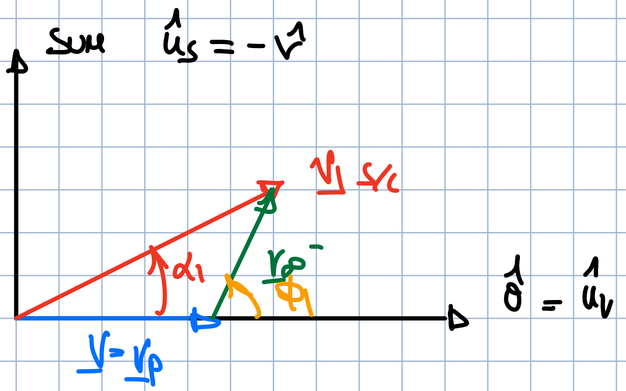
② characterize the planetocentric entry location.

$$\underline{v}_\infty^- = \underline{V}_1^{s/c} - \underline{V}_p$$

$$\underline{v}_\infty^- = [v_\infty^-]_v \, \hat{u}_v + [v_\infty^-]_s \, \hat{u}_s$$

$$[v_\infty^-]_v = V_1^{s/c} \cos\alpha - V_p$$ → Because we are considering a leading-side fly by.

$$[v_\infty^+]_s = V_1^{s/c} \sin\alpha$$



$$\boxed{v_\infty^- = \sqrt{\underline{v}_\infty^- \cdot \underline{v}_\infty^-} = \sqrt{{V_1^{s/c}}^2 + V_p^2 - 2 V_1^{s/c} V_p \cos\alpha_1}} \qquad (4.4)$$

by solving the triangle of velocities

We can compute v_∞^- given $\underline{V}_{s/c}^1$ (helio s/c entry velocity) and planet helio velocity (v_p) and the flight path angle α_1

We can compute everything about the hyperbola.

CHARACTERISE THE HYPERBOLA

eq (4.21) $\quad a = r_p \sqrt{v_\infty^2 + \frac{2\mu}{r_p}}$

eq (4.21) $\quad e = 1 + \frac{r_p v_\infty^2}{\mu}$

μ of planet!!
r_p, e, v_∞ on hyperbola
so planetocentric ref.

ϕ_1 angle between $\underline{v_\infty}$ and $\underline{V_p}$

$$\phi_1 = tg^{-1}\left(\frac{[v_\infty^-]_S}{[v_\infty^-]_V}\right) = tg^{-1}\left(\frac{V_1^{s/c}\sin\alpha_1}{V_1^{s/c}\cos\alpha_1 - V_p}\right)$$

$$\boxed{\phi_1 = tg^{-1}\frac{V_1^{s/c}\sin\alpha_1}{V_1^{s/c}\cos\alpha_1 - V_p}} \qquad (4.42)$$

③ CHARACTERISE THE HYPERBOLA AT EXIT

$$\boxed{\phi_2 = \phi_1 + \delta}$$

↳ turn angle.

Recall $\delta = 2\sin^{-1}\frac{1}{e}$ eq (4.

We have characterise the velocity at SOI.

$$v_\infty^+ = v_\infty \cos\phi_2 \, \hat{u}_V + v_\infty \sin\phi_2 \, \hat{u}_S$$

$v_\infty = v_\infty^+ = v_\infty^-$ → In modulus.

④ CHARACTERISE THE HELIOCENTRIC EXIT CONDITIONS

$$\underline{V_2}^{s/c} = \underline{V_{s/c}}^+ = \underline{V_p} + \underline{v_\infty}^+$$

↳ cartis

$$\underline{V}_2^{s/c} = \underline{V}_{s/c}^{+} = [V_2^{s/c}]_V \, \hat{u}_V + [V_2^{s/c}]_S \, \hat{u}_S$$

We can compute $\underline{V}_2^{s/c}$ given v_∞, V_p, $\phi_2 = \phi_1 + \delta$

$$\boxed{\begin{aligned} [V_2^{s/c}]_V &= V_p + v_\infty \cos\phi_2 = V_{\theta_2} \\ [V_2^{s/c}]_S &= v_\infty \sin\phi_2 = -V_{r_2} \end{aligned}} \qquad (4.44)$$

We can obtain the parameters of the heliocentric exit leg.

$h_2 = R_p V_{\theta_2}$ → h_1 of helio leg after fly-by

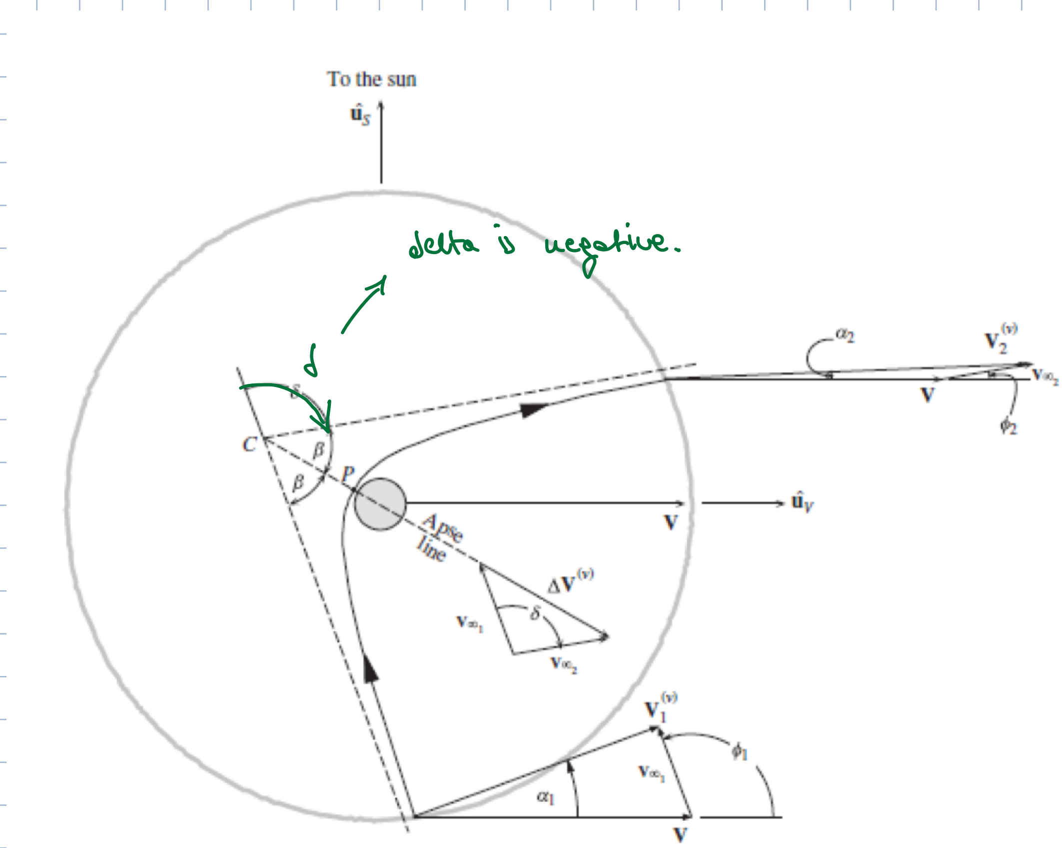
$$\begin{cases} R_p = \dfrac{h_2^2}{\mu_{sun}} \dfrac{1}{1+e_2\cos\theta_2} \\[2ex] V_{r_2} = \dfrac{\mu_{sun}}{h_2} e_2 \sin\theta_2 \end{cases}$$

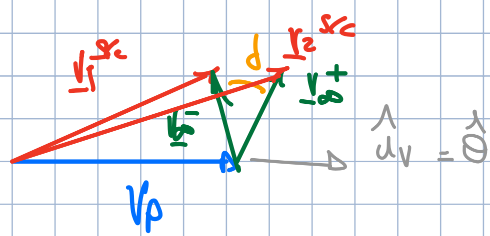
a, h_2, & heliocentric orbit

R_p = planet radius wrt Sun

NOTE fly-by is considered as impulsive manoeuvre (as seen from Sun) so R_p is constant during fly-by.

The same procedure is valid for trailing edge fly-by.





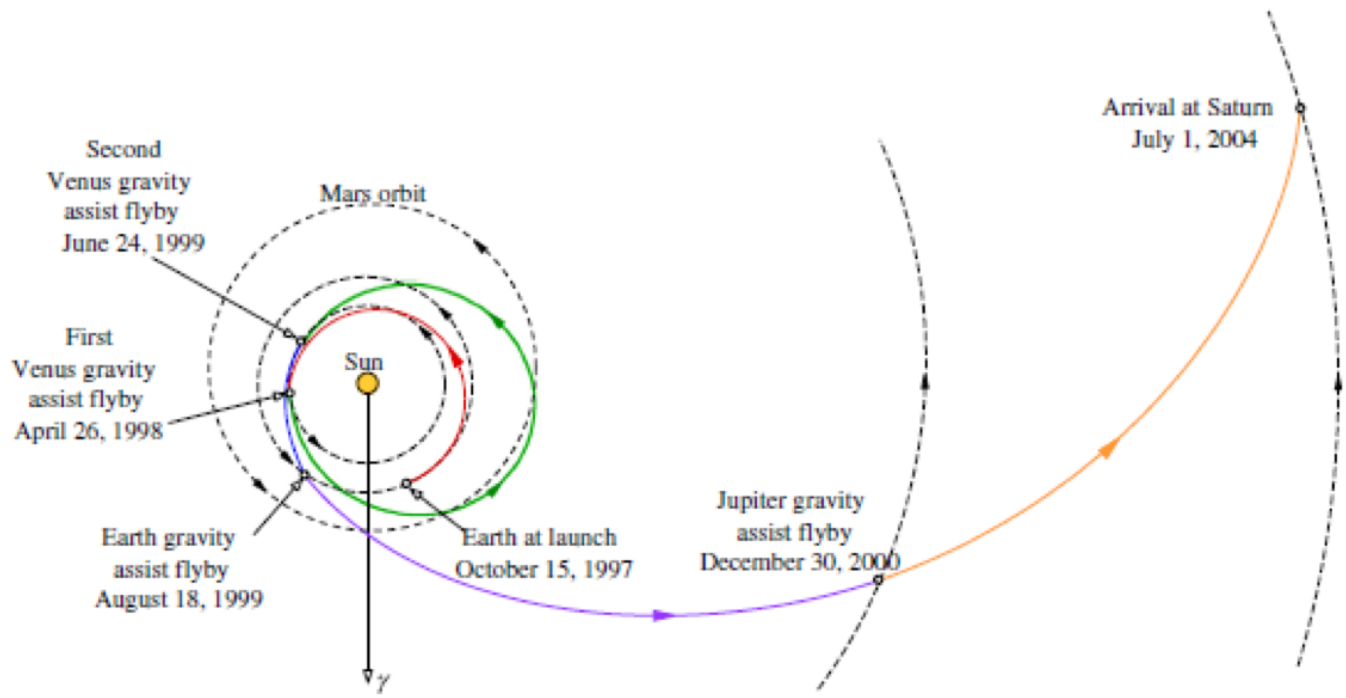


FIGURE 8.24

Cassini's 7-year mission to saturn.

The problem of the trajectory interplanetary design is that they can get really complicated → really fast.

INPUT/TOOLS

- planet ephemerides
- Lambert solver
- gravity/fly-by model

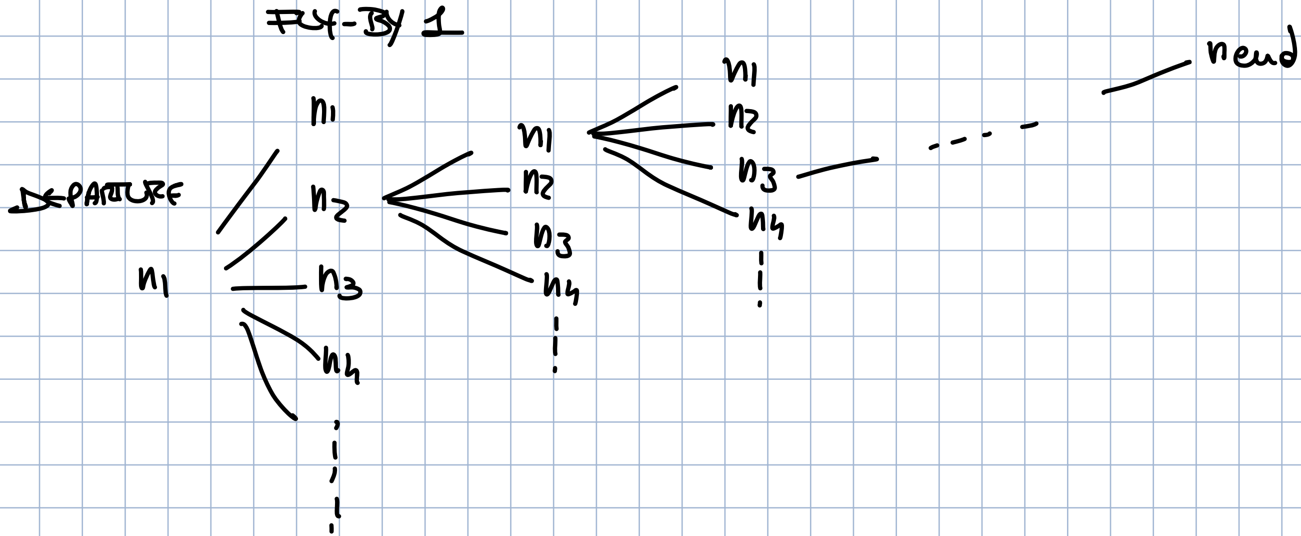
We do not know how many fly by we'll do when and with wich planet.

First problem

SEQUENCE SELECTION PROBLEM : sequence of fly-by.
COMBINATORIAL PROBLEM

n_1 - - - - $n_{planet_{fly\text{-}by}}$ - - - - - n_{end}

interger programming problem



So I need to find a sequence → Problem

Given a sequence.

n departure — Lambert leg (semplification) — n fly-by — LAMBER — n FLY-BY 2

t departure — t fly-by — t FLY-BY 2

ΔV departure — TOF — LAMBERT — DSM

[0, 1] α — usually Deep space manouevre DSM followed by → Lambert leg

αTOF

(done by propagation)

TOF = $t_{FLY\text{-}BY} - t_{departure}$

X : { $t_{departure}$ α_1, ΔV^1_{DSM}(3) $n_{FLY\text{-}BY}$ $t_{FLY\text{-}BY}$ α_2, ΔV^2_{DSM}(3) $n_{FLY\text{-}BY2}$ $t_{FLY\text{-}BY2}$

→ optimization vector

Every transfer we add we add dofs at the optimization vector

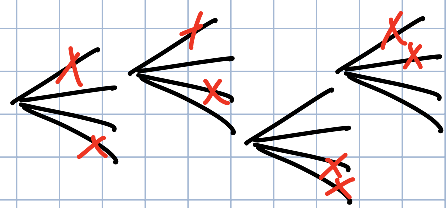
Find $x = \{ t_{DEP}, \alpha_1, \Delta V^{1}_{DSM}, t_{FB1}, \alpha_2, \Delta V^{2}_{DSM}, \dots \}$

$$\min_{x} \sum_{TOT} \Delta V$$

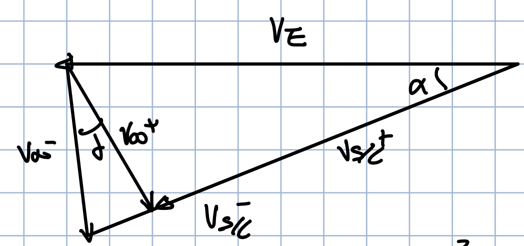
INTEGER – CONTINUUM OPTIMIZATION PB

there are a lot of algorithm →
greedy they do not consider the bad branches

– branch and bound



→ Check at website: global trajectory optimization competition.



$V_E = 29.$

$$V_{a}^{+2} = V_E^2 + V_{s/c}^{+2} - 2V_E V_{sc}^{+} \cos\alpha$$

$$V_{s/c}^{+2} + (\underbrace{-2V_E\cos\alpha}_{A})\, V_{s/c}^{+} + (\underbrace{V_E^2 - V_a^{+2}}_{B}) = 0$$

$$V_{s/c}^{+} = \frac{-A \pm \sqrt{A^2-4B}}{2}$$

29.752

29.709