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Spacecraft Attitude Dynamics

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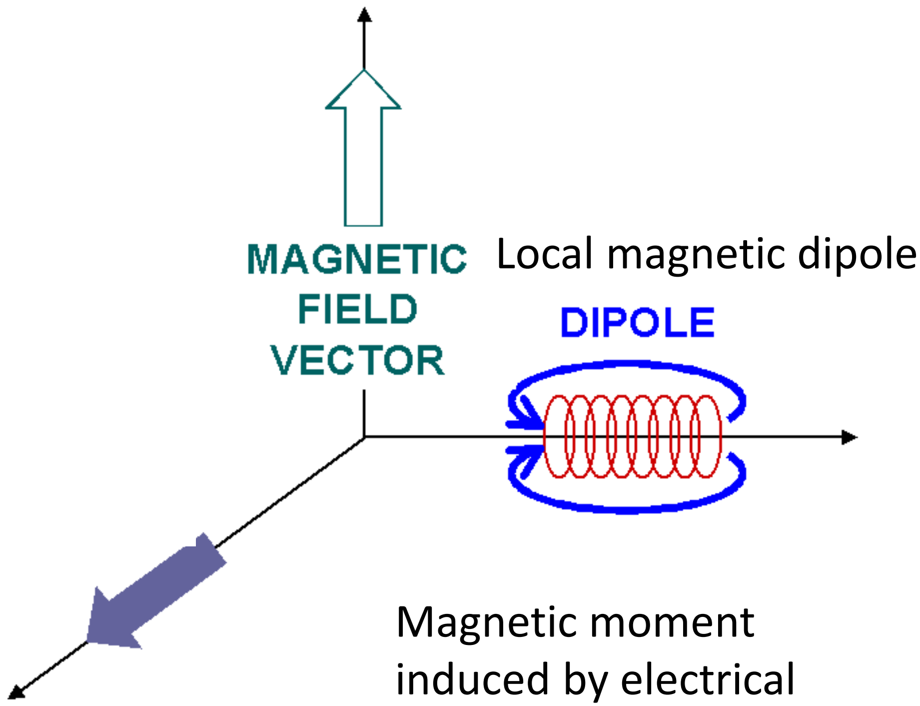
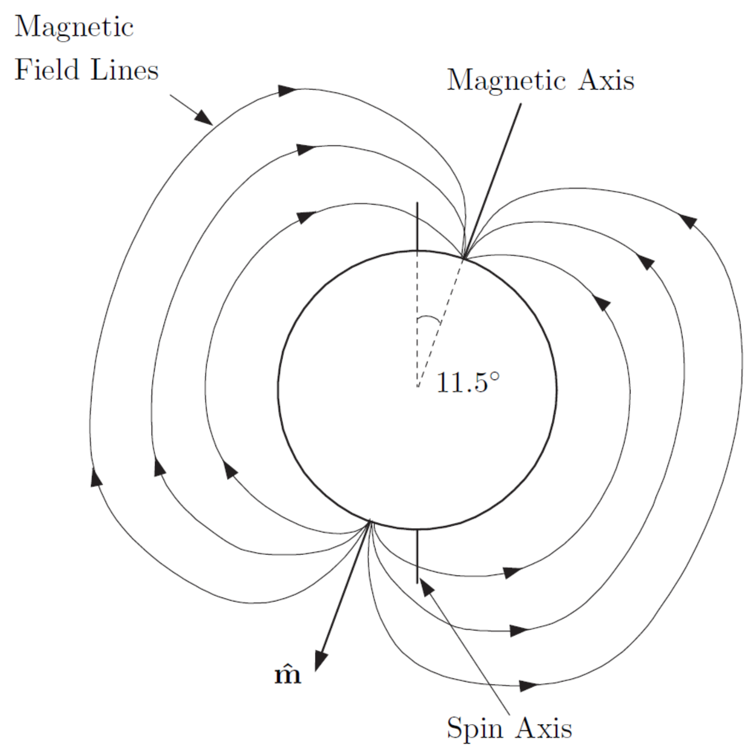
Magnetic Field Modelling

Magnetic field disturbance torque

$$\underline{M} = \underline{m} \wedge \underline{B}$$



Magnetic moment induced by electrical current



Magnetic B-field

Potential in terms of Schmidt quasi-normalized associated Legendre Polynomial

$$V(r,\theta,\varphi) = R\sum_{n=1}^{k}\left(\frac{R}{r}\right)^{n+1}\sum_{m=0}^{n}\left(g_n^m\cos(m\varphi) + h_n^m\sin(m\varphi)\right)P_n^m(\theta)$$

r, θ and φ are the spherical coordinates of the position of the satellite from the center of the Earth, co-latitude and East longitude from Greenwich.

IAGA – International Association of Geomagnetism and Aeronomy

IGRF – International Geomagnetic Reference Field

$$B_r = \frac{-\partial V}{\partial r}$$

B_r -> radial component, positive outward

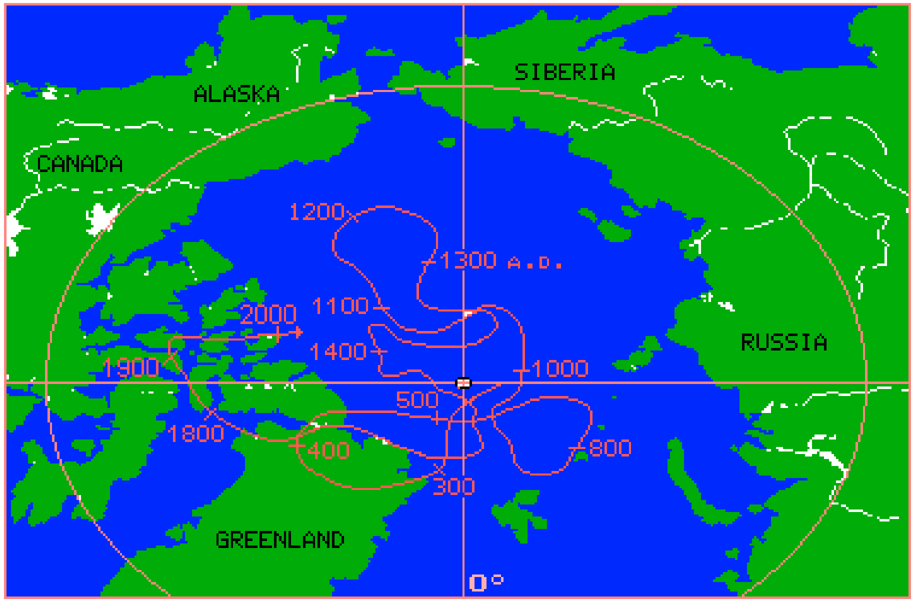
$$B_\theta = \frac{-1}{r}\frac{\partial V}{\partial \theta}$$

B_θ -> coelevation component, positive if directed towards south

$$B_\varphi = \frac{-1}{\mathrm{rsin\theta}}\frac{\partial V}{\partial \varphi}$$

B_φ -> azimuth component, positive towards east

The wandering magnetic North



wandering path of magnetic north
rotational north pole

Gaussian coefficients (nT)

		IGRF 1995		IGRF 2000	
n	m	g_n^m	h_n^m	g_n^m	h_n^m
1	0	-29682	-	-29615	-
1	1	-1789	5318	-1728	5186
2	0	-2197	-	-2267	-
2	1	3074	-2356	3072	-2478
2	2	1685	-425	1672	-458
3	0	1329	-	1341	-
3	1	-2268	-263	-2290	-227
3	2	1249	302	1253	296
3	3	769	-406	715	-492
4	0	941	-	935	-
4	1	782	262	787	272
4	2	291	-232	251	-232
4	3	-421	98	-405	119
4	4	116	-301	110	-304

		IGRF 1995		IGRF 2000	
n	m	g_n^m	h_n^m	g_n^m	h_n^m
5	0	-210	-	-217	-
5	1	352	44	351	44
5	2	237	157	222	172
5	3	-122	-152	-131	-134
5	4	-167	-64	-169	-40
5	5	-26	99	-12	107
6	0	66	-	72	-
6	1	64	-16	68	-17
6	2	65	77	74	64
6	3	-172	67	-161	65
6	4	2	-57	-5	-61
6	5	17	4	17	1
6	6	-94	28	-91	44

		IGRF 1995		IGRF 2000	
n	m	g_n^m	h_n^m	g_n^m	h_n^m
7	0	78	-	79	-
7	1	-67	-77	-74	-65
7	2	1	-25	0	-24
7	3	29	3	33	6
7	4	4	22	9	24
7	5	8	16	7	15
7	6	10	-23	8	-25
7	7	-2	-3	-2	-6
8	0	24	-	25	-
8	1	4	12	6	12
8	2	-1	-20	-9	-22
8	3	-9	7	-8	8
8	4	-14	-21	-17	-21
8	5	4	12	9	15

		IGRF 1995		IGRF 2000	
n	m	g_n^m	h_n^m	g_n^m	h_n^m
8	6	5	10	7	9
8	7	0	-17	-8	-16
8	8	-7	-10	-7	-3
9	0	4	-	5	-
9	1	9	-19	9	-20
9	2	1	15	3	13
9	3	-12	11	-8	12
9	4	9	-7	6	-6
9	5	-4	-7	-9	-8
9	6	-2	9	-2	9
9	7	7	7	9	4
9	8	0	-8	-4	-8
9	9	-6	1	-8	5

Magnetic field

$$\mathbf{B} = -\nabla V$$

Earth rotating-fixed frame

$$b_r = \sum_{n=1}^{k} \left(\frac{R}{r}\right)^{n+2} (n+1) \sum_{m=0}^{n} \left(g_n^m \cos m\,\phi + h_n^m \sin m\,\phi\right) P_n^m(\theta)$$

$$b_\theta = -\sum_{n=1}^{k} \left(\frac{R}{r}\right)^{n+2} \sum_{m=0}^{n} \left(g_n^m \cos m\,\phi + h_n^m \sin m\,\phi\right) \frac{\partial P_n^m(\theta)}{\partial \theta}$$

$$b_\phi = -\frac{1}{\sin\theta} \sum_{n=1}^{k} \left(\frac{R}{r}\right)^{n+2} \sum_{m=0}^{n} \left(-g_n^m \sin m\,\phi + h_n^m \cos m\,\phi\right) P_n^m(\theta)$$

Schmidt quasi-normalized associated Legendre Polynomial

$$P_n^m = \left[\frac{2(n-m)!}{(n+m)!}\right]^{1/2} P_{n,m}$$

$$P_{n,m}(v) = (1-v^2)^{1/2m} \frac{d^m}{dv^m}(P_n(v))$$

$$P_n(v) = \frac{1}{2^n n!} \left(\frac{d}{dv}\right)^n (v^2-1)^n$$

$$P_0(v) = 1$$
$$P_1(v) = v$$
$$P_2(v) = (3v^2-1)/2$$
$$P_3(v) = (5v^3-3v)/2$$

Davis, J., “Mathematical Modeling of Earth’s Magnetic Field” Virginia Tech, Technical Note, 2004.

Magnetic field $\mathbf{B} = -\nabla V$

Alternative normalization due to Gauss

$$B_r = \frac{-\partial V}{\partial r} = \sum_{n=1}^{k} \left(\frac{R}{r}\right)^{n+2} (n+1) \sum_{m=0}^{n} \left(g^{\mathrm{n,m}} \cos(\mathrm{m}\varphi) + h^{\mathrm{n,m}} \sin(\mathrm{m}\varphi)\right) P^{\mathrm{n,m}}(\theta)$$

$$B_\theta = \frac{-1}{r}\frac{\partial V}{\partial \theta} = -\sum_{n=1}^{k} \left(\frac{R}{r}\right)^{n+2} \sum_{m=0}^{n} \left(g^{\mathrm{n,m}} \cos(\mathrm{m}\varphi) + h^{\mathrm{n,m}} \sin(\mathrm{m}\varphi)\right) \frac{\partial P^{\mathrm{n,m}}(\theta)}{\partial \theta}$$

$$B_\varphi = \frac{-1}{\mathrm{r}\sin\theta}\frac{\partial V}{\partial \varphi} = \frac{-1}{\sin\theta} \sum_{n=1}^{k} \left(\frac{R}{r}\right)^{n+2} \sum_{m=0}^{n} m\left(-g^{\mathrm{n,m}} \sin(\mathrm{m}\varphi) + h^{\mathrm{n,m}} \cos(\mathrm{m}\varphi)\right) P^{\mathrm{n,m}}(\theta)$$

$$g^{\mathrm{n,m}} = S_{\mathrm{n,m}} g_n^m$$
$$h^{\mathrm{n,m}} = S_{\mathrm{n,m}} h_n^m$$

Magnetic field $\mathbf{B} = -\nabla V$

Alternative normalization due to Gauss

$$S_{0,0} = 1$$

$$S_{n,0} = S_{n-1,0}\frac{(2n-1)}{n} \quad \text{for } n \geq 1$$

$$S_{n,m} = S_{n,m-1}\left[\frac{(\delta_m^1 + 1)(n-m+1)}{(n+m)}\right]^{\frac{1}{2}} \quad \text{for } m \geq 1$$

$$P^{0,0} = 1$$

$$P^{n,n} = \sin\theta \, P^{n-1,n-1}$$

$$P^{n,m} = \cos\theta \, P^{n-1,m} - K^{n,m} P^{n-2,m}$$

$$K^{n,m} = 0 \quad \text{for } n = 1$$

$$K^{n,m} = \frac{(n-1)^2 - m^2}{(2n-1)(2n-3)} \quad \text{for } n > 1$$

Magnetic field $\mathbf{B} = -\nabla V$

Earth rotating fixed frame

$$b_r = \sum_{n=1}^{k} \left(\frac{R}{r}\right)^{n+2} (n+1) \sum_{m=0}^{n} (g_n^m \cos m\,\phi + h_n^m \sin m\,\phi)\, P_n^m(\theta)$$

$$b_\theta = -\sum_{n=1}^{k} \left(\frac{R}{r}\right)^{n+2} \sum_{m=0}^{n} (g_n^m \cos m\,\phi + h_n^m \sin m\,\phi) \frac{\partial P_n^m(\theta)}{\partial \theta}$$

$$b_\phi = -\frac{1}{\sin\theta} \sum_{n=1}^{k} \left(\frac{R}{r}\right)^{n+2} \sum_{m=0}^{n} (-g_n^m \sin m\,\phi + h_n^m \cos m\,\phi)\, P_n^m(\theta)$$

Inertial Frame

$$b_1 = (b_r \cos(\delta) + b_\theta \sin(\delta)) \cos(\alpha) - b_\varphi \sin(\alpha)$$
$$b_2 = (b_r \cos(\delta) + b_\theta \sin(\delta)) \sin(\alpha) + b_\varphi \cos(\alpha)$$
$$b_3 = (b_r \sin(\delta) - b_\theta \cos(\delta))$$

$$\delta = \frac{\pi}{2} - \theta$$
$$\alpha = \phi + \alpha_G$$

Body Frame

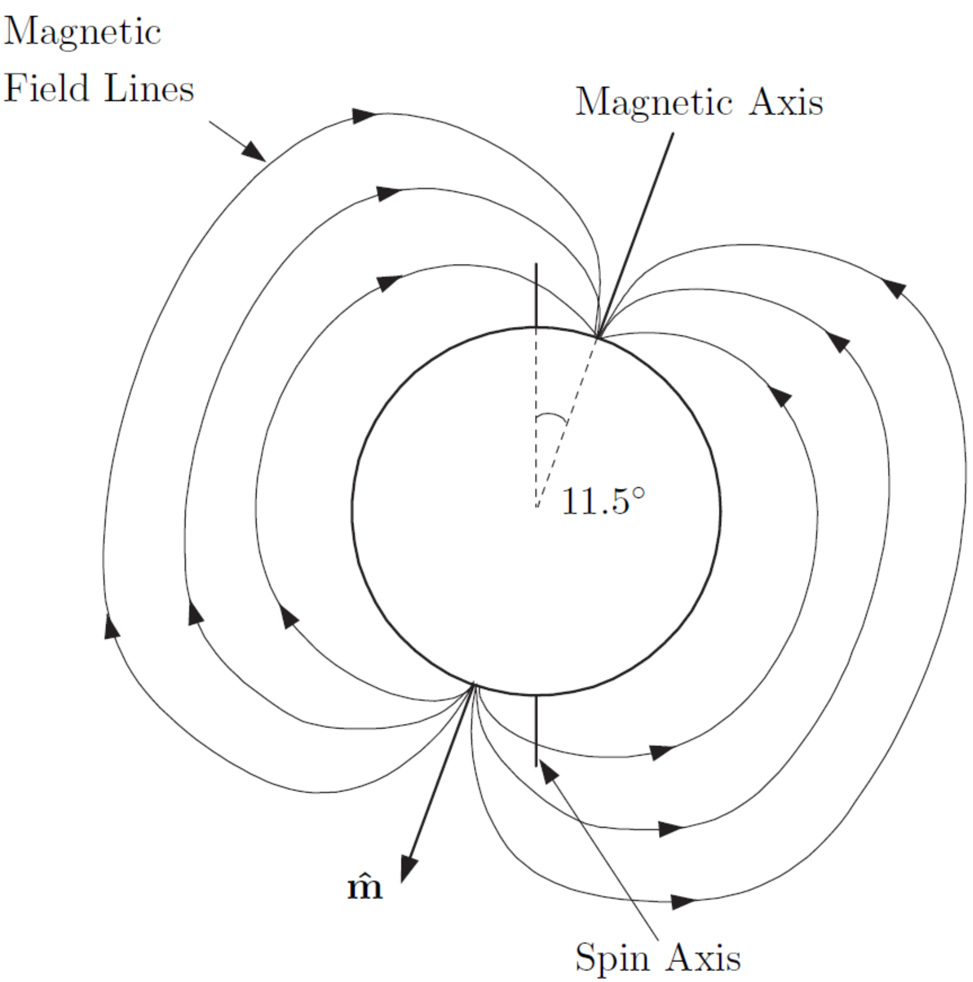
$$\underset{\sim}{b}_B = A_{B/E}\, \underset{\sim}{b}_E$$

Simple dipole model of the Earth's Magnetic Field *(n=1)*

$$\underline{b}_N = \frac{R^3 H_0}{r^3}\left[3(\underline{\hat{m}} \cdot \underline{\hat{r}})\underline{\hat{r}} - \underline{\hat{m}}\right]$$

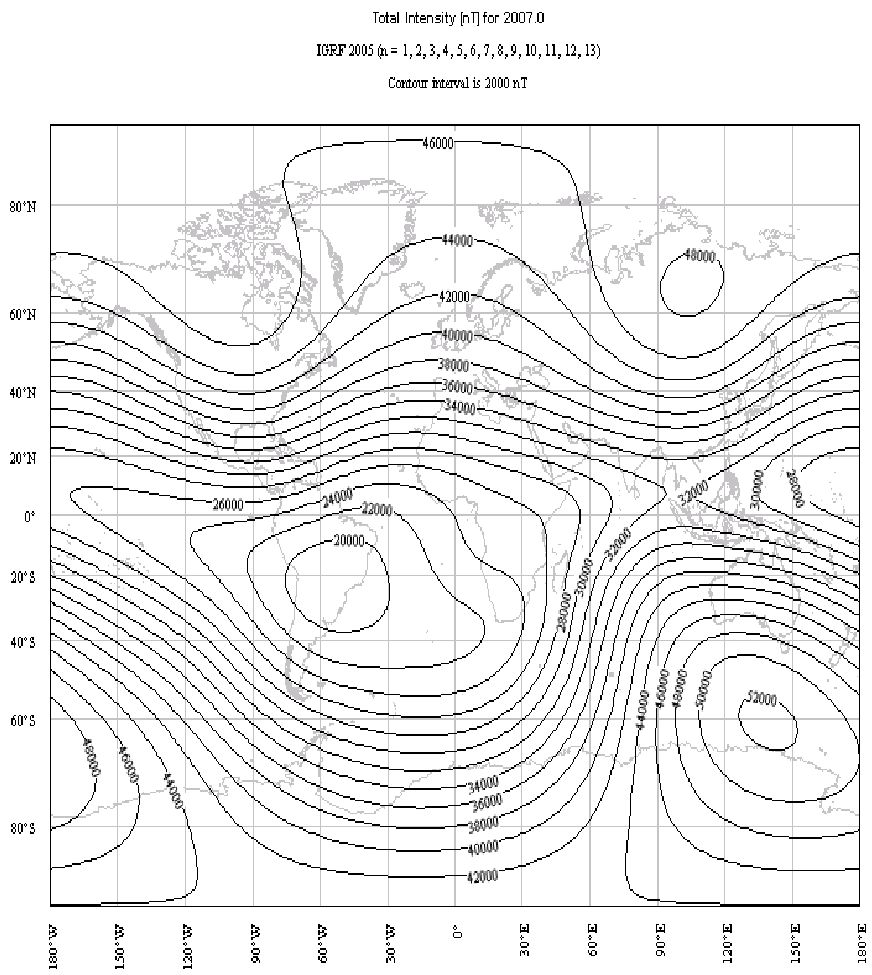
$$H_0 = \left((g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2\right)^{1/2}$$

$$\hat{m} = -\begin{bmatrix} \sin 11.5^\circ \cos \omega_\oplus t \\ \sin 11.5^\circ \sin \omega_\oplus t \\ \cos 11.5^\circ \end{bmatrix}$$

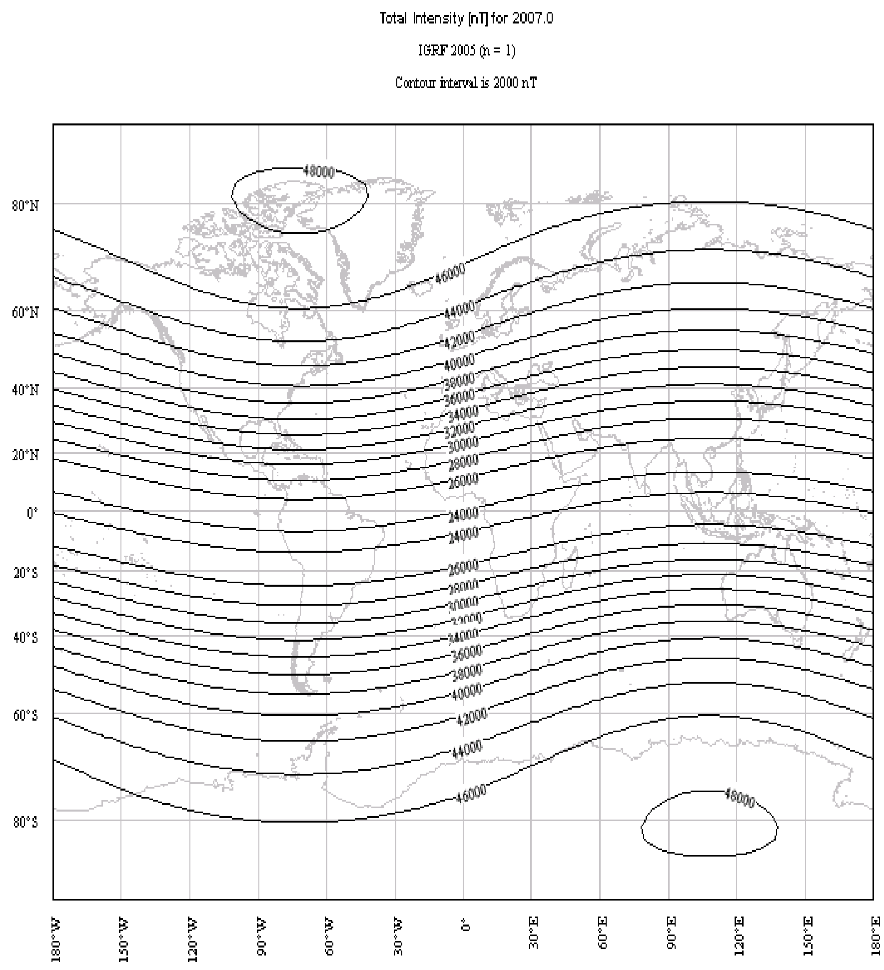


Magnetic field Model at 500km altitude

Order 13

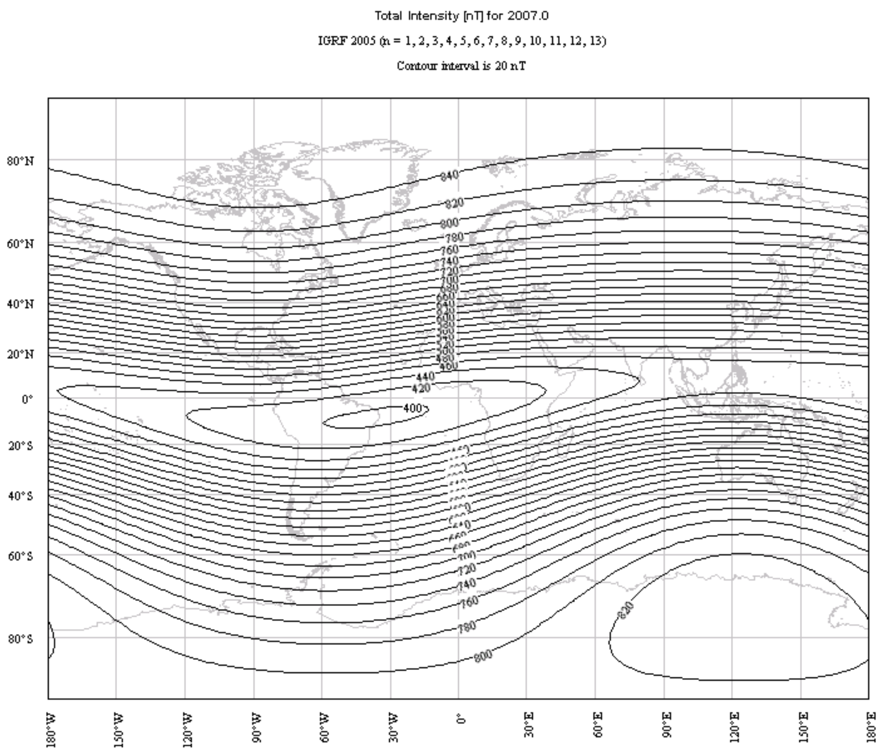


Dipole model

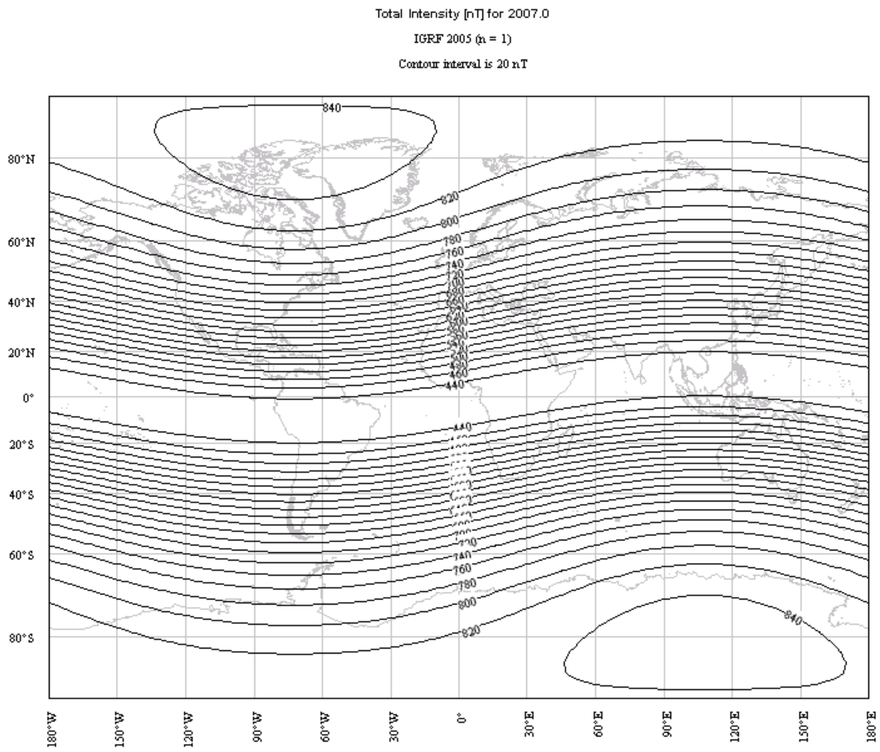


Magnetic field Model at 10000km altitude

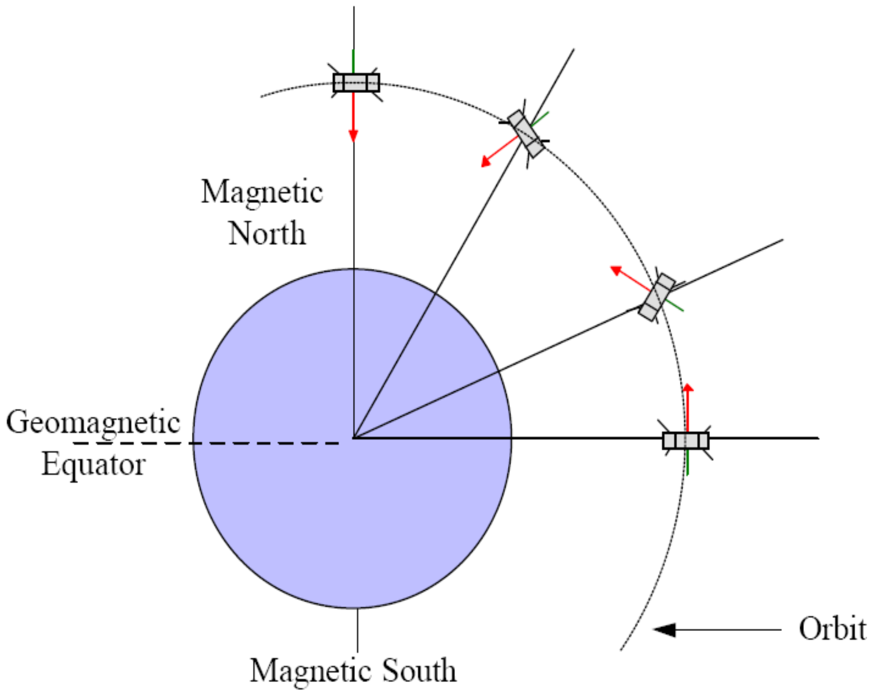
Order 13

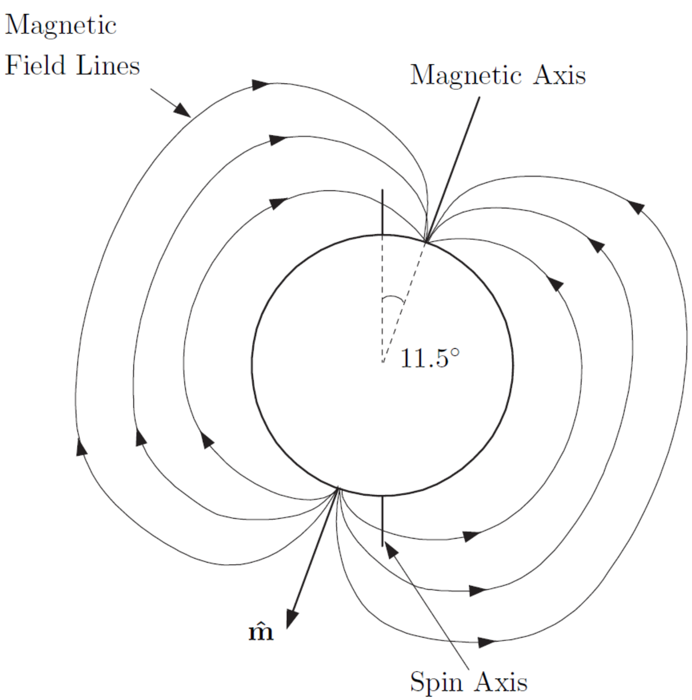


Dipole Model



Permanent magnet for passive pointing





Magnetic field disturbance torque

$\underline{M} = \underline{m} \wedge \underline{B}$ $\underline{m} = [0.1 \quad 0.1 \quad 0.1]^T \, Am^2$ worst case scenario

NASA SP-8018

Table I.–*Criteria for Magnetic Properties Control*

	Class I	Class II	Class III
Design	Formal specification on magnetic properties control; approved materials and parts lists; cancellation of moments by preferred mounting arrangements and control of current loops.	Advisory specifications and guidelines for material and parts selection. Avoidance of "soft" magnetic materials or current loops and awareness of good design practices.	Nominal control over current loops; guidelines for avoidance of "soft" magnetic materials.
Quality control	Complete magnetic inspection of parts and testing of subassemblies.	Inspection or test of suspect parts.	Test of subassemblies that are potentially major sources of dipole moment.
Test and compensation	Deperming either at subassembly or spacecraft level; test of final spacecraft assembly and compensation if required.	Deperming and compensation frequently used.	Test and compensation optional.

Note.–Class I–Magnetic torques dominant when compared with other torques.
II–Magnetic torques comparable to other torques.
III–Magnetic torques insignificant when compared with other torques.

Table IV.–*Factors for Estimating Spacecraft Dipole Moment (M).*

Category of magnetic properties control (see table I)	Estimate of dipole moment per unit mass for nonspinning spacecraft		Estimate of dipole moment per unit mass for spinning spacecraft	
	$A\text{-}m^2/kg$	(pole–cm/lb)	$A\text{-}m^2/kg$	(pole–cm/lb)
Class I	1×10^{-3}	(0.45)	0.4×10^{-3}	(0.18)
Class II	3.5×10^{-3}	(1.6)	1.4×10^{-3}	(0.63)
Class III	10×10^{-3} and higher	(4.5)	4×10^{-3} and higher	(1.8)