POLITECNICO
MILANO 1863

Spacecraft Attitude Dynamics

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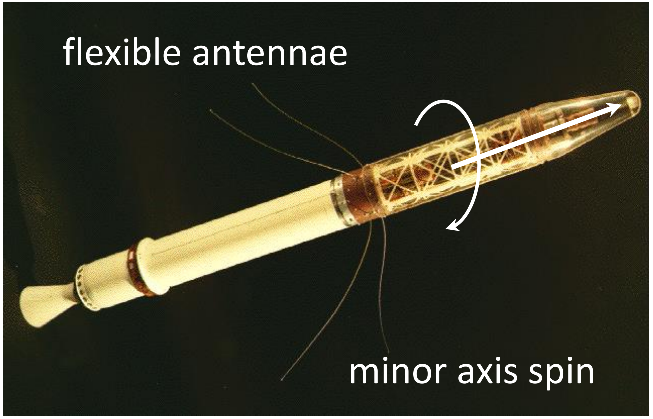
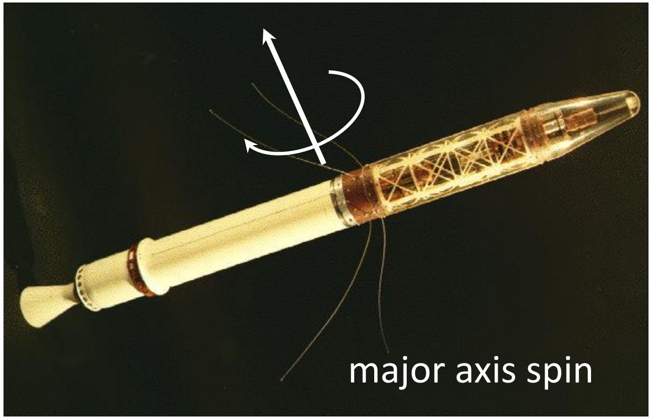
Spin and dual spin pointing stability

Explorer 1 flat spin

- •Explorer 1 (first US satellite, 1958) designed as a minor axis spinner !
- Via energy dissipation spacecraft experienced transition to major axis spin
- Energy dissipation caused by flexing of wire antennae on spacecraft body

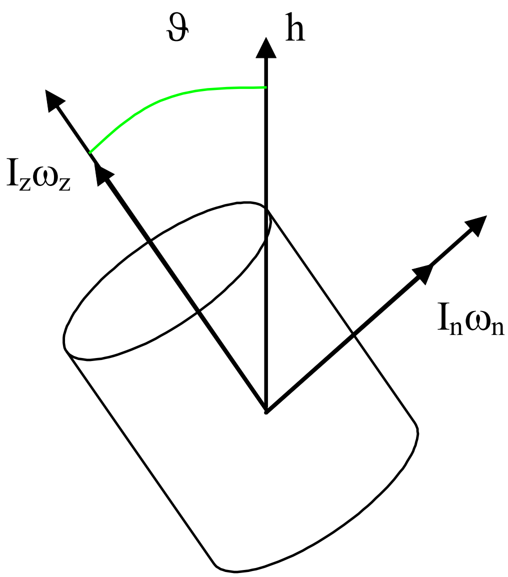




Satellite re-orientation for a symmetric spacecraft

$$h = I_n^2\omega_x^2 + I_n^2\omega_y^2 + I_z^2\omega_z^2$$

$$T = \frac{1}{2}(I_n\omega_x^2 + I_n\omega_y^2 + I_z\omega_z^2)$$



- If there is no external torque applied to the system then *h* must be constant.
- Kinetic energy can reduce through internal energy dissipation.

Satellite re-orientation for a symmetric spacecraft

- Under these conditions it can be shown that

because it decreases

$$2\dot{T} = 2I_z\omega_z\dot{\omega}_z + 2I_n\omega_n\dot{\omega}_n < 0 \quad \Longrightarrow \quad \dot{T} = \frac{I_z}{I_n}(I_n - I_z)\dot{\omega}_z\omega_z \leq 0$$

$$2h\dot{h} = 2I_z^2\omega_z\dot{\omega}_z + 2I_n^2\omega_n\dot{\omega}_n = 0 \quad \Longrightarrow \quad \omega_n\dot{\omega}_n = -\frac{I_z^2}{I_n^2}\omega_z\dot{\omega}_z$$

- This implies that the minor axis is always decreasing in velocity to zero or the major axis is always increasing to its maximum

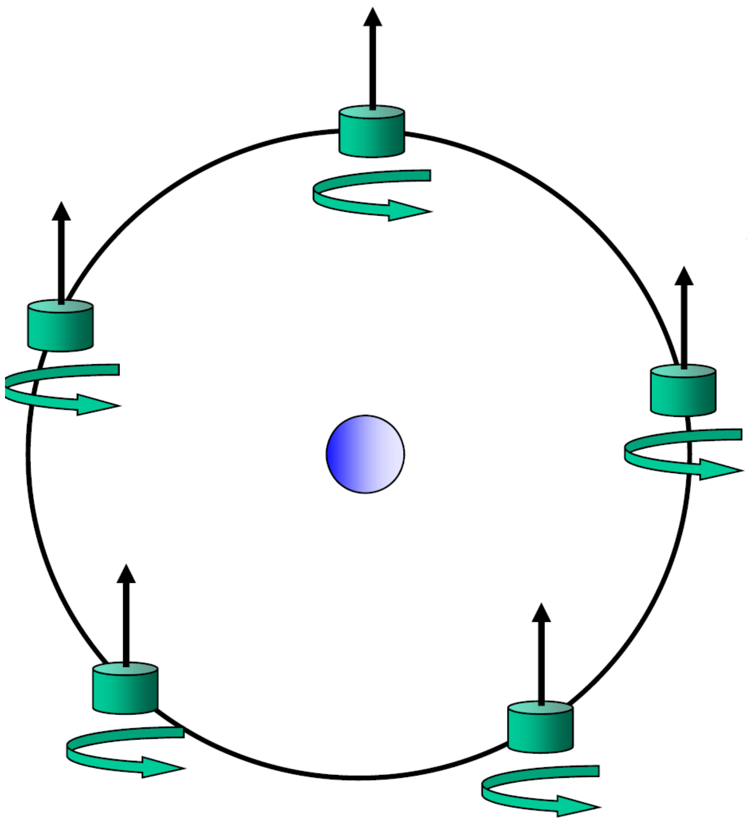
- If I_z is the minimum inertia axis

$$\omega_z\dot{\omega}_z < 0$$

$$\omega_z > 0 \qquad \dot{\omega}_z < 0$$

$$\omega_z < 0 \qquad \dot{\omega}_z > 0$$

Spin-stabilization

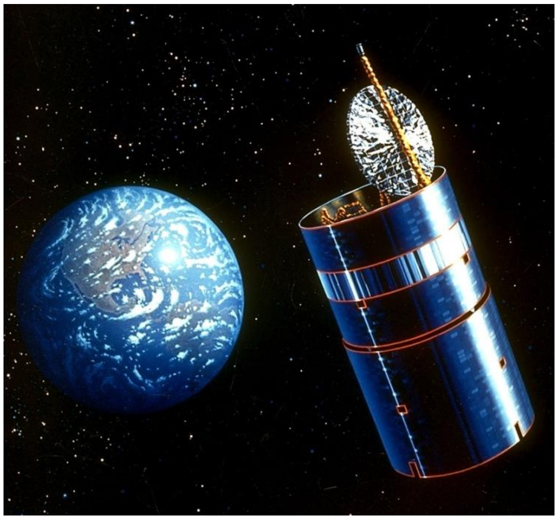


- Stable with respect to an inertial frame
- Poor for communication since the antenna is changing direction wrt to the Earth
- Spin stabilization useful in orbital maneuvering
- Can be combined with active control to change pointing direction during the orbit - expensive

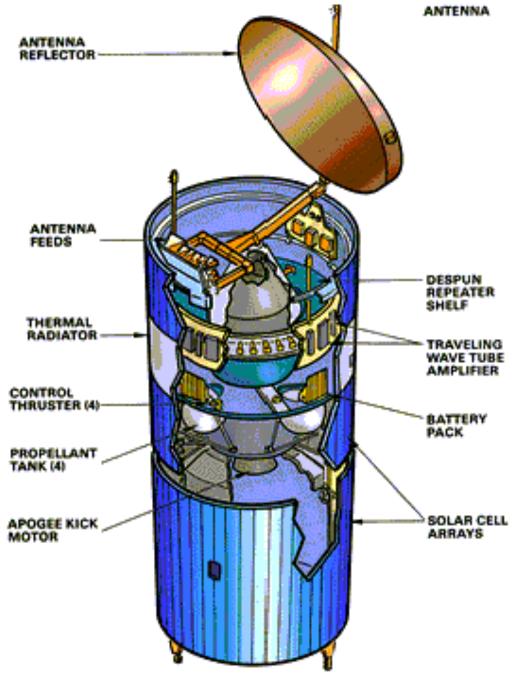
Dual-Spin stabilization

One portion of the space craft is not spinning

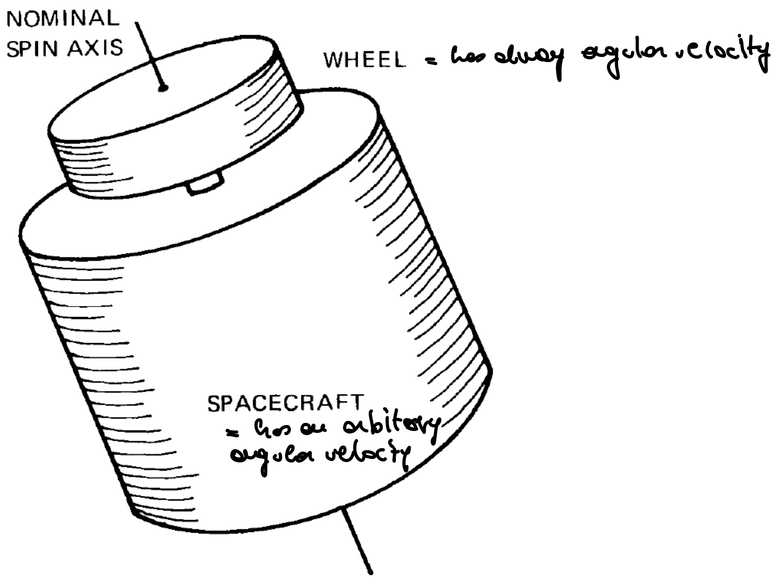
- Simplicity of spin-stabilised spacecraft, but de-spun platform at top
- Mount payload on de-spun platform for better pointing, but passive stability
- Popular for some GEO comsats

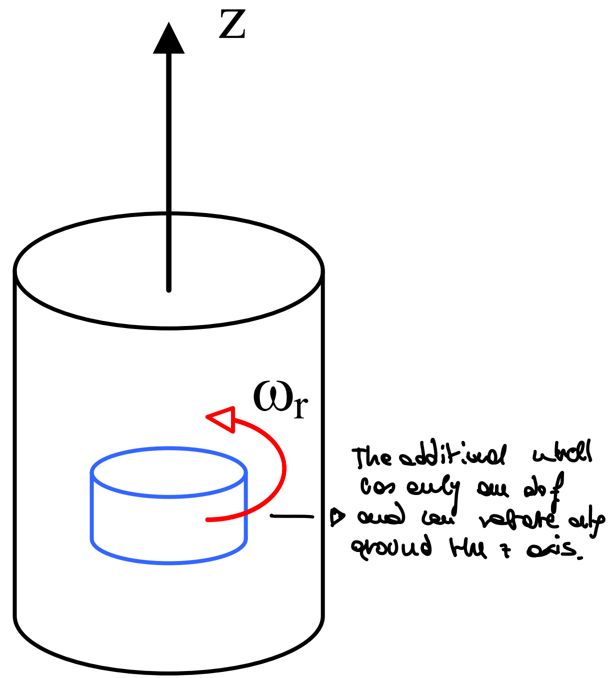


Boeing SBS 6



Equations of motion of a dual spin-spacecraft





Equations of motion of a dual spin-spacecraft

Computed already considering the presence of the wheel.

usually small compared to the s/c ones

$$\bar{h} = I_x \omega_x \underline{i} + I_y \omega_y \underline{j} + (I_z \omega_z + I_r \omega_r)\underline{k}$$

! relative angular velocity wrt the spacecraft.

$$\begin{cases} I_x \dot{\omega}_x + (I_z - I_y)\omega_z \omega_y + I_r \omega_r \omega_y = M_x \\ I_y \dot{\omega}_y + (I_x - I_z)\omega_x \omega_z - I_r \omega_r \omega_x = M_y \\ I_z \dot{\omega}_z + (I_y - I_x)\omega_y \omega_x + I_r \dot{\omega}_r = M_z \\ I_r \dot{\omega}_r = M_r \end{cases}$$

→ Relative torque between the wheel and the s/c

It is possible to rewrite the last two eq considering M_r in the case of no external torque

⇒ $M_x = 0$, $M_y = 0$, $M_z = 0$

$$I_x \dot{\omega}_x = (I_y - I_z)\omega_z \omega_y - I_r \omega_r \omega_y$$
$$I_y \dot{\omega}_y = (I_z - I_x)\omega_x \omega_z + I_r \omega_r \omega_x$$
$$I_z \dot{\omega}_z = (I_x - I_y)\omega_y \omega_x - I_r \dot{\omega}_r$$
$$I_r \dot{\omega}_r = -I_r \dot{\omega}_z + M_r$$

? what is that

A spinning spacecraft with an inertial wheel

$$\begin{cases} I_x\dot{\omega}_x + (I_z - I_y)\overline{\omega}_z\omega_y + I_r\overline{\omega}_r\omega_y = 0 \\ I_y\dot{\omega}_y + (I_x - I_z)\omega_x\overline{\omega}_z - I_r\overline{\omega}_r\omega_x = 0 \\ I_z\dot{\omega}_z + I_r\dot{\omega}_r = 0 \\ I_r\dot{\omega}_r = 0 \end{cases}$$

Where $\overline{\omega}_z$ is the main body axis spin rate and $\overline{\omega}_r$ is the spinning rate of the inertia wheel

lets assume that $I_y > I_z > I_x$

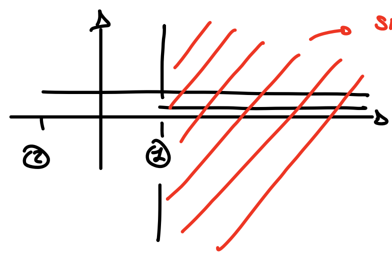
→ spin on the intermediate inertia axis

The general stability condition is

$$\begin{cases} (I_z - I_y)\overline{\omega}_z + I_r\overline{\omega}_r > 0 \\ (I_z - I_x)\overline{\omega}_z + I_r\overline{\omega}_r > 0 \end{cases} \quad \Rightarrow \quad \begin{cases} \overbrace{(I_y - I_z)}^{>0}\overline{\omega}_z < I_r\overline{\omega}_r & \Rightarrow (1) \\ \underbrace{(I_x - I_z)}_{<0}\overline{\omega}_z < I_r\overline{\omega}_r & \Rightarrow (2) \end{cases}$$

NOTE

The use of a dual spin permits to increase the number of possible combination in which the spin of the s/c is stable.



SATISFY BOTH STABILITY CONDITION EVEN IF I_z is the intermediate inertia axis

Stability diagrams

coefficient of the first eq of the Euler equation.

Non-dimensional coefficients (referred to body axes)

$$K_x = \frac{I_z - I_y}{I_x}$$

$$K_y = \frac{I_z - I_x}{I_y}$$

$$K_z = \frac{I_y - I_x}{I_z}$$

Bounded between ± 1 due to the property of the inertia matrix.

Non-dimensional coefficients (referred to orbit frame)

pointing out

K_y Yaw = K_x

K_r Roll = K_y

K_p Pitch = K_z

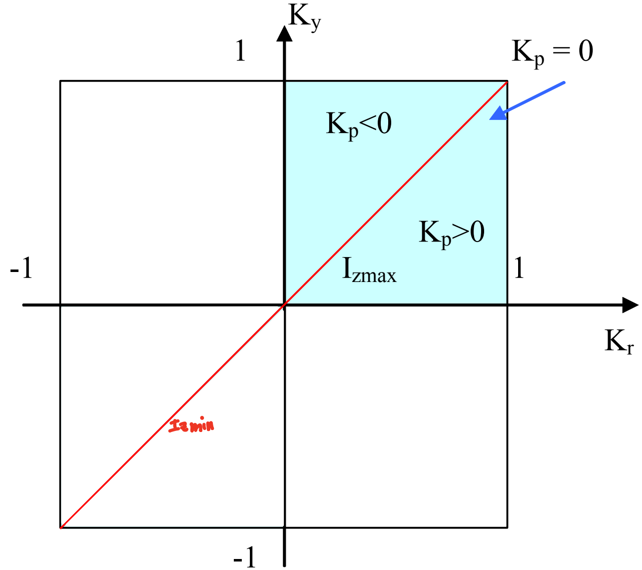
(→ orthogonal to the orbit plane.

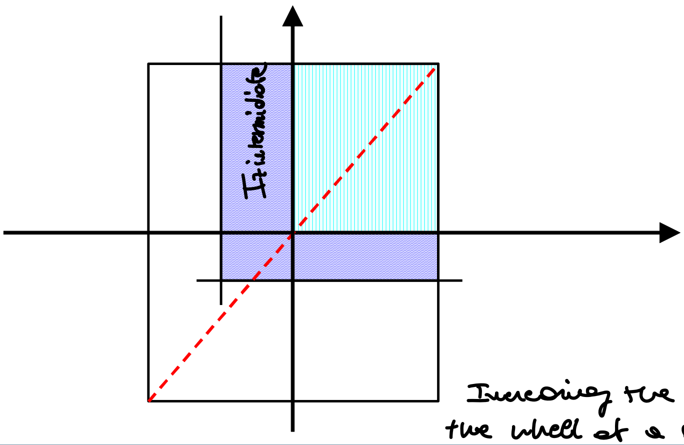
$$K_{yaw} = \frac{I_z - I_y}{I_x}$$

$$K_{roll} = \frac{I_z - I_x}{I_y}$$

$$K_{pitch} = \frac{I_y - I_x}{I_z}$$

Stability diagrams





Correspond to the major axis rule.

Simple spin

$$K_y > 0$$
$$K_r > 0$$

stable condition in the first quadrant.

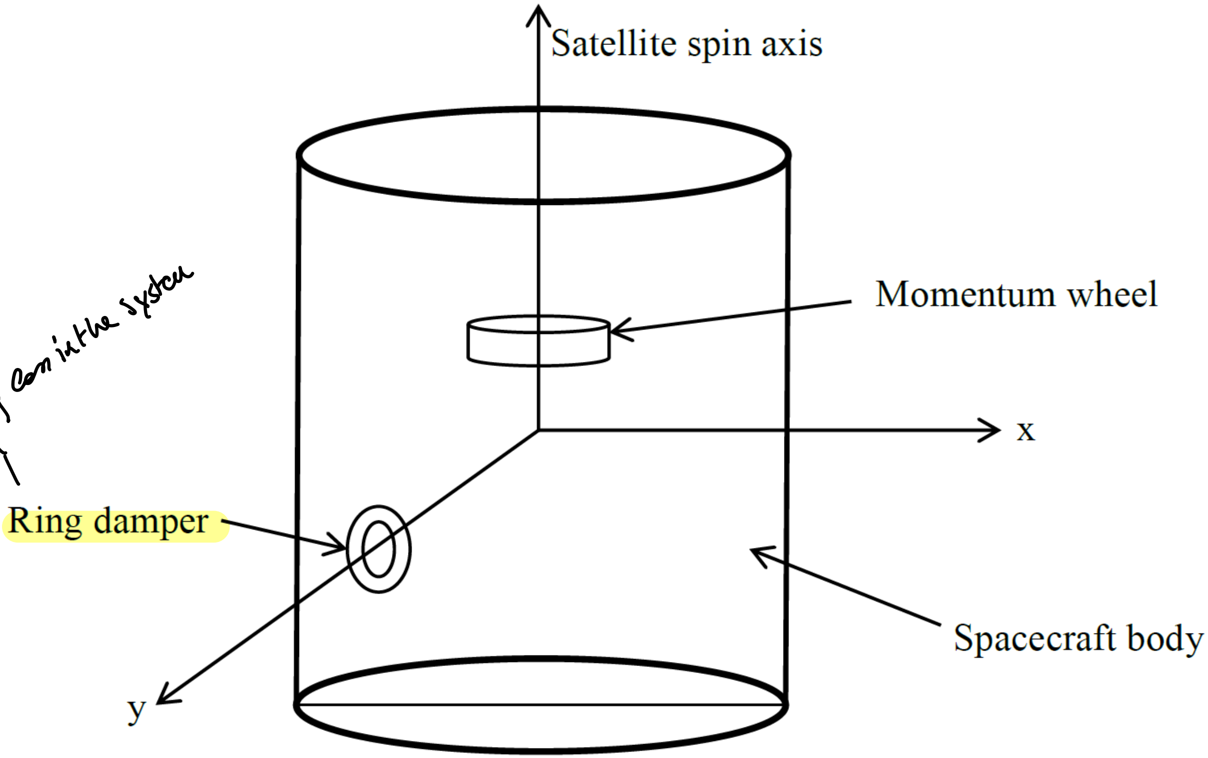
We are extending the possible combination for stability in terms of inertia properties

Dual spin

$$K_y > -\frac{I_r}{I_x}\frac{\overline{\omega_r}}{\overline{\omega_z}}$$
$$K_r > -\frac{I_r}{I_y}\frac{\overline{\omega_r}}{\overline{\omega_z}}$$

Increasing the stability region => If we can spin the wheel at a sufficient high rate the stability region could be as close as the all region.

Fluid-ring dampers



energy con in the system

How we can model this attitude

Fluid-ring dampers

$$\underline{h} = (I_x\omega_x + I_f\omega_f)\underline{i} + I_y\omega_y\underline{j} + (I_z\omega_z + I_r\omega_r)\underline{k}$$

$$\begin{cases} I_x\dot{\omega}_x + (I_z - I_y)\omega_z\omega_y + I_r\omega_r\omega_y + I_f\dot{\omega}_f = 0 \\ I_y\dot{\omega}_y + (I_x - I_z)\omega_x\omega_z - I_r\omega_r\omega_x + I_f\omega_f\omega_z = 0 \\ I_z\dot{\omega}_z + I_r\dot{\omega}_r + (I_y - I_x)\omega_x\omega_y - I_f\omega_f\omega_y = 0 \\ I_r\dot{\omega}_r = 0 \\ I_f\dot{\omega}_f + c(\omega_x + \omega_f) = 0 \end{cases}$$

No torque applied to the fluid ring

c → represent the friction

Equivalent Torque due to friction ⇒ internal moment.

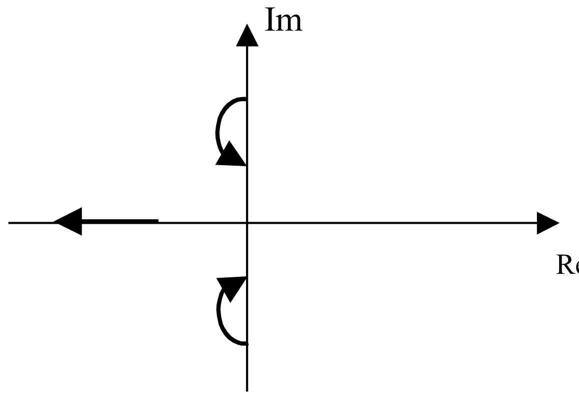
$$\begin{cases} I_x\dot{\omega}_x + (I_z - I_y)\bar{\omega}_z\omega_y + I_r\bar{\omega}_r\omega_y + I_f\dot{\omega}_f = 0 \\ I_y\dot{\omega}_y + (I_x - I_z)\omega_x\bar{\omega}_z - I_r\bar{\omega}_r\omega_x + I_f\omega_f\bar{\omega}_z = 0 \\ I_f\dot{\omega}_f + c(\omega_f + \omega_x) = 0 \\ I_z\dot{\omega}_z + I_r\dot{\omega}_r = 0 \\ I_r\dot{\omega}_r = 0 \end{cases}$$

First two equations: rotation around x and y axis.

Last two equations: rotation around z axis that is decoupled from the other equations

Linearizing about the equilibrium point (C,0,0,0) we have:

For $I_z > I_y > I_z$ root locus, as a function of c, has the following trace on the complex plane:



Increase in the coefficient c does not mean a continuous increase in the system damping

It can be shown that

Optimal value of c that maximizes damping

$$\boxed{c = \frac{I_f I_r \omega_r}{\sqrt{I_x I_y}}}$$

when c becomes too large the fluid it does not flow it stick to the wing (too viscous). That why there is an optimal value of the damping.