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MILANO 1863

Spacecraft Attitude Dynamics

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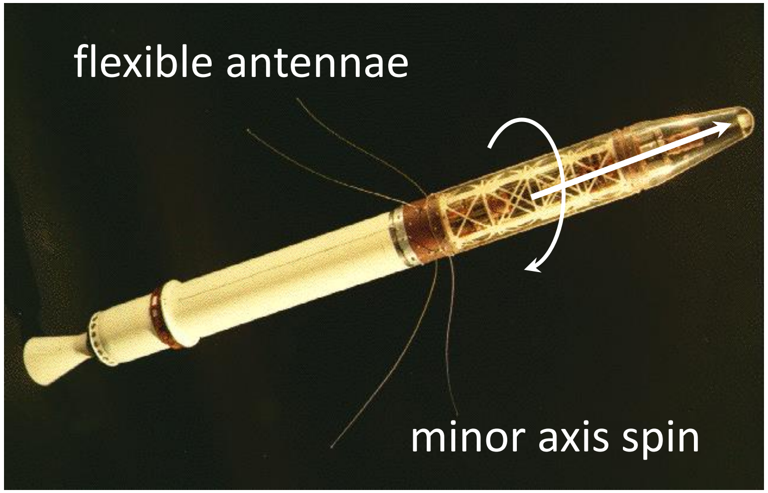
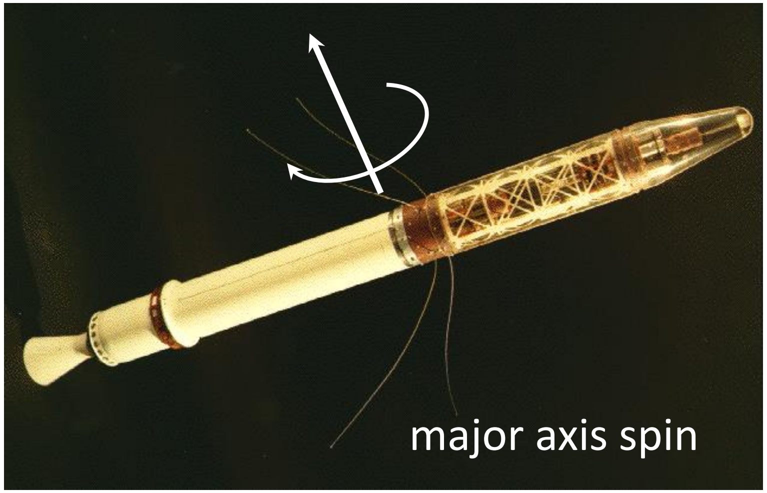
Spin and dual spin pointing stability

Explorer 1 flat spin

- Explorer 1 (first US satellite, 1958) designed as a minor axis spinner !
- Via energy dissipation spacecraft experienced transition to major axis spin
- Energy dissipation caused by flexing of wire antennae on spacecraft body

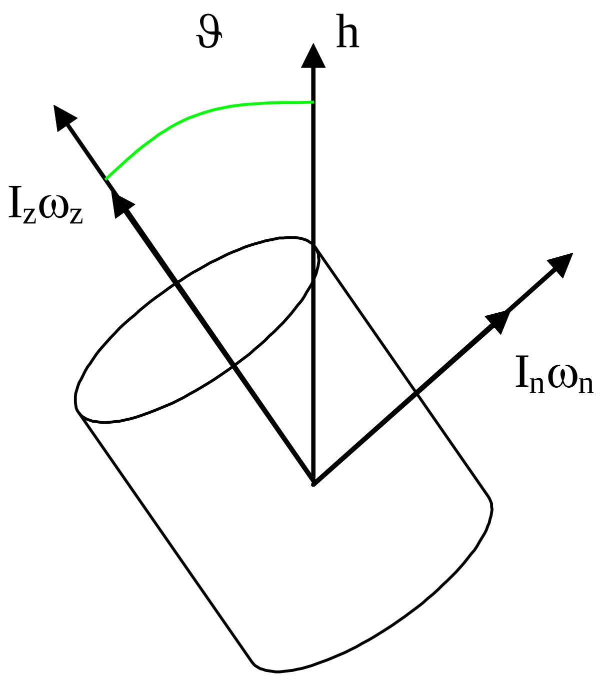




Satellite re-orientation for a symmetric spacecraft

$$h = I_n^2\omega_x^2 + I_n^2\omega_y^2 + I_z^2\omega_z^2$$

$$T = \frac{1}{2}(I_n\omega_x^2 + I_n\omega_y^2 + I_z\omega_z^2)$$



- If there is no external torque applied to the system then *h* must be constant.
- Kinetic energy can reduce through internal energy dissipation.

Satellite re-orientation for a symmetric spacecraft

- Under these conditions it can be shown that

$$2\dot{T} = 2I_z\omega_z\dot{\omega}_z + 2I_n\omega_n\dot{\omega}_n < 0 \quad \Rightarrow \quad \dot{T} = \frac{I_z}{I_n}(I_n - I_z)\dot{\omega}_z\omega_z \leq 0$$

$$2h\dot{h} = 2I_z^2\omega_z\dot{\omega}_z + 2I_n^2\omega_n\dot{\omega}_n = 0 \quad \Rightarrow \quad \omega_n\dot{\omega}_n = -\frac{I_z^2}{I_n^2}\omega_z\dot{\omega}_z$$

- This implies that the minor axis is always decreasing in velocity to zero or the major axis is always increasing to its maximum

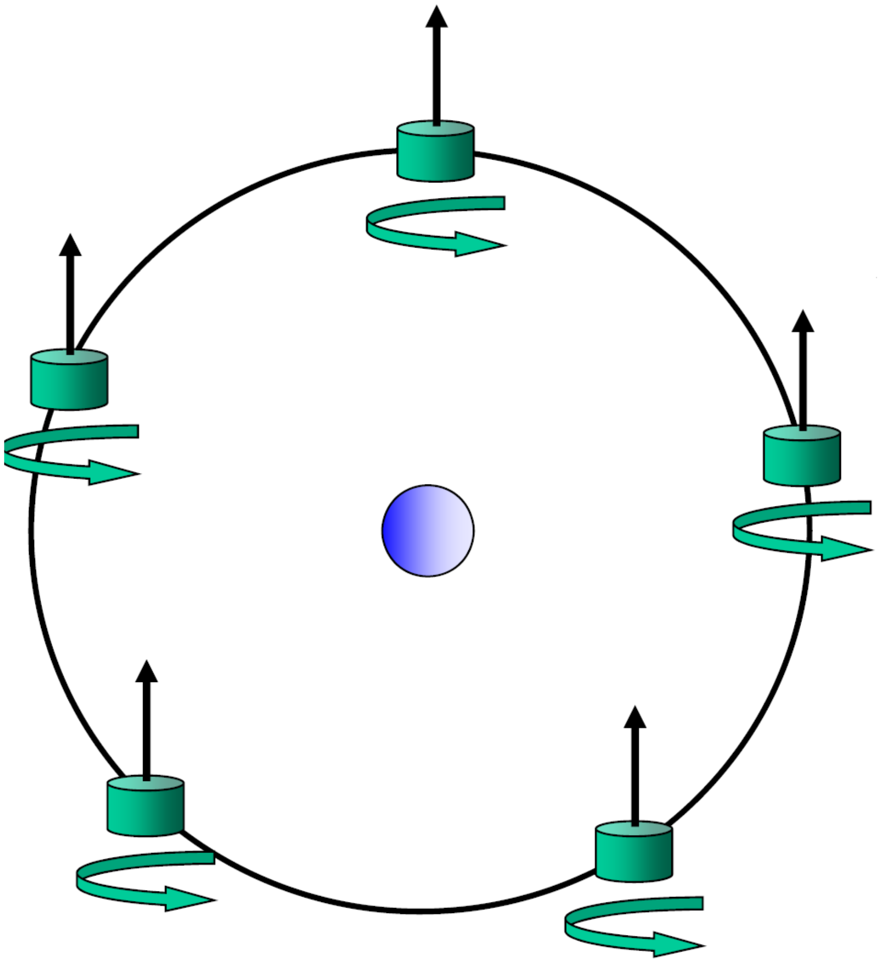
- If I_z is the minimum inertia axis

$$\omega_z\dot{\omega}_z < 0$$

$$\omega_z > 0 \qquad \dot{\omega}_z < 0$$

$$\omega_z < 0 \qquad \dot{\omega}_z > 0$$

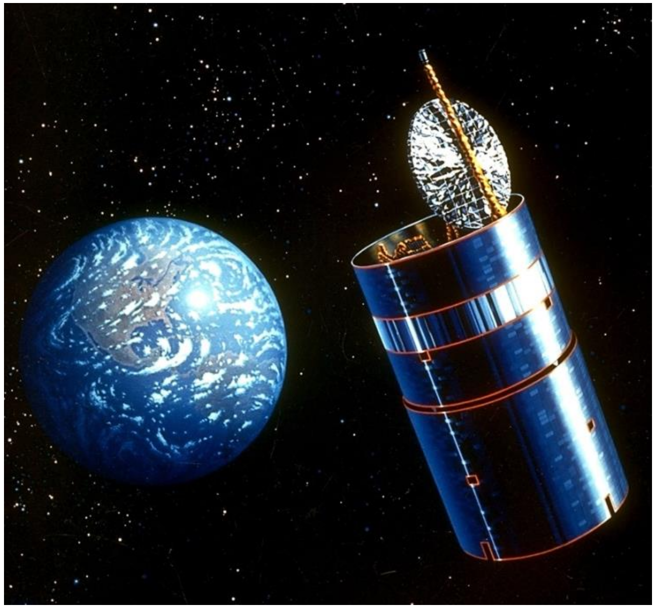
Spin-stabilization



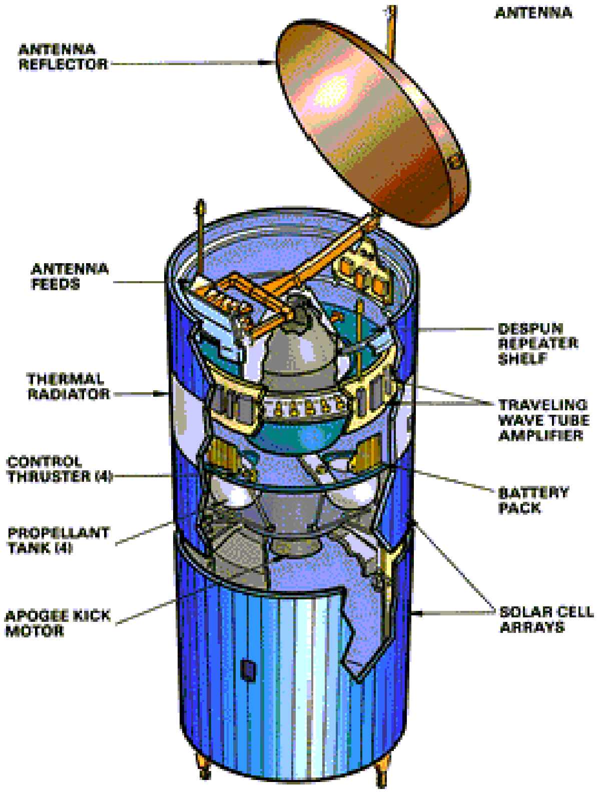
- Stable with respect to an inertial frame
- Poor for communication since the antenna is changing direction wrt to the Earth
- Spin stabilization useful in orbital maneuvering
- Can be combined with active control to change pointing direction during the orbit - expensive

Dual-Spin stabilization

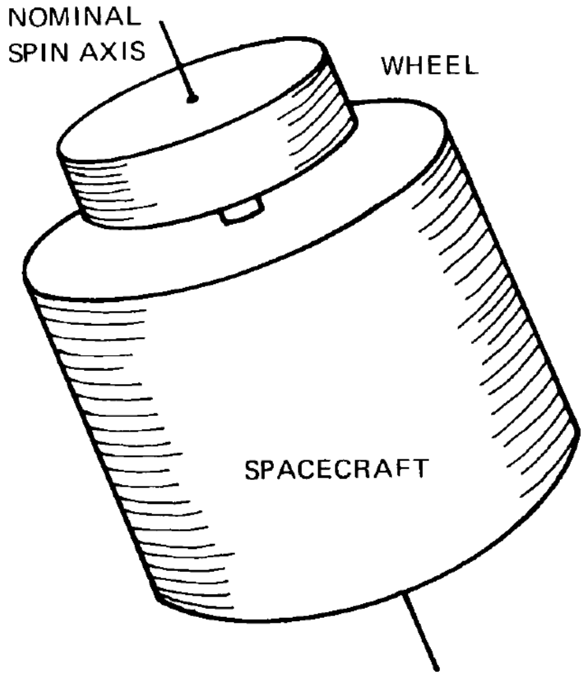
- Simplicity of spin-stabilised spacecraft, but de-spun platform at top
- Mount payload on de-spun platform for better pointing, but passive stability
- Popular for some GEO comsats

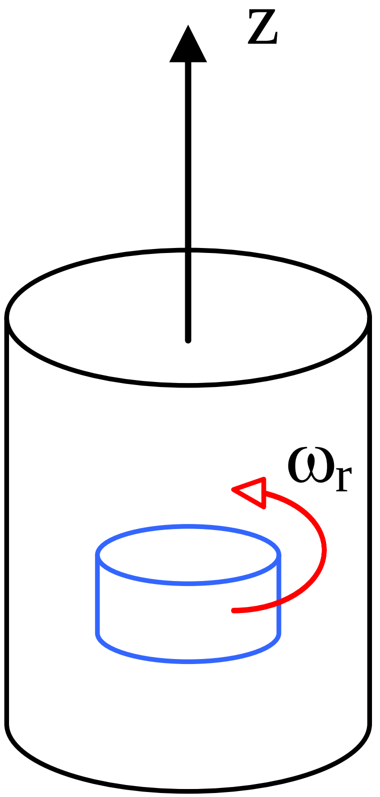


Boeing SBS 6



Equations of motion of a dual spin-spacecraft





Equations of motion of a dual spin-spacecraft

$$\overline{h} = I_x\omega_x\underline{i} + I_y\omega_y\underline{j} + (I_z\omega_z + I_r\omega_r)\underline{k}$$

$$\begin{cases} I_x\dot{\omega}_x + \left(I_z - I_y\right)\omega_z\omega_y + I_r\omega_r\omega_y = M_x \\ I_y\dot{\omega}_y + \left(I_x - I_z\right)\omega_x\omega_z - I_r\omega_r\omega_x = M_y \\ I_z\dot{\omega}_z + \left(I_y - I_x\right)\omega_y\omega_x + I_r\dot{\omega}_r = M_z \\ I_r\dot{\omega}_r = M_r \end{cases}$$

$$\begin{aligned} I_x\dot{\omega}_x &= \left(I_y - I_z\right)\omega_z\omega_y - I_r\omega_r\omega_y \\ I_y\dot{\omega}_y &= (I_z - I_x)\omega_x\omega_z + I_r\omega_r\omega_x \\ I_z\dot{\omega}_z &= \left(I_x - I_y\right)\omega_y\omega_x - I_r\dot{\omega}_r \\ I_r\dot{\omega}_r &= -I_r\dot{\omega}_z + M_r \end{aligned}$$

A spinning spacecraft with an inertial wheel

$$\begin{cases} I_x\dot{\omega}_x + (I_z - I_y)\overline{\omega}_z\omega_y + I_r\overline{\omega}_r\omega_y = 0 \\ I_y\dot{\omega}_y + (I_x - I_z)\omega_x\overline{\omega}_z - I_r\overline{\omega}_r\omega_x = 0 \\ I_z\dot{\omega}_z + I_r\dot{\omega}_r = 0 \\ I_r\dot{\omega}_r = 0 \end{cases}$$

Where $\overline{\omega}_z$ is the main body axis spin rate and $\overline{\omega}_r$ is the spinning rate of the inertia wheel

The general stability condition is

$$\begin{cases} (I_z - I_y)\overline{\omega}_z + I_r\overline{\omega}_r > 0 \\ (I_z - I_x)\overline{\omega}_z + I_r\overline{\omega}_r > 0 \end{cases} \Rightarrow \begin{cases} (I_y - I_z)\overline{\omega}_z < I_r\overline{\omega}_r \\ (I_x - I_z)\overline{\omega}_z < I_r\overline{\omega}_r \end{cases}$$

Stability diagrams

Non-dimensional coefficients (referred to body axes) →

$$K_x = \frac{I_z - I_y}{I_x}$$
$$K_y = \frac{I_z - I_x}{I_y}$$
$$K_z = \frac{I_y - I_x}{I_z}$$

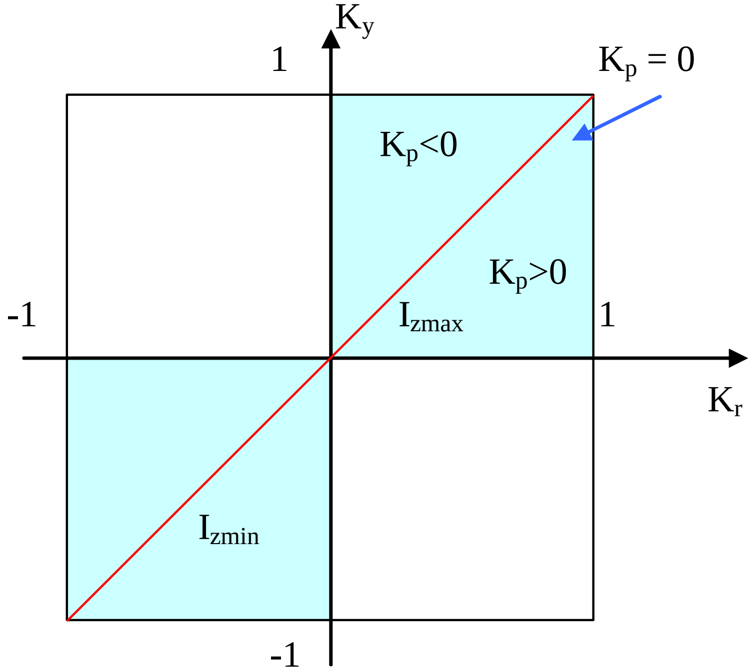
Non-dimensional coefficients (referred to orbit frame) →

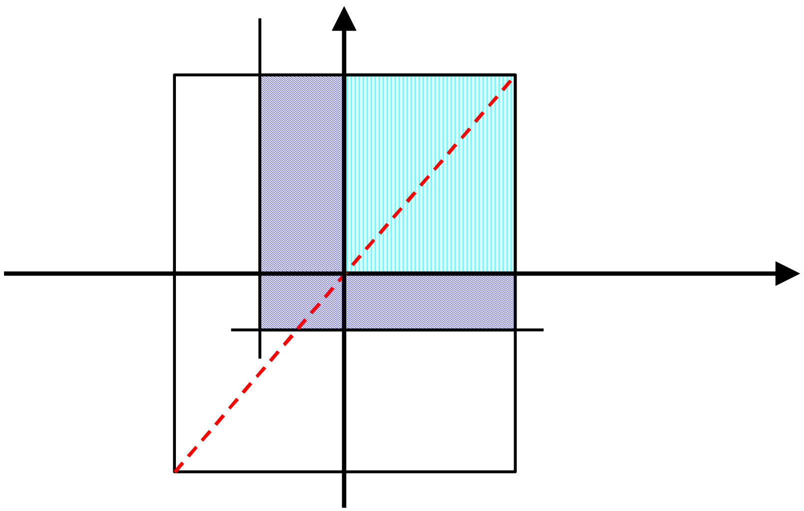
K_y	Yaw = K_x
K_r	Roll = K_y
K_p	Pitch = K_z

→

$$K_{yaw} = \frac{I_z - I_y}{I_x}$$
$$K_{roll} = \frac{I_z - I_x}{I_y}$$
$$K_{pitch} = \frac{I_y - I_x}{I_z}$$

Stability diagrams





Simple spin $\rightarrow$ $K_y > 0$, $K_r > 0$

Dual spin $\rightarrow$

$$K_y > -\frac{I_r}{I_x}\frac{\overline{\omega}_r}{\overline{\omega}_z}$$

$$K_r > -\frac{I_r}{I_y}\frac{\overline{\omega}_r}{\overline{\omega}_z}$$