POLITECNICO
MILANO 1863

# Spacecraft Attitude Dynamics

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WRT THE ATMOSFERE OF THE EARTH

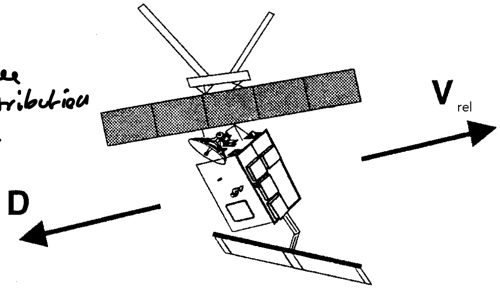
# Disturbance Torques – SRP and air drag

SOLAR RADIATION PRESSURE
DEPENDS ON ORIENTATION AND POSITION WRT THE SUN

# Force due to atmospheric drag – inertial frame

The drag force on an Earth orbiting spacecraft is

It is not the same order as the atmospheric drag → the rule of continuum aerodynamics are not precise anymore → we need to study the impact dynamics of the distribution of particles in a volume.




where

But we can approximate similarly to the continuum dynamics rule

$$\boxed{\bar{F}_i = -\frac{1}{2}\rho C_D {v_{rel}}^2 \frac{\mathbf{v}_{rel}}{\|\mathbf{v}_{rel}\|} A_{cross}}$$

$\rho(h,t)$ atmospheric density

$\mathbf{v}_{rel}$ relative speed

$A_{cross}$ cross sectional area perpendicular to $\mathbf{v}_{rel}$

$C_D \approx 2.2, 1.5 < C_D < 2.6$ drag coefficient (LEO)

↳ for a flat plate and it take in account for some therms omitted in the equation.

flat plate drag coefficient

# Air density example data set

$$\bar{F}_i = -\frac{1}{2}\rho C_D {v_{rel}}^2 \frac{\mathbf{v}_{rel}}{\|\mathbf{v}_{rel}\|} A_{cross}$$

↳ Need to be computed account for 3 surface of a cubical satellite.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| altit. (km) | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| density (g/cm$^3$) | 4,02e-04 | 8,34e-05 | 1,57e-05 | 3,18e-06 | 8,37e-07 | 2,33e-07 | 5,86e-08 | 1,40e-08 | 2,99e-09 | 5,17e-10 |
| altit. (km) | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
| density (g/cm$^3$) | 8,42e-11 | 1,84e-11 | 7,36e-12 | 3,78e-12 | 2,19e-12 | 1,37e-12 | 9,00e-13 | 6,15e-13 | 4,32e-13 | 3,10e-13 |
| altit. (km) | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 | 290 | 300 |
| density (g/cm$^3$) | 2,27e-13 | 1,68e-13 | 1,26e-13 | 9,58e-14 | 7,35e-14 | 5,68e-14 | 4,43e-14 | 3,48e-14 | 2,75e-14 | 2,18e-14 |
| altit. (km) | 310 | 320 | 330 | 340 | 350 | 360 | 370 | 380 | 390 | 400 |
| density (g/cm$^3$) | 1,74e-14 | 1,40e-14 | 1,13e-14 | 9,10e-15 | 7,39e-15 | 6,02e-15 | 4,92e-15 | 4,03e-15 | 3,31e-15 | 2,72e-15 |
| altit. (km) | 410 | 420 | 430 | 440 | 450 | 460 | 470 | 480 | 490 | 500 |
| density (g/cm$^3$) | 2,25e-15 | 1,86e-15 | 1,54e-15 | 1,28e-15 | 1,07e-15 | 8,89e-16 | 7,43e-16 | 6,22e-16 | 5,22e-16 | 4,39e-16 |
| altit. (km) | 510 | 520 | 530 | 540 | 550 | 560 | 570 | 580 | 590 | 600 |
| density (g/cm$^3$) | 3,71e-16 | 3,13e-16 | 2,66e-16 | 2,26e-16 | 1,93e-16 | 1,65e-16 | 1,41e-16 | 1,22e-16 | 1,05e-16 | 9,14e-17 |
| altit. (km) | 610 | 620 | 630 | 640 | 650 | 660 | 670 | 680 | 690 | 700 |
| density (g/cm$^3$) | 7,96e-17 | 6,97e-17 | 6,12e-17 | 5,41e-17 | 4,79e-17 | 4,27e-17 | 3,82e-17 | 3,44e-17 | 3,10e-17 | 2,82e-17 |
| altit. (km) | 710 | 720 | 730 | 740 | 750 | 760 | 770 | 780 | 790 | 800 |
| density (g/cm$^3$) | 2,57e-17 | 2,35e-17 | 2,16e-17 | 1,99e-17 | 1,84e-17 | 1,71e-17 | 1,59e-17 | 1,49e-17 | 1,39e-17 | 1,31e-17 |
| altit. (km) | 810 | 820 | 830 | 840 | 850 | 860 | 870 | 880 | 890 | 900 |
| density (g/cm$^3$) | 1,23e-17 | 1,16e-17 | 1,10e-17 | 1,04e-17 | 9,86e-18 | 9,37e-18 | 8,91e-18 | 8,48e-18 | 8,09e-18 | 7,72e-18 |
| altit. (km) | 910 | 920 | 930 | 940 | 950 | 960 | 970 | 980 | 990 | 1000 |
| density (g/cm$^3$) | 7,37e-18 | 7,04e-18 | 6,74e-18 | 6,45e-18 | 6,18e-18 | 5,92e-18 | 5,68e-18 | 5,44e-18 | 5,22e-18 | 5,02e-18 |

# Atmospheric density model

$$\rho(h,t) = \rho_0 \exp\left[-\frac{h-h_0}{H}\right]$$

$\rho_0$ reference density

$h, h_0$ actual and reference altitude

$H$ scale height

**TABLE 7-4. Exponential Atmospheric Model.** Although a very simple approach, this method yields moderate results for general studies. Source: Wertz, 1978, 820, which uses the *U.S. Standard Atmosphere* (1976) for 0 km, CIRA-72 for 25–500 km, and CIRA-72 with $T_\infty$ = 1000 K for 500–1000 km. The scale heights have been adjusted to maintain a piecewise-continuous formulation of the density.

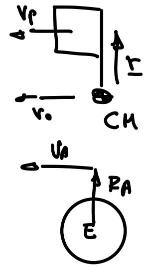
| Altitude $h_{ellp}$ (km) | Base Altitude $h_o$ (km) | Nominal Density $\rho_o$ (kg/m$^3$) | Scale Height $H$ (km) | Altitude $h_{ellp}$ (km) | Base Altitude $h_o$ (km) | Nominal Density $\rho_o$ (kg/m$^3$) | Scale Height $H$ (km) |
|---|---|---|---|---|---|---|---|
| 0–25 | 0 | 1.225 | 7.249 | 150–180 | 150 | $2.070\times10^{-9}$ | 22.523 |
| 25–30 | 25 | $3.899\times10^{-2}$ | 6.349 | 180–200 | 180 | $5.464\times10^{-10}$ | 29.740 |
| 30–40 | 30 | $1.774\times10^{-2}$ | 6.682 | 200–250 | 200 | $2.789\times10^{-10}$ | 37.105 |
| 40–50 | 40 | $3.972\times10^{-3}$ | 7.554 | 250–300 | 250 | $7.248\times10^{-11}$ | 45.546 |
| 50–60 | 50 | $1.057\times10^{-3}$ | 8.382 | 300–350 | 300 | $2.418\times10^{-11}$ | 53.628 |
| 60–70 | 60 | $3.206\times10^{-4}$ | 7.714 | 350–400 | 350 | $9.158\times10^{-12}$ | 53.298 |
| 70–80 | 70 | $8.770\times10^{-5}$ | 6.549 | 400–450 | 400 | $3.725\times10^{-12}$ | 58.515 |
| 80–90 | 80 | $1.905\times10^{-5}$ | 5.799 | 450–500 | 450 | $1.585\times10^{-12}$ | 60.828 |
| 90–100 | 90 | $3.396\times10^{-6}$ | 5.382 | 500–600 | 500 | $6.967\times10^{-13}$ | 63.822 |
| 100–110 | 100 | $5.297\times10^{-7}$ | 5.877 | 600–700 | 600 | $1.454\times10^{-13}$ | 71.835 |
| 110–120 | 110 | $9.661\times10^{-8}$ | 7.263 | 700–800 | 700 | $3.614\times10^{-14}$ | 88.667 |
| 120–130 | 120 | $2.438\times10^{-8}$ | 9.473 | 800–900 | 800 | $1.170\times10^{-14}$ | 124.64 |
| 130–140 | 130 | $8.484\times10^{-9}$ | 12.636 | 900–1000 | 900 | $5.245\times10^{-15}$ | 181.05 |
| 140–150 | 140 | $3.845\times10^{-9}$ | 16.149 | 1000– | 1000 | $3.019\times10^{-15}$ | 268.00 |

Eq. (7-31) requires knowledge of the actual altitude, found by subtracting the Earth's radius (6378.137 km) from the satellite's given radius ($h_{ellp}$ = 747.2119 km). Now, if we use values from Table 7-4, Eq. (7-31) becomes

$$\rho = 3.614\times10^{-14}\,\mathrm{EXP}\left[-\frac{747.2119-700}{88.667}\right] = 2.121\,985\,4\times10^{-14}\,\frac{\mathrm{kg}}{\mathrm{m}^3}$$

▲ Note that the units in the exponential cancel (all are km), and the result is less than the base value at 700 km, as we would expect.

# Relative orbital velocity in the inertial frame



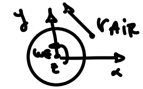

$\underline{v}_P = \underline{v}_O + \underline{\omega} \wedge \underline{r}$

$\underline{v}_A = \underline{\omega}_E \wedge \underline{R}_A$

$\underline{V}_{rel} = \underline{v}_P - \underline{v}_A$

$$\mathbf{v}_{orbit} = \frac{d\vec{R}}{dt}$$

$$\mathbf{v}_{atmosphere} = \omega_{Earth} \times \vec{R}$$




$$\underline{v} = \underline{\omega} \wedge \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 0 & 0 & \omega_E \\ x & y & z \end{vmatrix} = \begin{Bmatrix} -\omega_E y \\ +\omega_E x \\ 0 \end{Bmatrix}$$

Dependent on the orbit that we define

The atmosphere is rotating

$$\mathbf{v}_{rel} = \begin{bmatrix} \dot{x} + \omega_{\oplus} y \\ \dot{y} - \omega_{\oplus} x \\ \dot{z} \end{bmatrix}$$

$$\omega_{Earth} = [0 \quad 0 \quad \omega_{\oplus}]$$

point of intersection

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R \begin{bmatrix} \cos\theta \\ \sin\theta\cos i \\ \sin\theta\sin i \end{bmatrix}$$

$$R = \frac{a(1-e^2)}{1+e\cos\theta}$$

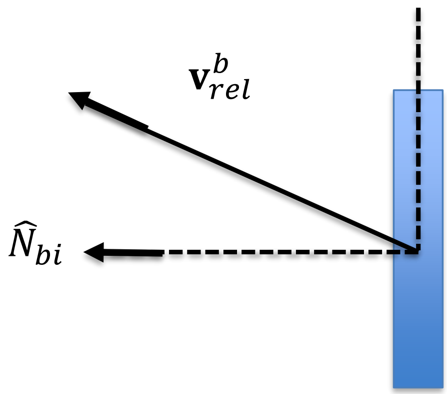
$$\dot{\theta} = \frac{n(1+e\cos\theta)^2}{(1-e^2)^{3/2}}$$

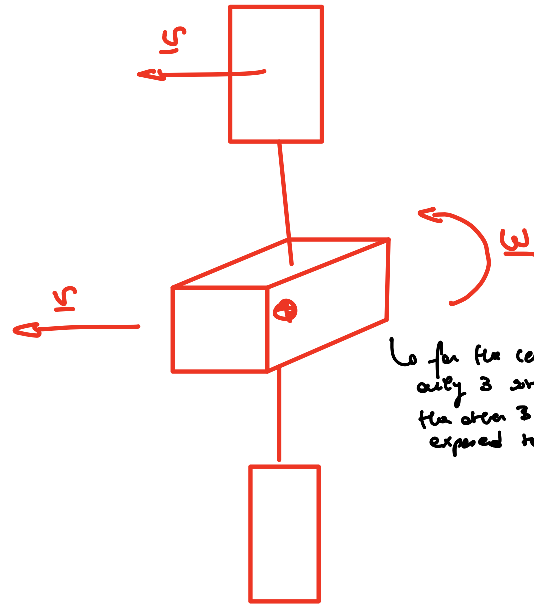
$$\omega_{\oplus} = 0.000072921 rad/\sec$$

# Relative velocity in body fixed coordinates

$$\mathbf{v}_{rel}^{b} = A_{B/N}\mathbf{v}_{rel} \qquad \mathbf{v}_{rel} = \begin{bmatrix} \dot{x} + \omega_{\oplus} y \\ \dot{y} - \omega_{\oplus} x \\ \dot{z} \end{bmatrix}$$

body




↳ for the central body only 3 surfaces becau[se] the other 3 won't be exposed to the air.

$$A_{cross} = A_i\left(\widehat{N}_{bi} \cdot \frac{\mathbf{v}_{rel}^{b}}{\left\|\mathbf{v}_{rel}^{b}\right\|}\right)$$

# Torque due to atmospheric drag

The aerodynamic force acting on a flat surface is defined by:

$$\bar{F}_i = -\frac{1}{2}\rho C_D {v_{rel}}^2 \frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|}(\hat{N}_{bi}\cdot\frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|})A_i$$

number of surfaces exposed to the Air

$$T_{aero} = -\sum_{i=1}^{n} \bar{r}_i \times \frac{1}{2}\rho C_D {v_{rel}}^2 \frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|}(\hat{N}_{bi}\cdot\frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|})A_i$$

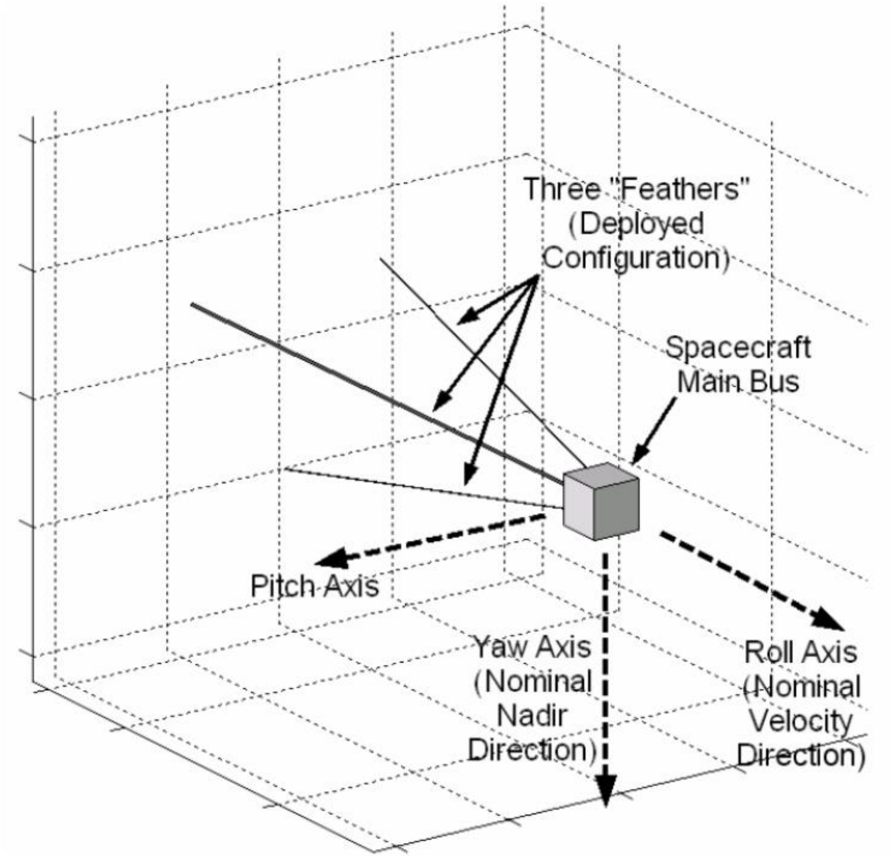
.

Total torque on a rigid body

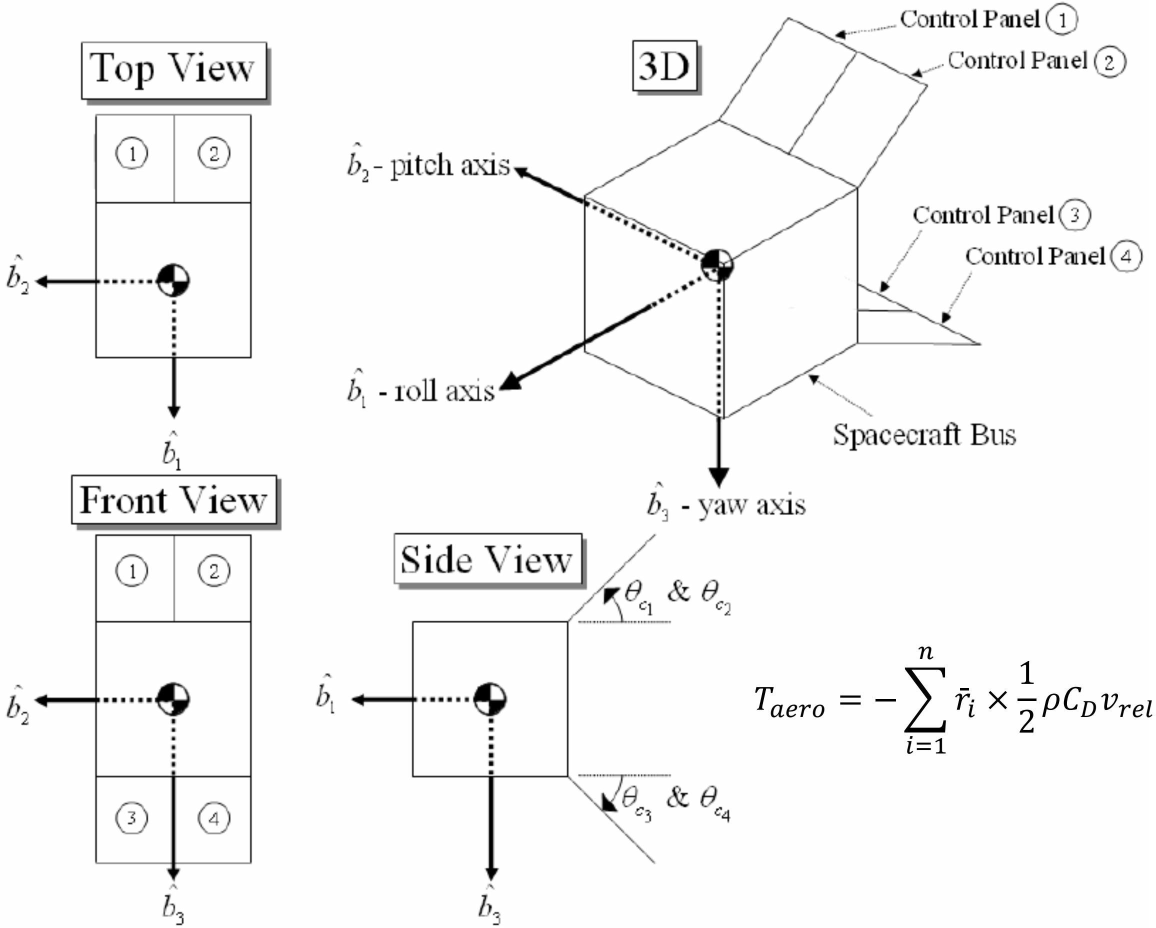
$$T_{aero} = \begin{cases} -\frac{1}{2}\rho C_D {v_{rel}}^2 \sum_{i=1}^{n} \bar{r}_i \times \frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|} \sum_{i=1}^{n} (\hat{N}_{bi}\cdot\frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|})A_i & \hat{N}_{bi}\cdot\frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|} > 0 \\ 0 & \hat{N}_{bi}\cdot\frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|} < 0 \end{cases}$$

# Shuttlecock concept

It is generating a passive stability but introduce more drag so we will have some orbit decay.



# CubeSat – air drag control




$$T_{aero} = -\sum_{i=1}^{n} \bar{r}_i \times \frac{1}{2}\rho C_D v_{rel}{}^2 \frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|} \sum_{i=1}^{n} (\hat{N}_{bi} \cdot \frac{\mathbf{v}_{rel}^b}{\|\mathbf{v}_{rel}^b\|}) A_i$$

# Force due to solar radiation pressure

for orbit around Earth the distance from the Sun ≈ cost

Depends on the relative position of the center body wrt to the Sun!

center body of our orbit
→ But if is general we can compute always the radiation emitted from the center body
It depends on the temperature

| Altitude from the surface of the Earth (Km) | Direct solar radiation (W/m²) | Radiation reflected by the Earth (W/m²) | Earth radiation (W/m²) |
|---|---|---|---|
| 500 | 1358 | 600 | 150 |
| 1000 | 1358 | 500 | 117 |
| 2000 | 1358 | 300 | 89 |
| 4000 | 1358 | 180 | 62 |
| 8000 | 1358 | 75 | 38 |
| 15000 | 1358 | 30 | 14 |
| 30000 | 1358 | 12 | 3 |
| 60000 | 1358 | 7 | 2 |

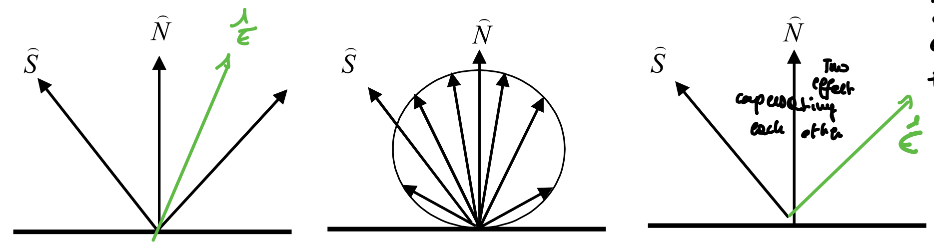
The average pressure due to radiation can be evaluated as

$$P = \frac{F_e}{c}$$

where $c$ is the speed of light and $F_e$ is the power per unit surface.

Note
it is important also the direction of the Radiation from the Earth and not only its modules.
for at least 1000 km it should be considered

$$\rho_s + \rho_d + \rho_a = 1$$




Specular reflection
→ perfect mirror
direction of the force is $\hat{N}$

Diffuse reflection
we decompose the vector

Absorption

→ The equivalent force has the same direction of $\hat{S}$.

We can use a combination of these effects and the total of the radiation that is reflected Diffuse and absorb should be 100% of the total incoming radiation.

# Force due to solar radiation pressure (from the Sun)

Force on a flat panel

$$\bar{F}_i = -PA\left[\rho_a(\hat{S}\cdot\hat{N})\hat{S} + 2\rho_s(\hat{S}\cdot\hat{N})^2\hat{N} + \rho_d(\hat{S}\cdot\hat{N})(\hat{S} + \frac{2}{3}\hat{N})\right]$$

$$\bar{F}_i = -PA(\hat{S}\cdot\hat{N})\left[(1-\rho_s)\hat{S} + (2\rho_s(\hat{S}\cdot\hat{N}) + \frac{2}{3}\rho_d)\hat{N}\right]$$

$$\bar{F}_i = -PA_i(\hat{S}_b\cdot\hat{N}_{bi})\left[(1-\rho_s)\hat{S}_b + (2\rho_s(\hat{S}_b\cdot\hat{N}_{bi}) + \frac{2}{3}\rho_d)\hat{N}_{bi}\right]$$

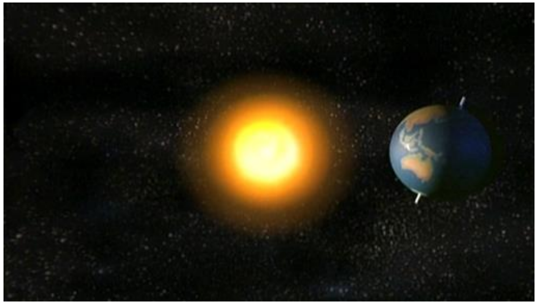
# Torque due to solar radiation pressure

$$\bar{F}_i = -PA_i(\hat{S}_b \cdot \hat{N}_{bi})\left[(1-\rho_s)\hat{S}_b + \left(2\rho_s(\hat{S}_b \cdot \hat{N}_{bi}) + \frac{2}{3}\rho_d\right)\hat{N}_{bi}\right]$$

Modelling the Sun in body coordinate

All the computation must be done in the principal axis frame because the torque in the principal axis frame will be the input of the system.

$$\hat{S}_b = A_{B/N}\hat{S}_i$$



NOTE

there is a difference between the maximum force and the maximum Torque because the geometry have a really important role

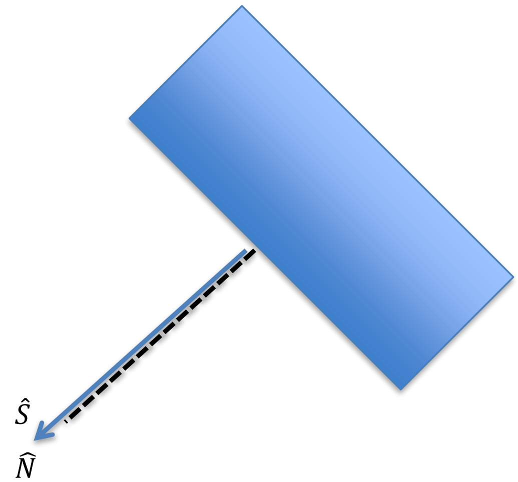
For a rigid body we have

$$T_{SRP} = \sum_{i=1}^{n} \bar{r}_i \times \bar{F}_i$$

$$T_{SRP} = \begin{cases} \sum_{i=1}^{n} \bar{r}_i \times \bar{F}_i & \hat{S}_b \cdot \hat{N}_b > 0 \\ 0 & \hat{S}_b \cdot \hat{N}_b < 0 \end{cases}$$

# Maximum SRP Force and corresponding torque

$$\bar{F}_i = -PA(\hat{S}\cdot\widehat{N})\left[(1-\rho_s)\hat{S} + \left(2\rho_s(\hat{S}\cdot\widehat{N}) + \frac{2}{3}\rho_d\right)\widehat{N}\right]$$




$\hat{S} = \widehat{N}$

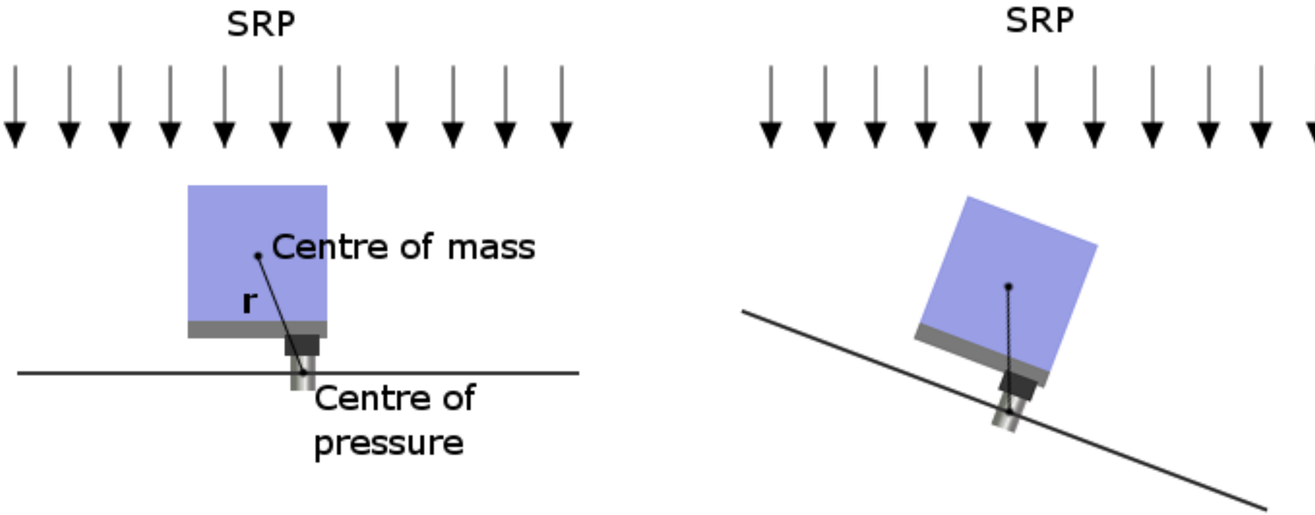
then

$$\bar{F}_i = -PA\left[(1+\rho_s) + \frac{2}{3}\rho_d\right]\widehat{N}$$

$$T_{SRP} = \sum_{i=1}^{n} \bar{r}_i \times \bar{F}_i$$

# Exploiting SRP for attitude control - CubeSail

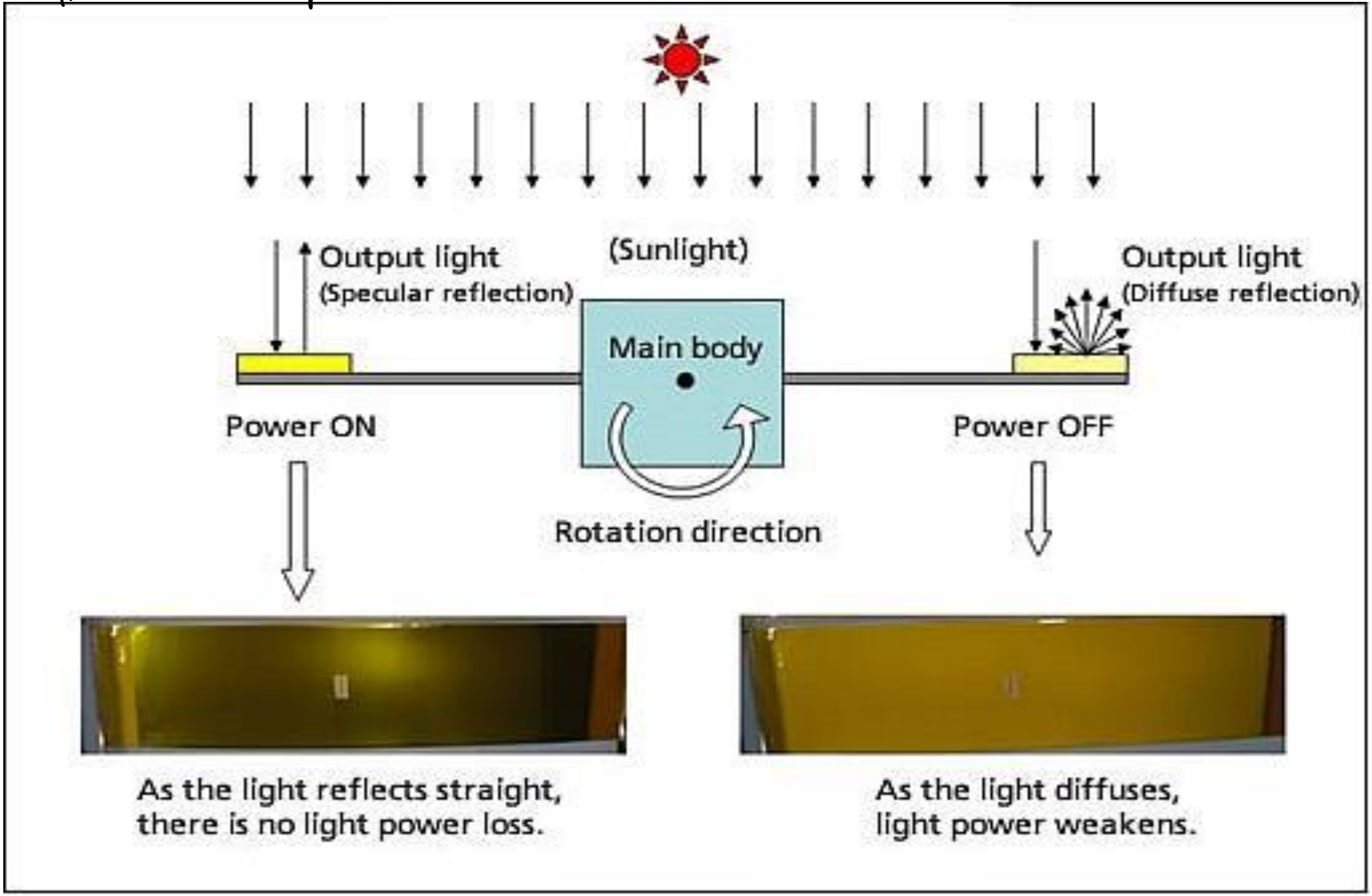
Using a sail is a way to generate some torque.



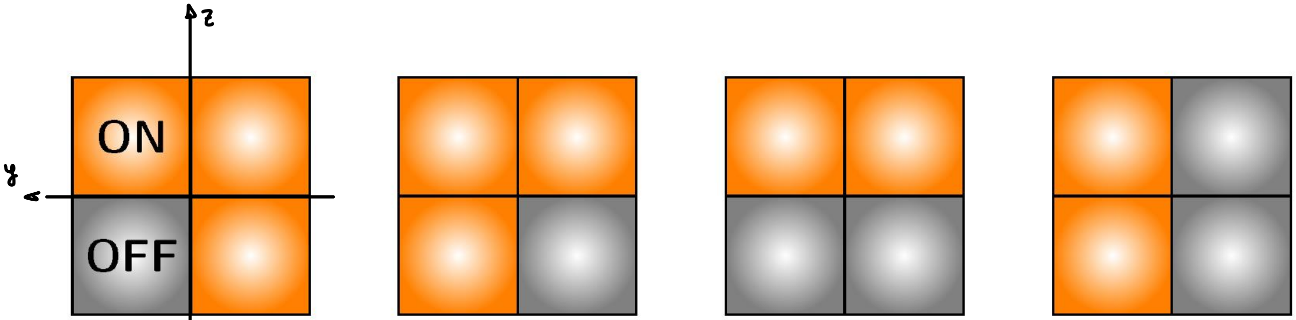

$$T_{srp} = \sum_{i=1}^{n} \bar{r}_i \times \bar{F}_i$$

# IKAROS – reflectivity control devices

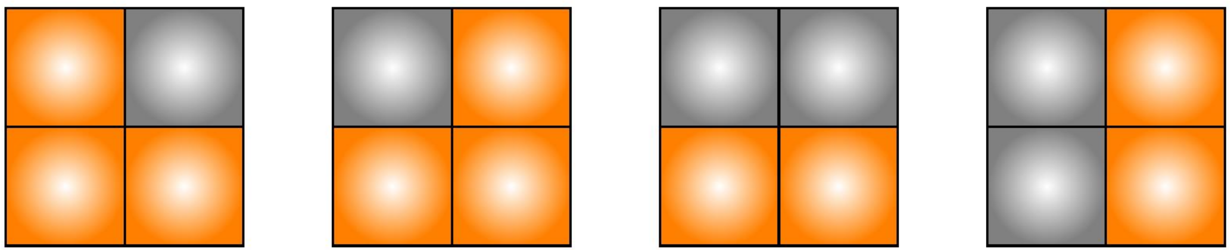
Different panel can give a net torque.



# Basic control logic → we are interested in seeing what happens for the Torque

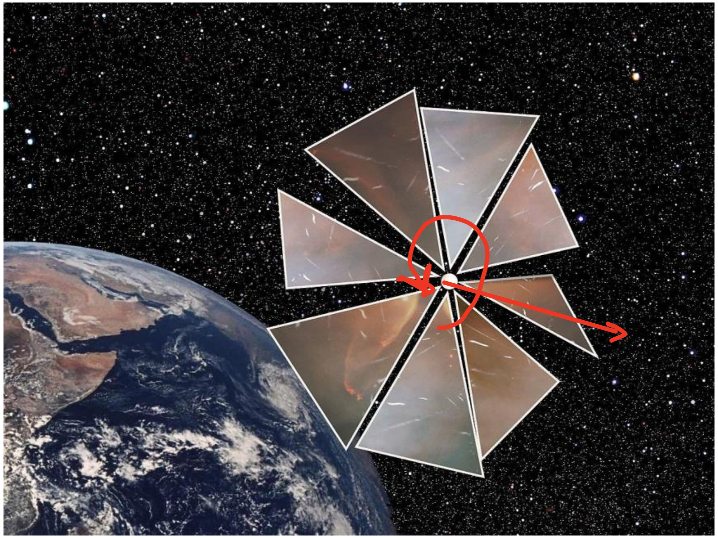



$u_y^{id} > 0 \quad u_z^{id} > 0$ $\quad$ $u_y^{id} > 0 \quad u_z^{id} < 0$ $\quad$ $u_y^{id} > 0 \quad u_z^{id} = 0$ $\quad$ $u_y^{id} = 0 \quad u_z^{id} < 0$



$u_y^{id} < 0 \quad u_z^{id} < 0$ $\quad$ $u_y^{id} < 0 \quad u_z^{id} > 0$ $\quad$ $u_y^{id} < 0 \quad u_z^{id} = 0$ $\quad$ $u_y^{id} = 0 \quad u_z^{id} > 0$

# Future concepts for attitude control with SRP



$$\bar{F}_i = -PA_i(\hat{S}_b \cdot \hat{N}_{bi})\left[(1-\rho_s)\hat{S}_b + \left(2\rho_s(\hat{S}_b \cdot \hat{N}_{bi}) + \frac{2}{3}\rho_d\right)\hat{N}_{bi}\right]$$