Selected slides from

Dynamics and control of space structures

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Control system design – review of classical techniques

2020/2021

Review of classical control design
Introduction

- The main goal of this first part of the course is to **review some basic relevant topics related to the design of feedback control systems using classical techniques** (in the frequency domain for single-input-single-output systems).

- This review is aimed at **preparing the background** to present and discuss the design of control systems using state-space techniques and other new topics introduced later in this course.

- The review is also useful as an **introductory material to students with "poor/weak" knowledge/competence of feedback control systems**.
- These students are invited to start from this material and reinforce their preparation by referring to specific textbooks.

Good **references** are the following:

[in Italian]
P. Bolzern, R. Scattolini, N. Schiavoni, *Fondamenti di controlli automatici*, McGraw Hill

[in English]
G.F. Franklin, J.D. Powell, A. Emami-Naeini, *Feedback control of dynamic systems*, Pearson

The review of classical control design is here carried out by **using a simple example (in order to privilege a practical approach to the subject)**, which highlights some aspects close to a realistic situation.

The system refers to a simplified configuration:

linear spacecraft attitude control (single-axis rotation).

Even though the spacecraft attitude dynamics is nonlinear, we restrict ourselves to the linear case since the lessons learned from control design of linear system are very important and **the linear analysis is typically the first step also in the case of full nonlinear systems**.

The attitude of a spacecraft must be typically stabilized to some desired attitude which is compatible to a required task (e.g., **point an object**). This task must be achieved when the spacecraft is subjected to external (environmental) disturbances. → Even if the spacecraft is passively stable some active control are used to get a better response from the spacecraft. Some time the attitude is stable but the satellite is very slow to come back to a stable condition after a perturbation

When pointing accuracy is high, passive stabilization is generally not enough and the spacecraft is equipped with an **active attitude control system***.

**it is important to design a spacecraft having passive stability (if possible), and then augment this with an active control scheme.*

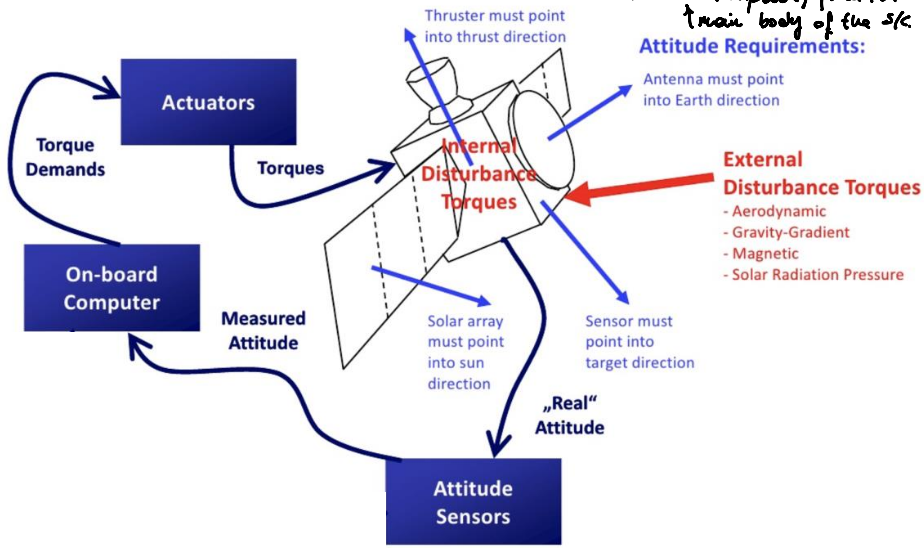
An active spacecraft attitude control system consists of attitude sensors, attitude actuators and typically a processor.

The **attitude sensors** take measurements which are used to compute the current spacecraft attitude and/or angular velocity.

The **attitude actuators** then supply torques to correct the difference between the measured and desired attitude.

The mathematical relationship between the measured attitude and the corrective torques is called a **control law.**
It is implemented as a program on the processor.

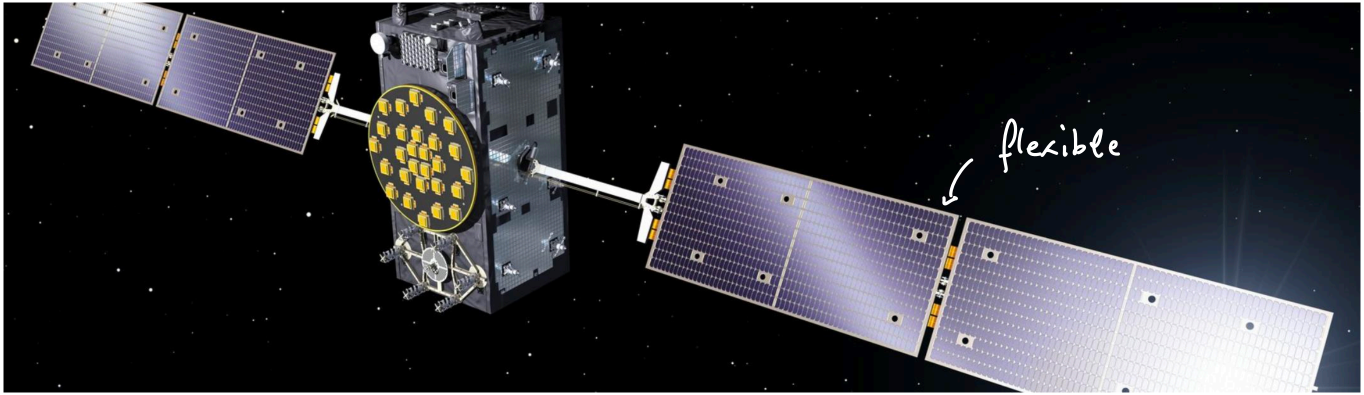
usually the antennae are mounted with a gimbal in order to be directed independly from the main body of the s/c



A **common spacecraft structure** consists of two principal parts.

The first one is the ***body of the spacecraft***, which contains all the payload instrumentation and control hardware.
This structure must be very rigid in order to withstand mechanical loads during the launch phase.

The second part consists of (large) ***flexible appendages*** (parabolic antennas, large synthetic-aperture radar, solar panels, booms, …) built from light materials in order to reduce weight.



LSAC – simplified configuration

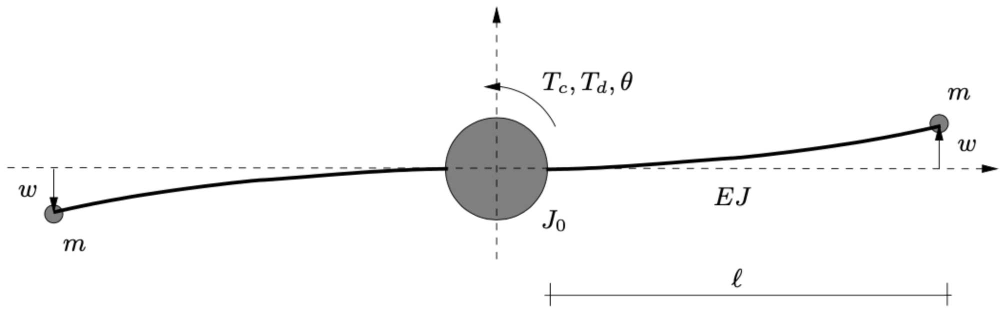
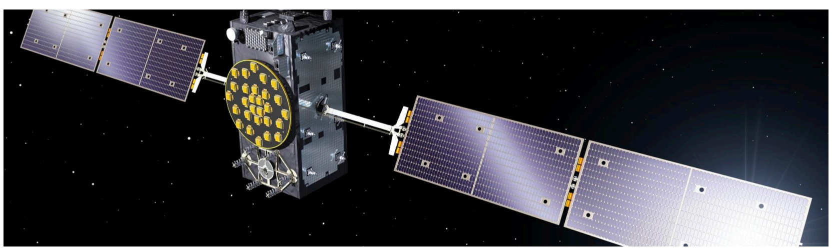


Figure 16.1 A simplified spacecraft rotating about a single axis.

simplified configuration

As a first approx we will ignored the flexibility of the all structure. Then we will consider it to get a more accurate model.



Central rigid body (moment of inertia J_0)
Two identical appendages (**massless beams of length l, bending rigidity EJ, carrying a tip mass m**)
Control torque $T_c(t)$
Sensor measuring rotation $\theta(t)$
Disturbances (e.g., solar radiation pressure) equivalent to a disturbance torque $T_d(t)$ on the central body

System modeling – **RIGID CASE (rigid appendages)**.

One degree of freedom: $\theta(t)$

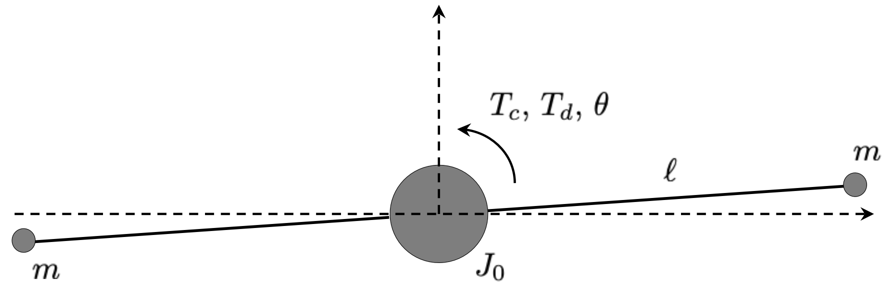
Lagrange's equations:

kinetic energy   potential energy

$$\mathcal{L} = T(\dot{\theta}, \theta) - V(\theta)$$

↳ Lagrange polinomial

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = Q_\theta}$$



$$T = \frac{1}{2} J_0 \dot{\theta}^2 + 2\frac{1}{2} m \dot{s}^2$$

$V = 0$ *(rigid case)*

$$\delta W_{nc} = \delta\theta T_c + \delta\theta T_d = \delta\theta Q_\theta$$

In the rigid case:

$\dot{s} = \ell\dot{\theta}$

Review of classical control design
LSAC – rigid case – equation of motion

Equation of motion
(time-domain representation)

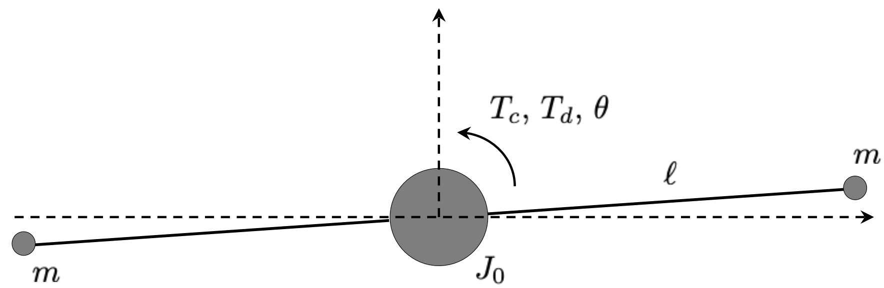
$$J\ddot{\theta}(t) = T_c(t) + T_d(t)$$

$$\theta(0) = 0$$

$$\dot{\theta}(0) = 0$$

Initial condition on the angle and angular state.

Linear time-invariant system
(LTI system)



where

$$J = J_0 + 2m\ell^2$$

is the total equivalent moment of inertia of the system.

Usually we switch to a different domain.

Laplace (frequency) representation

$$\theta(t) \rightleftarrows \theta(s)$$

Laplace transform:

$$\theta(s) = \mathcal{L}\left[\theta(t)\right]$$

$$\theta(t) = \mathcal{L}^{-1}\left[\theta(s)\right]$$

Definition of Laplace transform:

$$f(s) = \mathcal{L}\left[f(t)\right] = \int_{0_-}^{\infty} f(t)e^{-st}\mathrm{d}t$$

where s is a complex variable

$$s = \sigma + j\omega$$

and ω is the frequency (rad/s)

Laplace transforms can be used to study the complete response characteristics of feedback systems, including the transient response. This is in contrast to Fourier transforms, in which the steady-state response is the main concern.

Review of classical control design
[review] *Laplace transform - properties*

Properties of the Laplace transform:

Ripassare elettrotecnica per le trasformazioni !

$$\mathcal{L}\left[af(t)+bg(t)\right]=af(s)+bg(s)$$ (linearity - superposition)

→ vedere automatica

$$\mathcal{L}\left[\dot{f}(t)\right]=sf(s)-f(0)$$

(differentiation)

$$\mathcal{L}\left[\ddot{f}(t)\right]=s^2f(s)-sf(0)-\dot{f}(0)$$

$$\mathcal{L}\left[\int_0^t f(\tau)\mathrm{d}\tau\right]=\frac{1}{s}f(s)$$ (integration)

$$\lim_{t\to\infty} f(t)=\lim_{s\to 0} sf(s)$$ (final value theorem)

$$f(0)=\lim_{s\to\infty} sf(s)$$ (initial value theorem)

→ Important.

[review] *Laplace transform - table*

Common Laplace transforms:

(delta (impulse) function) $$\mathcal{L}\left[\delta(t)\right] = 1$$

(step function) $$\mathcal{L}\left[\mathrm{step}(t)\right] = \frac{1}{s}$$

(ramp function) $$\mathcal{L}\left[\mathrm{ramp}(t)\right] = \frac{1}{s^2}$$

(exponential function) $$\mathcal{L}\left[e^{-at}\mathrm{step}(t)\right] = \frac{1}{s+a} \qquad (a > 0)$$

(sine function) $$\mathcal{L}\left[\sin(\omega t)\mathrm{step}(t)\right] = \frac{\omega}{s^2+\omega^2}$$

(cosine function) $$\mathcal{L}\left[\cos(\omega t)\mathrm{step}(t)\right] = \frac{s}{s^2+\omega^2}$$

The laplace transform of a system dynamics

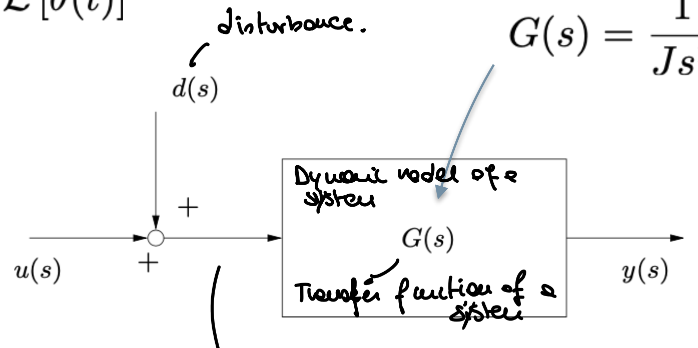
Laplace (frequency) representation $\theta(s) = \mathcal{L}\left[\theta(t)\right]$

output

$$s^2 J\theta(s) = T_c(s) + T_d(s)$$ input

$$\boxed{y(s) = G(s)u(s) + G(s)d(s)}$$

$$G(s) = \frac{1}{Js^2}$$



Block diagram of the open-loop system.

Input is the sum of the u(s) and d(s) (disturbances).

(control input)

where

- $y(s)$ is the **system (sensor) output** (Laplace transform)
- $u(s)$ is the **control (actuator) input** (L.T.)
- $y(s)$ is the **disturbance** (L.T.)
- $G(s)$ is the **system (open-loop) transfer function** (from control input to sensor output and from disturbance to sensor output)

The system output (sensor measurement) is typically affected by noise (sensor noise).

By considering additive noise:

$$y(s) = G(s)u(s) + G(s)d(s) + n(s)$$

If the output of the system is mesured by a sensor it will subjected to some kind of noise

where

$n(s)$ is the **output (sensor) noise** (Laplace transform)

noise.

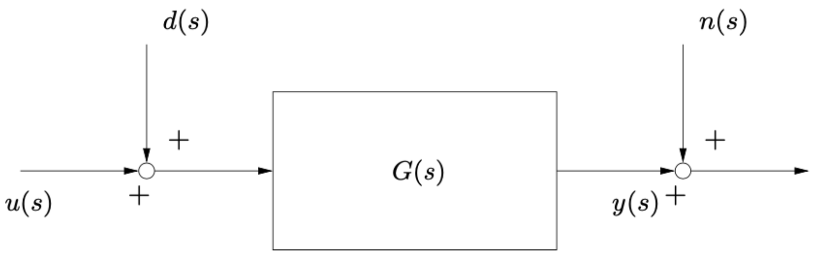
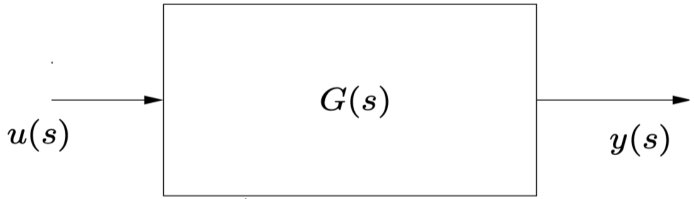


Figure 16.2 Block diagram of the open-loop system.

[review] *Transfer function*



out in

$$y(s) = G(s)u(s)$$

The transfer function is the ratio of the Laplace transform of the output of the system to its input assuming all zero initial conditions

$$G(s) = \frac{N(s)}{D(s)} = \frac{\beta_0 + \beta_1 s + \beta_2 s^2 + \cdots + \beta_\nu s^\nu}{\alpha_0 + \alpha_1 s + \alpha_2 s^2 + \cdots + \alpha_n s^n}$$

$\nu < n$	strictly proper
$\nu = n$	(simply) proper
$\nu > n$	improper

$$N(s) = 0 \;\Rightarrow\; \nu \text{ roots} - \text{zeros } z_i$$

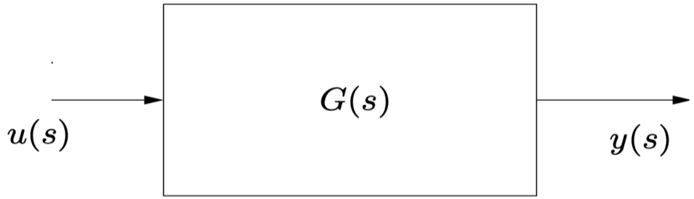
$$\mathcal{L}\left[af(t) + bg(t)\right] = af(s) + bg(s)$$

gain

$$G(s) = \frac{\mu}{s^g}\frac{\prod_i (s - z_i)}{\prod_i (s - p_i)}$$

type → How many poles and zeros we have at the origin.

Review of classical control design
[review] *Transfer function*



The system has the inherent capability to block frequencies coinciding with its zero locations.

The poles of the system determine its stability properties.

$$G(s) = \frac{\mu}{s^g} \frac{\prod_i (s - z_i)}{\prod_i (s - p_i)}$$

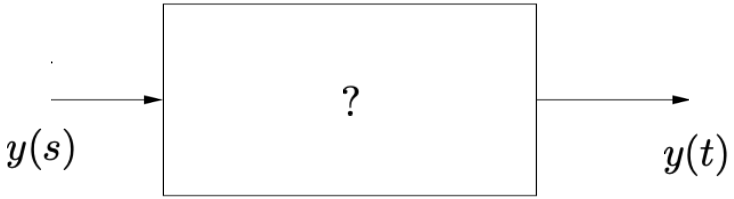
real zeros and poles: $G(s) = \frac{\tilde{\mu}}{s^g} \frac{\prod_i (1 + s\tau_i)}{\prod_i (1 + sT_i)}$

(→ can be both.

second order terms to represent complex conjugate poles and zeros

complex conjugate zeros and poles: $G(s) = \frac{\tilde{\mu}}{s^g} \frac{\prod_i (1 + 2\frac{\zeta_i}{\alpha_i} s + \frac{s^2}{\alpha_i^2})}{\prod_i (1 + 2\frac{\xi_i}{\omega_i} s + \frac{s^2}{\omega_i^2})}$

[review] *Convolution integral*



Consider the case: $u(t) = \delta(t)$

with the system initially at rest.

The response of the system to this input is called **impulse response.**

It is denoted as $h(t)$

Impulse: a very intense force for a very short time.

Delta function – properties:

$$\delta(t) = 0 \quad t \neq 0$$

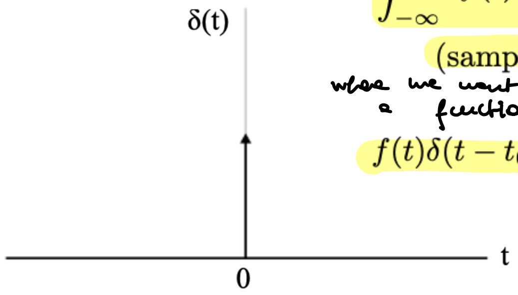
$$\int_{-\infty}^{+\infty} \delta(t)\mathrm{d}t = 1 \quad \text{(unit area)}$$

$$\int_{-\infty}^{+\infty} f(t)\delta(t-t_0)\mathrm{d}t = f(t_0)$$

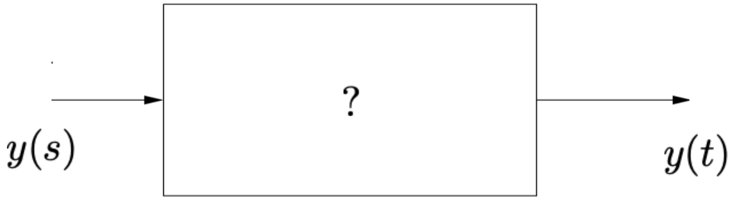
(sampling property)

where we want to extract the value of a function.

$$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$$



Review of classical control design
[review] *Convolution integral*



It is possible to express (compute) the system's response to any arbitrary input *u(t)* by using the impulse response *h(t)*.

We are able to solve for the response of a linear system to a general signal simply by decomposing the given signal into a sum of the elementary components and, by superposition, concluding that the response to the general signal is the sum of the responses to the elementary signals.

$$\boxed{y(t) = \int_0^\infty h(t-\tau)u(\tau)d\tau}$$

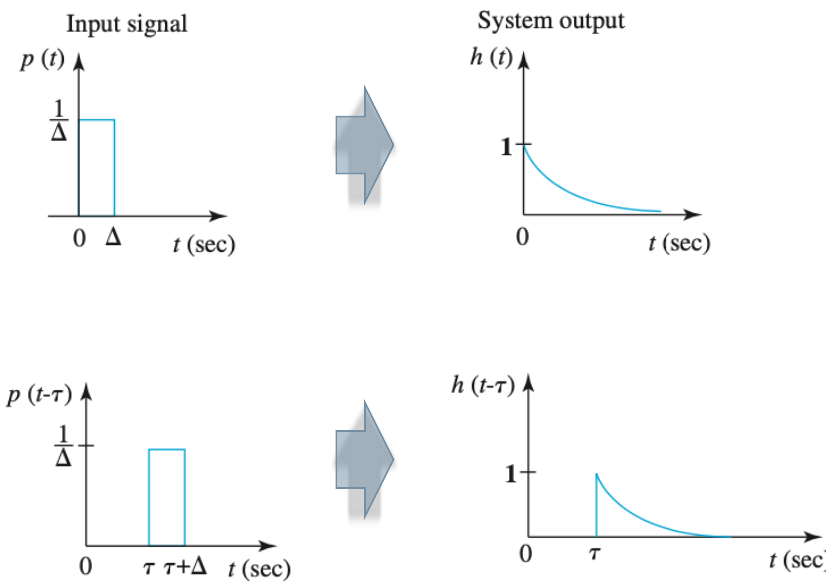
Convolution integral

Review of classical control design
[review] *Convolution integral*

Suppose the input signal to an LTI system
is a short pulse *p(t)*
and the corresponding output signal is *h(t).*

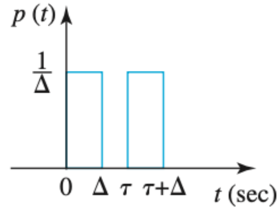
If the input is scaled to *u(0)p(t)*
then, by the scaling property of superposition,
the output response will be *u(0)h(t).*

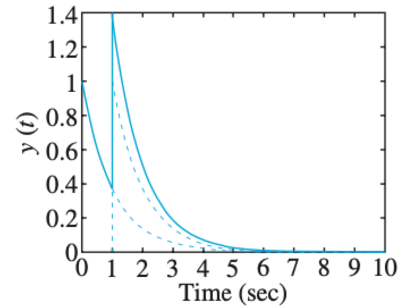
If we delay the short pulse in time by τ
the input has the form $p(t\text{-}\tau)$
and the output response will be delayed
by the same amount $y(t\text{-}\tau)$.



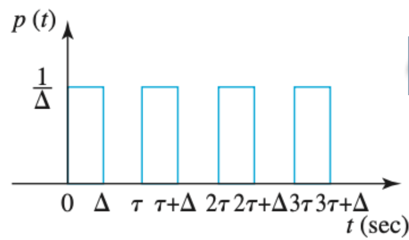
[review] *Convolution integral*

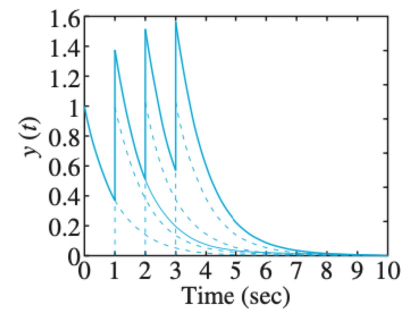
By superposition, the response to the two short pulses will be the sum of their individual outputs.





If we have four pulses as the input, then the output will be the sum of the four individual responses



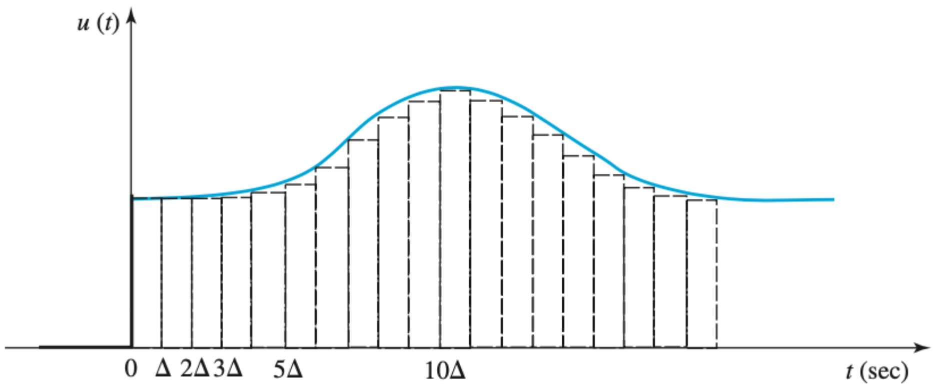


…and so on…

[review] *Convolution integral*

Any arbitrary input signal *u(t)* may be approximated by a series of pulses.

Combination of an infinite set of impulses with different magnitude.



[review] *Convolution integral*

Short pulse as a rectangular pulse having unit area

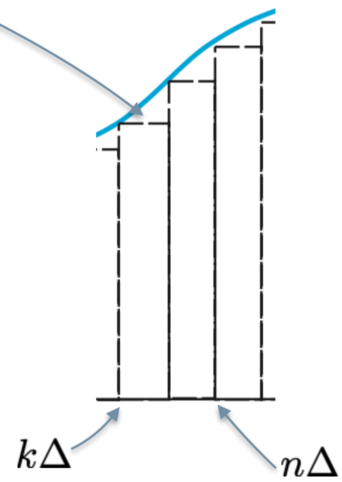
$$p_\Delta(t) = \begin{cases} \frac{1}{\Delta}, & 0 \le t \le \Delta \\ 0, & \text{elsewhere} \end{cases}$$

The response of the system is $h_\Delta(t)$

The response at time $n\Delta$ to $u(k\Delta)p_\Delta(k\Delta)$ is $u(k\Delta)h_\Delta(n\Delta - k\Delta)$

By superposition, the total response at time *t* to the series of the short pulses is given by

$$y(t) = \sum_{k=0}^{\infty} u(k\Delta)h_\Delta(t - k\Delta)$$



Taking the limit as $\Delta \to 0$

$$\lim_{\Delta \to 0} p_\Delta(t) = \delta(t)$$

$$\lim_{\Delta \to 0} h_\Delta(t) = h(t) = \text{the impulse response}$$

$$y(t) = \sum_{k=0}^{\infty} u(k\Delta) h_\Delta(t - k\Delta)$$

$$\boxed{y(t) = \int_0^{\infty} h(t-\tau)u(\tau)d\tau}$$

Equivalent forms:

$$y(t) = \int_0^{t} h(t-\tau)u(\tau)\mathrm{d}\tau$$

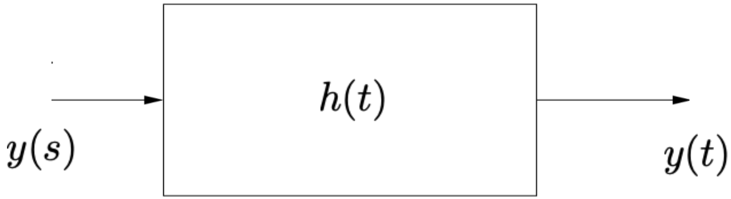
impulse response of a system

$$y(t) = \int_0^{t} h(t-\tau)u(\tau)\mathrm{d}\tau$$

shape of the output.

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)u(t-\tau)\mathrm{d}\tau$$

[review] *Frequency response*



Consider the case: $u(t) = \sin(\omega t) \quad (\omega \geq 0)$

with the system initially at rest.

It can be shown that

$$\boxed{y(t) = |H(j\omega)| \sin(\omega t + \phi)}$$

where

frequency response

$$H(j\omega) = \int_0^{+\infty} h(t) e^{-j\omega t} \mathrm{d}t$$

not a generic Laplace transform → $s = \sigma + j\omega$

Important to understand the performance of a given system.

Frequency response function:

$$H(j\omega) = H(s = j\omega)$$

$$H(j\omega) = |H(j\omega)| e^{j\phi}$$

$$\phi = \arg H(j\omega) = \tan^{-1} \frac{\mathrm{Im}\left[H(j\omega)\right]}{\mathrm{Re}\left[H(j\omega)\right]}$$

[review] *Bode diagram (plot)*

Transfer function can be always rappresented as

Plot of $G(j\omega) = G(s = j\omega)$ magnitude and phase

first order $\qquad$ second order

$$G(s) = \frac{\mu}{s^g} \frac{\prod_i (1 + s\tau_i)}{\prod_i (1 + sT_i)} \frac{\prod_i (1 + 2\frac{\zeta_i}{\alpha_i}s + \frac{s^2}{\alpha_i^2})}{\prod_i (1 + 2\frac{\xi_i}{\omega_i}s + \frac{s^2}{\omega_i^2})}$$

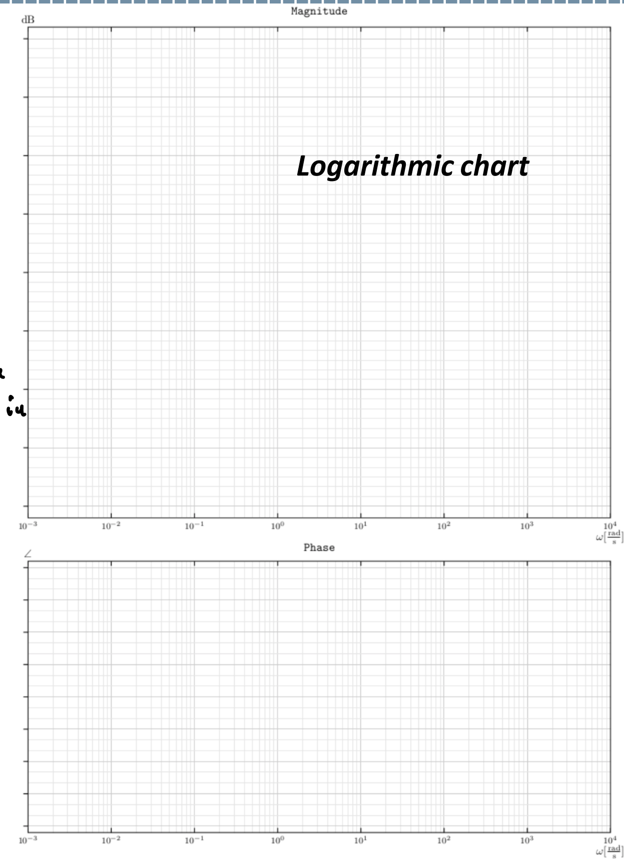
↳ number of singularity at the origin. g can be < 0 in that case we have an excess of zeros. in normal mechanical system correctly modelled we will have g > 0 so an excess of poles in the origin.

$$G(j\omega) = \frac{\mu}{(j\omega)^g} \frac{\prod_i (1 + j\omega\tau_i)}{\prod_i (1 + j\omega T_i)} \frac{\prod_i (1 + 2\frac{\zeta_i}{\alpha_i}j\omega - \frac{\omega^2}{\alpha_i^2})}{\prod_i (1 + 2\frac{\xi_i}{\omega_i}j\omega - \frac{\omega^2}{\omega_i^2})}$$

Magnitude: $\quad |G(j\omega)|_{\mathrm{dB}} = 20\log_{10}|G(j\omega)|$

Phase: $\quad \arg G(j\omega)$

They give an information of what the system is like when we change the frequency.



The Bode diagram can be obtained as a superimposition (overlap) of contributes. we have 4 different type of contribution.

$$G(j\omega) = \frac{\mu}{(j\omega)^g} \frac{\prod_i (1 + j\omega\tau_i)}{\prod_i (1 + j\omega T_i)} \frac{\prod_i (1 + 2\frac{\zeta_i}{\alpha_i} j\omega - \frac{\omega^2}{\alpha_i^2})}{\prod_i (1 + 2\frac{\xi_i}{\omega_i} j\omega - \frac{\omega^2}{\omega_i^2})}$$

The gain will be the linear combination of the gain of the single terms

Four classes of terms:

1. μ
2. $\frac{1}{(j\omega)^g}$
3. $(1 + j\omega T)^{\pm 1}$
4. $\left(1 + 2\xi\frac{j\omega}{\omega_i} - \frac{\omega^2}{\omega_i^2}\right)^{\pm 1}$

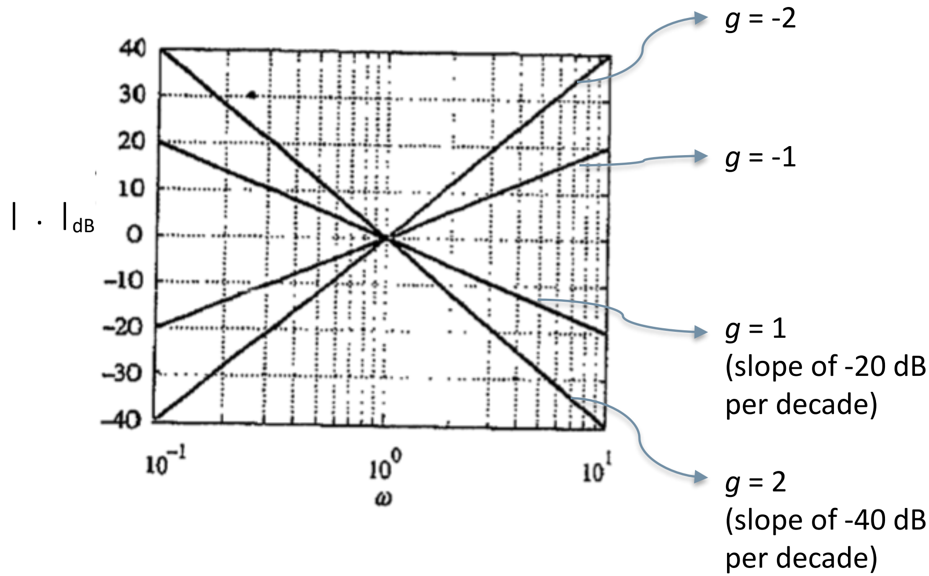
Magnitude:

steady state gain → magnitude of the frequency response when $f = \phi$

$$|G(j\omega)|_{\mathrm{dB}} = 20\log|\mu| - 20g\log|j\omega|$$
$$+ \sum_i 20\log|1 + j\omega\tau_i| + \sum_i 20\log|1 + 2\frac{\zeta_i}{\alpha_i} j\omega - \frac{\omega^2}{\alpha_i^2}|$$
$$- \sum_i 20\log|1 + j\omega T_i| - \sum_i 20\log|1 + 2\frac{\xi_i}{\omega_i} j\omega - \frac{\omega^2}{\omega_i^2}|$$

[review] *Bode diagram - magnitude*

$$-20g \log |j\omega|$$



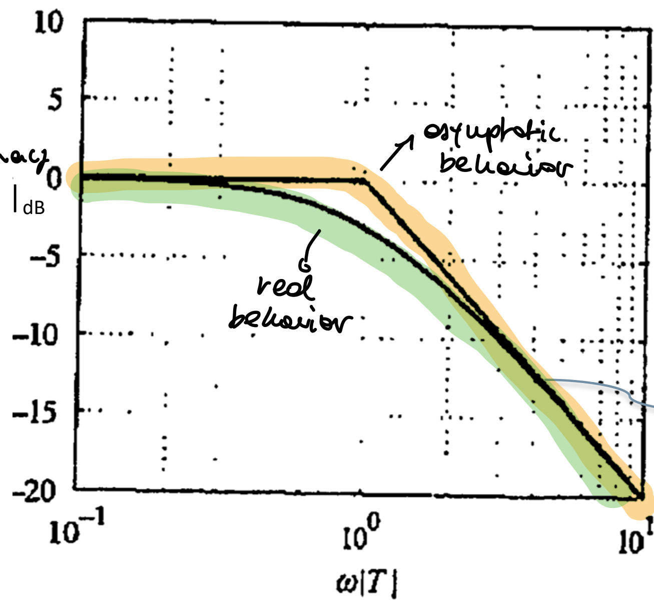
[review] *Bode diagram - magnitude*

$$- 20 \log |1 + j\omega T|$$
$$= -20 \log \sqrt{1 + \omega^2 T^2}$$

looking only at high or low value of frequency respect to the value of the singularity $|\cdot|_{dB}$

Asymptotic behavior:

$$\begin{cases} = -20 \log 1 = 0 & \omega \ll \dfrac{1}{|T|} \\ = -20 \log \omega |T| & \omega \gg \dfrac{1}{|T|} \end{cases}$$



slope of -20 dB per decade

[review] *Bode diagram - magnitude*

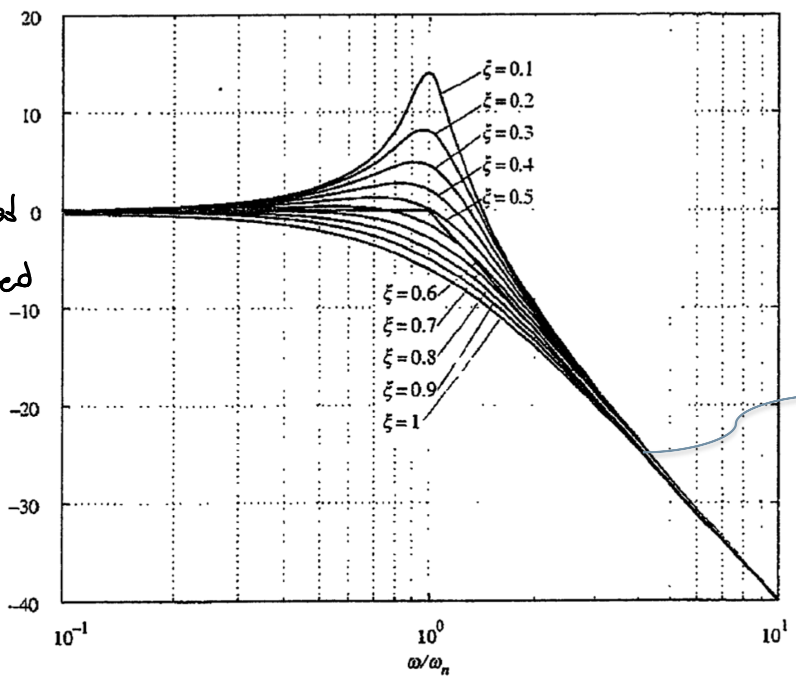
adimensional damping factor.

$$-20\log\left|1+2\frac{\xi_i}{\omega_i}j\omega-\frac{\omega^2}{\omega_i^2}\right|$$

Depending on the damping the real gain can be underestimated or under estimated.

$\xi < 0.5$ the gain is underestimated

$\xi > 0.6$ the gain is overestimated by the linear approx.



slope of -40 dB per decade

Phase:

$$\arg G(j\omega) = \arg\mu - g\arg(j\omega) + \sum_i \arg(1+j\omega\tau_i) + \sum_i \arg\left(1+2\frac{\zeta_i}{\alpha_i}j\omega - \frac{\omega^2}{\alpha_i^2}\right) - \sum_i \arg(1+j\omega T_i) - \sum_i \arg\left(1+2\frac{\xi_i}{\omega_i}j\omega - \frac{\omega^2}{\omega_i^2}\right)$$

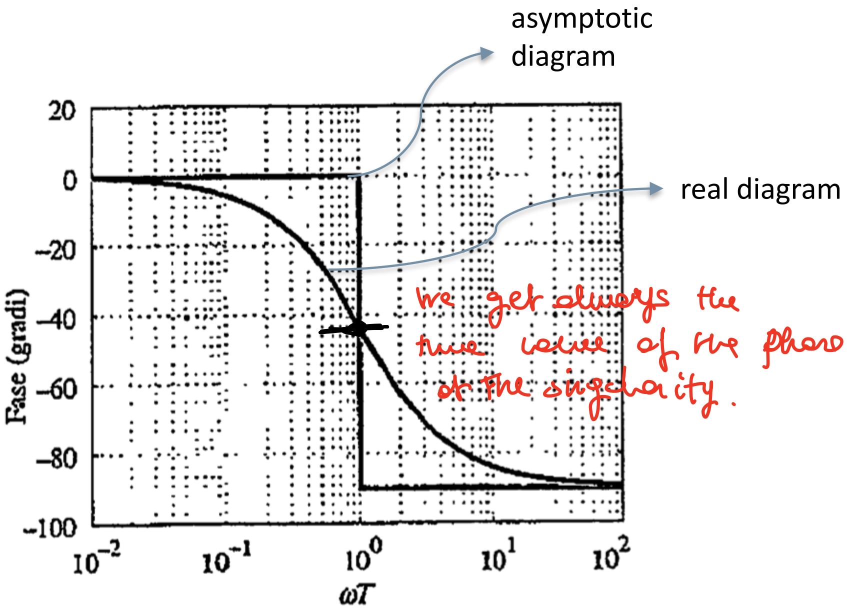
$$\arg\mu = \begin{cases} 0^\circ & ,\ \mu > 0 \\ -180^\circ & ,\ \mu < 0 \end{cases}$$ phase of the gain.

$$\arg\left(\frac{1}{j\omega}\right) = -90^\circ$$

Poles in the origin (g >0): phase delay
Zeros in the origin (g < 0): phase lead

$$-\arg(1 + j\omega T)$$

$$\simeq \begin{cases} -\arg(1) = 0°, & \omega \ll 1/|T| \\ -\arg(j\omega T) = \begin{Bmatrix} -90° & , T > 0 \\ +90° & , T < 0 \end{Bmatrix}, & \omega \gg 1/|T| \end{cases}$$



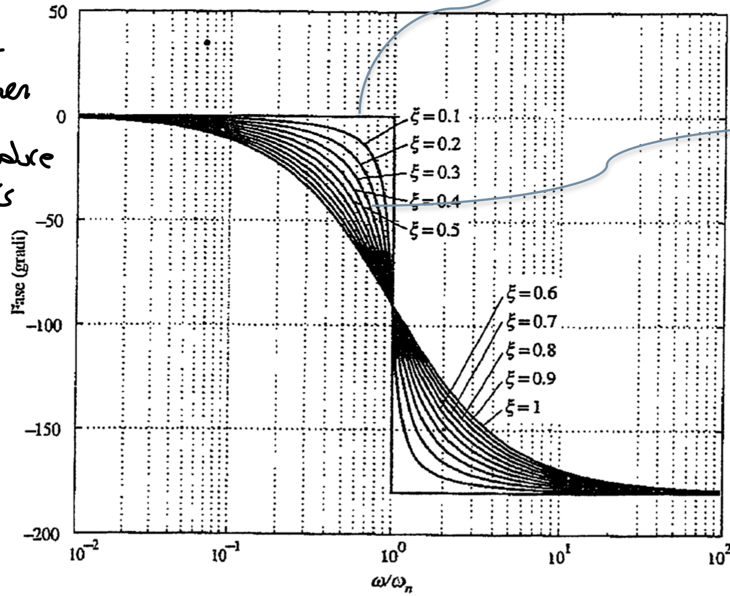
[review] *Bode diagram - phase*

$$-\arg\left(1 + 2\frac{\xi_i}{\omega_i} j\omega - \frac{\omega^2}{\omega_i^2}\right)$$

In case of the phase the damping coefficient as it is lower → closer to zero the error will be relative → the direction of the error is always the same.

asymptotic diagram

real diagram

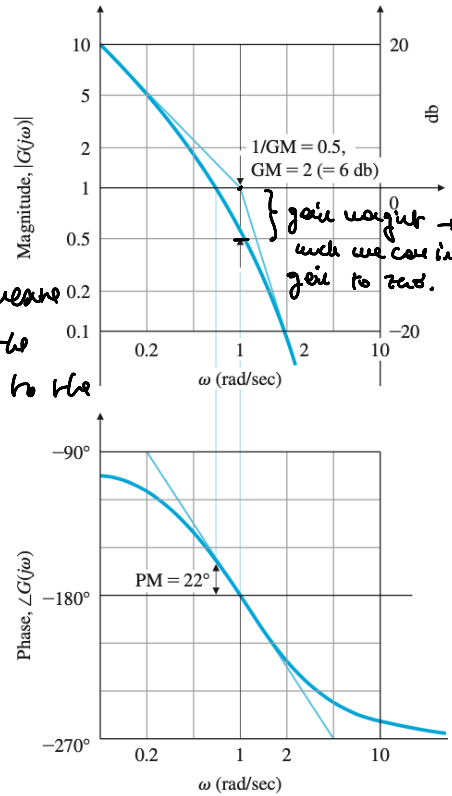


They measure the degree of stability of a closed-loop system.

The gain margin (GM) is the factor by which the gain can be increased (or decreased in certain cases) before instability results.

The phase margin (PM) is the amount by which the phase of G (jω) exceeds −180° when |KG(jω)| = 1.

gain and phase margin are a measure on how much the system is closed to the instability.



gain margin → how much we can increase the gain to zero.

LSAC – Control scheme and objectives

The goal of the present application is to design a feedback control system (regulator *R(s)*) such that the spacecraft is rotated by a desired angle ($\theta_r = r$, reference angle) from an initial attitude angle (θ_0 = 0) at time t=t_0=0.

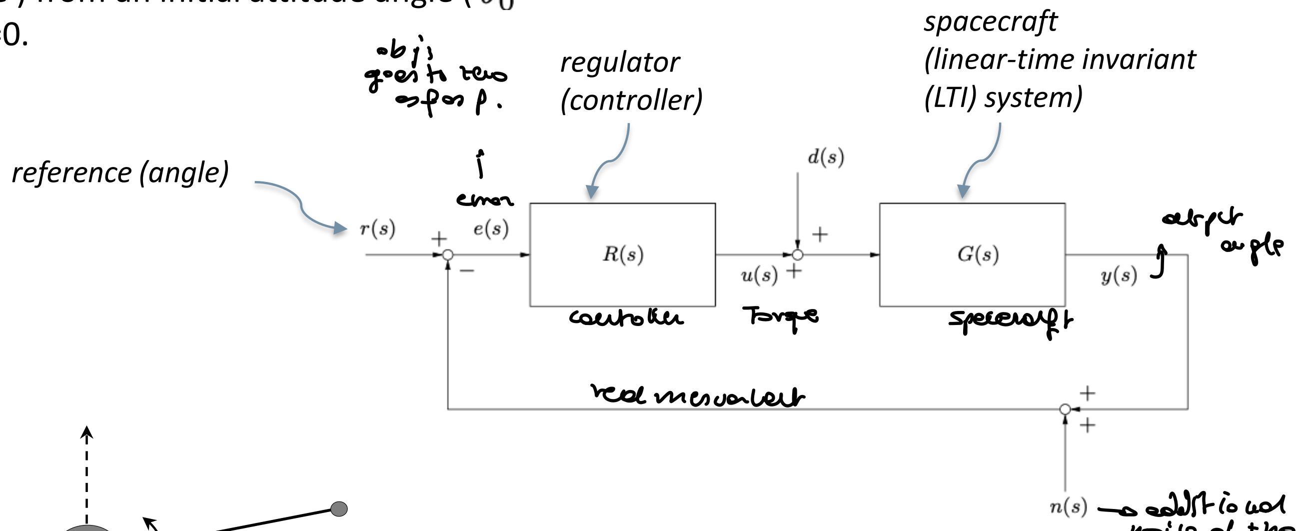
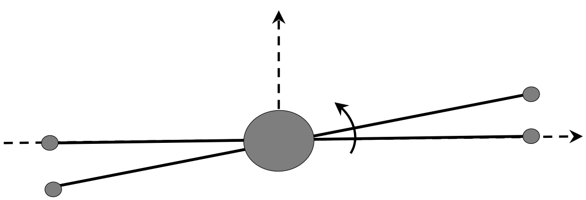


Figure 16.3 Block diagram of the closed-loop system.



Goals of the feedback control system:

- **a)** **closed-loop stability**
- **b)** **reference tracking** (with prescribed static and dynamic performance)
- c) capability of **rejecting disturbance**
- d) capability of **rejecting measurement noise**
- **e)** **robustness** against model uncertainties

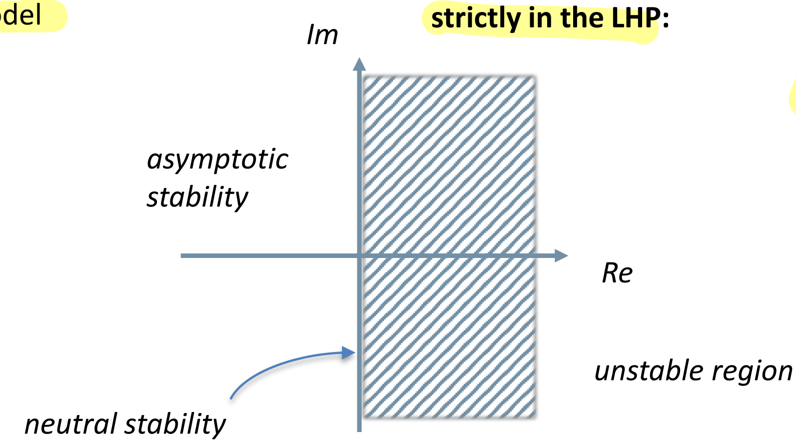
a) **Stability**

An LTI system whose transfer function is

$$G(s) = \frac{N(s)}{D(s)} = \frac{\beta_0 + \beta_1 s + \beta_2 s^2 + \cdots + \beta_\nu s^\nu}{\alpha_0 + \alpha_1 s + \alpha_2 s^2 + \cdots + \alpha_n s^n}$$

is stable if **all the poles p_i (roots of D(s)) are strictly in the LHP**:

$$\boxed{\mathrm{Re}\{p_i\} < 0}$$

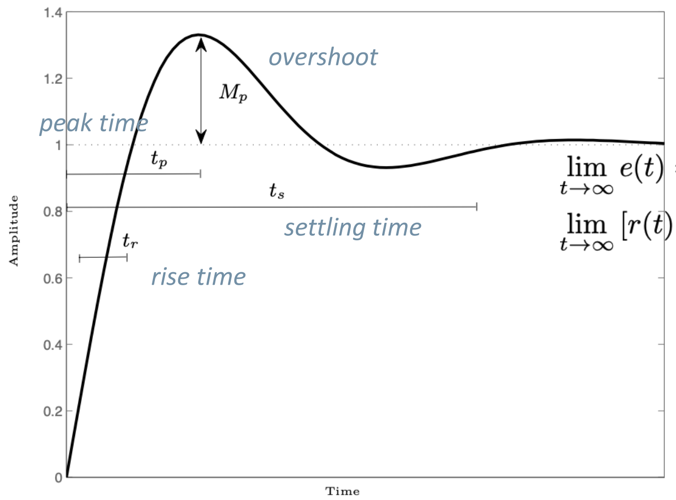


Goals of the feedback control system:

a) closed-loop stability
b) reference tracking (with prescribed static and dynamic performance)
c) capability of rejecting disturbance
d) capability of rejecting measurement noise
e) robustness against model uncertainties

b) **Reference tracking**

Steady-state behavior (static requirements)
Transient behavior (dynamic requirements)



$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} [r(t) - y(t)] = 0$$

Review of classical control design
LSAC – Control scheme and objectives

Goals of the feedback control system:

a) closed-loop stability
b) reference tracking (with prescribed static and dynamic performance)
c) capability of rejecting disturbance
d) capability of rejecting measurement noise
e) robustness against model uncertainties

c), d) **Rejection capabilities**

Good reference tracking is achieved when the system is subjected to disturbance forces.

Good reference tracking is achieved when the system is subjected to measurement noise.

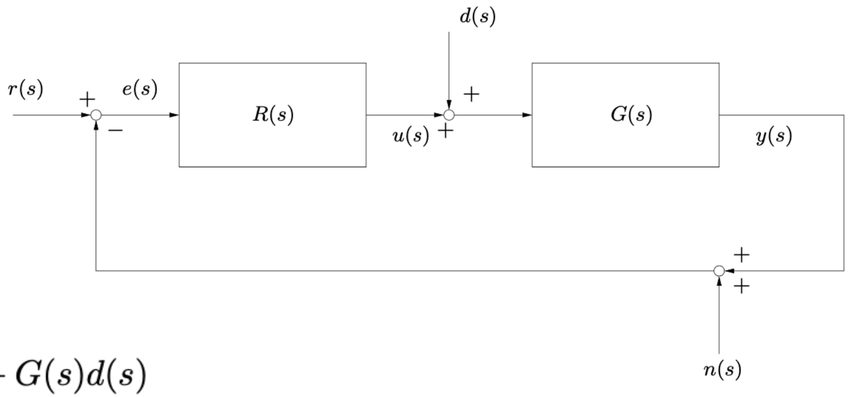
e) **Robustness**

The design of the controller must be sufficiently robust against modeling errors and/or variation of the system parameters (not any perturbation of the system but those falling within some uncertainty margins)

Review of classical control design
LSAC – Basic equations of control

$$u(s) = R(s)e(s)$$

$$e(s) = r(s) - [y(s) + n(s)]$$



$$\begin{aligned} y(s) &= G(s)R(s)e(s) + G(s)d(s) \\ &= G(s)R(s)\,[r(s) - y(s) - n(s)] + G(s)d(s) \\ &= \frac{R(s)G(s)}{1+R(s)G(s)}r(s) - \frac{R(s)G(s)}{1+R(s)G(s)}n(s) + \frac{G(s)}{1+R(s)G(s)}d(s) \end{aligned}$$

$$\boxed{y(s) = \frac{L(s)}{1+L(s)}r(s) + \frac{G(s)}{1+L(s)}d(s) - \frac{L(s)}{1+L(s)}n(s)}$$

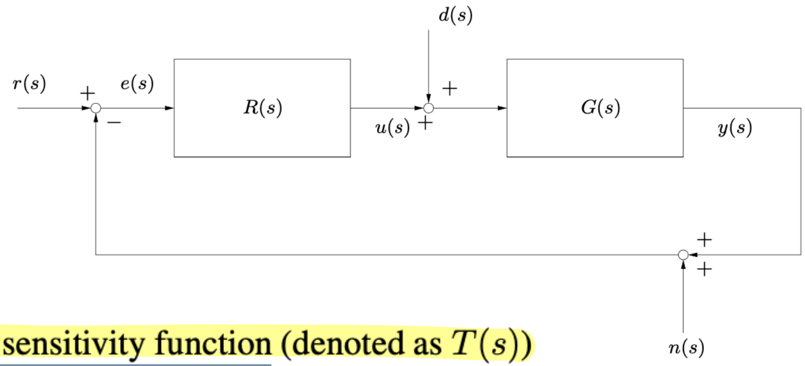
→ output

we want some design characteristic → for example disturbance rejection.

Loop transfer function

$$L(s) = R(s)G(s)$$

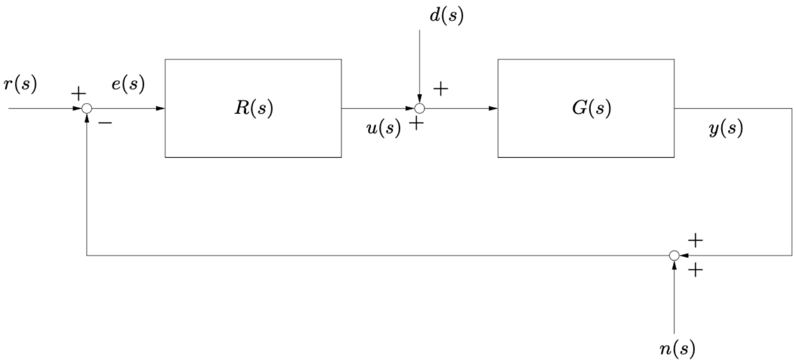
The objective is to have this stable



$$y(s) = \frac{L(s)}{1+L(s)} r(s) + \frac{G(s)}{1+L(s)} d(s) - \frac{L(s)}{1+L(s)} n(s)$$

- $\dfrac{L(s)}{1+L(s)}$, which is called complementary sensitivity function (denoted as $T(s)$)
- $\dfrac{G(s)}{1+L(s)}$, which is called disturbance sensitivity function
- $\dfrac{L(s)}{1+L(s)}$, which is called noise sensitivity function
- $\dfrac{1}{1+L(s)}$, which is called sensitivity function (denoted as $S(s)$)

$$\boxed{T(s) + S(s) = 1}$$



$$y(s) = \frac{L(s)}{1+L(s)}r(s) + \frac{G(s)}{1+L(s)}d(s) - \frac{L(s)}{1+L(s)}n(s)$$

a) **closed-loop stability**

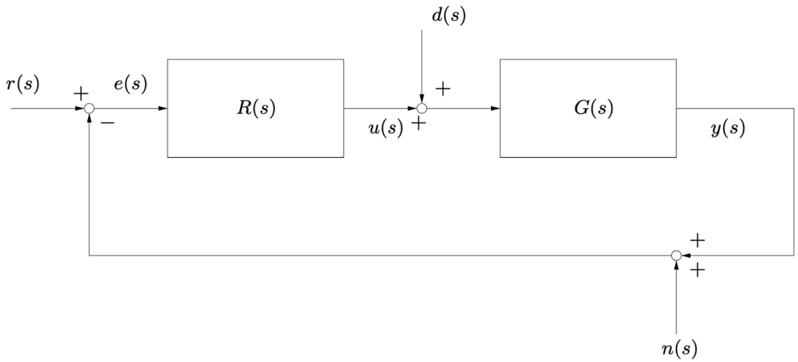
Re(closed-loop poles) < 0

closed-loop poles: roots of $\boxed{1 + L(s) = 0}$

(common denominator of the sensitivity functions of the system)

Review of classical control design
LSAC – Design guidelines

$$y(s) = \frac{L(s)}{1+L(s)}r(s) + \frac{G(s)}{1+L(s)}d(s) - \frac{L(s)}{1+L(s)}n(s)$$



b),c),d) **tracking and rejection capabilities**

$$\frac{L(s)}{1+L(s)} \to 1 \quad \text{for reference tracking}$$

$$\frac{G(s)}{1+L(s)} \to 0 \quad \text{for disturbance rejection}$$

$$\frac{L(s)}{1+L(s)} \to 0 \quad \text{for noise rejection}$$

$|L(j\omega)| \gg 1$ **good tracking and disturbance rejection**

$|L(j\omega)| \ll 1$ **good noise rejection**

They are incompatible!!!
(cannot be simultaneously met at all frequencies)
but…

A satisfactory design can be generally achieved since the frequency content of the reference *r(s)* and disturbance *d(s)* is typically well separated from the frequency content of the measurement noise *n(s)*.

ω

low-frequency

$$|L(j\omega)| \gg 1$$

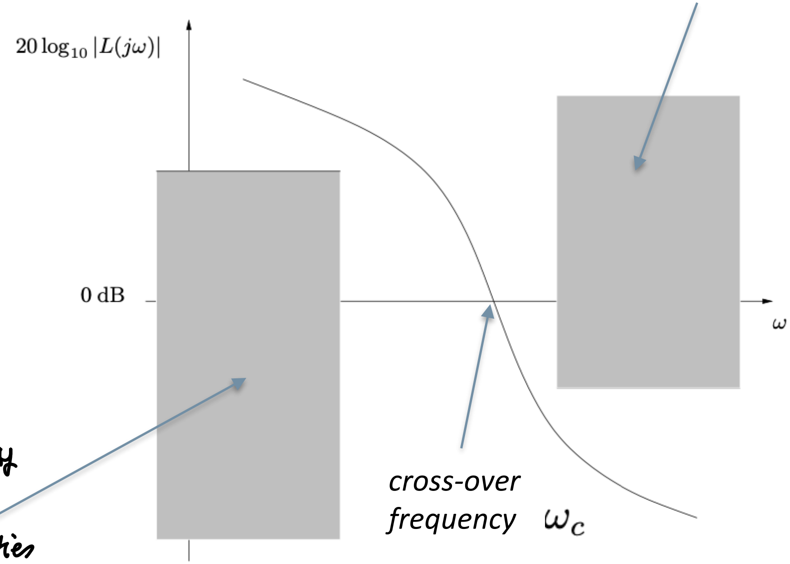
for lower frequency we want to have a good reference tracking

high-frequency

$$|L(j\omega)| \ll 1$$

for higher frequency we want to have a good noise rejection capabilities

noise rejection



cross-over frequency ω_c

tracking and disturbance rejection

Figure 16.5 Example of desired shape of magnitude of the loop transfer function $|L(j\omega)|$.

low-pass filtering

$$|T(j\omega)| \approx \begin{cases} 1 & \omega \leq \omega_c \\ 0 & \omega > \omega_c \end{cases}$$

$$|S(j\omega)| \approx \begin{cases} 0 & \omega \leq \omega_c \\ 1 & \omega > \omega_c \end{cases}$$

high-pass filtering

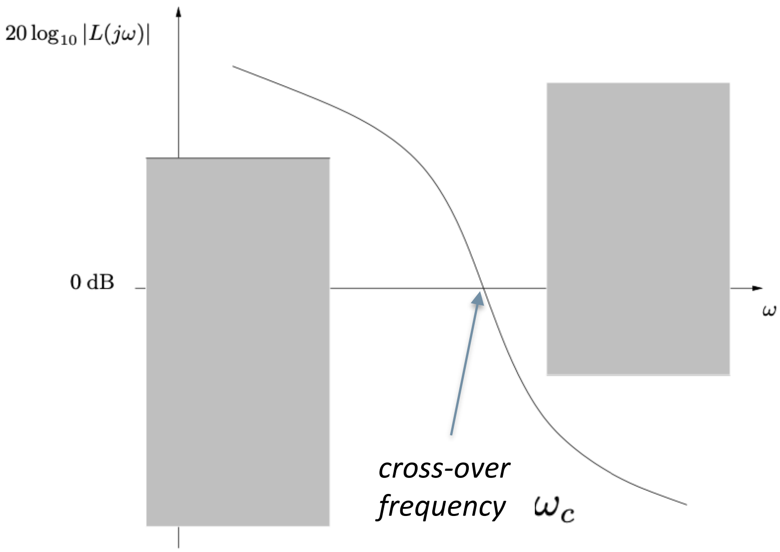


Figure 16.5 Example of desired shape of magnitude of the loop transfer function $|L(j\omega)|$.

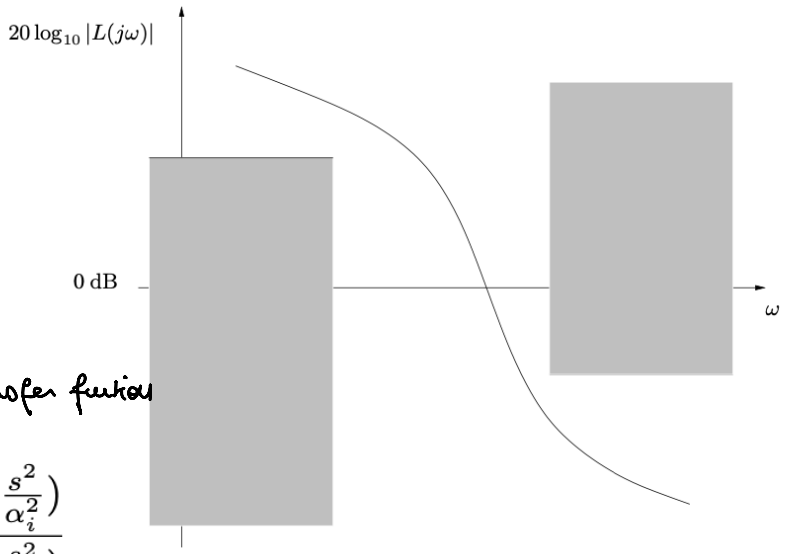
Review of classical control design
LSAC – Design guidelines

Tracking capabilities - static requirements
(typically achieved by imposing a minimum slope of L(s) at s=0):

steady-state error

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} se(s)$$

$$= \lim_{s\to 0} \frac{s}{1+L(s)} r(s)$$



General form of L(s) → general representation of a transfer function

$$L(s) = \frac{\mu}{s^g} \frac{\prod_i (1+s\tau_i)}{\prod_i (1+sT_i)} \frac{\prod_i (1+2\frac{\zeta_i}{\alpha_i}s + \frac{s^2}{\alpha_i^2})}{\prod_i (1+2\frac{\xi_i}{\omega_i}s + \frac{s^2}{\omega_i^2})}$$

Step reference (r(s) = 1/s):

$$e_{\mathrm{ss}} = \lim_{s\to 0} \frac{s}{1+L(s)} \frac{1}{s} = \lim_{s\to 0} \frac{1}{1+L(s)} = \frac{1}{1+\lim_{s\to 0} L(s)}$$

g = 0: $$e_{\mathrm{ss}} = \frac{1}{1+\mu}$$

g > 0: $$e_{\mathrm{ss}} = 0$$

to get $e_{ss} = 0$ for step reference

l_0 error $\neq 0$ I can make μ big to reduce e_{ss}

Ramp reference (r(s) = 1/s^2):

$$e_{\mathrm{ss}} = \lim_{s\to 0} \frac{s}{1+L(s)} \frac{1}{s^2} = \lim_{s\to 0} \frac{1}{s+sL(s)} = \lim_{s\to 0} \frac{1}{sL(s)}$$

g = 0: $$e_{\mathrm{ss}} = \infty$$

g = 1: $$e_{\mathrm{ss}} = \frac{1}{\mu}$$

g > 1: $$e_{\mathrm{ss}} = 0$$

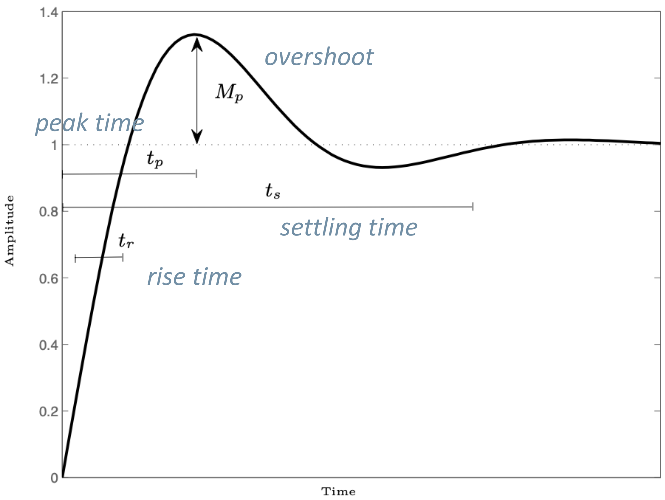
Tracking capabilities - dynamics requirements

By referring to a step reference:

prescribed maximum peak (rise) time
prescribed maximum settling time
prescribed maximum overshoot

Correspondence between the above transient characteristics and the **shaping of the loop function** (bandwidth, phase margin).

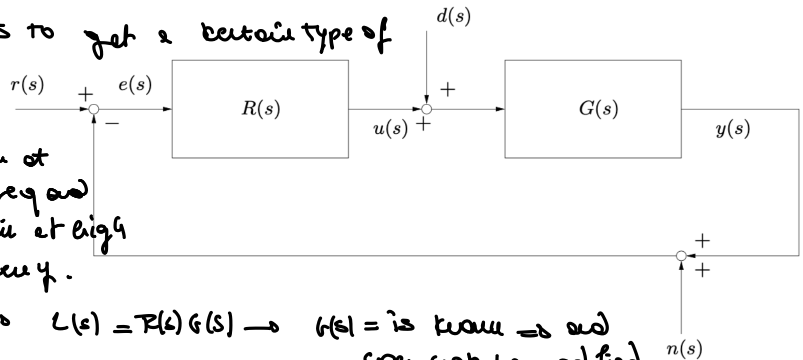
Correspondence between the above transient characteristics and the **location of the closed-loop zeros and poles**.



How to design R(s)? The object is to get a certain type of

Closed-loop transfer function (complementary sensitivity function): L(s)

high gain at low freq and low gain at high frequency.



$$\frac{y(s)}{r(s)} = T(s) = \frac{R(s)G(s)}{1 + R(s)G(s)}$$

↳ $L(s) = R(s)G(s) \rightarrow G(s)$ = is known and can not be modified

Characteristic equation (roots are the poles of *T(s)*):

$$1 + R(s)G(s) = 0$$

R(s) → where to put zero and pole of the regulators.

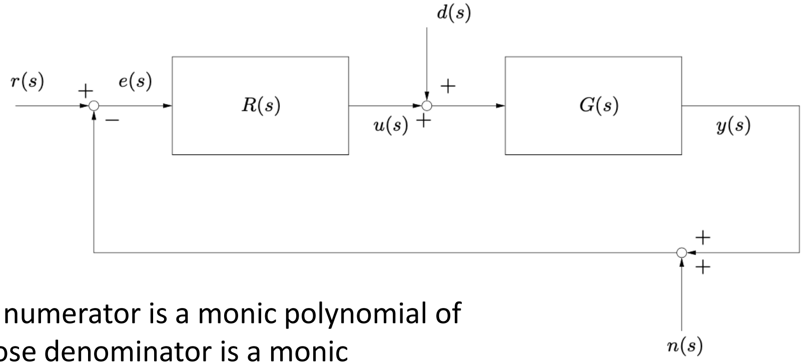
Let's study the roots of this equation as the gain k of the controller R(s) changes. The plot of all possible roots as k varies from zero to infinity is called root locus (positive values of k).

$$\text{Phase:} \quad \arg G(j\omega)$$

Phase: $\arg G(j\omega)$

~~T_c, T_d, θ~~

$L(s)$



$$L(s) = \frac{b(s)}{a(s)}$$

rational transfer function whose numerator is a monic polynomial of degree *b(s)* of degree *m* and whose denominator is a monic polynomial *a(s)* of degree n such that $n \geq m$.

$$b(s) = s^m + b_1 s^{m-1} + \cdots + b_m$$
$$= (s - z_1)(s - z_2)\cdots(s - z_m) \quad = \prod_{i=1}^{m}(s - z_i)$$

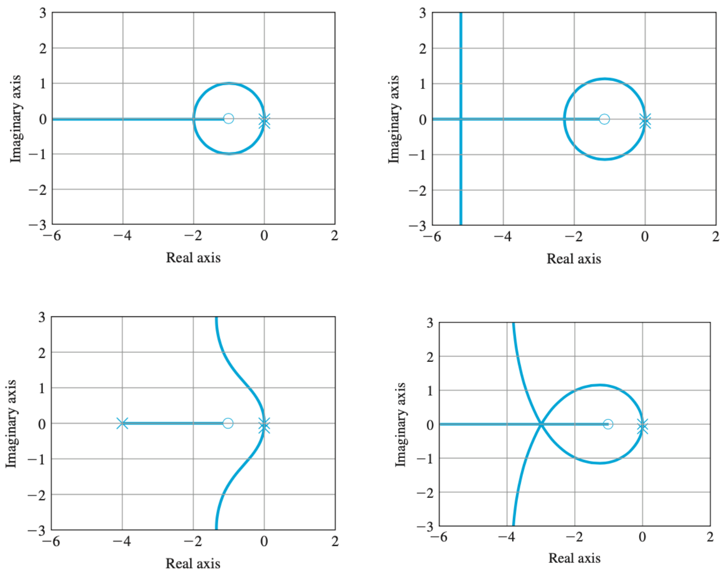
$$a(s) = s^n + a_1 s^{n-1} + \cdots + a_n$$
$$= \prod_{i=1}^{n}(s - p_i).$$

The roots of the characteristic equation are r_i from the factored form

$$a(s) + kb(s) = \prod_{i=1}^{n}(s - r_i)$$

[review] *Root locus*

Because the solutions to $a(s) + kb(s) = 0$ are the roots of the closed-loop system characteristic equation and are thus closed-loop poles of the system, **the root-locus method can be thought of as a method for inferring dynamic properties of the closed-loop system as the parameter *k* changes**.



Review of classical control design
[review] *Root locus*

There are some rules to construct a root locus…

RULE 1. *The n branches of the locus start at the poles of L(s) and m branches end on the zeros of L(s).*

RULE 2. *The loci are on the real axis to the left of an odd number of poles and zeros.*

RULE 3. […]

RULE 4. […]

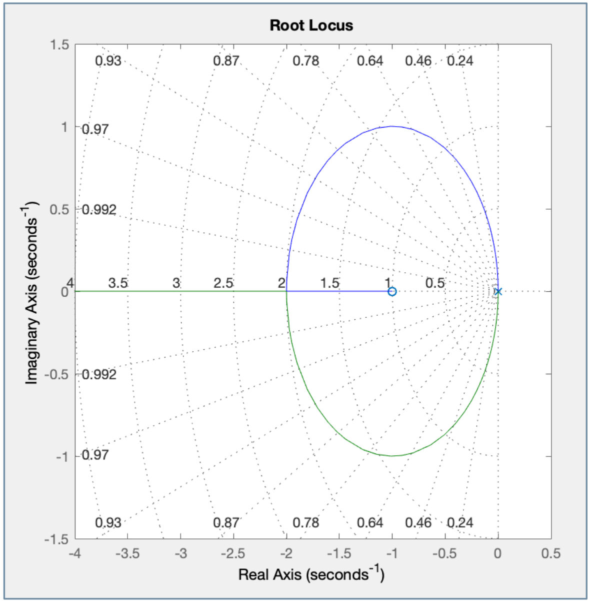
RULE 5. […]

MATLAB is your best friend…. 😉

LIAR! I want to remember how to do that by hand

```
s=tf('s');
sys=(s+1)/s^2;
rlocus(sys);
```

RIPASSO AUTOMATICA



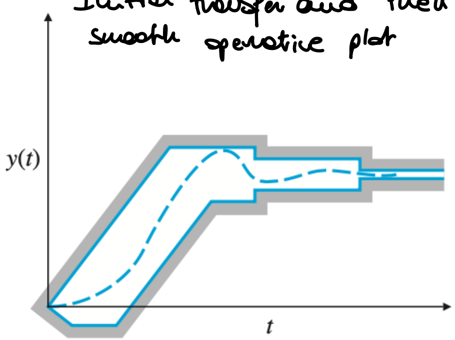
Outline of control systems design

Summary of control design steps (approximation of good practice):

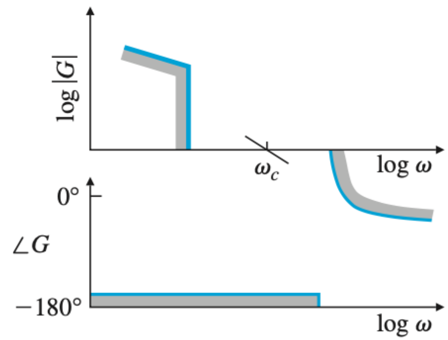
1. understand the process and its performance requirements
2. select type and number of sensors
3. select type and number of actuators
4. build a (linear) dynamic model of the system
5. try first with a simple control design (P, PD, PID, lead-lag compensators)
6. simulate the design including nonlinearities, additional dynamics, noise, parameter variations
7. consider modifying the plant and the control system (refined regulators, advanced control design techniques,…) to satisfy the requirements and optimize the performance

Step 1 – understand the process and its performance requirements

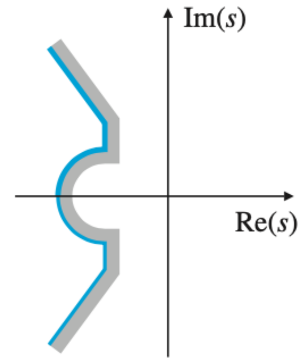
Initial transfer and then a smooth operative plot



step response inside some constraint boundaries



open-loop frequency response satisfying certain constraints



closed-loop poles to the left of some constraint boundary

Step 2 – select type and number of sensors → choosing the best option to get what we want

↳ select proper sensor to understand the characteristic of the system → sometimes one sensor could be better than 4 different sensor.

consider which variables are important to control and which variables can physically be measured

Number of sensors and locations:	Select minimum required number of sensors and their optimal locations
Technology:	Electric or magnetic, mechanical, electromechanical, electro-optical, piezoelectric
Functional performance:	Linearity, bias, accuracy, bandwidth, resolution, dynamic range, noise
Physical properties:	Weight, size, strength
Quality factors:	Reliability, durability, maintainability
Cost:	Expense, availability, facilities for testing and maintenance

Step 3 – select type and number of actuators

in order to control a dynamic system,
(obviously) you must be able to influence the response

Number of actuators and locations:	Select minimum required actuators and their optimal locations
Technology:	Electric, hydraulic, pneumatic, thermal, other
Functional performance:	Maximum force possible, extent of the linear range, maximum speed possible, power, efficiency, etc.
Physical properties:	Weight, size, strength
Quality factors:	Reliability, durability, maintainability
Cost:	Expense, availability, facilities for testing and maintenance

Step 4 – build a (linear) dynamic model of the system

take the best choice for process, actuator and sensor;
identify the equilibrium point of interest;
construct a small-signal dynamic model valid over the range of frequencies included in the specifications of Step 1

linear dynamic model

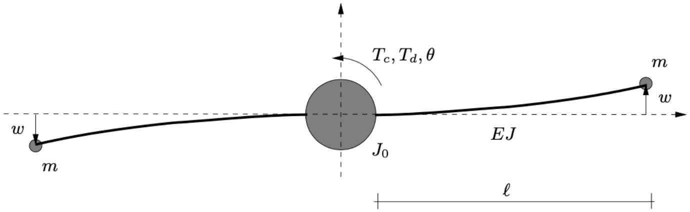
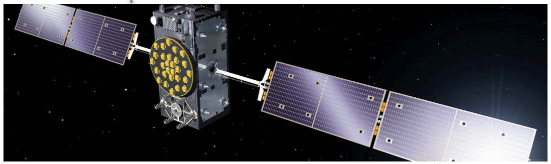


Figure 16.1 A simplified spacecraft rotating about a single axis.



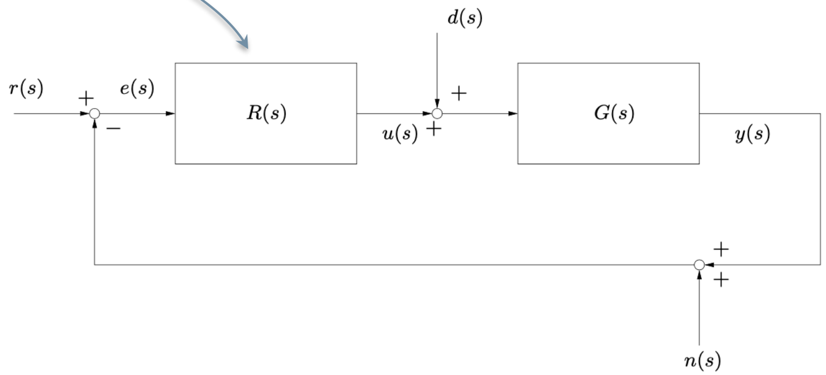
Step 5 – try first with a simple control design (P, PD, PID, lead-lag compensators)

try to meet the specifications with a simple controller of lead-lag variety
including integral action if steady-state error response requires it

$$R(s) = k_p$$

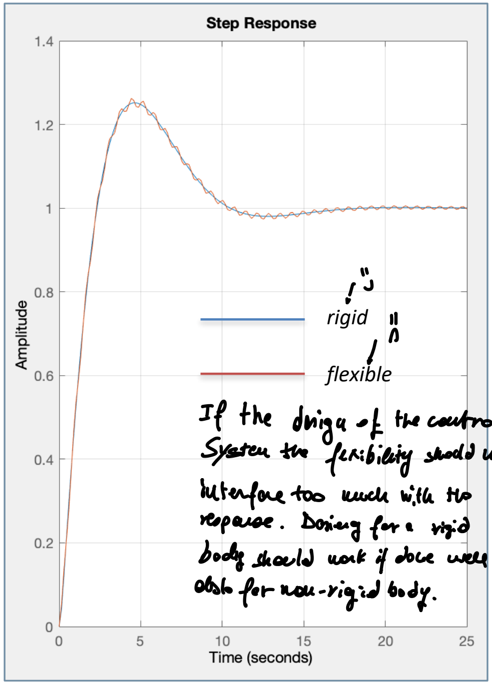
$$R(s) = k_p + sk_d$$

$$R(s) = k_p + sk_d + \frac{k_i}{s}$$



Step 6 – simulate the design including nonlinearities, additional dynamics, noise, parameter variations

based on the simple control design,
evaluate the source of the undesirable characteristics
of the system performance
(for example, you might find vibrational modes that prevent
the design from meeting the initial specifications)



Step 7 – consider *modifying the plant* and the control system
(refined regulators, advanced control design techniques,…)
to satisfy the requirements and optimize the performance

it is important to consider all parts of the design
(type and location of sensors, type and location of actuators,…)
not only the control logic

if trial-end-error compensators do not give entirely satisfactory performance
consider a design based on advanced control techniques
(we'll discuss this later on)

Some remarks on step 7.

- In many cases, **proper plant modifications** can provide additional damping or increase in stiffness, change in mode shapes, reduction of system response to disturbances, reduction of Coulomb friction, …

- The usual approach of designing the system and “throwing it over the fence” to the control group has proved to be inefficient and flawed.

- A better approach that is gaining momentum is to **get the control engineer involved from the onset of a project** to provide early feedback on whether or not it is difficult to control the system. The control engineer can provide valuable feedback on the choice of actuators and sensors and can even suggest modifications to the plant.

- It is often much more efficient to **change the plant design while it is on the drawing board** before “any metal has been bent.” Closed-loop performance studies can then be performed on a simple model of the system early on.

- Implicit in the process of design is the well-known fact that **designs within a given category often draw on experience gained from earlier models**. Thus, good designs evolve rather than appear in their best form after the first pass.

Classical input-output model:

$$u(s) \longrightarrow \boxed{G(s)} \longrightarrow y(s)$$

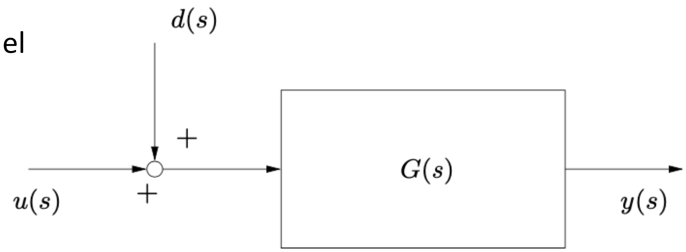
where

$y(s)$ is the **system (sensor) output**

$u(s)$ is the **control (actuator) input**

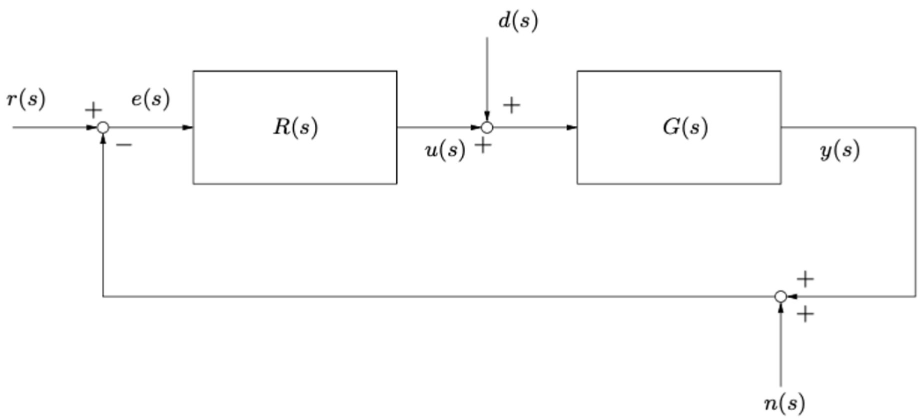
$G(s)$ is the **system (open-loop) transfer function** (from control input to sensor output)

Classical input-output model including disturbance:

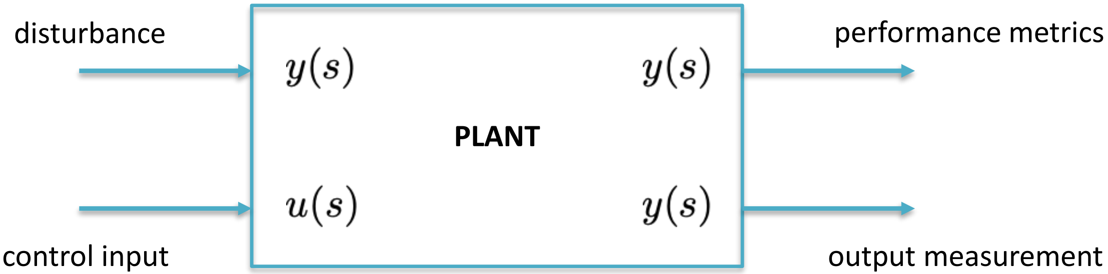


where $y(s)$ is the **disturbance**

Classical input-output model including feedback control:



Generalized input-output model:



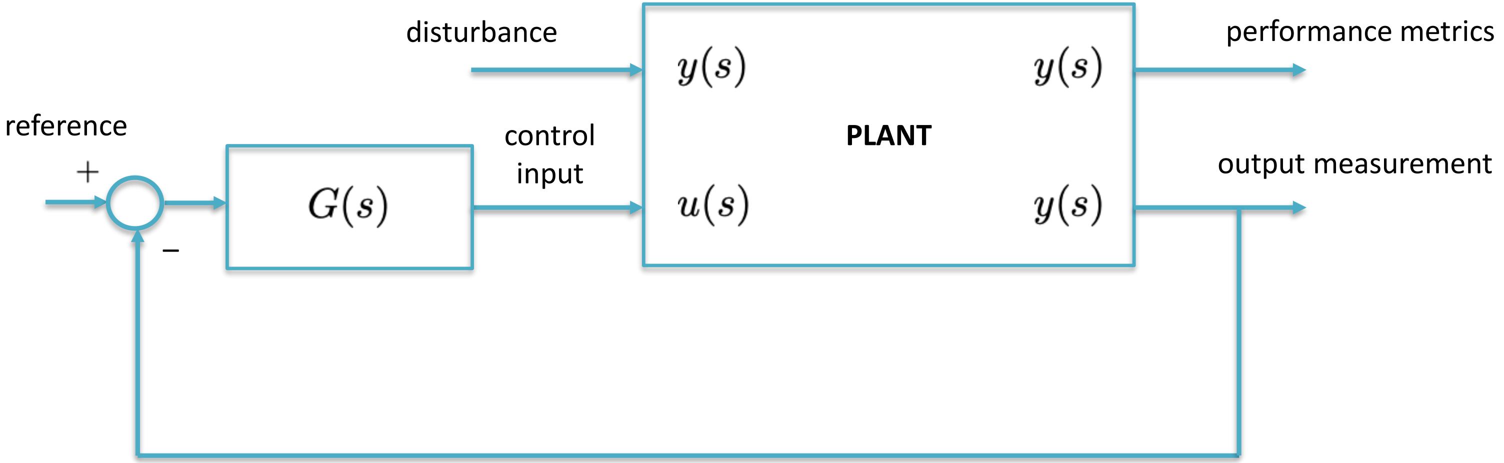
Four open-loop transfer functions:

$$G_{yu}(s)$$
$$G_{yd}(s)$$
$$G_{zu}(s)$$
$$G_{zd}(s)$$

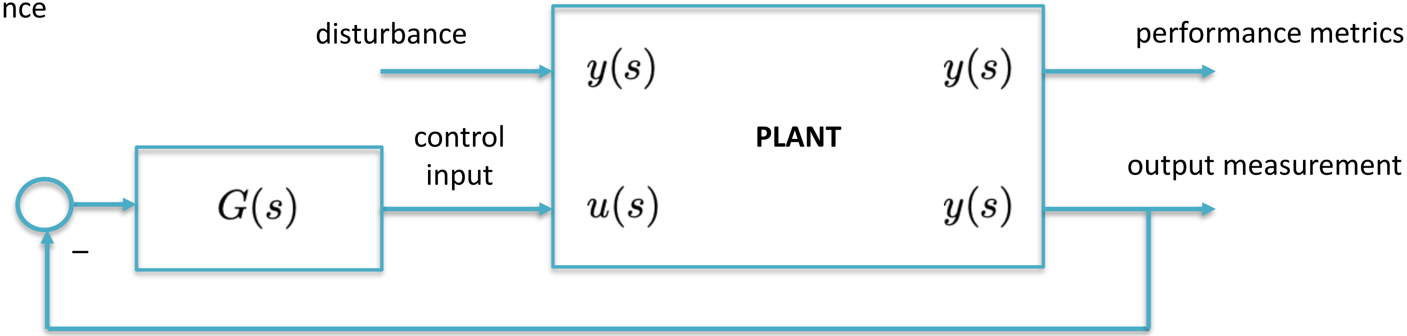
$$y(s) = G_{yu}(s)u(s) + G_{yd}(s)d(s)$$

$$N(s) = 0 \;\Rightarrow\; \nu \text{ roots} - \;\text{zeros}\; z_i$$

Generalized input-output model
with feedback control:



Generalized input-output model with feedback control – null reference



$$u(s) = -R(s)y(s)$$

$$y(s) = \underbrace{[1 + R(s)G_{yu}(s)]^{-1}\, G_{yd}(s)}\, d(s)$$

$$\neq$$

$$z(s) = \overbrace{\left[G_{zd}(s) - R(s)G_{zu}(s)\,[1 + R(s)G_{yu}(s)]^{-1}\right]}\, d(s)$$