Spacecraft Attitude Dynamics

prof. Franco Bernelli

Linear attitude control

General control problem

Assume we are using reaction wheels as actuators

$$M = A\dot{h}_r$$

rate of change of the spin of reaction wheel.

3xn

n×1 → reaction wheel

How to evaluate M in a general case?

If the satellite is inertial pointing, Euler equations assume a particularly simple form

Why this assumption? → In case of inertial pointing Euler equation for each axis become uncouple from the other

$$M_i = I\dot{\omega}_i \qquad i = 1,2,3$$

Linearizing the system, we have three second order decoupled equations:

$$M_i = I\ddot{\alpha}_i$$

second derivative of the rotation angle

To have complete control I need 3 actuator one for each axis

Three independent actuators for the three axes. Each equation represents a linear second order system, so that we can assume a simple PID control would allow obtaining the desired system performances:

The PID can be changed in a optimal control → as seen the previous lesson (LQR logic)

simplest control logic

$$M = f(\alpha) = \mathrm{PID}(\alpha)$$

we know how to tune the pid to get sufficient performance

General control problem

When the satellite is far from equilibrium, its dynamics should include also the coupling terms due to angular velocities

$$M = I\underline{\dot{\omega}} + \underline{\omega} \wedge I\underline{\omega}$$

To consider the coupling terms $\underline{\omega} \wedge I\underline{\omega}$ the control torque should be evaluated as:

$$M_c = I\underline{\dot{\omega}}$$

The nonlinear terms can be considered as a correction to the control torque

$$M = M_c + \underline{\omega} \wedge I\underline{\omega}$$

General control problem

In case of a satellite with a set of RWs the problem is formulated as:

The system can be divided into different equations

$$M = I\underline{\dot{\omega}} + \underline{\omega} \wedge I\underline{\omega} + A\underline{\dot{h}_r} + \underline{\omega} \wedge \mathrm{A}\mathrm{h}_r \quad \rightarrow \quad \text{nonlinear dynamics}$$

$$0 = I\underline{\dot{\omega}} + \underline{\omega} \wedge I\underline{\omega} + A\underline{\dot{h}_r} + \underline{\omega} \wedge \mathrm{A}\mathrm{h}_r \quad \rightarrow \quad \text{control equation}$$

based on the fact that we use only one angle for each axis → true for small angles

$$-M_c = I\underline{\dot{\omega}} = \mathrm{PID}(\underline{\omega}, \underline{\alpha}) \quad \rightarrow \quad \text{pseudocontrol function}$$

one for every axis. → linearized

$$M_c = -\underline{\omega} \wedge I\underline{\omega} - A\underline{\dot{h}_r} - \underline{\omega} \wedge \mathrm{A}\mathrm{h}_r \quad \rightarrow \quad \text{actuator equation}$$

$$\underline{\dot{h}_r} = A^*\left[-M_c - \underline{\omega} \wedge I\underline{\omega} - \underline{\omega} \wedge \mathrm{A}\mathrm{h}_r\right] \quad \rightarrow \quad \text{actuator command}$$

→ Real acceleration of the reaction wheel

→ computed pseudo control

→ actuator know how much they should accelerate

→ Mapping of the control into the input of the actuators.

If the equation are non linear we should consider the full nonlinear form of the Euler angle eq.

If we consider also large angular rotations:

- the error angles definition depends on the sequence of rotations considered;
- a general solution of the control problem must then be sought for.

Assuming we are able to extract the non linear terms of the dynamic equation and include them into the actuator command.

General control problem

↳ we are trying to find a more general solution to the control problem. To do so we need to go back to the notion of orientation and rotation.

Call A_S the satellite attitude matrix in an inertial frame, and assume the target attitude is given by matrix A_T:

→ Target Attitude

$$A_S = \begin{bmatrix} a_{11S} & a_{12S} & a_{13S} \\ a_{21S} & a_{22S} & a_{23S} \\ a_{31S} & a_{32S} & a_{33S} \end{bmatrix} = \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix} \qquad A_T = \begin{bmatrix} a_{11T} & a_{12T} & a_{13T} \\ a_{21T} & a_{22T} & a_{23T} \\ a_{31T} & a_{32T} & a_{33T} \end{bmatrix} = \begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix}$$

Each row of A_S and A_T represents one reference axis, either satellite or target. Our goal is

$$A_S A_T^T = I$$

when the spacecraft is pointing in the wrong direction $A_S \neq A_T$

In actual conditions, the error in the attitude is

$$A_S A_T^T = A_e \rightarrow$$

Rotation that the spacecraft has to do to point correctly at the target.

We want now to relate the attitude error A_e with the control torque

General control problem

target axis

$$A_e = A_S A_T^T = \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix} \begin{bmatrix} X_T^T & Y_T^T & Z_T^T \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} & 12 & 13 \end{array} \\ \begin{bmatrix} X_S X_T^T & \boxed{X_S Y_T^T} & \boxed{X_S Z_T^T} \\ Y_S X_T^T & Y_S Y_T^T & \boxed{Y_S Z_T^T} \\ Z_S X_T^T & Z_S Y_T^T & Z_S Z_T^T \end{bmatrix} \begin{array}{c} \\ 23 \\ \\ \end{array} \end{array} \rightarrow$$

↳ spacecraft actual axis

If we have a correct pointing the diagonal terms will be 1 and the off diagonal terms will be equal to zero because we have a dot product between two orthogonal vector

To reach the zero error, the extra diagonal terms must vanish to zero.

$X_S Y_T^T = 0$ means that X_S and Y_T must become orthogonal. This condition can be obtained if the satellite rotates around its z body axis

It is easier to design a control law to set off diagonal terms $= \phi$ → similar to other control methods seen rather than set diagonal terms $= 1$.

12

$$M_{zS} = f_z(X_S Y_T^T)$$

↳ input terms

If we want $X_S Z_T^T = 0$ the torque must be around the satellite body axis Y_S

$$M_{yS} = f_y(X_S Z_T^T)$$

↳ 13

To have $Y_S X_T^T = 0$ we need a torque around the satellite body axis X_S

$$M_{xS} = f_x(Y_S Z_T^T)$$

↳ 23

$12 \rightarrow X_S Y_T^T = \alpha_z$

$13 \rightarrow X_S Z_T^T = \alpha_y$

$23 \rightarrow Y_S Z_T^T = \alpha_z$

} if we have small erog

A similar result would be obtained by considering the terms below the diagonal

General control problem

To understand how the control functions can be designed, consider the case of small errors

$$A_S A_T^T = \begin{bmatrix} 1 & \alpha_z & -\alpha_y \\ -\alpha_z & 1 & \alpha_x \\ \alpha_y & -\alpha_x & 1 \end{bmatrix}$$

For small rotations:

$$M_z = PD(\alpha) = K_{pz}\alpha_z + K_{dz}\dot{\alpha}_z$$

α_z corresponds to $X_S Y_T^T$ in the case of large errors, or to a_{12e}. Then generalize the control function as:

$$M_{zs} = K_{pz}a_{12e} + K_{dz}\omega_z$$

Then evaluate K_p and K_d for the linear approximation of the dynamics and extend the validity of the control law

standard linear control to an extended control law → non linear model

$$\begin{cases} M_x = K_{px}\alpha_x + K_{dx}\dot{\alpha}_x \\ M_y = K_{py}\alpha_y + K_{dy}\dot{\alpha}_y \\ M_z = K_{pz}\alpha_z + K_{dz}\dot{\alpha}_z \end{cases} \rightarrow \begin{cases} M_{xs} = K_{px}a_{23e} + K_{dx}\omega_x \\ M_{ys} = K_{py}a_{31e} + K_{dy}\omega_y \\ M_{zs} = -K_{pz}a_{21e} + K_{dz}\omega_z \end{cases}$$

where the non linearity is expressed in the terms a_{23}, a_{31}, a_{21}

General control problem

The same can be designed adopting the terms below the diagonal, obtaining for each different case a different transient response in case of large initial errors:

$$M_{zs} = -K_{pz} a_{21e} + K_{dz} \omega_z$$

We can finally try to have an intermediate situation, for which:

$$\begin{cases} M_{xs} = f_x(Y_S Z_T^T - Z_S Y_T^T) \\ M_{ys} = f_y(X_S Z_T^T - Z_S X_T^T) \\ M_{zs} = f_z(X_S Y_T^T - Y_S X_T^T) \end{cases}$$

In this case

Extension of standard linear control to the direction cosine that represent large rotation.

$$a_{12e} - a_{21e} = 2\alpha_z$$

$$\begin{cases} M_{\mathrm{xs}} = \frac{K_{\mathrm{px}}}{2}\left(a_{23\mathrm{e}} - a_{32\mathrm{e}}\right) + K_{\mathrm{dx}} \omega_x \\ M_{\mathrm{ys}} = \frac{K_{\mathrm{py}}}{2}\left(a_{31\mathrm{e}} - a_{13\mathrm{e}}\right) + K_{\mathrm{dy}} \omega_y \\ M_{\mathrm{zs}} = \frac{K_{\mathrm{pz}}}{2}\left(a_{12\mathrm{e}} - a_{21\mathrm{e}}\right) + K_{\mathrm{dz}} \omega_z \end{cases}$$

General control problem

One further option → Using the quaternions

If the rotations are small has the same effect of α_{23}

$$\begin{cases} M_{xs} = \dfrac{K_{px}}{2}(4q_{1e}q_{4e}) + K_{dx}\omega_x = 2K_{px}q_{1e}q_{4e} + K_{dx}\omega_x \\ M_{ys} = \dfrac{K_{py}}{2}(4q_{2e}q_{4e}) + K_{dy}\omega_y = 2K_{py}q_{2e}q_{4e} + K_{dy}\omega_y \\ M_{zs} = \dfrac{K_{pz}}{2}(4q_{3e}q_{4e}) + K_{dz}\omega_z = 2K_{pz}q_{3e}q_{4e} + K_{dz}\omega_z \end{cases}$$

As the satellite approaches the target condition, the dynamics becomes automatically linear.

If at steady state we want a nonzero value for one angular velocity component, the control function must be modified as:

$$M_i = K_{\mathrm{pi}}\alpha_i + K_{\mathrm{di}}(\dot{\alpha}_i - \bar{\dot{\alpha}}_i) \qquad \Rightarrow \qquad \mathrm{M}_i = 2\mathrm{K}_{\mathrm{pi}}q_{\mathrm{ie}}q_{4\mathrm{e}} + K_{\mathrm{di}}(\omega_i - \bar{\omega}_i)$$

Linear State Observer

$$\ddot{\alpha}_x + (K_x - 1)n\dot{\alpha}_y + K_x n^2 \alpha_x = 0$$
$$\ddot{\alpha}_y + (1 - K_y)n\dot{\alpha}_x + K_y n^2 \alpha_y = 0$$

$$\underline{\dot{x}} = A\underline{x}$$
$$\underline{x} = [\alpha_x \quad \alpha_y \quad \dot{\alpha}_x \quad \dot{\alpha}_y]^T$$
$$y = C\underline{x}$$
$$y = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{C} \underline{x}$$

The possible output of our spacecraft is α_y and $\dot{\alpha}_y$
Center pointing → Horizon sensor → providing directly pitch and roll axis but it give no information on the yaw axis.
α_y and $\dot{\alpha}_y$ → used to control pitch and yaw behaviour.

If I use a full state observer the state of the observer is driven also by the estimate on the output.

$$\underline{\dot{\hat{x}}} = A\underline{\hat{x}} + L(\underline{y} - \underline{\hat{y}})$$
$$\underline{\hat{y}} = C\underline{\hat{x}}$$
$$e = \underline{x} - \underline{\hat{x}}$$
$$e \to 0$$

important to use full state observer because it can give some mitigation on errors → like noise.

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

Listen here

Active attitude control

Different mission phases require different control design

(i) De-tumbling → we are not trying to achieve any specific α we just want to get $\dot{\alpha} = 0$

(ii) Re-pointing → Rotation manoeuvre after we know what is the error

(iii) 3-axis stabilization/tracking → satellite can experience sudden disturbances.

General control strategy

{ How to map the computed control into the
input to give to our actuators

Ideal control input

Linear controller

$$\underline{u} = -K\partial\underline{x}$$

Nonlinear de-tumbling controller

$$\underline{u} = -k\underline{\omega}$$

Nonlinear re-pointing control

$$\underline{u} = -k_1\underline{\omega} - k_2\underline{q}_e$$

Real control inputs

$\underline{h}_r$	RW momentum
$\dot{\delta}$	Gimbal angle rate of a CMG
$\underline{m}$	Magnetic moment of a magnetic torquer
$\underline{F}_{on-off}$	On-off thruster impulses