ORBITAL MECHANICS

After s/c arrives at the planet:

20/01
08/02 } Dotto

15/01
29/01 } Colombo

1) RENDEZVOUS MISSION

s/c stay orbit @ planet 2

2) FLY BY MISSION

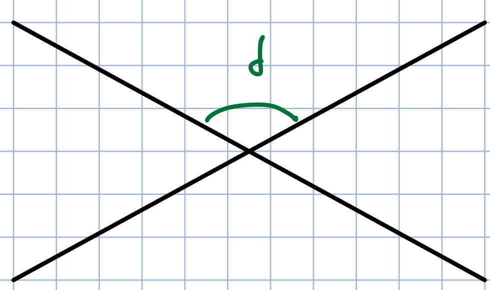
s/c continues after perigee of hyp on its heliocentric leg after fly-by of planet 2.

If rendez-vous:

- impact $\Delta \rightarrow r_p = R_{planet} \Rightarrow$ IMPACT
- stay in orbit @ planet $\Delta \rightarrow r_p > R_{planet} \Rightarrow$ hyperbola

To stay in orbit Δv is needed to go from hyperbola $\rightarrow$ close to orbit (optimal if @ hyperbola perigee)

If no Δv is given $\Rightarrow$ continue on hyperbola $\Rightarrow$ perform a fly-by exit SOI with the same v_∞ as you entered.



δ = TURN ANGLE

$$\boxed{\delta = 2\sin^{-1}\left(\frac{1}{e}\right)} \quad (4.31)$$

$v_\infty, r_p \Rightarrow e_{HYP}$ $\qquad e_{HYP} = 1 + \dfrac{r_p v_\infty^2}{\mu_2}$ all referred to the planet

The turn angle becomes $\quad \delta = 2\sin^{-1}\left(\dfrac{1}{1+\frac{r_p v_\infty^2}{\mu_2}}\right)$

Recall definition of AIMING RADIUS or IMPACT PARAMETER

$$\Delta = \bar{b} = \bar{a}\sqrt{e^2-1} \qquad (1.82)$$

and recall (1.78)

$$a_{Hyp} = \frac{p}{1-e^2} \longrightarrow \bar{a} = \frac{p}{e^2-1}$$

$$a_{Hyp} < 0$$

So Δ can be written

$$\Delta = \frac{p}{e^2-1}\sqrt{e^2-1} \longrightarrow \Delta = \frac{h^2}{\mu_2}\frac{1}{\sqrt{e^2-1}} \qquad (4.32)$$

but from equation (4.20)

$$h = \frac{\mu_2}{v_\infty}\sqrt{e^2-1}$$

Substitute (4.20) into (4.32)

$$\Delta = \frac{\mu_2^2}{v_\infty^2}\frac{(e^2-1)}{\sqrt{e^2-1}\,\mu_2} = \frac{\mu_2}{v_\infty^2}\sqrt{e^2-1}$$

Then substituting (4.21)

$$\Delta = \frac{\mu_2}{v_\infty^2}\sqrt{\cancel{1}+\left(\frac{r_p v_\infty^2}{\mu_2}\right)^2+2\frac{r_p v_\infty^2}{\mu_2}-\cancel{1}}$$

$$\Delta = r_p\sqrt{\frac{\mu_2^2}{v_\infty^4}\left(\frac{v_\infty^4}{\mu_2^2}+2\frac{v_\infty^2}{\mu_2 r_p}\right)}$$

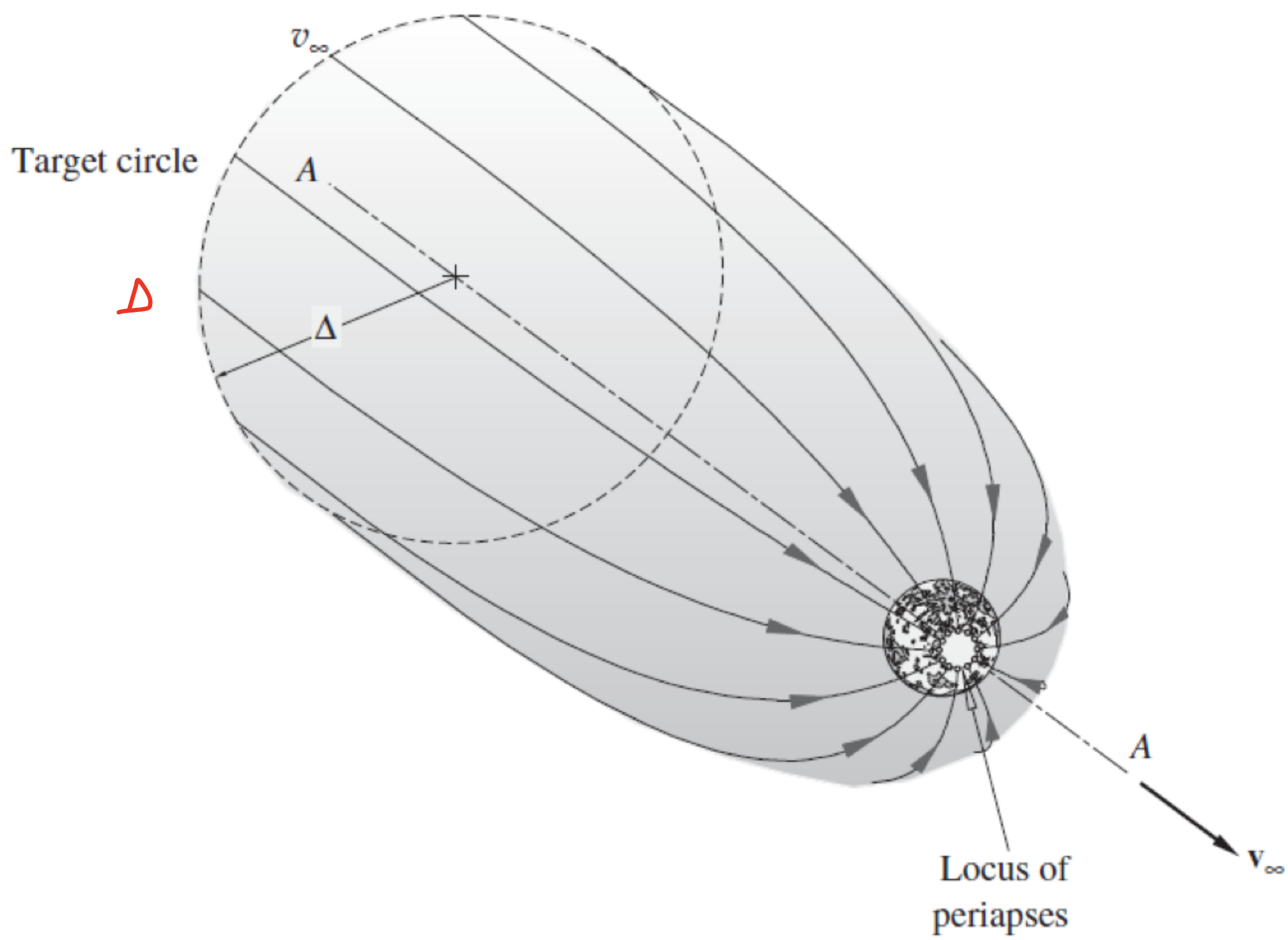
$$\boxed{\Delta = r_p\sqrt{1+\frac{2\mu_2}{v_\infty^2 r_p}}} \qquad (4.33)$$

given by the arrival condition of the helio eq

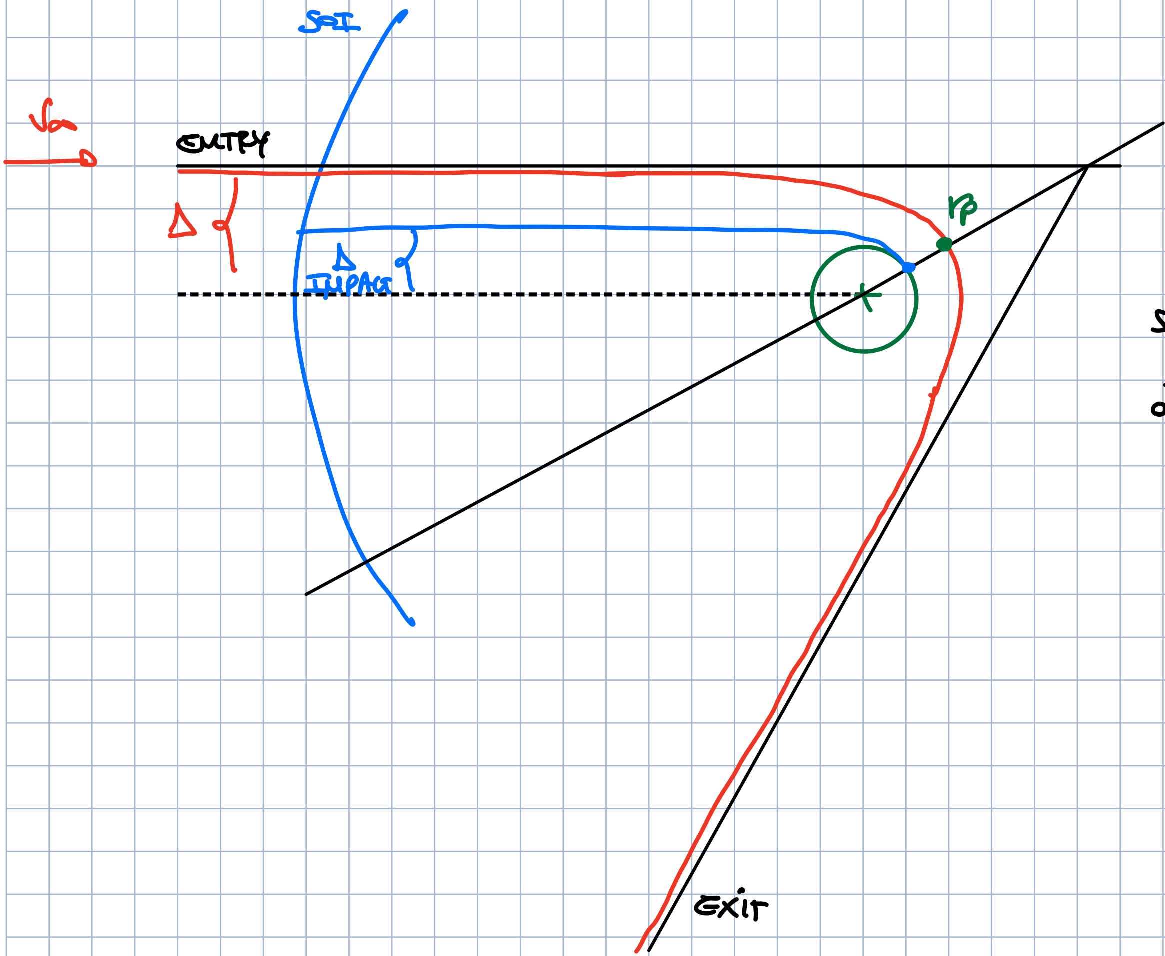
Relating the radius of the hyperbola and the velocity at ∞

→ by setting Δ at ∞ I define the "future" radius of perigee.
We will not see the planet at ∞ because it is very small →
we know the position of the planet thanks to the ephemerides
so we can set the Δ and this will define the radius of the
perigee.

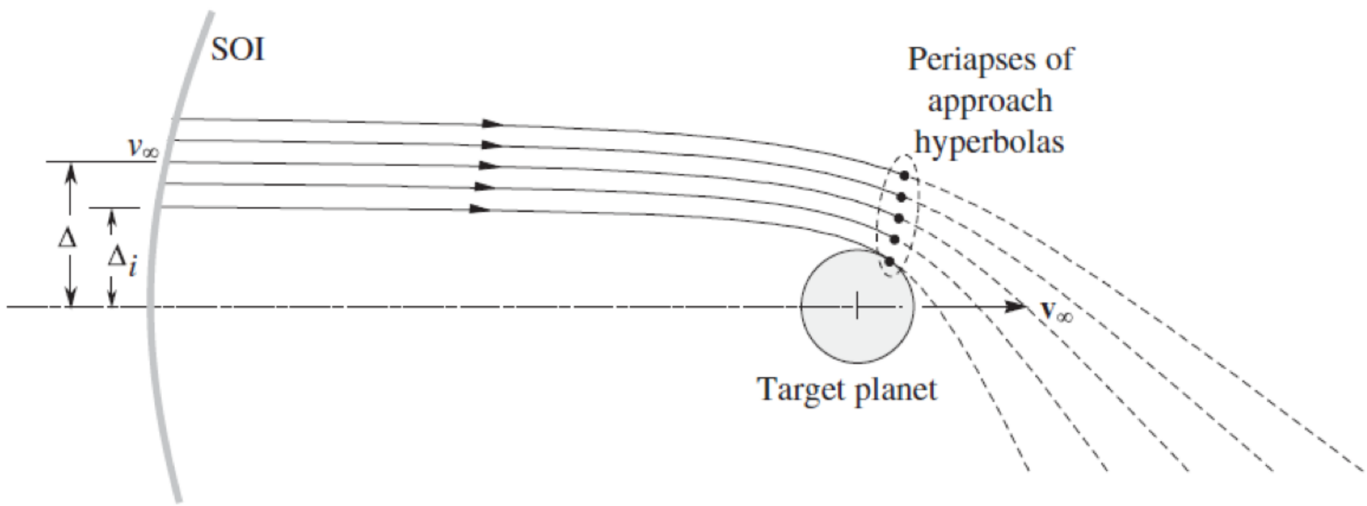
As for departure hyp also the approach hyperbola can lie in
∞ number of planes (passing through the center of planet)
entry eq parallel to v_∞ (ΔVA)



From (4.33) setting v_∞, Δ is directly related to r_p.



SELECTING Δ
define rp



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let's on rendez-vous on elliptical orbit @ planet 2
let's characterized the capture phase
r on hyperbola @ entry of SOI by considering that specific energy is conserved

$$\boxed{\varepsilon_{Hyp\,\infty} = \varepsilon_{Hyp\ perigee}} \qquad (4.34)$$

$$\frac{v_\infty^2}{2} - \cancel{\frac{\mu_2}{r_\infty}} = \frac{v_p^2}{2} - \frac{\mu_2}{r_p} \qquad v_p = \sqrt{v_\infty^2 + \frac{2\mu_2}{r_p}}$$

$\to 0$

To be captured on elliptical orbit we need a Δv (breaking manoeuvre)

Velocity at perigee on the capture orbit

$$h = r_p\, v_{p\,capture}$$

$$v_{p\,capture} = \frac{h}{r_p}$$

target eccentricity on capture orbit.

$$\Delta v = v_{p\,Hyp} - v_{p\,capt} = \sqrt{v_\infty^2 + \frac{2\mu_2}{r_p}} - \sqrt{\frac{\mu_2(1+e)}{r_p}}$$

$\Delta v = \text{fun}(r_p, e)$ $\quad$ Δv gets smaller for higher e

GIVEN v_∞ (heliocentric leg), e (target due to science requirements)

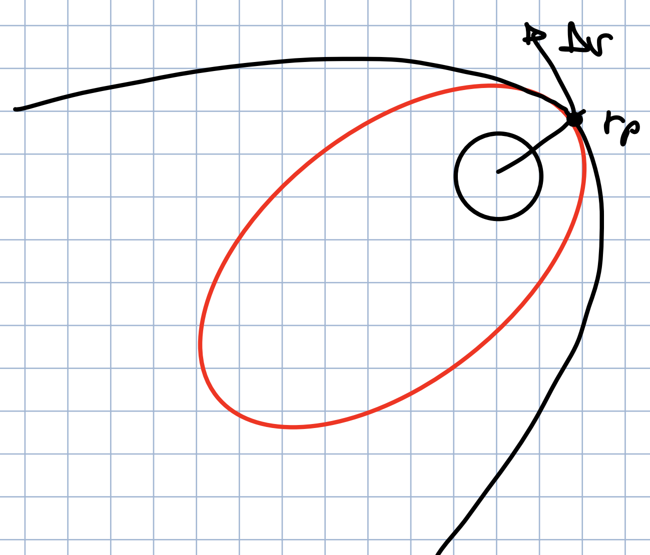
FIND minimum Δv.

$$r_{p\,Hyp} = r_{p\,capture}$$

$$\frac{\Delta v}{v_\infty} = \sqrt{1 + \frac{2\mu_2}{r_p v_\infty^2}} - \sqrt{\frac{\mu_2(1+e)}{r_p v_\infty^2}}$$

$$\xi = \frac{r_p v_\infty^2}{\mu_2} \qquad (4.35)$$

change of variable.



$$\frac{\Delta v}{v_\infty} = \sqrt{1+\frac{2}{\xi}} - \sqrt{\frac{1+e}{\xi}}$$

$\frac{d}{d\xi}\left(\frac{\Delta v}{v_\infty}\right)$ → v_∞ is set by helio leg

→ minimize Δv wrt ξ my design parameters (r_p can be chosen by choosing δ)

$$\frac{d}{d\xi}\left(\frac{\Delta v}{v_\infty}\right) = 0$$ to have minimum

If we calculate this derivative

$$\frac{\partial}{\partial\xi}\left(\frac{\Delta v}{v_\infty}\right) = \frac{1}{\xi^{3/2}}\left(-\frac{1}{\sqrt{\xi+2}} + \sqrt{\frac{1+e}{2}}\right)$$

minimum → $\frac{d}{d\xi}\left(\frac{\Delta v}{v_\infty}\right) = 0$

$$\boxed{\xi_{min\,\Delta v} = 2\frac{1-e}{1+e}} \quad (4.36)$$

To make sure it is a minimum we calculate

$$\frac{\partial^2}{\partial\xi^2}\left(\frac{\Delta v}{v_\infty}\right)^2 = \frac{1}{\xi^{5/2}}\left(\frac{2\xi+3}{(\xi+2)^{3/2}} - \frac{3}{4}\sqrt{1+e}\right)$$

In correspondence to $\xi_{min\Delta v}$ the second derivative is

$$\left.\frac{\partial^2}{\partial\xi^2}\left(\frac{\Delta v}{v_\infty}\right)^2\right|_{\xi_{min\Delta v}} = \frac{\sqrt{2}}{64}\frac{(1+e)^3}{(1-e)^{3/2}} > 0$$

$\xi_{min\Delta v}$ is a minimum

→ Target of capture orbit $0 \le e < 1$

Substitute Eq (4.25) into (4.26) we get the optimal r_p to be captured on elliptical orbit $e_{TARGET} = e$ (minimum Δv)

$$\boxed{r_p = 2\,\frac{1-e}{1+e}\,\frac{\mu_2}{v_\infty^2}} \quad (4.37)$$

optimal radius of pericentre to do a capture.

$\xi_{min\Delta v} \longrightarrow \frac{\Delta v}{v_\infty} \Rightarrow$ minimum Δv

$\parallel$

$2\frac{1-e}{1+e}$

$$\boxed{\Delta v_{min} = v_\infty\sqrt{\frac{1-e}{2}}} \quad (4.38)$$

→ given

given

Let's characterise the radius of the apogee:

Recall: $r_a = a(1+e)$
$r_p = a(1-e)$ $\rightarrow$ $\frac{r_p}{r_a} = \frac{1-e}{1+e} \rightarrow r_a = r_p \frac{1+e}{1-e}$

from equation (4.37)

$$\boxed{r_a = \frac{2\mu_2}{v_\infty^2}}$$

After setting e and r_p optimal to get Δv min the radius of the apogee depends only on how we entered at ∞ $\rightarrow$ This is due to the conservation of energy.

If we substitute E_p (4.37) into E_p (4.33) $\Rightarrow$ Δ to be targeted at ∞

$$\Delta = r_p \sqrt{1 + \frac{2\mu_2}{r_p v_\infty^2}} \quad \text{with} \quad r_{p\,min\Delta v} = 2\left(\frac{1-e}{1+e}\right)\frac{\mu_2}{v_\infty^2}$$

$$\boxed{\Delta_{min\Delta v} = \frac{2\mu_2}{v_\infty^2}\left(\frac{1-e}{1+e}\right)\sqrt{\frac{2}{1-e}} \quad \text{OR} \quad \Delta_{min\Delta v} = r_{p\,min\Delta v}\sqrt{\frac{2}{1-e}}} \quad (4.40)$$

At ∞ (SOI) v_∞ is given by helio leg

If I target $\Delta_{min\Delta v}$ $\rightarrow$ $r_{p\,min\Delta v}$ $\rightarrow$ If at $r_{p\,min\Delta v}$ I brake with Δv_{min} $\Rightarrow$ capture orbit of given e.

? How I can set the minimization problem if planet was also some atmosphere (can exploit to brake)?