09.20 Notes

Math 403/503

September 2022

1 Week 5

We talked about how any transformation $T : V \to W$ can be given as a matrix $A \epsilon F^{m,n}$ with respect to a basis $v_1, ..., v_n$ of V and $w_1, ..., w_n$ of W. We further talked about how $F^{m,n}$ = the space of mxn matrices supports addition and scalar multiplication, making it into a vector space. Matrices additionally support multiplication or composition when certain dimensions align.

So let V be a vector space with basis $v_1, ..., v_n$ and W be a vector space with basis $w_1, ..., w_n$. Let $S \epsilon L(V, w)$. Then S has a matrix, call it A. The entries of A will be denoted $(a_{i,j})$. So $S(v_j) = \sum_{j=1}^{m} a_{i,j} w_i$. Now additionally let $T \epsilon L(U, V)$. Then T has a matrix, call it B. The matrices of B will be denoted $(b_{j,k})$. So $Tu_k = \sum_{j=1}^{n} b_{j,k} v_j$. We know we can compose $S \circ T$ denoted $ST \epsilon L(U, V)$.

Question: What is the matrix of ST?
$ST_{uk} = S(Tu_k) = S(\sum_{j=1}^{n} b_{j,k} v_j)$
Using linearity of S...
$= \sum_{j=1}^{n} b_{j,k} S v_j = \sum_{j=1}^{n} b_{j,k} \sum_{i=1}^{m} a_{i,j} w_i = \sum_{i=1}^{m} (\sum_{j=1}^{n} a_{i,j} b_{j,k}) w_i$
Note: The inner summation of the final equation written are the entries of matrix ST. We therefore define the matrix product AB to be the matrix with entries $\sum_{j=1}^{n} a_{i,j} b_{j,k}$.

Example: Let $U = R^2 \to u_k = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $V = R^3 \to v_j = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $W = R^4 \to w_i = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$. $S \epsilon L(V, W)$ suppose it has a matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$. $T \epsilon L(U, V)$ suppose it has matrix $B = \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}$. Then

$ST\epsilon L(U,W)$. It's matrix is $AB = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 10 & 3 \\ 13 & 3 \\ 16 & 3 \end{bmatrix}$.

The rule to find the i,k entry of AB is: you multiply the i^{th} row of A by the k^{th} column of B (in sum dot product fashion)