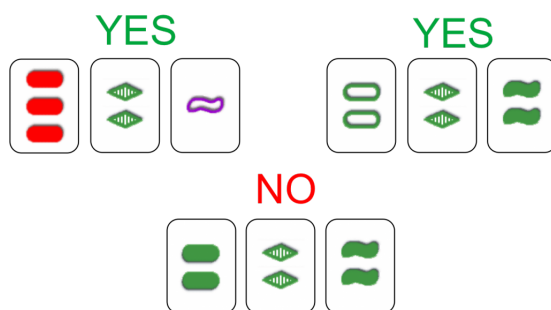


THE GAME OF SET AND FINITE GEOMETRIES

PAUL ELLIS, MANHATTANVILLE COLLEGE

1. HOW TO PLAY SET!

Each card has 4 attributes: color, shape, number, and shading. Three cards make a ‘set’ if each attribute is either the same on all three cards or different on all three cards.



If you see a set, then yell “SET!” and take the three cards. The dealer then puts three more cards on the table. The player who collects the most sets wins.¹

2. PROBLEMS ABOUT SET!

Problem 1. *How many cards are there in the SET deck?*

Problem 2. *How many sets are there? How many sets of each kind? (What are the different ‘kinds’?)*

Problem 3. *If you pick three cards at random, what is the probability they form a set?*

Problem 4. *Prove that if you take out 26 disjoint sets from the deck, the remaining 3 cards will also make a set.*

Problem 5. *What is the largest number of cards that can be on the table with no set appearing?*

¹Designed by Marsha Falco in 1974, SET evolved out of a coding system that she used in her job as a geneticist.

3. STEINER TRIPLE SYSTEMS

SET is an example of a Steiner Triple System (STS).² This is a finite *incidence geometry* consisting of “points” and “lines” with the following two properties:

- Given two distinct points, there is a unique line containing them.
- Every line has exactly three points.

Problem 6. *Explain how SET forms a Steiner Triple System.*

Problem 7. *Prove that the number of points in an STS is either $1 \bmod 6$ or $3 \bmod 6$.*

Problem 8. *Let $n = 3^k$. Construct an STS with n points.*

Problem 9. *Let $n = 2^k - 1$. Construct an STS with n points.*

Problem 10. *Can you construct an STS of a size not covered by the previous two problems?*

Problem 11. *(Kirkman’s schoolgirl problem)*

*Fifteen young ladies in a school walk out three abreast for seven days in succession: it is required to arrange them daily so that no two shall walk twice abreast.*³

Problem 12. *A solution to the previous problem is called a Kirkman Triple System (KTS). What is the relationship between STSs and KTSs?*

Problem 13. *In the Kirkman problem, there are $n = 15$ schoolgirls. For what other values of n are there solutions?*

Problem 14. *SET!*

²First described by W.S.B. Woolhouse in 1844 in the Prize question #1733 of *Lady’s and Gentlemen’s Diary*.

³proposed by Rev. Thomas Penyngton Kirkman in 1850 as Query VI in *Lady’s and Gentleman’s Diary*