A Game on $\mathbb{Z}_n$

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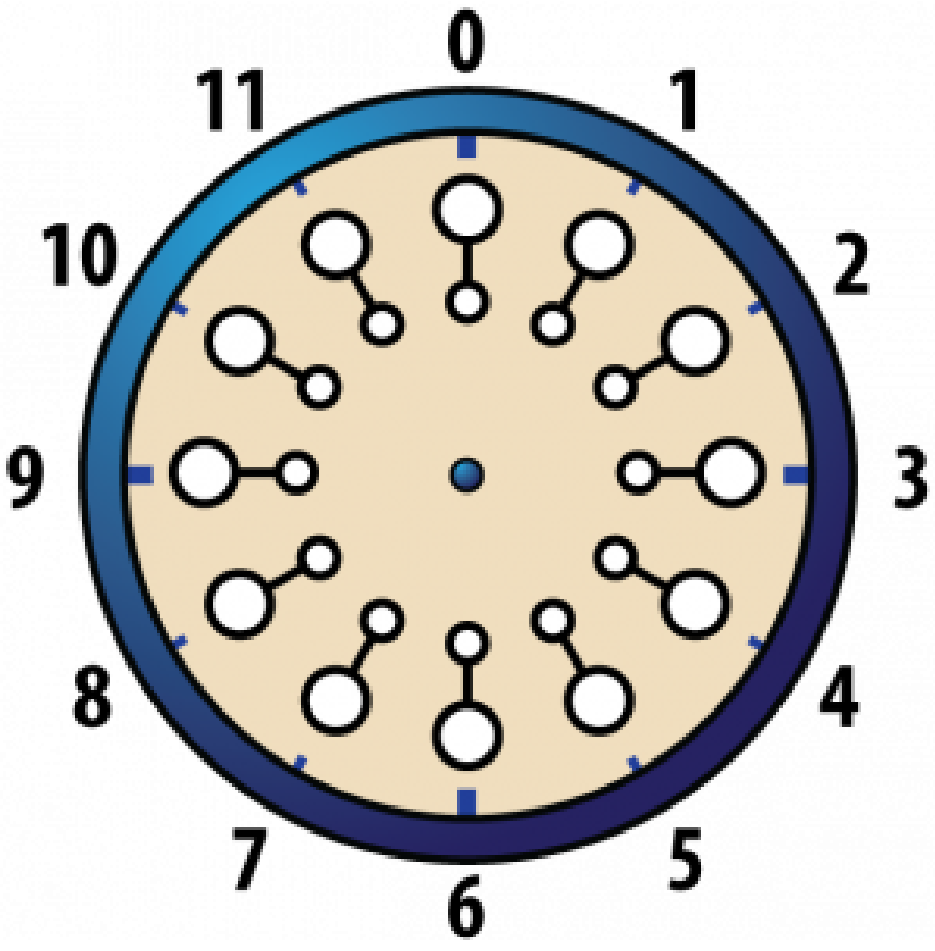
Modular Arithmetic - with clocks!



Some Practice

- $5 (\text{mod } 2) \equiv ?$
- $10 (\text{mod } 4) \equiv ?$
- $3 (\text{mod } 6) \equiv ?$

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- $5 (\text{mod } 2) \equiv 1$
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- $10 (\text{mod } 4) \equiv 2$
- $3 (\text{mod } 6) \equiv 3$

What is $\mathbb{Z}_n$?

$$Z_n = \langle 1, 2, 3, \ldots, n-2, n-1 \rangle$$

$$Z_5 = \langle 1, 2, 3, 4 \rangle$$

$$Z_{12} = \langle 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 \rangle$$

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- The game ends when all elements have been chosen or when the sum modulo n reaches 0.

Example Game

$$\mathbb{Z}_5 = \langle 1, 2, 3, 4 \rangle$$

Player 1	Player 2	Result
2		$2 (\text{mod } 5) \equiv 2$

Remaining choices: $1, \cancel{2}, 3, 4$

Example Game

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Player 1	Player 2	Result
2		$2 (\text{mod } 5) \equiv 2$
	4	$(2 + 4) (\text{mod } 5) \equiv 1$

Remaining choices: $1, \cancel{2}, 3, \cancel{4}$

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Player 1	Player 2	Result
2		$2 (\text{mod } 5) \equiv 2$
	4	$(2 + 4) (\text{mod } 5) \equiv 1$
1		$(1 + 1) (\text{mod } 5) \equiv 2$

Remaining choices: $\cancel{1}, \cancel{2}, 3, \cancel{4}$

Example Game

$$\mathbb{Z}_5 = \langle 1, 2, 3, 4 \rangle$$

Player 1	Player 2	Result
2		$2 (\text{mod } 5) \equiv 2$
	4	$(2 + 4) (\text{mod } 5) \equiv 1$
1		$(1 + 1) (\text{mod } 5) \equiv 2$
	3	$(2 + 3) (\text{mod } 5) \equiv 0$

And player two loses this round!

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