

Further Investigation:

- 1) List which games, if any, you won and if you played first/second. Do you notice a pattern?
- 2) What was your strategy when playing the games? Did you have one? Did it help you win games or not?
- 3) Can you list all of the PAIRS of elements in \mathbb{Z}_7 that add up to $0 \bmod 7$? What about the pairs in \mathbb{Z}_{10} for $0 \bmod 10$? \mathbb{Z}_8 for $0 \bmod 8$?
- 4) Did the game ever end before all the elements were played? How about if all elements were played? Does it matter?

Further Investigation Part 2: Building a Proof

- 1) Can Player 1 ever lose the game? Can Player 2 ever win the game? Under what conditions do these things occur?**

- 2) Let's play a game over \mathbb{Z}_a . You get to pick whether or not you want to go first or second. Which do you choose? Does it matter if I change the game to $\mathbb{Z}_a \times \mathbb{Z}_a$? What about any \mathbb{Z}_a where a is even? Odd?**

- 3) Provide some possible conjecture for this game: does it have a definite outcome for every instance it is played? Does someone always win and why?**