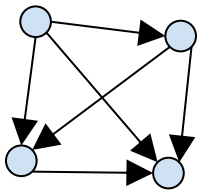
Tournament Winners

Imagine three friends play a game. Mia beats Alex, Alex beats Siki, and Siki beats Mia. Which one of the three, if any, should we consider to be overall winner? For this activity, a **tournament** will mean a series of games where every player plays everyone else once.

Activity 1: Play a Rock Paper Scissors Tournament!

- Introduce yourself to the other participants at your table. (Always a good start!)
- Two people at a time, play a Rock Paper Scissors match. (The first to win 2 times wins the match.)
- The winner gets to point at the other player with a yard stick. (Absolutely no violence. And no gloating – Rock Paper Scissors is a game of luck!)
- Keep playing until each of you has played against everyone else at the table. In the end you will have a diagram like the image.



1. Draw a diagram of your tournament results.

2. How many total Rock Paper Scissors matches did your group play?

3. Now that everyone has played, is there a clear overall winner?

4. Is the answer to the previous question different at any of the other tables?

Activity 2: Finding Winners

Let's say that a player A is a **strong winner** if they beat everyone else in the group. Let's say that a player A is a **weak winner** if for every other player B, player A beat player B, or player A beat someone who beat player B.

1. Does every tournament have a strong winner?

2. Did your group have any weak winners? If so, how many?

3. Can you fix your tournament so that the outcome matches each of the following criteria? In each case use your yard sticks to experiment and thei draw a dots-and-arrows diagram of your answer.
   0 strong winners, 1 weak winner

   1 strong winner, 1 weak winner

   0 strong winners, 2 weak winners

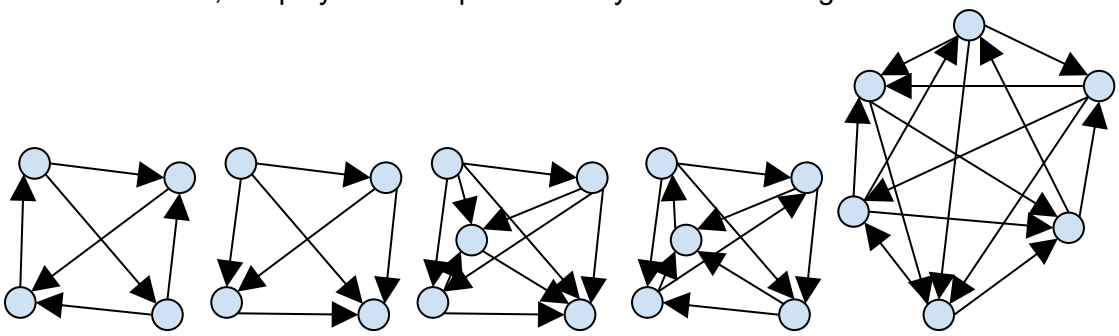
   0 strong winners, 3 weak winners

   1 strong winner, 0 weak winners

4. What do you observe about which outcomes are possible and which ones are impossible? Can you explain these observations?

Activity 3: Finding Weak Winners

1. For each of the following tournaments, write down the number of wins that each player has. Remember, the players are represented by dots in the diagram.



2. In each tournament above, look at the player with the most wins. Is that player necessarily a strong winner? Is that player necessarily a weak winner? If there is a tie, does this apply to all the players with the most wins?

3. **Claim**: Any player with the most wins will always be at least a weak winner.

    a. Let us call the player with the most wins player A. What would it mean if player A were *not* a strong winner?

    b. If player A were not a strong winner, explain why there is another player, call them player Z, who beat everyone A beat and who beat A as well.

    c. It player A has n wins, how many wins would player Z have?

    d. Explain why this means player A has to be a weak winner after all.

Activity 4: Bonus Questions

1. Find tournaments with four players that have 1 or 3 weak winners.

2. Explain why no tournament with four players can have exactly 2 or 4 weak winners.

3. Find a tournament with six players that has exactly 2 weak winners.

4. Suppose in a given tournament it is always the case that if A beat B, and B beat C, then A beat C. Explain why there must be a strong winner.

5. If player A is a weak winner but not a strong winner, is it true that player A must have beaten some other weak winner?