

Here are the first few fibonacci numbers:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765,
10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040,
1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169

Question 1. Which Fibonacci numbers are even? Which ones are odd? Why?

Question 2 Which Fibonacci numbers are divisible by 3?

Question 3. Which Fibonacci numbers are divisible by 4?

Question 4. Which Fibonacci numbers are divisible by 5?

Question 5. What patterns do you notice?

[Hint: It can be helpful to write the fibonacci numbers "mod" each divisor k]

[Hint: Try focusing just on divisibility by 2, 3, 5, 8, etc]

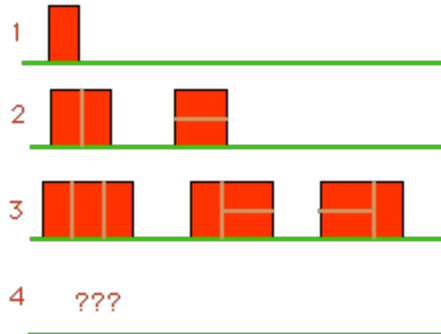
Now try it with another sequence: $2^n - 1$. Here are the first few of these numbers:

1, 3, 7, 15, 31, 63, 127, 255, 511, 1023, 2047, 4095, 8191, 16383, 32767, 65535,
131071, 262143, 524287, 1048575, 2097151, 4194303, 8388607, 16777215,
33554431, 67108863, 134217727, 268435455, 536870911, 1073741823

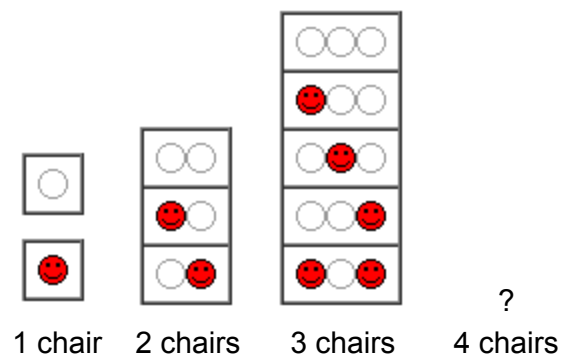
- Which of these numbers are divisible by 3?
- Which are divisible by 7?
- Which are divisible by 15?
- How does this pattern compare with the patterns in the fibonacci numbers? Can you explain why the pattern works?

Coloring problems

1. We have some bricks that are 1 unit by 2 units. We want to make a wall 2 units high, and n units wide. How many ways can it be done? The bricks can be vertical or horizontal.



2. Now you have a row of n chairs and you want to put students in some of the chairs. The students are taking an exam so they are not allowed to sit right next to each other. How many ways are there to do this?



Extensions

1. Look at the “running totals” of the Fibonacci numbers. Here are the first few:

$$1+1 = 2$$

$$1+1+2 = 4$$

$$1+1+2+3 = 7$$

$$1+1+2+3+5 = 12$$

...

Do you see any patterns in this new sequence? Can you explain it?

2. What if you only add up every *other* Fibonacci number

$$1+2 =$$

$$1+2+5 =$$

$$1+2+5+13 =$$

$$1+2+5+13+34 =$$

...

What do you get? Can you explain this pattern?

3. Look at the "diagonals" in Pascal's triangle:

1	1	
1 1	1 1	
1 2 1	1 2 1	
1 3 3 1	1 3 3 1	
1 4 6 4 1	1 4 6 4 1	...
1 5 10 10 5 1	1 5 10 10 5 1	
sum: 5	sum: 8	

Can you explain this pattern?