

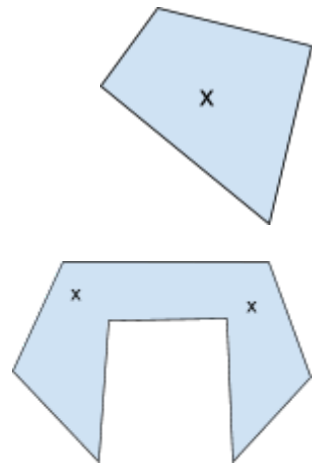
Taking in the Whole Room

How many security cameras do you need to see everything in a room?

- If the room is very simple, then just 1 camera is enough.
- If the room is more complicated, then you may need 2 or more cameras.

In the rest of this activity, we will make the following assumptions:

- The rooms are polygons with straight walls
- The rooms don't have any holes in the middle
- The cameras can see 360 degrees



Task 1.1

Design a complicated room that only needs 1 camera to see the whole thing.

Task 1.2

Design a room that needs lots of cameras to see the whole thing.

Task 2

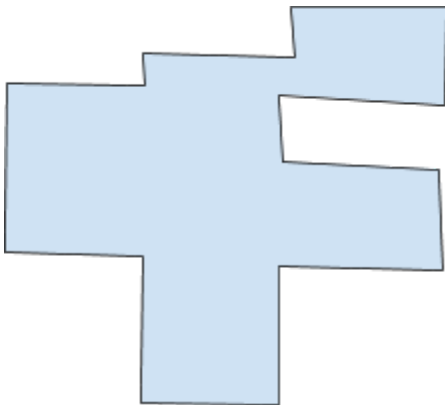
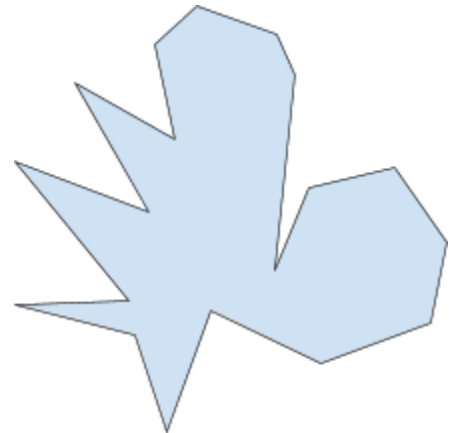
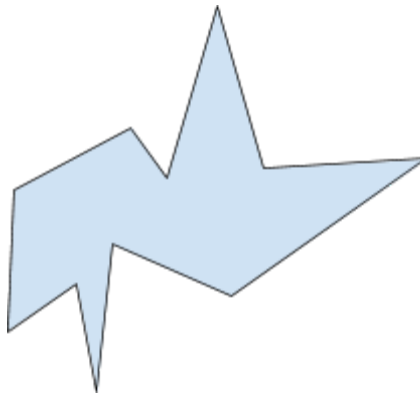
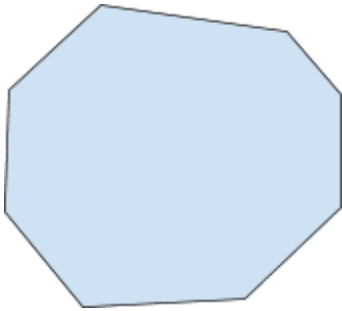
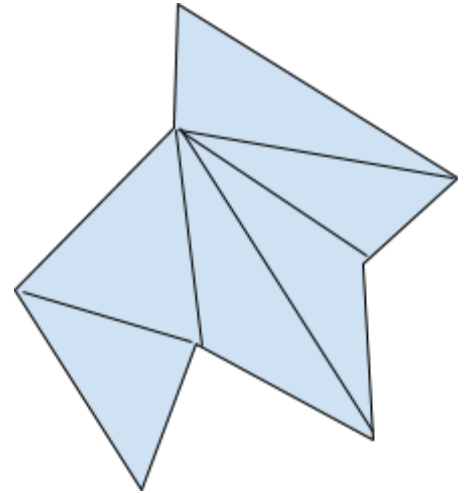
- Suppose you want to design a room that needs 2 cameras. How can you do this with the fewest number of walls?
- Suppose you want to design a room that needs 3 cameras. How can you do this with the fewest number of walls?
- Suppose you want to design a room that needs 4 cameras. How can you do this with the fewest number of walls? What about a larger number of cameras?

Triangulation

A **triangulation** of a polygon is any way to subdivide the polygon into triangles. (Don't add any new vertices on the walls or in the middle.)

Task 3

Triangulate each of the polygons below.



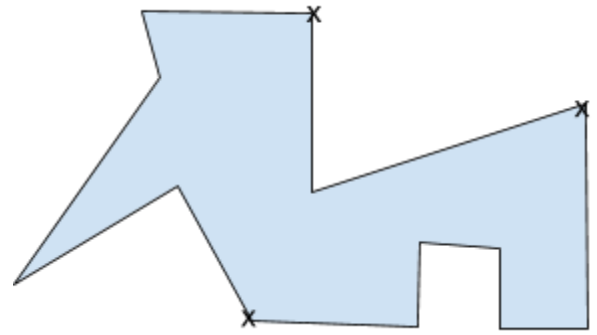
Your polygons:

Question

Is it always possible to triangulate a polygon? If so, explain why! (A mathematician would say “prove it”.)

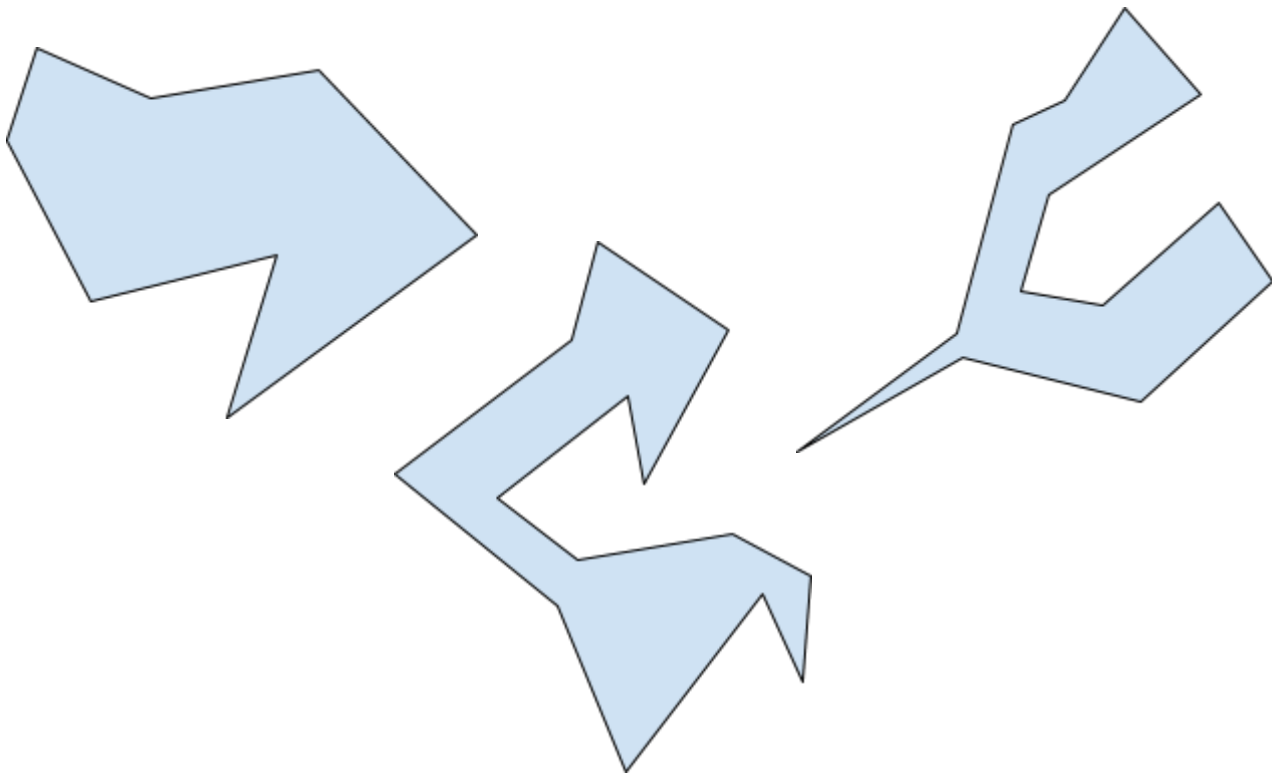
Cameras at the Vertices

Suppose you could only install cameras at the corners of the room (vertices of the polygon). Explain why this will always work (although it might not be optimal).



Task 4

For each of the polygons below, first triangulate the polygon, then place your cameras at vertices using as few cameras as you can.



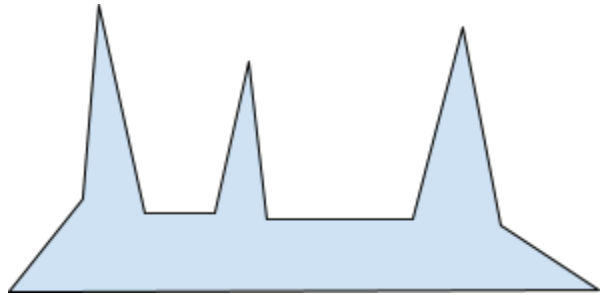
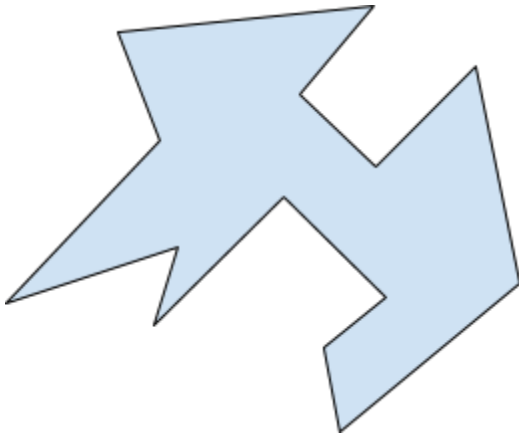
Question

If a polygon is triangulated, will you ever need more than 1 camera on the same triangle? Why or why not?

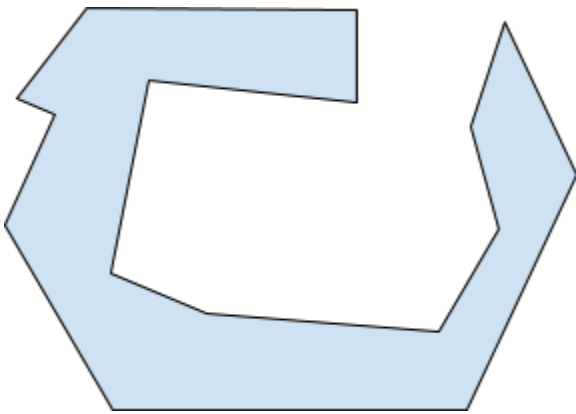
Task 5: Try a Camera Placement Strategy

Given a polygon, first triangulate it. Then, use three colors to assign each vertex 1 color, making sure that every triangle has all 3 colors. Finally, put your cameras at the vertices of a single color.

Try it!



Your polygons:



Question

Since this strategy always works, we only need at most _____ many cameras, where N is the number of vertices of the polygon. Fill in the blank and explain why.

Extensions

- What if you wanted to see the **outside** of the polygon with the cameras?
- What if the polygon has **holes**?
- Are there polygon rooms that need more than a third as many cameras as walls?
- Can you do better if all the angles in the polygon are right angles?
- What about three dimensional polyhedra?

References

- en.wikipedia.org/wiki/Art_gallery_problem
- www.ams.org/samplings/feature-column/fcarc-diagonals4
- math.iit.edu/~kaul/talks/LongArtGalleryTalk.pdf