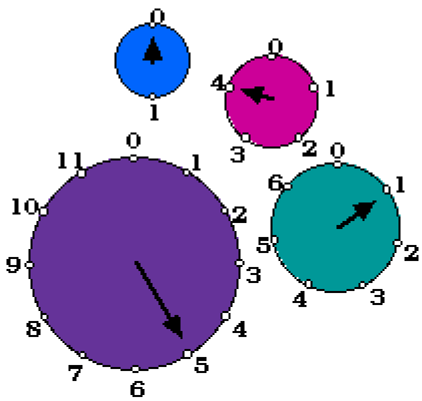
Clock Arithmetic

If we add numbers the way we add hours on the clock, days in the year, or degrees on a compass, we are doing **modular arithmetic**. A common form is clock arithmetic, where addition, subtraction, and multiplication are done on a **circular number line**.

Below are several different clocks. We can model modular arithmetic on these clocks. The biggest clock is similar to our own watches, with 12 hours shown. This is a modulo 12 (mod 12) clock. The smallest clock shown is a mod 2 clock.



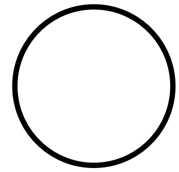
Draw a mod 3 clock and use it solve the following equations:

$2 + 2 = x$ $\quad$ $x + 2 = 1$ $\quad$ $x - 2 = 0$

Draw a mod 4 clock and use it solve the following equations:

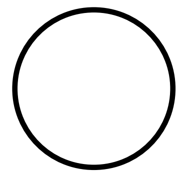
$1 + 2 + 3 = x$ $\quad$ $x + 3 = 1$ $\quad$ $1 - x = 3 + 2$ $\quad$ $2 * x = 1$

Draw a mod 5 clock and use it solve the following equations:



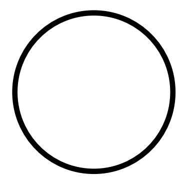
$1+\ 3*3=\ x$ $\qquad$ $3+2*4\ =x+\ 1$ $\qquad$ $2*x+1=4$

Draw a mod 11 clock and use it solve the following equations:



$10+10+3=\ x$ $\qquad$ $10*10=x$ $\qquad$ $x*10=7$

Come up with a different mod clock and a tough equation. See if your friend can solve it.



Addition Tables

It's often helpful to have a look-up table for calculations in clock arithmetic.

Complete the addition table for **mod 3** numbers.

+	0	1	2
0			
1		2	
2			

Make the addition tables for **mod 4, 5, 6, 7, and 8**. Make a list of patterns or conjectures you see as you work.

Multiplication Tables

Just like having tables for addition can be helpful, it's nice to have tables for multiplication.

Complete the multiplication table for **mod 3** numbers.

*	0	1	2
0			
1		1	
2			

Make the multiplication tables for **mod 4, 5, 6, 7, and 8**. Make a list of patterns or conjectures you see as you work.

Some Hard Equations

See if you can solve the following equations using clock arithmetic.

Note: some of the equations have many solutions, some have one or two, and some have no solutions. The best approach is to use your intuition and check your answers.

$x - 3 \cdot 5 = 6 \pmod 7$	$2^3 + x - 3 \cdot 5 = 1 - 4^4 \pmod 8$	$2x - 3^3 = 5^7 \pmod 7$
$2x + 11 = 3 + x \pmod 5$	$3x - 1 = 5 \pmod 6$	$x^2 = 4 \pmod 7$
$2^{10} = x \pmod 4$	$x^3 + 11x = 1 \pmod 6$	$(-1)^x = 2 \pmod 4$
$1 + 2 \cdot 5^{x-1} = 4 \pmod 7$	$x^2 + x + 1 = 0 \pmod 3$	$3x = 5x - 7 \pmod 8$
$x^5 + 5^x = 3 \pmod 7$	$(x+3)^2 = x^2 + 3^2 \pmod 5$	$5^x = 1 \pmod{31}$

Powers in Clock Arithmetic

Try listing all the powers of 2 in mod 9. What patterns do you see?

Hint: Reduce each answer (if possible) before calculating the next one.

Try calculating $\mathbf{2^{1000}}$ **(mod 9)**.

Try listing all the powers of 2 in mod 3, 4, 5, 6, 7, and 8. What patterns do you see?

Challenge Problems:

Can you calculate $\mathbf{2^{189}}$ **(mod 11)**? How can you know whether your answer is right?

Can you find a number whose powers mod 13 cycle through all the numbers?