**DO IT YOURSELF FRACTALS: CONNECTING GEOMETRY, MEASUREMENT, AND ALGEBRA**



**Mandy McDaniel**

**Boise State University**

**Teri Willard**

**Retired (Central Washington University)**

**November 10, 2018**

**Boise State University**

**Cutting a Set of Fractal Cards**

**Objective:** Experience a repetitive process that, when applied, will create cards exhibiting self-similarity. Each repetition of the process is called an iteration.

**Materials:** bright copy paper or construction paper, ruler with inch units, pencil, scissors, four sheets of 8 inch by 10 inch paper.

**Recommendation:** Use colored paper to make each card a different color.

**Card 1 (Stage 1 or Iteration 1)**

**1.** Select and fold one of the sheets of paper.



**2**. Measure **in**  • 8 inches = 2 inches. Draw a segment  • 5 inches = 2 inches **long**.



**3**. Cut and fold.



**4**. Unfold and crease each fold both ways.



**5**. Pop out the box.

**Card 2 (Stage 2 or Iteration 2)**

**1.** Select and fold another sheet of paper.



**2**. Measure **in**  • 8 inches = 2 inches. Draw a segment  • 5 inches = 2 inches **long**.



**3**. Cut and fold. Call the shaded portion the “inner rectangle.”



**4**. On the inner rectangle, measure **in**  • 4 inches = 1 inch. Draw a segment  • 2 = 1 inches **long**.



**5**. Cut and fold. The last diagram shows the new inner rectangle.



**6**. Unfold and crease each fold both ways.



**7**. Pop out the boxes.



**Card 3 (Stage 3 or Iteration 3)** Fill in the missing information and cut Card 3 without using pictures.

**1.** Select and fold another sheet of paper.

**2**. Measure **in**  • 8 inches = 2 inches. Draw a segment  • 5 inches = 2 inches **long**.

**3**. Cut and fold.

**4**. Measure **in**  • 4 inches = 1 inch. Draw a segment  • 2 inches = 1 inches **long**.

**5**. Cut and fold.

**6**. Measure **in**  • 2 inches = inches. Draw a segment  • 1 inches = inches **long**.

**7**. Cut and fold.

**8**. Unfold and crease each fold both ways.

**9**. Pop out the boxes.

**Card 4 (Stage 4 or Iteration 4)** Select your last sheet of paper. List your steps for cutting Card 4.

**Fractal Card Activities**

**Activity 1: Box Patterns for Fractals**

**Materials:** Stage 1 – 4fractal cards

**Objective:** Examine and extend patterns relating to the pop-out boxes on the fractal cards.

**Directions:** Extend the patterns for stage number 5 through *x*. Write expressions in *x* for columns 2 and 3.

**TABLE 1: Box Patterns**

|  |  |  |
| --- | --- | --- |
| Column 1 **Stage or Card Number, *x*** | Column 2 **Number of Boxes Added, *y1*** | Column 3Total Number of Boxes, ***y2*** |
| 1 | 1 | 1 |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| *x* |  |  |

**Question 1: Did you use your calculator to find the regression equation? Was the expression in the equation different from above? Why?**

**Question 2:** Write an equation using the expression from Column 2, *y*1 = (write your expression from Column 2, row *x*). On graph paper, label the *x* and *y*1 axes. Then graph each ordered pair (*x*, *y*1). Describe the shape of the graph.

**Question 3:** Write an equation using the expression from Column 3, *y*2 = (write your expression from Column 3, row *x*). On graph paper, label the *x* and *y*2 axes. Then graph each ordered pair (*x*, *y*2). Describe the shape of the graph.

**Activity 2: Fractal Patterns**

**Materials:** Stage 1 – 4fractal cards

**Objective:** Find your own two patterns. Then, examine and extend patterns relating to your fractal card patterns.

**Directions:** Fill in **Table 2**. Use the stage number as *x*. Your patterns will be *y*1 and *y*2.

**Describe your pattern for *y*1:**

**Describe your pattern for *y*2:**

**TABLE 2: My Patterns**

|  |  |  |
| --- | --- | --- |
| Column 1 **Stage or Card Number, *x*** | Column 2 ***y1*** | Column 3 ***y2*** |
| 1 | 1 | 1 |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| *x* |  |  |

**Write your equations below. Be prepared to share your patterns with the group.**

*y*1 =

*y*2 =

**Activity 3: Ratios for Fractal Cards**

**Directions**: Use the descriptions **A** – **G** to fill in **Table 3**. Use inches for all measurements. If your cards are not “perfect” use the measurements that would be ideal using the cutting pattern.

**A**. Stage Number (Card Number, always *x*)

**B**. Distance to measure **in** to make the newest cut (Write as a fraction, proper or improper, in simplest form to better see the pattern.)

**C**. Length of the newest cut (Write as a fraction, proper or improper, in simplest form to better see the pattern..)

**D**. Length of one face of the newest rectangular prism

**E**. The ratio of C to B (C ÷ B) (Write as a decimal.)

**F:** Area of **one** paper face of the newest, smallest box created at the given stage (Write as a decimal.)

**G**. Area of **all** box faces at the given stage (the paper covered faces only). Watch out!\*

**\* Hint for G:** Some box faces are part of two different boxes. Do not double count any areas. The last row of cells should be a constant value or a formula for the column in terms of *x*.

**TABLE 3: Ratios for Fractal Cards**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **A** | B | **C** | **D** | **E** | **F** | **G** |
| 1 | 2 | 2.5 = 5/2 | 4 | 1.25 | 10 | 20 |
| 2 |  |  |  |  |  | 25 |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| *x* |  |  |  |  |  |  |

For more information, read *Do–It–Yourself Fractal Functions* published in MATHEMATICS TEACHER | Vol. 110, No. 9 • May 2017, by Janet Shiver, Teri Willard, and Mandy McDaniel.

**Additional Ideas for Fractal Card Patterns**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **What Was Counted** | **Card 1** | **Card 2** | **Card 3** | **Card 4** | **Card *x*** |
| Edges added at each Stage | 11 | 22 | 44 | 88 |  |
| Edges added at each Stage (including invisible edges) | 12 | 24 | 48 | 96 |  |
| Total edges  (including invisible edges) | 12 | 36 | 84 | 180 |  |
| Paper planes added at each stage | 2 | 4 | 8 | 16 |  |
| Total paper planes | 2 | 6 | 14 | 30 | or |
| Invisible planes added | 4 | 8 | 16 | 32 | or |
| Total invisible planes | 4 | 12 | 28 | 60 | or |
| Planes (including the invisible ones) added | 6 | 12 | 24 | 48 |  |
| All Planes (including the invisible ones) | 6 | 18 | 42 | 90 |  |
| Vertices Added | 8 | 16 | 32 | 64 | or |
| Total Vertices | 8 | 24 | 56 | 120 | or |
| Right angles added each time (no invisible angles). | 16 | 32 | 64 | 128 | or |
| Right angles added each time including the invisible angles. | 24 | 48 | 96 | 192 |  |
| Total Right angles | 24 | 72 | 168 | 360 |  |
| Distance from the edge of previous rectangle | 2 | 1 | 1/2 | 1/4 |  |

**GEOMETRY VOCABULARY FOR FRACTAL CARDS**

|  |  |
| --- | --- |
| point | parallel lines |
| line | parallel planes |
| plane | perpendicular lines |
| rectangle | perpendicular planes |
| polygon | dihedral angle |
| quadrilateral | vertex |
| parallelogram | rectangular prism |
| two-dimensional | face |
| trapezoid | edge |
| line segment | three-dimensional |