Stat 202C   Project no.1 (15 points)

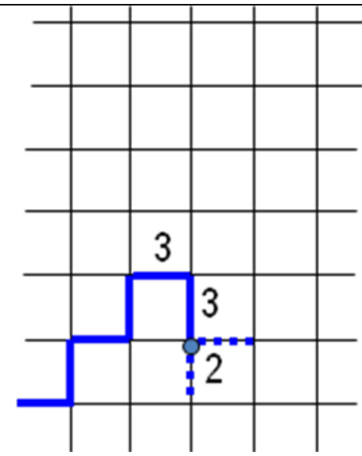
Due date: April 19 Friday at class.

Estimating the number of Self-Avoiding-Walks (SAWs) in an (n+1) x (n+1) grid.

Suppose we always start from position (0,0), i.e. lower-left corner. To solve this problem, we use Monte Carlo integration. We design a trial probability p(r) for a SAW r. Then we sample a number of M SAWs from p(r), and the estimation is calculated below.

$$
\begin{aligned}
K &= \sum_{r\in\Omega_{n2}} 1 = \sum_{r\in\Omega_{n2}} \frac{1}{p(r)} p(r) \\
&= E\left[\frac{1}{p(r)}\right] \\
&\approx \frac{1}{M}\sum_{i=1}^{M} \frac{1}{p(r_i)}
\end{aligned}
$$

$$
p(r) = \prod_{j=1}^{m} \frac{1}{k(j)}
$$



At each step, the trial probability p(r) can choose to stop (terminate the path) or walk to the left/right/up/down as long as it does not intersect itself. Each option is associated with a probability and these probabilities sum to 1 at each point.

1, What is the total number K of SAWs for n=10 [try $M=10^7$ to 10^8] ?   To clarify: a square is considered a 2x2 grid with n=1. Plot K against M (in log-log plot) and monitor whether the SIS process has converged. Try to compare at least 3 different designs for p(r) and see which is more efficient.

2, What is the total number of SAWs that start from (0,0) and end at (n,n)?

Here you can still use the same sampling procedure above, but only record the SAWs which successfully reach (n,n).   The truth for this number is what we discussed: $1.5687\text{x}10^{24}$.

3, For each experiment in 1, plot the distribution of the lengths of the SAWs in a histogram (Do you need to weight the SAWs in calculating the histogram?), and visualize the longest SAW that you find in print.

Submit a report: Your grade will be based on the quality of results and analysis of different designs.