CS147 - Lecture 13

Kaushik Patra
(kaushik.patra@sjsu.edu)

Binary Values & Operations ...

Boolean Values & Operations

- A Boolean variable assumes two values TRUE or FALSE.
  - TRUE is denoted as 1 and FALSE is denoted as 0

- There are three basic Boolean algebraic operations
  - AND (.) or conjunction (∧)
  - OR (+) or disjunction (∨)
  - NOT (') or negation (¬)

Boolean Function & Truth Table ...

Boolean Function & Truth Table

- A Boolean function is like any other algebraic function expressed as a function of a list of variable with a corresponding equivalent Boolean expression to evaluate the function value.

$$F(X,Y,Z) = X.Y' + Z$$

- Each part of the right hand side of the equation is called a '*term*'
  - X.Y' is a term and Z is another term.
  - X is also a term in X.Y' and thus Y'

Boolean Function & Truth Table

- Boolean functions are often represented in a truth table. For example the function F(X,Y,Z) = X.Y' + Z will have a truth table like the following.

X	Y	Z	F(X,Y, Z)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Boolean Function & Truth Table

- Accordingly, we can also create truth table for each basic Boolean operation.

Y = A.B		
A	**B**	**Y**
0	0	0
0	1	0
1	0	0
1	1	1

Y = A + B		
A	**B**	**Y**
0	0	0
0	1	1
1	0	1
1	1	1

Y = A'	
A	**Y**
0	1
1	0

Boolean Function & Truth Table

- '**Dual**' of a Boolean Function F is a function obtained by replacing '.' with '+' and '+' with '.' in the original function. Any function involving constant value 0 or 1, we interchange them too.
  - F(X,Y,Z) = XY + X'Z + YZ
  - Dual of F will be (X+Y)(X'+Z)(Y+Z)

- '**Complement**' of a Boolean function F is a function obtained by changing 1s to 0s and 0s to 1s in the truth table for the variable values of the original function F. This means like dual '.' is changed to '+' and vice-verse. However, unlike dual, variables are complemented too.
  - F(X,Y,Z) = X'YZ' + X'Y'Z
  - F'(X,Y,Z) = (X + Y' + Z)(X + Y + Z')

Basic Identities & Algebraic Manipulation ...

Basic Identities

1. X + 0 = X	2. X . 1 = X
3. X + 1 = 1	4. X . 0 = 0
5. X + X = X	6. X . X = X
7. X + X' = 1	8. X . X' = 0
9. (X')' = X	

10. X + Y = Y + X	11. XY = YX	Commutative
12. X+(Y+Z) = (X+Y)+Z	13. X(YZ) = (XY)Z	Associative
14. X(Y+Z) = XY + XZ	15. X + YZ = (X+Y)(X+Z)	Distributive
16. (X + Y)' = X'.Y'	17. (XY)' = X' + Y'	DeMorgan's

Boolean Algebraic Manipulation

- All the identity rules are used to simplify longer Boolean expression.

```
F = X'YZ + X'YZ' + XZ
  = X'Y(Z+Z') + XZ … by identity 14
  = X'Y.1 + XZ     … by identity 7
  = X'Y + XZ       … by identity 2
```

Boolean Algebraic Manipulation

- We can have some more commonly used theorems for Boolean expression simplification.
- Two columns shows the dual nature of Boolean algebra holds for these theorems too.

1. X + XY = X	4. X(X+Y) = X
2. XY+XY' = X	5. (X+Y)(X+Y') = X
3. X+X'Y = X + Y	6. X(X'+Y) = XY

Boolean Algebraic Manipulation

- The 'consensus' theorem is another useful method to simplify expression.
  - XY + X'Z + YZ = XY + X'Z
  - It's dual also holds true
    - (X+Y)(X'+Z)(Y+Z) = (X+Y)(X'+Z)

$$
\begin{aligned}
XY + X'Z + YZ &= XY + X'Z + YZ(X+X') \\
&= XY + X'Z + XYZ + X'YZ \\
&= (XY + XYZ) + (X'Z + X'YZ) \\
&= XY(1 + XYZ) + X'Z(1+Y) \\
&= XY.1 + X'Z.1 \\
&= XY + X'Z
\end{aligned}
$$

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