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Assignment 1

Question 1: (5 points)

The following sequence of real numbers has been obtained sampling an audio signal: 1.8, 2.2, 2.2, 3.2, 3.3, 3.3, 2.5, 2.8, 2.8, 2.8, 1.5, 1.0, 1.2, 1.2, 1.8, 2.2, 2.2, 2.2, 1.9, 2.3, 1.2, 0.2, -1.2, -1.2, -1.7, -1.1, -2.2, -1.5, -1.5, -0.7, 0.1, 0.9 Quantize this sequence by dividing the interval [-4, 4] into 32 uniformly distributed levels by placing the level 0 at -3.75, the level 1 at -3.5, and so on and level 31 at 4.00. Also, remember that quantization should result in least error

Write down the quantized sequence. (4 points)

Level	Quantization Buckets	Sampled Signals	Quantized Sequence	Bits
0	-3.75	1.8	22	00000
1	-3.5	2.2	24	00001
2	-3.25	2.2	24	00010
3	-3	3.2	28	00011
4	-2.75	3.3	28	00100
5	-2.5	3.3	28	00101
6	-2.25	2.5	25	00110
7	-2	2.8	26	00111
8	-1.75	2.8	26	01000
9	-1.5	2.8	26	01001
10	-1.25	1.5	21	01010
11	-1	1	19	01011
12	-0.75	1.2	20	01100
13	-0.5	1.2	20	01101
14	-0.25	1.8	22	01110
15	0	2.2	24	01111

16	0.25	2.2	24	10000
17	0.5	2.2	24	10001
18	0.75	1.9	23	10010
19	1	2.3	24	10011
20	1.25	1.2	20	10100
21	1.5	0.2	16	10101
22	1.75	-1.2	10	10110
23	2	-1.2	10	10111
24	2.25	-1.7	8	11000
25	2.5	-1.1	11	11001
26	2.75	-2.2	6	11010
27	3	-1.5	9	11011
28	3.25	-1.5	9	11100
29	3.5	-0.7	12	11101
30	3.75	0.1	15	11110
31	4	0.9	19	11111

How many bits do you need to transmit it? (1 points)

In order to cover levels 0-31 (32 Levels Total) we need 5 bits because $2^5 = 32$. As a result the total number of bits needed to transmit is:

$$total\ bits = 5\ [bits] * 32\ [levels] = 160\ bits$$

Question 2: (10 points)

A high-definition film color camera has 1080 lines per frame, 1920 pixels per line, with a 24 Hz capture frame rate. Each pixel is quantized with 12 bits per channel during the quantization process.

The capture pipeline employs the follow sequence:
1. YUV 4:2:0 color subsampling scheme
2. An optional feature, to the signal to standard definition CIF (352x288)
3. An obligatory MPEG2 compression phase
4. Disk write with a varying disk write speed (12 to 36 Mbytes per second).
Answer the following questions.

General Calculations:

$$bits/pixel = ((12 * 4) + 12 + 12)/4 = 18\ bits/pixel\ (P)$$

Assuming 1080 x 1920:

$$Bit\ Rate = N_i N_p N_{fps} P = 1080 * 1920 * 24 * 18 = 895,795,200\ bits/sec$$

$$Bit\ Rate = \frac{895,795,200\ bits}{1\ second} * \frac{1\ Mbyte}{8e+6\ bits} = 111.9744 \simeq 112\ Mbytes/sec$$

Assuming 352 x 288:

$$Bit\ Rate = N_i N_p N_{fps} P = 352 * 288 * 24 * 18 = 43,794,432\ bits/sec$$

$$Bit\ Rate = \frac{43,794,432\ bits}{1\ second} * \frac{1\ Mbyte}{8e+6\ bits} = 5.474 \simeq 5\ Mbytes/sec$$

If the second optional feature is off, what minimal compression ratio needs be achieved by the third compression step process? (4 points)

Solution:

Find the ratio of the calculated rate and the known disk write speed rate using the minimum speed of 12 Mbytes/second for the worst case scenario simulation.

$$Compression\ Ratio = \frac{Input\ Rate}{Output\ Rate} = \frac{112}{12} = 9.333:1$$

If the second optional feature is turned on to produce CIF format, how does your previous answer change? (3 points)

Solution:

Find the ratio of the calculated rate and the known disk write speed rate using the minimum speed of 12 Mbytes/second for the worst case scenario simulation. The answer will differ from the previous example because the pixels/frame is now altered before going into the compression phase. This means that there is less data that needs to be compressed which should lead to a smaller compression ratio. In this case there is no compression needed because the input bit rate doesn't exceed the minimum output bit rate.

$$Compression\ Ratio\ =\ \frac{Input\ Rate}{Output\ Rate}\ =\frac{5.474}{12}\ =\frac{1}{2}$$

If original pixels were square, how do the pixels stretch with the second optional feature turned on.? (3 points)

Solution:

$Original\ Image\ =\ 16:9\ (With\ Square\ Pixels)\ ->\ 352:198$

$Displayed\ at\ =\ 352:288$

As a result you can see that the width of the pixels remain unchanged but the height of the pixels got larger so the pixels themselves will be elongated vertically.

Question 3: (15 points)
Temporal aliasing can be observed when you attempt to record a rotating wheel with a video camera. In this problem, you will analyze such effects. Assume there is a car moving at 36 km/hr and you record the car using a film, which traditionally records at 24 frames per second. The tires have a diameter of 0.4244 meters. Each tire has a white mark to gauge the speed of rotation. (15 points)

General Calculations:

Car Speed:

36km/hr - 10 m/s

Tire Circumference:

$$2\pi r\ \rightarrow\ \pi d\ \rightarrow\ \pi(0.4244)\ =\ 1.33m$$

1 Full Rotation Takes:

$$\frac{10\ m}{1\ sec}=\frac{1.33\ m}{x\ sec}\ \rightarrow\ x\ =\ \frac{1.33}{10}\ \rightarrow\ 0.133\ seconds$$

Rotations per second:

$$\frac{1\,rotation}{0.133\,seconds} = \frac{x\,rotations}{1\,second} \rightarrow x = \frac{1}{0.133} \rightarrow 7.518\,rotations$$

If you are watching this projected movie in a theater, what do you perceive the rate of tire rotation to be in rotations/sec? (3 points)

Sample Rate:

$$24\,fps \rightarrow 1\,frame\,every\,0.0412\,seconds$$

Necessary Sample Rate to avoid aliasing:

$$2 * Max\,Frequency \rightarrow 2 * 7.518\,[rot/sec] \rightarrow 15.037$$

Answer:

Since we are sampling at 24 fps that is more than double the maximum frequency. In this situation we would not experience aliasing and we would see the rate of tire rotation at ***7.518 [rotations/second]***.

If you use your camcorder to record the movie in the theater and your camcorder is recording at one third film rate (ie 8 fps), at what rate (rotations/sec) does the tire rotate in your video recording (6 points)

Sample Rate:

$$8\,fps \rightarrow 1\,frame\,every\,0.125\,seconds$$

Necessary Sample Rate to avoid aliasing:

$$2 * Max\,Frequency \rightarrow 2 * 7.518\,[rot/sec] \rightarrow 15.037$$

Using Degree calculation:

$$First\,Frame\,Captured\,at\,0.125\,seconds$$

$$Degrees\,Rotated\,in\,first\,frame:$$

$$\frac{360\,[deg]}{0.133\,[s]} = \frac{x\,[deg]}{0.125\,[s]} \rightarrow x\,[deg] = \frac{360 * 0.125}{0.133} = 338.34\,degrees$$

Based on this the wheel has actually moved backwards ***21.66 degrees.*** It will continue this trend for the 2nd frame, 3rd frame, and so on seeming to move backwards at the rate above.

Rate of Rotation:

$$\frac{21.66\ [deg]}{x\ rotation} = \frac{360\ [deg]}{1\ rotation} \rightarrow x\ [rot] \ = \ \frac{21.66}{360} \ = 0.06016667\ rotations$$

There are 8 frames per second and 21.66 degrees per frame so that means the perceived rate of rotation is:

$$Rate\ of\ Rotation \ = \ \frac{0.06016667\ rotations/frame}{8\ frames/second} \ = \ 0.4813\ rotations/second$$

Aliased Frequency Formula:

$$f_a = \ |Sample\ Rate \ * \ RINT \ - \ Signal\ Rate|$$

$$RINT \ - \ Closest\ integer\ multiple\ of\ the\ sampling\ rate\ "Rs"\ to\ the\ frequency\ of\ the\ sampled\ signal$$

$$f_a = \ |\ 8 \ * \ 1 \ - \ 7.518| \ = \ 0.482\ rotations/second$$

Answer:

Since we are sampling at 8 fps that is less than double the maximum frequency. In this situation we would experience aliasing and we would see the rate of tire rotation at ***0.482 [rotations/second] backwards***. This can be seen both using the aliased frequency formula and the general degree per frame rotation.

The driver decides to participate in the race, and buys tires that safely allow a max speed of 180 km/hr. What must be the diameter of the tire if no temporal aliasing needs to be witnessed in the recording? (6 points)

General Calculations:

Car Speed: $180\ km/hr \ \rightarrow \ 50\ m/s$

Sample Rate: $24\ frames\ per\ second$

In order for there to be no temporal aliasing we need to maintain that:

$$f_{Max} < \frac{f_{sample}}{2}$$

Solution:

Using Nyquist relation:

$$Max\ Frequency < \frac{Sample\ Rate}{2} \rightarrow f_{Max} < \frac{24}{2}$$

$$f_{Max} < 12\ rotations/second$$

Using Proportion Math:

$$\frac{1\ rotation}{x\ seconds} = \frac{12\ rotations}{1\ second} \rightarrow x = \frac{1}{12} = 0.0833\ seconds\ for\ 1\ rotation$$

$$\frac{50\ m}{1\ second} = \frac{x\ [m]}{\frac{1}{12}\ seconds} \rightarrow x = \frac{50}{12} = 4.166\ meters$$

$$Circumference = \pi d \rightarrow 4.16 = \pi d \rightarrow d = \frac{4.166}{\pi} > 1.326\ meters$$

In order for there to not be any temporal aliasing the max frequency needs to be less than half of the sample rate. Which means that the max frequency needs to be less than 12 rotations/sec as calculated above. Using 12 rotations/sec as the max frequency I calculated the diameter of the tire using the given information. The final tire diameter that I ended with was ***1.326 meters.*** This means that the tire diameter needs to be ***greater than 1.326*** meters in order for the aliasing to not occur.