CNR with Bandwidth

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1 Review of existing model

The monoenergetic model follows the derivation in Spanne [1]:

$$\mathrm{CNR}(E) = \frac{|\mu_1(E) - \mu_2(E)|}{\sqrt{\mathrm{var}\{\mu_1(E)\} + \mathrm{var}\{\mu_2(E)\}}}, \tag{1}$$

where $\mu_1(E) \equiv \mu_{bg}(E)$ is the attenuation at the center voxel of a homogeneous spherical object of material "bg" and $\mu_2(E) \equiv \mu_{bg}(E) + \mu_c(E)$ is the attenuation of that same voxel with the addition of a small amount of contrast material "c." (In these notes, $i \in [1, 2]$ will index these two cases).

The variance is defined as

$$\mathrm{var}\{\mu_i(E)\} \propto \frac{1}{\bar{N}_i(E)}, \tag{2}$$

where

$$\bar{N}_i(E) = I_0(E) \, \exp\{-A_i(E)\}. \tag{3}$$

Here, $I_0(E)$ is the incident intensity (assumed monoenergetic for now), and A_i is given by

$$A_i(E) = \sum_j \mu_j(E) d_j, \tag{4}$$

where d_j is the length of each material, j, that is present in case i.

2 Bandwidth

We would like to introduce the spectral bandwidth, BW, into this model. We define BW as follows:

$$BW = \frac{\Delta E}{E} = 10^{-2} \text{ or } 10^{-4}, \tag{5}$$

where ΔE is the FWHM of the intensity function $I_0(E)$. We model the intensity function as:

$$I_0(E) = I_0 N(E|E, \sigma_{E,BW}), \tag{6}$$

where I_0 is a constant intensity and N is a normalized Gaussian function with mean E and standard deviation $\sigma_{E,BW}$:

$$\sigma_{E,BW} = \frac{E \cdot BW}{2\sqrt{2\ln 2}}, \tag{7}$$

calculated from the FWHM of the spectrum at energy E.

We can incorporate this expression into our model for $\bar{N}_i(E)$ as

$$\bar{N}_i(E) = I_0 \int_0^\infty dE' N(E'|E, \sigma_{E,BW}) \, \exp\{-A_i(E')\}, \tag{8}$$

and the variance and CNR are calculated as before.

3 Implementation

The mass attenuation data for "bg" (water) and "c" (Os, U or Pb) materials is taken from the NIST database. The sampled energy points for these databases are generally nonuniform and different for each material, with some materials requiring finer sampling around sharp absorption peaks. Accordingly, after the two materials are chosen, the attenuation data for the less densely sampled material is logarithmically interpolated onto the sampled energy points of the more densely sampled material, so they share the same array of E values from ~0-100 keV.

The integral in Eqn. 8 is then implemented with the following logic:

For each energy value E_k in the common energy array E_{arr},

- Calculate $\sigma_{E_k,BW}$ according to Eqn 7
- Define an n-length E' array: $E' \in [E_k - r\sigma_{E_k,BW}, E_k + r\sigma_{E_k,BW}]$
  - r defines the truncation width
  - Replace conflicting boundary values in E' with $\min(E_{arr})$ or $\max(E_{arr})$ as needed
- Calculate the n-length array $A_i(E')$ with Eqn. 4 using $\mu_i(E')$ logarithmically interpolated once more from the NIST data
- Calculate N, the n-length array of normalized Gaussian weights for E'
  - With mean E_k and standard deviation $\sigma_{E_k,BW}$
- $\bar{N}_i(E_k)$ is then calculated as

$$\bar{N}_i(E_k) = \mathrm{sum}(N \cdot \exp\{-A_i(E')\}), \tag{9}$$

  where $\cdot$ indicates an element-wise product.

The CNR is then calculated as before, with Eqns 1 and 2. I have been using $n = 50$ and $r = 4$. Many of these arrays can be pre-computed, which speeds up the actual implementation.

A sample implementation is shown in Figure 1 for $BW = 0$ (monoenergetic), $BW = 10^{-2}$ and $BW = 10^{-4}$. The inset shows detail near the finest structure of the monoenergetic plot, and the bottom plot shows the absolute percent difference as a function of energy.

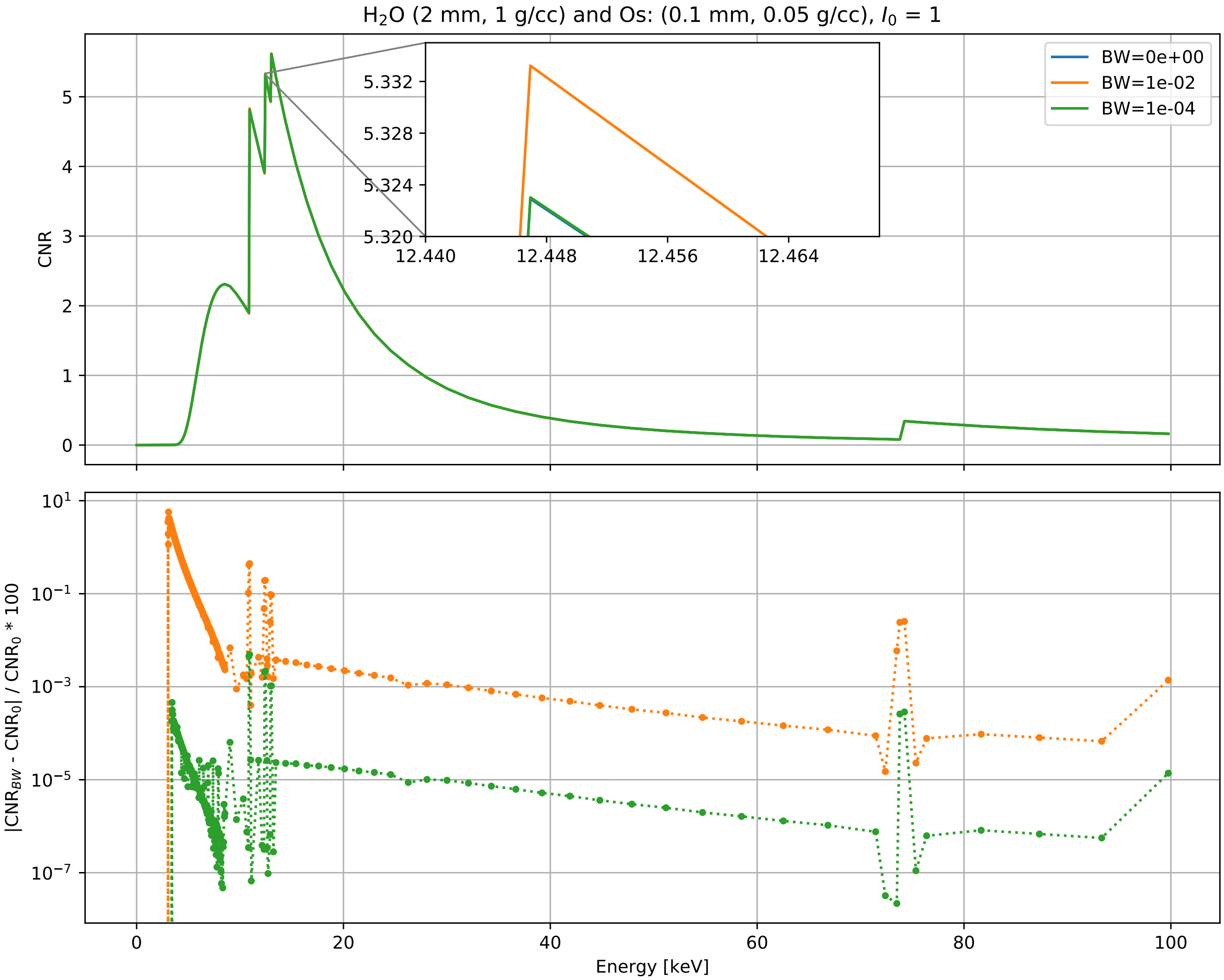

Figure 1: Results for CNR with bandwidth.

References

[1] P. Spanne, "X-ray energy optimisation in computed microtomography," *Physics in Medicine and Biology*, vol. 34, no. 6, pp. 679–690, 1989.