Update on Dual Energy Phase Contrast

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Model

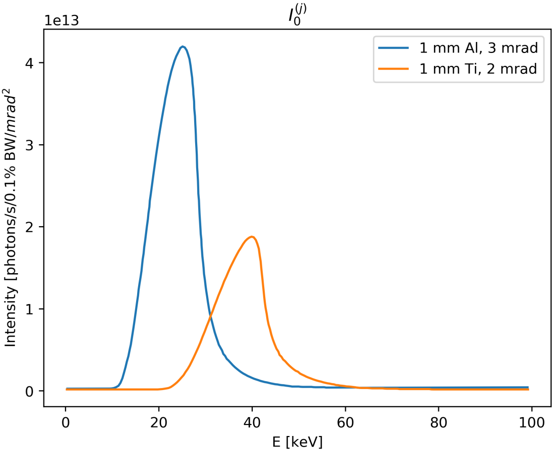
Measured intensity:

$$I_R^{(j)} = \int w(E) I_0^{(j)}(E) T(E) \left(1 + \frac{R_2}{k(E)} \nabla^2 \phi(E)\right) dE$$

with:

$w(E)$	detector response [$\approx E$]
$I_0^{(j)}$	entrance intensity for spectrum j [photons/s/0.1%BW/mrad^2]
$T(E)$	transmission factor [unitless]
R_2	sample-detector distance [$\approx$30 cm]
$k(E)$	wave number [$\frac{2\pi}{\lambda(E)} \approx 10^{11}$ m]
$\phi(E)$	phase factor [unitless]

Spectra: $I_0^{(j)}(E) \approx 10^{13}$



$\phi(E)$ and $T(E)$

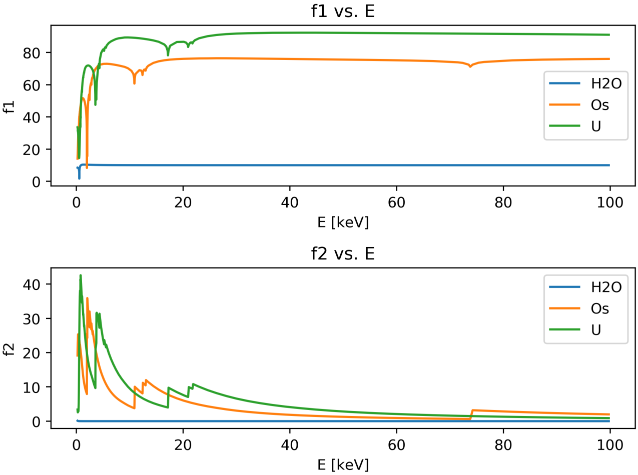
$$\phi(E) = r_e \lambda(E) \sum_i f_1^{(i)}(E) \int_L n_{a,i}(\vec{x}) dl$$

$$T(E) = \exp\left(-2 r_e \lambda(E) \sum_i f_2^{(i)}(E) \int_L n_{a,i}(\vec{x}) dl \right)$$

with:

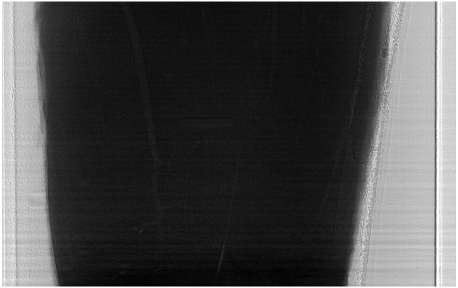
r_e	classical electron radius [$\approx 10^{-15}$ m]
$\lambda(E)$	wavelength [$\approx 10^{-11} - 10^{-9}$ m]
f_1, f_2	oscillation modes per atom [$\approx 10^1 - 10^2$ atom^{-1}]
$n_{a,i}(\vec{x})$	"atomic" number density [atoms / cm^3]

f_1 and f_2 from NIST

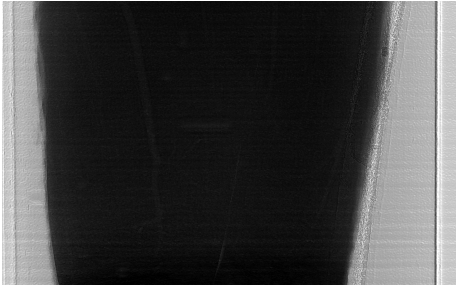


H_2O and Metal Phantoms

Al filter



No filter



$\int_L n_{a,i}(\vec{x})dl$

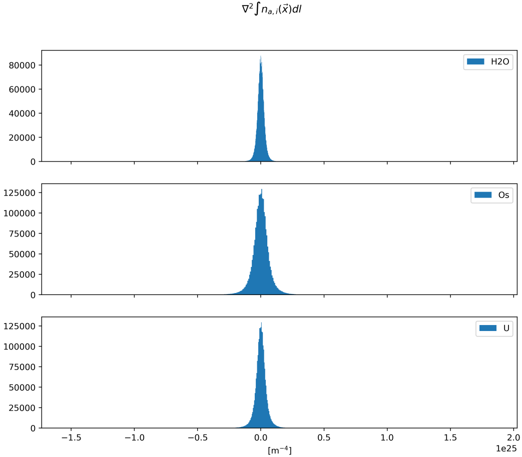
Normalized phantom images ($\mathrm{ph}_i(\vec{x})$) by number densities:

$$\int_L n_{a,i}(\vec{x})dl \approx \frac{\rho_i N_a}{A_i} \Delta L * \left(\frac{\mathrm{ph}_i(\vec{x})}{\mathrm{mean}\{\mathrm{ph}_i(\vec{x})\}} \right)$$
$$\approx 10^{25}\ [\mathrm{cm}^{-2}]$$

	H_2O	U	Os
ρ [g/cm^3]	1.0	19.1	22.59
A [g/mol]	18.03	238.03	190.23

$$N_a = 6.022e23 \text{ atoms/mole}$$
$$\Delta L = 2.38 \text{ mm}$$

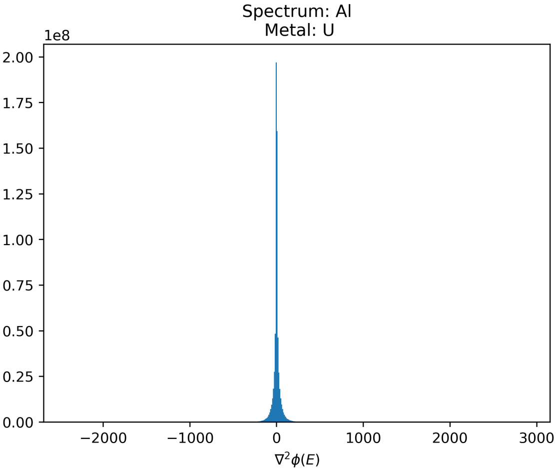
$\nabla^2 \int_L n_{a,i}(\vec{x}) dl$



	H_2O	Os	U
Mean	4e5	1e6	-1e6
Absolute value mean:	2e23	5e23	3e23
Max:	4e24	2e25	1e25

$$
\begin{aligned}
\nabla^2\phi(E) &= r_e\lambda(E)\sum_i f_1^{(i)}(E)\left[\nabla^2\int_L n_{a,i}(\vec{x})dl\right] \\
&\approx [10^{-15}\ \mathrm{m}][10^{-11} - 10^{-9}\ \mathrm{m}][10^2][10^6 - 10^{25}\ \mathrm{m}^{-4}] \\
&\approx 10^{-18} - 10^3\ \mathrm{m}^{-2}
\end{aligned}
$$

$\nabla^2\phi(E)$



Spectrum: Al
Metal: U
1e8
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
−2000
−1000
0
1000
2000
3000
∇²ϕ(E)

Issue: Small phase term

Recall:

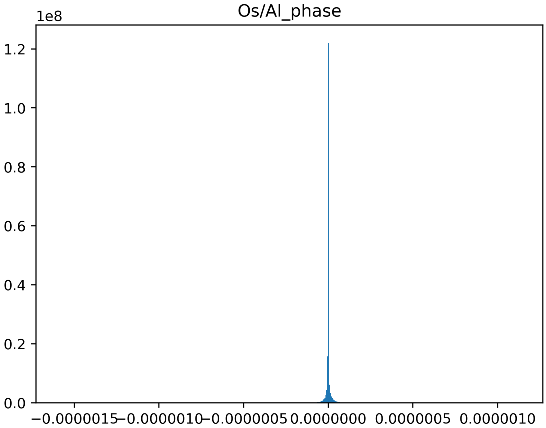
$$I_R^{(j)} = \int w(E) I_0^{(j)}(E) T(E) \left(1 + \frac{R_2}{k(E)} \nabla^2 \phi(E)\right) dE$$

Phase term:

$$\begin{aligned} & \frac{R_2}{k(E)} \nabla^2 \phi(E) \\ & \approx \frac{[10^1 \text{ m}]}{[10^{11} \text{ m}^{-1}]} \cdot [10^3 \text{ m}^{-2}] \\ & \approx 10^{-7} \end{aligned}$$

Issue: Small phase term

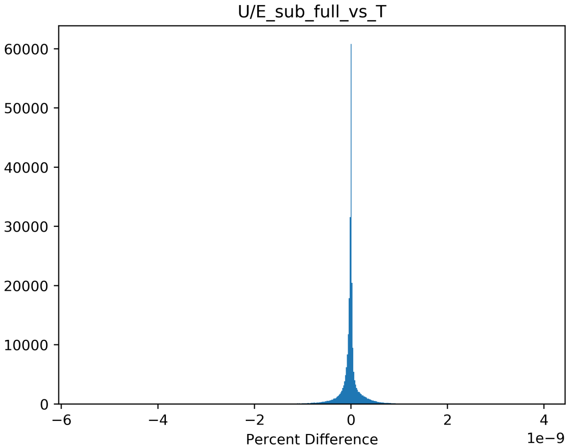
Actual magnitude of phase term:



Mean	7e-26
Absolute value mean:	6e-9
Max:	1e-6

Issue: Small phase term

Percent difference between subtraction image *with* phase and subtraction image *without* phase



Mean	-1e-11
Absolute value mean:	1e-10
Max:	4e-9