Spherical histogram notes

Algorithm for finding the spherical harmonic coefficients $\{c\}$.

- \bullet Choose N spherical bin points
- Find bin counts b_n from a list of K vectors
- For a given l, m, calculate the coefficient as

$$c_{l,m} = \frac{1}{K} \sum_{N} b_n Y_l^m(\theta_n, \phi_n) \tag{1}$$

We expect the true orientation distribution function to be even about the origin, hence the existing implementation which only uses the even-ordered l values.

However, we do not necessarily expect the raw ODF coming from the structure tensor orientation estimates to be even. We can force the raw ODF to be even by adding a new sample point on the sphere $(\theta_n + \pi, \phi_n)$ and giving it the bin value b_n for each existing point (θ_n, ϕ_n) . The final calculation of the SH coefficient thus looks like

$$c_{l,m} = \frac{1}{2K} \sum_{N} b_n \left[Y_l^m(\theta_n, \phi_n) + Y_l^m(\theta_n + \pi, \phi_n) \right]$$
 (2)

Currently, this is implemented as follows:

$$c_{lm} = sum(\boldsymbol{b} \cdot (\boldsymbol{Y}_{lm}^{+} + \boldsymbol{Y}_{lm}^{-}))/2K$$
(3)