A Resourceful Reframing of Behavior Trees

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Designers of autonomous agents, whether in physical or virtual environments, need to express nondeterminisim, failure, and parallelism in behaviors, as well as accounting for synchronous coordination between agents. Behavior Trees are a semi-formalism deployed widely for this purpose in the games industry, but with challenges to scalability, reasoning, and reuse of common sub-behaviors.

We present an alternative formulation of behavior trees through a language design perspective, giving a formal operational semantics, type system, and corresponding implementation. We express specifications for atomic behaviors as linear logic formulas describing how they transform the environment, and our type system uses linear sequent calculus to derive a compositional type assignment to behavior tree expressions. These types expose the conditions required for behaviors to succeed and allow abstraction over parameters to behaviors, enabling the development of behavior "building blocks" amenable to compositional reasoning and reuse.

Additional Key Words and Phrases: linear logic, behavior trees, programming languages, type systems

ACM Reference format:

Chris Martens, Eric Butler, and Joseph C. Osborn. 2016. A Resourceful Reframing of Behavior Trees. 1, 1, Article 1 (January 2016), 17 pages.

DOI: 10.1145/nnnnnn.nnnnnnn

1 INTRODUCTION

Specifying the desired behaviors of agents in environments is a major theme in artificial intelligence. Analysts often need to define particular policies with explicit steps, but the agents must also acknowledge salient changes in a potentially hostile or stochastic environment. This challenge arises in applications including robotics, simulation, and video game development. Games in particular bring challenges related to interaction with human decision-makers: even for games notionally working against the objectives of the player, the activity of game design is centrally concerned with helping the player learn something or have an emotional experience, and in this sense can be thought of as a cooperative system between agents with different knowledge states, not unlike human-robot teams. The behaviors of non-player characters (NPCs) in games must be designed in support of this goal.

Game designers must be able to specify that a given agent should patrol a hallway until it gets hungry (or its battery runs low) and goes home for a snack (or to recharge); but if the agent sees a one-hundred dollar bill on the ground on the way to where it recuperates, it should force a detour. In some designs, we would want an adversary (e.g., the player) to be able to trick the agent into running out of fuel by this mechanism; in other designs we would hope the agent ignores optional

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DOI: 10.1145/nnnnnnnnnnnnnnn

but attractive diversions and prioritizes severe need. We can easily imagine two distinct agents within the same game which are differentiated only by whether they can be misled in such a way.

Game character AI designers have somewhat contradictory goals that distinguish their project from, for example, game-playing AI whose objective is optimal play. On the one hand they want believable characters who react reasonably to player actions and to the behaviors of other non-player characters; but on the other hand they want to craft certain very specific *experiences* that nudge the player into trying new actions or approaching problems from a specific direction or that prevent the agent from performing awkward-looking sequences of 3D animations. Traditionally, game character AI was implemented with explicit state machines built by hand; more recently behavior trees, goal-oriented action planning, and utility-based systems have come into vogue.

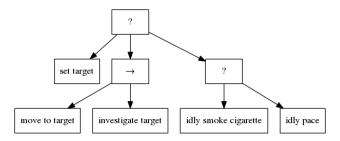


Fig. 1. A example behavior tree for a noise-investigation behavior. The tree is evaluated in preorder traversal. Leaf nodes specify actions in the world (such as moving to a target), which can succeed or fail. Interior nodes combine children into composite behaviors. The arrow (\rightarrow) is sequencing (run each child until first failure), and the question (?) is selection (run each child until first success).

Behavior trees are a scripting system for agents in virtual worlds, allowing designers of virtual agents to visually construct behavioral flowcharts based on conditions on the world around them. They are widely employed in the video games industry [16] for describing the "artificial intelligence" behavior of non-player characters, such as enemy units in combat simulators and members of virtual populations in open world-style games. Behavior trees have also been used for robot control [12]. They are often described as merging the utility of decision trees and state machines, allowing repeated or cyclic behaviors that modify and check state (internal or shared) as they execute. Figure 1 shows an example behavior tree for a hypothetical security guard character. The tree defines how to sequence and prioritize basic behaviors of listening for noises, investigating the source of the noise, or doing idle activities. During game simulation, behavior trees are typically re-executed with some frequency depending on the game, as often as once or more per time step. The example in Figure 1, for instance, needs to be executed twice to both acquire a target and investigate it.

Since behavior trees are often deployed in multi-agent simulations and with complex state-changing behavior, the ability for a designer to reason about the correctness of the tree quickly succumbs to its size and branching factor. Even for straightforward sequences of behaviors, the preconditions and postconditions are left unstated. For example, if an agent is told to move to door, open door, and go through door, we might reasonably expect that in all circumstances where the door is accessible, the agent will be on the opposite side of it by the time its behavior finishes. However, this is not possible to conclude unless we reason both about the conditions and effects of the individual actions and how the effects of earlier actions are expected to connect to the conditions of later ones. Such a sequence of actions could fail, for instance, if the player were to intervene and close the door immediately after the agent opened it. Furthermore, the success of behaviors may

depend on external conditions on the environment: an agent may expect another agent to have placed an important item that it needs, and the behavior is only correct on the condition that this dependency has been satisfied.

We describe an approach to reasoning compositionally about behavior trees in such a way that they may be constructed in small units, typechecked against an expected behavioral schema, and combined to form behaviors with new, compositionally-defined types. The approach requires the author to provide a linear logical specification of the atomic actions, i.e. the leaves of the tree; types for complex expressions formed from these leaves are derived from a linear logical interpretation of the behavior tree operations (sequencing, selection, and conditions). The present work can be seen as a way to regain some of the guarantees given by reasoning about a behavior from start to finish without losing the reactivity, which is the main benefit of using behavior trees over, for example, static plan generation [7].

Since behavior trees are a relatively simple formalism repeatedly realized in different incarnations, and since game developers are under somewhat notorious pressure to ship products, there is no authoritative, *standard* version of behavior trees. As alluded to above, a recurring issue with behavior trees is resolving the apparent tension between reacting to unexpected changes in the environment on the one hand and to performing authored behaviors over a longer duration on the other hand. The ad hoc extensions applied to behavior trees in the wild are often intended to resolve this tension. The approaches described in this paper could give a theoretical foundation for addressing these "hacks" employed in practice—and, potentially, for more principled and better-behaved adaptations of behavior trees towards the problem of designing complex agent and character behaviors.

Our contributions are a formal specification and operational semantics for our formulation of behavior trees, a type system and synthesis algorithm backed by an interpretation in linear logic, and an implementation of these systems in Standard ML. These results represent the first step of toward building a toolkit for robust authoring of virtual agent behaviors, combining support for correct human authorship and certified goal-driven synthesis of behaviors.

The rest of the paper is organized as follows: Section 2 discusses related work; Section 3 describes further how behavior trees are used in the games industry, Section 4 explains linear logical specifications and how they may be used to describe a possibility space for virtual worlds; Section 5 describes the syntax and operational semantics of our behavior tree language; Section 6 describes the type system and its guarantees; Section 7 describes our implementation; Section 8 discusses our current scope and future work; and 9 summarizes our contributions.

2 RELATED WORK

For the most part, efforts to provide robust formalisms to designers of virtual agents have been disjoint from formal and language-based approaches. We identify related work in two key areas: previous attempts to characterize virtual agent behaviors from a formal methods standpoint, and related models of computation that have been characterized with linear logic.

2.1 Formal accounts of behavior trees

Marzinotto et al. provide an account [12] of behavior trees in the context of robot control, citing a dearth of mathematical rigor prior to their contribution. Their work contributes the first mathematical definition of behavior trees and accounts for their expressive capabilities.

More recently, there has been some very recent work in applying synthesis and verification to AI behavior trees [4]. The formal basis for said work is model checking in linear temporal logic (LTL). Our work, by contrast, seeks a type-theoretic solution that supports modular reuse of behaviors.

2.2 Linear logical accounts of agents and processes

Linear Session Types [2] are an important touchstone for this work as another characterization of a pre-existing system, π -calculus, under a semantics derived from linear sequent calculus. Our work does not identify a direct logical correspondence between logical and operational notions in the same way, but similarly provides a basis for static reasoning about complex behaviors.

The CLF [18] logical framework and corresponding implementation Celf [17] form a basis for interpreting linear logic formulas as programs under a proof-construction-as-execution paradigm (logic programming). While operationally, this approach diverges from the semantics of behavior trees, the representation formalism informs out approach.

Finally, linear logic has been used to account for planning in a number of interesting ways: *deductive planning* [5] runs with the observation that, in addition to Masseron et al.'s observation that linear proof search can model planning [13], linear proofs *generalize* plans: they can characterize recursive and contingent (branching) plans, recovering some of the same expressiveness as behavior trees. Dixon et al. [6] apply deductive planning to an agent-based domain for dialogue-based environments. This work encourages us to consider integrating the generative abilities of planners with the reactivity of behavior trees in future work.

3 BACKGROUND: BEHAVIOR TREES IN GAMES

Behavior trees are widely used to define the behavior of non-player characters in digital game genres ranging from strategy and simulation to first-person shooters. The major game-making tools (Unreal Engine, Unity 3D, CryEngine, Amazon Lumberyard, and others) all either provide natively or have third-party implementations of the technique. The canonical examples of behavior trees' use in games come from the *Halo* series of first-person shooter games [9]. Notable in their formulation is that most of the tree is shared across the different types of enemy agents that appear in the game, which reflects the difficulty of authoring good and reasonable behavior policies in general. Behavior trees give authors a way to reuse some behaviors and override others from agent to agent.

Behavior trees are usually characterized as a *reactive* AI formalism, in this context meaning that agents are defined in terms of their reactions to a changing environment, rather than by a top-down plan that tries to achieve a goal by considering contingencies in advance. Certainly, even finite state machines can be made reactive by adding appropriate transitions, but scaling them to myriad potential game events quickly overwhelms authors. Behavior trees reduce that burden by asking a behavior author to structure the reactive behaviors in a tree, implicitly defining which behaviors supersede or interrupt which other behaviors by their position in a preorder traversal of that tree.

A behavior tree is a data structure describing how an agent decides on its next actions, and at the leaves some primitives for executing those actions. Behavior trees are repeatedly *evaluated* and on each evaluation they process their nodes in sequence. When a node is processed, it evaluates with some status: RUNNING, SUCCEEDED, or FAILED. Different sorts of nodes in the tree are specified in terms of the circumstances under which they evaluate to each return value.

A key question in behavior tree semantics is whether a tree which ends an evaluation with the RUNNING status should, on the next evaluation, continue from where it left off; the alternative is for it to begin its next evaluation from the root again. The latter approach is more *reactive* to changes in the environment or interruptions to behaviors, but in the former it is easier to specify and conceptualize behaviors which take some time and should not be interrupted. It is also easier to avoid behavior *oscillations* in the former evaluation strategy. For example, with the investigation example from Figure 1: with the latter approach, the agent can be interrupted by a new noise when moving to a target, while with the former approach, the agent will fully investigate a target

without distraction. Game designers have explored both semantics and even hybrids between these approaches; we leave our discussion of this issue until Sec. 5.

Leaf nodes of the tree can be domain-specific *conditions* (which succeed if the condition is currently satisfied or fail otherwise) or domain-specific *actions* (for example, setting the value of a variable or triggering some external action). These are the only operations which can interact with the environment. The actions in Figure 1 include setting a variable representing the agent's current target or physically navigating the agent towards said target. Failure may come from, for example, there being no navigable path to the target. In video games, these are often implemented using arbitrary program code working outside of the behavior tree formalism.

Non-leaf nodes come in three key variants (others are usually possible to define as syntactic sugar). First, *sequences* evaluate each of their child nodes from left to right, and are RUNNING if any child node is RUNNING, FAILED if any child is FAILED, or SUCCEEDED otherwise. Second, *selectors* also evaluate their child nodes left to right, but are RUNNING if any child is RUNNING, SUCCEEDED if any child has SUCCEEDED, and FAILED if all the child nodes are FAILED. Third, the *parallel* node evaluates each of its children independently of each other, and has SUCCEEDED if more than a certain number of its children succeeds, FAILED if more than a certain number fail, and RUNNING otherwise. It is also implicit in the definition of behavior trees that there is some external environment where state can be stored and persisted from evaluation to evaluation.

In practice, there are many other types of nodes that can alter the semantics of the tree in arbitrary ways, often violating the assumption of a preorder traversal: repeaters which evaluate their children over and over until they evaluate with some status, stateful versions of sequence and selector with *memory* that remember when they get stuck in RUNNING and only evaluate from that stuck node forwards in their next evaluation, and so on. We ignore such extensions in this work to simplify the presentation. Most of the extensions of behavior trees are meant to facilitate long-running actions, to limit the reactivity of behavior trees (e.g., to allow interruptions only at designer-defined times), and to ease the sharing of behavior tree or character state across situations, characters, and actions. Actions, conditions, and decorators often themselves involve arbitrary code in practice, so in our presentation of the formal semantics we require a linear logic formulation of the leaf nodes.

4 ACTION SPECIFICATIONS IN LINEAR LOGIC

As a first step towards a type system for general behaviors, we concretize *action specifications* for describing the behavior of an atomic action, such as "idly smoke cigarette" in Figure 1. Although in reality, this behavior may simply take the form of an observable effect (e.g., some animation), semantically, there are certain things we expect for it to make sense: for instance, that the agent has a supply of cigarettes (and perhaps that this action spends one). Other actions, like passing through a door, have more important requirements and effects, such as requiring being near the door and resulting in the door being open: these are aspects of the environment that may be created, or depended on, by other agents (or the same agent at another time).

There is a long line of successful work on describing actions in a protocols and virtual worlds using any of a class of related formalisms: multiset rewriting, Petri nets, vector addition systems, and linear logic. These systems have in common an approach to specification using *rules* (or *transitions* in some systems) that describe dependencies and effects, such that the cumulative effects of applying those rules may be reasoned about formally.¹

¹Planning domain description languages also share this approach, but most standards such as PDDL [14], do not have as clean of a compositional interpretation due to their allowance for the "deletion" of facts that do not appear as preconditions.

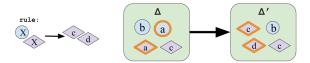


Fig. 2. One step of multiset rewriting execution, visualized. Each color/shape (purple diamond, blue circle) represents a distinct predicate; the contents of those shapes are terms (a, b, c) or term variables (X). This diagram represents a transition of the state $\Delta = \{diamond(a), circle(a), circle(b), diamond(c)\}$ along the rule $circle(X) \otimes diamond(X) \longrightarrow diamond(C) \otimes diamond(C)$ to the new state $\Delta' = \{diamond(c), diamond(d), circle(b), diamond(c)\}$. The thick orange borders on some atoms highlight which ones are replaced and added by the rule.

The following example uses a linear logic-based notation adapted from Ceptre [11] to describe action specifications for an Investigation world that could assign meaning to the actions used in Figure 1:

```
set_target : no_target -o has_target.
move_to_target : has_target -o has_target * at_target.
investigate : has_target * at_target * heard_noise -o no_target.
smoke : has_cigarette -o 1.
pace : 1 -o 1.
```

The "lolli" syntax A \neg o B describes the ability to transition from a world in which A obtains to one in which A no longer obtains and has been replaced with B. The atomic propositions include facts like at_door and door_open, which represent pieces of world state, the "tensor" p * q syntax conjoins them, and 1 is the unit of tensor. World configurations can be represented as multisets (or linear contexts) \triangle specifying which facts hold, such as {no_target, heard_noise, has_cigarette, has_cigarette}.

In general, predicates can take arguments (e.g., at(castle)) and rules can universally quantify over variables that stand in for term arguments, in which case states are always ground (contain no variables) and the application of rules identifies appropriate substitutions for variables for which the rule applies. Figure 2 visualizes a step of execution for an example.

Multiset rewriting has been used commonly to model nondeterminism and concurrency: rulesets can be nondeterministic whenever multiple rules may apply to a given state, and concurrency arises from the partial-order causal relationships between rules firing. If two rules operate on disjoint parts of the state, for instance, they can be considered to fire simultaneously, whereas rules that depend on the effects of previous rules firing must obey sequential ordering. See Figure 3 for a diagram of the causal relationships between actions for a particular program trace in which the agent sets a target, moves to the target, investigates a noise, and smokes a cigarette.

For the work described in this paper, however, we are less interested in the multiset rewriting interpretation of the rules. The specification under the multiset rewriting interpretation alone does not give us as authors any control over strategies for action selection or goal-driven search. Instead, it can be thought of as a description of the *space of possible actions* and a way of calculating their cumulative effects. Behavior trees, then, can be understood as directives for how to explore this space.

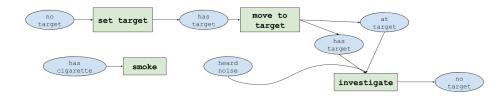


Fig. 3. A causal diagram for a possible trace of actions under the multiset rewriting interpretation of the Investigate specification.

Formally, we define action specifications under the following grammar:

$$arg ::= t \mid x$$
 $args ::= \cdot \mid arg, args$
 $S ::= p(args) \mid 1 \mid S \otimes S$
 $opdecl ::= name : xs. S - S$
 $\Sigma ::= \cdot \mid \Sigma, opdecl$

 Σ is a collection of specifications for operators op. Σ may also specify a collection of valid domain types over which the arguments of operators may range; for example, the operator move(Dir, N) may range over directions and natural numbers, perhaps meaning to move in that direction a certain number of units. The world state Δ is represented as a linear logic context, i.e. a multiset of atomic propositions p(args) representing available resources.

In the next section, we assume an arbitrary signature Σ for each action that computes a function on states, which does not depend on the linear logical interpretation. However, we revisit this idea in Section 6 to assign types to behavior tree expressions.

5 BTL: A FORMAL SEMANTICS FOR BEHAVIOR TREES

In this section we describe BTL, a formal calculus for describing synchronous agent behaviors with sequencing, branching, conditions, and loops.

The goals of this system are similar in many ways to the BTs used in practice: we aim to provide simple authoring affordances for scripting reactions to different circumstances in an implicit environment that is changing around the agent, and which the agent can change. We also adopt some goals that are not currently met by industry practice:

- Compositional reasoning. In order to be able to reason about BT nodes in terms of the behaviors of their subtrees, we need to know that subtree behaviors won't be interrupted in unknowable states.
- Debugging support—specifically, the ability for authors to state what they expect a behavior
 to accomplish and have this expectation checked algorithmically. The algorithm should be
 able to identify where in the tree a stated expectation is violated.
- Support for the expression of coordinated multi-agent behaviors. This requirement means
 that we question the notion of an action dependency necessarily meaning failure and
 instead (or additionally) require a blocking semantics (for instance, an agent may wait until
 another agent joins them in the same location to hand off a needed item).
- Support for the eventual integration of behavior synthesis, or algorithms that accept a
 propositional goal for the world state and generate BT subtrees corresponding to conditional
 plans that achieve the goal.

These nonstandard goals entail some tradeoffs of expressiveness. While it would be ideal to retain, for example, the "reactive" nature of BTs that allow them to break sequential actions to tend to urgent interruptions, we do not adopt this form of reactivity by default because it would preclude the ability to reason about sequential behaviors compositionally. In Section 8 we revisit these expressiveness tradeoffs and consider ways to re-incorporate additional features.

5.1 Expressions

The expressions of BTL are:

$$\alpha ::= op(args) \mid ?p. \alpha \mid Seq\{\alpha; \alpha\} \mid Sel\{\alpha + \alpha\} \mid Seq\{\} \mid Sel\{\} \mid Repeat\{\alpha\}\}$$

Intuitively, op(args) is an atomic action, invoking a pre-defined operator on a set of ground arguments (such as move(left)); Seq{ α ; α } is a sequence node; Seq{ α + α } is a selector node; Seq{ β is the unit of sequencers (does nothing); Sel{ β is the unit of selectors (always fails); β and executes α if it holds, failing otherwise; and Repeat{ α } is a repeater node, running α until failure.

5.2 Operational Semantics

We define an operational semantics for behavior trees in terms of what they may do to an abstract world state, using a big-step evaluation judgment $\alpha \triangleright \Delta \Downarrow \delta$, where Δ is a world state and δ is the result of evaluating a BTL expression, either a new world state (on successful execution) or FAIL.

The evaluation judgment requires a few preliminaries to define. First, we implicitly index the judgment by a signature Σ , which provides a specification for a *transition function* $t:\tau\to\Delta\to\delta$ for each operator (atomic action) available to an agent, which takes arguments of type τ , computes a transformation on a world state if the action can be performed, and returns FAIL otherwise. Concretely, our linear logical action specifications can play this role. Second, we assume a notion of a condition "holding for" a world state, expressed by the judgment $\Delta \Vdash p$. Again, while evaluation can be defined holding this judgment abstract, in we can fulfill this definition by expressing conditions in terms of a (positive) subset of linear logic formulas and interpreting \Vdash as affine provability.

Evaluating an operation consists of looking up its transtition function in Σ and applying that function to the current state; evaluating a condition requires that the current state satisfies the condition, and otherwise fails:

$$\frac{\Sigma(op) = t \quad t(args, \Delta) = \delta}{op(args) \triangleright \Delta \Downarrow \delta} \qquad \frac{\Delta \Vdash S \quad \alpha \triangleright \Delta \Downarrow \delta}{?S. \ \alpha \triangleright \Delta \Downarrow \delta} \qquad \frac{\Delta \not\Vdash S}{?S. \ \alpha \triangleright \Delta \Downarrow \mathsf{FAIL}}$$

A sequence evaluates by chaining the states computed by successful subtrees through successive subtrees in the sequence, and fails if any subtree fails:

$$\frac{}{\operatorname{Seq}\{\} \triangleright \Delta \Downarrow \Delta} \qquad \frac{\alpha \triangleright \Delta \Downarrow \Delta' \quad \operatorname{Seq}\{\alpha'\} \triangleright \Delta' \Downarrow \delta}{\operatorname{Seq}\{\alpha; \alpha'\} \triangleright \Delta \Downarrow \delta} \qquad \frac{\alpha \triangleright \Delta \Downarrow \operatorname{FAIL}}{\operatorname{Seq}\{\alpha; \alpha'\} \triangleright \Delta \Downarrow \operatorname{FAIL}}$$

A selector succeeds with the first successful subtree and fails if no options are possible:

$$\frac{}{\mathsf{Sel}\{\} \triangleright \Delta \Downarrow \mathsf{FAIL}} \qquad \frac{\alpha \triangleright \Delta \Downarrow \mathsf{FAIL} \quad \mathsf{Sel}\{\alpha'\} \triangleright \Delta \Downarrow \delta}{\mathsf{Sel}\{\alpha + \alpha'\} \triangleright \Delta \Downarrow \delta} \qquad \frac{\alpha \triangleright \Delta \Downarrow \Delta'}{\mathsf{Sel}\{\alpha + \alpha'\} \triangleright \Delta \Downarrow \delta}$$

Repeaters continue evaluating the underlying expression until failure:

$$\frac{\alpha \triangleright \Delta \Downarrow \Delta' \quad \mathsf{Repeat}\{\alpha\} \triangleright \Delta' \Downarrow \delta}{\mathsf{Repeat}\{\alpha\} \triangleright \Delta \Downarrow \delta} \qquad \frac{\alpha \triangleright \Delta \Downarrow \mathsf{FAIL}}{\mathsf{Repeat}\{\alpha\} \triangleright \Delta \Downarrow \Delta}$$

This definition of BTL adopts similar conventions and semantics to process algebras, such as the adoption of two key operators, sequential (conjunctive) and choice (disjunctive) composition, which have certain algebraic properties. In the case of BTL, evaluation respects the following structural congruence:

In other words, sequences form a monoid with the unit Seq{}; selectors form a monoid with the unit Sel{}; and sequencing distributes over selection. We state that this equivalence is respected by evaluation but omit the proof for brevity:

Conjecture 5.1. BTL operational semantics respects congruence: If $\alpha \triangleright \Delta \Downarrow \delta$ and $\alpha \equiv \beta$ then $\beta \triangleright \Delta \Downarrow \delta$.

While the system bears resemblance to models of concurrency such as CSP [8] and (CCS) [15], it differs in that interactions between BTL expressions and their environment happen implicitly through manipulation of a shared world state, not through channel-based communication (as in CSP) or explicit labels for inputs and outputs (as in CCS). The lack of such machinery is what makes behavior trees so attractive to authors; it reduces the burden of needing to specify how information is transmitted from one agent to another. However, it also makes the dependencies between agents tacit and therefore difficult to debug when things go wrong, which is what this paper aims to address.

Kleene algebra, particularly Kozen's variant with tests (KAT) [10], offers another touchstone for semantic insights; however, BTL does not quite satisfy the Kleene conditions: (1) order matters in selector semantics due to fallthrough, so selectors are not commutative; (2) the annihilation law does not hold; $Seq\{\alpha; Sel\{\}\}$ is not equivalent to $Sel\{\}$ due to the state changes that α may incur.

5.3 Example

Below is BTL implementation of the behavior tree described in Figure 1. This and all future examples use an n-ary form of Seq and Sel defined in the obvious way.

```
Sel{?heard_noise.set_target() +
    Seq{move_to_target(); investigate_target()} +
    Sel{idle_smoke() + idle_pace()}}
```

To illustrate how an BTL expression evaluates, we consider an evaluation of this tree in an environment where the agent already has a reachable target and has not heard a noise, i.e. the situation {has_target}. Starting evaluation at the root, the outer selector expression evaluates each child in succession until one succeeds. The first child will fail because the heard_noise condition does not hold. The second child, a sequence, will evaluate each of its children in succession. The first action, predicated on having a target, evaluates by modifying the world state such that the agent is in the same location as the target. Upon the movement action succeeding, the investigate_target() action will be evaluated; however, this node fails in the absence of having heard a noise, and that failure propagates to the root of the tree.

$$\frac{\Gamma; p \vdash p \ init}{\Gamma; p \vdash p \ init} \quad \frac{\Gamma; \Delta \vdash M : C}{\Gamma; \Delta, 1 \vdash C} \ 1L \quad \frac{\Gamma; \Delta \vdash T}{\Gamma; \Delta, A, B \vdash C} \ (\text{no } TL)$$

$$\frac{\Gamma; \Delta_1 \vdash A \quad \Gamma; \Delta_2 \vdash B}{\Gamma; \Delta_1, \Delta_2 \vdash A \otimes B} \otimes R \quad \frac{\Gamma; \Delta, A, B \vdash C}{\Gamma; \Delta, A \otimes B \vdash C} \otimes L$$

$$\frac{\Gamma; \Delta, A \vdash B}{\Gamma; \Delta \vdash A \multimap B} \multimap R \quad \frac{\Gamma; \Delta_1 \vdash A \quad \Gamma; \Delta_2, B \vdash C}{\Gamma; \Delta_1, \Delta_2, A \multimap B \vdash C} \multimap L$$

$$\frac{\Gamma; \Delta \vdash A \quad \Gamma; \Delta \vdash B}{\Gamma; \Delta \vdash A \otimes B} \otimes R \quad \frac{\Gamma; \Delta, A_i \vdash C}{\Gamma; \Delta, A_1 \otimes A_2 \vdash C} \otimes L_i$$

$$\frac{\Gamma, x : \tau; \Delta \vdash A}{\Gamma; \Delta \vdash \forall x : \tau} \forall R \quad \frac{\Gamma \vdash t : \tau \quad \Gamma; \Delta, N[t] : A \vdash C}{\Gamma; \Delta, \forall x : \tau : A \vdash C} \forall L$$

Fig. 4. A fragment of intuitionistic linear sequent calculus.

If instead we started in an environment {has_target, heard_noise}, then at the same point in the tree, the investigate_target action will succeed and change the world state by replacing has_target with no_target (in practice, this might have a more interesting effect like updating variables representing the agent's knowledge of its target). Because both children of the sequence evaluate to success, the sequence evaluates to success. Thus, the root selector will itself evaluate to success without evaluating the third branch, completing the evaluation of the entire tree, and resulting in the state {no_target}.

6 COMPOSITIONAL REASONING

Compositional reasoning for behavior trees means that understanding the effects of a whole BT can be done by understanding the effects of its subtrees. The type system we describe gives a precise account of the conditions under which a BT has *successful* execution and the consequences of that execution. Accounting for the range of behaviors possible under failure is outside the scope of this paper (see Section 8). However, these types are richer than sets of precondtions and postcondtions: they account for the "reactive" nature of BTs by requiring dependencies to be filled not prior to execution but just at the node of the tree where they are needed; types also describe resources that are *released* periodically if they are not needed for later use.

This "open" structure of behavior types makes the account of agents' behavior amenable to analysis in the presence of multiple agent executing in parallel: BTs may both incur and use changes in the environment.

6.1 A linear type system, take 1

Our guiding principle for assigning types to BTL expressions adopts a "formulas-as-processes" point of view to imagine the proof-theoretic semantics of what the formula admits provable under arbitrary environments. Consider linear logic formulas $A := p \mid \mathbb{1} \mid \top \mid A \otimes A \mid A \otimes A \mid A \longrightarrow A$ and an intuitionistic sequenct calculus defining their provability (following [3]) shown in Figure 4.

The following intuition guides the correspondence we seek:

• Firing a leaf action op(args) of type $S \multimap S'$ in an environment Δ corresponds to the \multimap -left rule in linear sequent calculus: to succeed, it requires that the current environment match the antecedent of the action and then changes the environment to replace it with the

consequent. Correspondingly, evaluating op(args) in an environment Δ, Δ' where $\Delta' \vdash S$ evaluates to Δ, S' in the operational semantics.

- The unit selector Sel{} always fails, having run out of options; this corresponds to the ⊤ unit of & in linear logic, which has no left rule, so everything is beneath it in the preorder.
- The unit sequence Seq{} does nothing, corresponding to the left rule of the unit 1 of ⊗. Correspondingly, the operational semantics of Seq{} take the environment Δ to itself.
- Selectors Sel $\{\alpha_1 + \alpha_2\}$ nearly correspond to making a choice, as in Linear Logic's & operator. There is a difference in that & is symmetric; A & B and B & A are interprovable, whereas order matters in BTL selectors. However, certain reasoning principles apply: if either $\alpha_1 \triangleright \Delta \Downarrow \Delta_1$ or $\alpha_2 \triangleright \Delta \Downarrow \Delta_2$, then one of Δ_1 or Δ_2 will be the result of evaluating Sel $\{\alpha_1 + \alpha_2\}$ against Δ .

For reasons described above, however, accounting for sequences will be more difficult. It might be tempting to think that \otimes is an appropriate interpretation, despite the relative lack of ordering constraints, for reasons similar to how & can approximate selectors. A conjectured rule:

$$\frac{\alpha_1:A_1 \quad \alpha_2:A_2}{\mathsf{Seq}\{\alpha_1;\alpha_2\}:A_1\otimes A_2} \ \mathit{BAD_RULE}$$

At this point we need to formulate the metatheorem we have so far been implicitly expecting to hold:

Conjecture 6.1. If $\alpha : A$ and $\Delta, A \vdash S$, then $\alpha \triangleright \Delta \parallel \Delta'$ and $\Delta' \vdash S$.

(Recall that S stands for a formula with no &s or \multimap s, representing a *successful* state in the course of a BTL expression's execution.) The proposed rule violates this conjecture; we show a counterexample next.

6.2 The trouble with sequences: an example

The following action specification describes a Doors world in which agents may pass through open doors, open unlocked doors, and unlock locked doors if they have keys:

```
walk_to_door : at_elsewhere -o at_door.
pass_through : door_open * at_door -o door_open * through_door.
open_door : door_unlocked * at_door -o door_open * at_door.
smash_door : door_locked * at_door -o door_open * at_door.
close_door : door_open * through_door -o door_unlocked * through_door.
```

For a counterexample to Conjecture 6.1, let $\alpha = \text{Seq}\{\text{open_door}; \text{walk_to_door}\}\$ and let $\Delta = \{\text{at_elsewhere}, \text{door_unlocked}\}\$. According to $BAD_RULE, \alpha : A = (\text{at_door}\otimes \text{door_unlocked} \longrightarrow \text{door_open})\otimes (\text{at_elsewhere} \longrightarrow \text{at_door})\$. By straightforward rule applications, $\Delta, A \vdash \text{door_unlocked}$, but it is not the case that $\text{Seq}\{\text{open_door}; \text{walk_to_door}\} \triangleright \Delta \parallel \text{door_unlocked}$.

In addition to the clear unsoundness of describing a sequential behavior with a commutative connective, there are also concerns regarding the granularity of concurrent execution. Consider a simple sequential behavior for opening and going through a door:

```
Seq{walk_to_door; open_door; pass_through; close_door}
```

A type we could reasonably expect to ascribe to this behavior is:

```
at_elsewhere ⊗ door_unlocked — through_door ⊗ door_unlocked
```

This formula corresponds to the assumption that if our starting environment has at_elsewhere and door_unlocked, each element in this sequence of actions will consume the output of the previous action as an input, resulting in through_door. Each successive action depends on the

effects of previous actions: opening the door assumes that the previous walk action brought us to the door; passing through assumes we successfully opened the door; and closing the door assumes we passed through and the door is still open.

However, in a general, maximally concurrent environment, we would not be allowed to make these assumptions: suppose, for example, another agent interferes and closes the door just after we open it. This relaxed assumption instead observes that we might forfeit *all* of the effects of previous actions, resulting in the following type:

```
at\_elsewhere \multimap at\_door \otimes (at\_door \otimes door\_unlocked \multimap at\_door \otimes door\_open \otimes (at\_door \otimes door\_open \multimap through\_door \otimes door\_open \otimes (door\_open \otimes through\_door \multimap through\_door \otimes door\_unlocked)))
```

This formula characterizes the behavior that, at each step, a sequence releases some resources into the world along with a "continuation" that could, in some cases, potentially reabsorb those resources, or require new ones along the way.

These two ascriptions correspond to different assumptions about how behaviors interact with other behaviors manipulating the environment. The former assumes an un-interruptable, "critical section" behavior to sequences and gives a stronger guarantee, allowing us to treat the sequence as a black-box behavior without worrying about internal failure. On the other hand, the latter permits interruption and "race condition"-like scenarios that are common in games and interactive simulations in practice, but offers less strict guarantees that reflect the complexity of reasoning about fine-grained interaction.

Our type system makes the latter assumption that processes may be interrupted, but we discuss the potential to accommodate both in Section 8.

6.3 Linear Behavior Interfaces

We constrain linear logical formulas to the following grammar of interfaces, expressed inductively as nested *stagings* of inputs and outputs (and choice between multiple possible interfaces):

$$N ::= S \mid S \multimap N \mid S \otimes N \mid N \otimes N \mid \top$$

This grammar mainly serves to prevent \multimap from appearing to the left of another \multimap while representing staged inputs and outputs as described above.

We assign types as linear logic formulas N to BTL expressions α with the judgment $\alpha :_{\Sigma} N$. where α is an expression, N is an interface type, and Σ is a specification giving types $S \multimap S'$ to the actions used at the leaves of the trees.

The typing rules are as follows, with Σ left implicit as an index to the judgment except when it is needed. Atomic operations, conditions, the units, and selectors, are straightforward, and conditions must assume, but then reproduce, the condition they depend on. Sequences are assigned a type based on a computation seq of the types of their components:

The seq operator is defined as follows:

```
\begin{array}{rclcrcl} & \operatorname{seq}(\mathbb{1},N) & = & N \\ & \operatorname{seq}(S_1,S_2) & = & S_1 \otimes S_2 \\ & \operatorname{seq}(S,S' \otimes N) & = & (S \otimes S') \otimes N \\ & \operatorname{seq}(S,N_1 \otimes N_2) & = & \operatorname{seq}(S,N_1) \otimes \operatorname{seq}(S,N_2) \\ & \operatorname{seq}(S_1,S_2 \multimap N) & = & S_1 \otimes (S_2 \multimap N) \\ & \operatorname{seq}(S \otimes N_1,N_2) & = & \operatorname{seq}(S,\operatorname{seq}(N_1,N_2)) \\ & \operatorname{seq}(S_1 \multimap N_1,N_2) & = & S_1 \multimap \operatorname{seq}(N_1,N_2) \\ & \operatorname{seq}(N_1 \otimes N_2,N) & = & (\operatorname{seq}(N_1,N) \otimes \operatorname{seq}(N_2,N)) \end{array}
```

It can be interpreted as pushing the requirements of the first formula to the outside of the whole formula, then conjoining its consequences with the specification of the second formula. The correctness of this definition, and of the type system in general, with respect to the operational semantics, is considered next.

6.4 Metatheory

We revisit Conjecture 6.1 and sketch a proof. First we establish a lemma about the seq operator:

LEMMA 6.2. If Δ , seq $(N_1, N_2) \vdash S$ and Δ is flat, i.e. consists only of propositions of the form S, then there exists S_1 such that Δ , $N_1 \vdash S_1$ and Δ , $S_1 \vdash N_2$.

PROOF. By induction on the definition of seq. We show the interesting cases.

- Case: $seq(S_1, S_2 \multimap N) = S_1 \otimes (S_2 \multimap N)$. Assume $\Delta, S_1, S_2 \multimap N \vdash S$. In this case, we can just tensor together the first state and feed it into the second. $\Delta, S_1 \vdash \bigotimes(\Delta) \otimes S_1$, and $\Delta, \bigotimes(\Delta) \otimes S_1, S_2 \multimap N \vdash S$ by untensoring that proposition to get to the assumption.
- Case: $seq(S_1 \multimap N_1, N_2) = S_1 \multimap seq(N_1, N_2)$. Assume $\Delta, S_1 \multimap seq(N_1, N_2) \vdash S$. Because the proof of this sequent concludes with an S, somewhere along the way we must discharge the \multimap , i.e. some part of Δ proves S_1 . Rewrite $\Delta = \Delta_1, \Delta'$ where $\Delta_1 \vdash S_1$. Somewhere in the proof there is an application of $\multimap L$ such that Δ' , $seq(N_1, N_2) \vdash S$ is a subproof, and by inductive hypothesis, there exists S' such that $\Delta', N_1 \vdash S'$ and $S', N_2 \vdash S$.

Now it suffices to show that $\Delta, S_1 \multimap N_1 \vdash S'$ (since we already have $S', N_2 \vdash S$). This can be established by reusing the part of Δ that discharges S_1 , using $\multimap L$ on $\Delta_1 \vdash S_1$ and $\Delta', N_1 \vdash S'$.

• Remaining cases are straightforward.

THEOREM 6.3. If $\alpha : A$, Δ is flat, and Δ , $A \vdash S$, then $\alpha \triangleright \Delta \Downarrow \Delta'$ and $\Delta' \vdash S$.

PROOF. By lexicographic induction on the typing derivation and proof. We show the sequence case here.

• Case:

$$\frac{\alpha_1:N_1\quad\alpha_2:N_2}{\mathsf{Seq}\{\alpha_1;\alpha_2\}:\mathsf{seq}(N_1,N_2)}$$

Known: Δ , seq $(N_1, N_2) \vdash S$. By lemma, there exists S' such that Δ , $N_1 \vdash S'$ and S', $N_2 \vdash S$.

By i.h., $\alpha_1 \triangleright \Delta \Downarrow \Delta'$ where $\Delta' \vdash S'$. By i.h., $\alpha_2 \triangleright \{S'\} \Downarrow \Delta''$ where $\Delta'' \vdash S$. By appropriate equivalence between the positive propostion S' and context Δ' , and by the sequence evaluation rule, Seq $\{\alpha_1; \alpha_2\} \triangleright \Delta \Downarrow \Delta''$ where $\Delta'' \vdash S'$.

6.5 Example

We now return to the "investigating a sound" example whose evaluation was shown in Section 5.3. The computed type for the example:

```
Sel{?heard_noise.set_target() +
    Seq{move_to_target(); investigate_target()} +
    Sel{idle_smoke() + idle_pace()}}
```

is:

$$(\text{heard_noise} \multimap (\text{heard_noise} \multimap \text{no_target} \multimap \text{has_target}) \\ \& \qquad (\text{has_target} \multimap (\text{has_target} \otimes \text{at_target} \otimes \\ \qquad (\text{has_target} \otimes \text{at_target} \otimes \text{heard_noise} \multimap \text{no_target})) \\ \& \qquad (\text{has_cigarette} \multimap \mathbb{1}) \\ \& \qquad (\mathbb{1} \multimap \mathbb{1})$$

7 IMPLEMENTATION

We implemented both an interpreter for BTL (with repeater nodes) and a type synthesis algorithm for BTL excluding repeaters, both following the descriptions in the paper. The implementation is written in 523 lines of Standard ML, including detailed comments, and we authored an additional 448 lines of examples, including those used in this paper.

The implementation is freely available on GitHub at (URL redacted for double-blind review).

8 DISCUSSION

A longer-term goal for this work is to be able to account for how behavior trees are used in practice, to integrate the type system into the behavior authoring process (perhaps through a combination of checking and synthesis), and to evaluate how it may make designers (particularly without programming background) more effective. We anticipate using the implementation of BTs in the Unreal Engine as a benchmark. Shorter term, there are a few theoretical concerns we still need to consider. We now describe a roadmap for this project.

8.1 Parallel Composition

Currently, the semantics of agents operating in the world concurrently is not specified by the language. To account for placing multiple world-manipulating agents into the environment, we might consider introducing a "parallel" operator to BTL:

$$\alpha ::= \ldots \mid \mathsf{Par}\{\alpha_1 \parallel \alpha_2\}$$

We may consider a few options for an operational semantics that warrant a different typetheoretic treatment. For instance, perhaps parallel behaviors split the state and operate in isolation until complete. This behavior could be captured with the rule:

$$\frac{\Delta = \Delta_1, \Delta_2 \quad \alpha_1 \triangleright \Delta_1 \ \big | \ \Delta_1' \quad \alpha_2 \triangleright \Delta_2 \ \big | \ \Delta_2'}{\mathsf{Par}\{\alpha_1 \ \big | \ \alpha_2\} \triangleright \Delta \ \big | \ \Delta_1', \Delta_2'}$$

$$\frac{\Delta/\alpha_1 \to \delta \Delta/}{\Delta/\operatorname{skip} \to \Delta/} \ step/\operatorname{skip} \qquad \frac{\Sigma + op[args] : A \multimap B \quad \Delta_A \vdash A}{\Delta, \Delta_A/op(args) \to \Delta, B/} \ step/op$$

$$\frac{\Delta/\alpha_1 \to^* \Delta'/}{\Delta/\alpha_1; \alpha_2 \to \Delta'/\alpha_2} \ step/; \qquad \frac{\Delta/\alpha_1 + \alpha_2 \to \Delta/\alpha_1}{\Delta/\alpha_1 + \alpha_2 \to \Delta/\alpha_1} \ step/+_1 \qquad \frac{\Delta/\alpha_1 \to^*}{\Delta/\alpha_1 + \alpha_2 \to \Delta/\alpha_2} \ step/+_2$$

$$\frac{\Delta}{\Delta, p/?p. \ \alpha \to \Delta, p/\alpha} \ step/? \qquad \frac{\Delta/\alpha \to^* \Delta'/}{\Delta/\alpha^* \to \Delta'/\alpha^*} \ step/^*$$

$$\frac{\Delta_1/\alpha_1 \to \Delta_1'/\alpha_1'}{\Delta_1, \Delta_2/\alpha_1 \|\alpha_2 \to \Delta_1', \Delta_2/\alpha_1' \|\alpha_2} \ step/\|_1 \qquad \frac{\Delta_2/\alpha_2 \to \Delta_2'/\alpha_2'}{\Delta_1, \Delta_2/\alpha_1 \|\alpha_2 \to \Delta_1, \Delta_2/\alpha_1 \|\alpha_2'} \ step/\|_2$$

Fig. 6. A small-step semantics for BTL without failure.

Additional rules may specify that if either subbehavior fails, the whole behavior fails.

However, in practice, behavior trees allow for finer-grained interactions between processes. The above specification precludes, for example, the below two behaviors succeeding:

These behaviors will only succeed if they interact when run; a1's action give(a2,0) will only succeed if a2's first action, move(a2,L), is permitted to succeed first. Figure 5 describes visually the behavior specification we would like to result in this interaction.

To account for such fine-grained concurrent behaviors formally, we require a small-step semantics over the judgment $\alpha/\Delta \to \alpha/\Delta'$. A sketch of this semantics that includes parallel

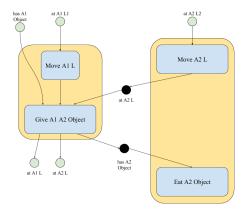


Fig. 5. Composing processes that interact.

composition is in Figure 6. However, note that this semantics does not properly handle failure; instead, it embodies the synchronous semantics of behaviors simply pausing (failing to evolve) if their conditions are not satisfied, instead permitting the possibility of a delayed transition if their conditions become satisfied as another behavior evolves. While this behavior may be useful in some scenarios, it is not universally desirable, so we need a way to account for this behavior, perhaps through a stack-based semantics with success and failure continuations. Likewise, the type system has a clear extension to count for arbitrarily "pausing" processes (\otimes is a straightforward interpretation), but accounting for failure in the type system is also left to work.

8.2 Theoretical extensions

In addition to accounting for parallel execution, we also need to consider repeater nodes. The operational semantics are fairly easy to specify, but guaranteeing convergence of computing fixed

points for a type-based characterization may prove difficult. Recursive types have been successfully integrated into linear logic [1], and we plan to investigate their use, although readability may remain a challenge.

Another step we would like to take is to introduce additional forms of lightweight verification on top of the type system. For instance, selectors are often designed with the intent that all possible cases are covered: each child of the selector is guarded by a condition, and the disjunction of the conditions characterizes an *invariant* of the state. Provided a proof that the invariant actually holds, it may be useful to simplify the type to omit the guards. This corresponds to provability between e.g. $(A \oplus B) \otimes ((A \multimap C) \otimes (B \multimap D))$ and $(C \oplus D)$.

Next, while we have established a correspondence between the type system and evaluation of *successful* behaviors, we believe we can formulate a conjecture to the effect that the situations in which types fail to yield a flat context (because there is some implication that cannot be discharged on the left, say) correspond to the failure cases of execution. We expect this proof will be more difficult than the former.

9 CONCLUSION

We have presented a formal semantics and type system for a fragment of behavior trees as they are used to describe characters in virtual environments. Our system includes a reference implementation and correctness proofs. This work represents substantial new ground broken towards a longer-term vision of authoring robust, designer-friendly specifications for reactive agent behaviors.

If our long-term vision is successful, we can enable several new things for behavior authors: integrating hand-authored trees with behavior synthesis algorithms through linear logic theorem proving (akin to planning); the development of *behavior libraries* consisting of reusable, parameterized behavior trees whose specification precludes examining the entire tree; and certified behaviors that are guaranteed to succeed under certain environments. These features would improve the effectiveness of developing agents in virtual worlds with varied applications in entertainment, arts, simulation modeling, research competitions and challenges, and computing education.

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