YAPO Monty Hall Problem

Here is yet another proof of (YAPO) the Monty Hall Problem. The Monty Hall problem has an almost 30-year history although equivalent problems go back to Martin Gardner at Scientific American in 1959. In 1990 Marilynn Vos Savant published its approximately current description

> Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice? (Whitaker, 1990, as quoted by vos Savant 1990a)

The primary characteristics of this proof is it avoids Bayes Theorem, relies on the probability sum equaling 1 and relies on a distinction between labeling and real-world action and uses that distinction to most simply solve the problem.

Let's set the labels of the doors as follows:

1. contestant action (pick a door);
2. label that door A;
3. Monty action (pick a door with a goat);
4. label that door B;
5. label the remaining door C;
6. finally, contestant picks whether to switch choice from A to C.

So, we alternate the action, such as pointing a finger at a door, with using a label for the result of that action. In the examples described here, the action precedes the label.

Now, some notation is that "cA" means "the car is behind the door with label A".

The problem states P(cA)=1/3, P(cB)=0 because Monty always picks a goat, and P(cC)= 1-P(cA)-P(cB)=2/3 since the car is somewhere. The labels of the doors make these probability statements perfectly clear.

So, the contestant should switch from door A to door C.

The argument to solving the Monty Hall problem that does not take care in when to label the doors has generated controversy of its own. Some call it “false”, “misleading”, or “shaky”. Let’s discuss the article that uses the description “shaky”.

The “shaky” alternative line of reasoning is that you label the doors 1, 2, and 3 and

> The problem states P(c1)=1/3, P(c2)=0 because Monty always picks a goat, and P(c3)= 1-P(c1)-P(c2)=2/3 since the car is somewhere.

The alternative choice of labeling implicitly assumes we set all door labels 1,2,3 at the start and then does all the actions of picking doors, “say door 1” in Savant’s description above. It leads to confusion because it is not clear when Monty picks a door with a goat that the door is door 2. It could be the door labeled 3 because the label 2 and 3 were applied to the doors before any door picking. Therefore, P(c2) may not be 0 given labeling assumed in this line of reasoning. Thus, the conclusion does not follow as is and we must go back to consider what if it is door 2 and what if it is door 3.

Going back to the “shaky” argument, we’ll quote the article by *Rosenthal, Jeffrey S. (September 2005a). "Monty Hall, Monty Fall, Monty Crawl" (PDF). Math Horizons: 5–7.*

> This [*“shaky”*] solution is actually correct, but I consider it “shaky” because it fails for slight variants of the problem. For example, consider the following:
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> > Monty Fall Problem: In this variant, once you have selected one of the three doors, the host slips on a banana peel and accidentally pushes open another door, which just happens not to contain the car. Now what are the probabilities that you will win the car if you stick with your original selection, versus if you switch to the remaining door?
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> In this case, it is still true that originally there was just a 1/3 chance that your original selection was correct. And yet, in the Monty Fall problem, the probabilities of winning if you stick or switch are both 1/2, not 1/3 and 2/3. Why the difference? Why doesn’t the Shaky Solution apply equally well to the Monty Fall problem?

Now consider how proper labeling clarifies this variant. For the Monty Fall problem, let's set the labels of the doors as follows:

1. contestant action (pick a door);
2. label that door A;
3. Monty action (accidentally pushes open another door);
4. label that door B;
5. we all see it has a goat
6. label the remaining door C;
7. finally, contestant picks whether to switch choice from A to C.

The problem states P(cA)=1/3. But P(cB) is not 0 because Monty accidentally picks a door. In fact, there is no additional information by the accidental pick that influences the value of P(cB). All we know is that the random variable happens to be a goat but this does not affect P(cB). So, P(cB) is its initial value of 1/3. Finally, P(cC)= 1-P(cA)-P(cB)=1/3 since the car is somewhere.

So, the contestant gets no benefit in a switch from door A to door C. Careful labeling correctly handles this variant.

Another variant in the article that makes the non-label-careful argument “shaky” is the Monty Crawl Problem.

> Monty Crawl Problem: As in the original problem, once you have selected one of the three doors, the host then reveals one non-selected door which does not contain the car. However, the host is very tired, and crawls from his position (near Door #1) to the door he is to open. In particular, if he has a choice of doors to open (i.e., if your original selection happened to be correct), then he opens the smallest number available door. (For example, if you selected Door #1 and the car was indeed behind Door #1, then the host would always open Door #2, never Door #3.) What are the probabilities that you will win the car if you stick versus if you switch?

> This Monty Crawl problem seems very similar to the original Monty Hall problem; the only difference is the host’s actions when he has a choice of which door to open. However, the answer now is that if you see the host open the higher-numbered unselected door, then your probability of winning is 0% if you stick, and 100% if you switch. On the other hand, if the host opens the lower-numbered unselected door, then your probability of winning is 50% whether you stick or switch. Why these different probabilities? Why does the Shaky Solution not apply in this case?

For the Monty Crawl problem, let's set the labels of the doors as follows: For the case when Monty opens the further door,

1. contestant action (pick a door);
2. label that door A;
3. Monty action (crawls to push open the further door)
4. label that door C;
5. we all see it has a goat
6. label the remaining door B;
7. finally, contestant picks whether to switch choice from A to B.

Assuming we all recognize Monty’s predicament, P(cB)=1 because Monty’s crawling past door B implies that B does not have a goat and thus has the car.

Similar reasoning to this and to Monty Fall shows that if Monty does not crawl to the further door then the probability is 50% and there is no benefit to switching.

One last time, quoting the Rosenthal article,

> The original Monty Hall problem implicitly makes an additional assumption: if the host has a choice of which door to open (i.e., if your original selection was correct), then he is equally likely to open either non-selected door. This assumption, callously ignored by the Shaky Solution, is in fact crucial to the conclusion (as the Monty Crawl problem illustrates).

The “choice of which door to open” is, however, itself confusing because there really is no reason to assume “he is equally likely to open either non-selected door” in the case where there are goats behind both doors 2 and 3. A devious Monty could make a random choice that picks door 2 80% of the time and door 3 20% of the time. The above labeling of YAPO Monty Hall shows this in fact is not crucial to the conclusion.

Lastly, I look forward to all constructive or polite feedback. I’m very curious to find out whether the additional wrinkle of proper labeling generates more clarification, controversy or criticism.