EVOLUTION OF WEALTH INEQUALITY IN SIMULATED NETWORKS*

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Abstract. We use a network-based simulation to identify the emergent social structures of segregation based on distribution of cooperative behavior and income when individuals have a preference for fairness and interact with agents when playing a Prisoner's Dilemma game. We find that regardless of the initial wealth distribution (as measured by the Gini coefficient), the winning frequency by play strategy is robust across changes in distribution of cooperative strategies.

Key words. Cooperation, Inequality, Networks

AMS subject classifications. 62SC05, 91-05, 91-08, 91-10

1. Introduction.

Information asymmetry and contractual incompleteness drive modern microeconomics literature. Emergent trends in microeconomics literature has seen a greater focus on the study of persistent inequality. The existing literature on economic inequality falls into two broad categories: one attempts to articulate the *objective* impact and the other seeks to articulate a *normative* approach to understanding social welfare [11]. Regardless of which methodology is chosen, economic performance is driven by the surrounding institutions.[1] With the importance of institutions apparent, to what extent does cooperative behavior matter for long term changes in wealth equality? Given the same institutions, do long run levels of inequality differ among a society that is more or less cooperative? In what ways do they differ? If divergent behaviors exist for differing levels of starting wealth inequality, it is then plausible that appropriating economic systems from one society to another has deeper ramifications than existing cultural struggles.

We use a network-based simulation to identify the emergent social structures of segregation based on distribution of cooperative behavior and income when individuals have a preference for fairness and interact with agents when playing a Prisoner's Dilemma game. Further, from our initial values, we are able to track how wealth inequality changes over time to see the effect of initial conditions on wealth inequality.

Many social scientists study individual action and preference as "within-individual". That is, the actions taken and the preferences held by an agent are endogenous. Game theorists have long modeled these decisions as optimizing a function of payoffs to designate a best-response function.[2] Many behavioral researchers in the social sciences have shown preferences are, instead, "atomized" [7]. This modern research has shown that our behaviors are tied to the behaviors of those around us and in an agent's community, or in-group. Creating group dynamics in human subject experiments is simple and their effects on behavior are significant [12]. For work that examines individual action, deriving the social network is challenging — often forcing researchers

*See code here: https://github.com/scottcohn97/stat535

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[1] A particularly interesting example of this is the effects on institutions on land tenure in West Bengal. [2, 3]

[2] When confronted with a multitude of strategies in a game, the best-response is the strategy that maximizes a player's payoff while taking another player's action as given.

to consider preferences to be endogenous. Our use of a network structure allows preferences to be atomized. The decision to cut a tie with an agent is based on, in part, the actions that another player takes. This conditional approach makes deriving analytical solutions to the game impossible, further supporting the use of simulated behavior.

This study aims to investigate the plausibility of the effects that differing levels of cooperation and initial wealth distributions may create long run wealth hierarchies illustrated by cutting ties to form subgraphs. A number of Monte Carlo simulations are run to establish the effects of these treatments on these segmentations.

2. Methods. Studying emergent behaviors on networks poses a challenging analytical problem. The presence of stochastic elements in choice functions makes analytical solutions impossible to derive. These challenges occur to a lesser extent in simulations. Human subject experiments on behavior from the lab or field leave open data challenges due to human and monetary costs of increasing treatment granularity. Simulated data occurs only a cost in terms of computational power. Further, increased granularity aids in the understanding of behavior at the margins. Limitations on the interpretation of results will be discussed in Section 4.

2.1. Prisoner's Dilemma. Game theorists and a broad spectrum of social scientists have turned to the Prisoner's Dilemma (PD) game to understand the evolution of cooperative behavior. The PD has two players: a row player and a column player. These "players" can represent individual actors (often called agents) or firms. Each player has two options: *Cooperate* or *Defect*. The applied narrative may change these labels, but their function remains; one choice represents a cooperative behavior and the other a defection. For each player there exists an action that when taken independent of the other player would yield a higher payoff. These actions are typically called *strategies*, and this higher-paying strategy is referred to as the *dominating strategy*. The key to the utility of the PD is that if both players were to choose their dominating strategy, their payoff would be *worse* than if they had acted differently.

TABLE 1
Prisoner's Dilemma Game

	Cooperate	*Defect*
Cooperate	(a, a)	(c, b)
Defect	(b, c)	(d, d)

Table 1 represents a PD payoff matrix in its canonical form. By definition, the payoffs follow strict inequalities:

$$b > a > d > c.$$

The intermediate condition of $a > d$ suggests that mutual cooperation is superior for both players. This is referred to as the *social optimum*. However, the $b > a$ and $d > c$ relations illustrate the dominance of the defection strategy for players. Put simply, if player's work together they maximize their shared payoff. If they both act in their own self-interest they get a lower shared payoff. And, if one cooperates and the other defects then the cooperating player gets a substantially lower payoff the the defector.

If the game is played once, it is a *one-shot* game. Played multiple times, as it is in our simulation, it is called an *iterated* Prisoner's Dilemma. Iteration confers the option of an evolution of player strategy with the same opposing player.

2.2. Gini Coefficient. The Gini coefficient is a measure of wealth or income dispersion across agents in a network. We use an alternative way of explaining the Gini coefficient not simply as a result of the Lorenz curve (as it is traditionally used) but instead as a result of the relative mean differences among members of a population to model emergent properties of wealth inequality among agents [4]. This method negates downward bias present in the Lorenz-based calculation that can be substantial for small-sized networks. Let n denote the number of agents in the system and $\{i, j\}$ denote the indices of each agent's wealth y. To calculate the Gini, we first calculate the absolute difference between wealth, y, with the sum of absolute difference denoted as Δ.

$$\Delta = \sum_{i=j+1}^{i=n} \sum_{j=1}^{j=n-1} |y_i - y_j| \tag{2.1}$$

Let $\bar{y}$ be the mean wealth in the network. Then, we calculate the Gini, G,

$$G = \frac{\Delta}{n(n-1)} \cdot \frac{1}{\bar{y}} \tag{2.2}$$

where $G \in [0, 1]$. A Gini coefficient closer to 0 shows greater wealth equality and a Gini closer to 1 shows higher wealth inequality.

3. The Model. We start with 100 agents on a complete graph, where a node represents an agent and the edge a connection between two agents. A Gini coefficient is selected from one of the treatment values and an initial wealth distribution that is equivalent to the Gini is distributed using a Log-normal distribution. For aggregate wealth level represented by a random variable Y, then

$$\ln Y \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{3.1}$$

where $G\left(\{y_1, y_2, \ldots, y_n\}\right) = G_{\texttt{Treatment}}$ for $G(\cdot)$ in equation 2.2. Thus, at Round 0, each agent has degree equal to $N-1$ and a starting endowment. Further, agents are randomly assigned one of four strategies from Robert Axelrod's Prisoner's Dilemma tournament: *Always Cooperate*, *Always Defect*, *Tit-for-Tat* (TFT), or *Random* [1]. The distribution of these strategies among agents represents the "cooperativeness" of our simulated society. Higher proportions of *Always Cooperate* and TFT in the total population signal a more cooperative society. Conversely, higher levels of *Always Defect* make up a less cooperative society. We fix *Random* at 25% of the population for all of our experiments. In repeated play, *Always Cooperate* will choose to cooperate no matter who they are playing with. *Always Defect* will choose to defect in every round. TFT always starts by cooperating, then repeats the strategy played in the previous round by the other agent. For example, if agent A has strategy TFT and agent B has strategy *Always Defect* then agent A will start by cooperating and then only play Defect because that is the strategy of agent B. Conversely, if agent B is *Always Cooperate* then agent A will cooperate in every round. *Random* chooses randomly to defect or cooperate at each round uniformly. These strategies remain fixed for the entirety of the simulation.

At each time interval, the same sequence occurs. The sequence is shown in algorithm 3.

Algorithm 3.1 Simulation Algorithm

Result: Run Simulation
begin
  Initialize parameters and network represented by a complete graph $G = \{V, E\}$
  Print graph and Gini
  Runs = 0
  while *Runs* $< N$ **do**
    for *all edges* $(a_i, a_j) \in E$ **do**
      a_i plays PD with a_j; update the wealth for both agents
    end
    w = average wealth of the population after all games are played
    for *all edges* $(a_i, a_j) \in E$ **do**
      if *threshold is met* **then**
        Cut edge between a_i and a_j with probability p given by a function $f(a_i, a_j, w)$
      end
    end
    Runs = Runs + 1
  end
  Return new graph and Gini
end

Each agent will play 10 rounds of an iterated PD game with every agent they are connected to. An agent with degree 5 will play 10 rounds of a PD with each connection, totalling 50 rounds for a single agent in one round. The agent will observe their payoff from each set of interactions and if a threshold is not met, they will cut the tie with an agent. Updates are synchronous as one interaction pair does not affect another interaction within the round. The threshold is probabilistic in respect to both actors. If agents i and j both accrue income, π, that is higher than the mean network income, they break their tie with $p = 0.01$. If they both earn less than the average global income, they break their tie with probability $p = 0.99$. Else, they break their tie with $p = 0.5$.

$$P\,(\text{Break tie} \mid \cdot) = \begin{cases} 0.01 & \pi_i \text{ and } \pi_j > \bar{\pi} \\ 0.99 & \pi_i \text{ and } \pi_j < \bar{\pi} \\ 0.50 & \text{Otherwise} \end{cases} \tag{3.2}$$

The agent will then have the income from that round added to their initial endowment. A measure of the inequality is measured via the Gini. Then the next iteration of the same sequence occurs. We stop running the simulation after 25 runs. We find more simulations have no impact on the results.

The payoffs for the iterated PD are in Table 2.

Table 2
Prisoner's Dilemma Game

	Cooperate	*Defect*
Cooperate	(4, 4)	(0, 6)
Defect	(6, 0)	(2, 2)

TABLE 3
Initial strategy distributions

	Always Cooperate	*Always Defect*	*Random*	*Tit-For-Tat*
`std`	25%	25%	25%	25%
`hc1`	15%	10%	25%	50%
`hc2`	35%	10%	25%	30%
`hc3`	35%	5%	25%	35%

We use two treatments with four factors each, creating 16 experimental conditions. The starting wealth makes up a Gini that takes on one of the following values: $\{0.1, 0.3, 0.7, 0.9\}$. We use one standard (`std`) strategy distribution and three high cooperate (`hc`) distributions in our game initiation. See Table 3.

We employ the following hypotheses:

H1: A small initial Gini will lead to a bifurcation of wealth.
H2: A large initial Gini will have a greater variance of wealth.
H3: Agents assigned the *Random* strategy will have a higher mean wealth at the end of the simulation than agents assigned the *Always Cooperate* strategy.
H4: Agents assigned the *Always Defect* strategy will have a higher mean wealth at the end of the simulation than agents assigned the *Random* strategy.
H5: Agents assigned *Tit-for-Tat* strategy will have a higher mean wealth at the end of the simulation than those assigned the *Always Defect* strategy.
H6: At the end of the simulation, the only connected players will be those with *Always Defect* or *Tit-for-Tat* as their strategy.

4. Results. We proceed in three steps. We first investigate the effects that the initial level of wealth equality has on long-run wealth. In the second step, we compare the success of the agents in our simulation to our hypothesized results. In the third step, we examine the effects of the initial Gini when varied compared against varying distributions of strategy.

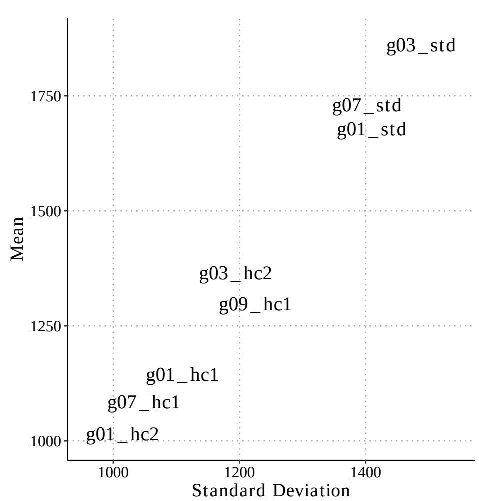


FIG. 1. *Standard deviation and mean of treatments.*

4.1. Hypothesis 1. We start by looking at the impact of the starting Gini coefficient. The initial endowments were distributed randomly to agents given the initial Gini coefficient. In hypothesis 1, we predicted that a small starting Gini would lead to a greater bifurcation of wealth. We find that in all trials, there was clear wealth bifurcation that fell along strategy lines. See Figures 4 – 7 in Appendix A. In all trials, regardless of Gini, the *Always Defect* and *Random* strategies performed the best.

4.2. Hypothesis 2. In hypothesis 2, we predicted that a large starting Gini would lead to a greater variance in the final wealth distribution. We find no evidence of this. Again, strategy distribution is a stronger predictor of results. In Figure 1, we see no clustering by Gini, but rather by strategy treatment.

4.3. H3 — H6. In Robert Axelrod's seminal book on the cooperative strategies, he finds TFT is the most winning strategy in a round-robin PD game tournament [1]. While we did not include all of Axelrod's strategies, we observe persistent winning behavior among select strategies. Contrary to Axelrod's findings, *Always Defect* wins every time. TFT and *Always Cooperate* remain the most losing strategies, with TFT only marginally outperforming *Always Cooperate*. The slight winning of TFT over *Always Cooperate* is likely to be an artifact of replicating a defect strategy after round 1 when playing against *Always Defect*, or resembling the play style of a *Random* player in a TFT-*Random* pairing. We also found that due to the dominance of the *Always Defect* strategy, the relationships between those agents and the TFT-*Always Cooperate* agents were almost always disconnected. As a result, the only connected agents at the end of the simulation were *Always Defect* and *Random* agents. In summary, the *Always Defect* agents had the highest mean wealth, followed by the *Random* agents, the *Tit-for-Tat* agents, and the *Always Cooperate* agents.

5. Other results. In addition to varying the starting wealth endowment, we also varied the distribution of cooperative strategies. We added three "high cooperative" treatments (`hc`) in addition to the standard (`std`) uniform distribution. The breakdown of these treatment factors is in Table 3. We find the winning-ability of the strategies is robust to different strategy distributions. Even at 5% of the population, *Always Defect* is still the dominant strategy when agents are able to cut ties.

6. Conclusion. In this paper, we analyzed the role of cooperation and starting wealth inequality in simulated networks where agents could take retributive action by cutting ties when met with unfavorable outcomes. We find that regardless of the initial wealth distribution (as measured by the Gini coefficient), the *Always Defect* strategy is the most winning strategy, followed by *Random*, then TFT and *Always Cooperate*. This result is robust across changes in distribution of cooperative strategies — increasing the prevalence of TFT/*Always Cooperate* and decreasing the prevalence of *Always Defect*. Our simulation supports theories of how path dependence can lead to greater wealth for non-cooperators and mitigate the earnings of enduring cooperators — especially when cooperators exhibit preferences for fairness over loss.

Appendix A. Figures.

FIG. 2. *Network at Round 1 with equal distribution of strategies and Gini = 0.3*

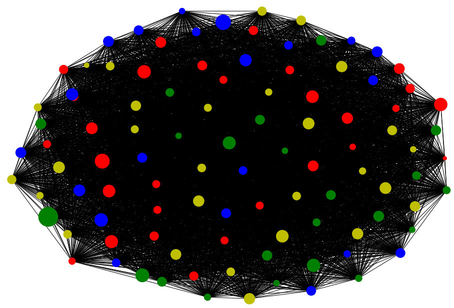
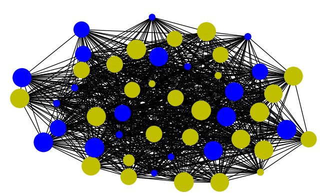
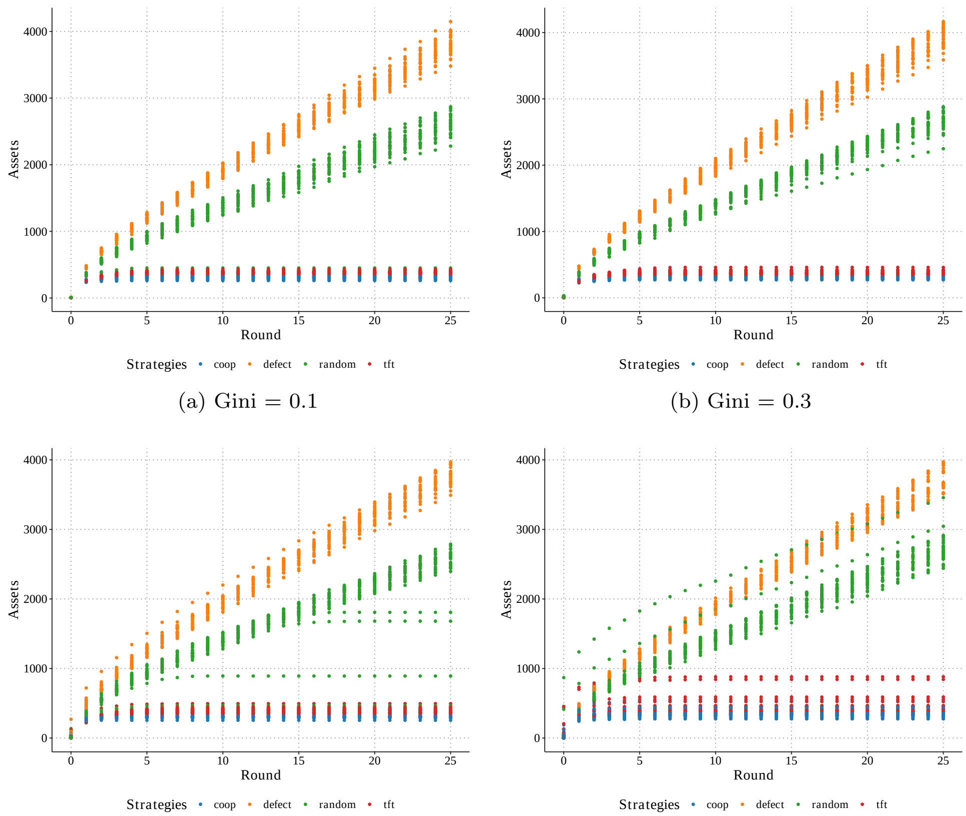


FIG. 3. *Network at Round 25 with equal distribution of strategies and Gini = 0.3*



In Figures 2 and 3, red is *Always Cooperate*; blue is *Always Defect*; yellow is *Random*; and green is TFT.



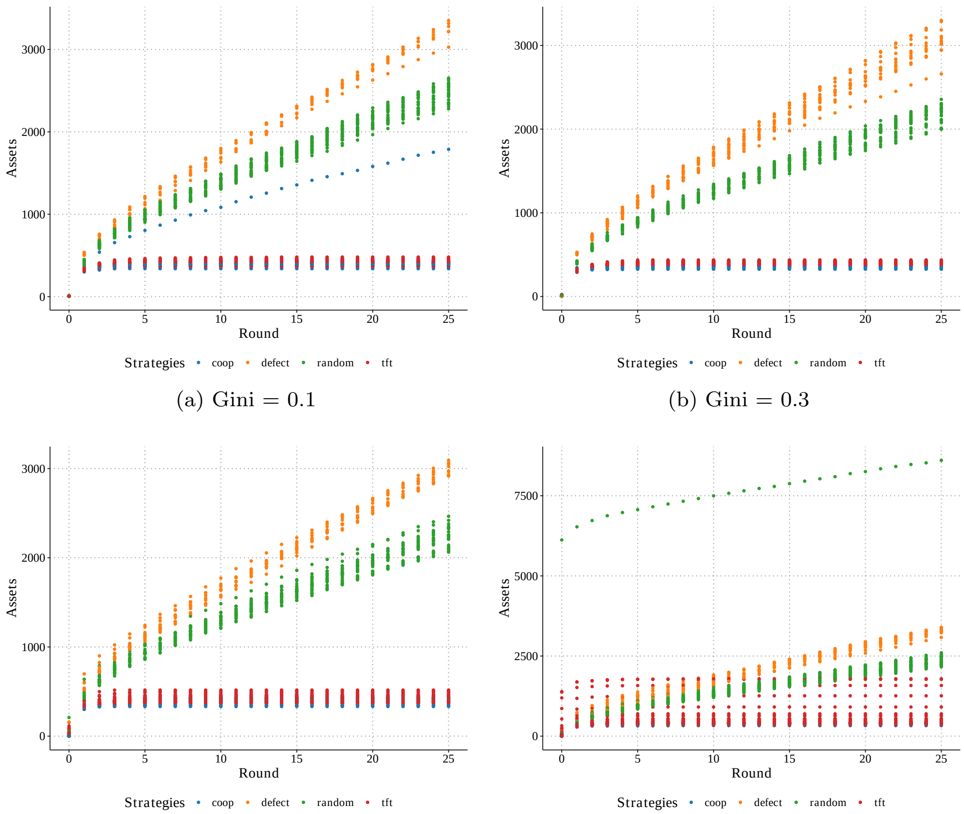
(a) Gini = 0.1

(b) Gini = 0.3

(c) Gini = 0.7

(d) Gini = 0.9

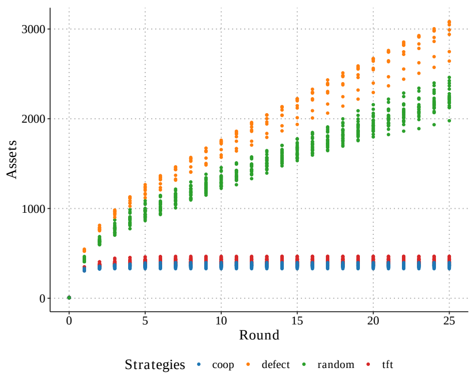
FIG. 4. *Wealth disparity under uniform distribution of strategies.*



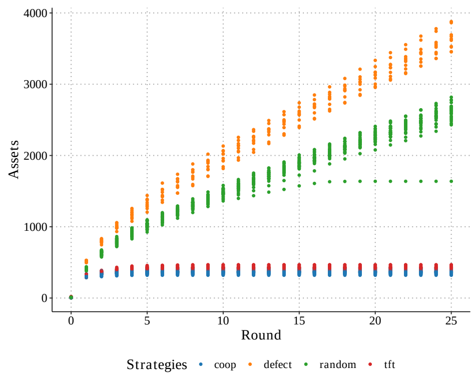
(a) Gini = 0.1 (b) Gini = 0.3

(c) Gini = 0.7 (d) Gini = 0.9

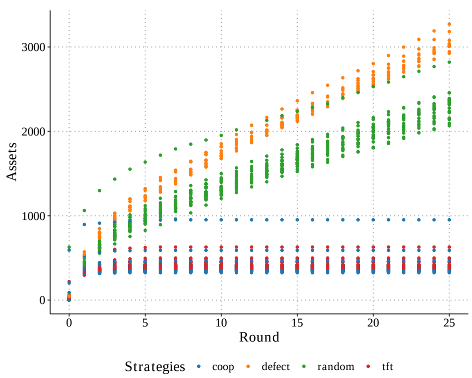
FIG. 5. *Wealth disparity under* `hc1` *distribution of strategies.*



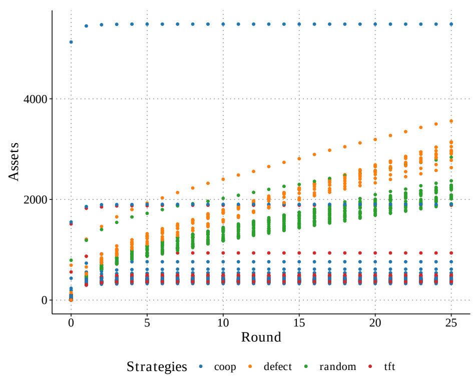
(a) Gini = 0.1



(b) Gini = 0.3

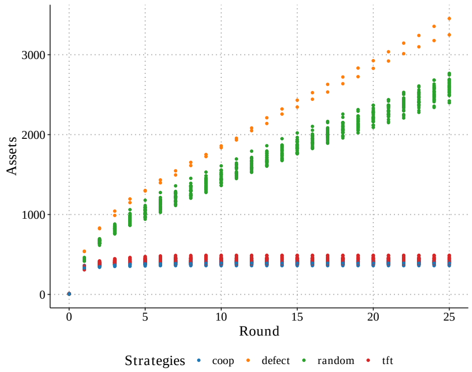


(c) Gini = 0.7

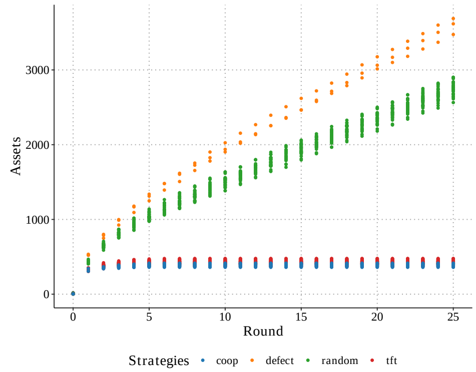


(d) Gini = 0.9

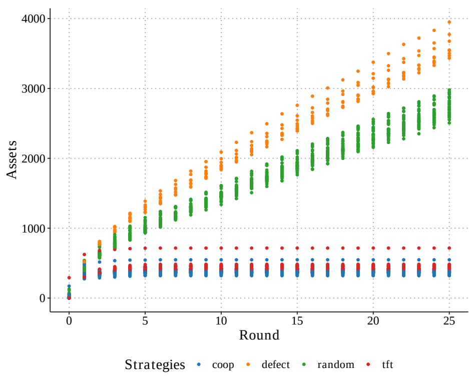
FIG. 6. *Wealth disparity under* `hc2` *distribution of strategies.*



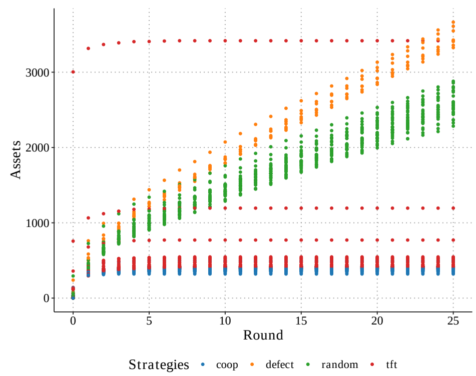
(a) Gini = 0.1



(b) Gini = 0.3



(c) Gini = 0.7



(d) Gini = 0.9

FIG. 7. *Wealth disparity under* `hc3` *distribution of strategies.*

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