

Evolution of Wealth Inequality in Simulated Networks

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To plant early, or to plant late?

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Introduction

Motivation

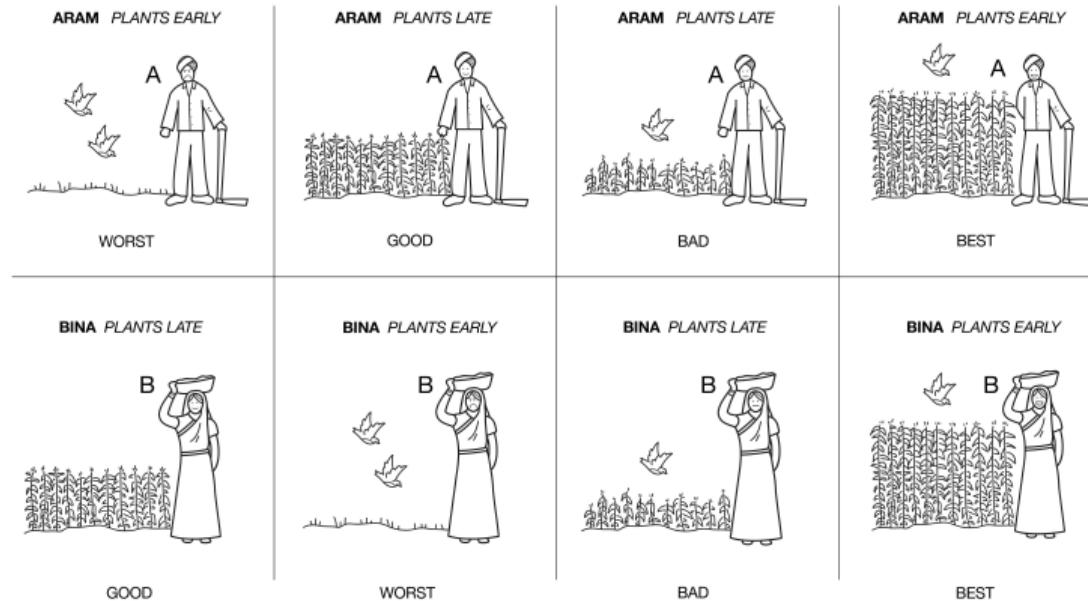


Figure: Palanpur Farmer's Game (Cartoonist: Anmei Zhi, Smith '20)

Introduction

Research Question

Problem

How does cooperation evolve, or even persist, in a world of self-maximizing egoists?

- Scenario known as the Prisoner's Dilemma (PD)
- If you risk being suckered, which strategy is best?

Research Question

How do initial levels of societal wealth affect long-run wealth hierarchies?



Figure: Palanpur in Uttar Pradesh, India

Prisoner's Dilemma

A more rigorous treatment

- Payoffs follow strict inequalities

$$b > a > d > c$$

- Dominance of defection

$$b > a \text{ and } d > c$$

- Use PD to see change in wealth
- Not concerned about intra-game equilibria

Table: Prisoner's Dilemma Game

	Cooperate	Defect
Cooperate	(a, a)	(c, b)
Defect	(b, c)	(d, d)

Prisoner's Dilemma

Strategies

- Always Cooperate
- Always Defect
- Random
- Tit-for-Tat
- ...and more [2]

Table: Prisoner's Dilemma Game

	Cooperate	Defect
Cooperate	(4, 4)	(0, 6)
Defect	(6, 0)	(2, 2)

How do we measure societal wealth?

- Gini coefficient [3]
- Pairwise comparison of one's own wealth or income with that of someone else
- $G \in [0, 1]$
- 0 : Everyone has identical incomes
- 1 : All income goes to one person

Example

US = 0.38; Denmark = 0.25; Korea = 0.32;
Chile = 0.49; China = 0.61

$$\Delta = \sum_{i=j+1}^{i=n} \sum_{j=1}^{j=n-1} = |y_i - y_j| \quad (1)$$

$$\text{Gini} = G = \frac{\Delta}{n(n-1)} \cdot \frac{1}{\bar{y}} \quad (2)$$

- n : Number of agents
- \bar{y} : Mean wealth
- y_i : Wealth of agent i
- Δ : Sum of absolute difference

Why Simulation?

- Stochastic
- Interactions between agents make analytical solutions impossible
- Increased granularity in treatments

Hypotheses

We'll focus on the first two.

H1: A small initial Gini will lead to a bifurcation of wealth.

H2: A large initial Gini will have a greater variance of wealth.

H3: Agents assigned the *Random* strategy will have a higher mean wealth at the end of the simulation than agents assigned the *Always Cooperate* strategy.

H4: Agents assigned the *Always Defect* strategy will have a higher mean wealth at the end of the simulation than agents assigned the *Random* strategy.

H5: Agents assigned *Tit-for-Tat* strategy will have a higher mean wealth at the end of the simulation than those assigned the *Always Defect* strategy.

H6: At the end of the simulation, the only connected players will be those with *Always Defect* or *Tit-for-Tat* as their strategy.

Algorithm 1: Simulation Algorithm

```
begin
    Initialize parameters and network represented by a complete graph  $G = \{V, E\}$ ;
    Print graph and Gini;
    Runs = 0;
```

```
    while  $\text{Runs} < N$  do
```

```
        for all edges  $(a_i, a_j) \in E$  do
```

```
            |  $a_i$  plays PD with  $a_j$ ; update the wealth for both agents;
```

```
        end
```

```
        w = average wealth of the population after all games are played
```

```
        for all edges  $(a_i, a_j) \in E$  do
```

```
            if threshold is met;
```

```
            then
```

```
                | Cut edge between  $a_i$  and  $a_j$  with probability  $p$  given by a function  $f(a_i, a_j, w)$ ;
```

```
            end
```

```
        end
```

```
        Runs = Runs + 1;
```

```
    end
```

```
    Return new graph and Gini;
```

```
end
```

The Model

Microdynamics

- Synchronous updating
- Discrete time
- Stochastic
 - Initiation
 - Tie breaking
- 25 rounds
- 100 agents
- Treatments:
 - Low: {0.1, 0.3}
 - High: {0.7, 0.9}

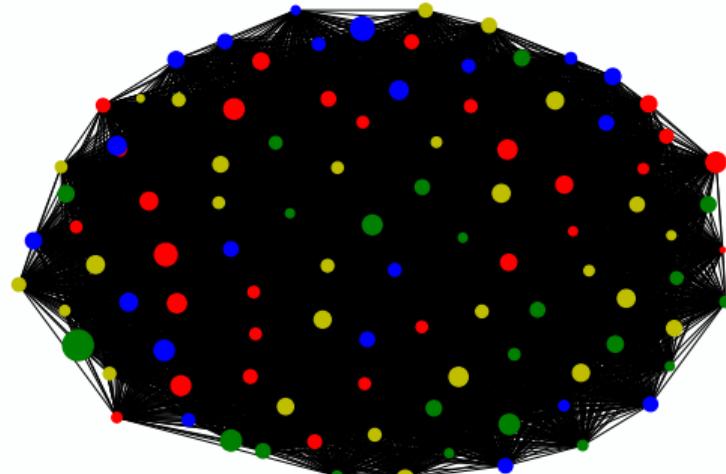


Figure: Game initiation ($G = 0.3$)

R: Always Cooperate, B: Always Defect
Y: Random, G: TFT

Results

Hypothesis 1

A small initial Gini will lead to a bifurcation of wealth.

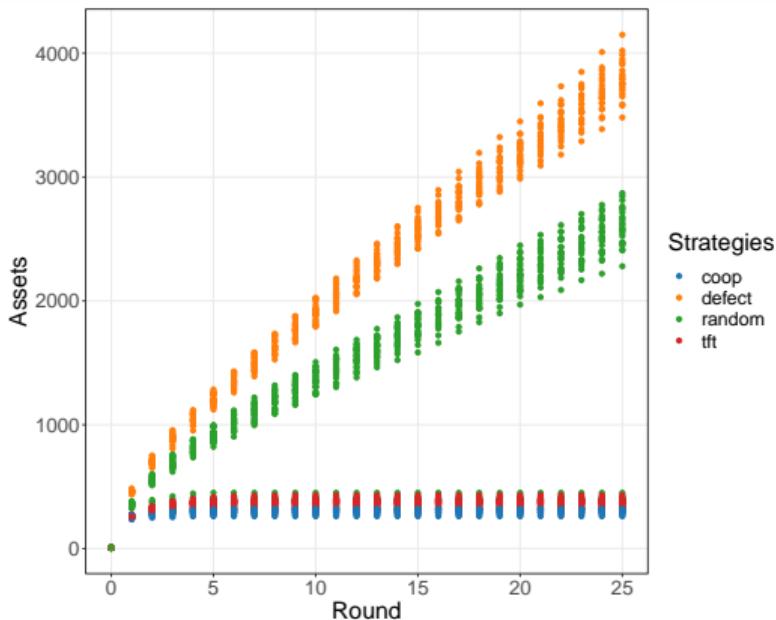


Figure: Starting Gini = 0.1

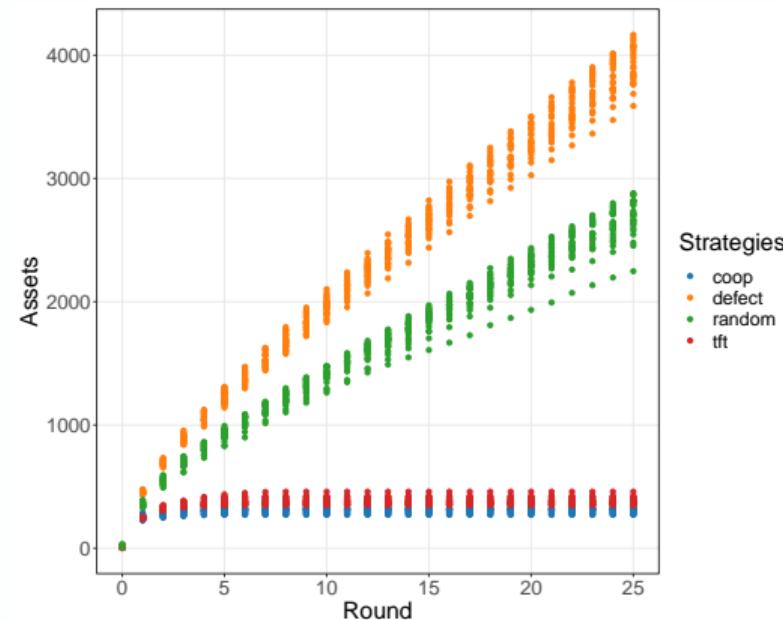


Figure: Starting Gini = 0.3

Results

Hypothesis 2

A large initial Gini will have a greater variance of wealth.

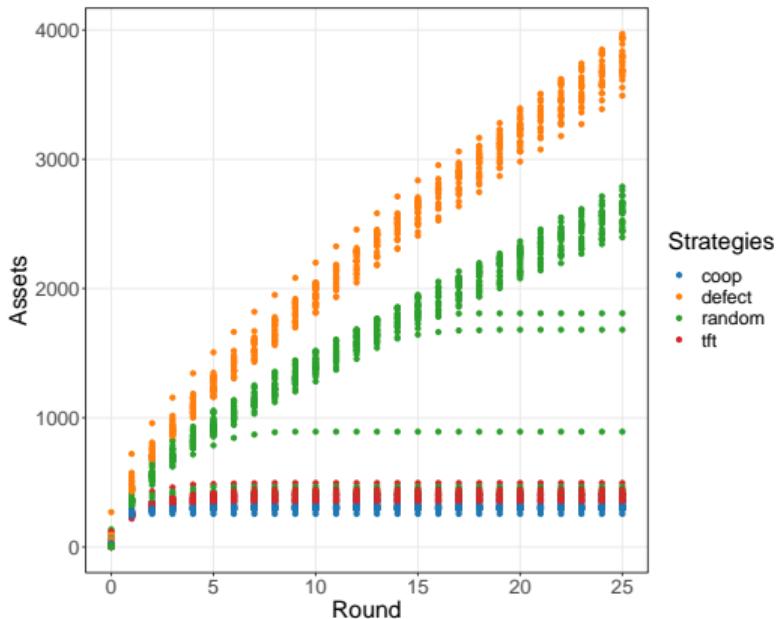


Figure: Starting Gini = 0.7

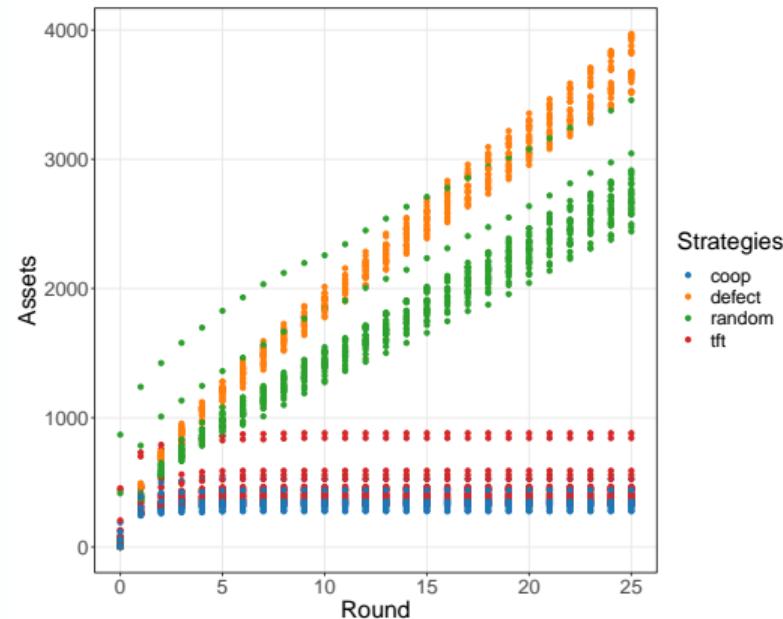


Figure: Starting Gini = 0.9

Results

Graphs Pre- and Post-

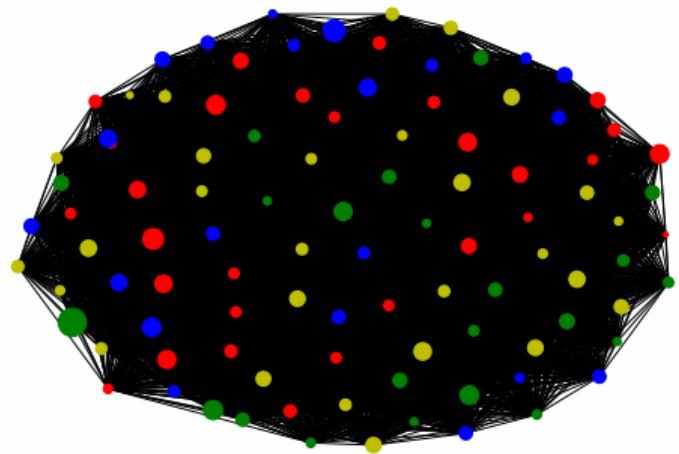


Figure: Game initiation ($G = 0.3$)

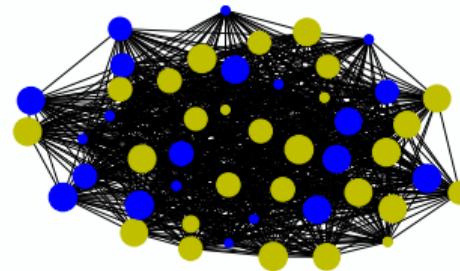


Figure: Final graph ($G = 0.3$)

Conclusion

- In both treatments, the Gini converges to about 0.5.
- Our results seem to be primarily driven by strategy, rather than starting wealth position
- Contrary to Axelrod's 1981 findings [2], *Always Defect* dominates
- Outlook
 - Our tie-cutting thresholds may be too strong
 - *Always Defect* never cuts ties.

Questions?

For Further Reading I

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