Evolution of Wealth Inequality in Simulated Networks

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Outline

Introduction

Motivation

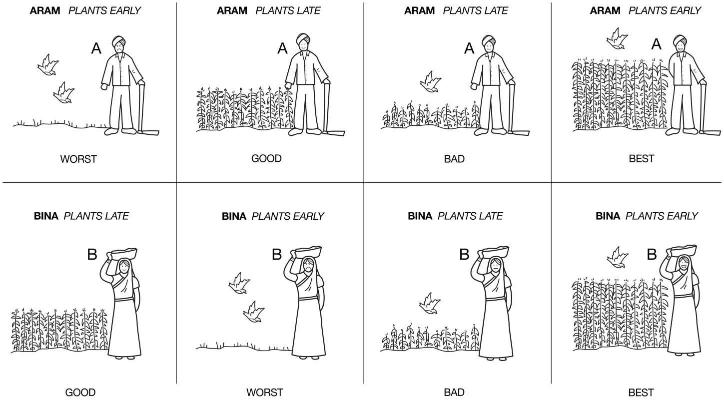

Figure: Palanpur Farmer's Game (Cartoonist: Anmei Zhi, Smith '20)

Problem

How does cooperation evolve, or even persist, in a world of self-maximizing egoists?

- Scenario known as the Prisoner's Dilemma (PD)
- If you risk being suckered, which strategy is best?

Research Question

How do initial levels of societal wealth affect long-run wealth hierarchies?

Figure: Palanpur in Gujarat, India

Prisoner's Dilemma

A more rigorous treatment

- Payoffs follow strict inequalities

$$b > a > d > c$$

- Dominance of defection

$$b > a \text{ and } d > c$$

- Use PD to see change in wealth
- Not concerned about intra-game equilibria

Table: Prisoner's Dilemma Game

	Cooperate	*Defect*
Cooperate	(a, a)	(c, b)
Defect	(b, c)	(d, d)

Prisoner's Dilemma

Strategies

- Always Cooperate
- Always Defect
- Random
- Tit-for-Tat
- …*and more* [2]

Table: Prisoner's Dilemma Game

	Cooperate	*Defect*
Cooperate	(4, 4)	(0, 6)
Defect	(6, 0)	(2, 2)

How do we measure societal wealth?

- Gini coefficient [3]
- Pairwise comparison of one's own wealth or income with that of someone else
- $G \in [0, 1]$
- 0 : Everyone has identical incomes
- 1 : All income goes to one person

Example

US = 0.38; Denmark = 0.25; Korea = 0.32; Chile = 0.49; China = 0.61

$$\Delta = \sum_{i=j+1}^{i=n} \sum_{j=1}^{j=n-1} = \left| y_i - y_j \right| \tag{1}$$

$$\text{Gini} = G = \frac{\Delta}{n(n-1)} \cdot \frac{1}{\bar{y}} \tag{2}$$

- n: Number of agents
- $\bar{y}$: Mean wealth
- y_i: Wealth of agent i
- Δ: Sum of absolute difference

Why Simulation?

- Stochastic
- Interactions between agents make analytical solutions impossible
- Increased granularity in treatments

Hypotheses

We'll focus on the first two.

H1: A small initial Gini will lead to a bifurcation of wealth.

H2: A large initial Gini will have a greater variance of wealth.

H3: Agents assigned the *Random* strategy will have a higher mean wealth at the end of the simulation than agents assigned the *Always Cooperate* strategy.

H4: Agents assigned the *Always Defect* strategy will have a higher mean wealth at the end of the simulation than agents assigned the *Random* strategy.

H5: Agents assigned *Tit-for-Tat* strategy will have a higher mean wealth at the end of the simulation than those assigned the *Always Defect* strategy.

H6: At the end of the simulation, the only connected players will be those with *Always Defect* or *Tit-for-Tat* as their strategy.

Algorithm 1: Simulation Algorithm

```
begin
    Initialize parameters and network represented by a complete graph G = {V, E};
    Print graph and Gini;
    Runs = 0;
    while Runs < N do
        for all edges (a_i, a_j) ∈ E do
            a_i plays PD with a_j; update the wealth for both agents;
        end
        w = average wealth of the population after all games are played
        for all edges (a_i, a_j) ∈ E do
            if threshold is met;
             then
                Cut edge between a_i and a_j with probability p given by a function f(a_i, a_j, w);
            end
        end
        Runs = Runs + 1;
    end
    Return new graph and Gini;
end
```

The Model

Microdynamics

- Synchronous updating
- Discrete time
- Stochastic
  - Initiation
  - Tie breaking
- 25 rounds
- 100 agents
- Treatments:
  - Low: $\{0.1, 0.3\}$
  - High: $\{0.7, 0.9\}$

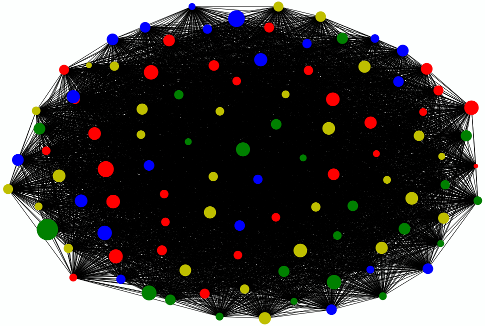

Figure: Game initiation (G = 0.3)

R: Always Cooperate, B: Always Defect
Y: Random, G: TFT

Results

Hypothesis 1

A small initial Gini will lead to a bifurcation of wealth.

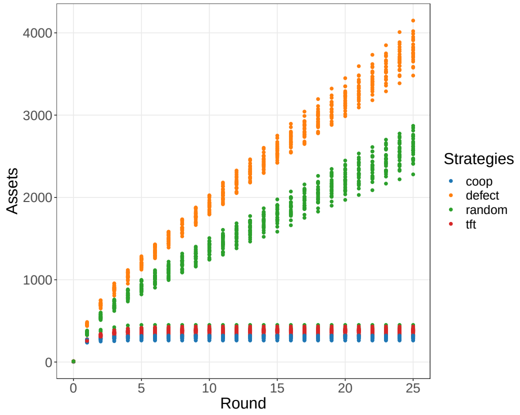

Figure: Starting Gini = 0.1

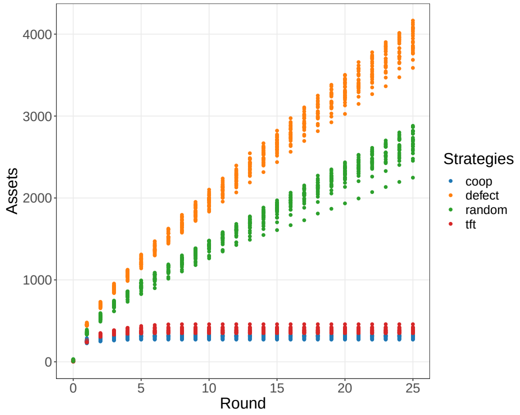

Figure: Starting Gini = 0.3

Results

Hypothesis 2

A large initial Gini will have a greater variance of wealth.

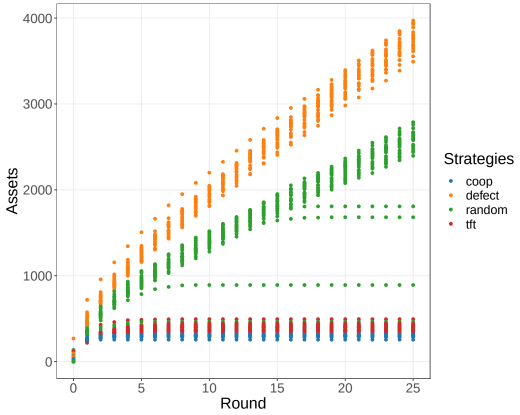

Figure: Starting Gini = 0.7

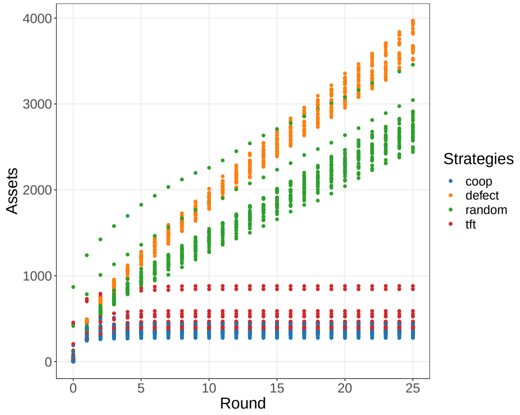

Figure: Starting Gini = 0.9

Results

Graphs Pre- and Post-

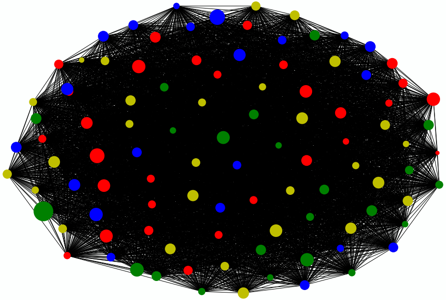

Figure: Game initiation (G = 0.3)

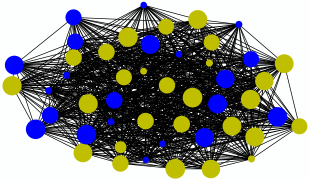

Figure: Final graph (G = 0.3)

Conclusion

- In both treatments, the Gini converges to about 0.5.
- Our results seem to be primarily driven by strategy, rather than starting wealth position
- Contrary to Axelrod's 1981 findings [2], *Always Defect* dominates
- Outlook
  - Our tie-cutting thresholds may be too strong
  - *Always Defect* never cuts ties.

Questions?

For Further Reading I

A. Sen and JE. Foster
On economic inequality.
Oxford University Press, 1997.

R. Axelrod and W.D. Hamilton.
The Evolution of Cooperation.
Science, 211(4489):1390–1396, 1981.

S. Bowles and W. Carlin.
Inequality as experienced difference: A reformulation of the Gini coefficient.
Economics Letters, 186(1):1087–1089, 2020.

D. Engelmann and M. Strobel.
Inequality aversion, efficiency, and maximin preferences in simple distribution experiments.
American Economic Review, 94(4):857–869, 2004.