Orderbook spreads and depth in a dynamic general equilibrium model

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ABSTRACT

This paper studies the optimal orderbook presented by market makers in a limit-order market with both liquidity providers and liquidity demanders. The model characterizes the equilibrium spread and price impact of the order book and allows for comparative statics on both of these definitions of market liquidity. As disagreements about asset values increase, spreads widen and price impact increases. Market liquidity, as measured both by spread and price impact, increases in the fee spread taken by the exchange. We also characterize the impact on market liquidity of changes in the relative demand for purchases of the asset. As an application, stocks which are aggressively marketed to liquidity demanders will, in equilibrium, see wider spreads but thicker orderbooks.

JEL classification: G14, D53.

1 Introduction

In most equity markets throughought the world, market makers provide liquidity to those wishing to purchase or sell assets by placing limit orders to sell or buy that remain on the book until either executed or canceled. The liquidity of these markets is often characterized in two specific ways. The spread is the difference between the highest bid (or buy) limit order and the lowest ask (or sell) limit order. The spread represents the cost associated with obtaining immediate execution of a trader's desired position. More liquid markets are characterized by narrower spreads.

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The price impact in a limit order market is the extent to which the price moves as greater quantities of shares are purchased or sold. One market is more liquid than another if its price impact is smaller. That is, if purchasing 100 shares in market A leads to a change in price of \$0.50 between the price at which the first share is purchased and the price at which the 100th share is purchased and in market B that price change is only \$0.20, then market B is more liquid than market A.

There is evidence that a lack of liquidity is perceived by traders to be risky in the aggregate (Pástor & Stambaugh 2003), increase fees associated with seasoned equity offerings (Butler et al. 2005), and lead to less informative prices (Kerr et al. 2020). These issues suggest that understanding the factors that determine liquidity in equilibrium is an important area of study.

This paper presents a model of liquidity that allows for an equilibrium characterization of both spreads and price impact. It does this without the presence of noise traders, meaning that the preferences of all participants are fully modeled. This allows us to study how changes in market characteristics like the relative quantity of market makers, the fees associated with providing or removing liquidity, or differences in opinion about the value of the asset change the nature of the orderbook and the liquidity of the market.

The model is based on a dynamic search model where buyers arrive at the market looking to immediately purchase an asset for which limit offers have been established. Buyers' valuations of the asset imply that for at least some offered prices they are willing to purchase the asset, although they are not willing to purchase the asset for any offered price. If they do not purchase the asset, they wait for the next period to see if a more attractive offer is available to them.

Sellers in the market are liquidity providers and place limit orders to sell the asset based on their internal valuation of the asset's value. They place limit orders at prices that are optimal given the publicly known demand parameters as well as the understanding that higher priced limit orders will receive lower priority than lower priced limit orders.

In this model we characterize the equilibrium orderbook, or distribution of offers, in terms of the exogenous parameters. This characterization allows us to answer equilibrium questions which are more difficult to answer in partial equilibrium frameworks. We show, for example, that increases in the fee charged to liquidity takers leads to liquidity providers placing more attractive offers and hence more liquid markets *ceteris paribus*. Intuitively, this occurs because, holding the internal valuation of the stock constant for both buyers and sellers, an increase in the take fee makes the asset less desirable for buyers which increases their bargaining power relative to sellers. We also find that increases in the popularity or salience of a particular stock to buyers has mixed results on the liquidity in the orderbook. The increase in popularity decreases spreads, but increases equilibrium price impact. Furthermore, we show that increases in the difference between buyers' and sellers' valuations of the asset lead to less liquid markets. This happens because as buyers' desire for the asset increases relative to sellers', the sellers' bargaining position increases and they can sell at more favorable prices for them.

After reviewing the literature, we will construct the model and equilibrium in section 2, followed by analysis of the properties of equilibrium in section 3. Section 4 concludes.

1.1 Literature

Two standard models of the orderbook are Glosten & Milgrom (1985) and Kyle (1985). Glosten & Milgrom (1985) models market makers as submitting limit orders so as to reflect the information content in the arriving order being a buy or sell. The arrival process of market orders is given exogenously and the best bid and offer prices are then derived optimally in a competitive environment. Our paper differs from Glosten & Milgrom (1985) in that we model the preferences of both liquidity demanders and liquidity suppliers. In so doing, we can trace out the shape of the order book, as opposed to describing just the best bid and offer. In modeling both liquidity suppliers and demanders we can say more about each side of the market's motivation, at the cost of increased complexity.

Kyle (1985) models a game of demand curve submission that can be interpreted as a primitive to forming the orderbook. Like in Glosten & Milgrom (1985), liquidity demanders are exogenous and divided between informed and uninformed traders. Informed traders in this model however, understand the impact that their orderflow will have on the price at which their orders are executed and so they shade their bids to maximize the expected profit from their trades. This model becomes tractable through a set of parametric assumptions (linearity and normality). Our model differs from that paper's model in its setup, the parameterization of liquidity demanders and that we assume that liquidity providers submit limit orders instead of demand/supply curves. We also motivate traders based on disagreements about asset values, rather than differential information.

Back & Baruch (2004) establishes a connection between Glosten & Milgrom (1985) and Kyle (1985) by nesting each of these in a continuous time model and establishing conditions under which the equilibrium of their model converges to that of Glosten & Milgrom (1985) and Kyle (1985). Their analysis focuses on (for Glosten-Milgrom) characterizing optimal trade timing for a single informed trader, and to show conditions under which this version of the model will converge to the equilibrium in Kyle. Although we discuss some of the connections between our results and those of Glosten & Milgrom (1985) and Kyle (1985), our modelling framework is sufficiently different from those models that we do not attempt to study conditions under which our model would converge to those.

Goettler et al. (2005) build a dynamic model of limit order markets where all traders' preferences are fully modelled, as in our paper. In that paper, traders arrive with random valuations and liquidity needs and must decide whether to buy or sell the asset. Information is disseminated over time in such a way that those who have submitted previous orders on the book did so with information that becomes outdated over time. Orders are cancelled exogenously when certain market conditions are met. Goettler et al. (2005) differs in several key ways from the model presented in this paper. First, in their model traders arrive at a constant rate and the orderbook is built up over time, while in ours traders arrive according to a Poisson rate given exogenously and we characterize the steady state orderbook. In Goettler et al. (2005) information changes over time in ways that lead to accumulation of orders based on changes in that information and the properties of the exogenous order cancellation function, while in our model orders are placed and executed endogenously. Goettler et al. (2005) allows for traders to endogenously determine whether they will buy or sell and whether they will do so with limit orders or market orders, where our model designates traders as market makers or liquidity demanders exogenously. Finally, Goettler et al. (2005) must be solved numerically, while our model has closed form solutions.

Foucault et al. (2005) build a model of the orderbook that relies on differences in patience of market participants, with patient traders submitting limit orders and impatient traders submitting market orders. Traders arrive according to a Poisson process and buyers and sellers have exogenously given differences in valuations, as in this paper. Unlike in this paper, Foucault et al. (2005) assume that all limit orders that are submitted must be price improving (i.e. narrow the spread) and that buyers and sellers alternate with certainty. These assumptions (which can be relaxed somewhat, as they show) allow them to study the expected waiting time for a limit order to be fulfilled. Unlike that paper, our model obtains a closed form characterization of both depth and spread (in steady state) where they characterize a distribution of spreads and extensively study executions times and resiliency.

Several search theoretic papers study liquidity in asset markets from a macro perspective. Lagos & Rocheteau (2009) studies liquidity in over-the-counter markets. This model makes predictions about liquidity measures like the bid-ask spread as in our paper, although they do not attempt to model the mechanics of price priority in limit orderbooks as is done in our paper. Cui & Radde (2016) also abstracts from the specific properties of the limit orderbook, embedding a financial sector that entails search with frictions into a dynamic general equilibrium consumption-saving-investment model. Vayanos & Wang (2007) builds a search-based model of trading with the friction that traders can search for only one asset and then studies the equilibria that result. They show that in one equilibrium short-horizon investors congregate in one market and that this market is more liquid than the other. The question of time horizon does not enter into our model since all traders have the same trading horizon.

2 Model

Suppose that the number of sellers placing limit orders is exponentially distributed with mean α and that the number of buyers placing market orders is exponentially distributed with mean β. These quantities are assumed to be independently distributed. Each market participant is assumed to want to buy or sell one unit of the asset. Given these assumptions, the joint distribution of the number of buyers n_b (who submit market orders) and the number of sellers n_a (who submit limit orders) is

$$f(n_a, n_b) = \alpha e^{-\alpha n_a} \beta e^{-\beta n_b}. \tag{1}$$

In the analysis that follows, it will be useful to understand the ratio of buy market orders to ask limit orders. Define $r = n_b$ and $q = \frac{n_b}{n_a}$. The joint pdf of r and q is

$$f(r, q) = \frac{e^{-r\left(\frac{\alpha}{q}+\beta\right)} r\alpha\beta}{q^2}. \tag{2}$$

Integrating over r, we get the pdf of q which is

$$f(q) = \frac{\alpha\beta}{(\alpha + q\beta)^2}. \tag{3}$$

In much of the analysis to follow, a useful parametrization will be to let $\phi = \beta/\alpha$, be the ratio of the expected size of the (market) buy side relative to the expected depth of the (limit) ask side of the market. Expressed in terms of ϕ, the probability density function of q in equation (3) becomes

$$f(q) = \frac{\phi}{(1 + \phi q)^2}. \tag{4}$$

Integrating this over q shows that q has cumulative distribution function

$$F(q) = \frac{\phi q}{1 + \phi q}. \tag{5}$$

Consider a seller who places a limit order at price a. The seller gets value d_A while holding the asset. Suppose he transacts with probability $G(a)$, and if so, receives a maker fee f_m. Then his Bellman function is:

$$\rho V(a) = d_A + G(a)(a + f_m - V(a)). \tag{6}$$

Solving this for $V(a)$ gives

$$V(a) = \frac{d_A + (a + f_m)G(a)}{\rho + G(a)}. \tag{7}$$

The probability of transacting $G(a)$ will be determined in equilibrium. Under the assumption of competitive limit order markets, each trader will have, in expectation, equal profits, so changing prices will not affect equilibrium profits. This implies that

$$\frac{\partial V(a)}{\partial a} = \frac{\rho G(a) + G(a)^2 + ((a + f_m)\rho - d_A)G'(a)}{\rho + G(a)} = 0. \tag{8}$$

We solve this differential equation under the assumption that the lowest (best) ask, denoted a_0 is executed with probability 1. That is, that $G(a_0) = 1$. This solution is

$$G(a) = \frac{d_A - (a_0 + f_m)\rho}{(a_0 + f_m)\rho + d_A - a(1+\rho)}. \tag{9}$$

Here, $G(a)$ gives conditions on the shape of the orderbook (the distribution of prices) that ensure equal profitability for each price. This expression is agnostic as to the underlying processes that generate $G(a)$. Ultimately, these probabilities will be a function of the realized flow of marketable buy orders and limit sell orders.

Inserting the solution for $G(a)$ into equation (7) allows us to solve for the value function and get

$$V(a) = \frac{a_0 + d_A + f_m}{1+\rho} \tag{10}$$

The key feature of the order book is its order of execution, starting at the best price for market orders and moving upward. This implies a relationship between the number of market orders that arrive and the marginal price that the last-to-arrive market order will face, which we explore now.

The probability of transacting (from a seller's perspective) is $G(a)$. The fraction of sellers that will transact (in aggregate) is q (when $q <= 1$). So these two must relate as follows:

$$G(a) = 1 - F(q). \tag{11}$$

Using equations (3) and (9) together with equation (11) gives

$$\frac{d_A - (a_0 + f_m)\rho}{(a_0 + f_m)\rho + d_A - a(1+\rho)} = 1 - \frac{q\phi}{1+q\phi}. \tag{12}$$

Solving this equation for the ask price a gives

$$\begin{aligned} a &= \frac{(1+\rho)a_0 + q\phi(\rho(a_0+f_m)-d_A)}{1+\rho} \\ &= a_0 + bq \end{aligned} \tag{13}$$

where $b = \phi(\rho(a_0+f_m)-d_A)/(1+\rho)$.

The price a is positively related to the fraction q of limit orders executed as expected, as long as $b > 0$, which will occur in a world where the rebate to market makers f_m is positive and dividends are small relative to the asset price. We make this assumption throughout the rest of the paper, since $b < 0$ implies that $a_0 + f_m < \frac{d_A}{\rho}$. The left hand side of this inequality is the value that a seller gets from selling the asset today and the right hand side is the value to the seller of holding the asset forever. If the right hand side were greater than the left hand size, then the seller would have no desire to sell at the price a_0 and thus would not offer that price in equilibrium. Because of this, we take $b > 0$ for the rest of the paper. Since no more than 100 percent of the limit orders on the book can be filled, the maximum value for q is $q = 1$. This means that the maximum price that a limit order trader would want to place is found by plugging $q = 1$ into equation (13) which yields

$$a_1 = a_0 + \frac{\phi((a_0+f_m)\rho - d_A)}{1+\rho}. \tag{14}$$

This implies, in turn, that the probability that the highest price is transacted is perceived to be $G(a_1) = \frac{1}{1+\phi}$. Note that this probability is not zero. In equilibrium, liquidity providers on the sell side will not offer a price that has no chance of transacting. It is interesting to note, however, that if a liquidity provider were to offer the price $a_1 + \epsilon$, then the probability of that order transacting would be zero. This is because, as will be shown when we consider the buyers' side of the problem, at the price $a_1 + \epsilon$, buyers would rather wait for future opportunities than purchase the asset now. So we can also interpret a_1 as the maximum offer that buyers would be willing to accept.[1]

The distribution of prices $a(q)$, truncated to $q \leq 1$, is

$$F(a(q)) = \frac{\frac{q\phi}{1+q\phi}}{\frac{\phi}{1+\phi}} = \frac{q(1+\phi)}{1+q\phi} \tag{15}$$

Using equation (11) and solving for q, and then plugging this into equation (15), we get

$$F(a) = \frac{\left(\frac{a-a_0}{b}\right)(1+\phi)}{1+\left(\frac{a-a_0}{b}\right)\phi}. \tag{16}$$

This equation characterizes the distribution of prices in this model, which can be used to show the shape of the orderbook, as a function of the endogenous best ask price a_0.

2.1 The buyers' problem

Impatient buyers submit market orders, but do not know exactly the price that they will receive because they can't guarantee that other orders won't be executed prior to theirs. On receiving the asset, an impatient buyer receives utility $d_B + \delta_B$. The term δ_B is meant to capture the impatience present in the buyer that induces them to place marketable orders instead of limit orders.

A buyer who places a marketable order faces one of three distinct possibilities: $q < 1$ so all buyers get to transact (at various prices), $q > 1$ so some buyers are unable to transact, but this buyer's order is early enough in the queue that she is able to transact, or $q > 1$ and this buyer's order is late enough in the queue that she is unable to transact.

We work under the assumption that if a particular q is drawn (from pdf $\frac{\phi}{(1+q\phi)^2}$), it is equally likely that a buyers' position s is anywhere from $0 \leq s \leq q$. Thus, the distribution

[1] This could mean, for example, that buyers are really submitting marketable limit orders at the price a_1.

on s is uniform on $[0,q]$, with pdf $1/q$. We assume that upon transacting, a buyer whose marketable order crosses the spread and transacts with a seller's limit order pays the "take fee" f_t. The value function of the buyers takes the form

$$\begin{aligned}\rho W_B = &\int_0^1 \frac{\phi}{(1+q\phi)^2}\left(\int_0^q \frac{d_B+\delta_B-a(s)-f_t}{q}ds\right)dq \\ &+\int_1^\infty \frac{1}{q}\frac{\phi}{(1+q\phi)^2}\left(\int_0^1\left(\frac{d_B+\delta_B}{\rho}-a(s)-f_t-W_B\right)ds\right)dq\end{aligned} \tag{17}$$

From equation (13), we see that $a(s) = a_0 + bs$, which allows us to rewrite equation (17) (after evaluating integrals)

$$W_B = -a_0 + \frac{d_B+\delta_B}{\rho} + \frac{(b+2a_0\rho)\phi - b\log\left((1+\phi)\left(1+\frac{1}{\phi}\right)^{\phi^2}\right) - 2\phi(d_B+\delta_B)}{2\phi\left(\rho+\phi\log\left(1+\frac{1}{\phi}\right)\right)} \tag{18}$$

To interpret these terms, we first consider the term $\phi\log\left(1+\frac{1}{\phi}\right)$. This is the probability that the market order of a buyer successfully transacts, either because there is enough liquidity for all market orders to transact or because there is not enough liquidity, but this trader arrives early enough in line to have her order executed.

To see this, note that from equation (5) the probability that the orderbook is deep enough to satisfy all incoming orders is $P(q \le 1) = \frac{\phi}{1+\phi}$. When there is not enough liquidity to satisfy all incoming market orders, the probability that a particular trader i's market order is executed is

$$P(q \ge 1, i \text{ is served}) = \int_1^\infty \frac{\phi}{(1+q\phi)^2}\frac{1}{q}dq = \phi\left(\log\left(1+\frac{1}{\phi}\right) - \frac{1}{1+\phi}\right). \tag{19}$$

Summing these two probabilities gives the probability that a buyers' (liquidity takers') order is executed denoted X_t,

$$\begin{aligned} X_t &= P(q \leq 1) + P(q \geq 1, \text{trader is served}) \\ &= \frac{\phi}{1+\phi} + \phi\left(\log\left(1+\frac{1}{\phi}\right) - \frac{1}{1+\phi}\right) \\ &= \phi \log\left(1+\frac{1}{\phi}\right). \end{aligned} \tag{20}$$

Now consider the expected fraction of the orderbook (i.e. the fraction of market makers' orders) that will be executed. This is

$$X_m = \int_0^1 \frac{\phi}{(1+q\phi)^2} q dq + \int_1^\infty \frac{\phi}{(1+q\phi)^2} 1 dq = \frac{1}{\phi}\log(1+\phi). \tag{21}$$

Given these definitions, we can write the value to buyers as

$$W_B = \frac{X_t}{\rho + X_t}\left(\frac{d_B + \delta_B}{\rho} - a_0 - f_t\right) + \frac{b}{2(\rho + X_t)}(X_m - X_t) \tag{22}$$

We can use this expression to calculate the buyers' maximum willingness to pay. A buyer will be indifferent between a specific price and waiting for a future opportunity to buy if their current value on the market W_B is equal to the value of owning the asset at the maximum price. That is, if

$$\frac{d_B + \delta_B}{\rho} - (a_1 + f_t) = W_B. \tag{23}$$

Plugging in equation (22) gives the condition that

$$\frac{\rho}{\rho + X_m}\left(\frac{d_B + \delta_B}{\rho}\right) - (a_1 + f_t) + \frac{X_t}{\rho + X_t}(a_0 + f_t) - \frac{b}{2(\rho + X_t)}(X_m - X_t) = 0 \tag{24}$$

Equation (13) under the condition that $q = 1$, implies that $a_1 = a_0 + b$. Plugging this into equation (24) and substituting the definition of b, also from equation (13) leads to (after simplifying)

$$a_0 = \frac{\phi\left(\rho + \frac{X_m + X_t}{2}\right)}{1 + \rho + \phi\left(\rho + \frac{X_m+X_t}{2}\right)}\left(\frac{d_A}{\rho} - f_m\right) + \frac{(1+\rho)}{1 + \rho + \phi\left(\rho + \frac{X_m+X_t}{2}\right)}\left(\frac{d_B + \delta_B}{\rho} - f_t\right) \tag{25}$$

From this, we see that the best offer available is a convex combination of the lifetime use value to sellers of the asset, minus the rebate that they would receive if they sell the asset[2] and the lifetime value to buyers of owning the asset minus the take fee.

To finish the solution for the orderbook, we then plug this solution for a_0 into the definition of b in equation (13) to get a value for b in terms of the exogenous variables. This value is

$$b = \frac{\rho\phi}{1 + \rho + \phi\left(\rho + \frac{X_m+X_t}{2}\right)}\left(\left(\frac{d_B + \delta_B}{\rho} - f_t\right) - \left(\frac{d_A}{\rho} - f_m\right)\right) \tag{26}$$

Figure 1 depicts the orderbook with the ask price a on the horizontal axis and the fraction of the orderbook sold on the horizontal axis.

3 Market liquidity in differing market environments

Having modeled both liquidity demanders and liquidity providers, on one side of the market we use this section to interpret the predictions of the model and understand the comparative statics of changes in the market environment on equilibrium liquidity provision.

There are two prevailing notions of the the liquidity of a limit order market. The first, as modeled in Glosten & Milgrom (1985), is the size of the spread. The second, as modeled, *inter alia*, in Kyle (1985), is the price impact of a trade of a set size. This notion of liquidity can be studied using equation (14).

[2] If sellers were to forego selling the asset they would also forego the make fee, which is why this fee is subtracted.

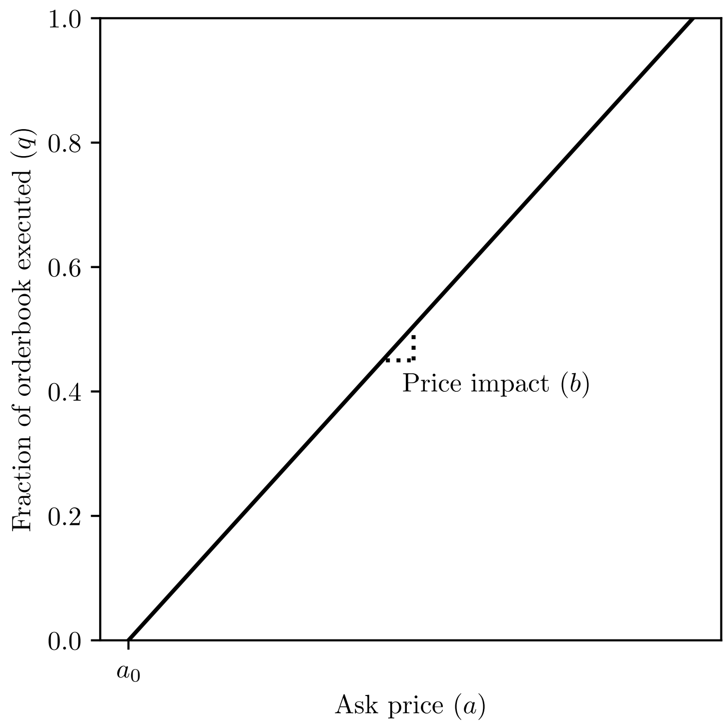

Figure 1: The orderbook

We define the spread to be

$$
\begin{aligned}
S &= a_0 - \left(\frac{d_A}{\rho} - f_m\right) \\
&= \frac{1+\rho}{1+\rho+\phi\left(\rho + \frac{X_m+X_t}{2}\right)}\left(\left(\frac{d_B+\delta_B}{\rho} - f_t\right) - \left(\frac{d_A}{\rho} - f_m\right)\right)
\end{aligned} \tag{27}
$$

Since we only model one side of the market here, we choose as our measure of the spread the difference between sellers' valuation of the asset and the lowest price at which they offer it for sale. If the market for market makers were perfectly competitive, they would sell the asset at their valuation. The frictions inherent in the model generate a positive spread.

The second notion of liquidity we will call *price impact*. From equation (13), b represents the change in the ask that occurs because of a small change in the fraction of the orderbook consumed.[3] We use this as our measure of price impact.

$$
PI = b = \frac{\rho\phi}{1+\rho+\phi\left(\rho + \frac{X_m+X_t}{2}\right)}\left(\left(\frac{d_B+\delta_B}{\rho} - f_t\right) - \left(\frac{d_A}{\rho} - f_m\right)\right) \tag{28}
$$

Notice that each of these measures of liquidity rely on the difference in the market value of buyers and sellers as well as the difference between the make rebate and the take fee. Define

$$
\begin{aligned}
\Delta V &= \frac{d_B+\delta_B}{\rho} - \frac{d_A}{\rho} \\
\Delta f &= f_t - f_m
\end{aligned} \tag{29}
$$

[3] Note that this is expressed in terms of q, the *fraction* of the orderbook that is removed by liquidity demanders. To express this in terms of the number of shares one would use the expression $q = \frac{n_b}{n_a}$ to note that a change in q represents a change in n_b. Thus bn_a will give the price impact of a small change in the *quantity* of shares removed from the orderbook. In what follows we stick to expressing this in terms of fractions of shares in order to not carry around the extra n_a term.

and express our measures of liquidity as

$$\begin{aligned} S &= \frac{1+\rho}{1+\rho+\phi\left(\rho+\frac{X_m+X_t}{2}\right)}\left(\Delta V-\Delta f\right) \\ PI &= \frac{\rho\phi}{1+\rho+\phi\left(\rho+\frac{X_m+X_t}{2}\right)}\left(\Delta V-\Delta f\right) \end{aligned} \tag{30}$$

From this we see that market liquidity is negatively related to the difference in valuations between buyers and sellers. As buyers' valuations increase relative to sellers' valuations, the relative market power of sellers increases which leads to both larger spreads and larger price impact.

As the exchanges's revenue from fees Δf increases there is a change in the relativer value of the asset for each party, which changes each sides' bargaining position in equilibrium. As f_t increases, buyers' net value of the asset decreases, which means that they are less inclined toward the asset in general and will only purchase the assets available if they are offered at better prices, *ceteris paribus*. Knowing this, sellers shade their offers down in equilibrium.

The term

$$\phi\left(\rho+\frac{X_m+X_t}{2}\right) \tag{31}$$

gives the component of equilibrium market liquidity that is determined by the distributions of liquidity demanders (buyers) and liquidity providers (sellers). This term is increasing in ϕ which implies that as ϕ increases the spread narrows. An increase in ϕ corresponds to a larger ratio of the expected number of liquidity providers (offers) relative to the expected number of liquidity demanders. Thus, as ϕ increases, liquidity provision becomes more competitive which leads to narrower spreads.

However, as the ratio ϕ increases, price impact also increases. Increased competitiveness amongst market makers leads to better initial offers from liquidity providers, but prices increase more rapidly after those initial offers, relative to a model with lower ϕ. These

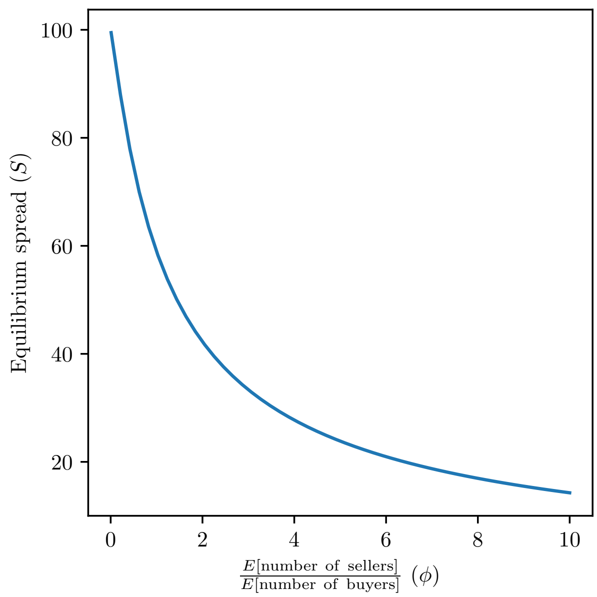

Figure 2: Spread as a function of ϕ

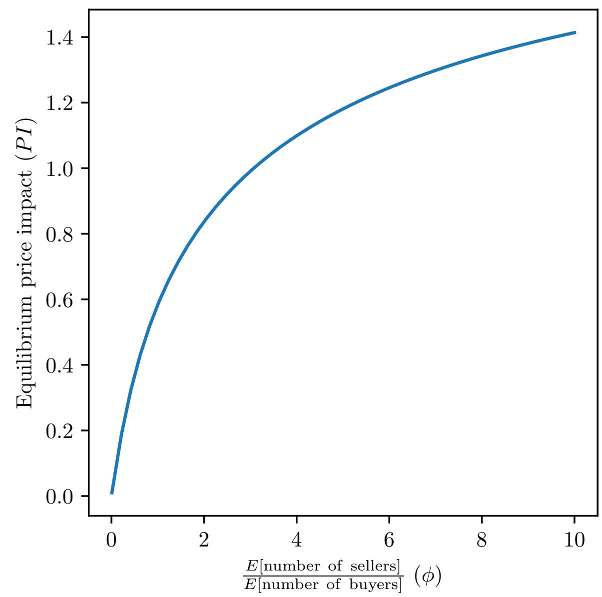

Figure 3: Price impact as a function of ϕ

phenomena are depicted in figures 2 and 3.

This result has implications for stocks that become faddish through extensive marketing. Such marketing, to the extent that it leads to increases in the relative number of liquidity demanders seeking to purchase the stock, will lead to increases in asset spreads, but thicker orderbooks in equilibrium. Future work could test this implication for stocks like Gamestop, AMC and various cryptocurrencies.

3.1 Comparison with the Glosten & Milgrom (1985) and Kyle (1985) models

One of the key differences between the model in this paper and the workhorse model of Glosten & Milgrom (1985) and Kyle (1985) is the absence of noise traders in the present model. This absence means that variables like the fraction of traders who are believed to be informed (as in Glosten & Milgrom (1985)) or the relative size of noise trader demand relative to fundamental uncertainty (as in Kyle (1985)) have no analog in our paper. However, the issue of spread size addressed in Glosten & Milgrom (1985) and price impact, addressed in Kyle (1985), can be compared.

Several key pieces of intuition about the formulation of the spread arise from Glosten & Milgrom (1985). These include the fact that competition amongst liquidity providers narrows spreads and that increases in the fraction of traders who are thought to be informed increases spreads. We see a similar result in terms of spread. An increase in the competitiveness of the offer side (as parameterized by an increase in ϕ), leads to narrower spreads. Glosten & Milgrom (1985) also contains the result that increases in the variability of the underlying asset value (as characterized by the difference in the value of the asset in the good state of the world vs. the bad state of the world) lead to increases in the spread. While not directly analogous, in our model differences in the valuation of market makers vs liquidity demanders also lead to increases in the spread and a reduction in market liquidity.

Kyle (1985) characterizes the price in the market as

$$P(y) = p_0 + \frac{\sqrt{\Sigma_0}}{2\sigma_u} y \tag{32}$$

where y is the quantity of shares demanded jointly by noise traders and insiders. This implies a marginal price, or price impact of $\frac{\sqrt{\Sigma_0}}{2\sigma_u}$, where Σ_0 is the variance of the underlying asset value and σ_u^2 is the variance of noise trader demand. If one interprets the difference in values between buyers and sellers in our model as being positively related to the underlying fundamental uncertainty about asset values, then this model and Kyle's model give results that are consistent in that price impact is linearly increasing in this measure.

4 Conclusion

This paper builds a model of limit order books that characterizes both spreads and price impact in general equilibrium when traders have differing opinions of the fundamental asset value. It makes predictions about market liquidity under various changes to the market, some of which are novel to this model. Future work could calibrate this model to existing markets and test the predictions made by the model. Finally, a next step enabled by this project is the calculation of the welfare consequences of market microstructure policies. Since the model does not involve noise traders, the costs and benefits to *all* market participants can be quantified in a way that allows policy makers to improve insight into the welfare effects of their decisions.

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