Comparison metrics

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DSFP Session 16

Adapted from a lesson at the Winter 2020 LSST-DESC collaboration meeting's "DE School"

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Overview

The problem set aims to be specific to Bayesian model comparison, but this lecture is to give you some context for what metrics mean in the context of working with probabilities as the end goal.

How do we know if/when any estimated probabilities are good enough?

Conditional probability & forward models

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Probability distribution function (PDF)

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A tangible example of astrophysical parameter estimation under Bayesian statistics

Context: photo-*z*s

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Spectroscopic redshift determination

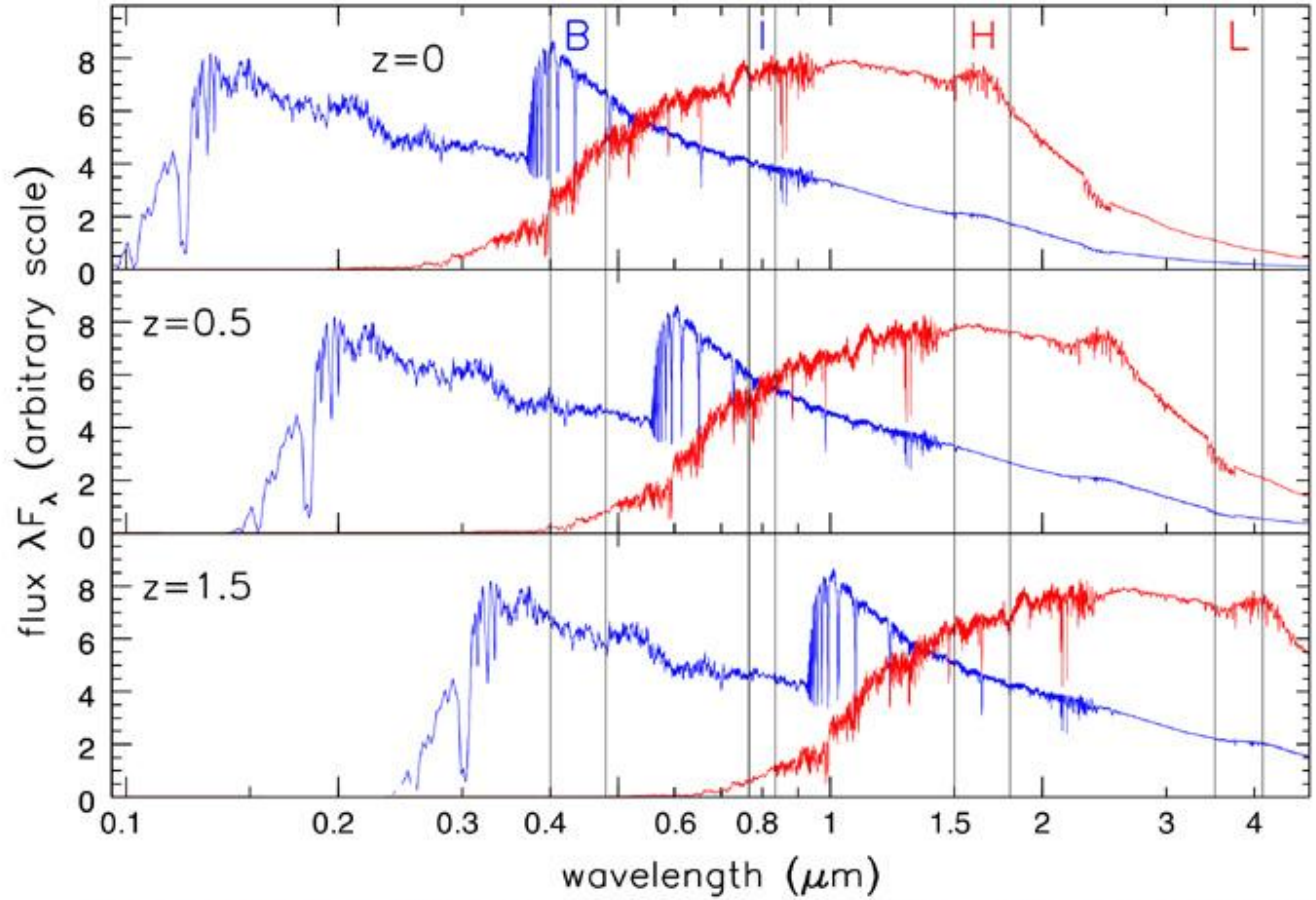
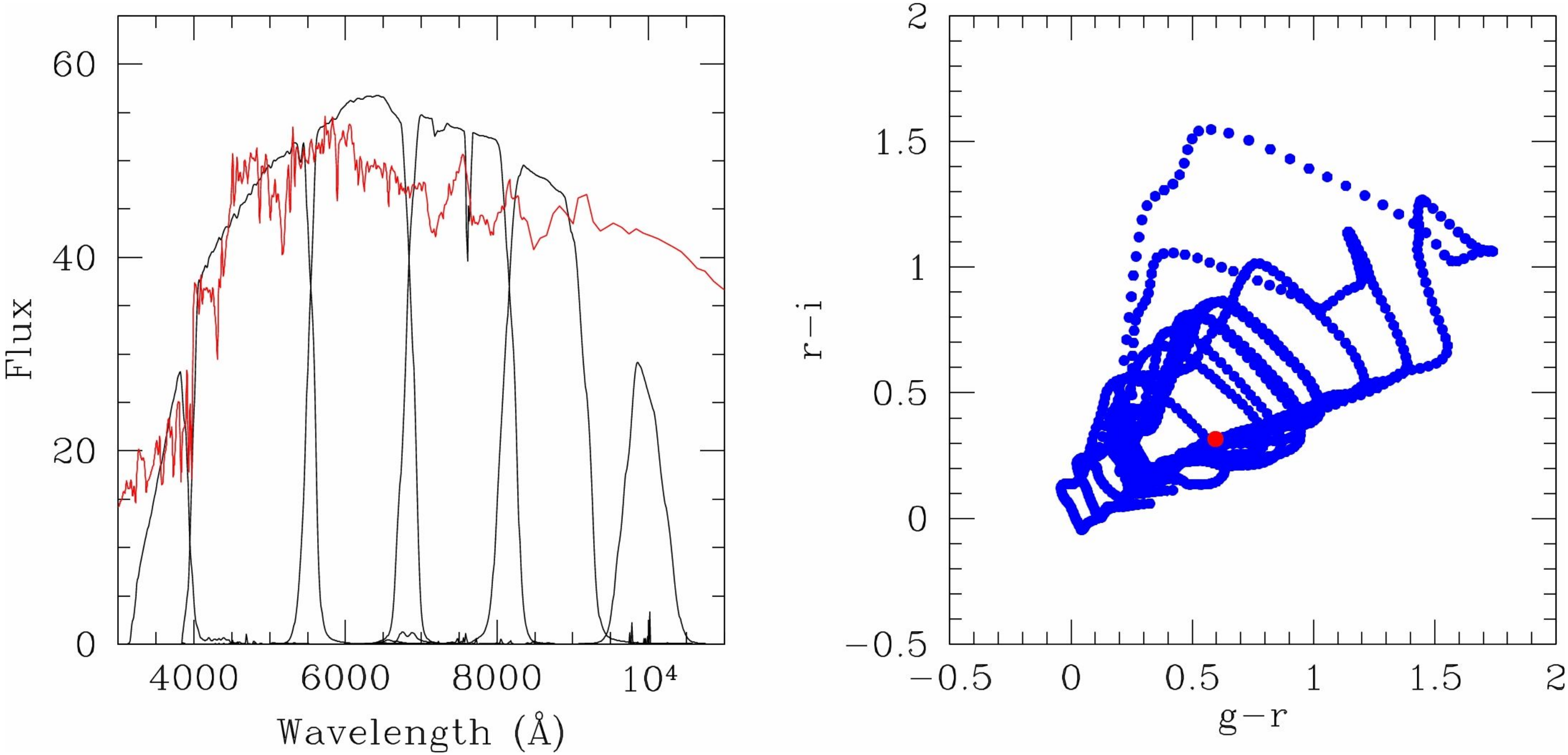


Fig 8.12 (S. Charlot) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Photometric redshift (photo-z) estimation



credit:LSST

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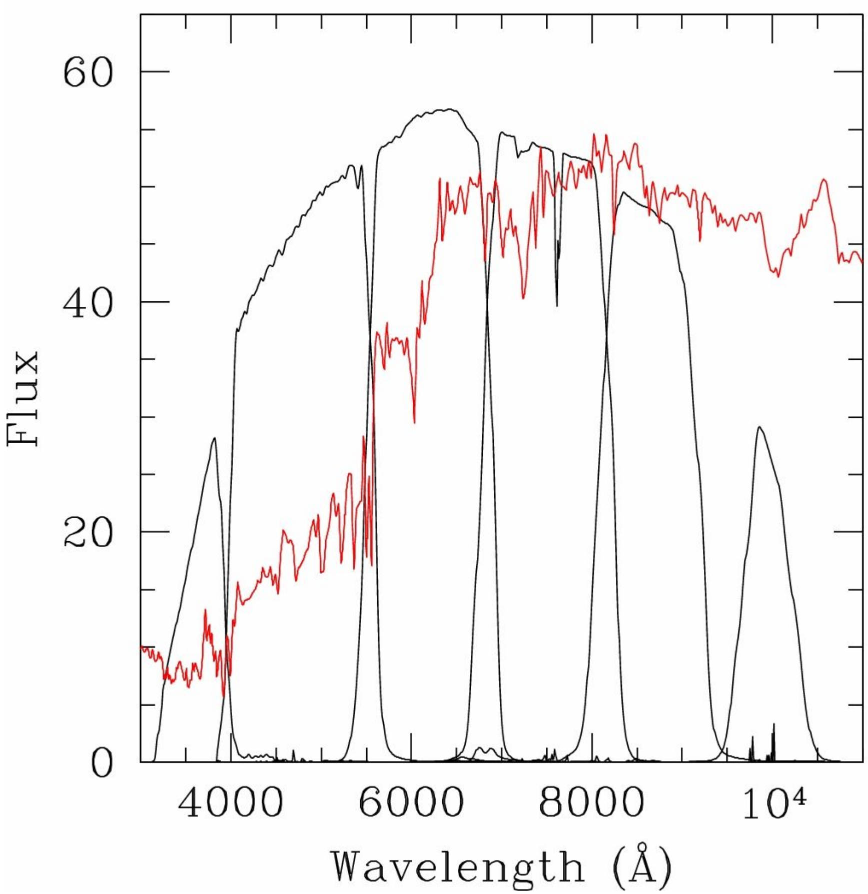

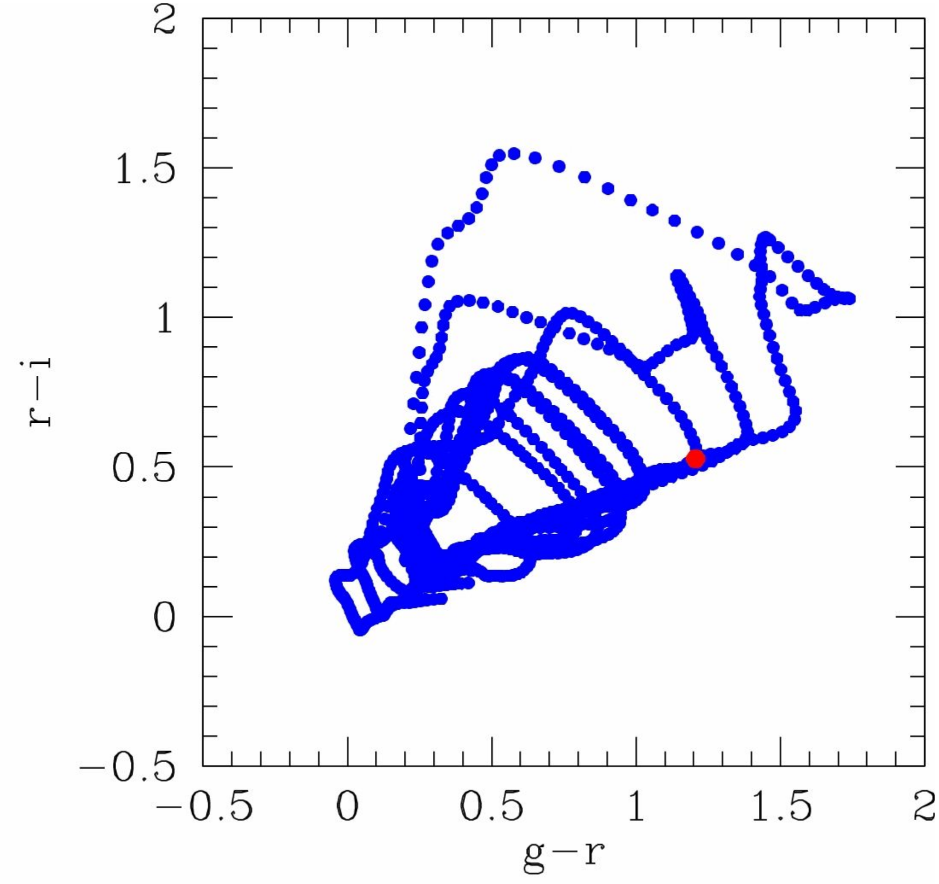

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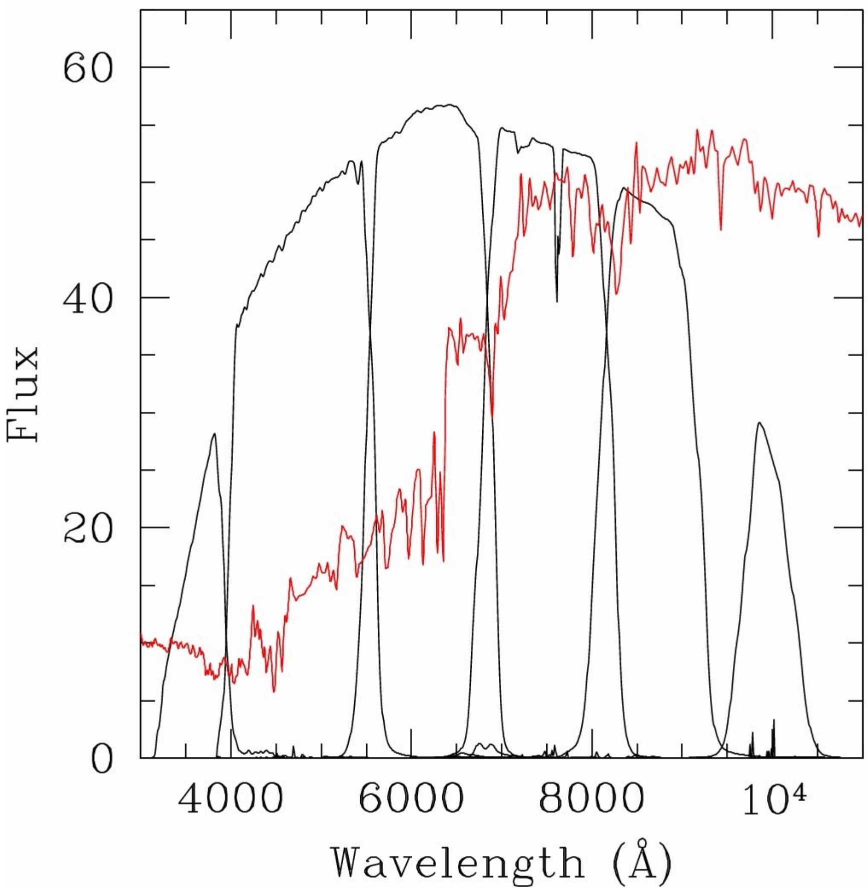

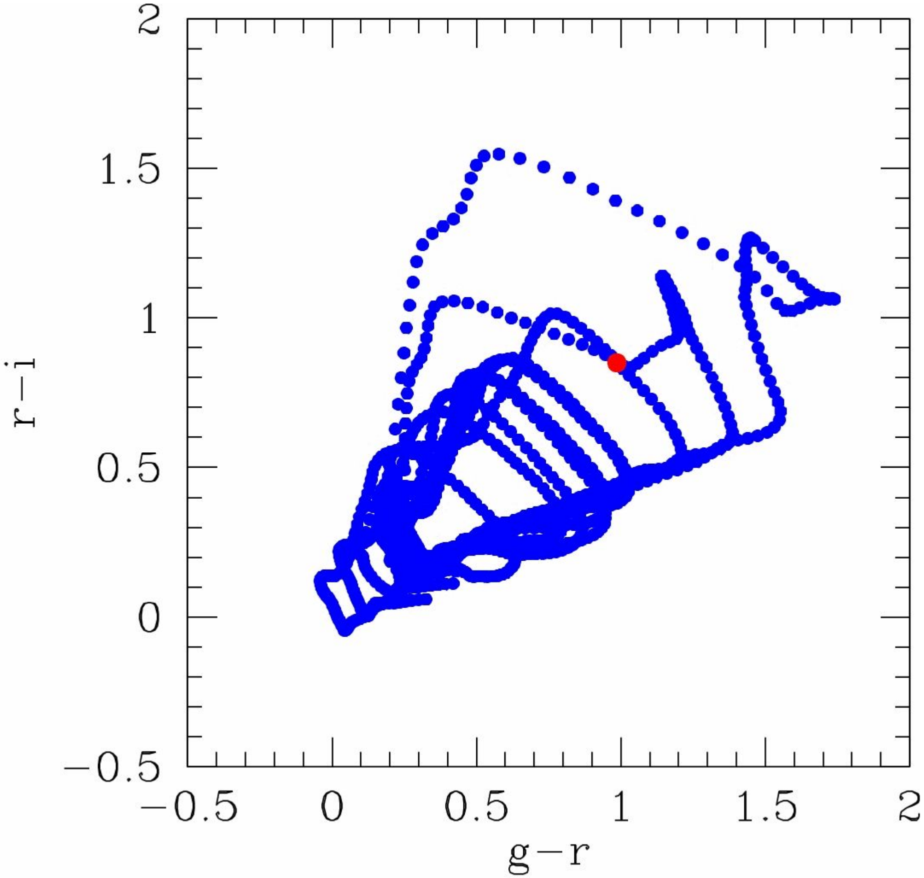

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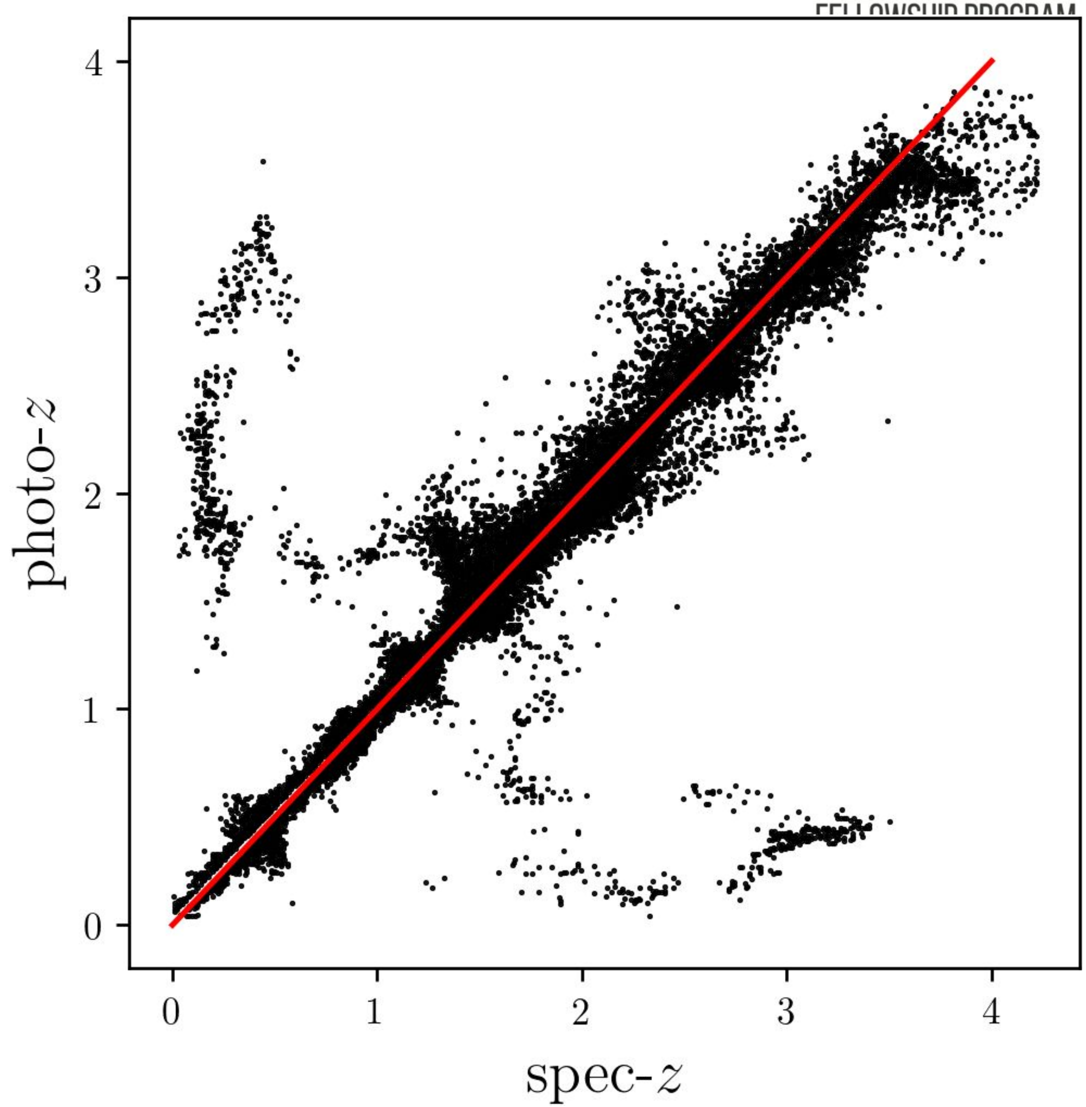

arXiv:1501.07897

Traditional photo-*z* metrics

Intrinsic scatter

$$\sigma_z \;<\; 0.02(1+z)$$

Bias

$$\langle|z-\hat{z}|\rangle \;<\; 0.003(1+z)$$

Catastrophic outlier rate

$$\mathrm{N}_{|z-\hat{z}|>3\sigma_z} \;<\; 0.1\mathrm{N}_{\mathrm{LSST}}$$

https://docushare.lsstcorp.org/docushare/dsweb/Get/LPM-17

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What do these metrics miss?

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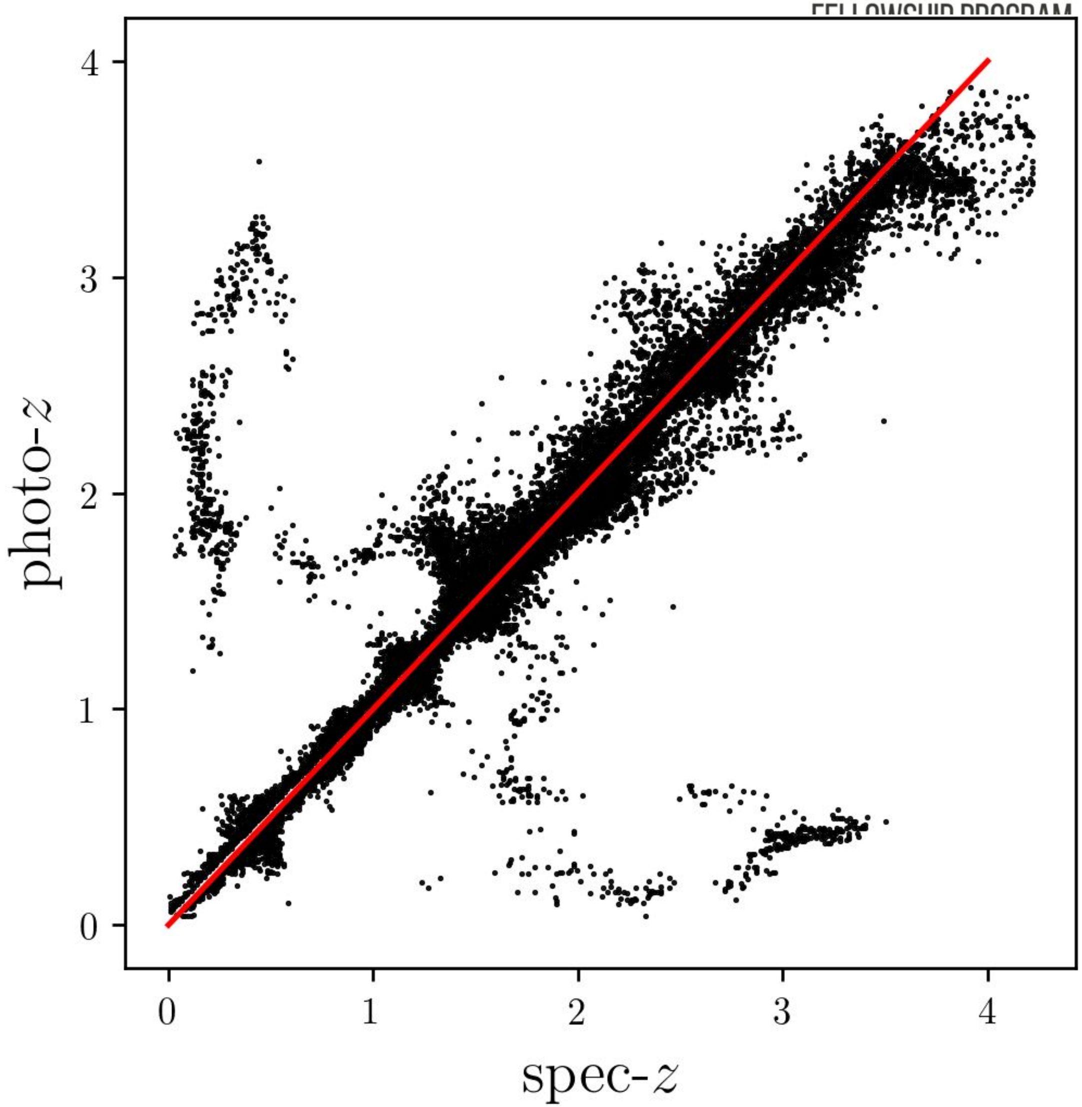

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We want to constrain the physical parameters θ that determined the observed data x, i.e. $p(\theta \mid x)$.

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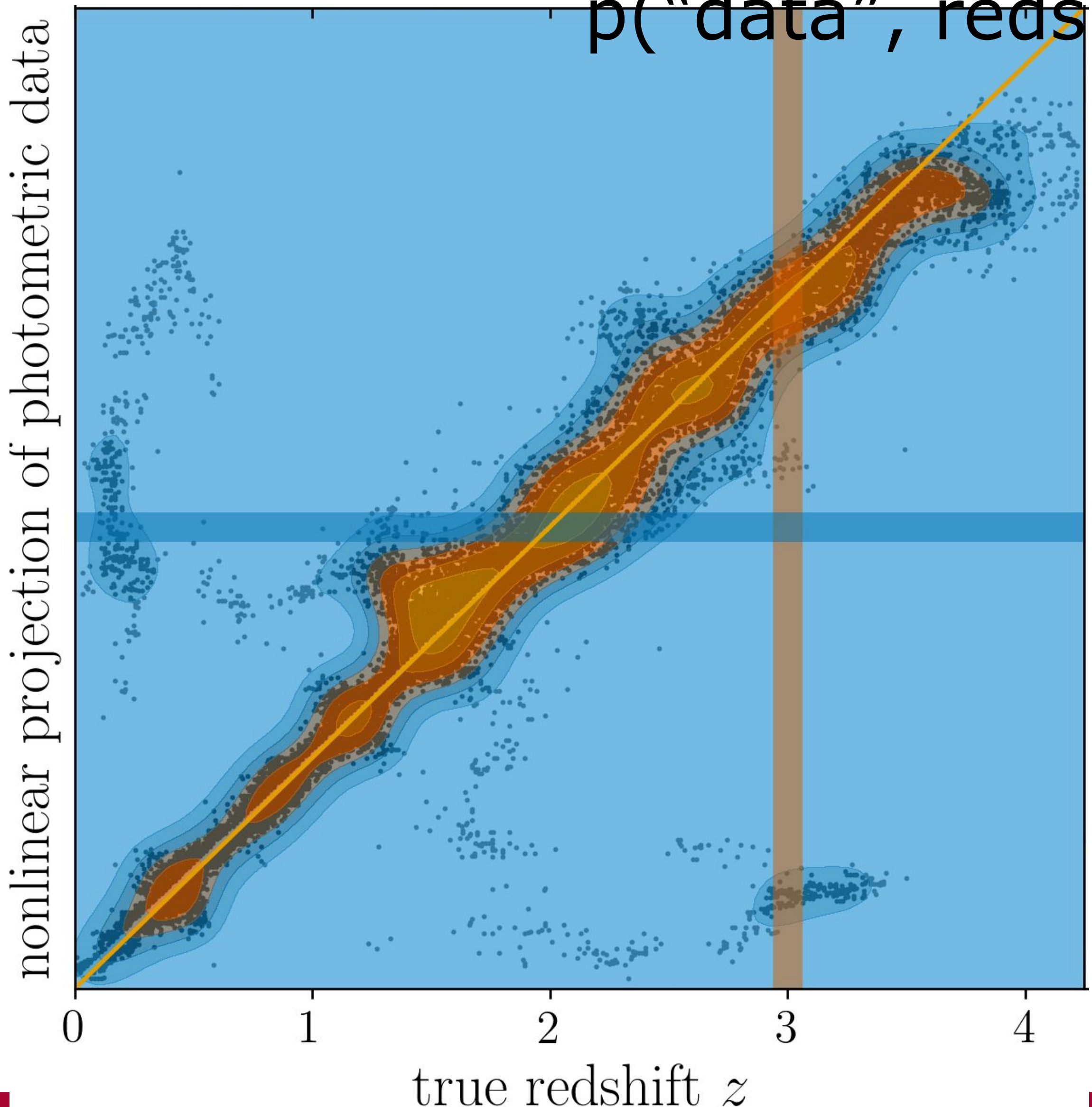
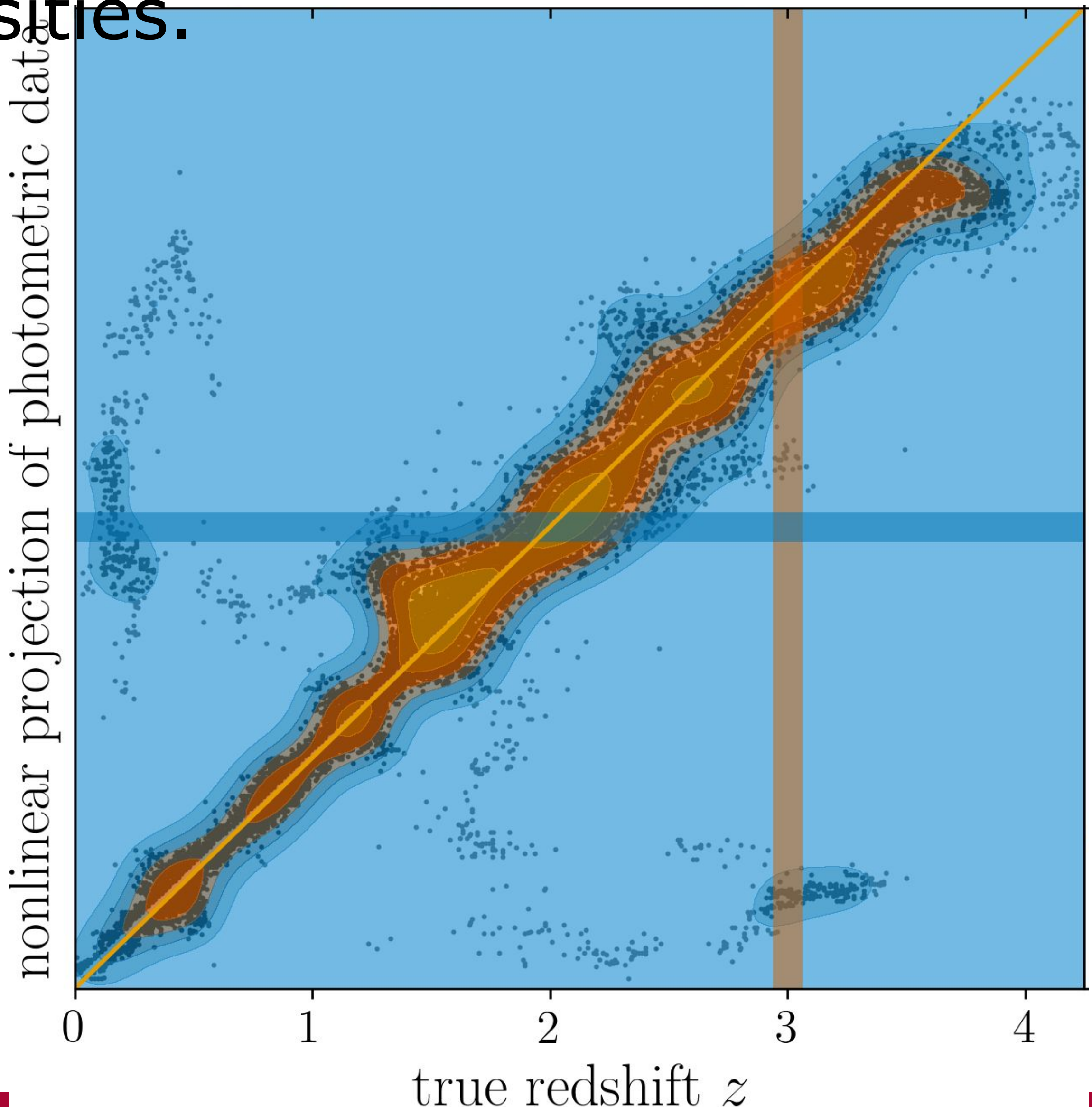
Photo-zs sample the joint PDF p("data", redshift)



arXiv:2007.12178

Sketch the projected probability densities.



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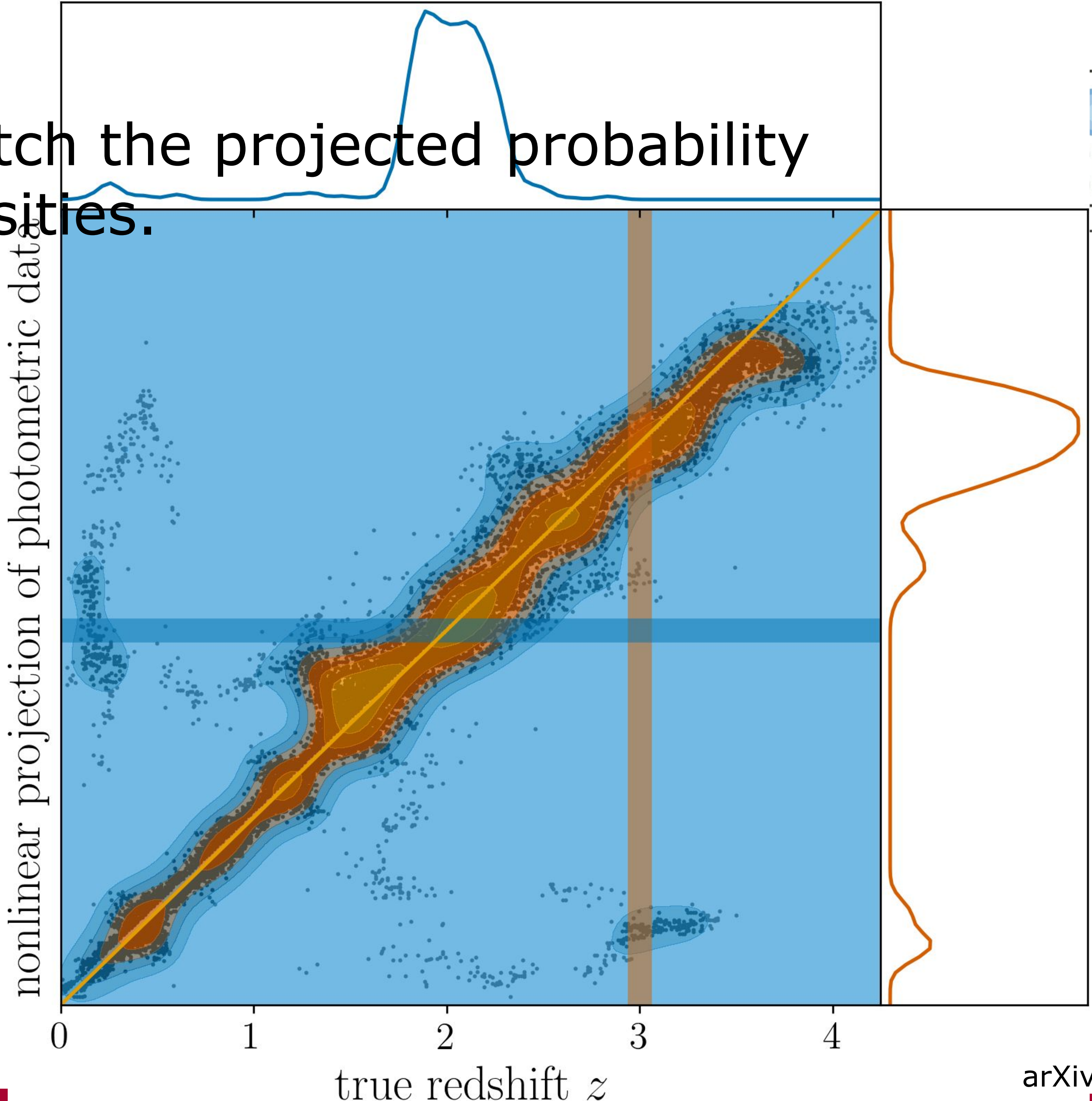

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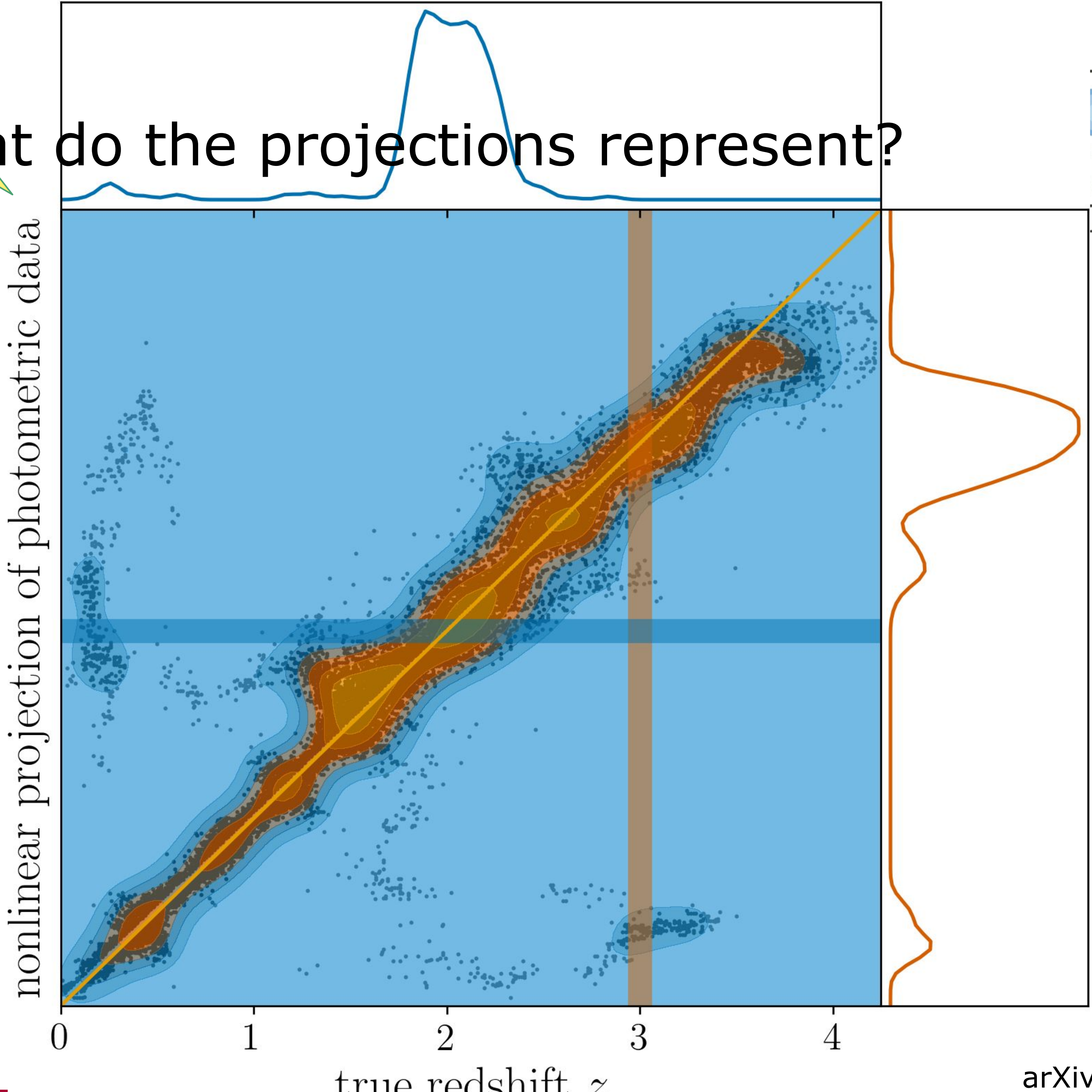
What do the projections represent?



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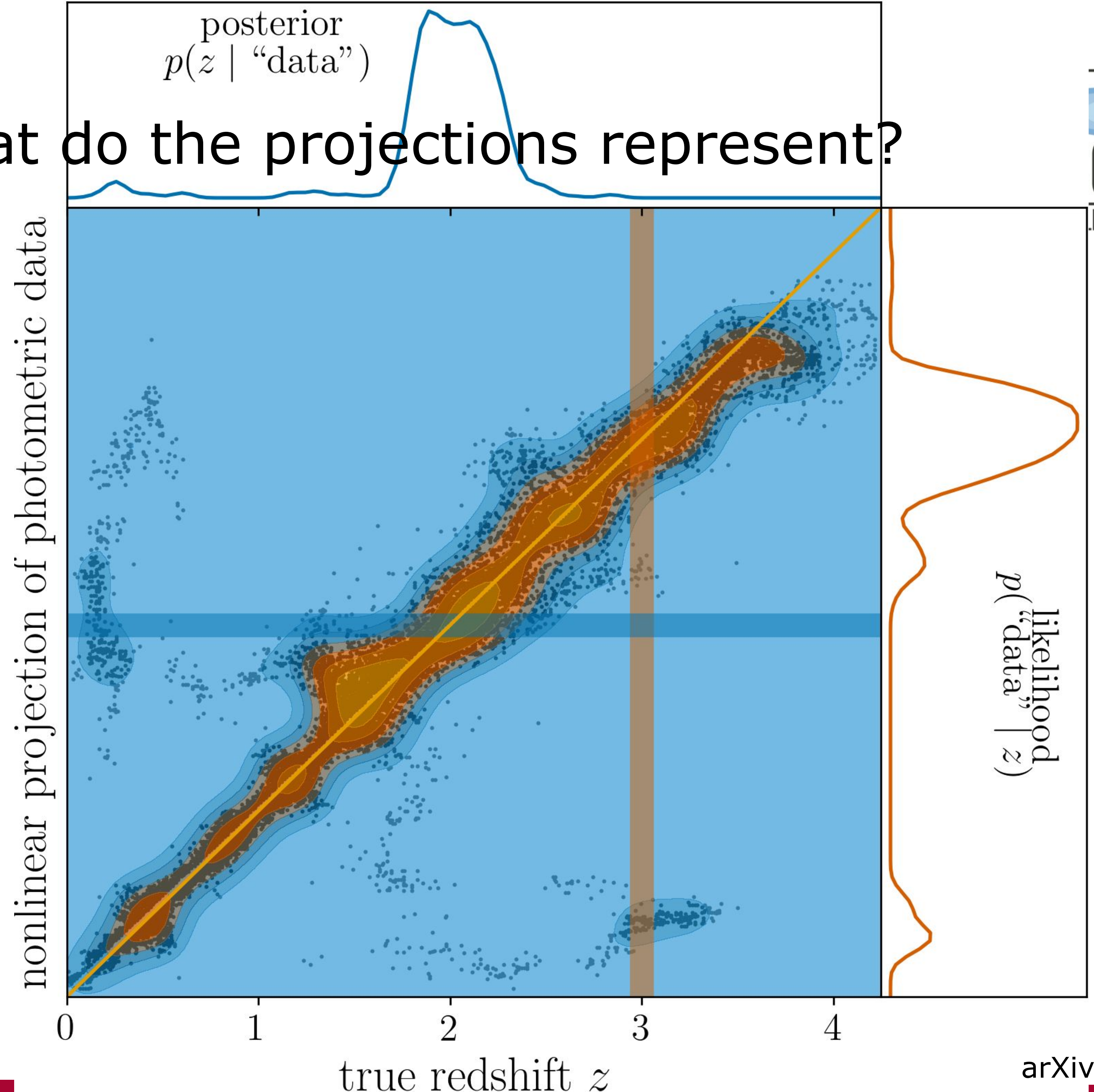

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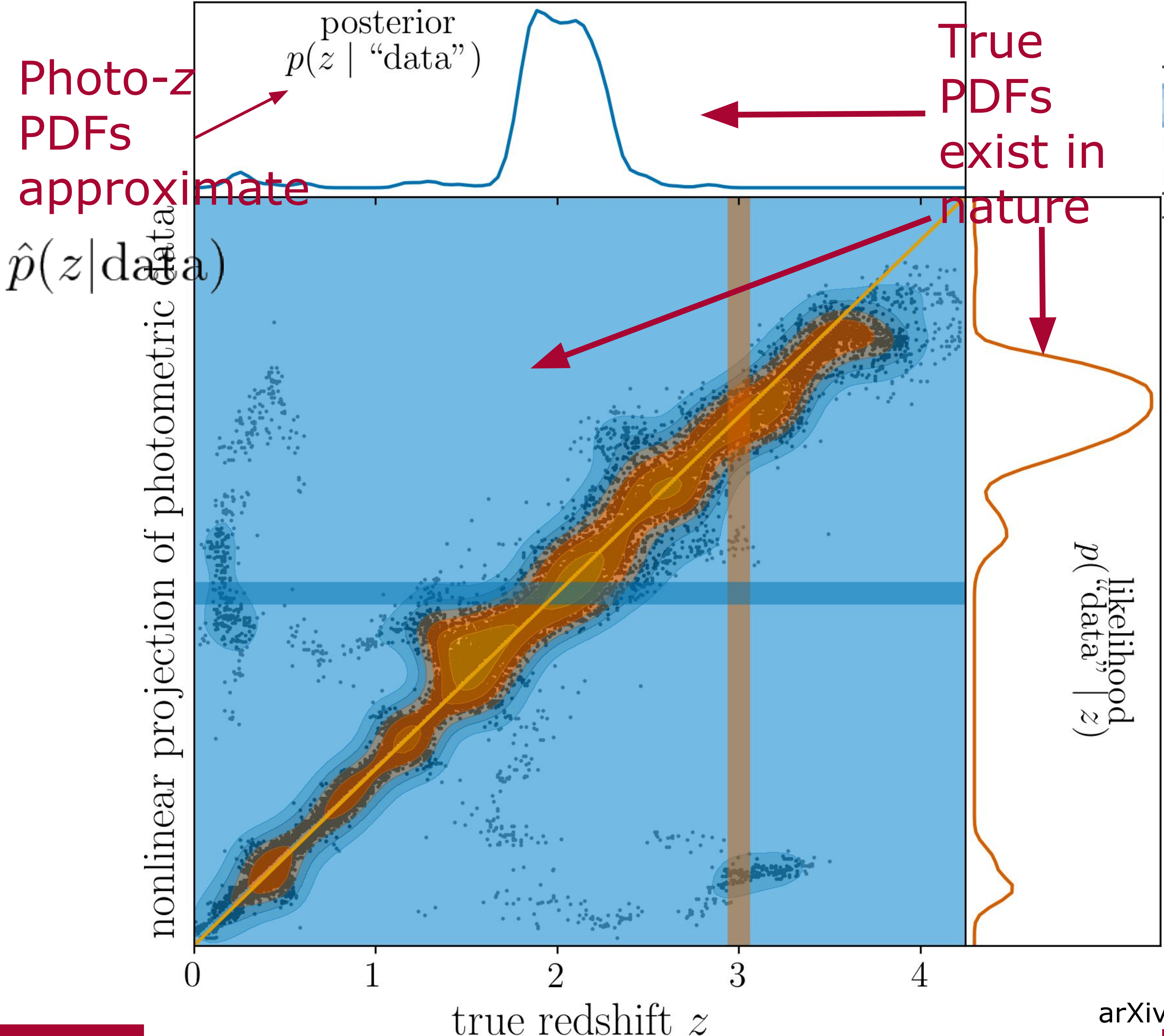
Photo-z PDFs approximate



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How do we compare PDFs?

Quantitative metrics of 1D PDFs

Root-mean-square Error (RMSE)

$$\mathrm{RMSE} = \sqrt{\int (p_{\mathrm{true}}(z) - \hat{p}_{\mathrm{est}}(z))^2 dz}$$

Kullback-Leibler Divergence (KLD)

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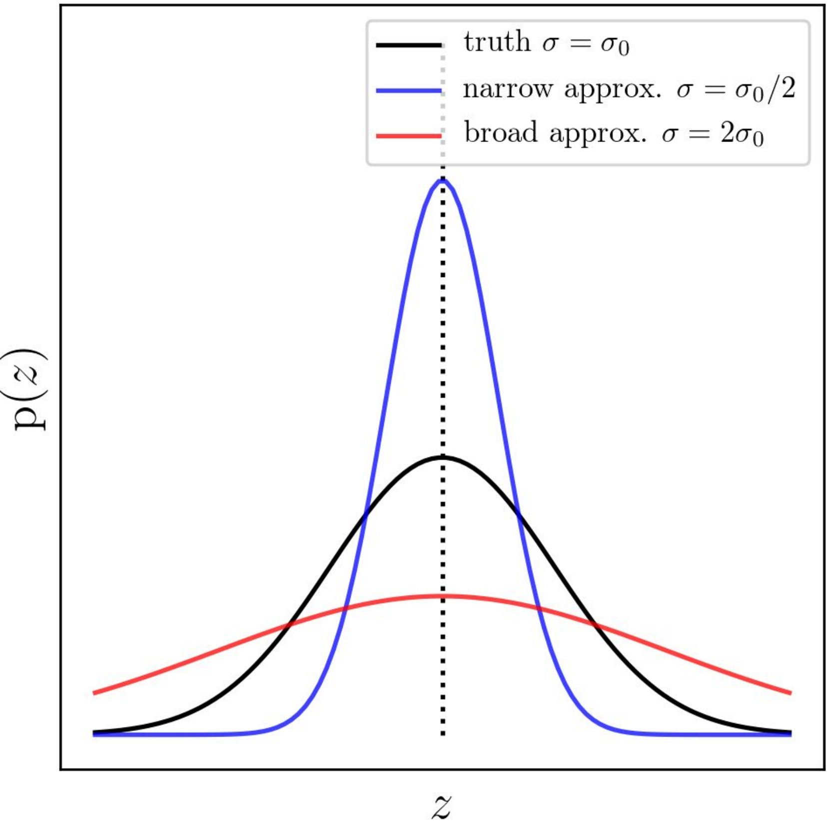
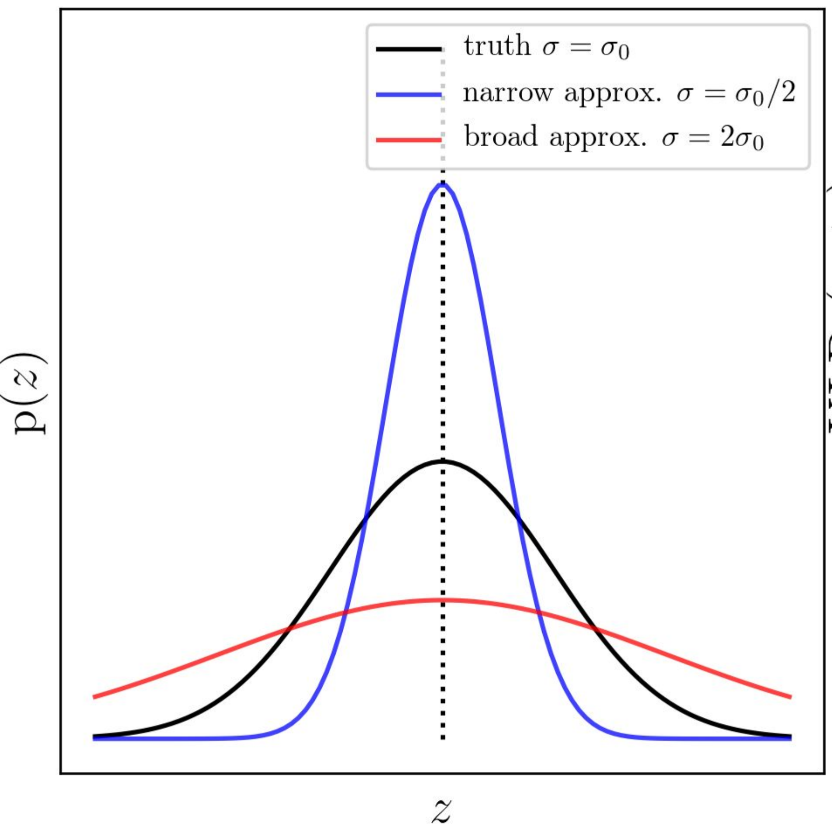
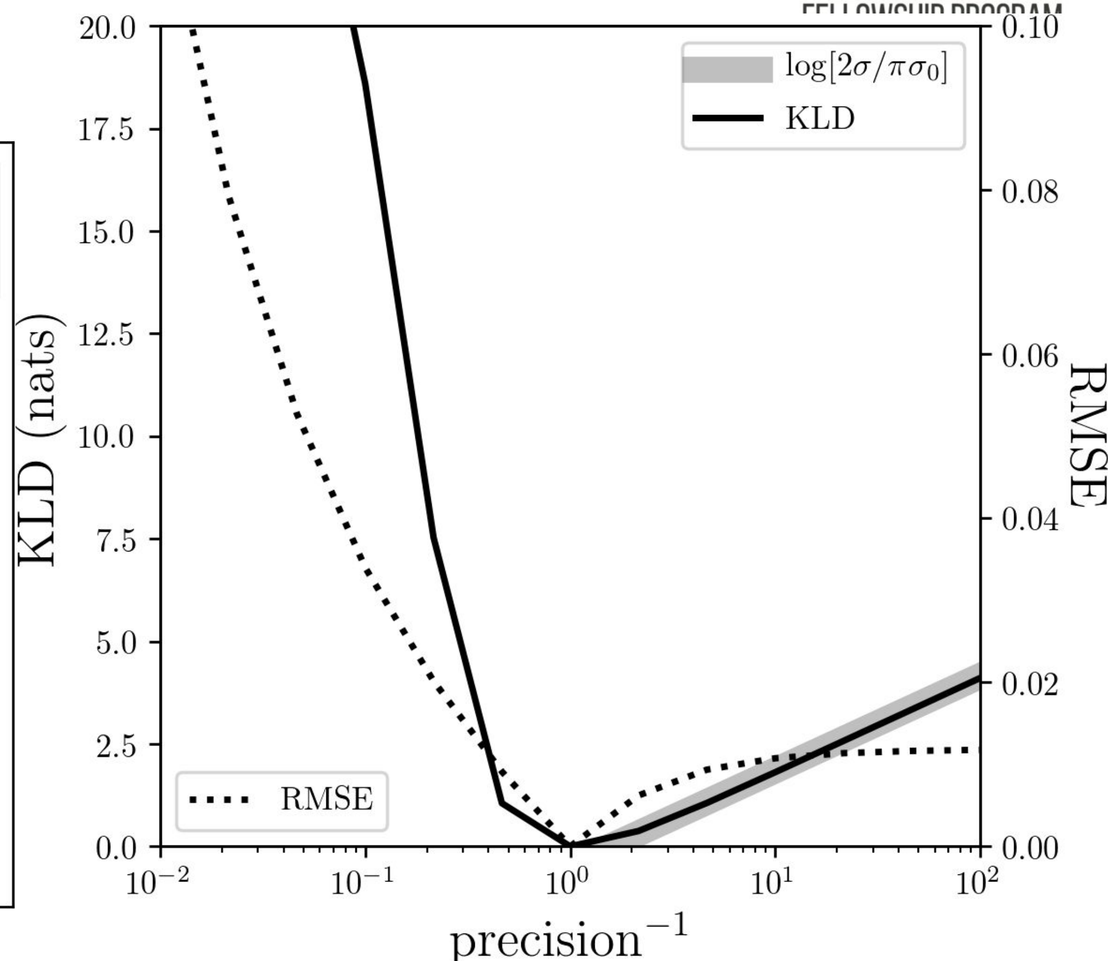
Gaussian example: $\mathrm{precision}^{-1} = \sigma/\sigma_0$



Interpret the asymptotic behavior of the metrics.





$$\text{precision}^{-1} = \sigma/\sigma_0$$

arXiv:1806.00014

Gaussian example: $\text{tension} = \frac{|\mu - \mu_0|}{\sqrt{(\sigma_0^2 + \sigma^2)}}$

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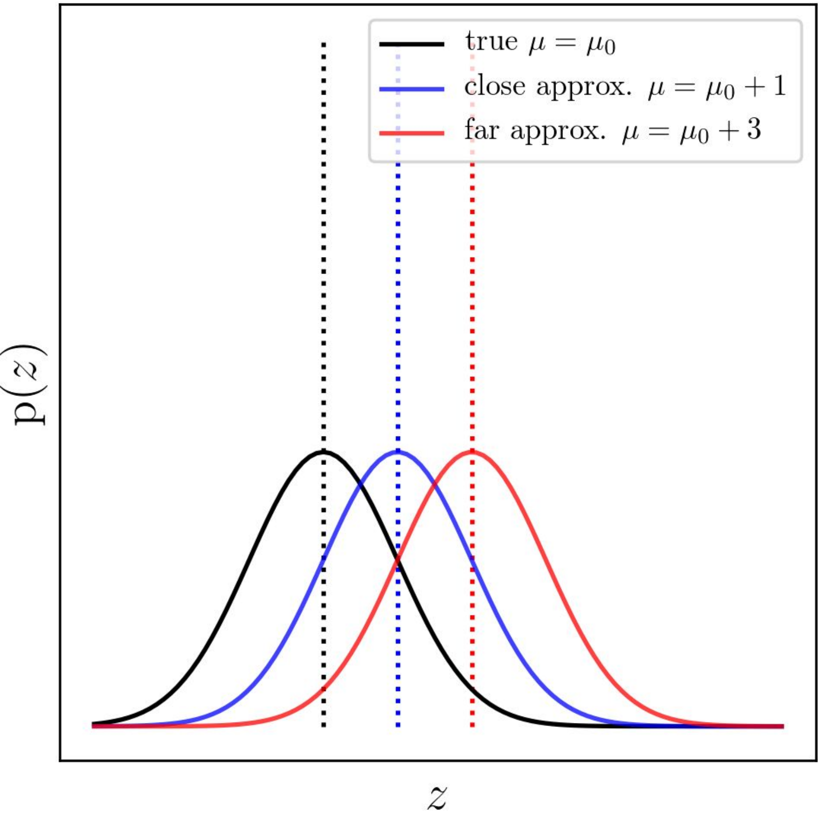

credit:A.I.Malz

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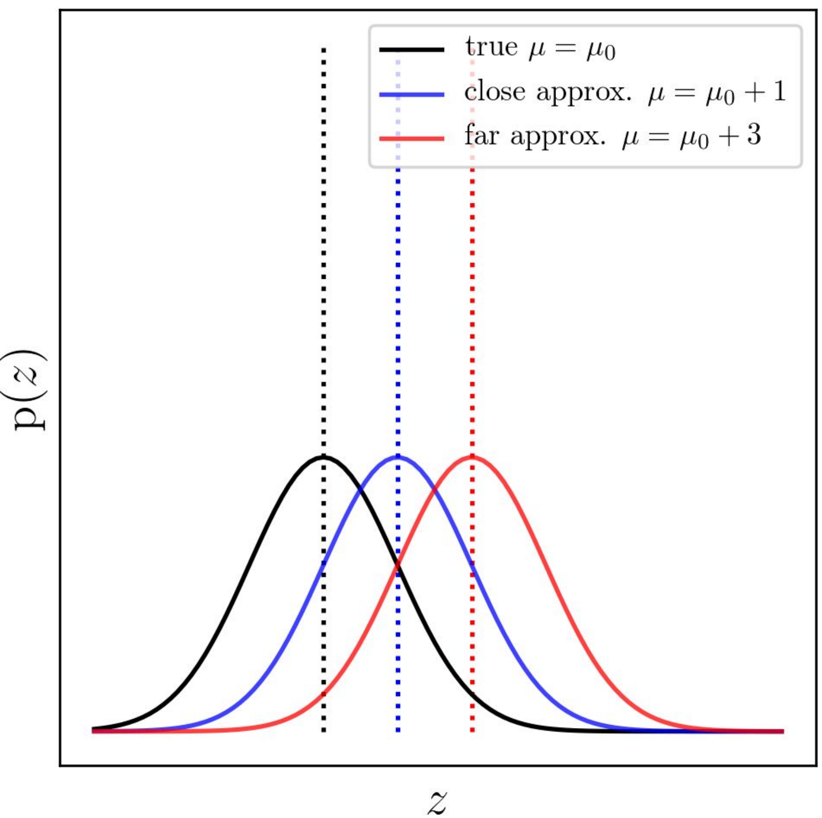

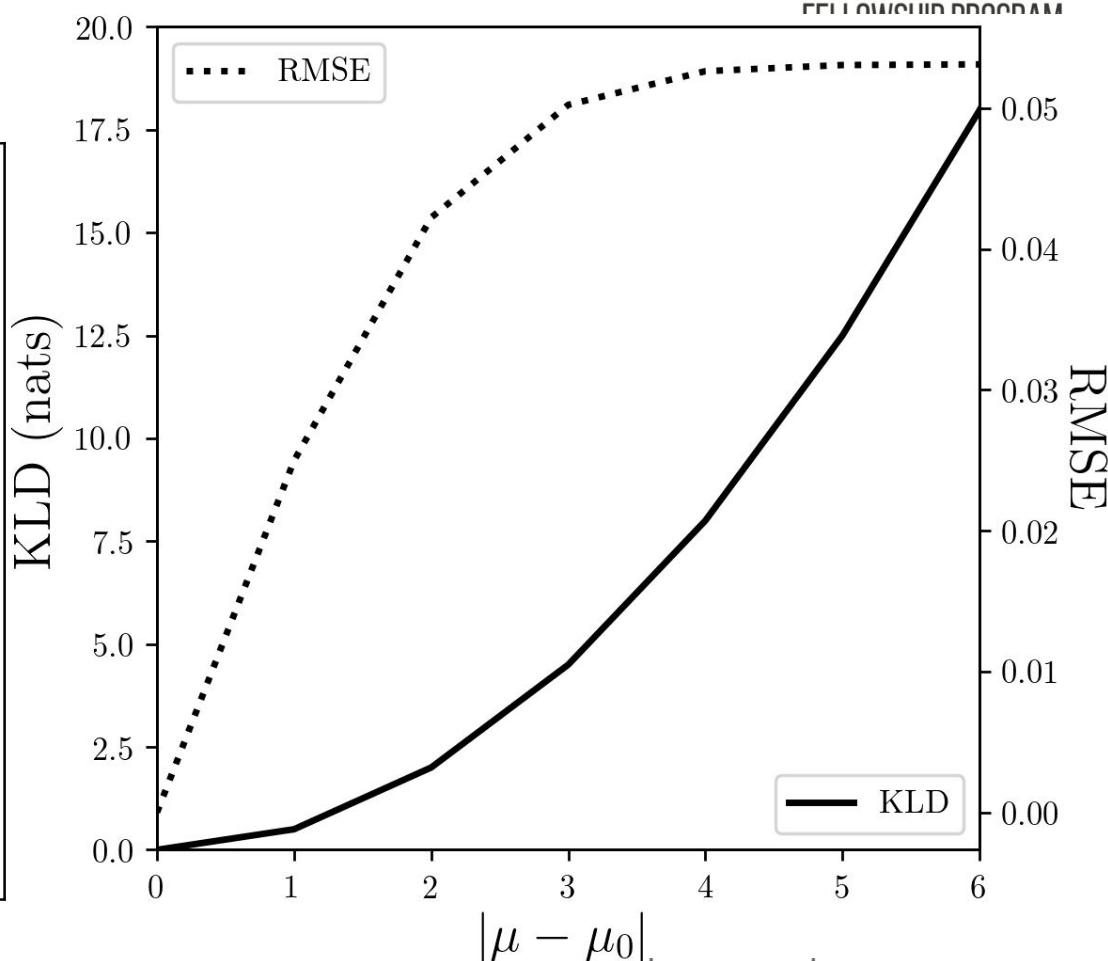

$$\text{tension} = \frac{|\mu - \mu_0|}{\sqrt{\left(\sigma_0^2 + \sigma^2\right)}}$$

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What does this have to do with Bayesian model comparison?

Example: PZ DC1

The PZ DC1 experiment

Motivation: identify the best photo-z posterior code for LSST-DESC

Data: cosmological redshifts & photometry catalog painted on N-body simulation

Control: idealized, shared prior information

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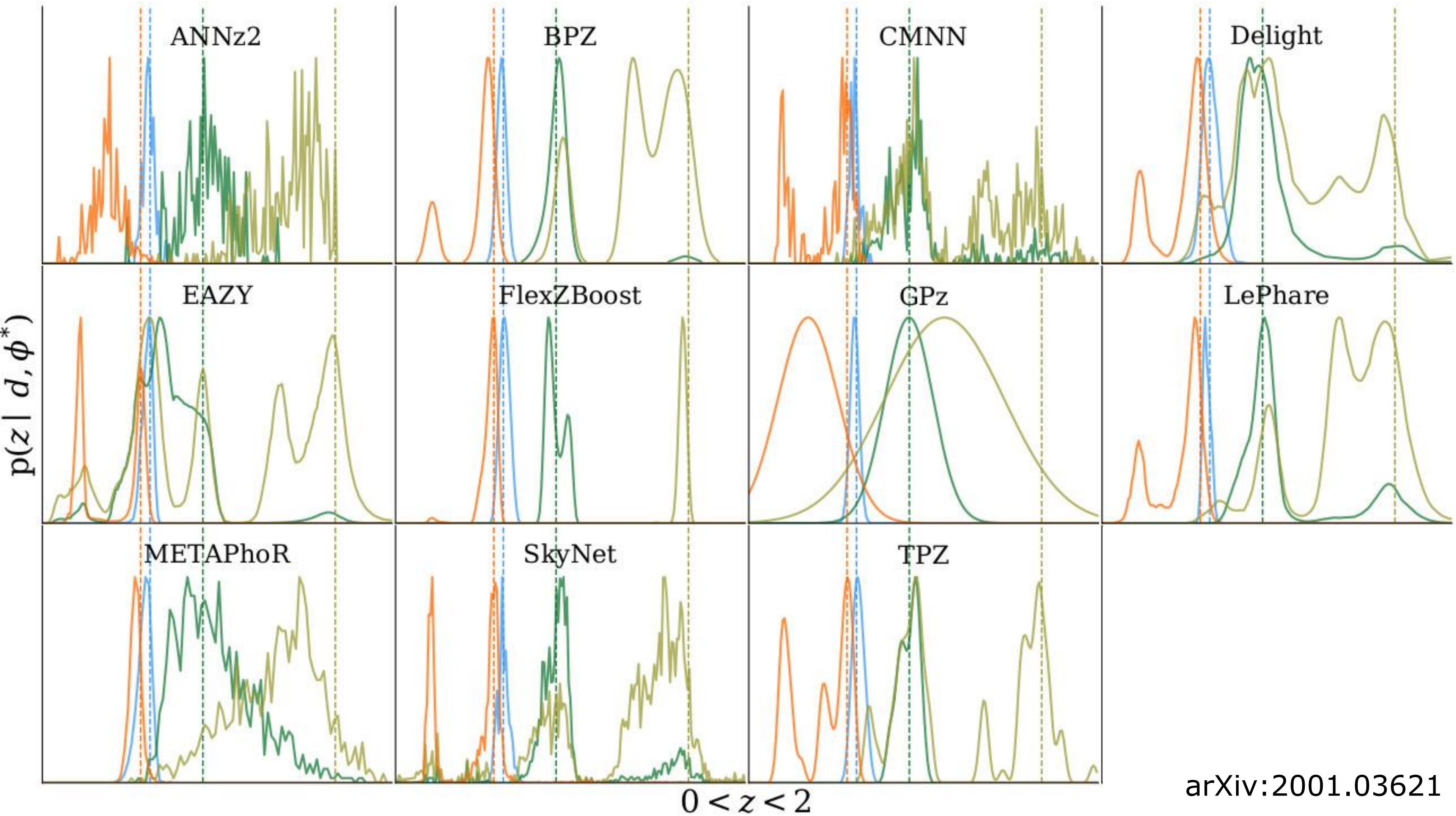

arXiv:2001.03621

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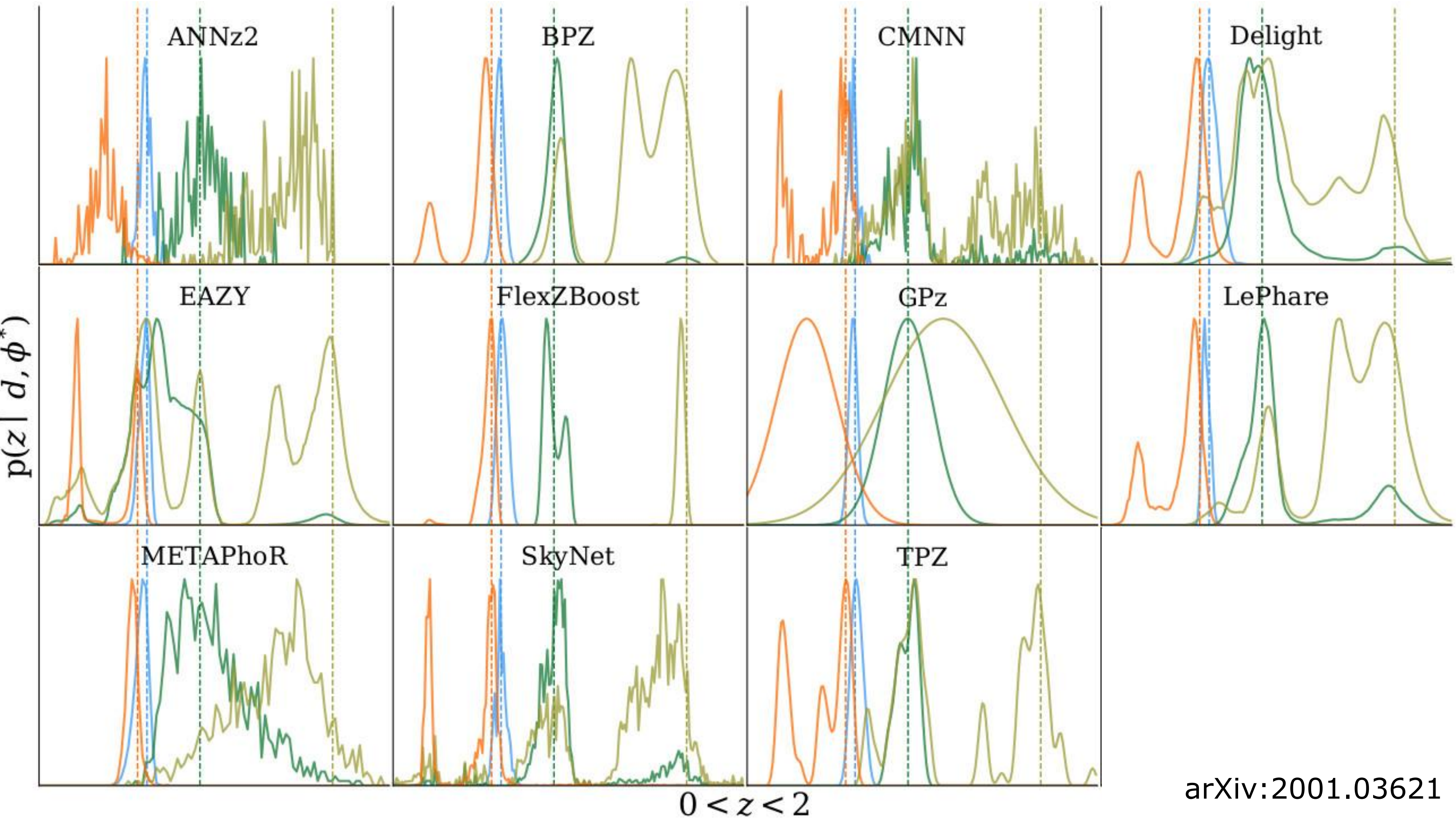

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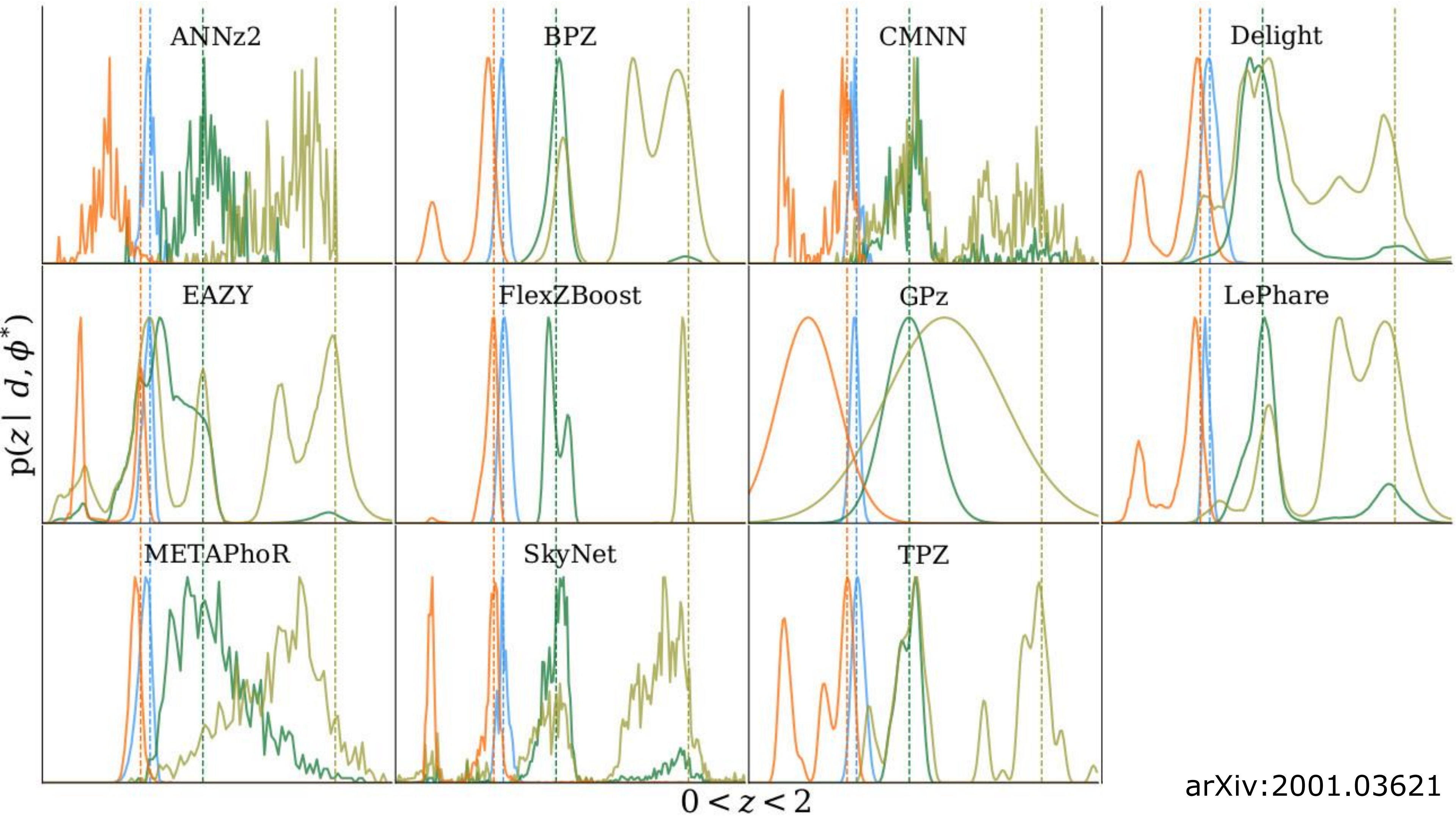
The photo-z posterior estimator is a model, distinct from the assumed prior.



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Probability Integral Transform (PIT)

$$P(\mathrm{PIT} \equiv \mathrm{CDF}[\hat{p}, z_{true}])$$

Sketch the PIT histogram of an ideal photo-*z* posterior estimator.

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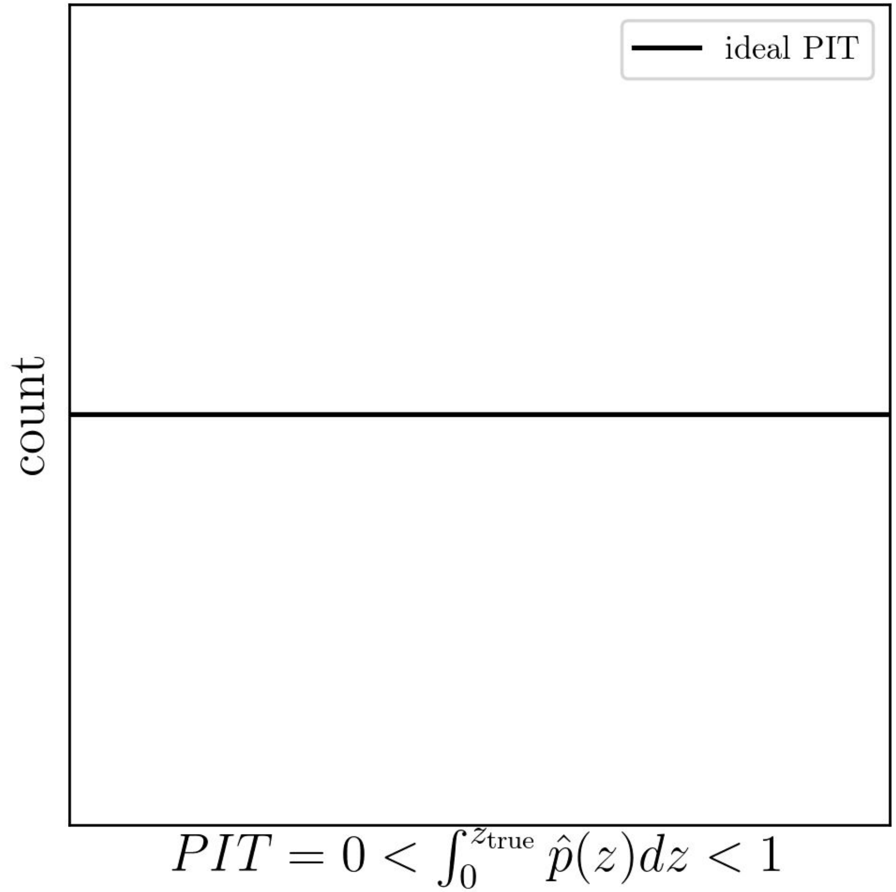

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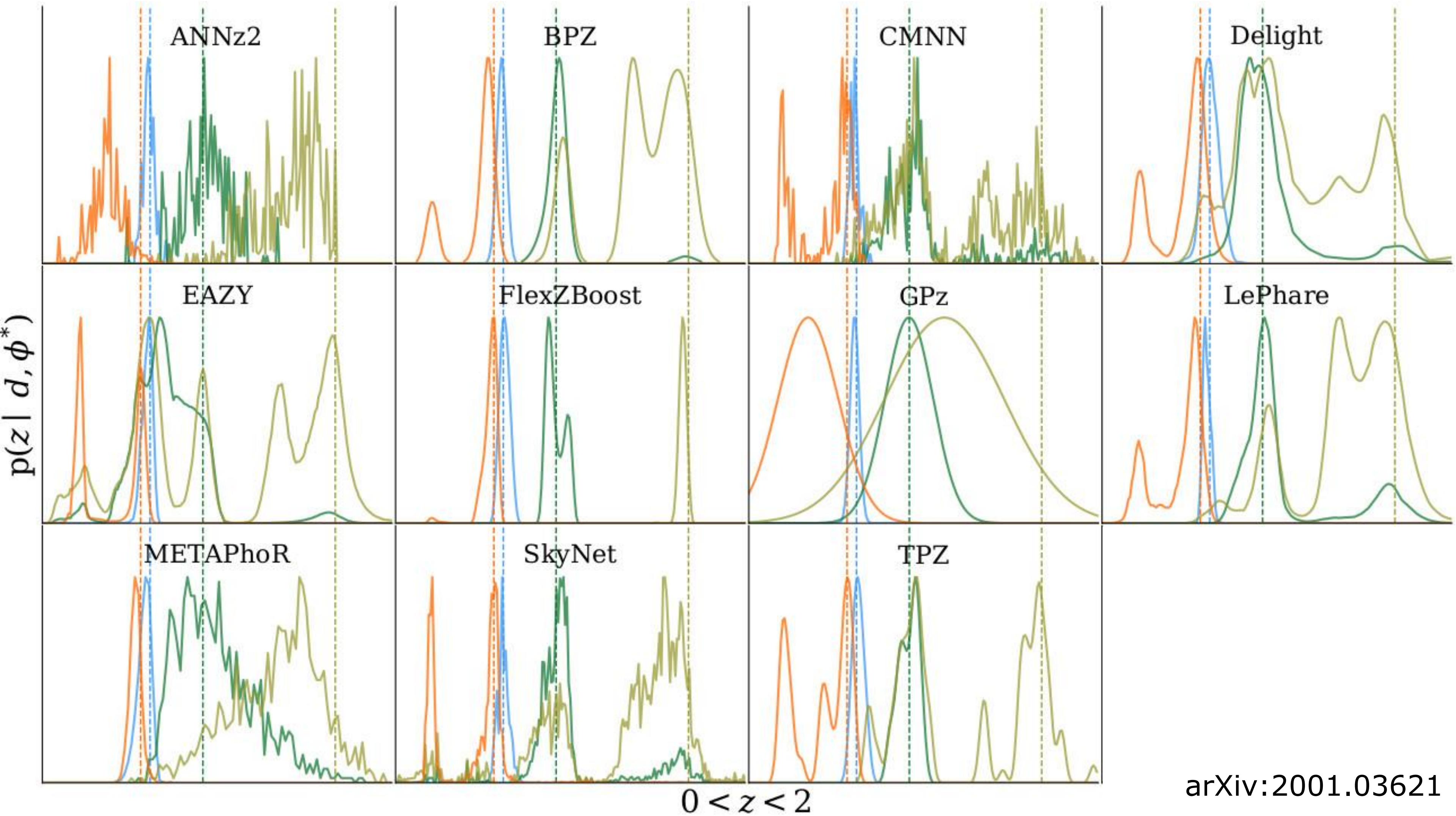
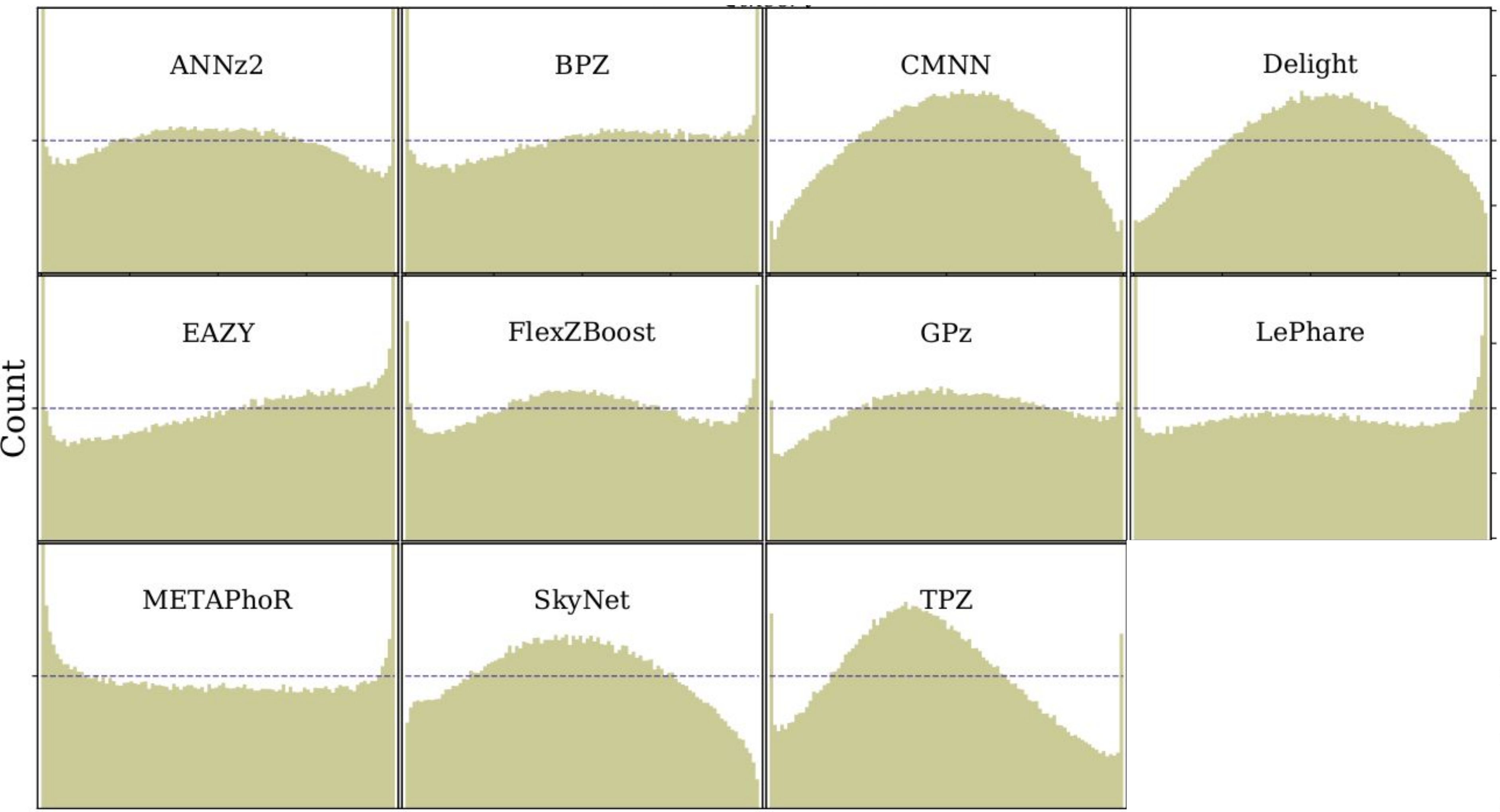
What might the PIT look like for these photo-*z* posterior estimators?



arXiv:2001.03621

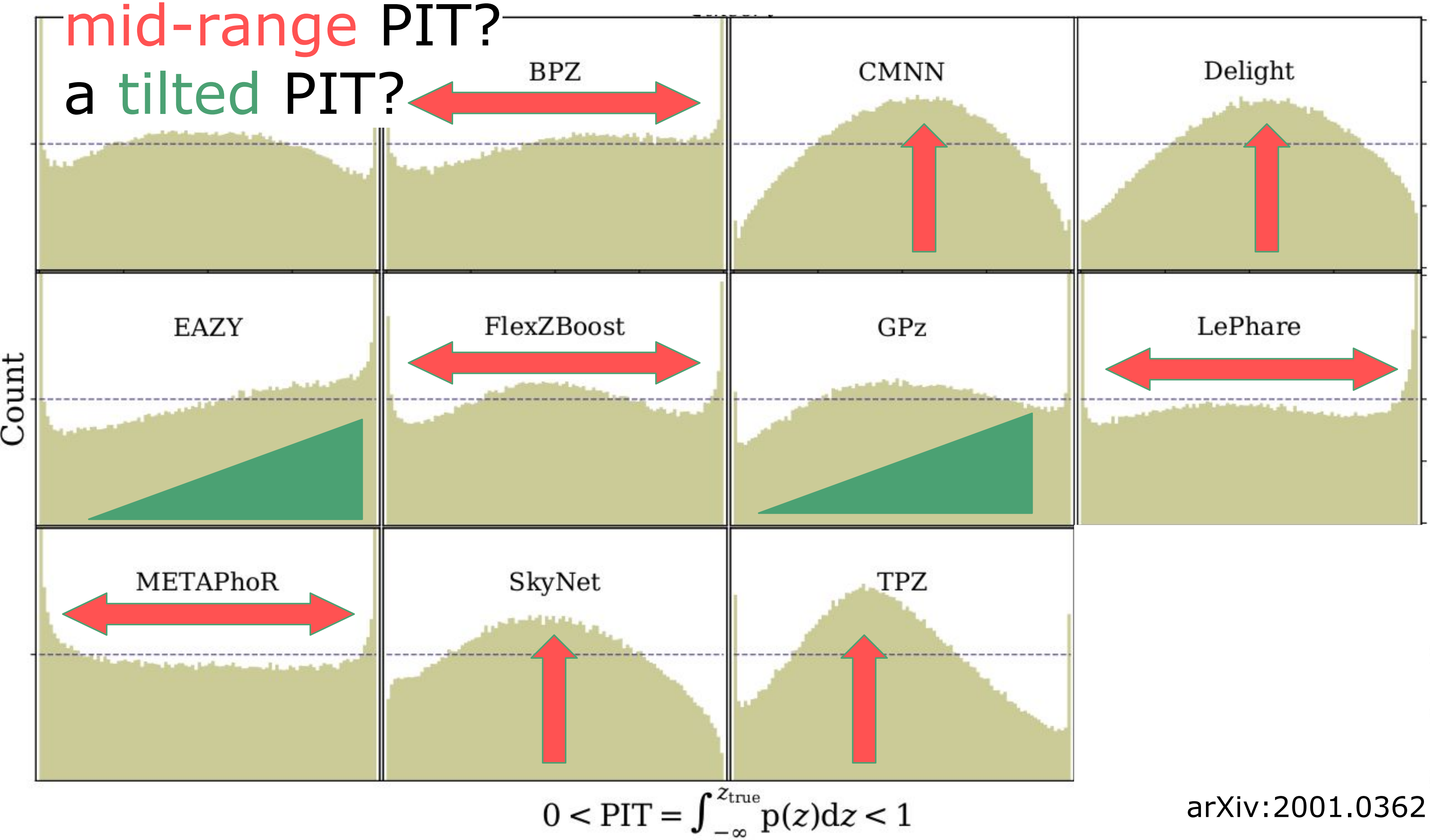
Photo-z posterior ensemble metrics



$$0 < \mathrm{PIT} = \int_{-\infty}^{z_{\mathrm{true}}} \mathrm{p}(z)\mathrm{d}z < 1$$

arXiv:2001.03621

What causes over-representation at extreme PIT? over-representation at mid-range PIT?
a tilted PIT?



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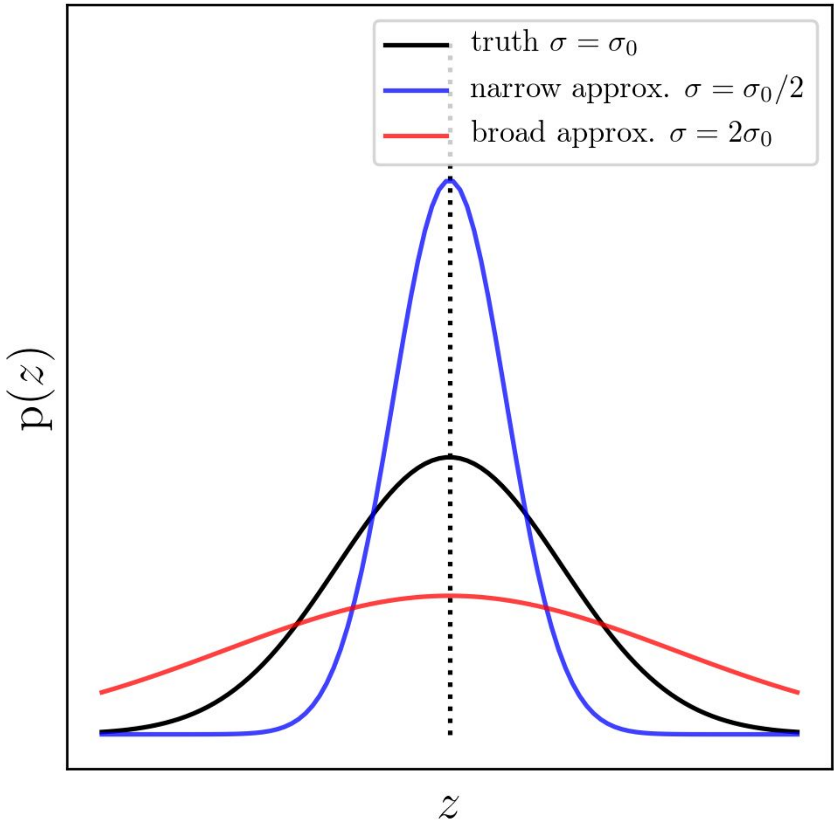
(Hint: Imagine the PIT histograms in the Gaussian example.)



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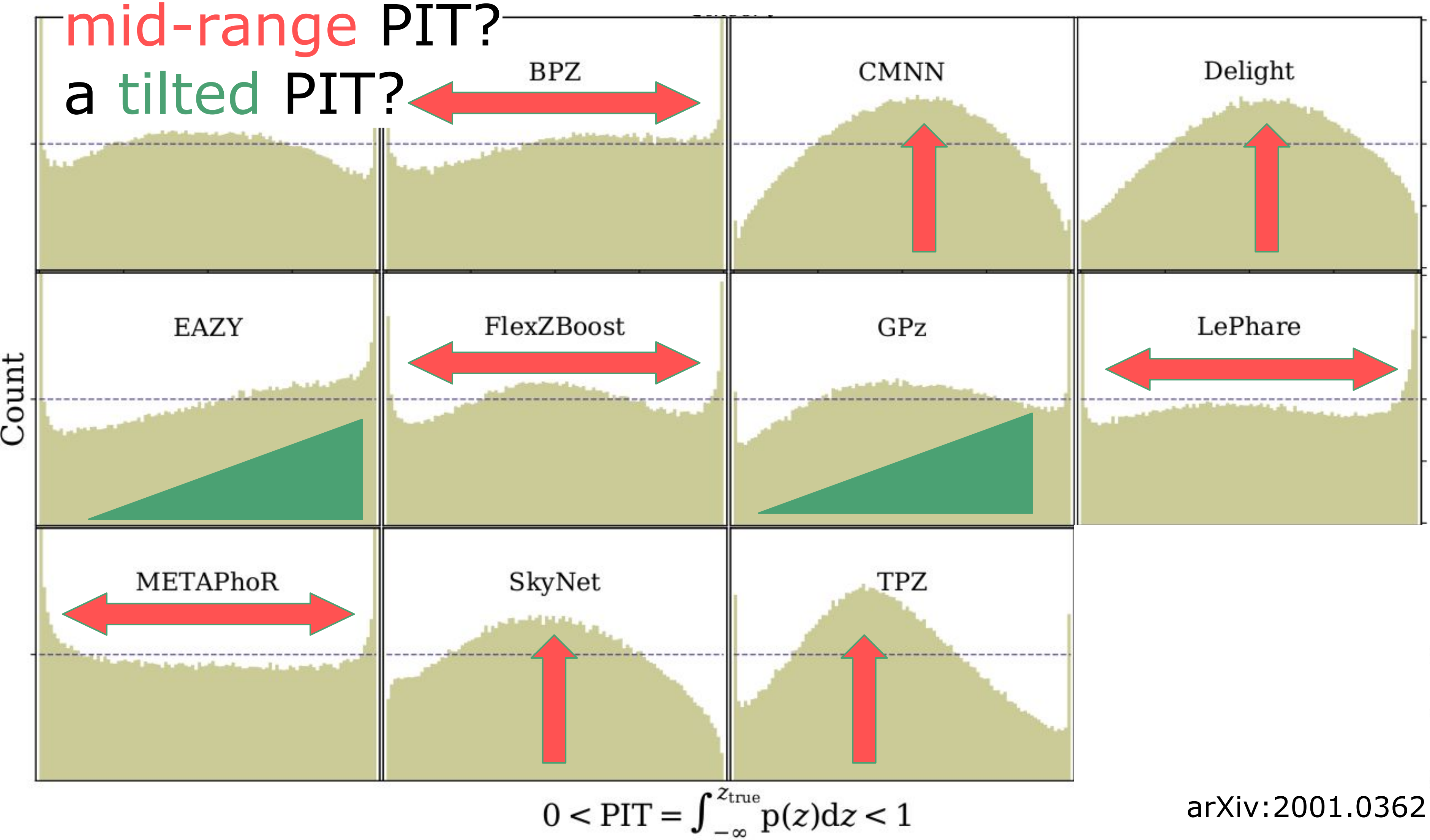

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Quantile-quantile (QQ) Plot $\sim \int p(PIT) dPIT$

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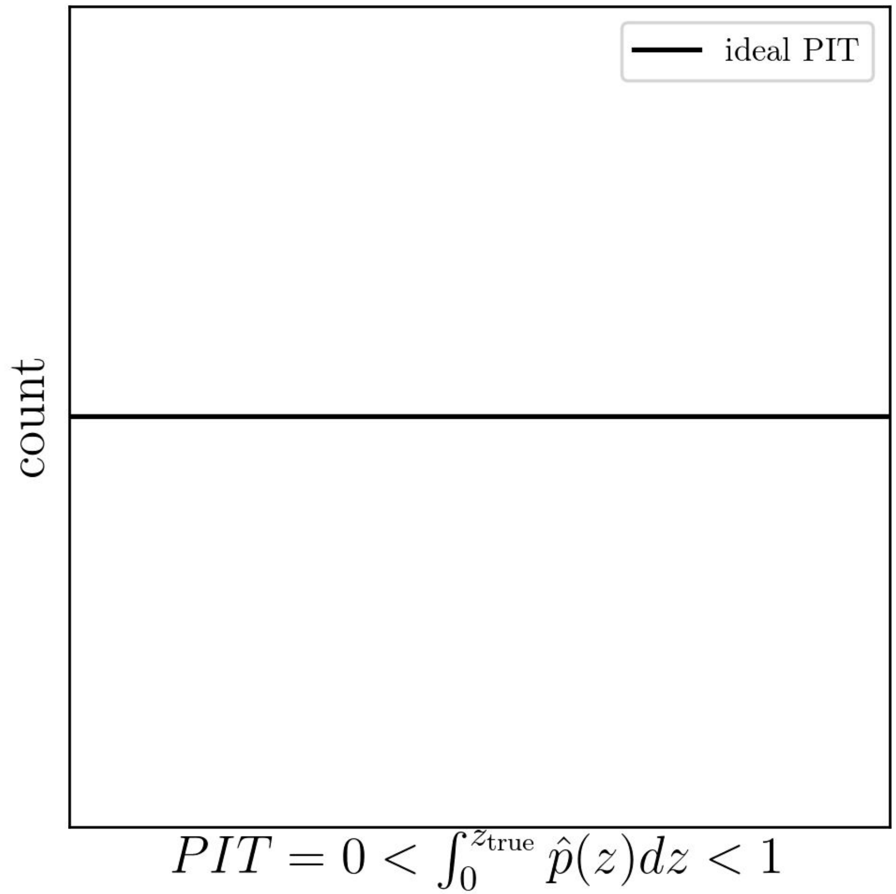
Sketch the Q-Q Plot of a hypothetical perfect photo-*z* estimator.



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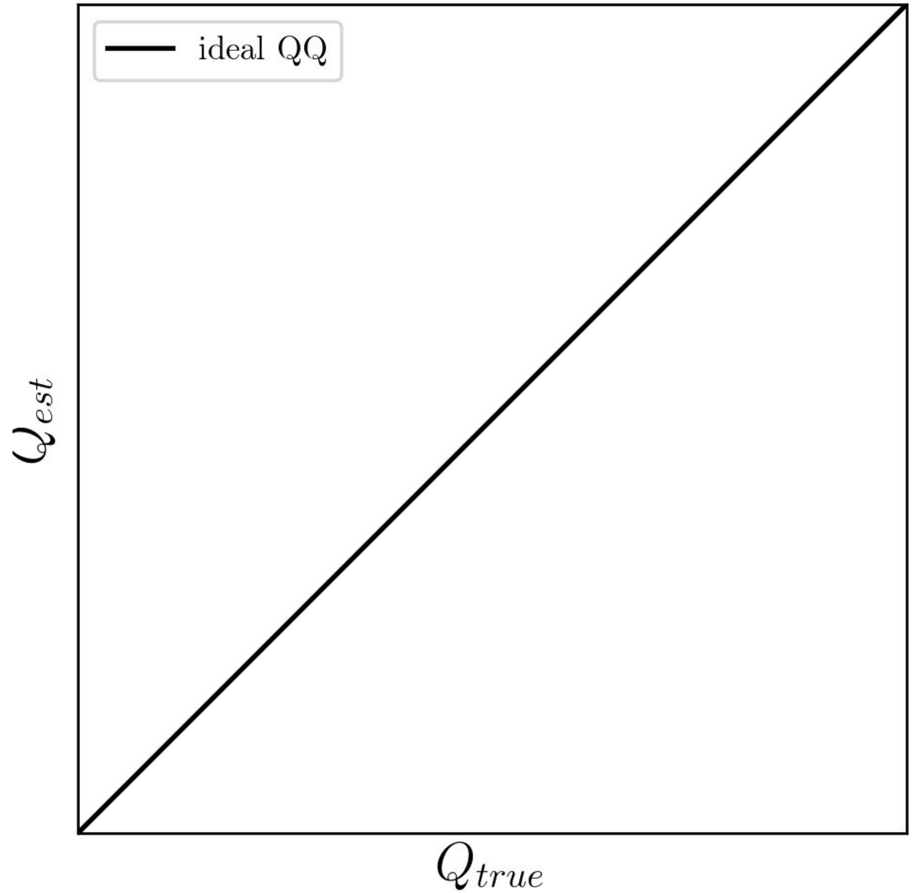

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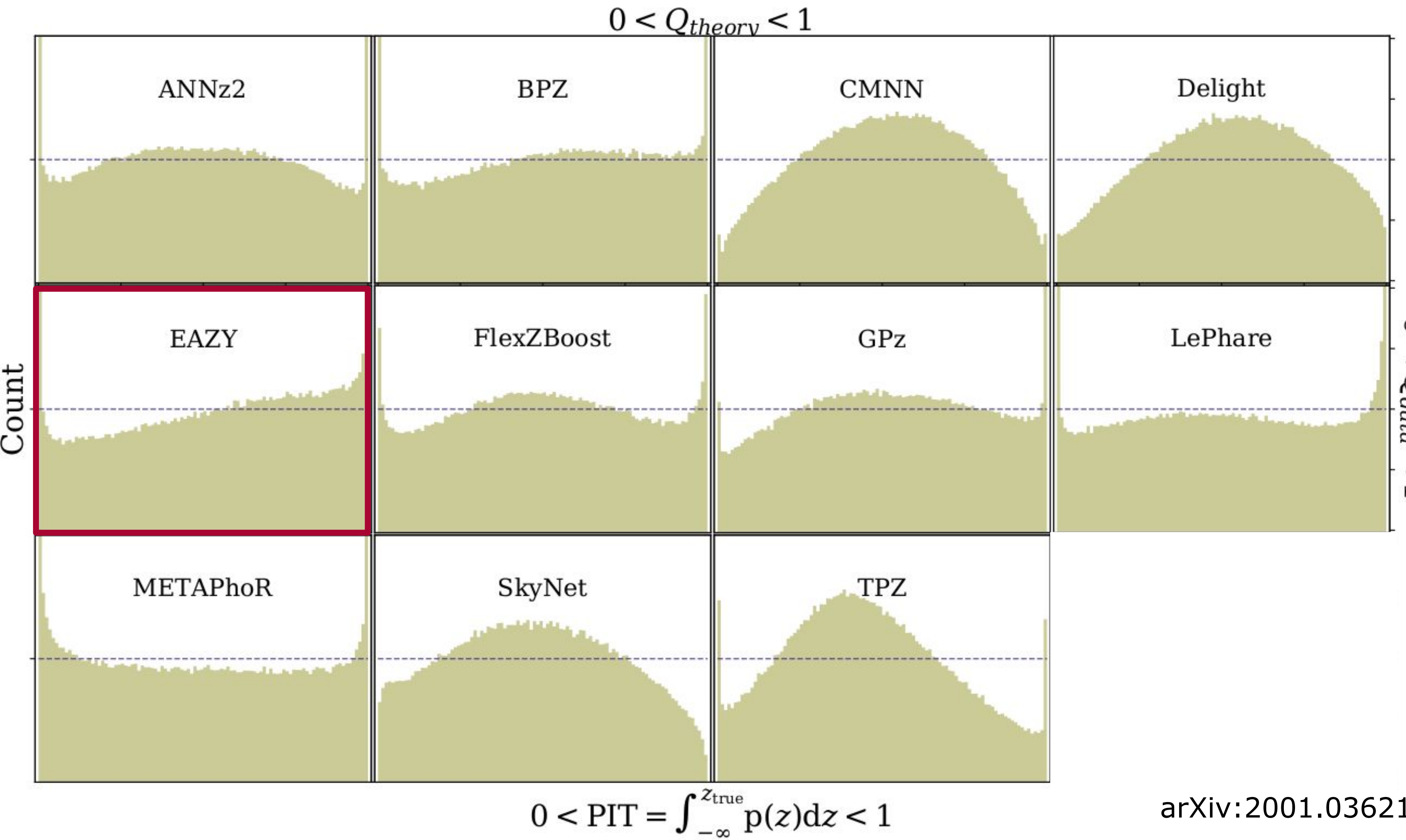
Sketch the QQ plot: $\sim\int p(PIT)dPIT$



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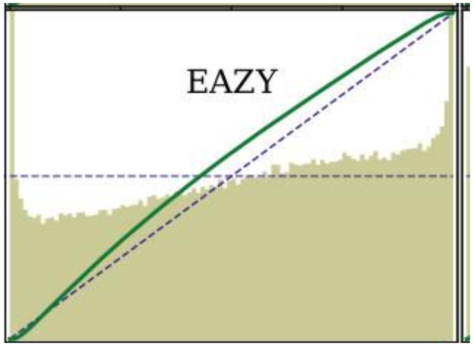
Sketch the QQ plot: ~$\int p(PIT)dPIT$



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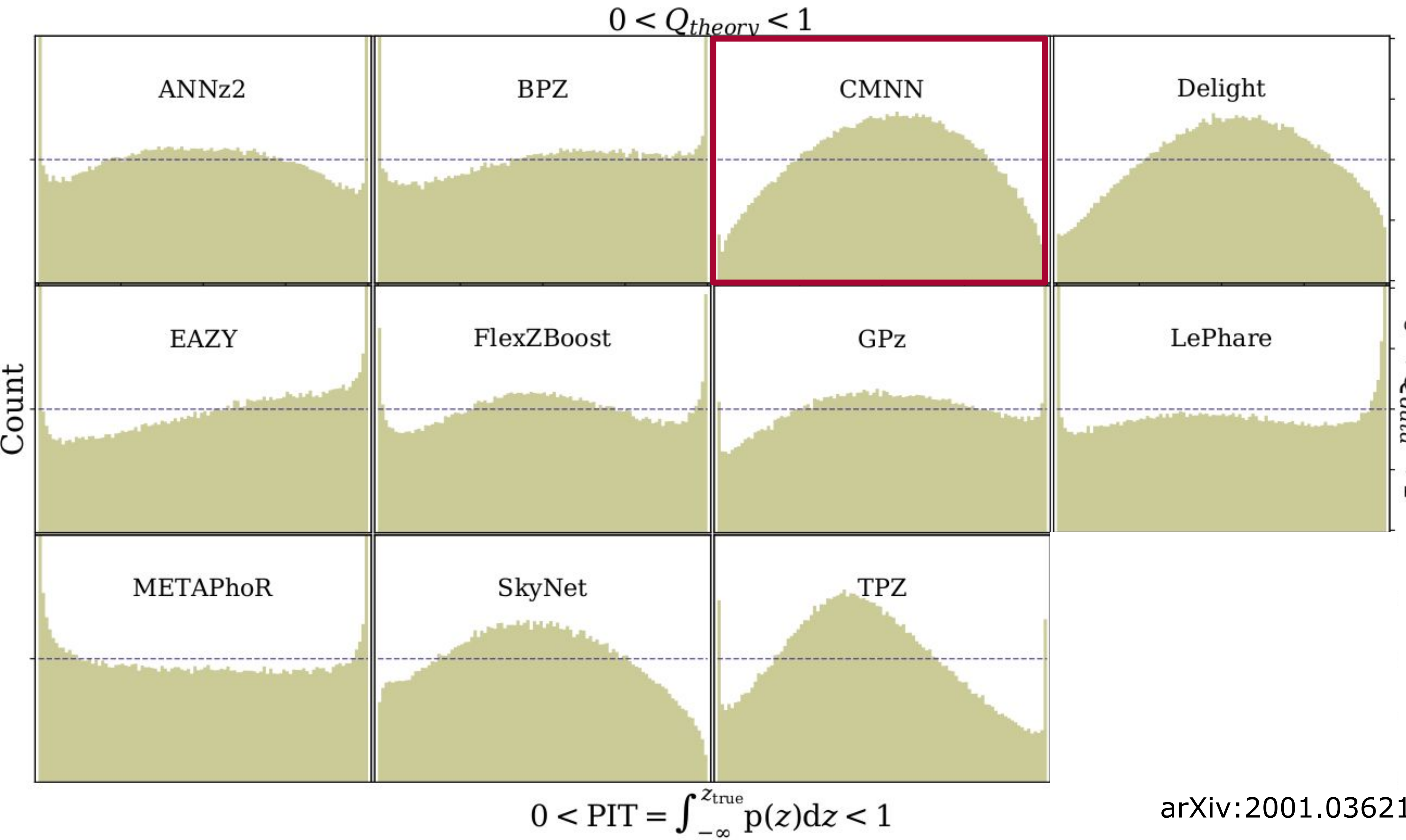

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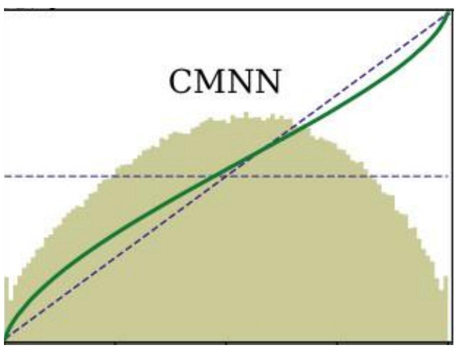

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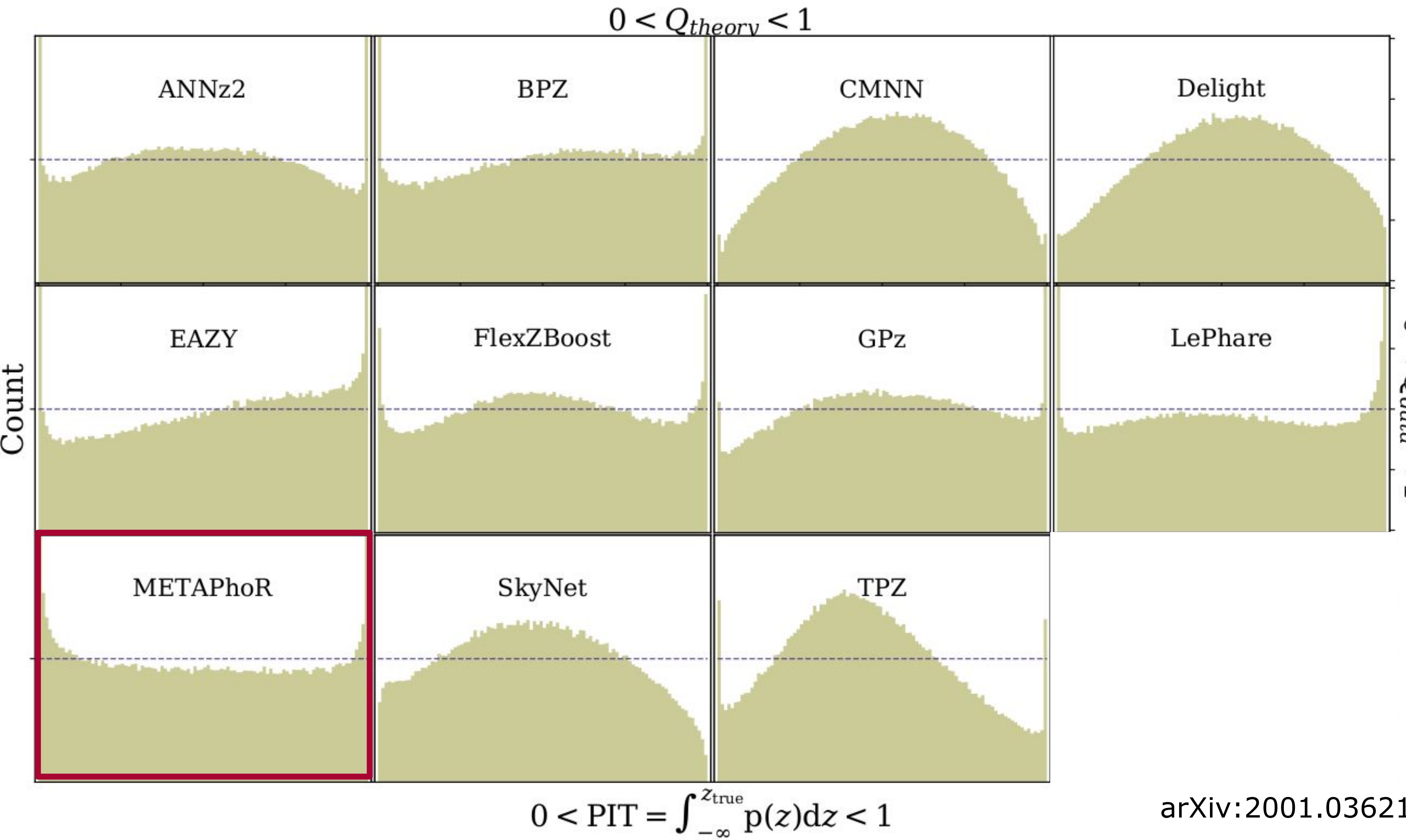

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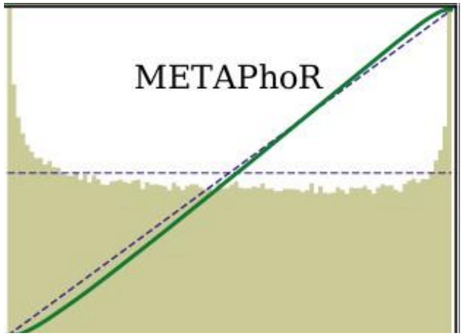
Sketch the QQ plot.~$\int p(PIT)dPIT$



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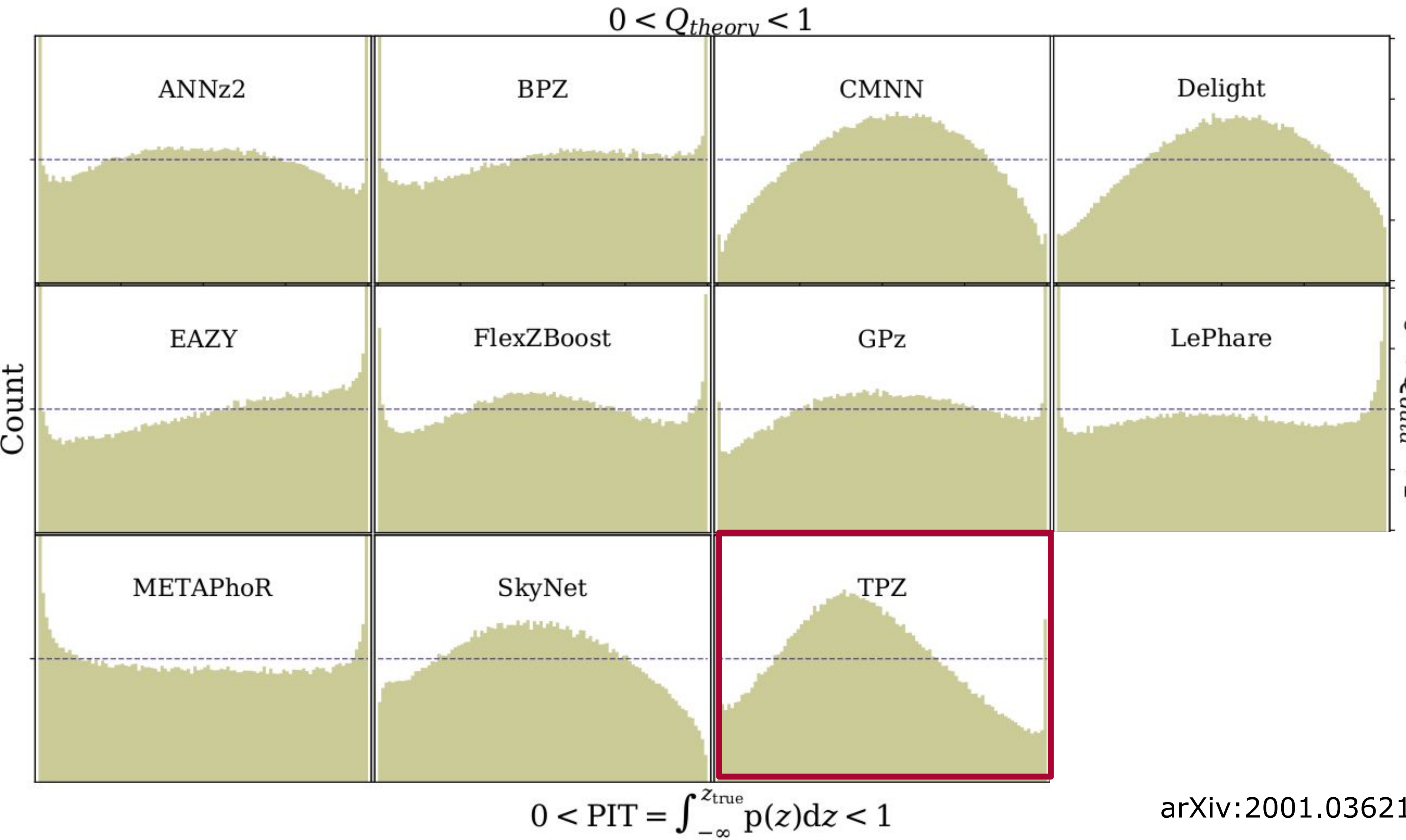

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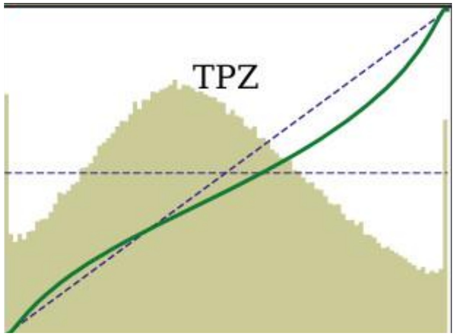

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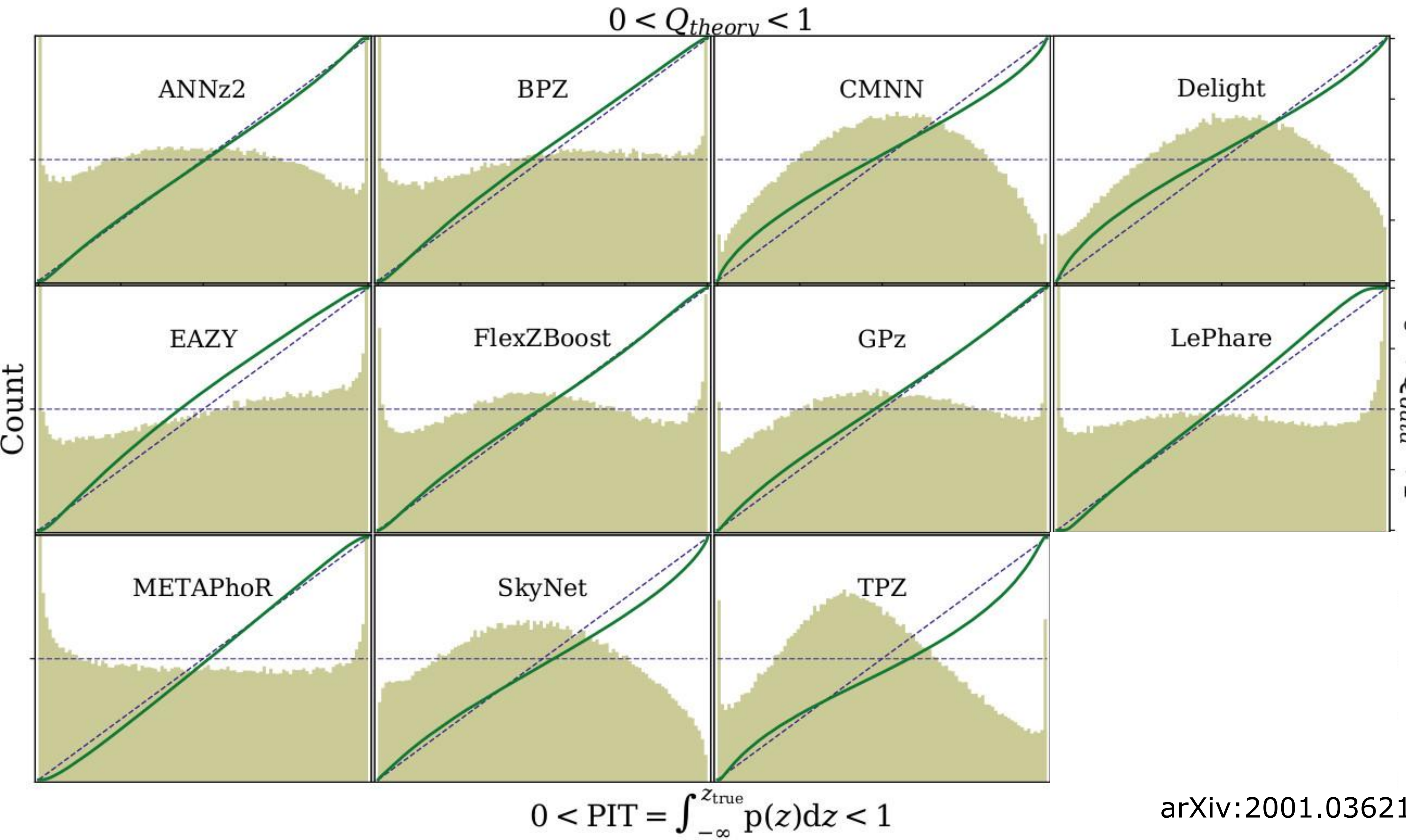

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Where is this going?

These metrics tell us about ensembles of 1D PDFs that correspond to different models.

In the problem session, we'll go over samples from 2D PDFs that correspond to different models (and data).