Introduction to Astrometry

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Outline

- What is astrometry and why are accurate positions important?
- Astrometric formalism
- Complications - camera distortions, effects from sensor anomalies, atmospheric distortion, DCR

What is astrometry?

- Astrometry is the measure of the position of celestial bodies.

- Astrometry provides the exact positions of the motions of objects, which can be used to study their properties.

- Getting accurate positions in single-frame observations is essential for downstream processing of the data.

Astrometry's Importance for Data Processing

(A non-comprehensive list)

- As Yusra will discuss, a lot of data processing involves adding or subtracting images
  - In order to do this, the images need to be aligned using the astrometric solution.
- Misalignment leads to
  - Dipoles artifacts when you subtract images that are not perfectly aligned - this leads to artifacts when searching for variable objects.
  - Bias in forced photometry measurements
  - Smearing out the shape of objects.
- On a rougher scale, the astrometric solution is needed to match to other catalogs, which is need for photometric calibration.

Astrometry's Importance for Science

(A non-comprehensive list)

- Solar System - trajectories of solar system objects
- Galactic - determining distances to stars using parallax, measuring absolute luminosities, general relativity, kinematics of stellar groups
- Extragalactic - position and shapes of galaxies are used for lensing, etc.

Important Terms

- Right Ascension (RA) - Azimuthal position; measured in degrees or hours, minutes, and seconds. I only use degrees/radians. Ranges from 0 to 360 degrees.
- Declination (Dec) - Altitude measured from directly overhead of the equator of the Earth. Measured in degrees. Ranges from -90 to +90 degrees.
- Proper motion - Observed motion of celestial objects (relative to very distant objects).
- Parallax - Apparent motion of objects due to Earth's rotation around the Sun.
- International Celestial Reference System (ICRS) - base of reference system
    - defined by positions of a specific set of extragalactic objects

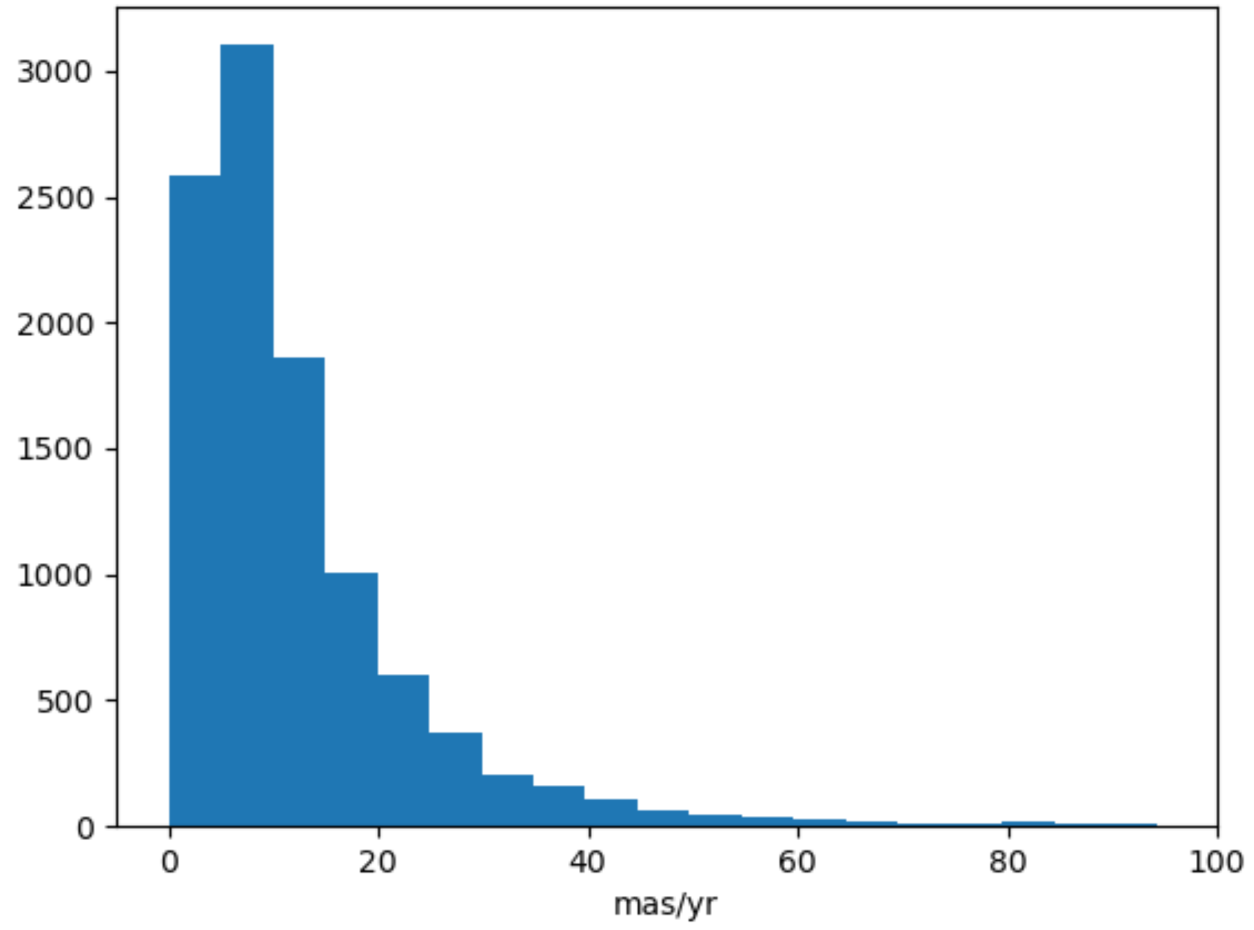
The World Coordinate System (WCS)

- The WCS gives the mapping between pixel coordinates and sky coordinates
    - The WCS can be a series of mappings, each of which has its own format (i.g. a polynomial function or a fixed offset)
    - The standard file format for astronomical images is FITS (Flexible Image Transport System).
    - There is a FITS standard for encoding WCS equations, but Rubin WCSs will not necessarily conform to this.
- This is one of the main products of the astrometric solution!

What scales are we talking about?

For a sun-like object at...	100pc	1kpc	10kpc
Parallax amplitude	10 mas*	1 mas	0.1 mas
Proper motion of 20 km/s (disk star)	41 mas/yr	4 mas/yr	.4 mas/yr
Proper motion of 200 km/s (halo star)	410 mas/yr	41 mas/yr	4 mas/yr
Angular Diameter	100 μas	100 μas	1 μas

Proper motion distribution from a subset of the Gaia catalog

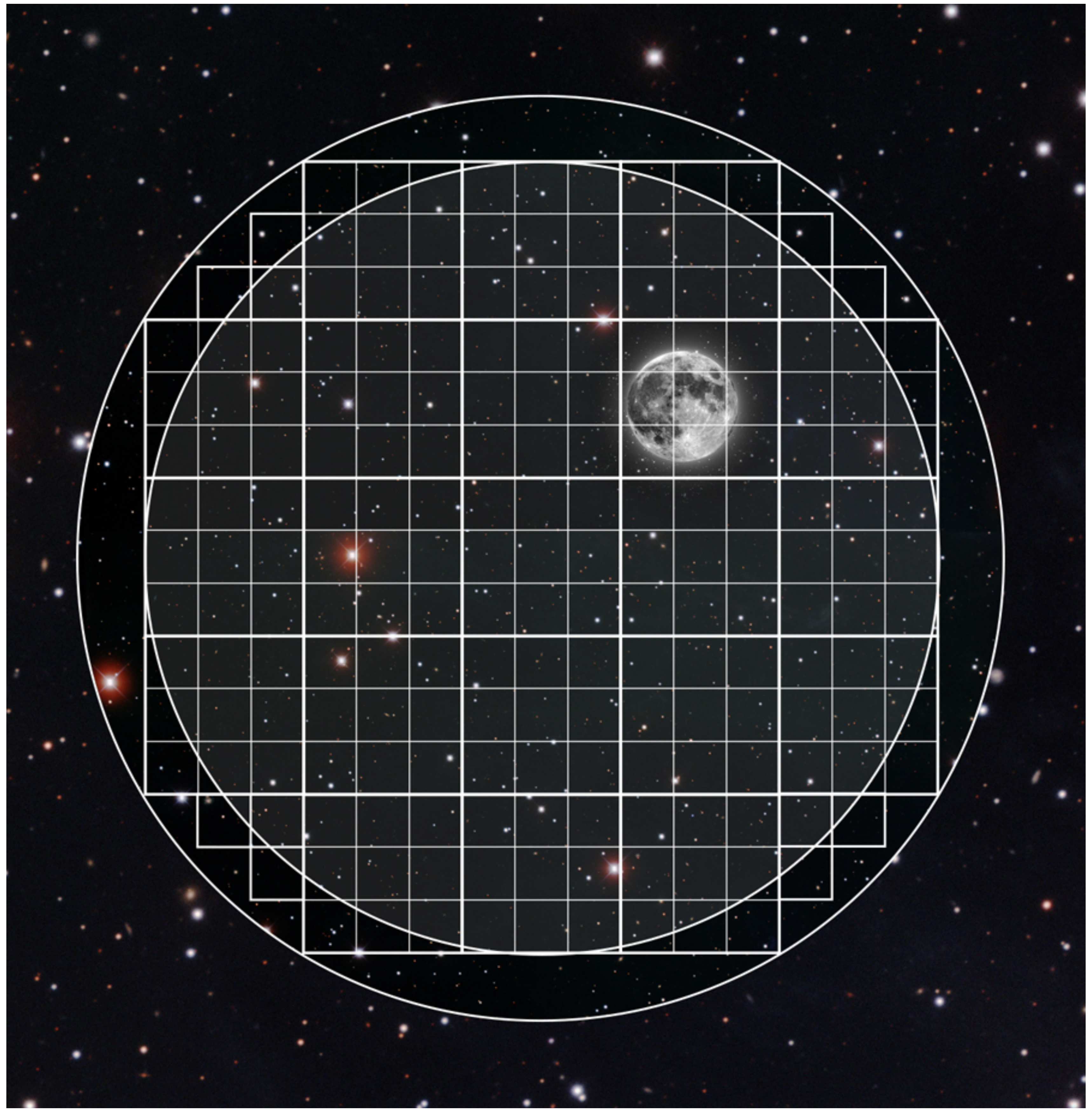


* 1 mas = 1 milliarcsecond = 1 / (1000 * 60 * 60) degrees

What scales are we talking about?

- LSST Science Requirements:
    - The RMS of the relative repeatability must not exceed 10 mas
    - Proper motion accuracy: 0.2 mas/yr
    - Parallax accuracy: 1.0 mas

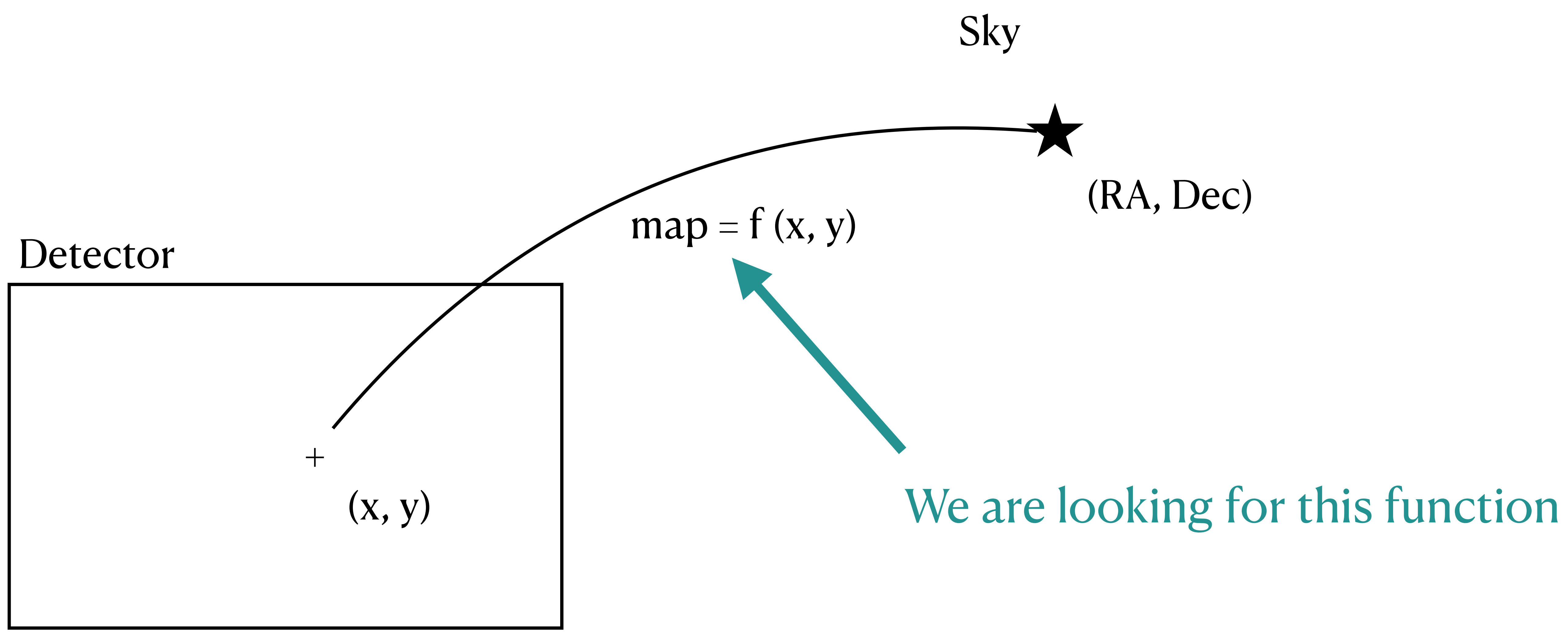
- LSST Field of View Diameter: 3.5 degrees
- Pixel size: 200 mas / pixel
- LSSTCam PSF 1-sigma: ~300 mas
- Single silicon atom: .01 mas



How do we get to the required precision?

Fitting an astrometric model

1. Identify isolated point sources on individual visits and measure their positions.
2. Match point sources that occur in multiple visits to each other and to a reference catalog if possible.
3. Use these matched observations to fit a map from pixel coordinates (x, y) to world coordinates (RA, Dec).



Step 1: Measuring Positions

- Positions are measured using a centroiding algorithm
- The centroiding algorithm is weighted using the PSF (following Cramer-Rao theorem that Eric mentioned yesterday).
- Solve for x_0 and y_0 such that M_x and M_y equal zero:

$$\begin{bmatrix} M_f \\ M_x \\ M_y \end{bmatrix} = \int dxdy \begin{bmatrix} 1 \\ x - x_0 \\ y - y_0 \end{bmatrix} I(x, y)\ W(x - x_0, y - y_0)$$

where M_f, M_x, and M_y are the moments in flux, x, and y; $I(x, y)$ is the flux of the source at (x, y), and $W(x - x_0, y - y_0)$ is the weight, which in this case will be the model PSF.

Step 2: Matching sources to a reference catalog

- Given an initial guess for the WCS, match sources that are within a certain radius of each other. Only want isolated stars, so through out cases where there is ambiguity about what source belongs to which group.
- An external reference catalog allows the WCS to be tied to absolute positions.
    - Gaia makes this easy — the latest release has ~1.8 billion stellar positions
    - Proper motions and parallax for ~1.46 sources.
    - Limiting magnitude in g-band of ~21.
    - Covers the whole sky
    - Gaia removes position degeneracy, but individual measurements are not the "Truth"— LSST will actual be more precise than Gaia over its lifetime and can go > 4 mag deeper.

Step 3: Fit The WCS!

- Recall what makes up a WCS—a series of mappings (i.e. functions) from x and y to RA and Dec.

  - So we're looking for something like

$$\mathrm{RA}(x, y) = F^{RA}_{visit} \circ F^{RA}_{detector}(x, y)$$

$$\mathrm{Dec}(x, y) = F^{Dec}_{visit} \circ F^{Dec}_{detector}(x, y)$$

- So, how do we know what these functions should be, and how do we fit them?

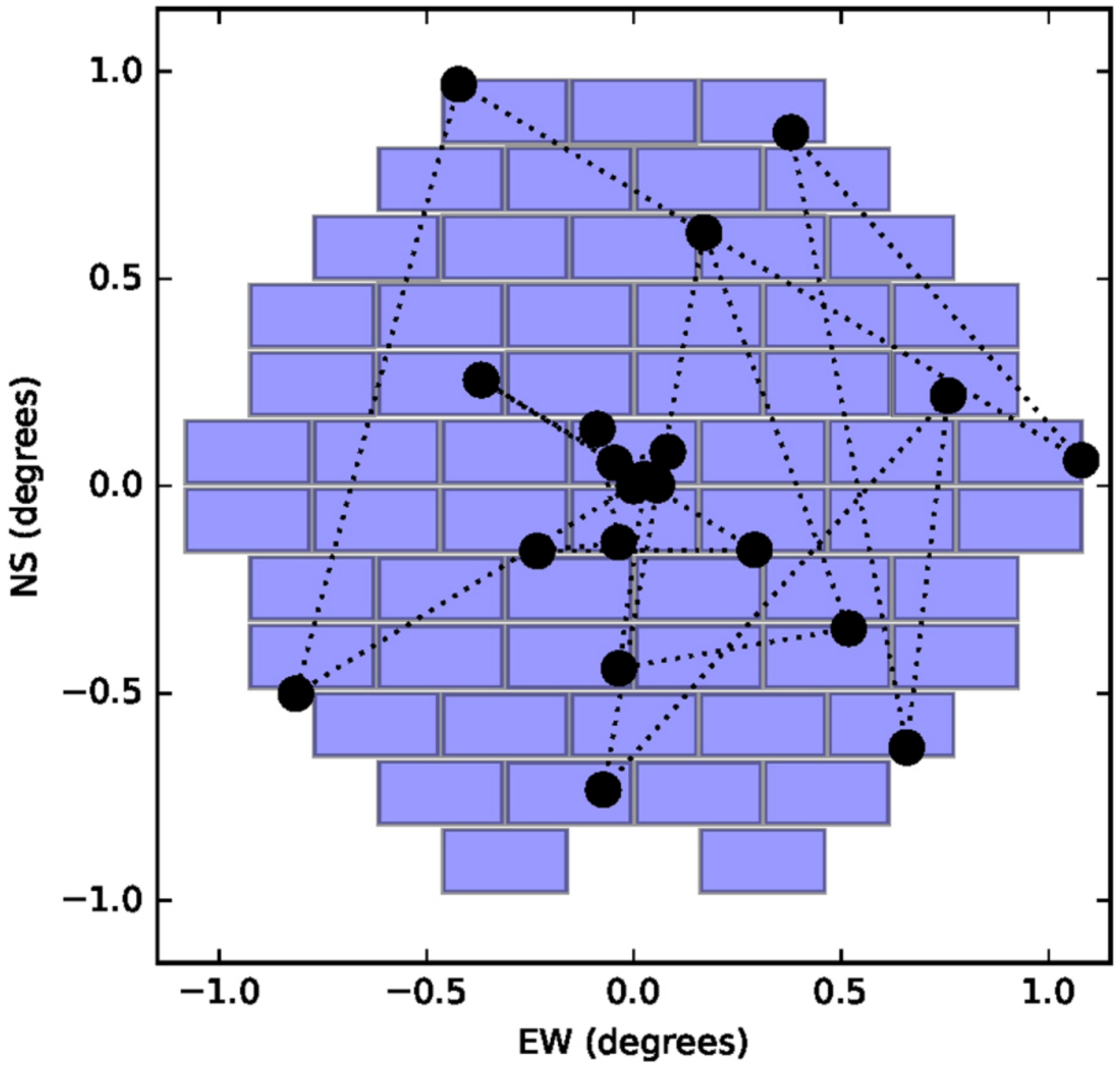
What is the WCS trying to model?

What affects a photon on the trip from its origin to the pixel?

- Gravitational deflection by solar system members
- Aberration due to Earth's motion
- Refraction in Earth's atmosphere
    - * Wavelength dependent* - (DCR)
- Stochastic deflection by turbulence in the atmosphere
- Projection of the spherical sky onto a flat focal plane about the axis of the telescope
- Distortions due to telescope optics
    - * Wavelength dependent*
- Position of the CCDs in the focal plane
- Deflection of photo-electrons by lateral electric fields in CCDs
- Departures of the CCD pixels from a perfect square grid

How can we fit such a complicated model?

- Fitting the astrometric model relies on the fact that there is a true position for each star (potentially involving proper motion)
    - External consistency - a star has the same true RA and Dec as its match in the Gaia catalog (and there are ~10000 Gaia stars in every LSSTCam exposure)
    - Internal consistency - a star should have the same RA and Dec when measure on different parts of the focal plane in different exposures.
- Because we have thousands of stars per exposure, we can easily constrain a smoothly-varying model for the WCS that has 10s of parameters.



Bernstein+ 2017

What is the WCS trying to model?

A lot can be explained by simple polynomials

- Gravitational deflection by solar system members
- Aberration due to Earth's motion
- Refraction in Earth's atmosphere
    - **Wavelength dependent - (DCR)**
- Stochastic deflection by turbulence in the atmosphere
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Per-exposure polynomial model

Per-ccd polynomial model

*** Note that we don't try to separate out these effects in the model**

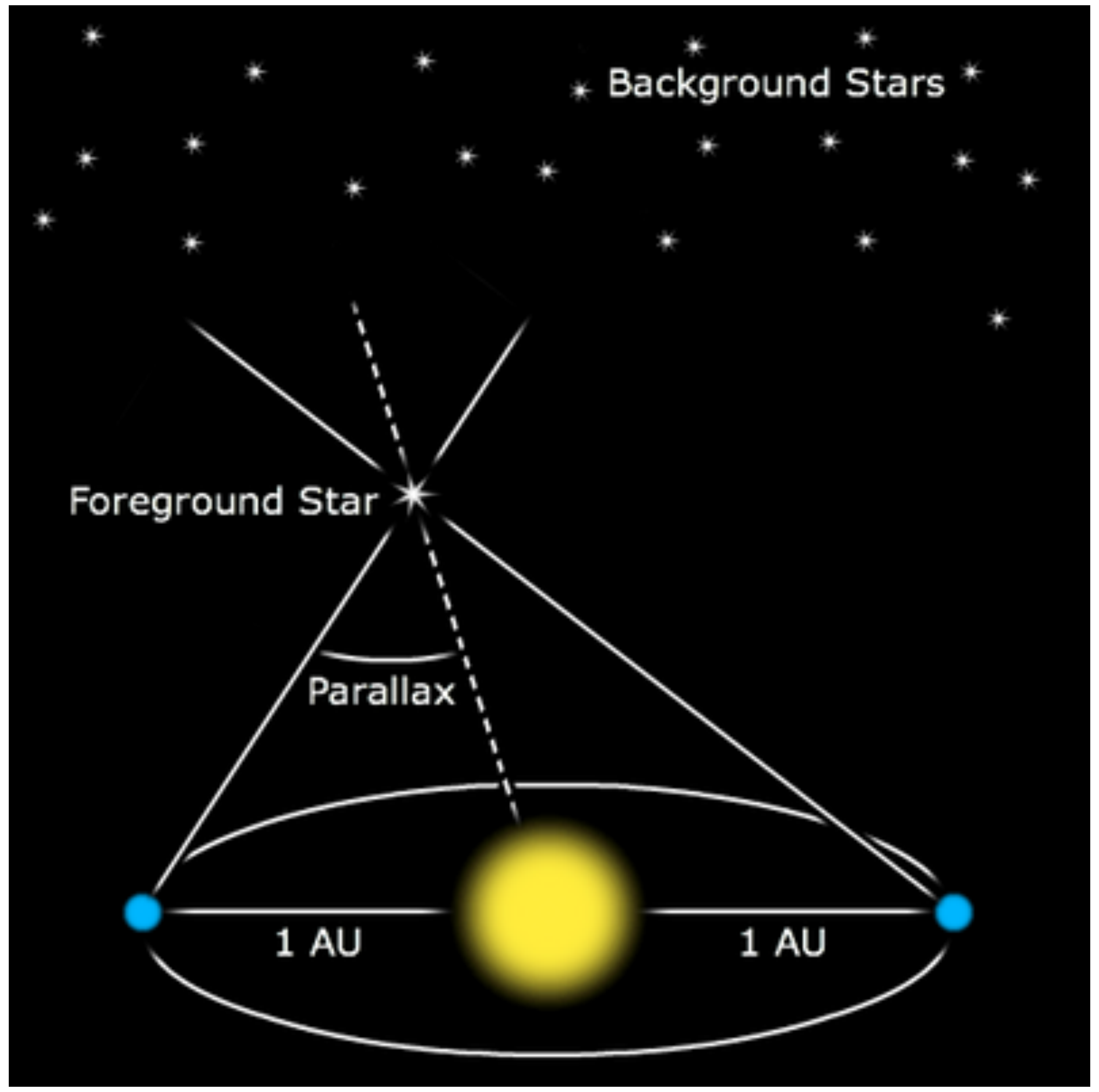
WCS model-fitting equation

$$\chi^2 = \sum_{visit} \sum_{det} \sum_{i} (\mathbf{S}_k - \mathbf{F}_{visit} \circ \mathbf{F}_{det}(x_i, y_i))\ \mathbf{C}_i\ (\mathbf{S}_k - \mathbf{F}_{visit} \circ \mathbf{F}_{det}(x_i, y_i))^T$$

- $\chi^2 = -2\log L + \text{const}$ is the quantity we are trying to minimize.
- $\mathbf{S}_i = [\text{RA}_i, \text{Dec}_i]$ is the true position of source i.
- (x_i, y_i) are the pixel coordinates of source i.
- $\mathbf{C}_i$ is the covariance matrix of source i.
- $\mathbf{F}_{det}$ is the model mapping from pixel coordinates to focal-plane coordinates.
- $\mathbf{F}_{visit}$ is the model mapping from focal-plane coordinates to the sky.

Add proper motion, parallax

- Nearby stars can have significant parallax and proper motion, so we need to add this to the model.
    - Proper motion has RA and Dec components.
    - Parallax is inversely proportional to the distance.
    - These can be fit, or you can just use the values from a reference catalog.



- Now $\mathbf{S}_k$ becomes

$$\mathbf{S}_k = \mathbf{S}_k(v_{\mathrm{RA}}, v_{\mathrm{Dec}}, \varpi, t, \lambda)$$

- $v_{\mathrm{RA}}, v_{\mathrm{Dec}}$ are the proper motions in RA and Dec.
- ϖ is the parallactic angle
- t is the time of the observation
- λ is the position of the observatory (relative to ICRS)

How to fit the solution?

- The polynomial model turns into a lot of variables:
  - For example, given a 4th order polynomial per detector…

    N_parameters = 15 * 2 (for RA and Dec) * N_detectors = 5670 for LSSTCam

  - And a 6th order polynomial per visit…

    N_parameters = 28 * 2 * N_visits = 5600 for 100 visits

The Gauss-Newton Method

- We are trying to minimize the chi-squared
- At the minimum, the derivative equals zero
- Use Gauss-Newton to find this point

- Analogous to the Newton-Raphson method (1d version here):

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

or, $f'(x_0)\Delta(x) = -f(x_0)$

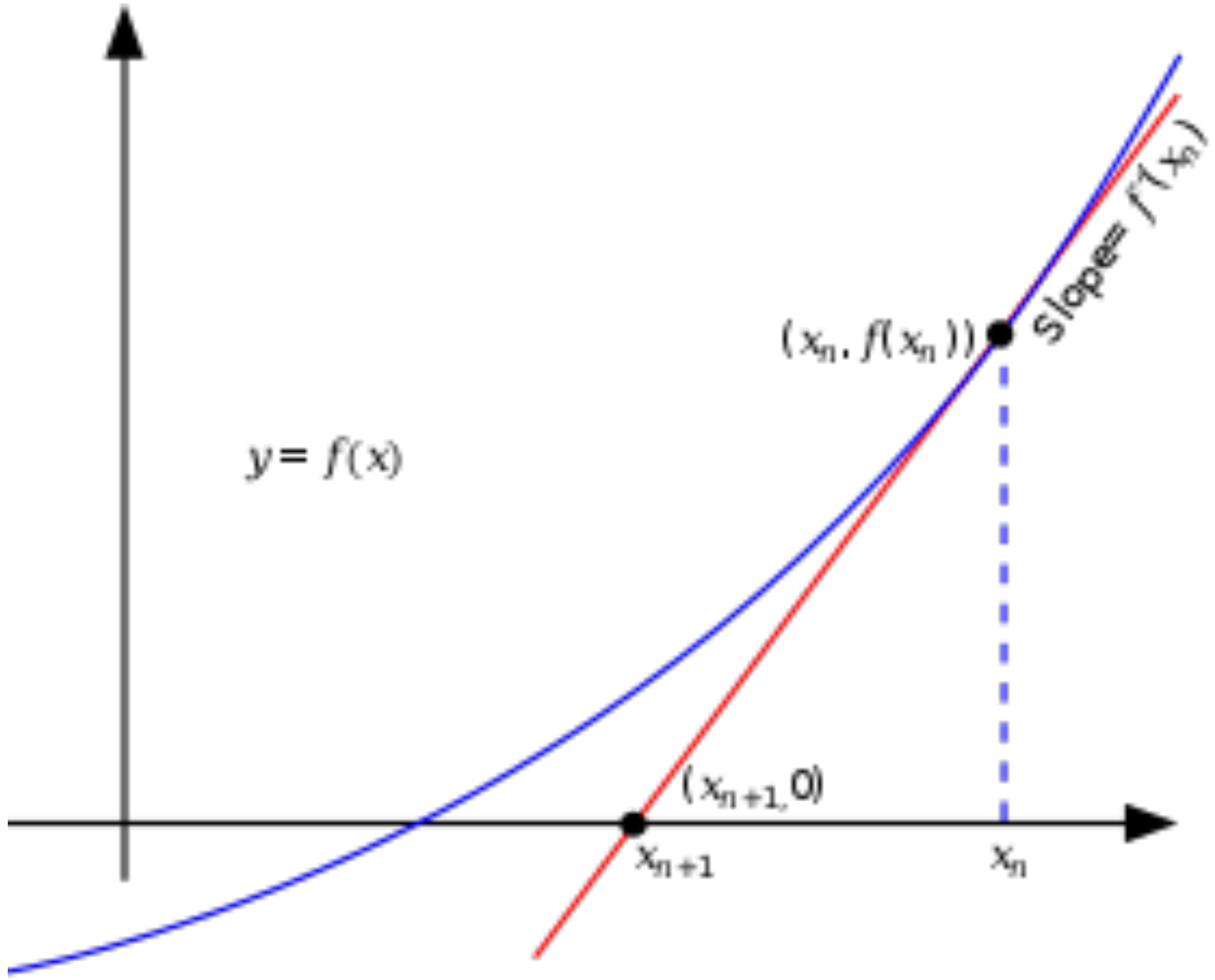
- or, the multidimensional version:

$$\boldsymbol{J}(\boldsymbol{f}(\boldsymbol{x}))\Delta(\boldsymbol{x}) = -\boldsymbol{f}(\boldsymbol{x})$$

- Gauss-Newton uses the same method, since we are try to find where the derivative of the chi-squared function equals zero. In our case, $\boldsymbol{f}(\boldsymbol{x})$ is actually the Jacobian of the chi-squared, and the derivative of that is the Hessian matrix:

$$\boldsymbol{H}(\chi^2)\Delta(\boldsymbol{x}) = -\boldsymbol{J}(\chi^2)$$

The Newton-Raphson Method:



What effects do we still need to account for?

- Gravitational deflection by solar system members
- Aberration due to Earth's motion
- Refraction in Earth's atmosphere
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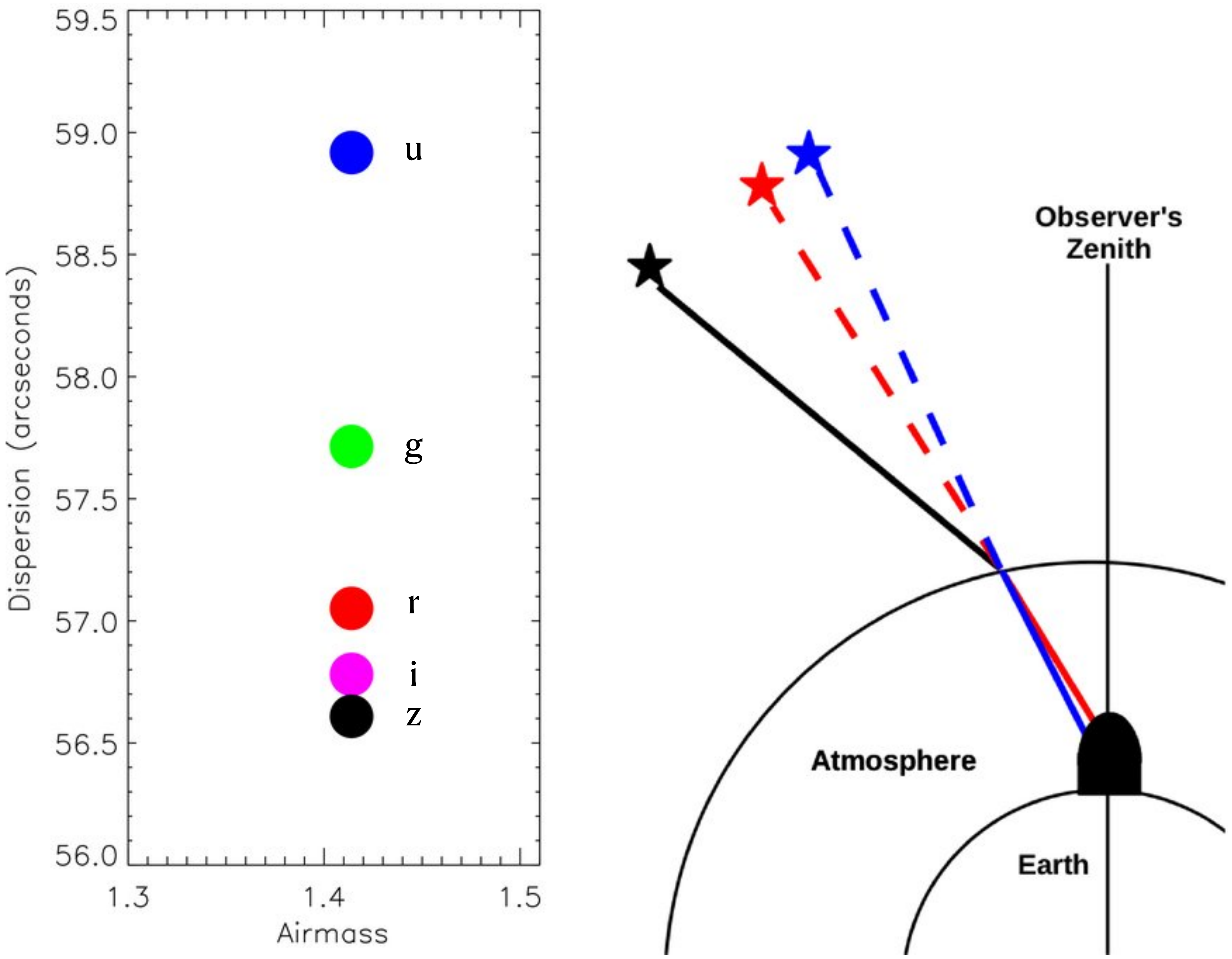
Differential Chromatic Refraction

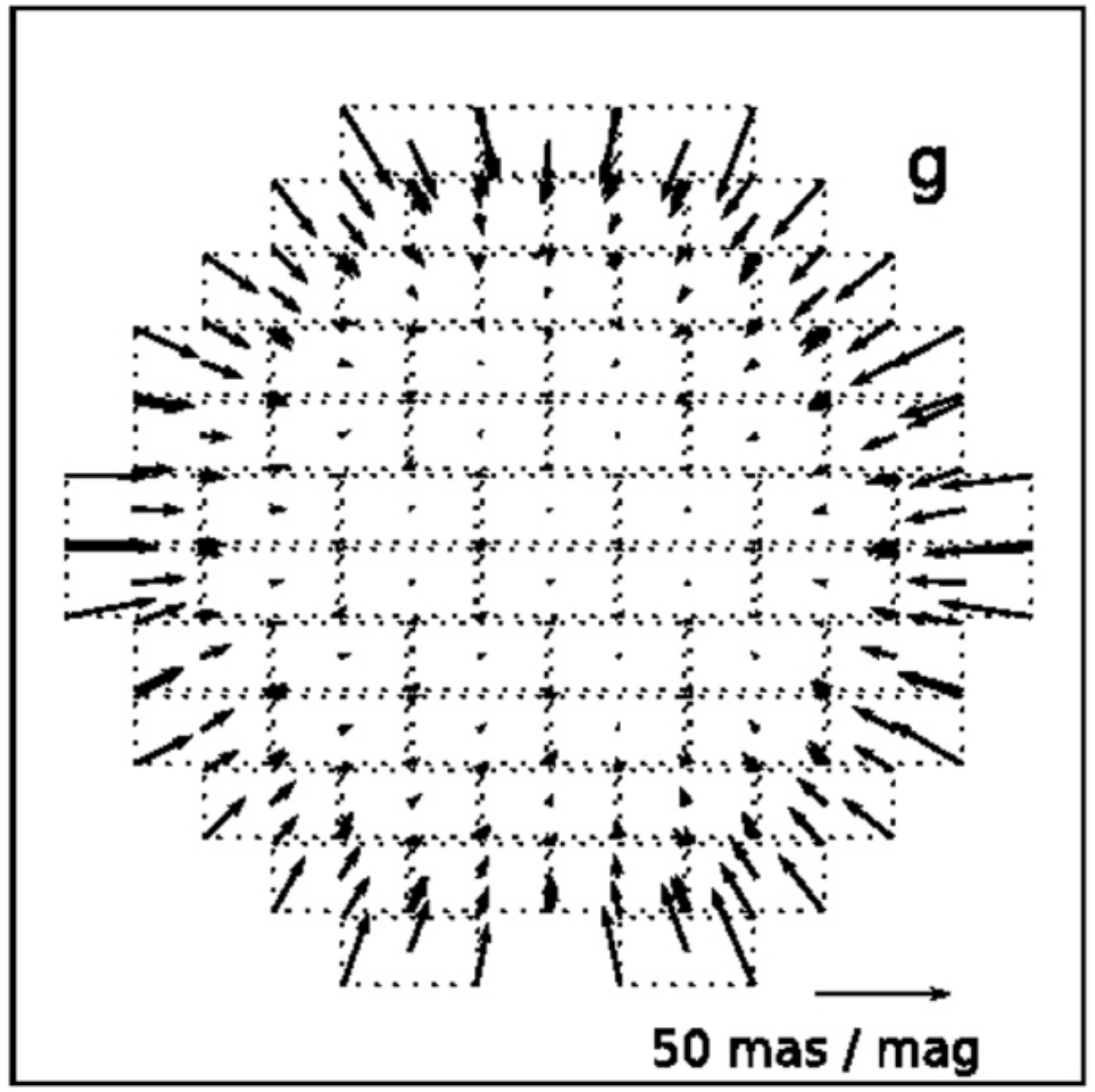
- A star's position will depend on the slope of its spectrum across the filter band
- It isn't part of the WCS because it is different for each star.
- It can be accurately fit using stellar color.

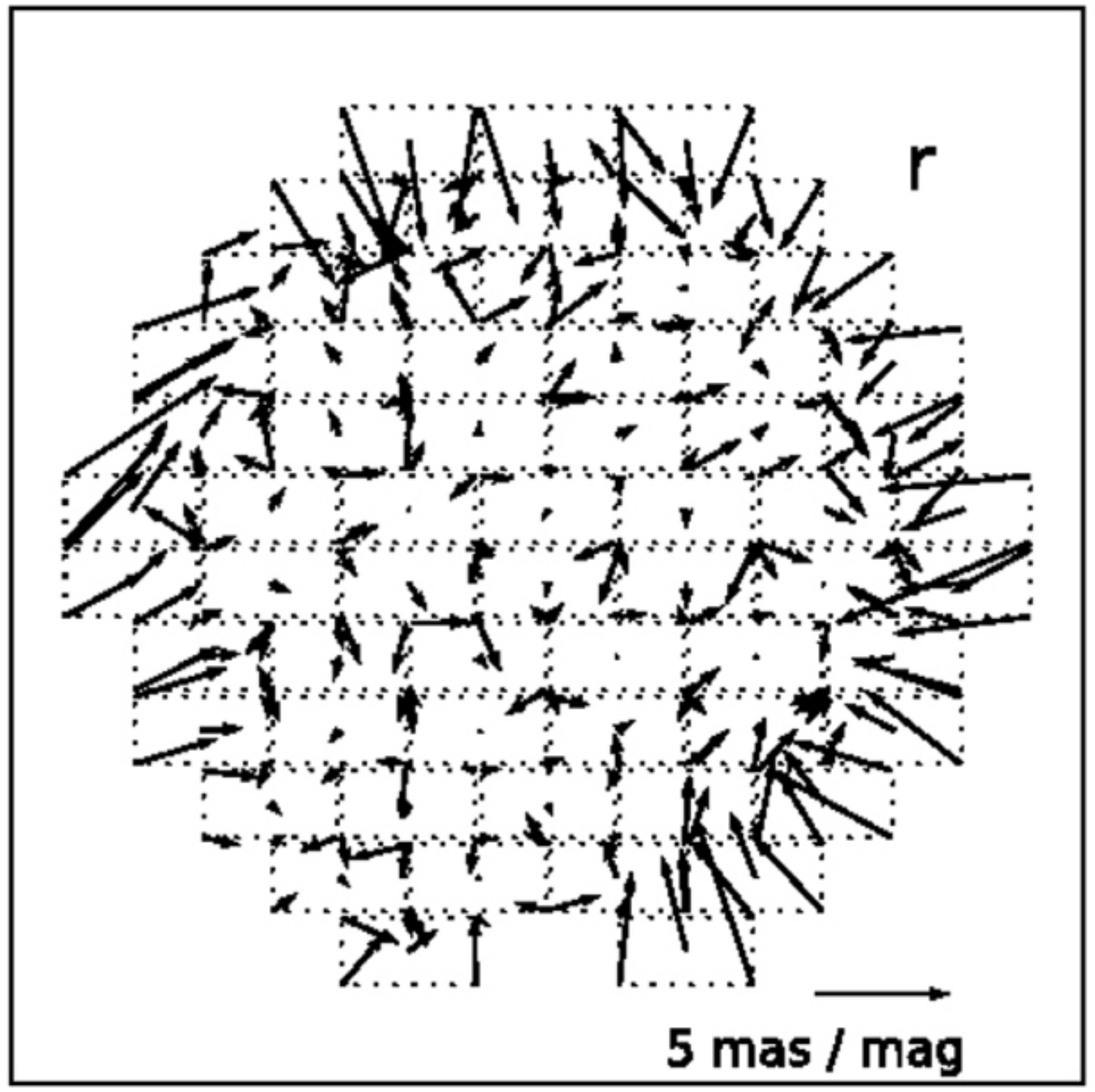


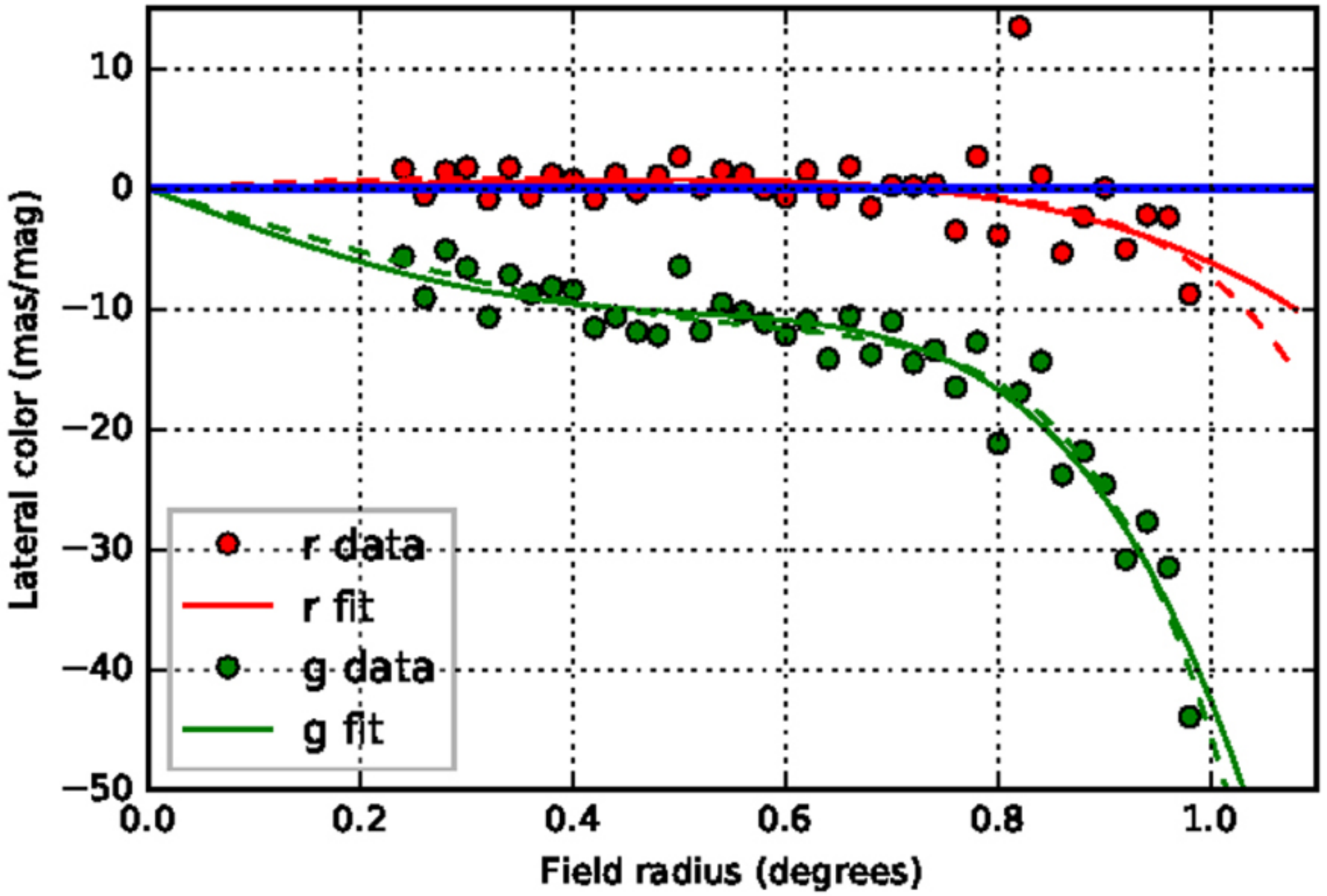
Kaczmarczik+ 2009

Color from glass in the optics

- Similar to DCR, but depends on focal plane position and points toward center.

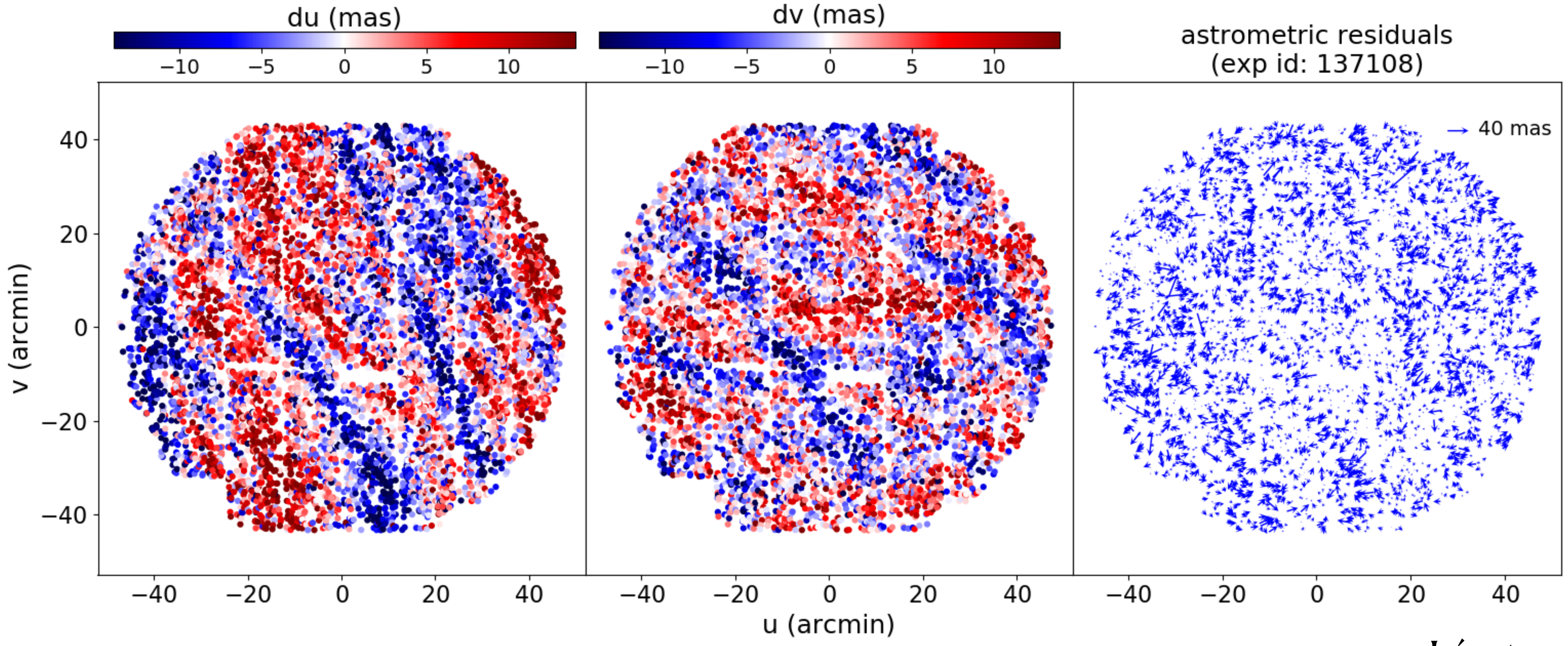






Bernstein+ 2017

Atmospheric Turbulence

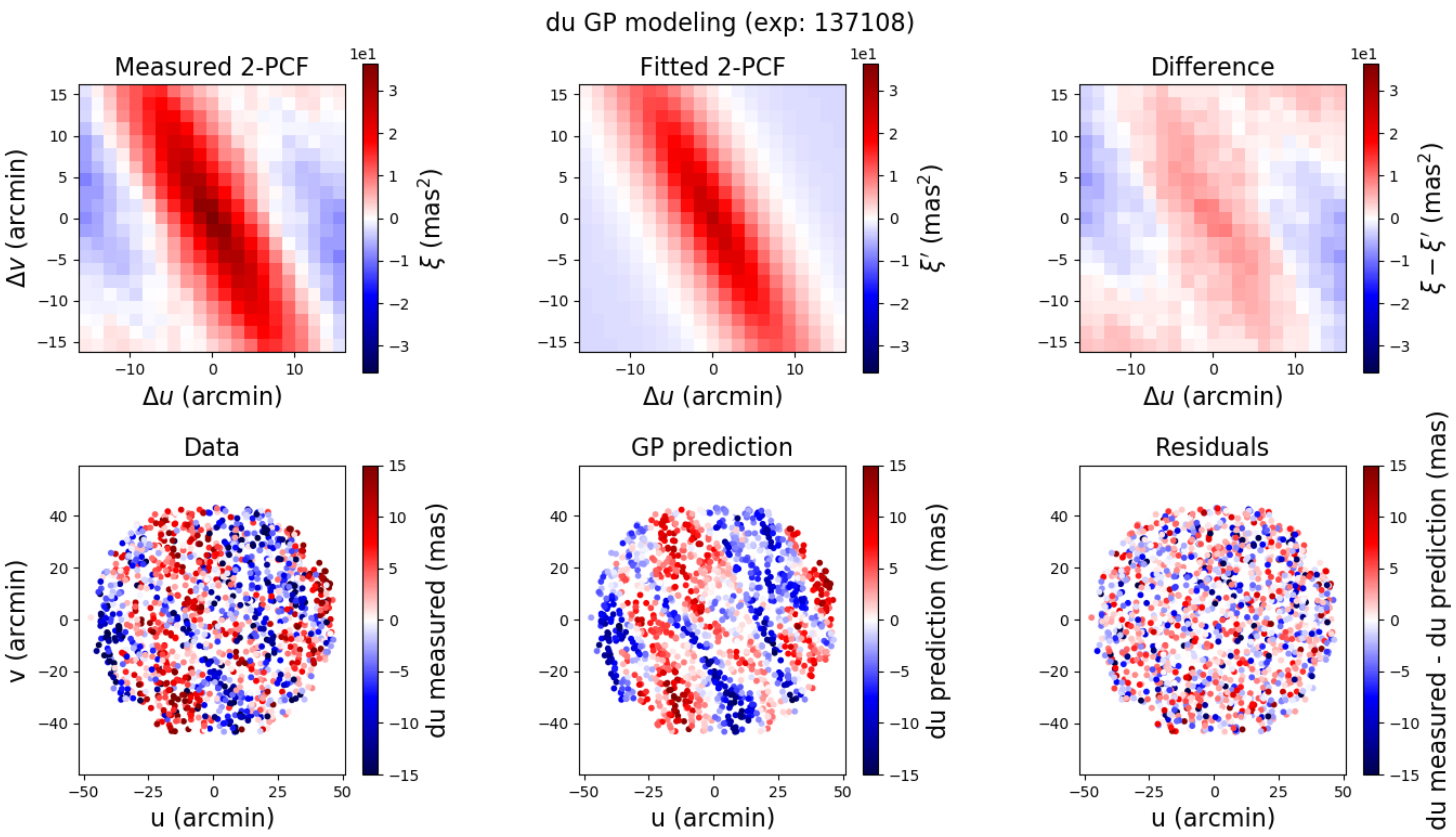


Léget 2021

- HSC has 300 second exposures, and atmospheric turbulence gets somewhat averaged out—LSST will have much shorter exposure times, and turbulence is expected to be a bigger problem.
- It's correlated (though not well fit by a polynomial), so we can try to fit it!

Modeling Turbulence with Gaussian Processes

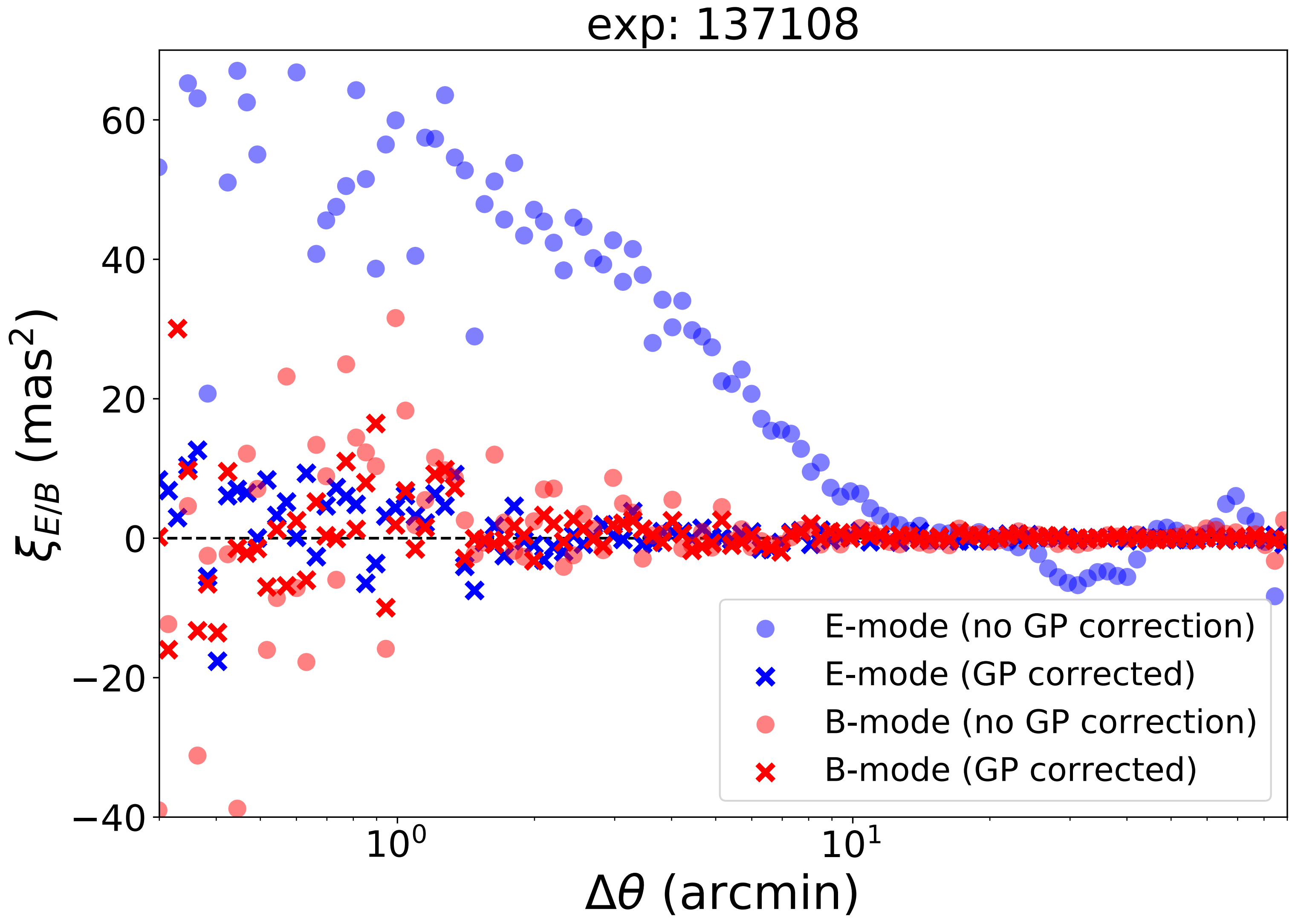
- Fortino+ 2020 and Léget+ 2021 propose using Gaussian Processes to model the effects of atmospheric turbulence.
- Gaussian Processes models data as a realization of a Gaussian field.



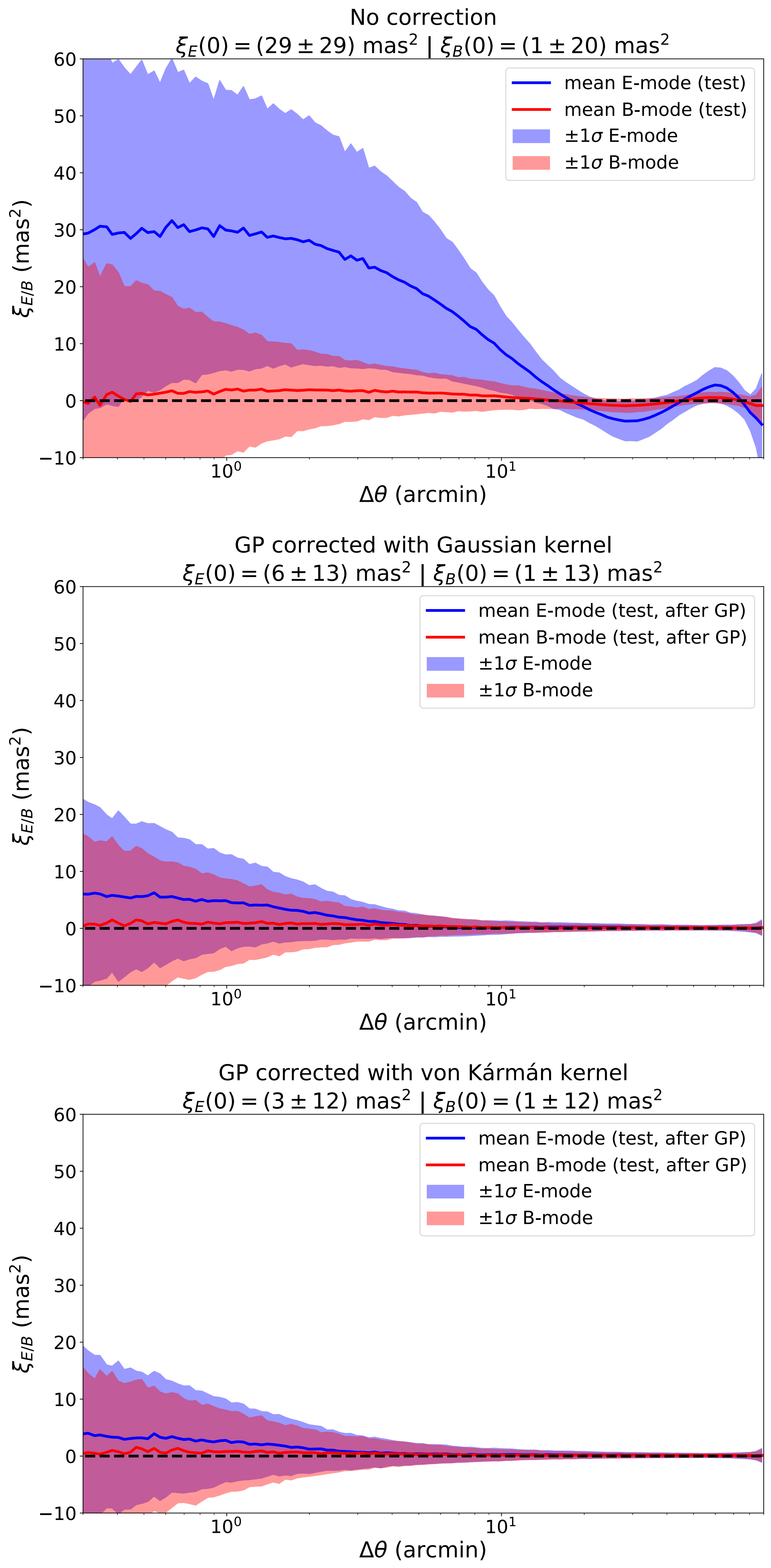
Léget 2021

Correlation in Astrometric Residuals After Gaussian Processes Correction

Correlation is measured by calculating E and B-modes (like in lensing)

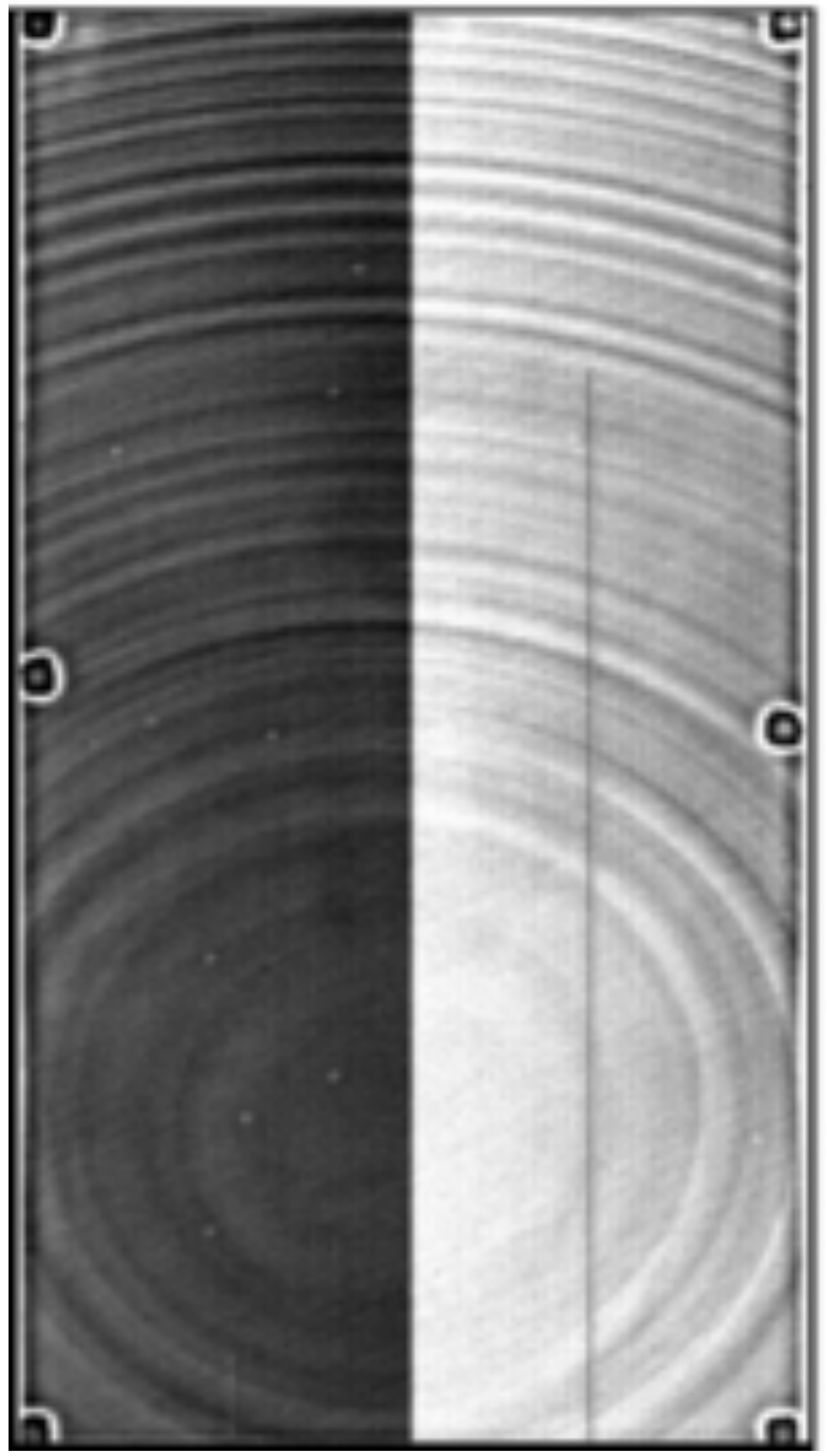


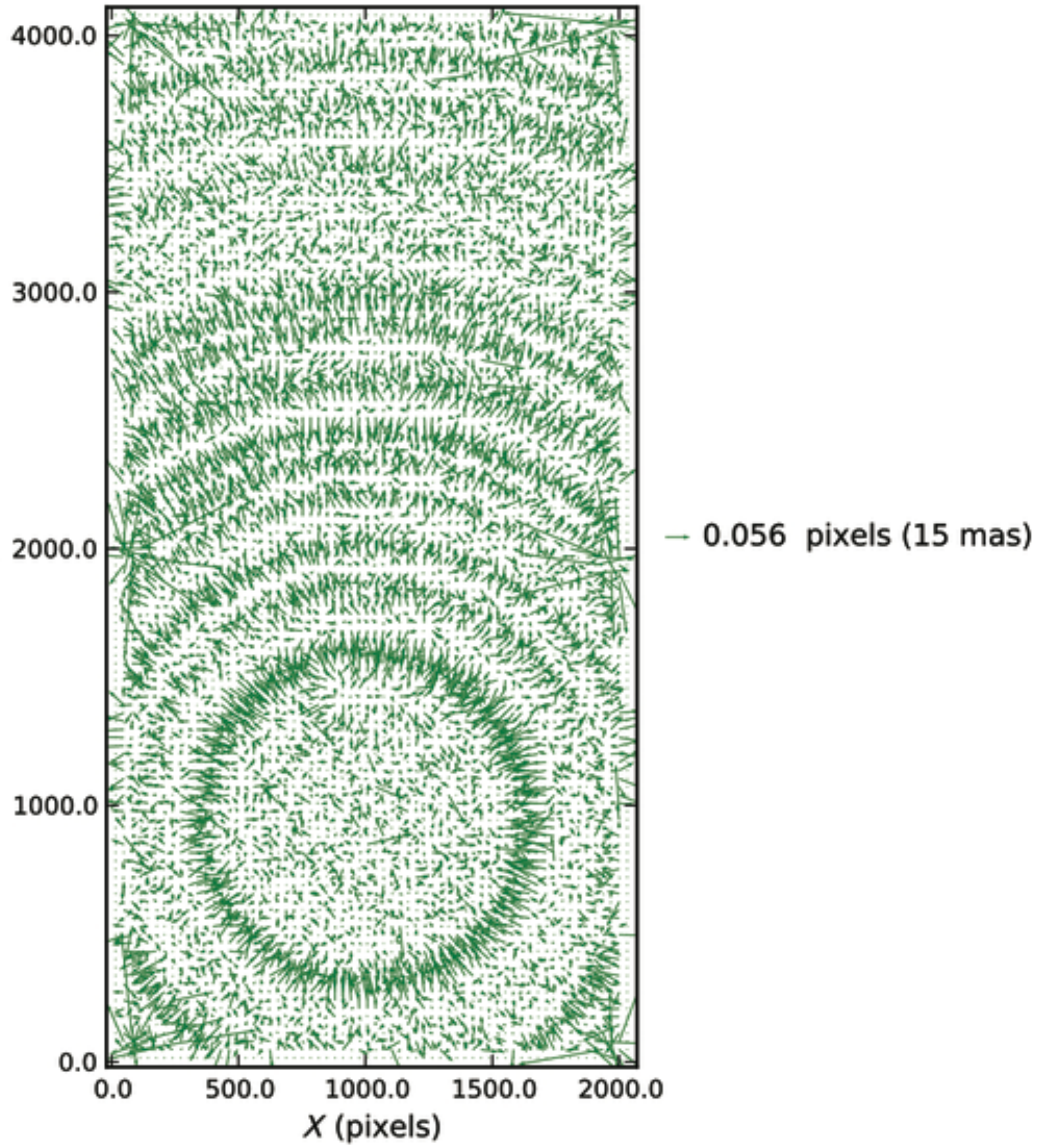
Average over all Exposures



Stray Electric Fields and imperfect pixels

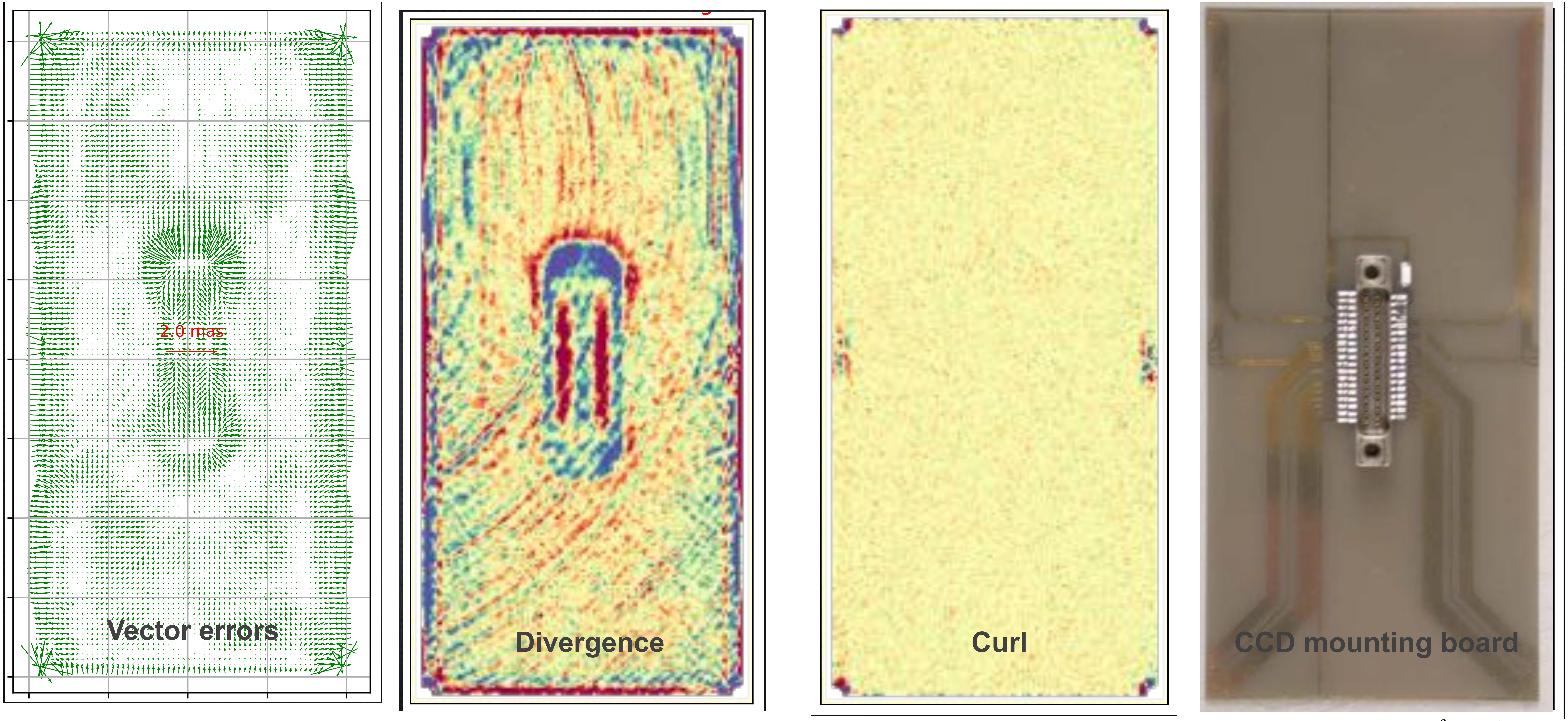
- These are static but subtle effects that become apparent after combining hundreds of exposures.
- They are not well-modeled by a polynomial, so they don't get accounted for in the regular camera model.
- Given enough data, they can be subtracted off with a separate mapping.





Plazas+ 2014

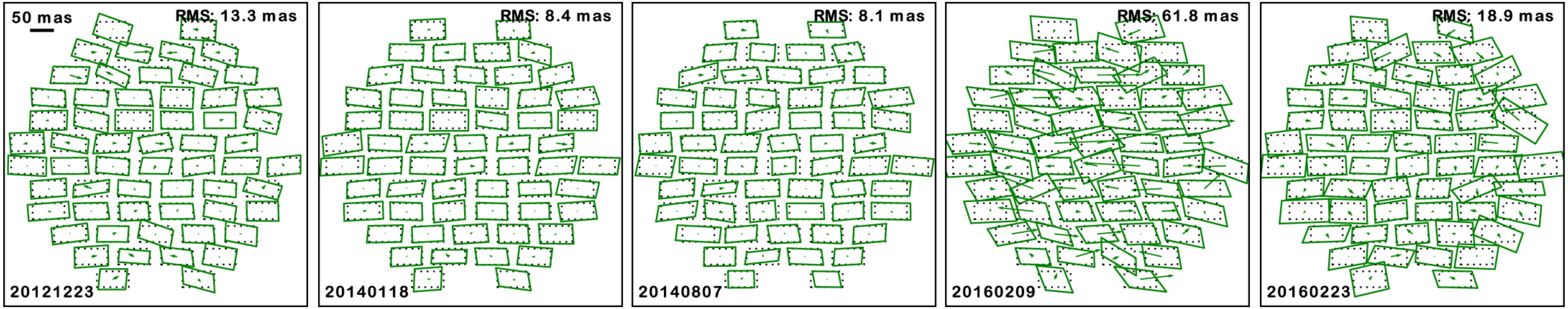
Mean Astrometric Residuals from all DECam CCDs



from Gary Bernstein

Changes in the camera over time

- Can be dealt with by splitting the “static” part of the camera model into different time periods.



Bernstein+ 2017

Conclusions

- Astrometry covers the measurement of the coordinates and movement of celestial bodies.
- Aligning images is an essential step in data processing.
- Using the positions of isolated stars that can be tied to an outside reference system, you fit a map that can then give the sky position for any coordinates of your image.
- It's essential to consider the factors affecting the path of a photon in order to minimize errors.

Further reading

A good overview of concepts:

- *Astrometric Calibration and Performance of the Dark Energy Camera,* G. Bernstein et al., 2017

Centroid fitting:

- Check out Gary Bernstein's talk on astrometry for Session 11.

The effect of tree-rings and other transverse fields:

- *On-Sky Measurements of the Transverse Electric Fields' Effects in the Dark Energy Camera CCDs,* A.A. Plazas et al., 2014

Using Gaussian Processes to improve the solution:

- I*mproving the astrometric solution of the Hyper Suprime-Cam with anisotropic Gaussian processes,* P.F. Léget, et al. 2021
- *Reducing Ground-based Astrometric Errors with Gaia and Gaussian Processes,* W. Fortino, et al., 2021

Differential Chromatic Refraction:

- *Astrometric Redshifts for Quasars,* M. Kaczmarczik, et al. 2009