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Fluctuation Theory

1 Sum Rules

After some effort,

$$\begin{aligned}
&\int d\Delta y \langle \delta E_T(\Delta y) \delta E_T(0) \rangle \\
&\qquad = \int d\Delta\eta \langle \delta\epsilon(\Delta\eta)\delta\epsilon(0) + \delta\tilde{P}_z(\Delta\eta)\delta\tilde{P}_z(0) \rangle \\
&\qquad = -\chi_{EE} - \chi_{pp}. \qquad (1.1)
\end{aligned}$$

$$\begin{aligned}
&\int d\Delta y \langle \delta E_T(\Delta y) \delta E_T(0) \rangle \cosh(\Delta y) \\
&\qquad = \int d\Delta\eta \{ \langle \delta\epsilon(\Delta\eta)\delta\epsilon(0) - \delta\tilde{P}_z(\Delta\eta)\delta\tilde{P}_z(0) \rangle \cosh(\Delta y)] \\
&\qquad\quad + \langle \delta\tilde{P}_z(\Delta\eta)\delta\epsilon(0) - \delta\epsilon(\Delta\eta)\delta\tilde{P}_z(0) \rangle \sinh(\Delta y) \} \\
&\qquad = -\chi_{EE} + \chi_{pp}. \qquad (1.2)
\end{aligned}$$

The susceptibilities can also be identified as:

$$\chi_{EE} = C_V, \quad \chi_{pp} = (P + \epsilon)T. \qquad (1.3)$$

Also, the last 2 terms in Eq. (1.2 are equal

$$\langle \delta\tilde{P}_z(\Delta\eta)\delta\epsilon(0) \rangle = -\langle \delta\epsilon(\delta\eta)\delta\tilde{P}_z(0) \rangle. \qquad (1.4)$$

Equivalently, one can add Eq.s(1.1) and (1.2) to get

$$\begin{aligned}
\int & d\Delta y \langle \delta E_T(\Delta y/2)\delta E_T(-\Delta y/2)\rangle \cosh^2(\Delta y/2) \\
&= \int d\Delta\eta \{ \langle \delta\epsilon(\Delta\eta/2)\delta\epsilon(\Delta\eta/2)\rangle \cosh^2(\Delta\eta/2) \\
&\qquad\qquad - \langle \delta\tilde{P}_z(\Delta\eta/2)\delta\tilde{P}_z(-\Delta\eta/2)\rangle \sinh^2(\Delta\eta/2) \\
&\qquad\qquad + \langle \delta\tilde{P}_z(\Delta\eta/2)\delta\epsilon(-\Delta\eta/2) - \delta\epsilon(\Delta\eta/2)\delta\tilde{P}_z(-\Delta\eta/2)\rangle \\
&\qquad\qquad\quad \sinh(\Delta\eta/2)\cosh(\Delta\eta/2)\} \\
&= -\chi_{EE}. \qquad (1.5)
\end{aligned}$$

or subtract them to get

$$\begin{aligned}
\int & d\Delta y \langle \delta E_T(\Delta y/2)\delta E_T(-\Delta y/2)\rangle \sinh^2(\Delta y/2) \\
&= \int d\Delta\eta \{ \langle \delta\tilde{P}_z(\Delta\eta/2)\delta\tilde{P}_z(-\Delta\eta/2)\rangle \cosh^2(\Delta\eta/2) \\
&\qquad\qquad - \langle \delta\epsilon(\Delta\eta/2)\delta\epsilon(-\Delta\eta/2)\rangle \sinh^2(\Delta\eta/2) \\
&\qquad\qquad - \langle \delta\tilde{P}_z(\Delta\eta/2)\delta\epsilon(-\Delta\eta/2) - \delta\epsilon(\Delta\eta/2)\delta\tilde{P}_z(-\Delta\eta/2)\rangle \\
&\qquad\qquad\quad \sinh(\Delta\eta/2)\cosh(\Delta\eta/2)\} \\
&= -\chi_{pp}. \qquad (1.6)
\end{aligned}$$