I. EQUATIONS

A. Hydrodynamics

In the following we will consider hydrodynamic evolution of the thermodynamic fluctuations using linear approximation. The stress-energy tensor of viscous fluid in the Landau frame:

$$T^{\mu\nu} = (\varepsilon + P(\varepsilon,\rho))u^\mu u^\nu - g^{\mu\nu}P + T^{\mu\nu}_\eta, \tag{1}$$

where ε, ρ are energy- and particle-densities respectively and $T^{\mu\nu}_\eta$ is:

$$T^{\mu\nu}_\eta = -\eta\left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}(\partial_l u^l)\right), \tag{2}$$

η is a coefficient related to viscosity, $\Delta^{\mu\nu} = u^\mu u^\nu - g^{\mu\nu}$ a projector on orthogonal to the velocity space. If baryon number is allowed to move diffusively, baryon current can be written in the form:

$$j^\mu = \rho u^\mu - D\Delta^{\mu\nu}\partial_\nu\rho, \tag{3}$$

where D is the diffusion constant. Hydrodynamic equations will be equivalent to the energy-momentum conservation:

$$\partial_\nu T^{\mu\nu} = 0, \tag{4}$$

$$\partial_\nu j^\nu = 0, \tag{5}$$

assuming the solution $\rho(t,\vec{r}), \varepsilon(t,\vec{r}), u^\mu(t,\vec{r})$ for some boundary and initial conditions $T^{\mu\nu}$ can be expanded into the series around (this) solution in the respective perturbations N, E, U^μ. We will focus mainly on $\delta T^{\mu\nu}$ linear in perturbations. They should satisfy following equations:

$$\partial_\nu \delta j^\nu = 0, \tag{6}$$

$$\partial_\nu \delta T^{\mu\nu} = 0. \tag{7}$$

This linear system of equations has trivial solution. Stable hydrodynamics would require that any perturbation relaxes over time to it. This in particular would apply to the thermodynamic fluctuation of baryon number, momentum or energy. Following the theory outlined in [?] we want to consider correlations of conserved charges at freeze-out. In particular, influence of non-diagonal susceptibilities at finite baryon density. For this Eqs.(6)-(7) can be considered in the following form:

$$\partial_0 \vec{H} = \hat{L}\vec{H}, \tag{8}$$

i.e. as an evolutionary equation for the $\vec{H} = \{N, E, U^\mu\}$ with a linear operator L, exact form of which depends on the background solution. Assuming correlations to be temporary and spatially uncorellated:

$$\partial_0\langle\vec{H}_1 \otimes \vec{H}_2\rangle = \langle\vec{H}_1 \otimes \hat{L}\vec{H}_2\rangle + \langle\vec{H}_2 \otimes \hat{L}\vec{H}_1\rangle + (u^\mu\partial_\mu + (\partial_\mu u^\mu))\langle\delta H(t,\vec{r_1})\delta H(t,\vec{r_2})\rangle\delta(\vec{r_1} - \vec{r_2}), \tag{9}$$

where indices 1 and 2 correspond to the spatial coordinates and

$$\langle\delta H(t,\vec{r_2})\delta H(t,\vec{r_2})\delta(\vec{r_1} - \vec{r_2})\rangle = \chi_{H\otimes H}(\rho,\varepsilon), \tag{10}$$

is the corresponding equilibrium susceptibility.

II. BJORKEN EXPANSION AS A BACKGROUND

To understand how correlations evolve during hydrodynamic evolution of strongly interacting matter a boost invariant Bjorken solution is used as a background. In this case $\rho(\tau) = \rho_0\frac{\tau_0}{\tau}, t = \tau\cosh\eta, z = \tau\sinh\eta$. Equal time spacial correlations are assumed of the form:

$$C_{AB}(\eta_1-\eta_2,\tau)=\langle\delta A(\tau,\eta_1)\delta B(\tau,\eta_1)\rangle\frac{\delta(\eta_1-\eta_2)}{\tau}+c_{AB}(\tau,\eta_1,\eta_2), \tag{11}$$

where $c(t,r_a,r_b)$ describes an ideal-gas correlations of the conserved charge. System of linear relaxation equations written in the previous section applied to correlations become:

$$\partial_\tau\langle\vec{H}_1\otimes\vec{H}_2\rangle=\langle\vec{H}_1\otimes\hat{L}\vec{H}_2\rangle+\langle\vec{H}_2\otimes\hat{L}\vec{H}_1\rangle+(\partial_\tau+\frac{1}{\tau})\langle\delta A(\tau,\eta_1)\delta B(\tau,\eta_1)\rangle\delta(\eta_1-\eta_2) \tag{12}$$

this is a system of $5*4/2=10$ equations. Defining green‘s functions of linear operator L:

$$\partial_\tau\delta\varepsilon=-\frac{1}{\tau}\delta\left(\varepsilon+P\right)-\partial_\eta\frac{\varepsilon+P-\frac{8\eta_s}{3\tau}}{\tau}\frac{\delta u^z}{\cosh\eta} \tag{13}$$

$$\partial_\tau(\varepsilon+P-\frac{4\eta_s}{3\tau})\delta u^\eta=-\frac{2}{\tau}(\varepsilon+P-\frac{4\eta_s}{3\tau})\delta u^\eta-c_\rho^2\frac{\partial_\eta\delta\varepsilon}{\tau}-\partial_\rho P\frac{\partial_\eta\delta\rho}{\tau}+\frac{4\eta_s\partial_\eta^2\delta u^\eta}{3\tau^2} \tag{14}$$

$$\partial_\tau\delta\rho=-\frac{\delta\rho}{\tau}+\frac{\rho+D\partial_\tau\rho}{\tau}\partial_\eta\delta u^\eta+\frac{D}{\tau^2}\partial_\eta^2\delta\rho. \tag{15}$$

as solutions for the source $j_a(\tau,\eta)=\delta(\tau-\tau_j)\frac{\delta(\eta_1-\eta_2)}{\tau}$, where τ appears due to $dl=c\tau d\eta$. They satisfy following useful relation:

$$\int dzG_{AB}(z,z_j,t,t_j)=\Theta(t-t_j)\delta_{AB}, \tag{16}$$

in the Milne coordinates:

$$\int d\eta\frac{t}{\cosh^2\eta}G\left(\frac{t}{\cosh\eta},\Delta\eta\right)=\Theta(t-t_j)\delta_{AB}. \tag{17}$$

Conservation laws equate to:

$$\int dtdz\delta(t-t_0)\delta T^{00}=const, \tag{18}$$

$$\int dtdz\delta(t-t_0)\delta T^{0z}=const. \tag{19}$$

which transforms into:

$$\int d\eta\frac{t_0}{\cosh^2\eta}\delta T^{00}\left(\frac{t_0}{\cosh\eta},\eta\right)=const, \tag{20}$$

$$\int d\eta\frac{t_0}{\cosh^2\eta}\delta T^{0z}\left(\frac{t_0}{\cosh\eta},\eta\right)=const. \tag{21}$$

However there are currents that are conserved in the η,τ frame:

$$\partial_t T^{tt}+\partial_z T^{tz}=\left(\partial_\tau+\frac{1}{\tau}\right)(\cosh\,\eta T^{tt}-\sinh\eta\,T^{tz})+\frac{1}{\tau}\partial_\eta(-\sinh\eta\,T^{tt}+\cosh\eta\,T^{tz})=0, \tag{22}$$

$$\partial_t T^{tz}+\partial_z T^{zz}=\left(\partial_\tau+\frac{1}{\tau}\right)(\cosh\,\eta T^{tz}-\sinh\eta\,T^{zz})+\frac{1}{\tau}\partial_\eta(-\sinh\eta\,T^{tz}+\cosh\eta\,T^{zz})=0 \tag{23}$$

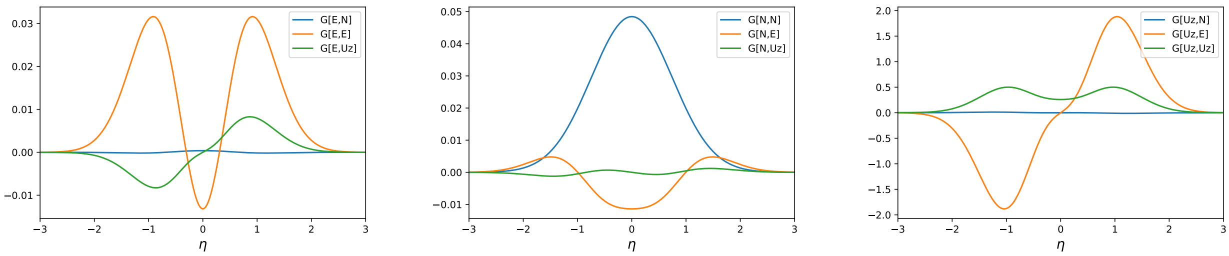

Figure 1. 1n0

$$\rho_\tau = \delta\varepsilon \cosh\eta + \left(\varepsilon + P - \frac{4}{3\tau}\eta_s\right) u^\eta \sinh\eta \tag{24}$$

$$\rho_\eta = \delta\varepsilon \sinh\eta + \left(\varepsilon + P - \frac{4}{3\tau}\eta_s\right) u^\eta \cosh\eta \tag{25}$$

are conserved charges in this theory (laboratory frame energy and momentum). In our numerical simulations we have observe fluctuation smaller then tenth of a percent which serves as a crosscheck. The solution can be written (and proven by direct substitution),

$$c_{AB} = \int d\tau_j \tau_j d\eta_j G_{AA^{\cdot}}(\eta_1 - \eta, \tau_1, \tau_j) G_{BB^{\cdot}}(\eta_2 - \eta, \tau_2, \tau_j)(\partial_{\tau_j} + \frac{1}{\tau_j})\langle \delta A(\tau_j, \eta_1)\delta B(\tau_j, \eta_1)\rangle \tag{26}$$

III. RESULTS

A. Green‘s functions

In order to obtain solution a set of green‘s functions for point sources in (τ, η) must be produced. Taking advantage of the boost invariance of the Bjorken flow we consider only $(\tau, 0)$ and obtain solution for a source at any η using Lorentz transformations. Figs.(1)-(3) show a response to the gaussian source with $\sigma^2 = 0.01, \tau_0 = 1fm/c$ at $\tau = 11fm/c$ in arbitrary units. One can observe the increase of momentum and energy response to the density perturbations as the background density increase from one normal nuclear density to five. The hydrodynamic evolution resembles two sound wave modes (linear in wave number) and diffusive mode(quadratic in wave number). Two peak structure of the energy responce becomes more prominent as the source width decreased in order to approach $\delta-$function. For better stability of the numeric schemes employed and taking into account experimental acceptance we will stick to the width value considered.

As an EoS we have used Taylor expansion for the $Q/B = 0.4$ as obtained by the HotQCD collaboration [**?**] up to the $O\left(\left(\frac{\mu_B}{T}\right)^6\right)$. All trajectories start at $T = 230\ MeV$ and evolve for $10fm/c$. The final temperature lies within 150 ± 1 MeV. The initial baryon density was chosen at $1, 2, 5n_0$, where $n_0 = .16fm^{-3}$ - normal nuclear density. The spread of correlations considering speed of sound signal propagation $\delta\eta = 2c_s \ln(\tau/\tau_0)$ for QCD matter roughly estimates at $\delta\eta = 2.5$.

B. NN correlations

Density correlations show notable sensitivity to other non-diagonal susceptibilities of conserved charges as baryon density increases. This might indicate that at high baryon density like the one reached in future experiments covariance of particle number fluctuations as a function of momentum difference will be sensitive to the presence of the energy fluctuations as well as energy density cross correlation. Possibly signal in fluctuations of energy spreads alot further compared to particle number fluctuations and can become another proxy for the critical point existence.

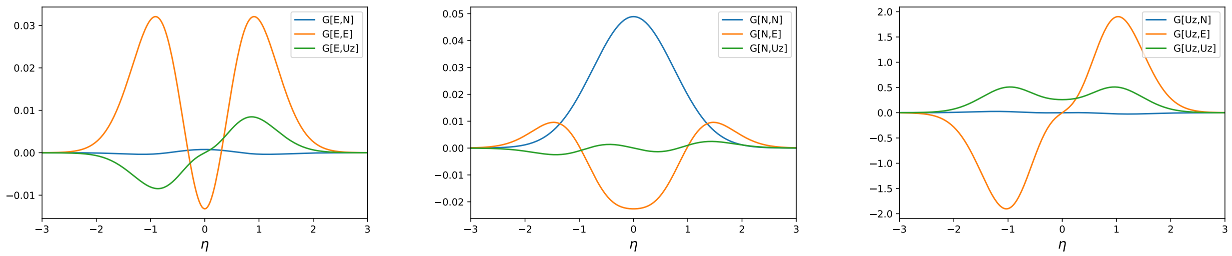

Figure 2. 2n0

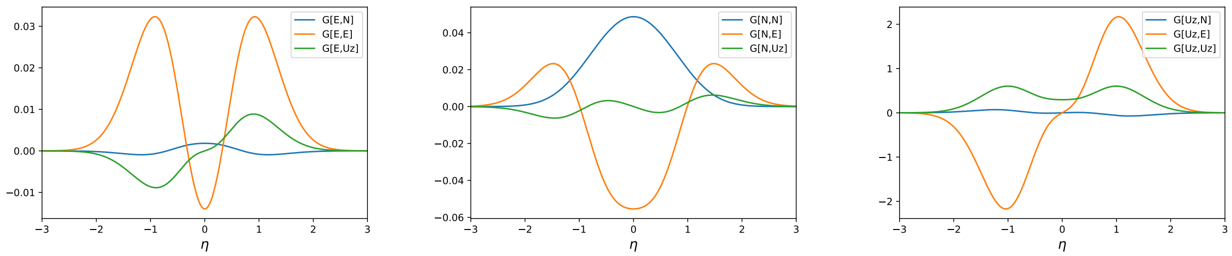

Figure 3. 5n0

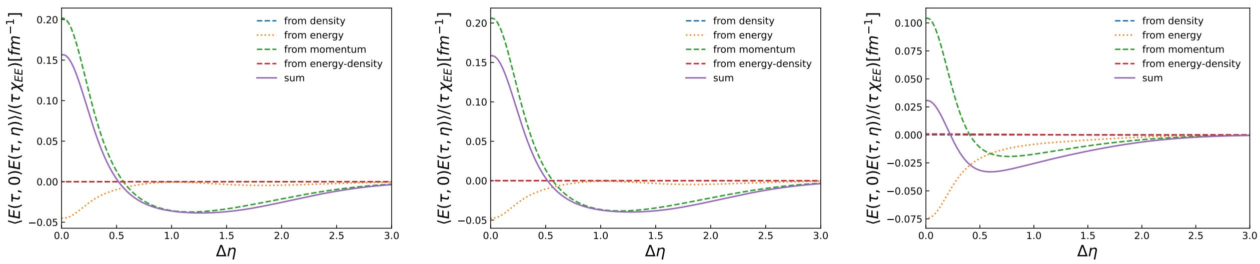

Figure 4. 1 2 5n0

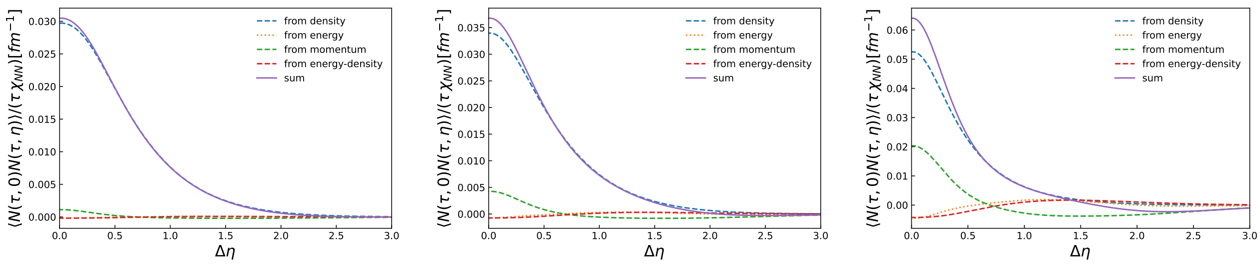

Figure 5. 1 2 5n0

IV. PARTICLIZATION

Two particle spectra can be written in the following form:

$$f(p_1, y_1, \beta + \delta\beta) f(p_2, y_2, \beta + \delta\beta) \approx f_1 f_2 + \delta\beta_1 \delta\beta_2 \partial_{\beta_1} f_1 \partial_{\beta 2} f_2, \tag{27}$$

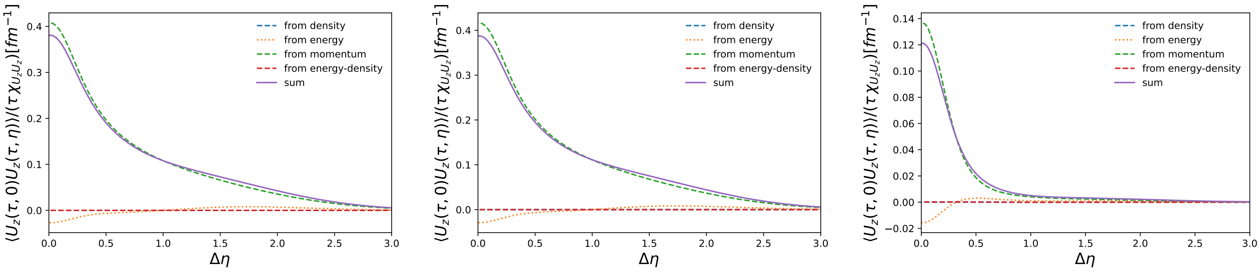

Figure 6. 1 2 5n0

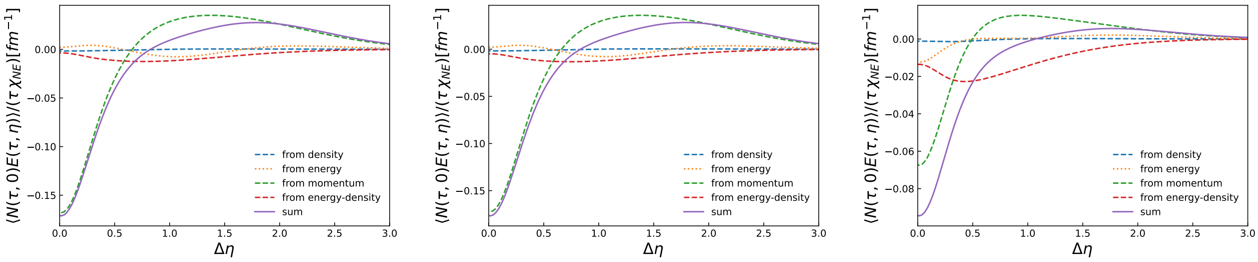

Figure 7. 1 2 5n0

here $\beta = T, \mu_B, u^\mu$. Since from hydrodynamics we obtain $\varepsilon, \rho_B, u^\eta$ they should be expanded into series again:

$$\delta\beta_i = \delta\lambda_j \partial_{\lambda_j}\beta_i \tag{28}$$

Particle number correlations:

$$\langle n_1 n_2\rangle - \langle n_1\rangle\langle n_2\rangle = \delta\beta_1\delta\beta_2 \int dp_1 dp_2 \partial_\beta f_1 \partial_\beta f_2 \tag{29}$$

$$p_0\left(\frac{d^3N}{dp^3}\right) = \frac{2d_i}{(2\pi)^3} = \frac{2d}{(2\pi)^2}\frac{R^2}{2} m_t K\left(\frac{u^\tau m_t}{T}\right)\exp\left(\frac{\mu}{T}\right) \tag{30}$$

$$\frac{\partial f}{\partial \mu} = \frac{f}{T} \tag{31}$$

$$\frac{\partial f}{\partial u^\tau} = \frac{m_t}{T}\frac{f}{K_1}K_1' \tag{32}$$

$$\partial_T f = -\partial_\mu f \frac{\mu}{T} - \partial_{u^\tau} f \frac{u^\tau}{T} \tag{33}$$