$$\partial_t g(r,t) - D\left[\partial_r^2 + \frac{1}{r}\partial_r - \frac{m^2}{r^2}\right]g(r,t) = 0,$$

$$f(r,t) = g(r,t)\cos(m\theta)$$