Correlations in rapidity

Adam Bzdak

AGH University of Science and Technology, Kraków

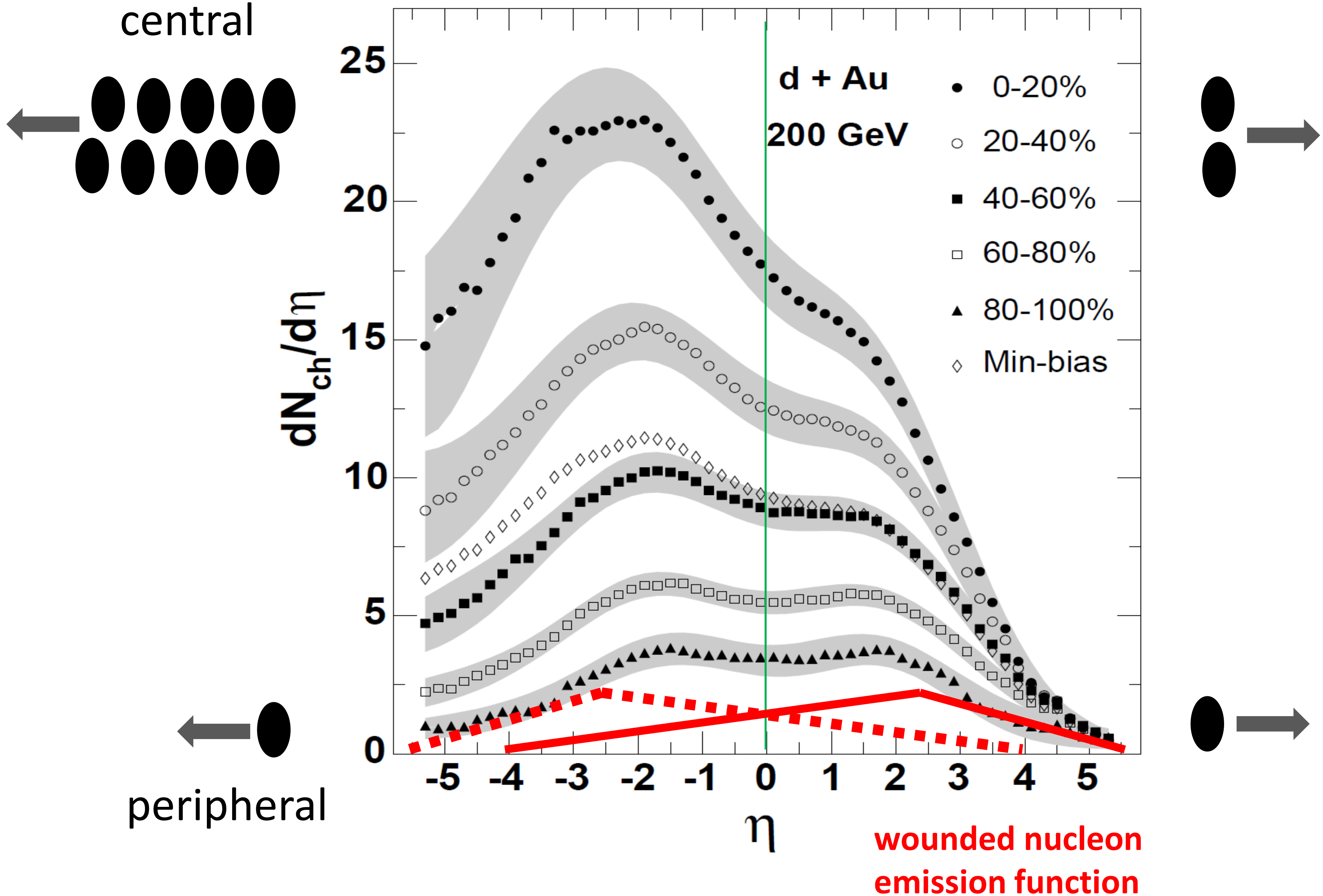
AGH

Outline

- wounded source emission function
- forward-backward correlations
- longitudinal fluctuations and correlations
- transverse-momentum multiplicity correlations
- conclusions

PRC 72 (2005) 031901

PHOBOS d+Au



Wounded nucleon (constituent) model

A.Bialas, W.Czyz
APPB 36 (2005) 905

$$\frac{dN}{d\eta} = w_L F(\eta) + w_R F(-\eta)$$

$w_{L,R}$ – number of left- and right-going constituents

$$F(\eta) = \frac{1}{2}\left[\frac{N(\eta) + N(-\eta)}{w_L + w_R} + \frac{N(\eta) - N(-\eta)}{w_L - w_R}\right]$$

wounded constituent emission function

$$N(\eta) \equiv \frac{dN}{d\eta}$$

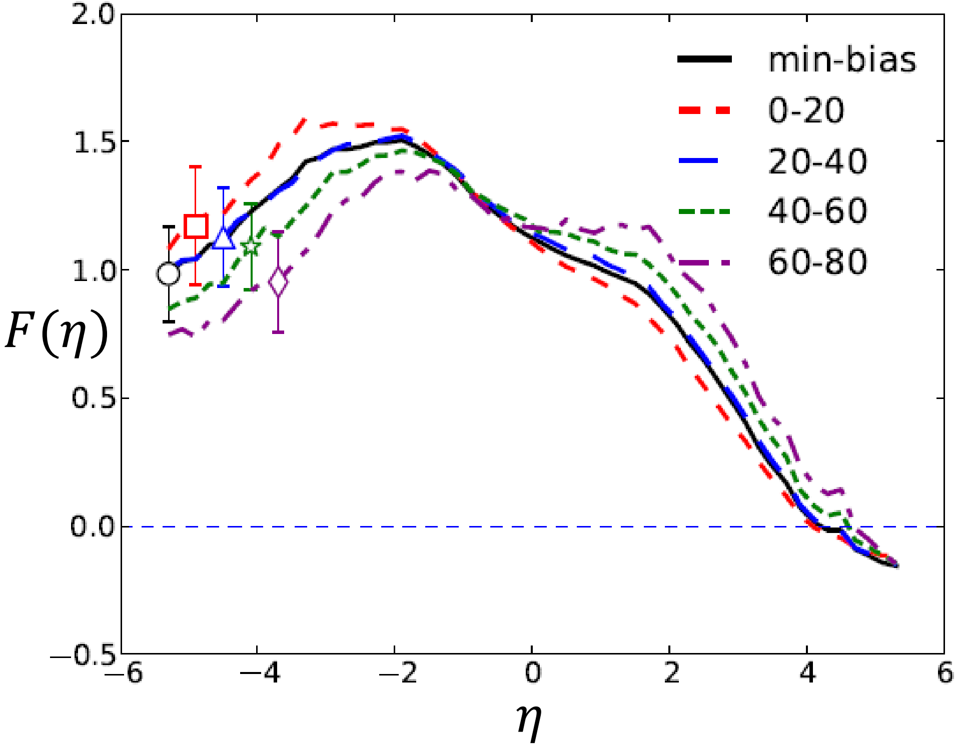
M.Barej, AB, in preparation

Wounded nucleon and quark emission functions

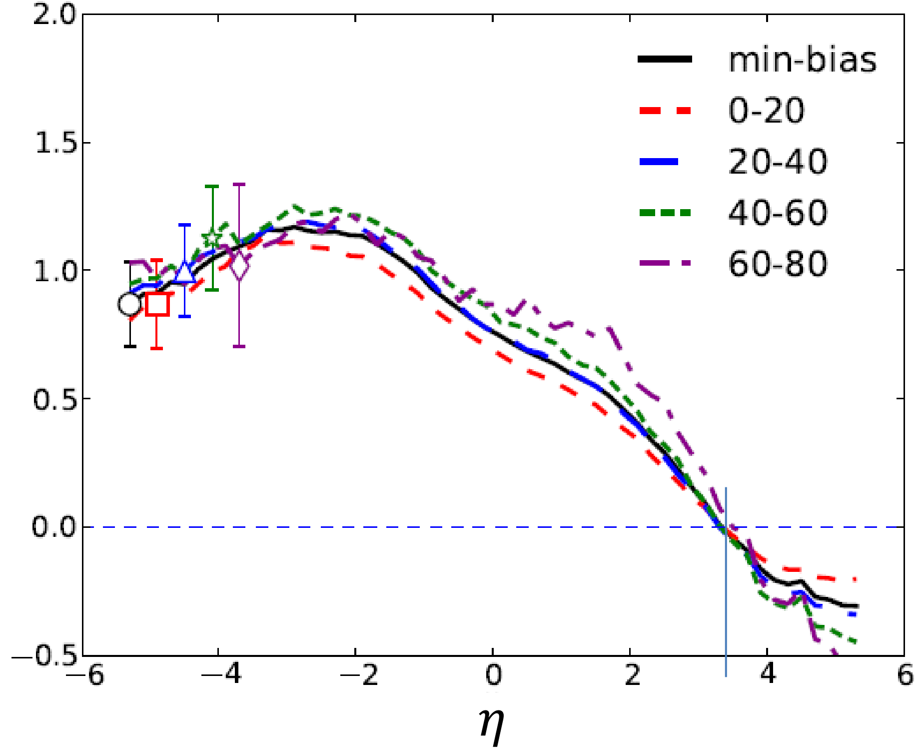
M.Barej, AB,
in preparation

$\sqrt{s} = 200$ GeV

wounded nucleons



wounded quarks



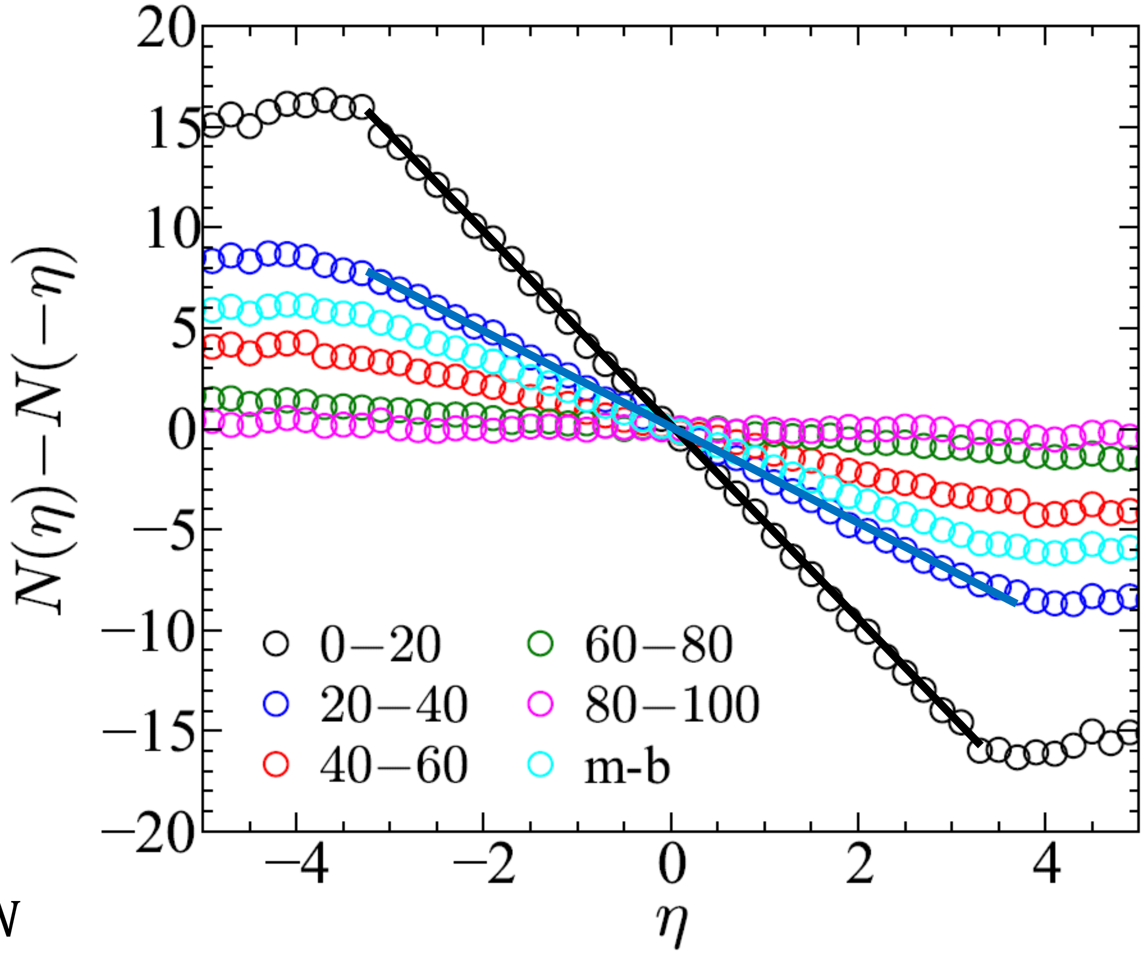
close to $\eta = 0, F(\eta) \sim \eta$

To be checked in p+Au and He3+Au

Antisimetrization of $N(\eta)$

A.Bialas, W.Czyz
APPB 36 (2005) 905

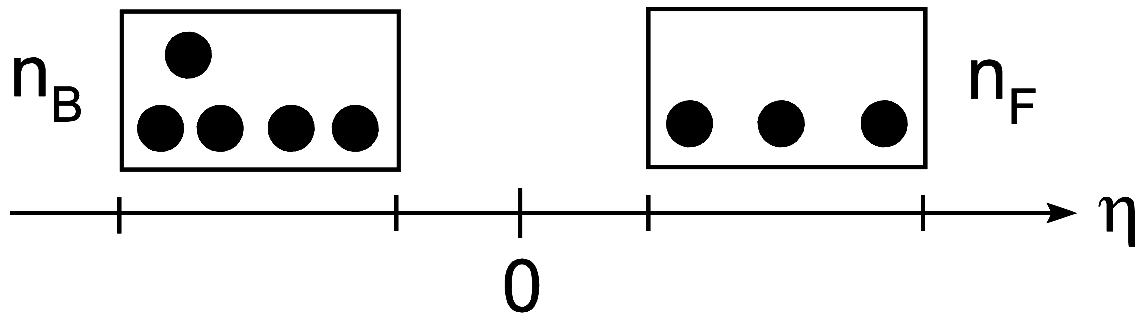
PHOBOS d+Au



$$N(\eta) \equiv \frac{dN}{d\eta}$$

homework for PHENIX

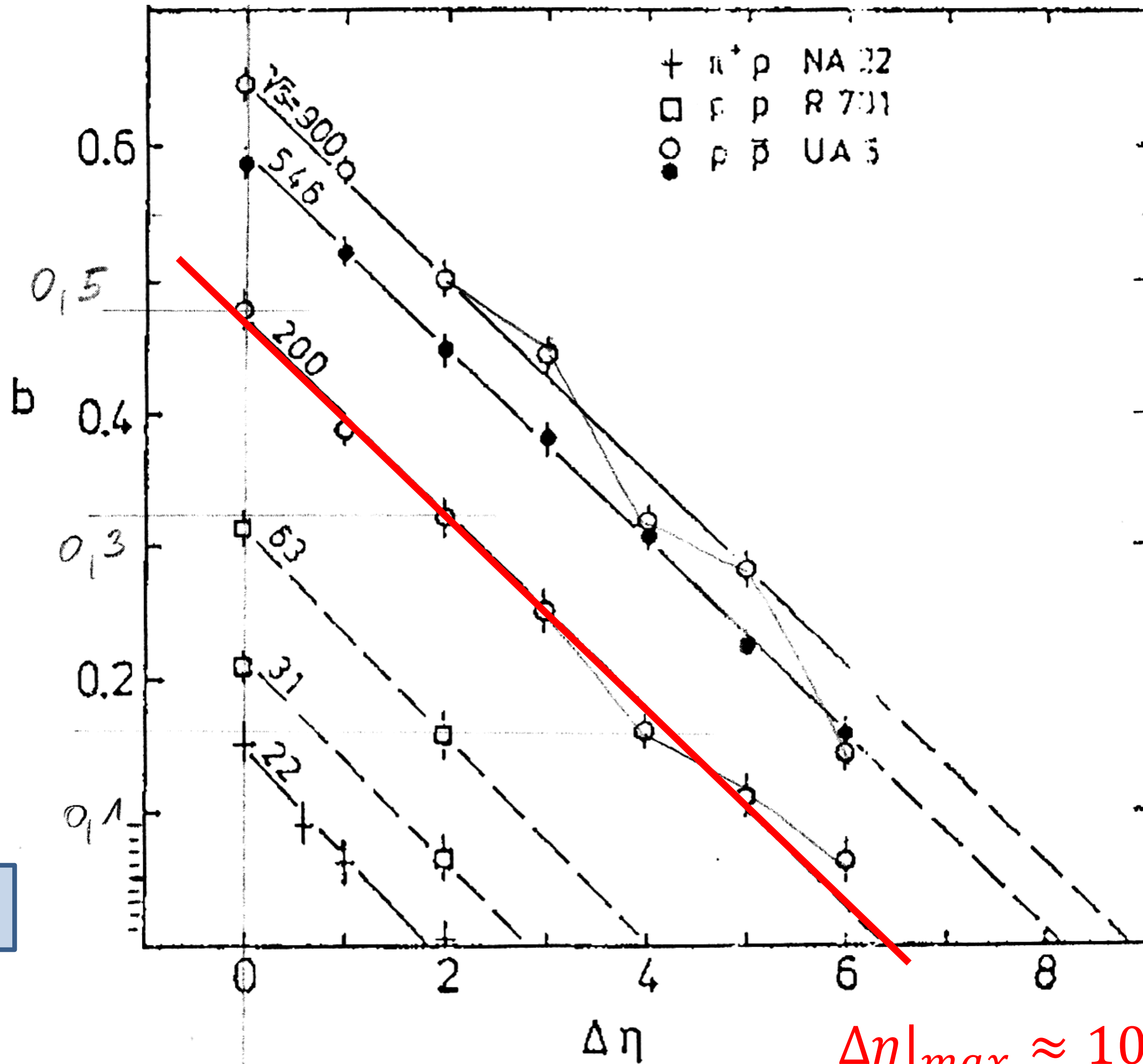
Forward-backward multiplicity correlations

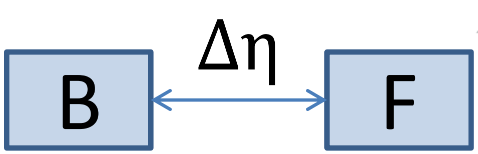


$$b = \frac{\langle n_B n_F \rangle - \langle n_B \rangle^2}{\langle n_B^2 \rangle - \langle n_B \rangle^2}$$

b = 1, maximum correlation
b = 0, no correlation
b = -1, maximum anticorrelation

correlation coefficient b for various energies and rapidity separations Δη between bins B and F

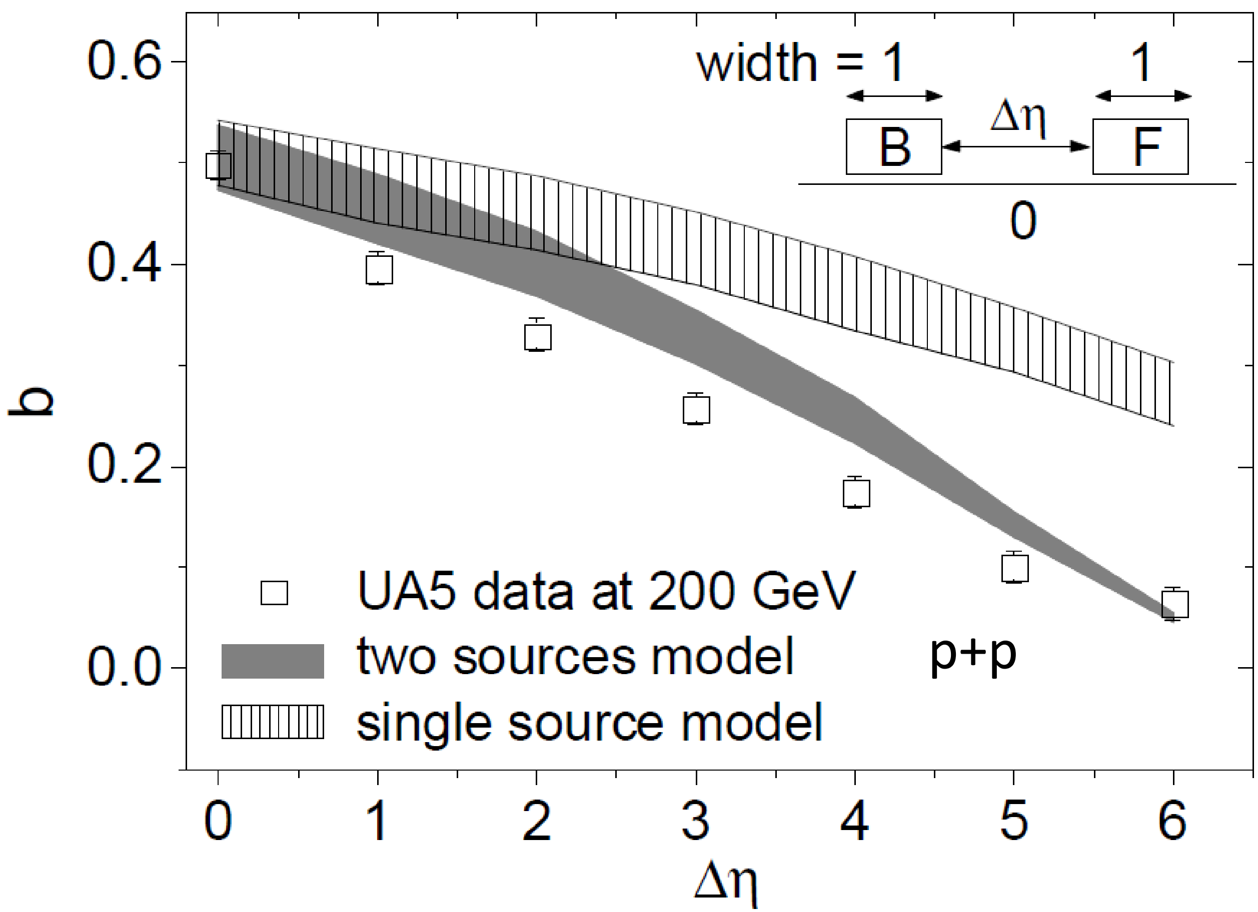




$\Delta\eta|_{max} \approx 10$
at 200 GeV

Forward-backward multiplicity correlation

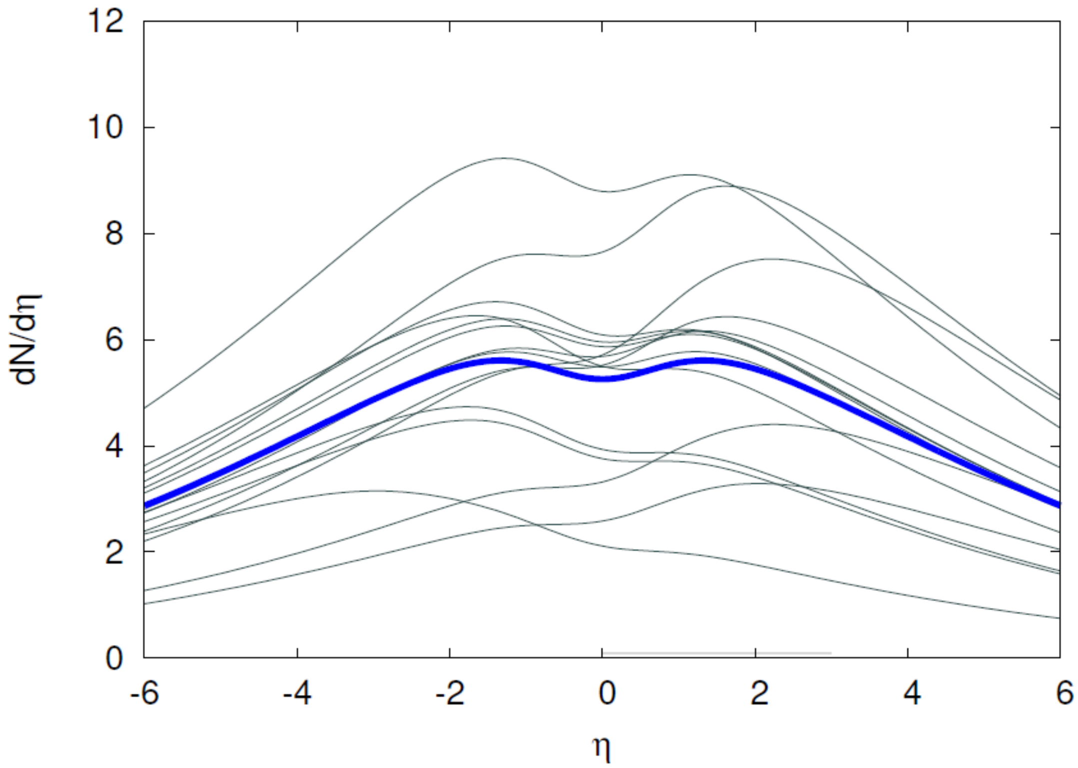
Good in e^+e^- and maybe proton-proton collisions. Difficult in A+A.

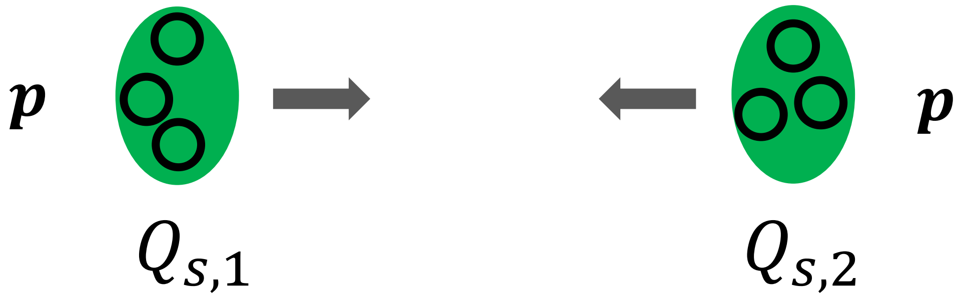


AB, APPB 41 (2010) 2471

p+p is **not** symmetric in rapidity, it is more like p+A

The shape of the fireball fluctuates in rapidity





New source of rapidity correlations

AB, D. Teaney,
PRC 87 (2013) 024906

$$\rho_{\text{event}}(y) = \langle \rho(y) \rangle \left[1 + a_0 + a_1 \frac{y}{Y} + \cdots \right]$$

single particle distribution in an event (neglecting statistical fluctuations)

average single particle distribution

a_0 is rapidity independent fluctuation of fireball as a whole

multiplicity distribution

a_1 is an event-by-event rapidity asymmetry

e.g. asymmetry in the number of left- and right-going constituents (nucleons, quarks, diquarks, etc.) in p+p, p+A and A+A

Y - measurement is from $-Y$ to Y

A.Bialas, AB, K.Zalewski,
PLB 710 (2012) 332

Long (and short) range rapidity correlations

AB, D. Teaney,
PRC 87 (2013) 024906

$$\rho_{\text{event}}(y) = \langle \rho(y) \rangle \left[1 + \sum_{i=0} a_i T_i(y/Y) \right]$$

orthogonal polynomials

$$\frac{C_2(y_1, y_2)}{\langle \rho(y_1) \rangle \langle \rho(y_2) \rangle} = \sum_{i,k} \langle a_i a_k \rangle T_i(y_1/Y) T_k(y_2/Y)$$

$$\boxed{\frac{C_2(y_1, y_2)}{\langle \rho(y_1) \rangle \langle \rho(y_2) \rangle} \sim \langle a_0^2 \rangle + \langle a_1^2 \rangle \frac{y_1 y_2}{Y^2} + \cdots}$$

PRC 95, 064914 (2017)

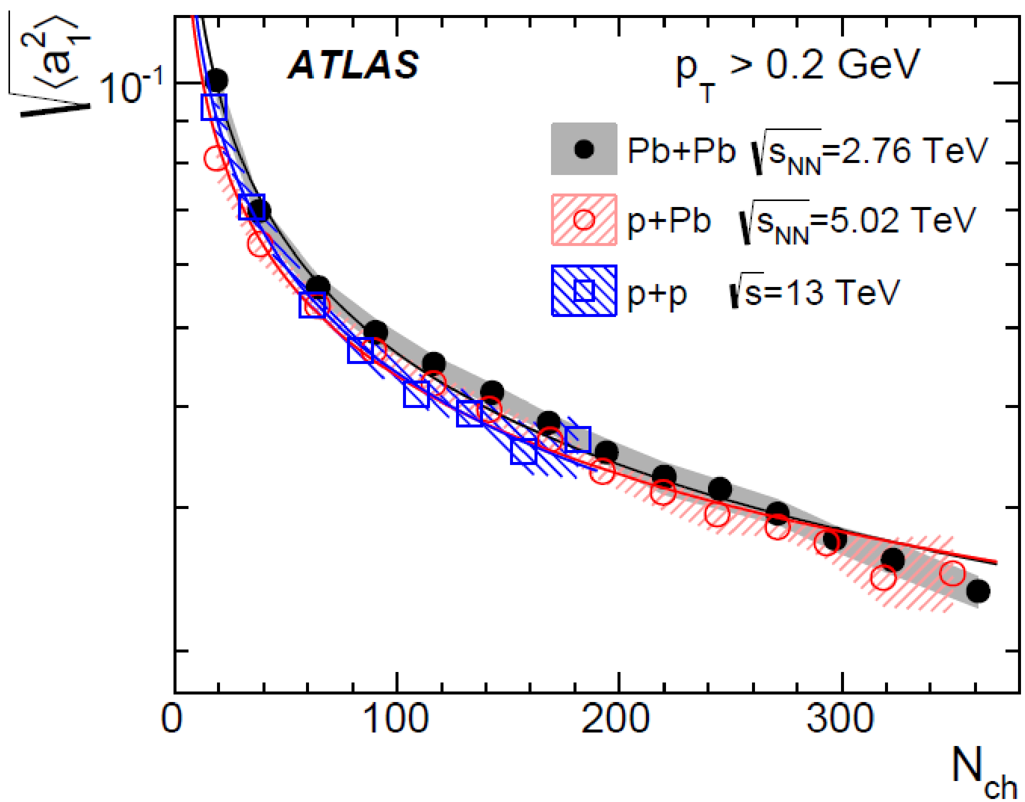
The ATLAS Collaboration

Abstract

Two-particle pseudorapidity correlations are measured in $\sqrt{s_{\mathrm{NN}}} = 2.76$ TeV Pb+Pb, $\sqrt{s_{\mathrm{NN}}} = 5.02$ TeV p+Pb, and $\sqrt{s} = 13$ TeV pp collisions at the LHC, with total integrated luminosities of approximately 7 μb^{-1}, 28 nb^{-1}, and 65 nb^{-1}, respectively. The correlation function $C_{\mathrm{N}}(\eta_1, \eta_2)$ is measured as a function of event multiplicity using charged particles in the pseudorapidity range $|\eta| < 2.4$. The correlation function contains a significant short-range component, which is estimated and subtracted. After removal of the short-range component, the shape of the correlation function is described approximately by $1 + \langle a_1^2 \rangle \eta_1 \eta_2$ in all collision systems over the full multiplicity range. The values of $\sqrt{\langle a_1^2 \rangle}$ are consistent between the opposite-charge pairs and same-charge pairs, and for the three collision systems at similar multiplicity. The values of $\sqrt{\langle a_1^2 \rangle}$ and the magnitude of the short-range component both follow a power-law dependence on the event multiplicity. The η distribution of the short-range component, after symmetrizing the proton and lead directions in p+Pb collisions, is found to be smaller than that in pp collisions with comparable multiplicity.

a_1 as a function of the number of produced particles in $|\eta| < 2.5$ and $p_t > 0.2$ GeV.

PRC 95, 064914 (2017)

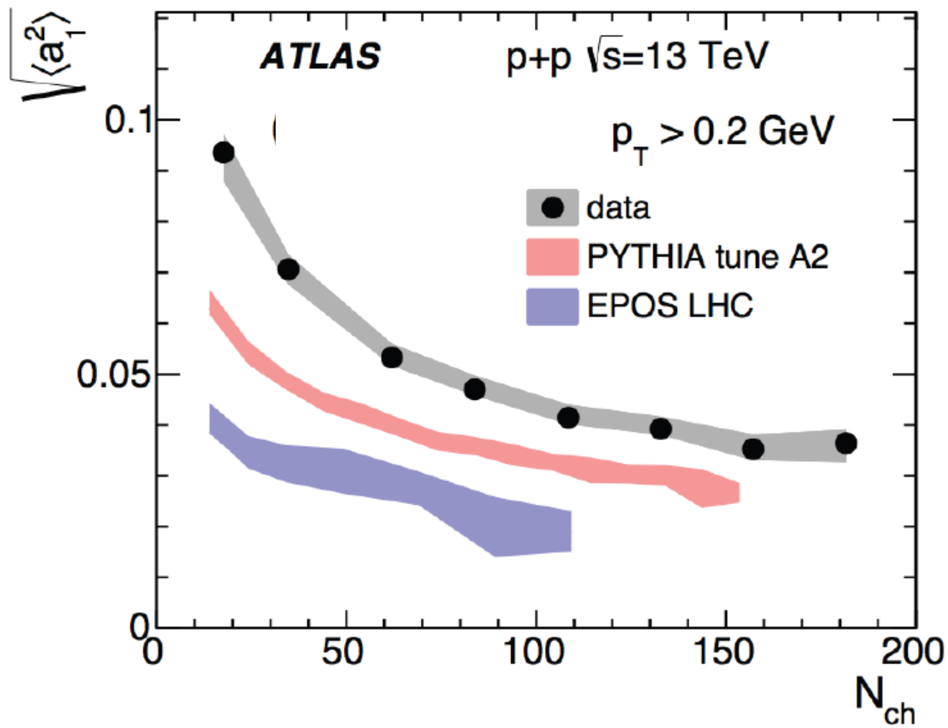


surprising scaling

$$\sqrt{\langle a_1^2 \rangle} \sim \frac{1}{N_{\mathrm{ch}}^{0.5}}$$

Particle sources and their fluctuations seem to be similar in peripheral Pb+Pb, min-bias p+Pb and very central p+p.

PYTHIA and EPOS vs p+p data



taken from
Jiangyong Jia (QM17)

Related papers:

J.Jia, S.Radhakrishnan , M.Zhou, PRC 93, 044905 (2016)
P.Bożek, W.Broniowski, A.Olszewski, PRC 92 (2015) 5, 054913
A.Monnai, B.Schenke, PLB 752 (2016) 317
B.Schenke, S.Schlichting, PRC 94, 044907 (2016)
P.Bożek, W.Broniowski, PRC 93, 064910 (2016)
R.He, J.Qian, L.Huo, 1702.03137
W.Ke, J.Moreland, J.Bernhard, S.Bass, 1610.08490

Multi-particle rapidity correlations

AB, P. Bożek, PRC 93, 024903 (2016)

For example the genuine 4 and 6-particle correlation functions

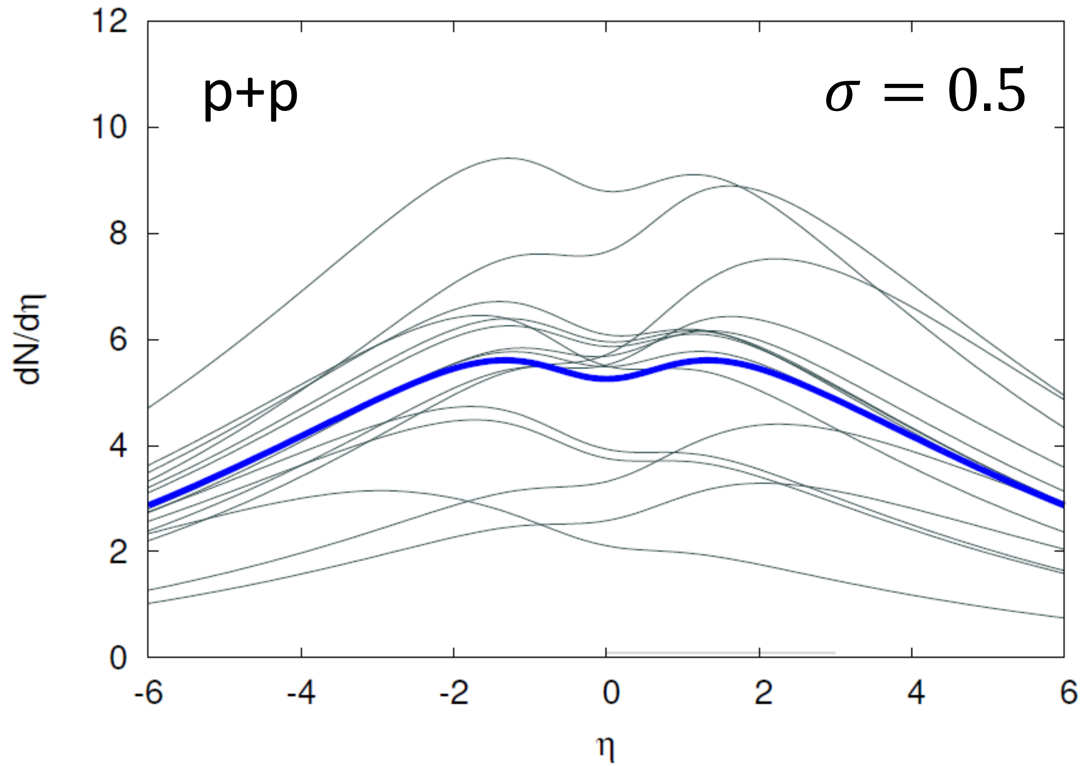
$$\frac{C_4(y_1,\ldots,y_4\ )}{\langle\rho(y_1)\rangle\ldots\langle\rho(y_4)\rangle}=\cdots+\left[\langle a_1^4\rangle-3\langle a_1^2\rangle^2\right]\frac{y_1y_2y_3y_4}{Y^4}+\cdots$$

$$\frac{C_6}{\langle\rho\rangle\ldots\langle\rho\rangle}=\cdots+\left[\langle a_1^6\rangle-15\langle a_1^2\rangle\langle a_1^4\rangle-10\langle a_1^3\rangle^2+30\langle a_1^2\rangle^3\right]\frac{y_1y_2y_3y_4y_5y_6}{Y^6}+\cdots$$

I denote these coefficients by $\langle a_1^4\rangle_{[4]}$ and $\langle a_1^6\rangle_{[6]}$

CGC application

AB, K. Dusling,
PRC 93, 031901 (2016)



$$Q_1^2 = Q_{o,1}^2 e^{+\lambda y}$$

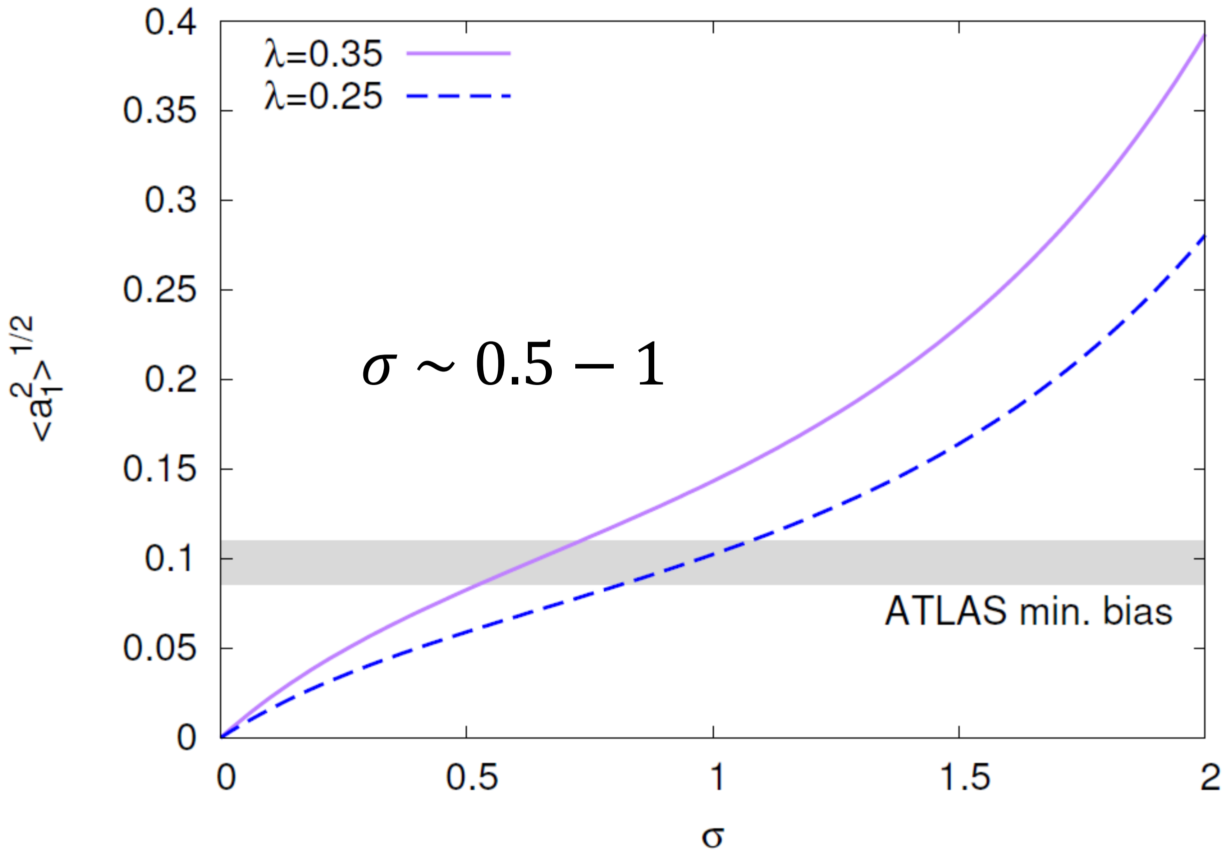
$$Q_2^2 = Q_{o,2}^2 e^{-\lambda y}$$

$$\frac{dN}{dy} \propto S_\perp \mathrm{Min}[Q_1^2, Q_2^2] \left(2 + \ln \frac{\mathrm{Max}[Q_1^2, Q_2^2]}{\mathrm{Min}[Q_1^2, Q_2^2]}\right)$$

$$P[\rho] = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\rho^2}{2\sigma^2}\right] \quad \text{where} \quad \rho \equiv \log\left(\frac{Q^2}{\bar{Q}^{\,2}}\right)$$

C. Marquet, G. Soyez, Bo-Wen Xiao, PLB 639 (2006) 635

AB, K. Dusling, PRC 93 (2016) 031901



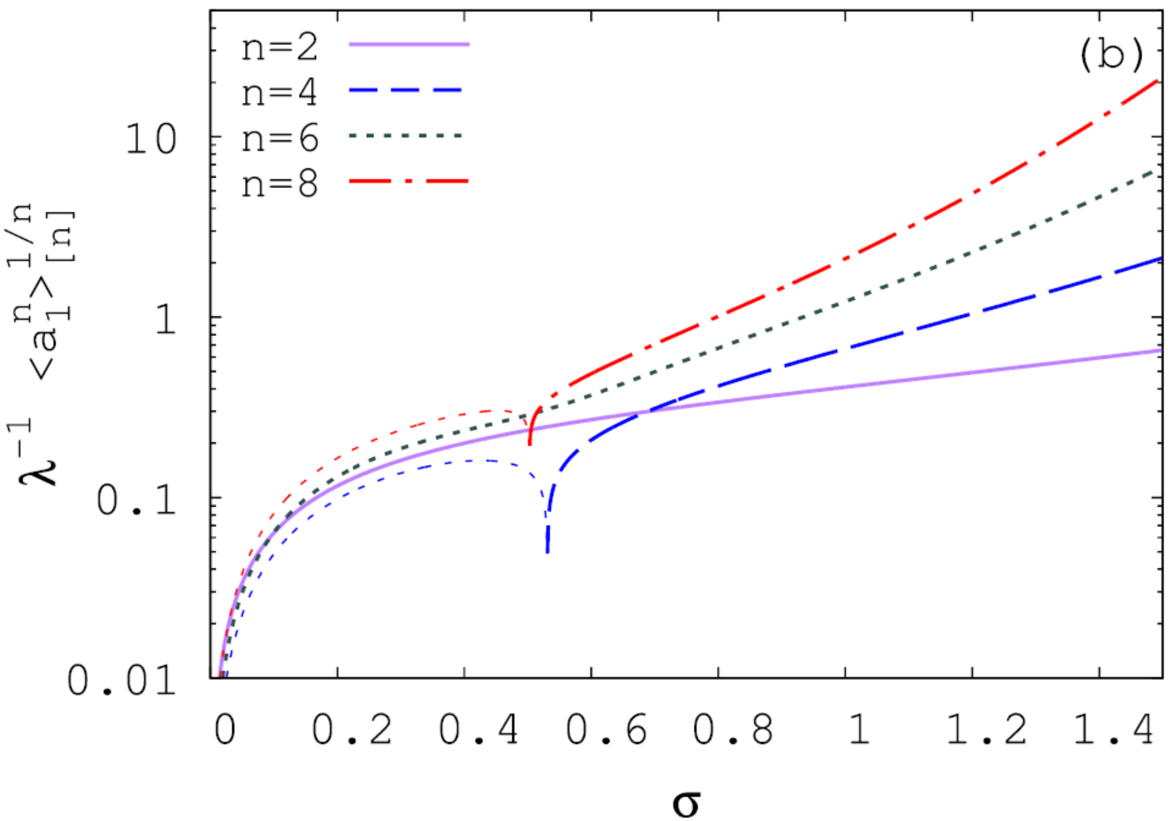
$$\langle a_1^2 \rangle \simeq \frac{\lambda^2\sigma^2}{2} \frac{4\pi\left(1+2\sigma^2\right)\exp\left[\sigma^2\right]\mathrm{Erfc}\left[\sigma\right] - 8\sqrt{\pi}\sigma}{\left(\sqrt{\pi}\left(\sigma^2-2\right)\mathrm{Erfc}\left[\frac{\sigma}{2}\right] - 2\sigma\exp\left[-\frac{\sigma^2}{4}\right]\right)^2}$$

See also:

L.McLerran, M.Praszalowicz, Annals Phys. 372 (2016) 215

L.McLerran, P.Tribedy, NPA 945 (2016) 216

AB, K. Dusling, PRC 94, 044918 (2016)



$$\left\langle a_1^n \right\rangle = \frac{\left[\lambda\sigma\sqrt{\pi}\exp\left(\frac{\sigma^2(n-2)}{4}\right)\right]^n}{\sqrt{\pi}} \frac{n!U\left(\frac{1+n}{2};\frac{1}{2};\frac{n^2\sigma^2}{4}\right)}{\left[\sqrt{\pi}(\sigma^2-2)\,\mathrm{Erfc}\left(\frac{\sigma}{2}\right)-2\sigma\exp\left(-\frac{\sigma^2}{4}\right)\right]^n}$$

U – confluent hypergeometric function

Erfc – complementary error function

AB, 1703.03003 (PRD, in press)

Similar technique for $p_t - p_t$ correlations

$$P_t(y) = \frac{1}{N} \sum_{i=1}^{N} p_t^{(i)},$$

average p_t in an event

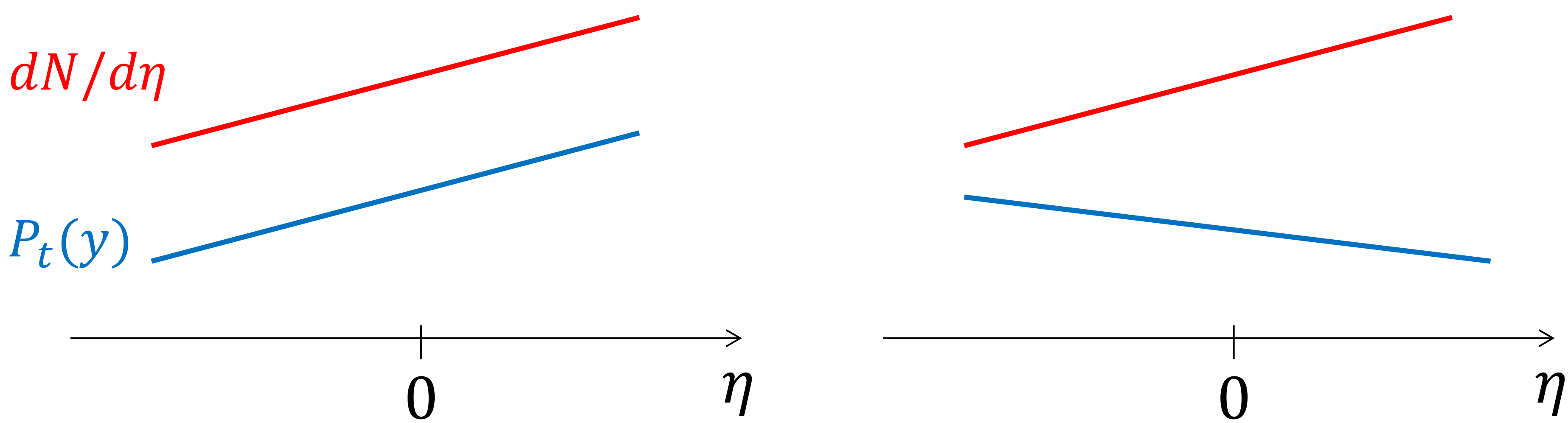
$$\frac{P_t(y)}{\langle P_t(y) \rangle} = 1 + b_0 + b_1 y + \ldots,$$

$$\frac{C_{[P,P]}(y_1, y_2)}{\langle P_t(y_1) \rangle \langle P_t(y_2) \rangle} = \langle b_0^2 \rangle + \underline{\langle b_1^2 \rangle} y_1 y_2 + \ldots,$$

$$C_{[P,P]}(y_1, y_2) \equiv \langle P_t(y_1) P_t(y_2) \rangle - \langle P_t(y_1) \rangle \langle P_t(y_2) \rangle .$$

or more interesting $N - p_t$ correlations

Hydro CGC



$a_1 > 0, b_1 > 0$ $\quad$ $a_1 > 0, b_1 < 0$

$\langle a_1 b_1 \rangle > 0$ $\quad$ $\langle a_1 b_1 \rangle < 0$

P.Bozek, AB, V.Skokov, PLB 728 (2014) 662
K.Deja, K.Kutak, PRD 95 (2017), 114027
F.Duraes, A.Giannini, V.Goncalves, F.Navarra, PRC 94, 024917 (2016) [different CGC conclusion]

AB, 1703.03003 (PRD, in press)

Rapidity $N - p_t$ correlations

$$C_{[N,P]}(y_1, y_2) \equiv \langle N(y_1) P_t(y_2) \rangle - \langle N(y_1) \rangle \langle P_t(y_2) \rangle \,,$$

$$\frac{C_{[N,P]}(y_1, y_2)}{\langle N(y_1) \rangle \langle P_t(y_2) \rangle} = \langle a_0 b_0 \rangle + \underline{\langle a_1 b_1 \rangle} \, y_1 y_2 + \ldots$$

In general

$$\frac{C_{[N,P]}(y_1, y_2)}{\langle N(y_1) \rangle \langle P_t(y_2) \rangle} = \sum\nolimits_{i,k} \langle a_i b_i \rangle \, T_i(y_1) T_k(y_2),$$

Conclusions

Universality of wounded source emission function?

Consistent with forward-backward rapidity correlation

New source of long-range rapidity correlations and ATLAS data

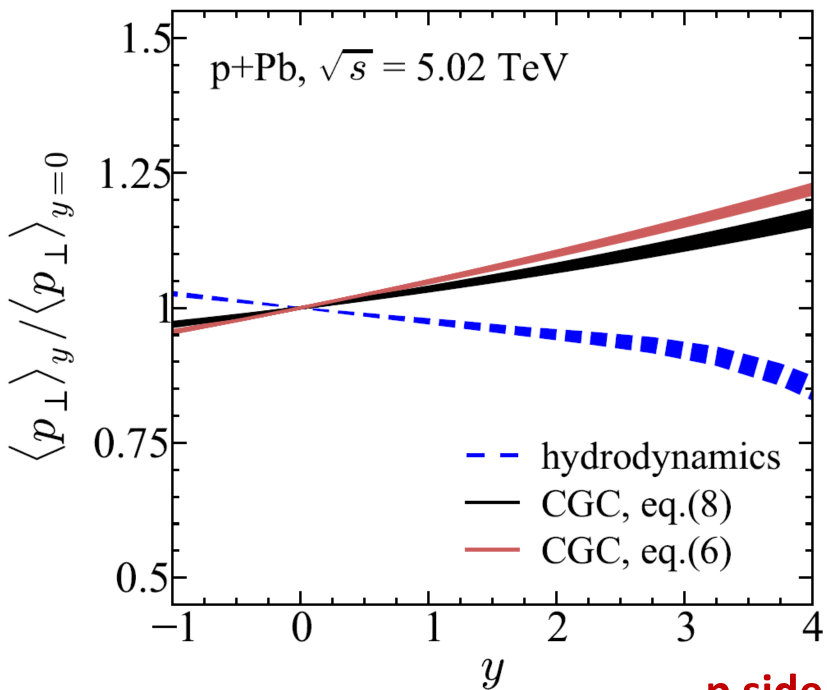
Transverse-momentum multiplicity correlations

It would be great to have $\langle a_1^2 \rangle$, $\langle b_1^2 \rangle$ and $\langle a_1 b_1 \rangle$ in p+p, p+Pb and Pb+Pb collisions

Backup

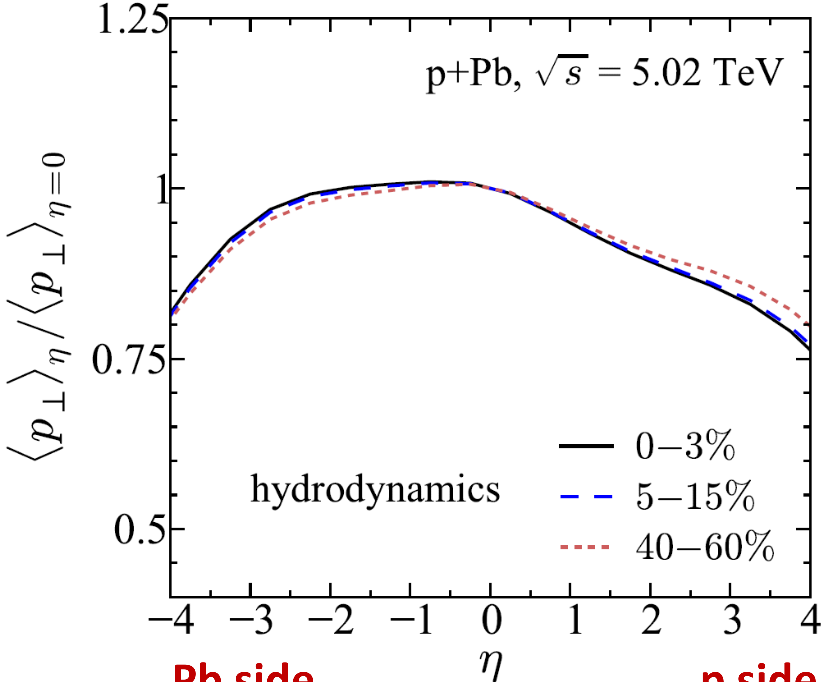
$\langle p_T \rangle$ versus η on proton side

$\langle p_t \rangle \sim Q_s^A$, Q_s^A is growing with y



p side

less stuff on proton side

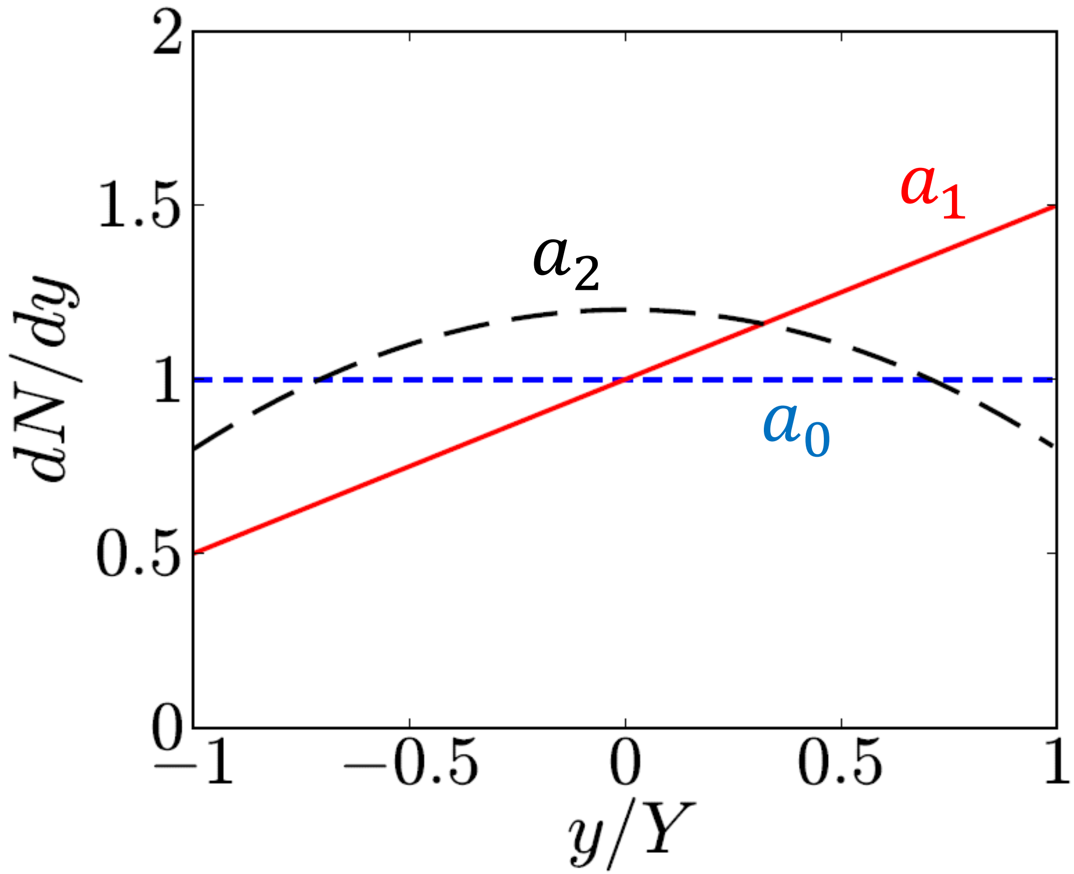


Pb side

p side

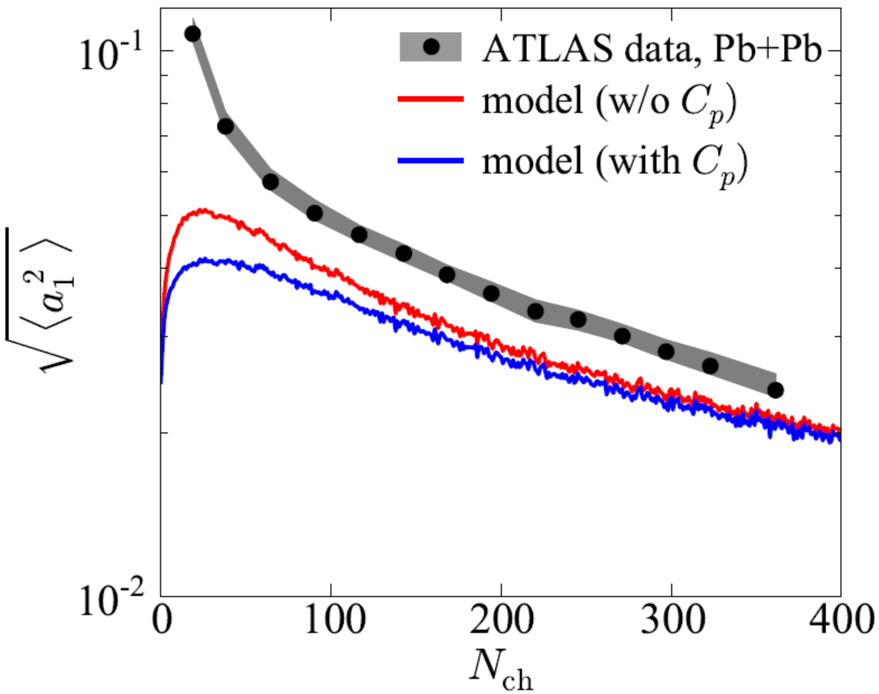
P.Bozek, AB, V.Skokov, PLB 728 (2014) 662

Fireball shape in rapidity can fluctuate

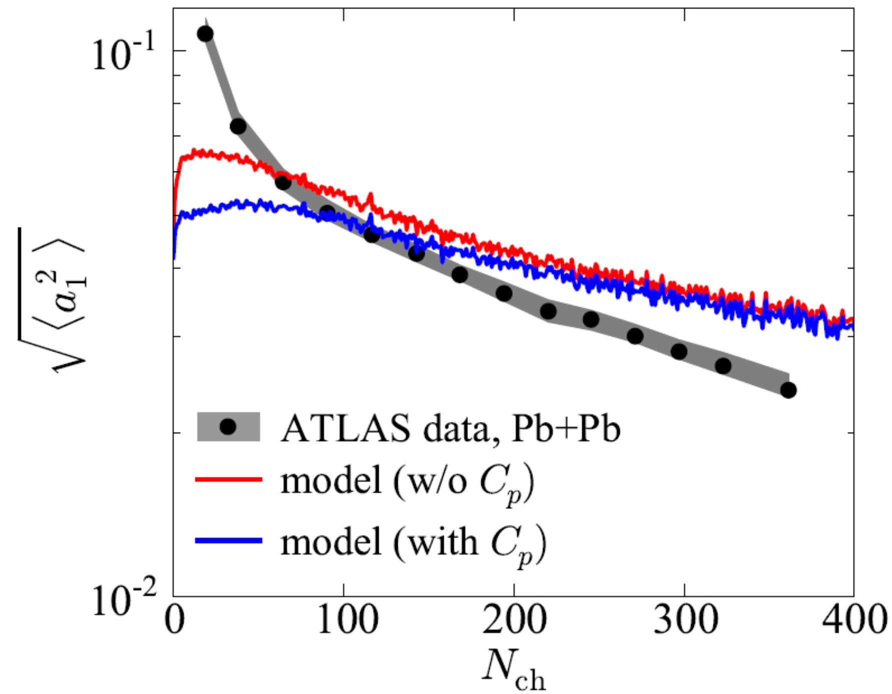


here $\mathrm{dN/dy} \equiv \rho_{\mathrm{event}}(y)$

So let’s expand in the orthogonal polynomials wounded nucleon model



wounded quark model

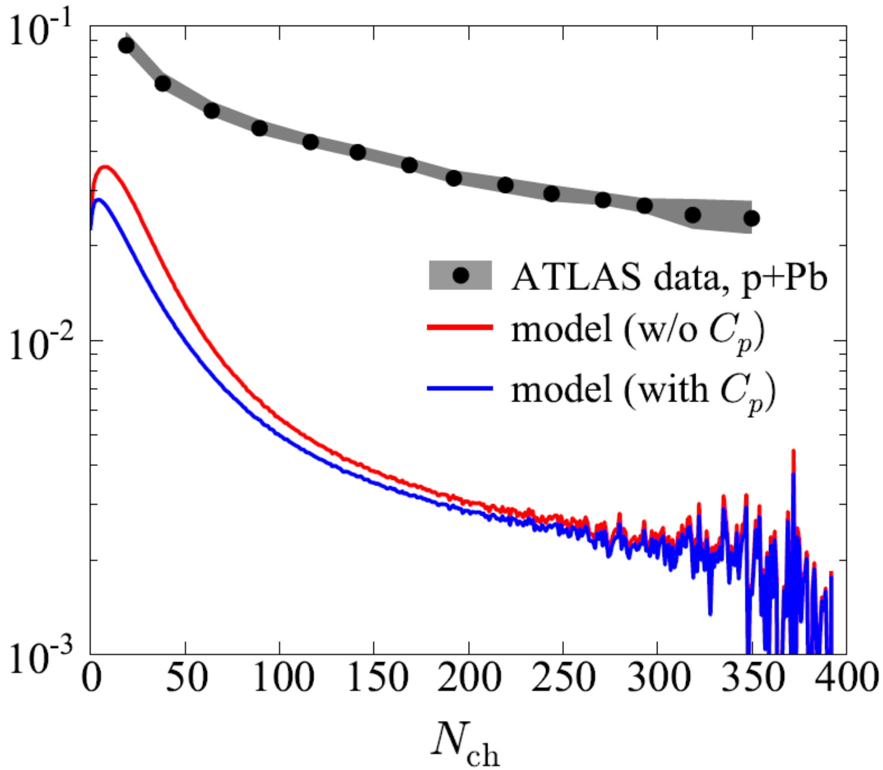


Contribution from fluctuation in the number of left and right going nucleons (quarks) at a given number of produced particles N_{ch}

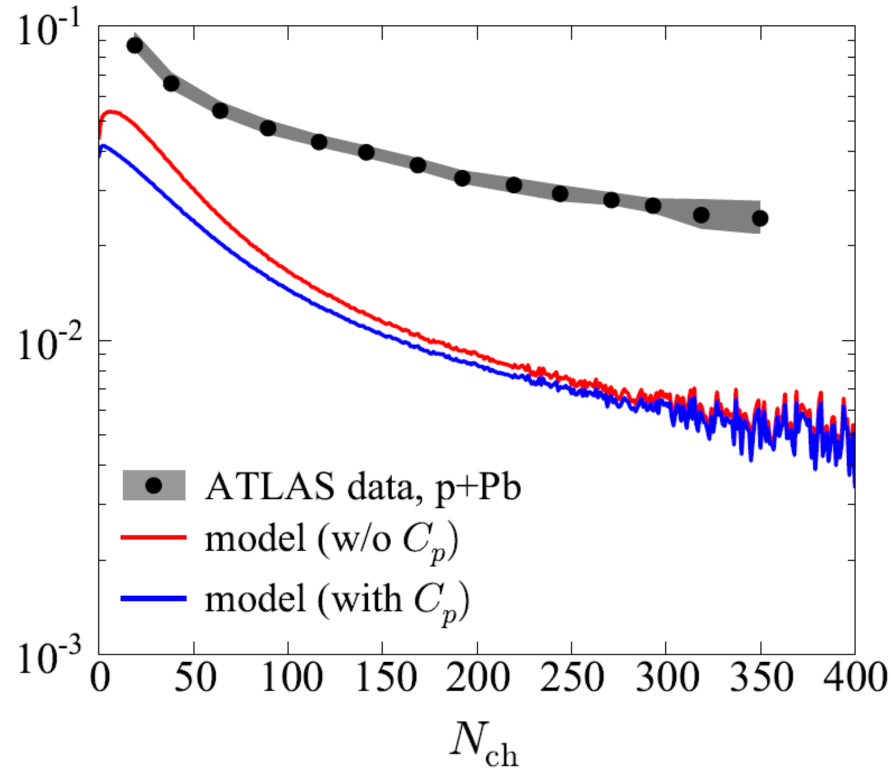
$$\langle a_1^2 \rangle = \frac{b^2}{a^2} \frac{\langle (w_L - w_R)^2 \rangle}{\langle w_L + w_R \rangle^2}$$

symmetric
A+A coll.

wounded nucleon model

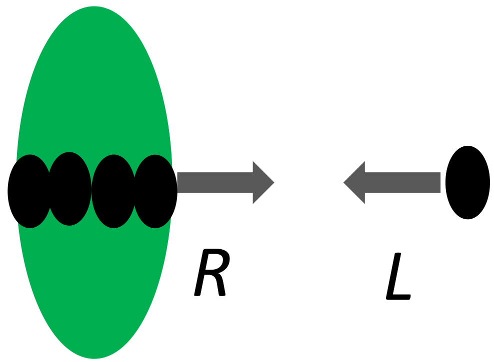


wounded quark model



$$\langle a_1^2 \rangle \approx 4\,\frac{b^2}{a^2}\,\frac{\langle w_R^2 \rangle - \langle w_R \rangle^2}{(1+\langle w_R \rangle)^4}$$

and slightly more complicated for the wounded quark model



Multi-particle correlation functions

AB, P. Bożek, PRC 93, 024903 (2016)

$$C_2(y_1,y_2) = \rho_2(y_1,y_2) - \rho(y_1)\rho(y_2)$$

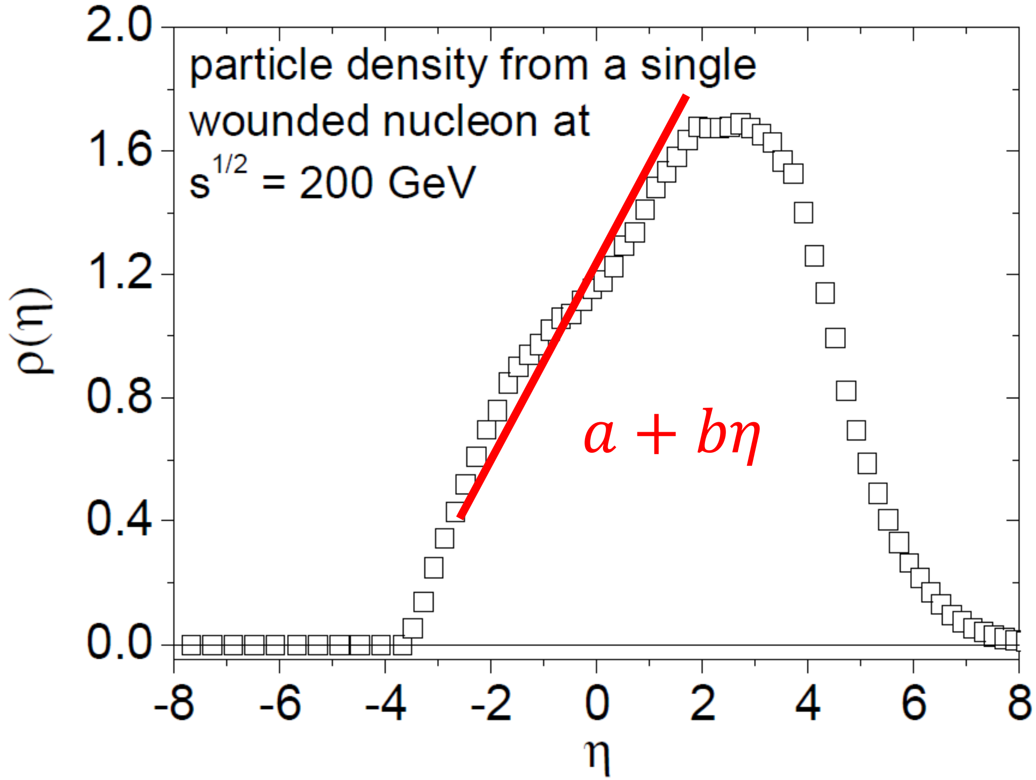
$$\begin{aligned}\rho_3(y_1,y_2,y_3) = {} & \rho(y_1)\rho(y_2)\rho(y_3) + \rho(y_1)C_2(y_2,y_3) + \rho(y_2)C_2(y_1,y_3) + \\ & \rho(y_3)C_2(y_1,y_2) + C_3(y_1,y_2,y_3),\end{aligned}$$

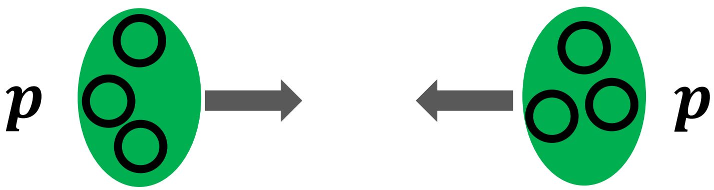
$$\begin{aligned}\rho_4(y_1,y_2,y_3,y_4) = {} & \rho(y_1)\rho(y_2)\rho(y_3)\rho(y_4) + \rho(y_1)\rho(y_2)C_2(y_3,y_4) + \rho(y_1)\rho(y_3)C_2(y_2,y_4) + \\ & \rho(y_1)\rho(y_4)C_2(y_2,y_3) + \rho(y_2)\rho(y_3)C_2(y_1,y_4) + \rho(y_2)\rho(y_4)C_2(y_1,y_3) + \\ & \rho(y_3)\rho(y_4)C_2(y_1,y_2) + \rho(y_1)C_3(y_2,y_3,y_4) + \rho(y_2)C_3(y_1,y_3,y_4) + \\ & \rho(y_3)C_3(y_1,y_2,y_4) + \rho(y_4)C_3(y_1,y_2,y_3) + C_2(y_1,y_2)C_2(y_3,y_4) + \\ & C_2(y_1,y_3)C_2(y_2,y_4) + C_2(y_1,y_4)C_2(y_2,y_3) + C_4(y_1,y_2,y_3,y_4).\end{aligned}$$

$$\rho_5 = \rho\rho\rho\rho\rho + \underbrace{\rho C_4}_{5} + \underbrace{\rho\rho C_3}_{10} + \underbrace{\rho\rho\rho C_2}_{10} + \underbrace{\rho C_2 C_2}_{15} + \underbrace{C_2 C_3}_{10} + C_5$$

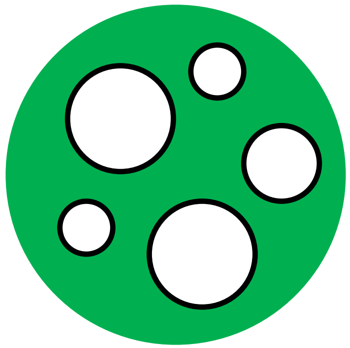
$$\begin{aligned}\rho_6 = {} & \rho\rho\rho\rho\rho\rho + \underbrace{\rho C_5}_{6} + \underbrace{\rho\rho C_4}_{15} + \underbrace{\rho\rho\rho C_3}_{20} + \underbrace{\rho\rho\rho\rho C_2}_{15} + \underbrace{\rho C_2 C_3}_{60} + \underbrace{\rho\rho C_2 C_2}_{45} + \underbrace{C_2 C_4}_{15} + \\ & \underbrace{C_3 C_3}_{10} + \underbrace{C_2 C_2 C_2}_{15} + C_6,\end{aligned}$$

Wounded nucleon (quark, quark-diquark) model





A.Bialas, M.Bleszynski, W.Czyz, NPB 111 (1976) 461
A.Bialas, W.Czyz, APPB 36 (2005) 905
A.Bialas, AB, PRC 77 (2008) 034908



Proton as a set of domains in which Q_s fluctuate independently

Superposition of independent log-normal distributions can be approximated by log-normal

$$\sigma^2 = \ln\left[\frac{1}{N_\mathrm{d}}\left(e^{\sigma_\mathrm{d}^2} - 1\right) + 1\right]$$

N_d - number of domains

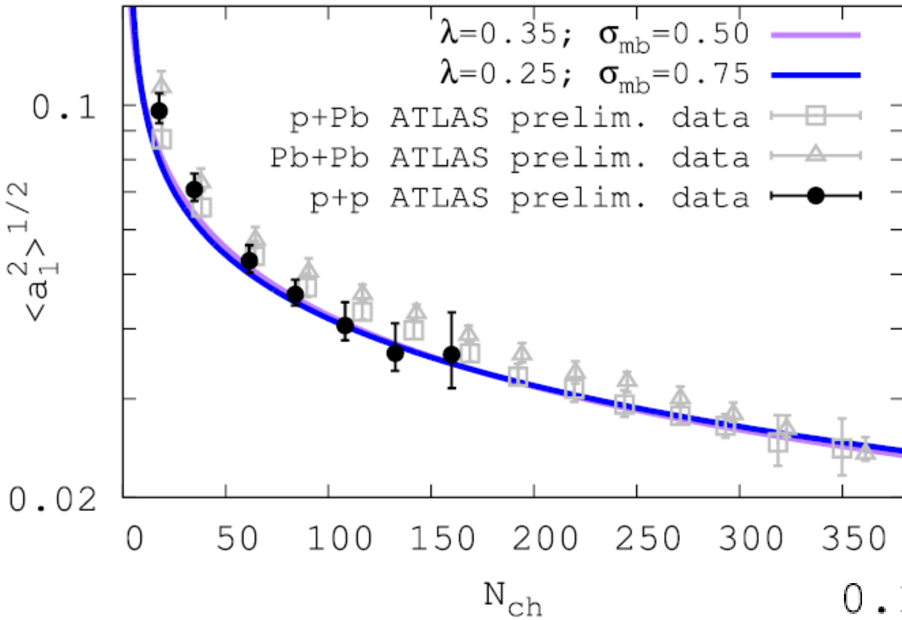
σ_d - log-normal width of a domain

σ - effective width from all domains

$$\sigma^2 \approx \frac{\sigma_\mathrm{d}^2}{N_\mathrm{d}}$$

$$\sigma^2 = \frac{N_\mathrm{ch}^\mathrm{mb}}{N_\mathrm{ch}}\sigma_\mathrm{mb}^2$$

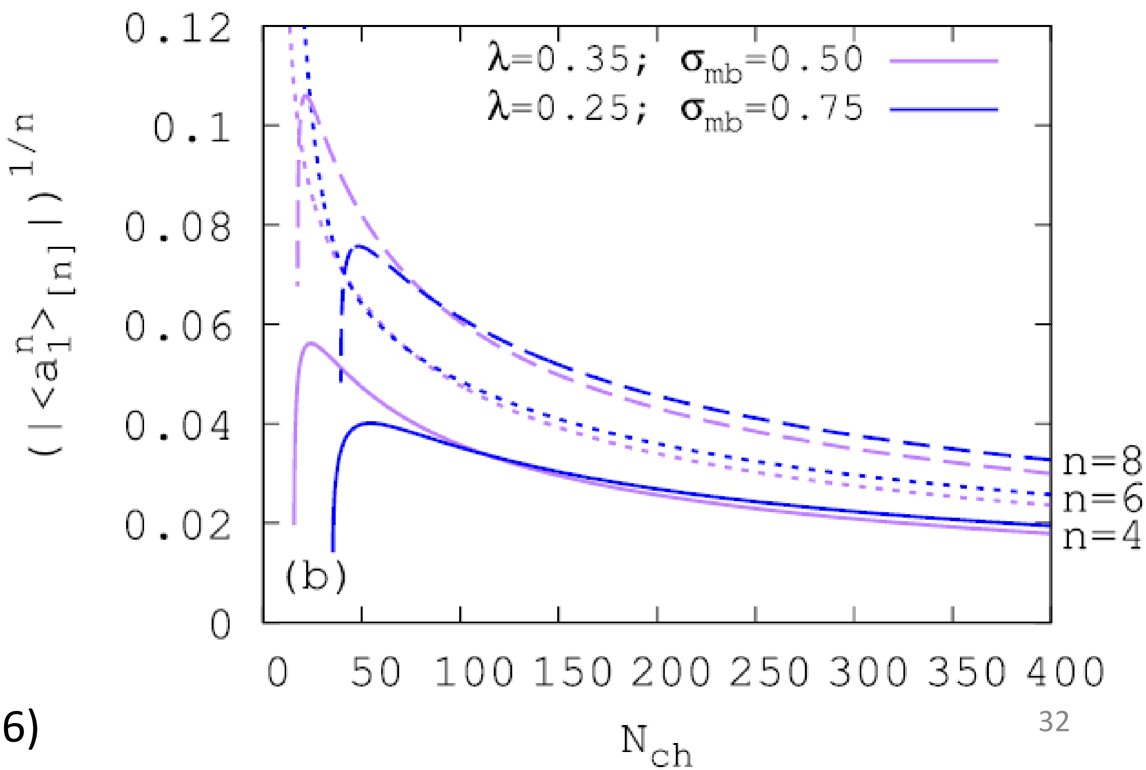
mb – minimum bias



postdiction

p+Pb and Pb+Pb is a puzzle

“prediction”

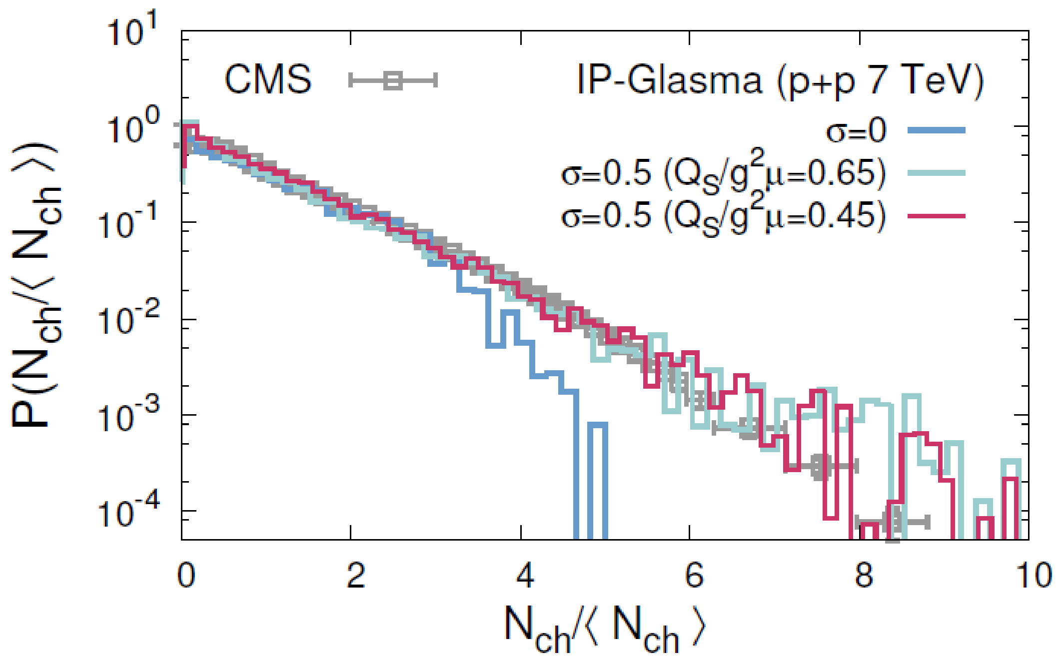


AB, K. Dusling, PRC 94, 044918 (2016)

We conclude that $\sigma \sim 0.5 - 1$

$$P[\rho] = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\rho^2}{2\sigma^2}\right] \quad \text{where} \quad \rho \equiv \log\left(\frac{Q^2}{\bar{Q}^2}\right)$$

Tails of multiplicity distributions are effected



L. McLerran, P. Tribedy, NPA 945 (2016) 216

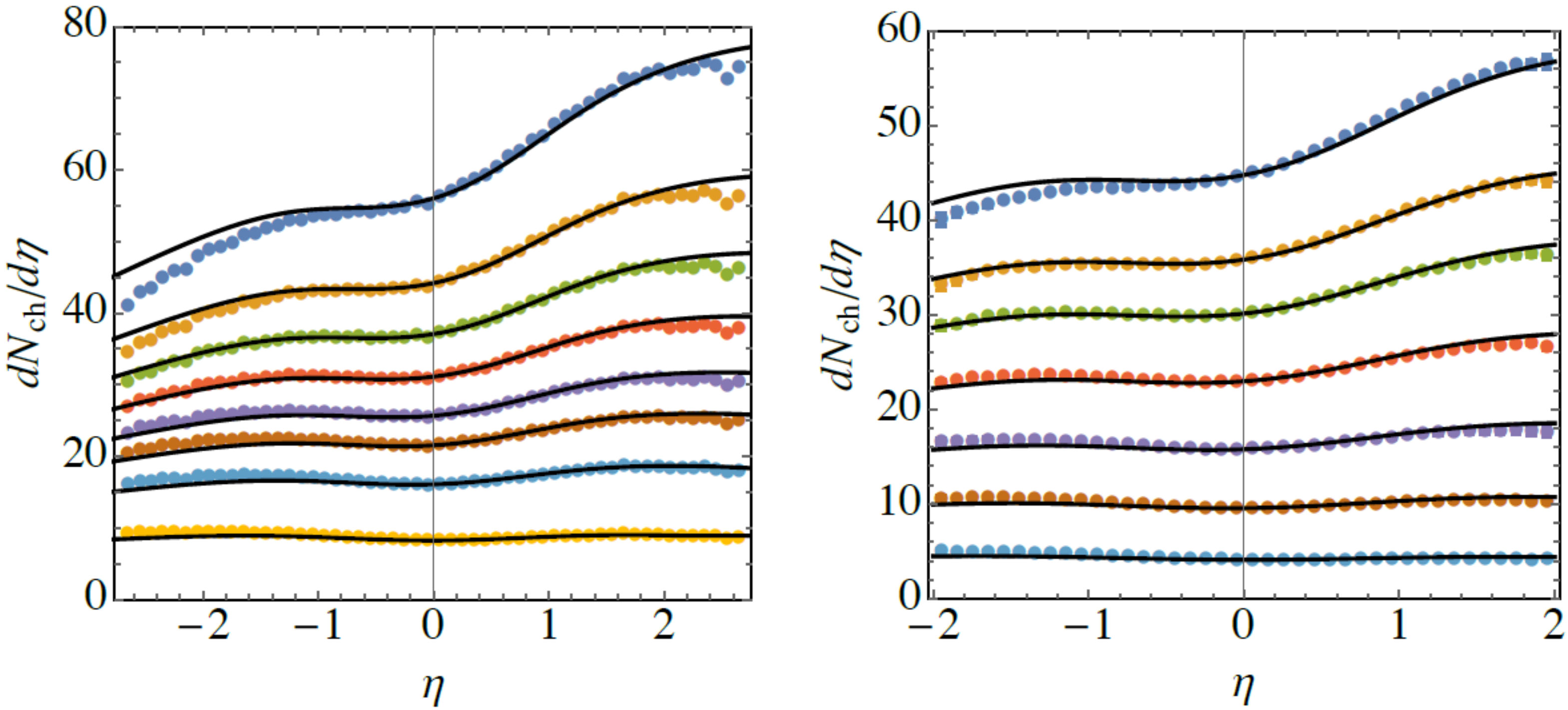
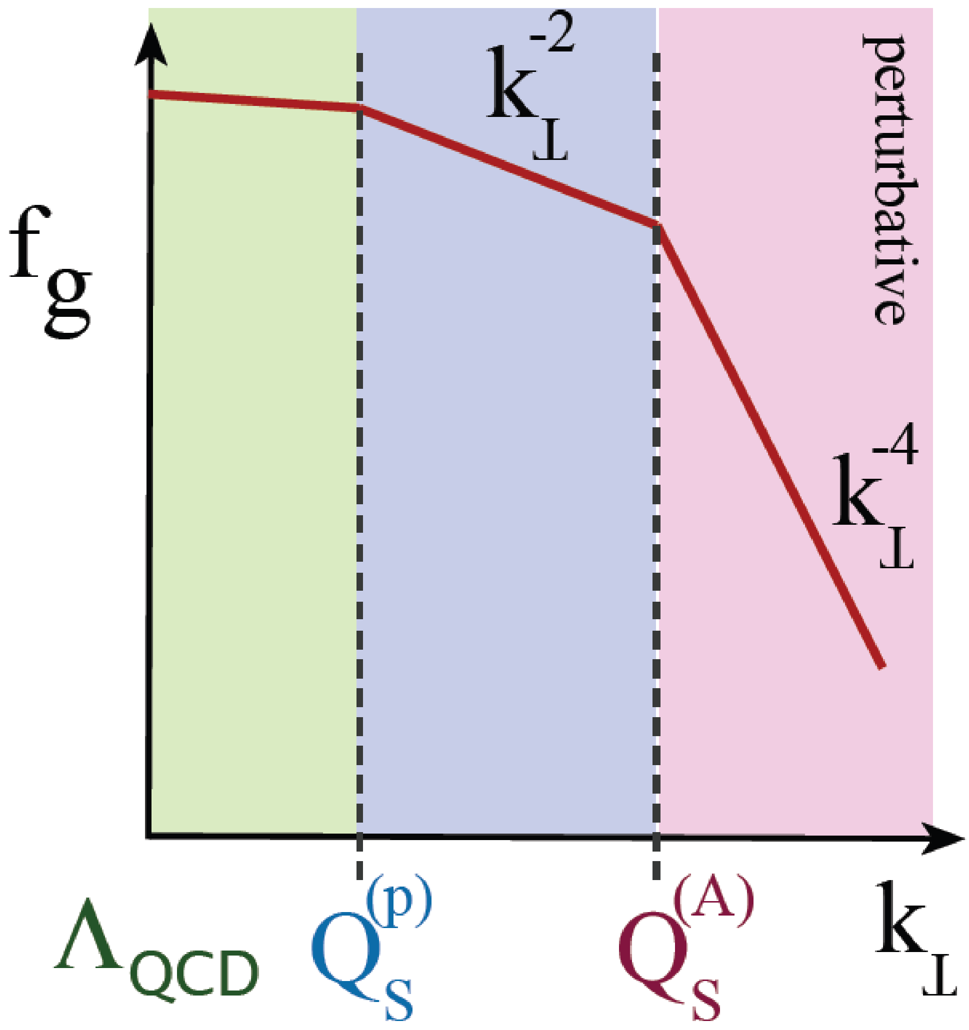


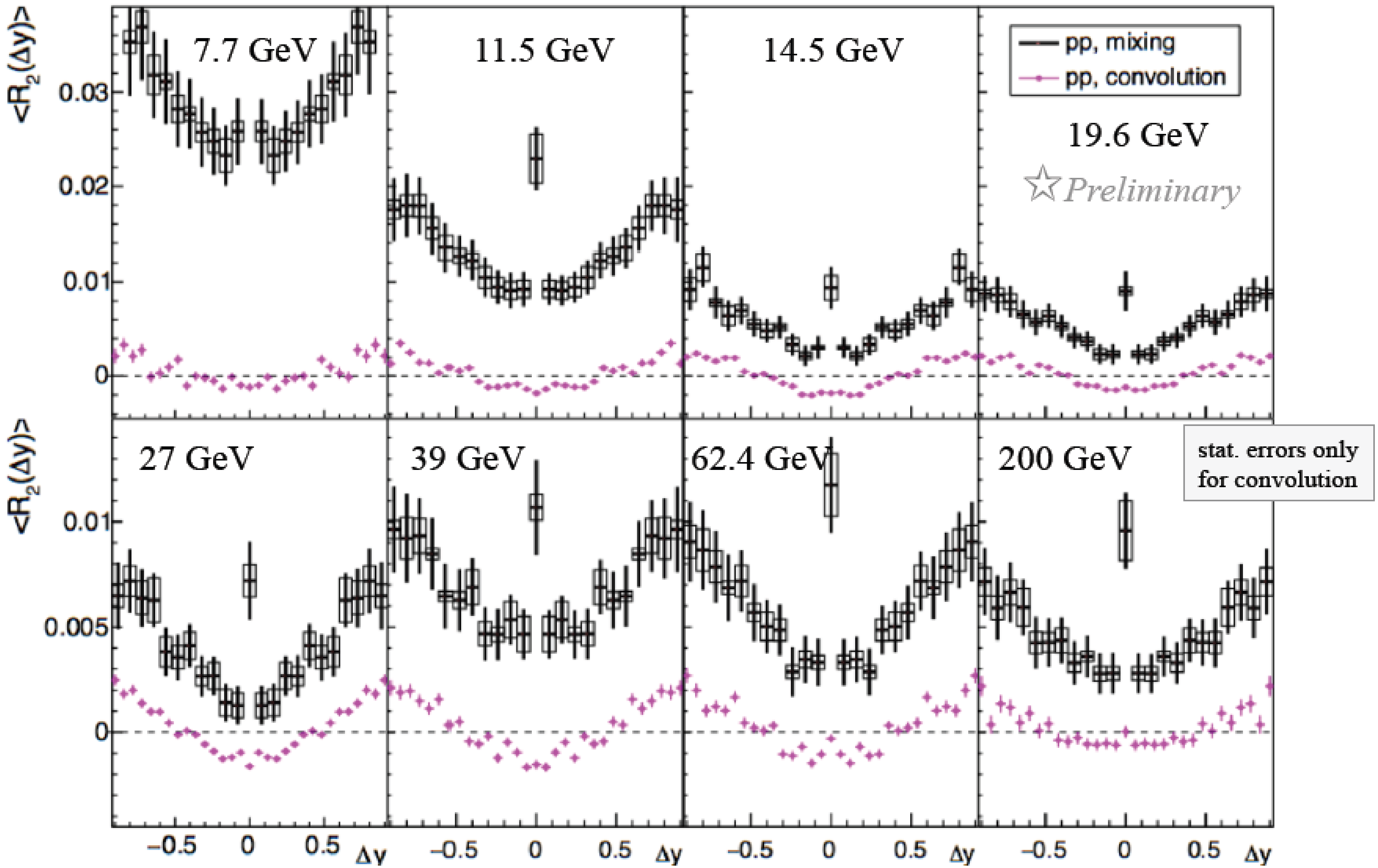
Figure 6: Multiplicity dependence on pseudo-rapidity η for the fluctuating case with $\sigma = 1.55$. Left plot corresponds to ATLAS whereas the right one to ALICE. Different curves correspond to the centrality classes defined in Tables 1 and 2.

L.McLerran, M.Praszalowicz, Annals Phys. 372 (2016) 215

CGC in asymmetric systems, two scales



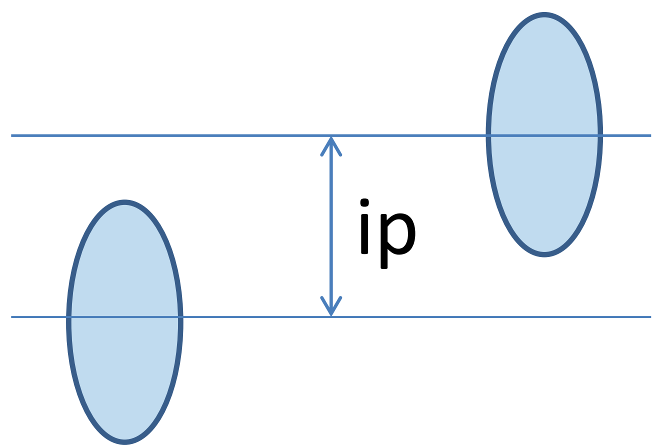
R2(Δy) for LS protons vs. $\sqrt{s_{NN}}$, 0-5% central, convolution & mixing



Better control of finite multiplicity effects from convolution
LS proton anticorrelation for Δy~0. Weak beam energy dependence.

b is difficult to study in A+A collisions because of impact parameter fluctuations

large number of particles in B and F



small number of particles in B and F