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# Determination of proton-deuteron scattering lengths

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**Abstract**

An energy dependent phase-shift analysis of p + d elastic scattering below the deuteron breakup threshold has been conducted. The analysis indicates the existence of a singularity in the doublet S-wave effective range function, resulting in a very small value for the scattering length $^2a_{pd}$. The inferred value of the quartet S-wave scattering length $^4a_{pd}$ is in good agreement with theoretical predictions. 

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One of the most serious manifestations of gaps in our understanding of the three-nucleon system is the discrepancy between theoretical and phenomenological values of the p–d S-wave scattering lengths. Because the dominant channel in the $^3$He ground state is the doublet S-wave channel, one expects that the $^3$He binding energy and the $^2S_{1/2}$ scattering length calculated with a given potential model should be strongly correlated; the approximately linear relations between calculated values of the trinucleon binding energy and various parameters characterizing the three-nucleon system, including the doublet S-wave scattering length, are known as the ''Phillips lines'' [1].

At present, ab initio calculations of these quantities do not yield the correct values for either the n–d or p–d system. However, experimental values for the triton binding energy and the n–d S-wave scattering lengths can be replicated by theory if the NN potential model is supplemented by a three nucleon interaction whose strength is governed by a meson range cut-off parameter $\Lambda$. The value of $\Lambda$ is not well-determined experimentally and in practice it is treated as an adjustable parameter [2]. Applying the same procedure to the p–d system, one can adjust $\Lambda$ to reproduce both the $^3$He binding energy and, using a modern NN potential like AV18 which includes charge dependent forces, the binding energy difference between $^3$He and $^3$H [3]. The experimental

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scattering lengths are *not* reproduced, however, and the question of why they are not is an important one for nuclear physics.

A consensus has emerged that these discrepancies may be the result of a singularity in the doublet S-wave effective range function near the two-body threshold that hinders one's ability to infer the scattering length on the basis of measurements made at scattering energies greater than a few hundred keV in the center-of-mass frame. It is only at lower energies that the curvature in the effective range function becomes evident. The doublet S-wave effective range function in the n–d system is known to exhibit such a singularity [4]. The existence of an analogous singularity in the p + d system was first posited by Chen et al., who solved the configuration space Faddeev equations at a series of finite energies [1]. These results were verified by other authors using a variety of potentials as input to momentum space calculations [5–7]. Kievsky et al. [8] have investigated the sub-breakup p + d continuum using the hyperspherical harmonic formalism with the AV14 and AV18 potentials, with and without the Urbana three-body force (UR3NF). The scattering lengths they calculated were consistent with those found by previous theoretical investigations; i.e., they were very nearly zero. By contrast, experimental results for ${}^2a_{\rm pd}$ obtained from previous analyses are significantly larger [4,9,10].

The discrepancy between theoretical and phenomenological results for the quartet S-wave scattering length is especially troubling. The requirement that the mixed-symmetry space and isospin components of the ${}^4S_{3/2}$ amplitudes form a completely antisymmetric combination implies that the interaction in this channel is largely determined by n + p scattering and the properties of the deuteron. Theoretical results for the p–d scattering lengths are given in Table 1. In the calculations of Ref. [7], the potential was adjusted to replicate the n-d scattering length ${}_2a_{\rm nd}$. In the other calculations cited, the potential was adjusted to reproduce the ${}^3$He binding energy. Experimental scattering lengths inferred from previous phase shift analyses are shown in Table 2.

Table 1
Theoretical determinations of the p–d S-wave scattering lengths

| Reference | ${}^2a_{\rm pd}$ | ${}^4a_{\rm pd}$ |
|---|---|---|
| [11] | $\approx 0.0$ | 13.76 |
| [5] | 0.257 | 13.68 |
| [7] | $-0.1$ | |
| [8] | 0.024 | 13.8 |

Table 2
Experimental determinations of the p–d S-wave scattering lengths

| Reference | ${}^2a_{\rm pd}$ | ${}^4a_{\rm pd}$ |
|---|---|---|
| [4] | $1.3 \pm 0.2$ | $11.4^{+1.8}_{-1.2}$ |
| [9] | $2.73 \pm 0.10$ | $11.88^{+0.4}_{-0.1}$ |
| [10] | $4.0^{+1.0}_{-0.67}$ | $11.11^{+0.25}_{-0.24}$ |

The phase shift analysis reported here is possible because numerous high quality measurements of p–d observables have been performed in the last few years. In particular, differential cross sections at energies much lower than previously available were recently measured at TUNL by Black et al. [12]. These data were obtained at $E_{\rm cm} = 163$ and 211 keV with unpolarized proton beam from the TUNL polarized ion source [13] accelerated through the TUNL minitandem accelerator [14] and incident on a target prepared by plasma-assisted chemical vapor deposition of deuterated methane gas. More complete experimental details are found in [12]. In addition, high precision measurements of $\mathrm{d}\sigma/\mathrm{d}\Omega$, and vector and tensor analyzing powers were obtained at TUNL by Brune et al. [15]. At center of mass energies between 1.3 MeV and the break-up threshold, similar sets of observables were measured by groups from Wisconsin [16] and Tokyo [17]. These new measurements, especially the precise data at the lowest energies, allowed us to fruitfully perform an energy-dependent phase shift analysis between $E_{\rm cm} = 0$ and 2 MeV.

The data base used in the phase shift analysis is listed in Table 3. The cross sections from Refs. [20] and [12] were measured only relatively. However, as the combination of the constraints provided by the effective range expansion and the scale set by the Coulomb force provides the absolute normalization, the normalization was left as an adjustable parameter in the analysis for all cross sections. The cross section renormalization factor was in all cases less than 5%.

Theoretical investigations imply that there is significant $J$-dependence to the low energy p + d elas-

Table 3
Data used in the present energy-dependent phase shift analysis

| $E_{cm}$ [MeV] | Observables | Reference |
|---|---|---|
| 2.0 | $A_y$, $\imath T_{11}$, $T_{20}$, $T_{21}$, $T_{22}$ | [16] |
| 2.0 | $\sigma(\theta)$ | [18] |
| 2.0, 1.7 | $\sigma(\theta)$, $A_y$, $\imath T_{11}$, $T_{20}$, $T_{21}$, $T_{22}$ | [17] |
| 1.3 | $\sigma(\theta)$, $A_y$ | [17] |
| 0.67, 0.60, 0.53 | $\sigma(\theta)$, $A_y$ | [19] |
| 0.47, 0.40, 0.33, 0.27 | $\sigma(\theta)$ | [19] |
| 0.432 | $T_{20}$, $T_{21}$, $\sigma(\theta)$, $\imath T_{11}$, $A_y$ | [15,8,20] |
| 0.211, 0.163 | $\sigma(\theta)$ | [12] |

tic phase shifts. It has also been found, through attempts to fit data by performing phase shift analyses at single energies, that a large number of partial waves contribute to the reaction mechanism, at least near the three-body breakup threshold. Therefore, our analysis included all $J$-split phases up to $J^\pi = \frac{11}{2}^+$ and $J^\pi = \frac{9}{2}^-$. For the purpose of determining the S-wave scattering lengths, all of the input parameters except for the S and P wave phase shifts, and the $\eta^+_{1/2}$ and the negative parity $\epsilon$ mixing parameters were fixed to theoretical values calculated using the AV18 + UR3NF potential. By varying the number of parameters in the fit, it was found that the extracted values of the scattering lengths were not significantly different regardless of whether one fixed all but this limited set of parameters or permitted all of the parameters to vary freely. The principal effect of restricting the set of freely varying parameters is to increase the $\chi^2$/degree of freedom for the fit from about 2.5 to 4.0.

Except for the doublet S-wave, the eigenphase-shifts ${}^{2S+1}\delta_L^J$ at different energies (momenta) were constrained to satisfy the effective range expansion, according to which the Coulomb-modified effective range function, defined by

$$ {}^{2S+1}K_L^J(k) = C_L^2 k^{2L+1}\left(\cot\left({}^{2S+1}\delta_L^J\right) + \frac{2\eta h(\eta)}{C_L^0}\right) $$

is presumed to have a Taylor series expansion in powers of $k^2$. The Coulomb parameters $\eta$, $C_L^2$, and $h(\eta)$ are defined as in Ref. [21]. The ${}^2S_{1/2}$ phase shifts were left unconstrained in the analysis. The reason that an effective range expansion was not imposed on the doublet S-wave is that it is difficult to fit a (possibly) singular function without biasing the result. The mixing parameters were constrained to be polynomial functions of the square of the center-of-mass momentum. In contrast to Ref. [10], they were not required to vanish at the two-body threshold as this is not required by either the three-body dynamics nor by any symmetry of the system, a result demonstrated in Delves' analysis of threshold behavior [22]. The polynomial order and allowed fitting range for the variable phase shifts and mixing parameters were fixed prior to the analysis by comparison with theoretical values calculated using the AV18 + UR3NF potential. The cut-off parameter $\Lambda$ in this calculation was adjusted to reproduce the $^3$He binding energy.

Theoretical predictions for the ${}^2S_{1/2}$ effective range function are shown in Fig. 1, along with the result of this analysis and the phenomenological curve of [19]. Although there is a fair amount of scatter in the values obtained in this analysis, the tendency of the function to diverge at small $k^2$ is evident. The theoretical effective range function also exhibits a curvature, though at smaller $k^2$ values. It is well fit [8] using an effective range expansion containing a pole located near threshold ($E_{pole} = -3$ keV). In the present analysis, the apparent singularity is at much larger $k^2$ – corresponding to $E_{cm} \approx 120$ keV – than that predicted by theory. A fit to the

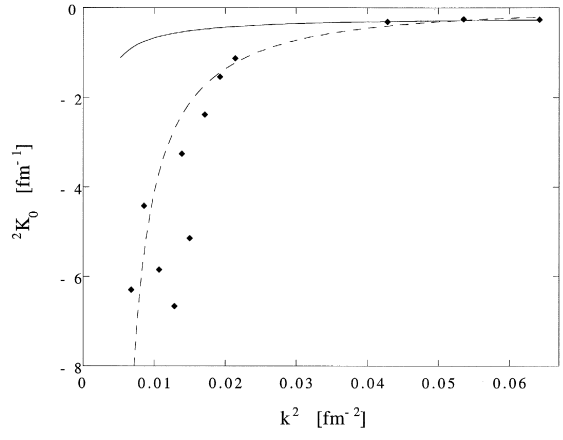


Fig. 1. ${}^2S_{1/2}$ effective range function. Diamonds denote the result of this analysis. The solid line is the theoretical prediction using the AV18+UR3NF force model and the dashed line is the fit to the extracted values using Eq. (1).

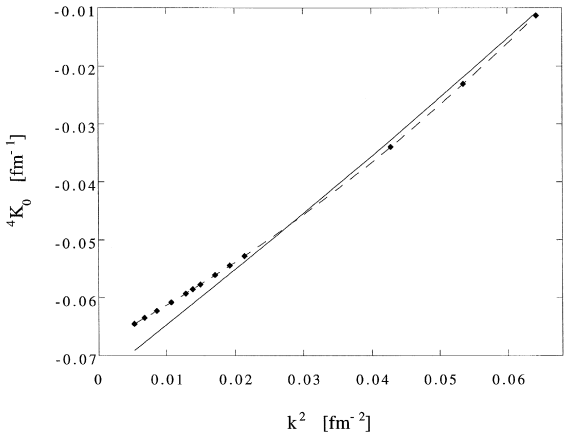


Fig. 2. $^4S_{3/2}$ effective range function. The plotting symbols are the same as those of Fig. 1. In this case, the effective range expansion is a second-order polynomial in $k^2$.

curve using the effective range expansion form [23,24,21]

$$^2K_0(k^2) = \frac{(-1/a_0) + \frac{1}{2}r_0k^2 + P_0k^4}{1-(k/k_0)^2} \qquad (1)$$

gives the doublet scattering length $^2a_0 = -0.13 \pm 0.04$ fm. The pole is located at $k_0^2 = 3.8 \times 10^{-3} \pm 0.3 \times 10^{-3}$ fm. This result is the first experimental confirmation of this theoretically predicted singularity in p + d elastic scattering.

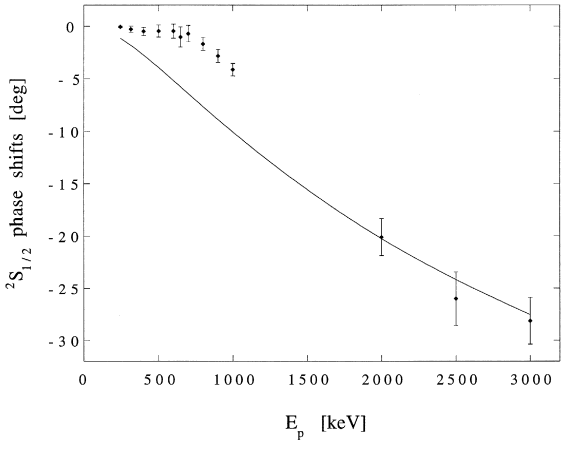


Fig. 3. $^2S_{1/2}$ phase shifts. Diamonds denote the result of this analysis. The error bars on these points represent the total uncertainty in the fitted values. The solid line is the theoretical prediction using the AV18+UR3NF force model.

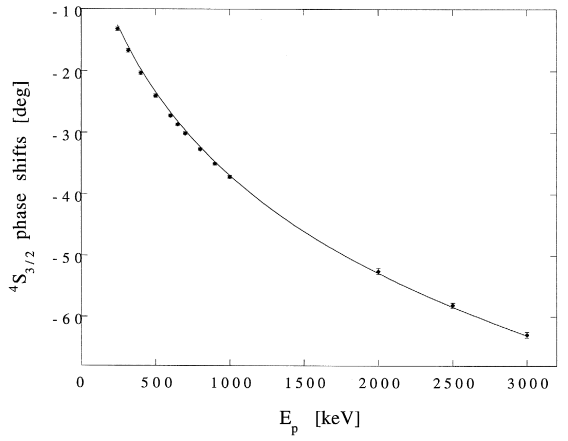


Fig. 4. $^4S_{3/2}$ phase shifts. The plotting symbols are the same as those of Fig. 3.

The phenomenological quartet S-wave effective range function obtained in this analysis is shown in Fig. 2. We find that in this channel, the scattering length $^4a_0 = 14.7 \pm 2.3$ fm. Comparisons between doublet and quartet S-wave phase shifts obtained in this analysis and theoretical calculations of these phase shifts are shown in Fig. 3 and Fig. 4, respectively. The cross sections measured by Black, et al. at proton center of mass energies of 163 keV and 211 keV are shown in Fig. 5, along with calculations of these observables employing the AV18 + UR3NF

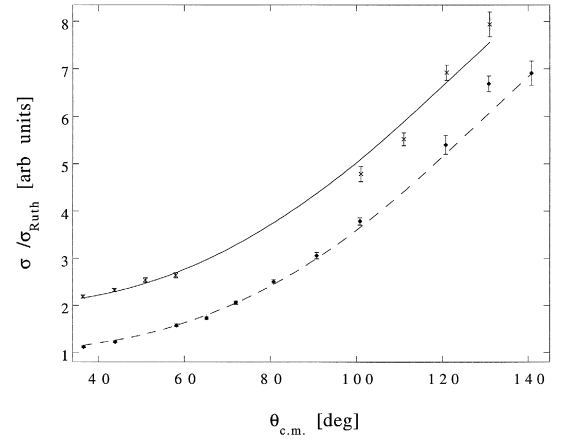


Fig. 5. Experimental cross sections at 163 keV (×'s) and 211 keV (diamonds) proton center of mass energy. The solid and dashed lines are the AV18+URNF predictions for 163 keV and 211 keV, respectively. The values plotted were divided by the Rutherford cross sections and arbitrarily re-normalized.

potential. The cross sections at these energies calculated using the parameters from the present phase shift analysis also compare well with the data and differ from the AV18 + UR3NF calculations by, at most, 2% at the furthest back angles.

While this analysis, which is based to a large extent on recent high-precision, low-energy p–d scattering data, is primarily sensitive to the $^4S_{3/2}$ phase shift, the extracted $^2S_{1/2}$ effective range function clearly reveals the theoretically predicted singularity. The parameterization of the effective range function from the present work differs, however, from that of theory. In particular, the location of the pole is found inside the physical region, in contrast to theoretical parameterizations [1,8]. Further interest in the pole position comes from a recent search for excited states in $^3$H and $^3$He [25]. Virtual states have been found below the d + N threshold which have a close relation to poles in the $^2S_{1/2}$ effective range expansion. Clearly, new high-precision cross section measurements, especially at energies below $E_{cm} =$ 163 keV, are now necessary to pin down the pole position.

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