The mean-field contribution to the energy-momentum tensor,

$$\begin{aligned}
T_\kappa^{\mu\nu} = \quad & \kappa(\rho\partial_\sigma\rho)\Big\{\langle\partial^\mu u^\sigma\rangle u^\nu + \langle\partial^\nu u^\sigma\rangle u^\mu\Big\} + \kappa\langle\partial^\mu\rho\rangle\langle\partial^\nu\rho\rangle - \frac{\kappa}{2}\rho^3\Big\{u^\mu\partial^\nu\left[\frac{\vec{\nabla}\cdot\vec{v}}{\rho}\right] + u^\nu\partial^\mu\left[\frac{\vec{\nabla}\cdot\vec{v}}{\rho}\right]\Big\} \\
+ \quad & \kappa\Big\{\frac{2}{3}\rho^{1/2}\partial^2(\rho^{3/2}) - \frac{1}{2}\rho^2(\vec{\nabla}\cdot\vec{v})^2 + \rho^2 u^\alpha\partial_\alpha\partial_\beta u^\beta\Big\}g^{\mu\nu} \\
- \quad & \kappa\Big\{\frac{2}{3}\rho^{1/2}\partial^2(\rho^{3/2}) - \frac{3}{2}\rho^2(\vec{\nabla}\cdot\vec{v})^2\Big\}u^\mu u^\nu \ , \qquad (1)
\end{aligned}$$

with

$$u^\mu = \gamma(1,\vec{v}) \ , \qquad \gamma = \frac{1}{\sqrt{1-v^2}} \qquad (2)$$

The rank-2 symmetric projection tensor is

$$\langle A^\mu B^\nu\rangle = \frac{1}{2}\{\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}\}A_\alpha B_\beta, \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu. \qquad (3)$$

For a comoving frame $u^\mu = (1,\vec{0})$, the property of projection tensor,

$$\langle A^0 B^\mu\rangle = \langle A^\mu B^0\rangle = 0, \qquad (4)$$

since $\Delta^{\mu 0} = 0$. So the projection tensor in the first two terms of $T_\kappa^{\mu\nu}$ only project out the vector component of the associated quantities.

The conservation equation of density ρ,

$$D\rho = -\rho\vec{\nabla}\cdot\vec{v} \ . \qquad (5)$$

Using these properties, from Eq.(1),

$$\begin{aligned}
T_\kappa^{00} = \quad & 0, \qquad (6) \\
T_\kappa^{ij} = \quad & \kappa(\partial^i\rho)(\partial^j\rho) - \kappa\{\rho\nabla^2\rho + \frac{1}{2}(\vec{\nabla}\rho)^2\}\delta^{ij} \ , \qquad (7) \\
T_\kappa^{i0} = \quad & -\frac{\kappa}{2}(\rho\partial_j\rho)\{\partial^i u^j + \partial^j u^i\} + \frac{\kappa}{2}(\vec{\nabla}\cdot\vec{v})\{\rho\partial^i\rho\} - \frac{\kappa}{2}\rho^2\{\partial^i(\vec{\nabla}\cdot\vec{v})\} \ . \qquad (8)
\end{aligned}$$

Furthermore, the additional contribution to the energy density due to (1),

$$u_\mu u_\nu T_\kappa^{\mu\nu} = 0 \ . \qquad (9)$$

Important identities :

1.

$$\begin{aligned}
\nabla^2\rho = \quad & \nabla_\mu\nabla^\mu\rho \\
= \quad & \partial^2\rho - D^2\rho + \frac{1}{\rho}(D\rho)^2 \\
= \quad & \partial^2\rho - D^2\rho + \rho(\vec{\nabla}\cdot\vec{v})^2 \ . \qquad (10)
\end{aligned}$$

Here, $\partial^2 = \partial^\mu\partial_\mu$, $\partial_\mu = u_\mu D + \nabla_\mu$, $D = u^\mu\partial_\mu$ and $\nabla_\mu = \Delta_{\mu\nu}\partial^\nu$.

2.

$$(\vec{\nabla}\rho)^2 = (\nabla_\mu\rho)(\nabla^\mu\rho) = (\partial\rho)^2 - (D\rho)^2 \ , \qquad (\partial\rho)^2 = (\partial^\mu\rho)(\partial_\mu\rho) \ . \qquad (11)$$

3. With the help of Eq.(10), (11) and (5) we obtain the coefficient of the $g^{\mu\nu}$ term in Eq.(1),

$$\begin{aligned}\rho\nabla^2\rho+\frac{1}{2}(\vec{\nabla}\rho)^2 = & \quad \rho\partial^2\rho+\frac{1}{2}(\partial\rho)^2-\rho D^2\rho+\frac{1}{2}(D\rho)^2 \\ = & \quad \frac{2}{3}\rho^{1/2}\partial^2(\rho^{3/2})-\frac{1}{2}\rho^2(\vec{\nabla}\cdot\vec{v})^2+\rho^2 u^\alpha\partial_\alpha\partial_\beta u^\beta \ ,\end{aligned} \tag{12}$$

where $\frac{2}{3}\rho^{1/2}\partial^2(\rho^{3/2})=\rho\partial^2\rho+\frac{1}{2}(\partial\rho)^2$ and in a comoving frame $u^\alpha\partial_\alpha\partial_\beta u^\beta=D(\vec{\nabla}\cdot\vec{v})$.

4. Similarly, the coefficient of the $u^\mu u^\nu$ term in Eq.(1) turns out to be,

$$\rho D^2\rho-2(D\rho)^2+\rho\nabla^2\rho+\frac{1}{2}(\vec{\nabla}\rho)^2=\frac{2}{3}\rho^{1/2}\partial^2(\rho^{3/2})-\frac{3}{2}\rho^2(\vec{\nabla}\cdot\vec{v})^2 \ . \tag{13}$$