



GCE AS/A level

0981/01

MATHEMATICS M2
Mechanics

A.M. TUESDAY, 10 June 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as 9.8 ms^{-2} .

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The diagram shows a piston, of mass 0.8 kg , enclosed in a horizontal tube and attached to a light spring of natural length 0.2 m and modulus of elasticity 625 N . The other end of the spring is fixed to the end of the tube at point B .



Initially, the piston is held at rest at a point A with the spring compressed a distance of 0.1 m , so that AB is the compressed length of the spring.

- (a) Calculate the elastic energy stored in the spring. [2]

The piston is then released. During the subsequent motion, it is subjected to a resistance to motion of constant magnitude 46 N .

- (b) Determine the velocity of the piston when the spring reaches its natural length. [5]

2. A particle of mass 5 kg moves under the action of a horizontal force given by $F = 30t^{-2} - 30 \text{ N}$ at time $t \text{ s}$, where $t > 0$. It also experiences a constant resistance to motion of magnitude 120 N .

- (a) Show that the motion of the particle satisfies the differential equation

$$\frac{dv}{dt} = 6t^{-2} - 30,$$

where $v \text{ ms}^{-1}$ is the velocity of the particle at time $t \text{ s}$. [2]

- (b) Calculate the value of t when the acceleration of the particle is 24 ms^{-2} . [2]

- (c) Given that the velocity of the particle is 18 ms^{-1} when $t = \frac{1}{3}$, find an expression for v in terms of t . Hence find the values of t when $v = 10$. [6]

3. A vehicle of mass 4000 kg is travelling up a slope inclined at an angle α to the horizontal, where $\sin \alpha = \frac{2}{49}$. The engine of the vehicle is working at a constant rate of 90 kW .

- (a) Calculate the resistance to the motion of the vehicle at the instant when its speed is 4.8 ms^{-1} and its acceleration is 1.2 ms^{-2} . [6]

- (b) Determine the maximum velocity of the vehicle when the resistance to motion has magnitude 12800 N . [4]

4. At time $t = 0$, an aeroplane A has position vector $(3\mathbf{i} + 5\mathbf{j} + 20\mathbf{k})\text{ m}$ and is flying with constant velocity $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})\text{ ms}^{-1}$.
At time $t = 0$, another aeroplane B has position vector $(-2\mathbf{i} + x\mathbf{j} + 15\mathbf{k})\text{ m}$, and is flying with constant velocity $(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})\text{ ms}^{-1}$.
- Find expressions for the position vector of A and the position vector of B at time $t\text{ s}$. [3]
 - Determine an expression for AB^2 , where AB is the distance between A and B at time $t\text{ s}$. [4]
 - Given that the shortest distance between A and B occurs at $t = 5$, calculate the value of x . [3]
5. A player kicks a ball from a point A on horizontal ground so that 2.5 seconds later the ball just clears a bar at a point B . The point B is 3 m above the ground. The horizontal distance of B from A is 42 m.
- Calculate the horizontal and vertical components of the initial velocity of the ball. [4]
 - Find the magnitude of the velocity of the ball and the angle that the direction of the velocity makes with the horizontal as it passes the point B . [6]
 - Determine the horizontal distance from B to the point where the ball first hits the ground again. [3]
6. A particle of mass 3 kg moves on a horizontal plane. At time $t = 0$, the particle has position vector $-2\mathbf{i} + 3\mathbf{j}\text{ m}$, where \mathbf{i} and \mathbf{j} are unit vectors along the x -axis and y -axis respectively. At time $t\text{ s}$, the particle moves with velocity $\mathbf{v}\text{ ms}^{-1}$ given by
- $$\mathbf{v} = 4\sin 2t\mathbf{i} + 15\cos 5t\mathbf{j}.$$
- Find the magnitude of the force acting on the particle at time $t = \frac{3\pi}{2}\text{ s}$. [5]
 - Determine the position vector of the particle at time $t\text{ s}$. [4]
 - Calculate the time and the distance of the particle from the origin when it crosses the y -axis for the first time. [4]
7. One end of a light rod of length l metres is freely jointed to a fixed point O and the other end is attached to a particle of mass $m\text{ kg}$. The particle is projected so that it describes a vertical circle. The speed of the particle at the highest point, $u\text{ ms}^{-1}$, is a quarter of its speed at the lowest point of the circle.
- Show that $u^2 = \frac{4}{15}gl$. [3]
 - When the rod is inclined at an angle θ to the **downward** vertical,
 - find an expression for the tension in the rod in terms of m , g and θ .
 - determine the value of θ when the tension in the rod becomes zero. [9]

END OF PAPER