

## GCE AS/A level

0981/01

# MATHEMATICS M2 Mechanics

A.M. TUESDAY, 10 June 2014

1 hour 30 minutes

#### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

#### **INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer all questions.

Take g as  $9.8 \,\mathrm{ms}^{-2}$ .

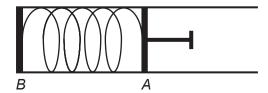
Sufficient working must be shown to demonstrate the mathematical method employed.

### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The diagram shows a piston, of mass  $0.8 \, \text{kg}$ , enclosed in a horizontal tube and attached to a light spring of natural length  $0.2 \, \text{m}$  and modulus of elasticity 625 N. The other end of the spring is fixed to the end of the tube at point B.



Initially, the piston is held at rest at a point A with the spring compressed a distance of  $0.1 \,\mathrm{m}$ , so that AB is the compressed length of the spring.

(a) Calculate the elastic energy stored in the spring. [2]

The piston is then released. During the subsequent motion, it is subjected to a resistance to motion of constant magnitude 46 N.

- (b) Determine the velocity of the piston when the spring reaches its natural length. [5]
- **2.** A particle of mass 5 kg moves under the action of a horizontal force given by  $F = 30t^{-2} 30$  N at time t s, where t > 0. It also experiences a constant resistance to motion of magnitude 120 N.
  - (a) Show that the motion of the particle satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 6t^{-2} - 30,$$

where  $v \, \text{ms}^{-1}$  is the velocity of the particle at time  $t \, \text{s}$ .

(b) Calculate the value of t when the acceleration of the particle is  $24 \,\mathrm{ms}^{-2}$ . [2]

[2]

- (c) Given that the velocity of the particle is  $18 \,\mathrm{ms^{-1}}$  when  $t = \frac{1}{3}$ , find an expression for v in terms of t. Hence find the values of t when v = 10.
- 3. A vehicle of mass 4000 kg is travelling up a slope inclined at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = \frac{2}{49}$ . The engine of the vehicle is working at a constant rate of 90 kW.
  - (a) Calculate the resistance to the motion of the vehicle at the instant when its speed is  $4.8 \,\mathrm{ms^{-1}}$  and its acceleration is  $1.2 \,\mathrm{ms^{-2}}$ .
  - (b) Determine the maximum velocity of the vehicle when the resistance to motion has magnitude 12800 N. [4]

- 4. At time t = 0, an aeroplane A has position vector (3i + 5j + 20k) m and is flying with constant velocity (-i + 2j + k) ms<sup>-1</sup>.
  At time t = 0, another aeroplane B has position vector (-2i + xj + 15k) m, and is flying with constant velocity (3i 4j + 2k) ms<sup>-1</sup>.
  - (a) Find expressions for the position vector of A and the position vector of B at time t s. [3]
  - (b) Determine an expression for  $AB^2$ , where AB is the distance between A and B at time t s. [4]
  - (c) Given that the shortest distance between A and B occurs at t = 5, calculate the value of x.
- **5.** A player kicks a ball from a point *A* on horizontal ground so that 2·5 seconds later the ball just clears a bar at a point *B*. The point *B* is 3 m above the ground. The horizontal distance of *B* from *A* is 42 m.
  - (a) Calculate the horizontal and vertical components of the initial velocity of the ball. [4]
  - (b) Find the magnitude of the velocity of the ball and the angle that the direction of the velocity makes with the horizontal as it passes the point B. [6]
  - (c) Determine the horizontal distance from B to the point where the ball first hits the ground again. [3]
- **6.** A particle of mass 3 kg moves on a horizontal plane. At time t = 0, the particle has position vector  $-2\mathbf{i} + 3\mathbf{j}$  m, where  $\mathbf{i}$  and  $\mathbf{j}$  are unit vectors along the *x*-axis and *y*-axis respectively. At time t s, the particle moves with velocity  $\mathbf{v}$  ms<sup>-1</sup> given by

$$\mathbf{v} = 4\sin 2t \,\mathbf{i} + 15\cos 5t \,\mathbf{j}.$$

- (a) Find the magnitude of the force acting on the particle at time  $t = \frac{3\pi}{2}$  s. [5]
- (b) Determine the position vector of the particle at time ts. [4]
- (c) Calculate the time and the distance of the particle from the origin when it crosses the y-axis for the first time. [4]
- 7. One end of a light rod of length l metres is freely jointed to a fixed point O and the other end is attached to a particle of mass m kg. The particle is projected so that it describes a vertical circle. The speed of the particle at the highest point, u ms<sup>-1</sup>, is a quarter of its speed at the lowest point of the circle.

(a) Show that 
$$u^2 = \frac{4}{15}gl$$
. [3]

- (b) When the rod is inclined at an angle  $\theta$  to the **downward** vertical,
  - (i) find an expression for the tension in the rod in terms of m, g and  $\theta$ .
  - (ii) determine the value of  $\theta$  when the tension in the rod becomes zero. [9]

#### **END OF PAPER**