

[DRAFT] Beyond Connecting the Dots: Mastering the Hidden Connections in Everything that Matters

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Chapter 1

Preface

Ludwig von Bertalanffy(1) first proposed, in 1937, that the same basic structures operated across all disciplines, and if one learned how these structures operated one could transfer much of their learning from one discipline to another. When moving from one discipline to another, one would simply have to learn the structures that were operating, and the labels on the elements of the structures. On first reading this may seem most profound, or maybe even preposterous.

However, if you think about it, maybe there is some truth to it after all. What follows is the introduction to a live Systems Thinking book presented from a cross discipline models perspective. Live in the sense that the models are presented in a form that allows you to actually interact with them.

von Bertalanffy wrote “Allegemein Systemlehre” which was translated into English as “General Systems Theory”(2) and I expect we’ve still not recovered from the translation error. What he intended was a “General Theory of Systems” or “General Systems Teaching,” a way to support learning about the structures which operated across all disciplines. Today there are a set of structures referred to as Systems Archetypes which I believe are just what Bertalanffy had in mind.

In the words of von Bertalanffy, “The student in ‘system science’ receives a technical training which makes systems theory – originally intended to overcome current overspecialization – into another of the hundreds of academic specialties”(1)

Systems Thinking is not a method though more of a way of looking at the world around us and understanding based not from understanding things though more from understanding relations and interactions between things. And while there are many who believe that Systems Thinking or a Systems Perspective provides the best foundation for creating effective approaches of dealing with challenges and shaping a better tomorrow. Yet even with that view, over the past 75 years it has not become widely adopted, even though during that period dozens of approaches have been developed with claim to embrace the Systems Thinking world view. I believe Pogo had it right when he said, “We have met

the enemy and he is us.” I have repeatedly commented to people that the greatest impediment to the adoption of Systems Thinking is Systems Thinkers.

This should provide you with a sense of why this book has to be different. Now let me offer you a view of how it will be different.

It is our intent to provide a basis for recovering from this overspecialization by offering an extensive series of models from everyday life that will show the value of looking at things through a different lens. We will then build on this to develop an understanding without all the terminology and complexity that typically drives people away from Systems Thinking.

References

- Davidson, Mark. 1983. Uncommon Sense: The Life and Thought of Ludwig von Bertalanffy <http://www.amazon.com/Uncommon-Sense-Thought-Bertalanffy-1901-1972/dp/087477165X/>

Chapter 2

Chapter 1 - It's The Pattern That Connects

Notes to Reviewers

Chapter Intent

Develop an awareness that the diverse world around us has a commonality that can be meaningfully represented by just a few interacting elements with rather simple attributes. The basic operation and interaction with embedded models must also be experienced and supporting aspects of Insight Maker explained.

Figure Captions

Each figure is followed by a sequenced figure caption line that starts with ** and these lines are also an internet link. These lines are inserted so I can easily get back to wherever that graphic originated should I need to create a revised version of it. These statements will be deleted by the post processor and replaced with figure captions which are embedded in the Markdown formatting.

Insight Maker References

I'm doing the best I can representing the version of Insight Maker I won't be able to see for a couple of months. The interactive Insight Maker models are embedded from Insight Maker and the model is owned by me. This means that when one looks at it in this chapter it doesn't look like it will look in the final book. Scott is creating a version of Insight Maker that will operate in a touch tablet environment. That version of Insight Maker will be embedded in the book and each book owner will own the models in the book. That means they will look different. As such I have to code something so I'll know what

to include later and reviewers can look at and connect with the written words associated with it. Getting through this seems to be a tall order.

Macros

There are certain aspects of the text formatting we don't have figured out and have resigned ourselves to the fact that we won't have this figured out for some time. As such macros are being coded to be replaced in the content post processing phase. I sorry that it's likely to make the text a bit more difficult to read.

- *model attribute*
- ξ
- *equation*
- *model primitive*
- *ui reference*

Relation to Table of Contents

What follows was presented in the Table of Contents as three separate chapters though the writing seemed to get away from me. The may be split into several chapters or the table of contents may be corrected. Presently it's a bit difficult to tell.

What you learn, and your capacity to learn, serves as the basis for everything you do in your life. Yet, have you ever thought about how you really learn about the world around you? Yes, there are some things you memorize early in life, like the times tables, and you learn to remember these, though is that really learning? Do you remember that if you put your hand on something very hot it will burn you, or is that something you learned? And if you learned that, how was it that that learning happened?

Consider the following

- I have a box that's about 3' wide, 3' deep and 6' high
- It's a rather heavy box
- The has a couple of doors on it
- When you open the doors it's cooler inside the box than outside
- One compartment is much colder than the other
- When you open the door a light comes on
- There's food inside the box
- The box is in the kitchen

- There are sticky notes all over the front of the box
- There's a collection of papers and stuff on top of the box
- If you move the box you'll probably find a lot of dust under it
- The box is plugged into an electrical outlet
- From time to time you can hear the box running

At some point in this sequence you probably became convinced that what was being described was a refrigerator. Now stop for a moment and ask yourself just how was it that you realized what was being described was a refrigerator? Yes it would have been easier if I had just shown you a picture of a refrigerator, though that would have spoiled it, wouldn't it.

As long as you knew beforehand what a refrigerator was, the statements could have been given to you in any order, and still at some point you would have finally realized what was being described. If you had never seen, nor heard about, a refrigerator before you would still be wondering what was being described and what to call it.

You have also most likely come to understand that all refrigerators are not identical. Some have one door with a separate compartment inside. Some have two doors and a drawer. Some are much smaller than others. Some can fit under a counter and some even fit on top of a counter. Some can be so large you can walk into them.

If you see any of these you quickly decide it's a refrigerator. How does that happen? Gregory Bateson, one of the great thinkers of our time, said, "It's the pattern that connects." If you reflect on this statement you should come to realize there are actually different ways to interpret what it means. In this particular case the pattern connects you to the following purpose

- The box keeps food from readily spoiling by keeping it cold
- Part of the box is a freezer which keeps food from spoiling for even longer

and you understand it to be a refrigerator. Though now that we've arrived at this point we still haven't addressed the question of how you know. You probably were not actually taught that it's the above purpose that defines the essence of a refrigerator. Most people were not, though they have essentially learned it over time.

Models

Models are the way we look at, and understand the world around us. All we have are our models. They are the way we understand everything. This is so because we build our understanding based on what we already understand. The world around us simply has too much detail for us to pay attention to everything. A refrigerator has many pieces though how many do you really



Figure 1. From the description you knew it was a refrigerator - but how?



Figure 2. Many kinds of refrigerators, or freezers - But how do you know?

pay attention to? Probably not many unless you build or repair refrigerators. We filter out much of the detail around us so we don't become overloaded and we choose what to pay attention to. Sometimes we do this consciously and sometimes subconsciously. In the midst of what we choose to pay attention to there are patterns. Whether we realize it or not it is these patterns that we pay attention to and attempt to make sense of. We understand these patterns by linking them to extend patterns we already understand. And much of the world around us we simply ignore for if we didn't we would just become overwhelmed.

Remember

A model is a simplified version of some aspect of the world around us to help us understand something.

Learning

When we experience something that experience falls somewhere between complete novelty, meaning that we can't connect it with anything in our past experience, and complete confirmation, meaning that it represents something we perceive as already completely understood. The things we experience which

lie somewhere between complete novelty and complete confirmation provide a basis for learning. They represent a basis for connecting to understood patterns, extending our understanding, and what results is learning. {Cite: Jantach, Eric. 1980. The Self-Organizing Universe: Scientific and Human Implications. Pergamon Press. <http://www.amazon.com/The-Self-Organizing-Universe-Implications-Innovations/dp/0080243118/>}

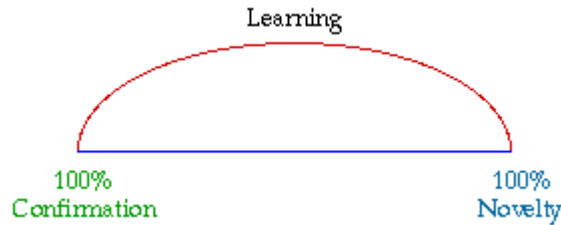


Figure 3. Experience between novelty and confirmation as a basis for learning

Consider running into a refrigerator that looks like no refrigerator you've never seen before. From an initial view you are likely not to perceive it as a refrigerator. As you inspect it to find it serves the purpose you've come to understand for refrigerators or if someone tells you it's a refrigerator you then expand or extend your awareness of the range of patterns that constitute a refrigerator.

A Basis for Flawed Learning

While reading the previous paragraphs did it dawn on you that much of this pattern recognition/connection/extension learning doesn't happen consciously? We connect with patterns and extend our knowledge at times without even being consciously aware that it is happening. And when it happens in an subconscious manner there isn't really any critical validation that happens along with the learning. Because this ongoing learning happens without critical validation there are things we learn and come to believe which are actually incorrect. We have perceived patterns and extended our learning in a flawed manner. The really annoying thing is that we then act on these beliefs, and when we produce results that don't go the way we planned we wonder why. Or even worse, we don't actually learn from the results and correct our flawed models.

When we act on flawed beliefs when attempting to solve problems we typically create more problems than we fix. It has been said repeatedly that the majority of today's problems are the direct result of yesterday's solutions. Wouldn't this provide a sense that we might really benefit from a better way to think about the world around us, develop better understanding, and develop solutions that don't come back to haunt us in the future?

Ladder of Inference

The best explanation of how we build our beliefs was developed by Chris Argyris {Argyris, Chris. 2013. Chris Argyris. Wikipedia.http://en.wikipedia.org/wiki/Chris_Argyris} and is referred to as the Ladder of Inference. When we consciously traverse this ladder we're in good shape. Though when we traverse this ladder subconsciously we often create learning that is not accurate. We then act on this inaccurate learning and produce inappropriate or undesirable results. {Bellinger, Gene. 2013. Ladder of Inference: Short Circuiting Reality. SystemsWiki. http://www.systemswiki.org/index.php?title=Ladder_of_Inference:ShortCircuiting_Reality}



Figure 4. How we form beliefs and influence our actions

Because we live in the moment, even though we may think about the past, or the future, we tend to relate to things in the moment. It is this living in the moment that is mostly responsible for us tending to think in terms of cause and effect, i.e., A caused B. Even when we consider things that have evolved over time we tend to ignore the passage of time, and more often than not simplify situations to one dimensional cause and effect, e.g., Wall Street is responsible for the Financial Crisis; Corporations aren't hiring more people because their taxes are too high; Obama is spending the US into ruin; etc. And as you might have gathered from these examples the simplification often produces beliefs that may not even be true. It's our enduring need to make sense of things, along with our lack of a better way to consider things, that drives us to fabricate meaning, even if it is at times invalid.

As such, many of the models we develop to make sense of thing are actually flawed. And as previously stated, when we act based on models that simply aren't correct, flawed in some way, the results of our actions are generally simply not what we intended or expected.

A Better Way

Based on the understanding I hope you've developed to this point it should be obvious that we could benefit from a better way to develop models that are more likely to be correct as well as surface flaws in many of our current mental models.

Ludwig von Bertalanffy first proposed, in 1937, that the same basic structures operated across all disciplines, and if one learned how these structures operated one could transfer much of their learning from one discipline to another. {Davidson, Mark. 1983. *Uncommon Sense: The Life and Thought of Ludwig von Bertalanffy*. J.P. Tarcher, Inc. <http://www.amazon.com/Uncommon-Sense-Thought-Bertalanffy-1901-1972/dp/087477165X/>} When moving from one discipline to another, one would simply have to learn the structures that were operating, and the labels on the elements of the structures. On first reading this may seem most profound, or maybe even preposterous. However, if you think about it, maybe there is some truth to it after all.

I'm not asking you to believe the previous statement just because it was provided here. Though if you give me a few minutes the experience that follows may allow you to arrive at a sensibility of the statement from your own perspective.

Consider the images in Figure 5 and ask yourself what it is that all these different items actually have in common.

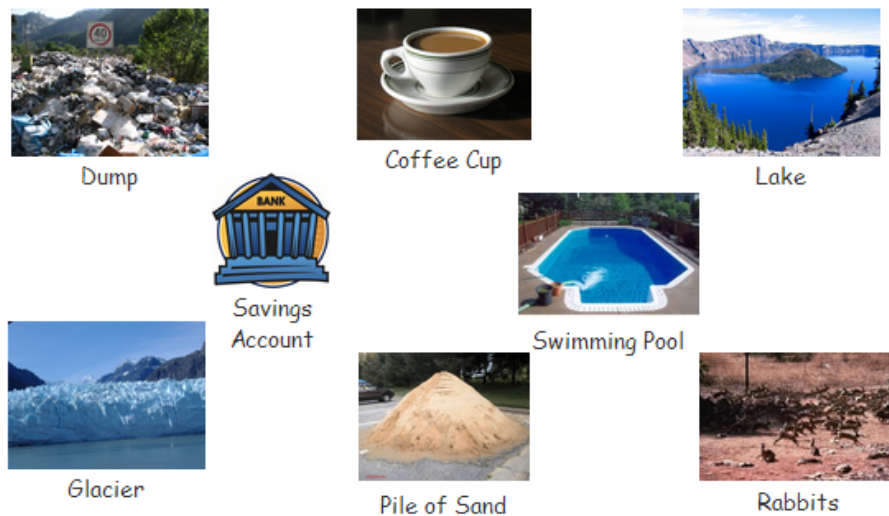


Figure 5. What do these items have in common?

Each of these items represents a collection of stuff. Admittedly each image represents different stuff though stuff just the same. Because in each case this stuff collected over time it's really more appropriate to refer to the the

collections as accumulations. And as you will come to realize it is extremely important to remember that accumulations take time to accumulate, and often even longer to get rid of when you find out you don't want them.

The shorter term often used to refer to an accumulation is "stock." Just where this term originate I'm unsure and what you call an accumulation of stuff isn't nearly as important as remembering it's a bunch of stuff that collected over time. How much time is different for each one of the accumulations and now it's probably time to talk about how accumulations happen over time.

For each of the accumulations in Figure 5, how they change is a bit different, as are the time frames concerned. Time frame being the time it takes for some real noticeable change in the accumulation. Let me describe each in some detail.

Coffee Cup

You usually fill a coffee cup from a coffee pot and it takes a few seconds. Then you take a few minutes to drink the coffee as it's usually too hot to drink when you initially get it.

Dump

Generally a dump accumulates by the truckload after the garbage is picked up at houses or businesses in your community. If the dump were just getting started you'd probably notice it grow with each additional truck load. As it gets bigger and bigger it's gets more difficult to notice that it's growing, even though it is. While the dump is likely to grow almost every day we are probably more likely to think about the growth of the dump in months and years. And does it ever really go away? Usually when it gets to be too much a new dump is started somewhere else and the current dump is buried. Though when it's buried it doesn't really go away. It's still there and we'll probably talk more about dumps later on.

Glacier

A glacier is a long term accumulation of snow which packs down and turns to ice. Glaciers get bigger in the winter when snow falls and they get smaller in summer when some portion of the glacier melts. The time frame one usually uses to think about glaciers is years or even decades.

Lake

Lakes are bigger than a pond and smaller than an ocean and usually filled with fresh water, not salty that is. The lake is filled by rivers and streams that flow into it as well as rain water. One might think of this in terms of gallons per hour or gallons per minute in the case of a large inflow such as at Niagara Falls where the water flows into Lake Ontario in the USA. Water leaves the lake through rivers and streams as well as evaporation into the air. For a lake one

might think about the water flowing into our out of the lake in hours though when considering the level of the lake itself the change might be considered over days or weeks. It sort of depends on what you're interested in.

Pile of Sand

The pile of sand probably showed up in a truck that dumped it right where it is. While it may have taken the truck a while to drive from the wherever it started it probably only took a couple of minutes to dump the truck once it arrived. And the sand is probably referred to in cubic yards, which is how much sand it takes to fill a box that's 1 yard wide, 1 yard deep, and 1 yard high. How long it takes for the sand to go away depends on how it's taken away. If you use a wheel barrow then you have to shovel the sand into the wheel barrow and take it to wherever you're going to use it. At this rate it may take days to move it. If you move it with a small piece of machinery, a Bobcat or a Backhoe, then will will probably only take a few minutes to an hour to get it moved.

Rabbits

A population of rabbits gets larger with new rabbit births and gets smaller with rabbit deaths. Have you ever heard the phrase "multiply like rabbits?" What it means is that it doesn't take very long for a few rabbits to become many rabbits, as long as there is a good food supply and not to many predators like wolves and coyotes. The time frame for considering a rabbit population is probably months to years.

Savings Account

A savings account is a bank account where if you put money and if you keep it there the bank will periodically give you money just for keeping it there. They won't give you very much, though some. If you keep putting money in your savings account every so often and never take it out one day you'll be rich. Yet, for some reason that doesn't happen to too many people. We'll have to talk about that sometime later in the book. One generally thinks about the money associated with a savings account in dollars, the interest rate as a percentage, and the time frame in months and years.

Swimming Pool

Swimming pools usually hold thousands of gallons of water and you usually have a couple of options to fill one. You might use a garden hose, which will take days, or a hose from a fire hydrant, which will take a few hours, or from a tanker truck, which probably takes a few loads. In each case the water filling the pool is probably measured in gallons per hour. Once you fill the pool you loose a little water when people get in and out of the pool, thought not too much. Most of the water loss from a pool is though evaporation due to the sun

and when you backwash the filter used to keep the pool clean. The change in amount of water is usually measured in gallons per hour.

Exercise

Take a few minutes and identify half a dozen situations you're familiar with where there are stocks that accumulate over time. What are the quantities for those stocks, e.g., gallons, pounds, kilograms, etc.. What are the flows that increase and decrease them and what are the time frames over which you think about the accumulation of that stock?

At this point you may be wondering why so much time was spent making you walk through all these examples for the accumulation of stuff. Since we said this was an interactive book you're probably wondering where the interaction is.

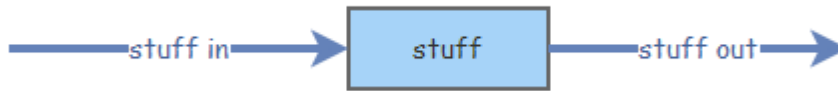


Figure 6. The Accumulation of Stuff

All the accumulations depicted in Figure 5 can be represented in a general form by the model in Figure 6. Remember we defined a model as a simplified version of some aspect of the world around us to help us understand something. It doesn't get much simpler than this does it?

Some amount of stuff flowing in causes stuff to increase over time and stuff flowing out causes stuff to decrease over time. With both of these happening at the same time stuff increases if stuff in is larger than stuff out. And if stuff out is greater than stuff in then the accumulation of stuff gets smaller. The most critical aspect of this to remember is that it takes time for stuff to increase or decrease. How fast the change happens depends on the amount of stuff in the flows.

Lets take a specific instance. Figure 7 represents Figure 6 in Insight Maker, an interactive modeling environment. We'll talk about how this was done shortly. Now suppose we have a swimming pool and we start filling it with a hose that fills at 50 gallons an hour. If we let the hose run for 24 hours how much water will be in the pool? Admittedly the math is pretty straight forward though the idea here is to show how you can use a model to show changes over time.

If I set up the model in Figure 7 with stuff = 0, stuff in = 50 and stuff out = 0, set the Time Settings for 24 hours, and then click the Run button, the model produces the graph in Figure 8.

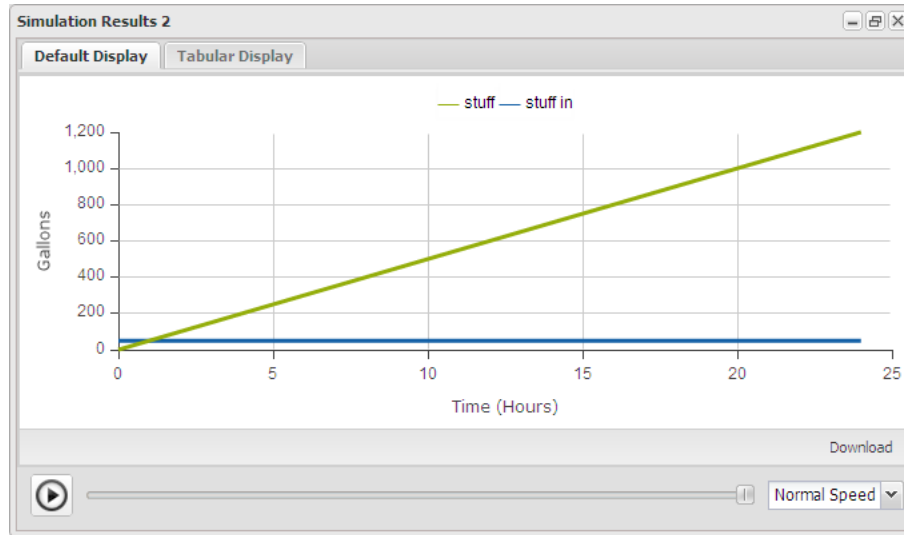


Figure 8. Adding water to the swimming pool

This graph indicates that the after 24 hours the swimming pool with have 1,200 gallons of water in it. I know, it's about as interesting as watching paint dry. Actually, as you will come to find out, that's a good thing because this is really easy. A more interesting question might be, if the swimming pool holds 20,000 gallons of water how long with it take to fill with water at 50 gallons per hour? We'll get to this shortly.

Modeling Notes

As various models are developed and used I'll present aspects of the modeling environment that you'll need to do the exercises. I won't spend a lot of time on pieces you're not going to use immediately so please don't let any of the displays overwhelm you. I think it's far easier to remember things when you actually use them.

Canvas

The center area is the work area where you create models. This area may be scrolled if necessary. I'll talk about how to actually create models in the next chapter.

Stock

A rectangle indicates a quantity of something that accumulates, and accumulation takes time. Stocks don't change in the blink of an eye, well unless you blink for a long time.

Flow

A directed arrow representing the flow of something into or out of a stock. Remember that a stock can only be changed by a flow. Hand waving and magic don't work. The flow has to be explicit to cause a stock to change, and it takes time.

Toolbar

Notice in the upper right corner there is a small down arrow. If you click on this arrow it will open the toolbar displayed in Figure 9. The toolbar contains all the tools you will use to build and modify models. Yes, you get to do everything on a single screen, with a few pop up windows of course.

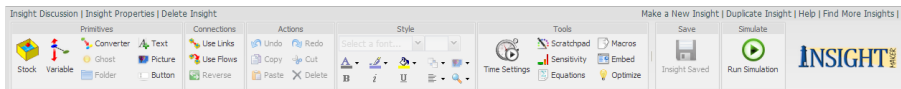


Figure 9. Toolbar

Parameter Tab

Just below the arrow you clicked to open the top toolbar is a right pointing double caret. If you click this the parameter tab will close and the right pointing double caret will now point left and can be used to open the parameter tab. This tab serves two different purposes.

If there are no elements of the model selected on the canvas the parameter tab will be similar to Figure 10 and contain the model description, tags, and parameter sliders used to set parameter values just before running the model.

If there is a single element selected on the canvas then the parameter tab will present the list of parameters that can be set for that element. Figure 11 shows the parameters for the stuff element of the model. This is where I set the stuff in to 50 before running the model. Please don't be overwhelmed by this long list of parameters. We'll cover them one at a time as they are actually used in a model.

You should note in Figure 11 under the User Interface section it indicates that there should be a slider for stuff and it can be set for values from 0 to 100. Each element has some of the same parameters and some unique to it. Click one of the flows and see what its parameters are.

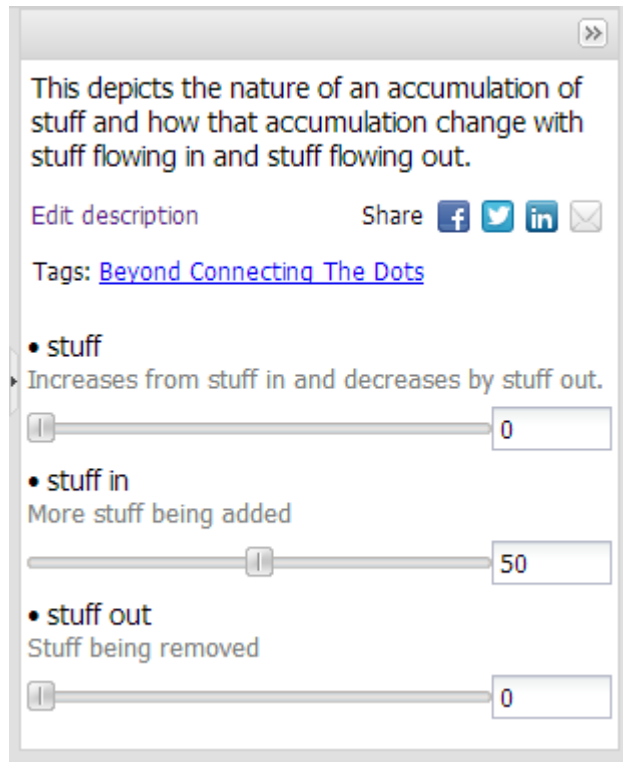


Figure 10. Configuration Panel

Just a couple more pieces and you can go interact with the model some more and get away from this boring description.

Time Settings

In Figure 8 I talked about the swimming pool filling for 24 hours. It's the Time Settings tool that allows you to define this for the model. Figure 12 shows the elements you can set before running a model.

This is where I told the model to start at Time = 0 and runs for 24 time units. It steps one unit at a time and the unit is in Hours. Don't worry about Time Step for now. We'll get into that later.

Simulation Results

When you click the Run button the model is stepped through the defined time period and produces a display of the results. There are various options for the type of display and which elements are displayed as in Figure 13.

Stock

General

(name)	stuff
Note	Increases from stuff in and decre...

Configuration

Allow Negatives	Yes
Initial Value =	0

Behavior

Delay	10
Stock Type	Store

User Interface

Image	None
Show Value Slider	Yes
Slider Max	100
Slider Min	0

Validation

Max Constraint	100
Max Constraint	No
Min Constraint	0
Min Constraint	No
Units	Unitless

?

A stock stores a material or a resource. Lakes and Bank Accounts are both examples of stocks. One stores water while the other stores money. The Initial Value defines how much material is initially in the Stock.

Examples of valid Initial Values:

Static Value
10

Mathematical Equation
 $\cos(2.78)+7*2$

Referencing Other Primitives
 $5+[My Variable]$

Figure 11. Element Parameters

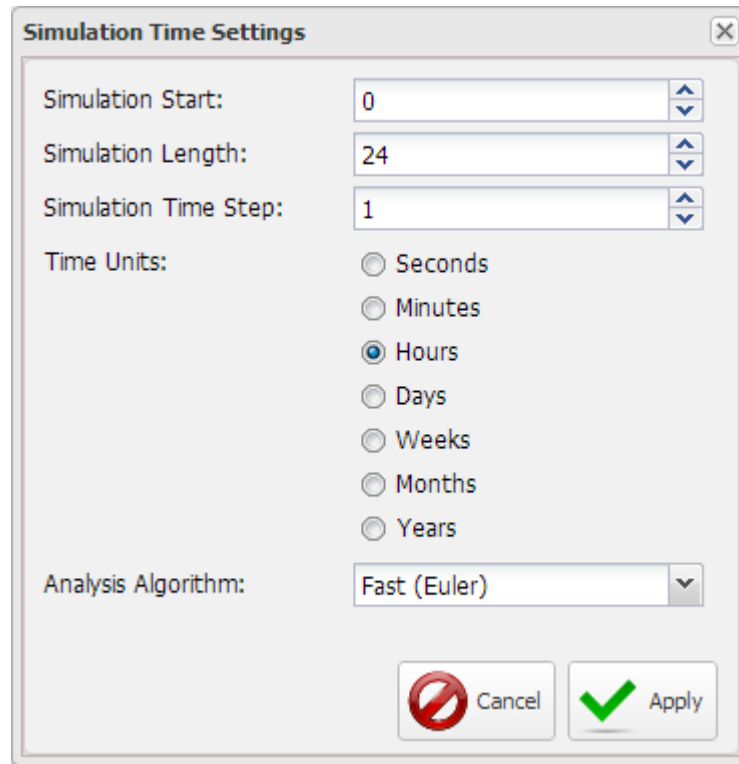


Figure 12. Time Settings

Configure Simulation Results

A default configuration is put together when the model is constructed on the canvas. If you click the Configure button in the upper right corner of the Simulation Results window the Chart/Table Configuration window will open. It is in this window you indicate what type of display you want and which items of the model are to be displayed. The only part you need to be concerned about at the moment is the Y-Axis Label field. That's where I indicated that the items displayed were in Gallons. You will need to change this shortly in the next exercise.

Note that if you change items in the configuration they will be immediately reflected in the Simulation Results window when you click Apply. You don't need to run the model over again to see a different configuration of the data. This makes it very convenient when you decide you need another display for one or two of the items.

I hope you haven't found this short introduction to the modeling environment too overwhelming. As I said I will try to introduce different parts of the environment just as you need them to interact with the models presented.

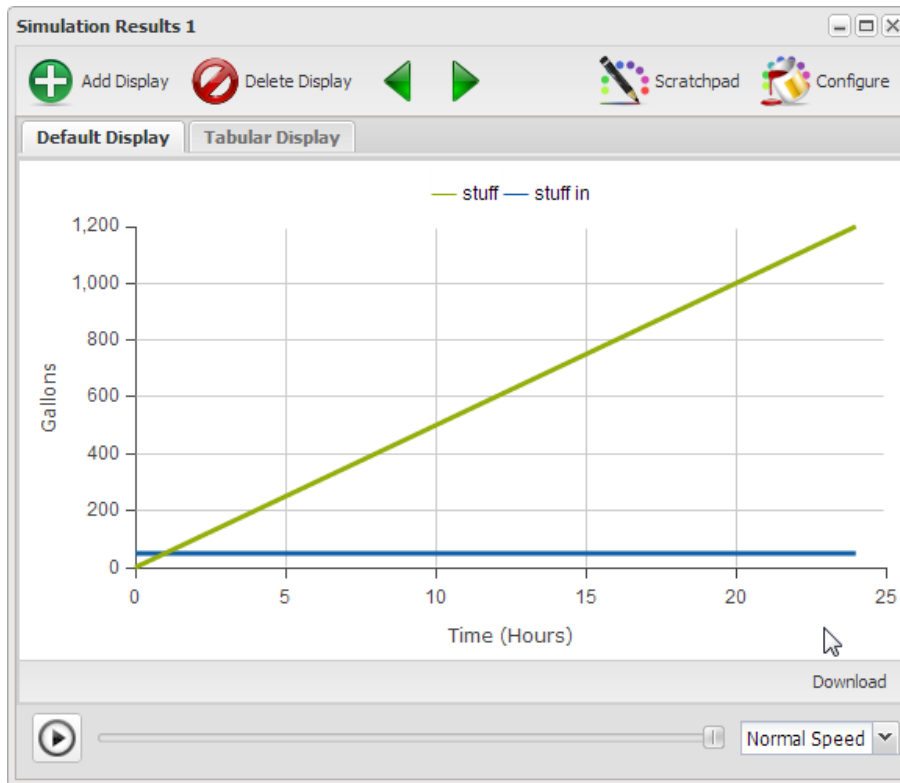


Figure 13. Simulation Results

Too much explaining and not enough hands on interaction gets to be real boring in a hurry. I encourage you to actually do the exercises presented. By interacting with the various aspects of the modeling environment you will develop a level of comfort and expertise which will serve you well throughout the rest of the book.

Exercise

Go back and consider the various pictures in Figure 5. Pick a couple of them to model. The only parts you need to set up are the Time Settings, how long will it run and the Time Units. You can also set the values for stuff, stuff in and stuff out on the Configuration Panel. After you run the model open the Chart/Table configuration window and set the Y-Axis Label appropriate for

The image shows a 'Chart/Table Configuration' dialog box with a close button (X) in the top right corner. It is divided into three main sections: 'General Settings', 'Chart Settings', and 'Secondary Y-Axis'.

General Settings

- Title:** A text field containing 'Default Display'.
- Type:** A dropdown menu set to 'Time Series'.
- Data:** A dropdown menu showing 'stuff' and 'stuff in' as options, with a small 'X' icon next to each.
- Add New:** A checked checkbox followed by the text 'Add newly created primitives to the data'.

Chart Settings

- X-Axis:** A label 'X-Axis:' followed by a 'Label:' text field containing 'Time (%u)'.
- Y-Axis:** A label 'Y-Axis:' followed by a 'Label:' text field containing 'Gallons'.
- Legend Position:** A dropdown menu set to 'Automatic'.
- Display Options:** Three checkboxes: 'Show Markers' (unchecked), 'Show Lines' (checked), and 'Use Areas' (unchecked).

Secondary Y-Axis

- Axis:** A label 'Axis:' followed by an empty 'Label:' text field. A mouse cursor is pointing at this field.
- Data:** A label 'Data:' followed by a dropdown menu set to 'Select which data to display'.

Buttons

At the bottom right, there are two buttons: 'Cancel' (with a red prohibition icon) and 'Apply' (with a green checkmark icon).

Figure 13. Chart/Table Configuration

what you're modeling. I encourage you to be adventurous. Make new Displays, Table Displays, etc. You can't brake anything, it's just an opportunity to become comfortable with the environment and learn.

Now that you've become intimately familiar with almost the simplest model possible lets go back and look at a couple of the pictures in Figure 5 and think about how the accumulations change in a bit more detail.

Rabbit Population Growth

If you modeled the accumulation of rabbits you may have already realized that the model of Figure 7 is missing something. Yes, if you add rabbits to rabbits you get even more rabbits. Though if you have more rabbits don't they create even more rabbits? Figure 14 is a model that reflects the the notion that rabbits create more rabbits.

Modeling Notes

I've stuck a couple new pieces in here and it's probably a good idea if I explain the pieces before talking about how it works. The previous model had a stock, something that accumulates, and flows, the movement of stuff into or out of a stock. And the real important thing to remember is that accumulations take time to change. Stocks only change in the blink of an eye if you blink for a very long time.

Variable

A constant or equation used to influence some part of the model. Remember that a variable and a stock are different. A stock is an accumulation that changes over time as a result of one or more flows. A variable may change though it doesn't represent an accumulation. Rabbit Birth Rate is a variable, and in this model a constant value.

Link

A link is used to communicate a value of one element to another. The link doesn't actually represent something moving like a flow does.

= & i

If you mouse over the elements of the model you'll notice an = and an i appear. The i indicates there is additional info available so if you click it a note window will open with a description of the element. This is because the info was entered when the model was created. The = indicates there is a value or equation associated with the element. If you click on the = it will open the *Equation Editor* window. We'll talk more about this when you start building a model.

Based on the previous modeling notes the model depicted in Figure 14 indicates that if you start with some population of Rabbits and each time period the current number of Rabbits times the Rabbit Birth Rate will result in a number of Births. This number of Births will then be added to the accumulation of Rabbits and figure into the calculation for the next period. If you mouse over the elements of the model and click on the = sign you can look at the definitions for the elements.

The Time Settings for the model were set up to run from 0 to 12 months. If you click the Run button you might be surprised when the model produces the graph in image in Figure 15.

Figure 15 really shouldn't be a surprise. If you look at the Configuration Panel you'll see that it indicates 0 Rabbits and 0 Rabbit Birth Rate. If there are no Rabbits how could anything happen? And if we had some Rabbits with the Rabbit Birth Rate was 0 what would you expect the result to be?

Suppose we start with 10 Rabbits, half of which are male and half of which are female. My research indicates that a female rabbit can give birth to between 18 and 26 Rabbits a year. I'll average this out $(18 + 26) / 2 = 22$ and then I'll round this up to 24 just because it will make the math easier. If a female Rabbit can produce 24 Rabbits in a year, that's 2 per months, though it actually takes two Rabbits. With all these assumptions we get about 1 new Rabbit per month for each Rabbit. If you plug Rabbits = 10 and Rabbit Birth Rate = 1 into the model and run it you should get Figure 16.

Forty thousand Rabbits in a year? That seems a bit bizarre doesn't it? This result actually points out the real value of modeling, which is learning. You build a model based on what you think you understand. You then populate it with assumptions about the values and you run it. The result then either seems to make sense or seems really bizarre. In that case what the model is telling you is that either the structure is wrong, the assumptions are wrong, or both, because the world can't possibly be this bizarre. As a result you investigate the model and your assumptions and as you understand better the model gets better. At some point the model finally serves its purpose, to be a simplification of some aspect of the world which leads to a better understanding. I hope you

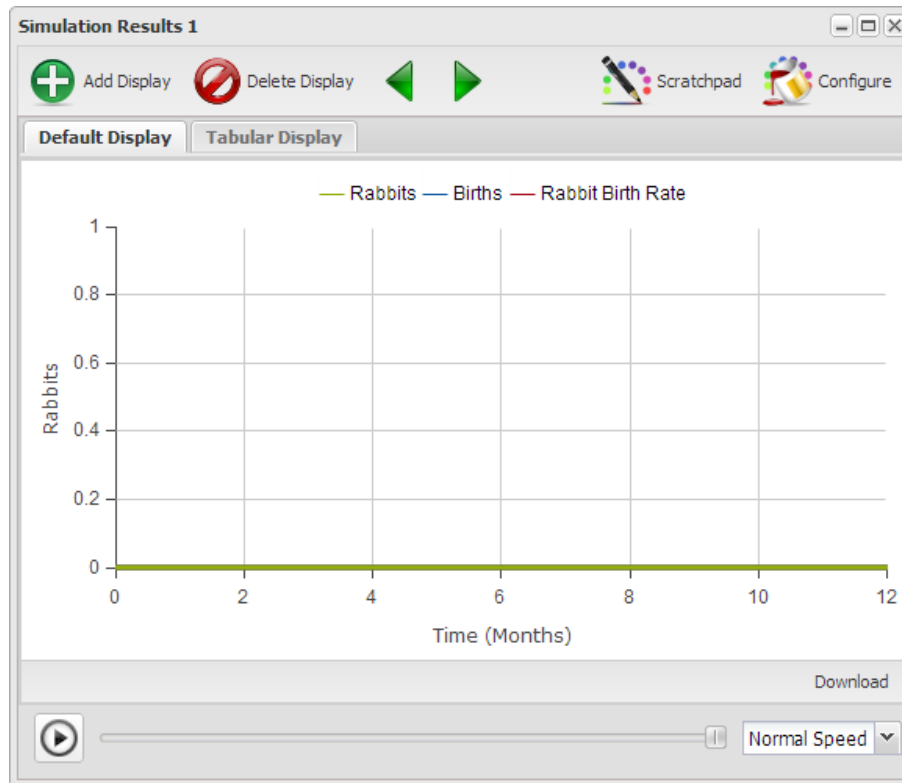


Figure 15. Rabbit Population Growth with No Rabbits

come to find, as I have, that going round and round with a model can be a delightful learning process.

After that sidetrack lets get back to our 40,000 Rabbits that can't possibly exist after a year. I'm pretty sure I can be certain how many Rabbits I started with at the beginning. And when I check my formula for $\text{Births} = \text{Rabbits} * \text{Rabbit Birth Rate}$ it seems to be in order. This sort of means my assumption for Rabbit Birth Rate must be too big. And if you think about what the model is doing it's probably not too difficult to figure out that the model assumes that a Rabbit can be born this month and then give birth to another Rabbit next month. If a Rabbit has to mature for six months before it gives birth to Rabbits then the Rabbit Birth Rate might be something more like 20%. Using this estimate for Rabbit Birth Rate the model produces Figure 17.

Is this right? A good thing to remember at this point is that's actually the wrong question. A better question might be, "What have I learned, and is there more I can learn?" The graph in Figure 17 sure seems more reasonable than what the model presented in Figure 16 though I don't think we have a high degree of confidence in the current Rabbit Birth Rate. And there are a number

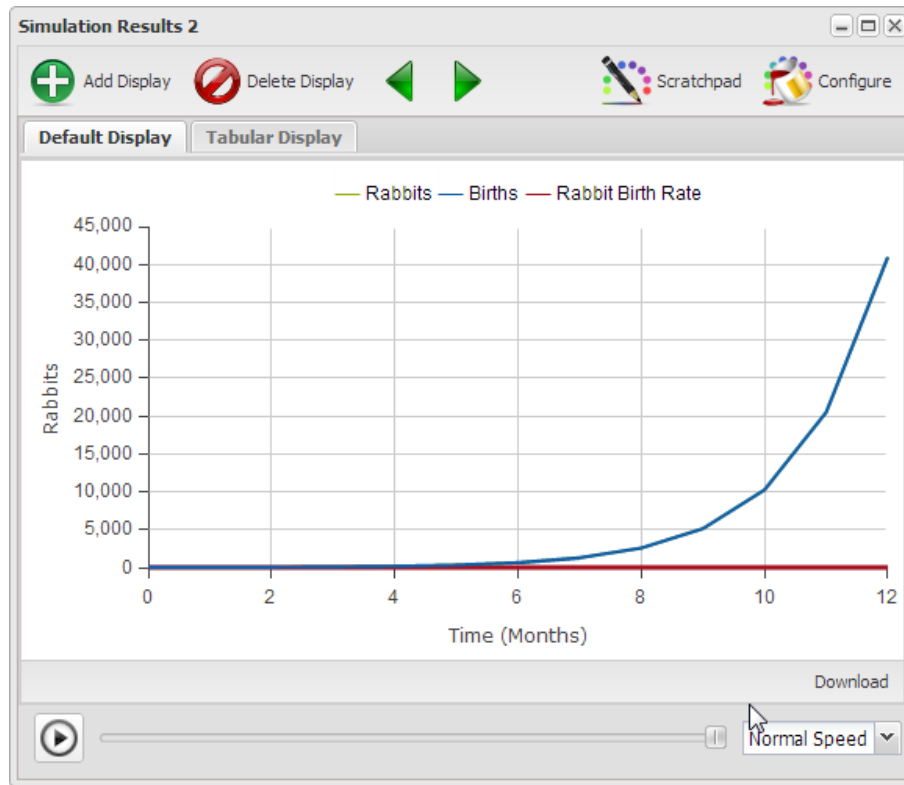


Figure 16. Rabbit Population Growth with 10 Rabbits

of other questions we could ask about our Rabbits. What is the Rabbit Death Rate? Do they have enough food to eat? Are they living out in the open where Coyotes and Foxes can get at them? Does their owner have a passion for Rabbit Stew? These might each be a basis for building a better model, though at this point we're going to leave the Rabbits alone and move on to something else.

The most important learning I hope you take away from this model is that when what flows into the accumulation increases as the accumulation increases the accumulation can get real big in a hurry. This is actually called exponential growth and we'll talk in more detail about this in due quite soon.

Filling A Swimming Pool

Long long ago, meaning back in Figure 7 and Figure 8 I was talking about filling a swimming pool with a hose and how much water was in the pool after a period of time. A more useful question might be, If the pool holds 20,000 gallons of water and the hose fills the pool at 50 gallons per hour, how long will it take to fill the pool. I know, you can do the math faster than it will take to

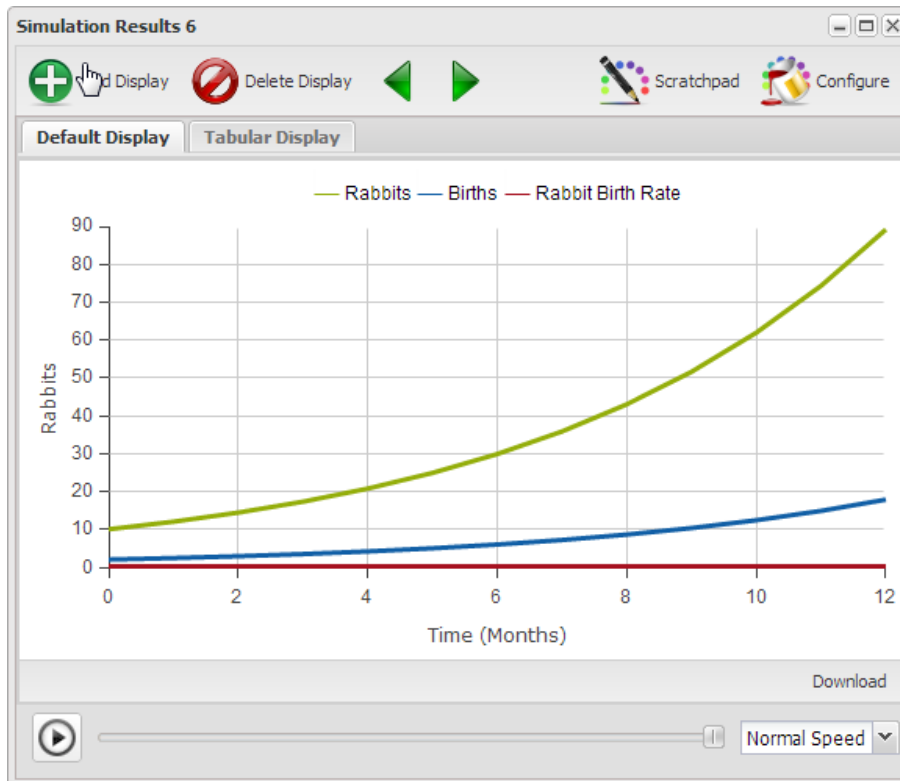


Figure 17. Rabbit Population Growth with 10 Rabbits and 20% Birth Rate

build the model. Please bear with me a bit as there's another aspect of models right around the corner you will find very useful on an ongoing basis.

I begin with a Swimming Pool that needs to be filled with a hose. I know how many gallons of water it takes to fill the pool and I don't want to put too much water in the pool. I create a model where I compare the amount of the water in the Swimming Pool with the the Full Level and use that to decide whether water is flowing in the hose or not. If you mouse over Hose and click the = sign you'll see the following equation.

```
IfThenElse([Swimmng Pool] < [Full Level], [Full Level]-[Swimmng Pool]), 0)
```

This says that if the Swimming Pool isn't full then I need to add enough water to fill the pool. And if the Swimming Pool is full then I add 0.

Modeling Note

Isn't it curious that the structure of this model looks just like the one for the Rabbit Population growth in Figure 14. I'll talk about this after we figure out how long it's going to take to fill the Swimming Pool.

With the Time Settings set for the model to run for 24 hours. Set the Swimming Pool to 0, meaning empty, and the Full Level to 20,000, on the Configuration go ahead and click the Run button. You should end up with the graph as shown in Figure 18.

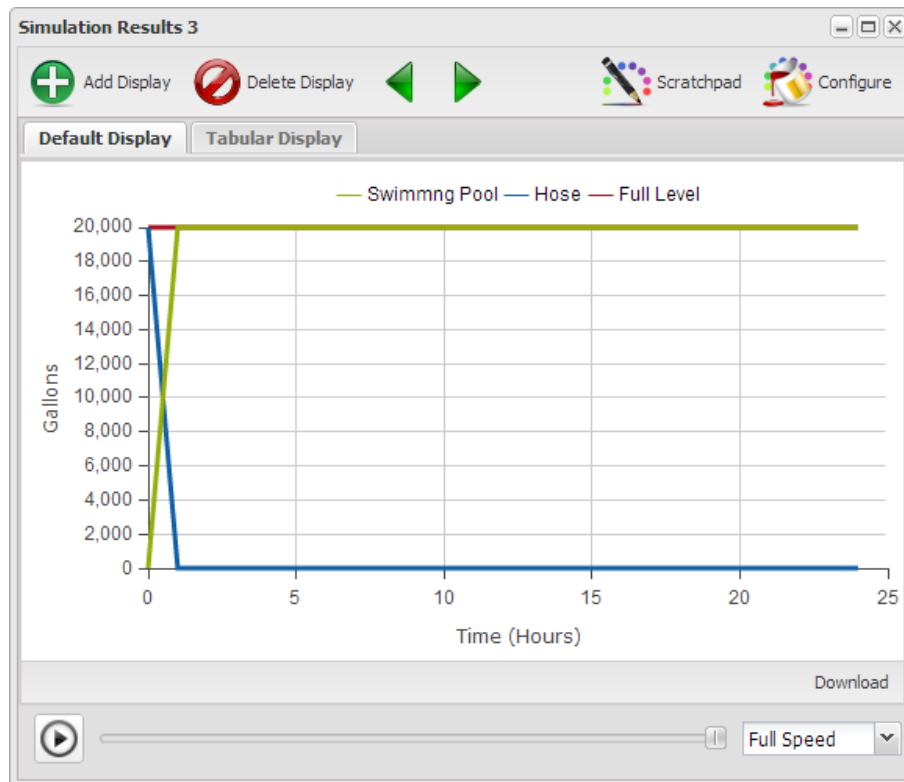


Figure 19. How long to fill the pool

This is really great. I can fill the Swimming Pool in just 1 day, or can I? Either it's a really really big hose or I've done something wrong because I don't think I can really fill the Swimming Pool with a Hose in one day if it takes 20,000 gallons.

Modeling Note

I sincerely hope you come to understand that when your models don't do what you expect them to do it's not a problem – it's opportunity for learning. This is the real reason why we do modeling. Just think of it as, the more things don't go the way you expect them too, the more opportunities you have to learn.

As I look back at the formula I put in for the Hose I notice I didn't take into account my initial statement that the Hose could only deliver 50 gallons per hour. And, might it be useful if I could see what happened with different Hose capacities?

Figure 20 is a revised version of the model with Hose Capacity as a variable so you can set the capacity of the hose before you run the model.

The new formula for Hose takes into account both the current amount of water in the Swimming Pool, Full Level and Hose Capacity

```
IfThenElse([Swimmng Pool] < [Full Level], min([Full Level]-[Swimmng Pool],[Hose Capacity]), 0)
```

With Hose Capacity = 50 if you run the model it should produce Figure 21.

Was this what you expected? Probably not. Over a period of 24 hours we've not even come close to filling the Swimming Pool.

Open the Time Settings and set the Simulation Length to 600 hours and Run the model again. Your run should produce the an equivalent of Figure 22.

Figure 22 indicates we need to wait 400 hours to fill the pool. That's a little over 16.5 days. I think we need a bigger hose.

While there are a number of things we could do to improve the model at this point I think we've gone far enough with this one.

Exercise

Do a number of runs for the model in Figure 20 with different values for Full Level and Hose Capacity.

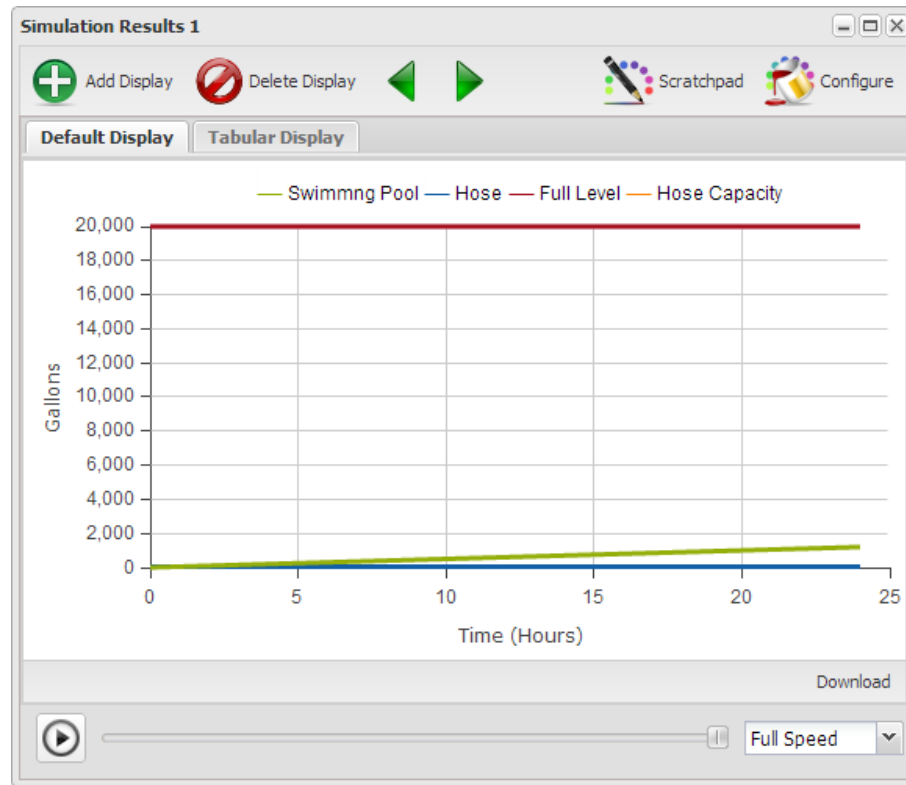


Figure 21. How long to fill the pool at 50 gallons per hour

Similar Structures / Different Behavior

If you compare Figure 14 and Figure 20 you should find the two of them to be quite similar. And yet the behavior of the two models are distinctly different.

In Figure 23 I've redrawn the Rabbits model so it's easier to see how similar they are. The Swimming Pool and Rabbits both represent accumulations. Hose and Births both represent flows into the stock. Hose Capacity and Rabbit Birth Rate are both factors which govern the rate of flow. Full Level is a target value which the Rabbits model doesn't have. The difference that makes a difference is what happens in the connection between the accumulation, or stock, and the flow.

The link between the stock and the flow provides information from one point to the other and is generally referred to as feedback, mostly probably because the information travels in the opposite direction as the flow.

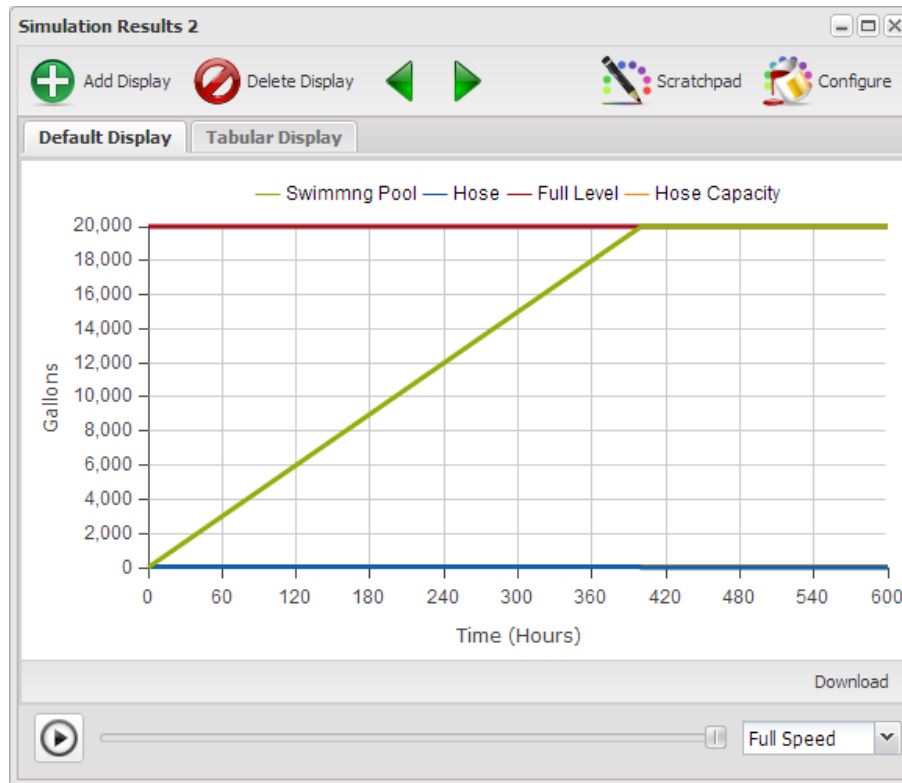


Figure 22. Filling the pool takes how long?

Balancing

In the Swimming Pool model the Hose flow depends on the difference between the value of Swimming Pool and Full Level. This difference influences the Hose flow to increase the Swimming Pool until it reached Full Level. The structure tries to bring about a balance between Swimming Pool and Full Level so the difference is zero, and then there's no more Hose flow.

Reinforcing

In the Rabbits model Births depends on the value of Rabbits. The number of Rabbits influences Births to increase the number of Rabbits which increases the Births. One might consider a Reinforcing structure to be a Balancing structure that's out of control.

Would you believe that no matter how complicated a model may look it's really only some number of these two structures connected together? In the next chapter you will begin actually building some models and investigating the implications of the Balancing and Reinforcing structures.

Exercise

I put some values in the Figure 23 model elements so when you run the model it will produce the graph in Figure 24. Can you figure out why the values assigned are responsible for the curves produced?

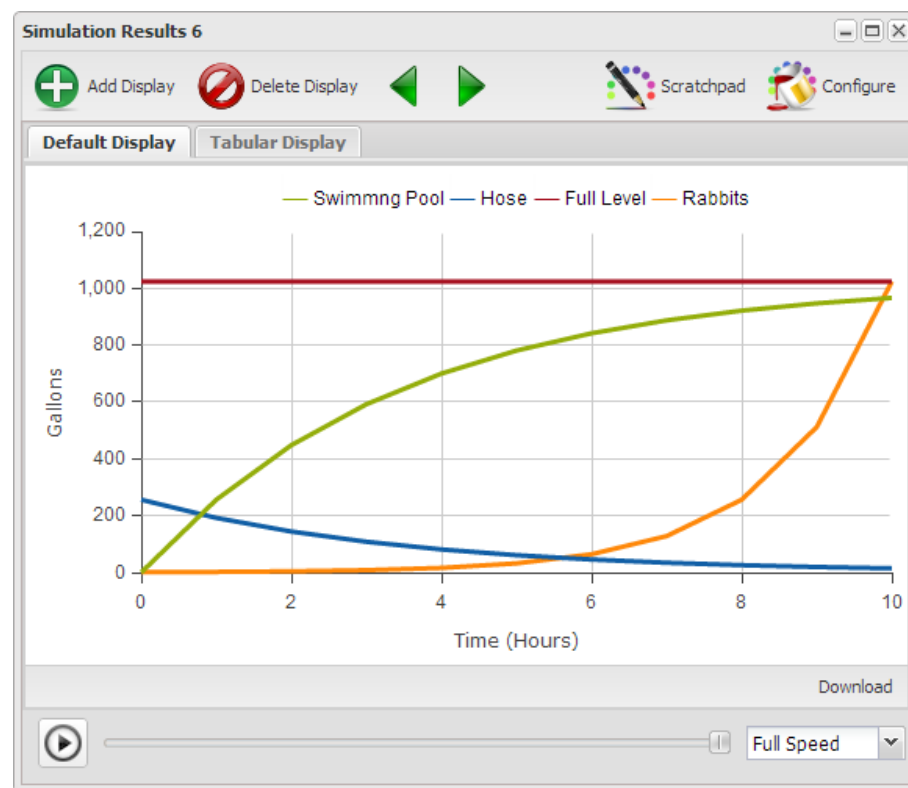


Figure 24. Balancing and Reinforcing Results

Summary

- Models are simplified versions of the world around us.
- We build models to help us understand.

- We build simple models and add to them as we learn with them.
 - Reinforcing and Balancing structures are the basic building blocks for all models.
 - These building blocks can aid understanding aspects of our interactions with the world around us.
-

Chapter 3

Models and Truth

All models are wrong, but some are useful – George E.P. Box

In discussing the relationship between models and truth it is useful to first take a step back and talk in general about the different types of models. Modeling is wide a ranging field and there are many distinctions that modelers and mathematicians make when discussing models. Some distinctions – such as Bayesian versus Frequentist statistical models¹ – have been the subject of century’s old philosophical arguments between mathematicians which continue to this very day.

These arguments and distinctions are of little interest to us – we will present our own classification scheme that once we complete it will really clarify the core dichotomy that is at the heart of modeling – but it can be useful to briefly discuss them in order to obtain a deeper understanding of the choices we make when we develop the construction of a model.

Deterministic versus Stochastic Models

There are two views of the world. One view says the fate of the universe is governed by strict predictable laws. The universe is like a giant machine and given its current state, its future states through the rest of time are predetermined. Another view, is that the universe is governed by chance and randomness. Random quantum mechanical fluctuations pile up and amplify each other leading to an infinite range of diverging possibilities. Which is the truth? We certainly do not know and it is possible that this will be one of the questions that physicists will never cease exploring. Albert Einstein had a

¹Briefly, this debate refers to two divides within the statistical community that hinge on how probability is interpreted. A Frequentist claims that probability is dependent on the long term sequence of outcomes. Bayesian’s take probability as a subjective degree of belief. Frequentist statistics are what are generally taught in introductory statistic courses and receive the most use.

particular view though. He was a strong partisan in favor of the deterministic view, famously remarking that “God doesn’t play dice with the world.”

When creating a model of a process, we must make a similar choice about chance. Do we build a model in deterministic way such that each time we run it we get the same results? Or do we conversely incorporate elements of uncertainty so that each time it is run we may obtain a different trajectory of outcomes?

Mechanistic versus Statistical Models

When beginning a model of a system, there are many questions that you should ask yourself. Two of them are:

1. Do I know (or have a hypothesis of) the mechanisms that drive the system?
2. Do I have data on the behavior of the system?

If the first question is answered in the affirmative, then you can build a mechanistic model that replicates your understanding (or hypothesis of) the true system. If the second question is answered in the affirmative, you can use statistical algorithms such as linear regression to create a model of the system based purely on the data.

If neither question is answered affirmatively, well there isn’t much of anything you can build.

Aggregated versus Disaggregated

When building a model, the question of scale becomes very important. Imagine we are concerned about the affects of Global Climate Change and whether there will be enough water resources in the future assuming a warming climate. At what resolution do we build the model?

- At the most aggregate we could simply estimate total World-wide water demands and supplies into the future.
- Maybe that is too coarse; clearly having excess water in Norway has little impact on the situation in Egypt. We could instead create a finer resolution model that separately looked at the water demand and consumption in each country.
- Maybe that is still too coarse, maybe we should reduce our model to look at a specific cities and population clusters within countries.
- At the extreme disaggregated level, we might even want to model individual people: all 7 billion of them.

Clearly, there is no simple answer to this question and the best choice is highly context sensitive and depends on the needs of the specific model and application.

Prediction, Inference and Narrative

The three scales presented above can be used to classify models. We can even use them to classify the models we have discussed in this book. Most of these models would be classified as deterministic (random chance is generally not explicit incorporated in the models), mechanistic (we generally assume mechanisms rather than estimating relationships from data), and aggregated (the agent based models are an exception to this).

Outside of modelers, however, these choices scales are of little importance. Let's take off our modeler hats for a moment, and instead look at modeling as a client. In this case, we would hire a modeler to build a model to fulfill some specific purpose. The choices the modeler make (aggregated versus disaggregated, stochastic versus deterministic, what software they use, etc.) are all really secondary to them fulfilling that purpose. Let's look back at Box's quote at the beginning of this chapter. We know all models are wrong, what we should really care about is their functionality: how useful they are.

So instead of using a scale based method to classify models, we can use a functional one based on the purpose or desired usage of a model. There are three main purposes for models: prediction, inference and narrative.

Prediction : Models used for inference are the most straightforward. They attempt to forecast some outcome given information about variables related in some way to that outcome. A weather forecast is an example of a model being used prediction. When you apply for a credit card at a bank, they run a predictive model to determine your risk of default. When you apply for life-insurance, similarly, the company has an actuarial model to predict how much they should charge you for a given payout.

Inference : Models used for inference are the most common in academic research. Often, academic research question boil down to the simple template: "Does X affect Y ?" These are inferential type questions. So an academic may make a hypothesis, for instance; "The wealthier a high-school student's family is, the higher the student's test scores will be." They may then build a model to support or refute this hypothesis and their response will generally be phrased in terms of a p -value indicating the significance of the evidence in support or against the hypothesis.

Narrative : Models are often used to tell a story. When the Obama administration wanted to persuade law makers and the public to support there proposed stimulus, they famously published the graph shown in Figure 2. A lot of complex modeling and mathematics went into constructing this figure, however its sole purpose is to tell a story: things were bad, but the stimulus would make them less bad. We will return to this figure later on.

All models can be classified in terms of these three primary usages and it can be useful to discuss modeling projects in terms of them. There is an even simpler

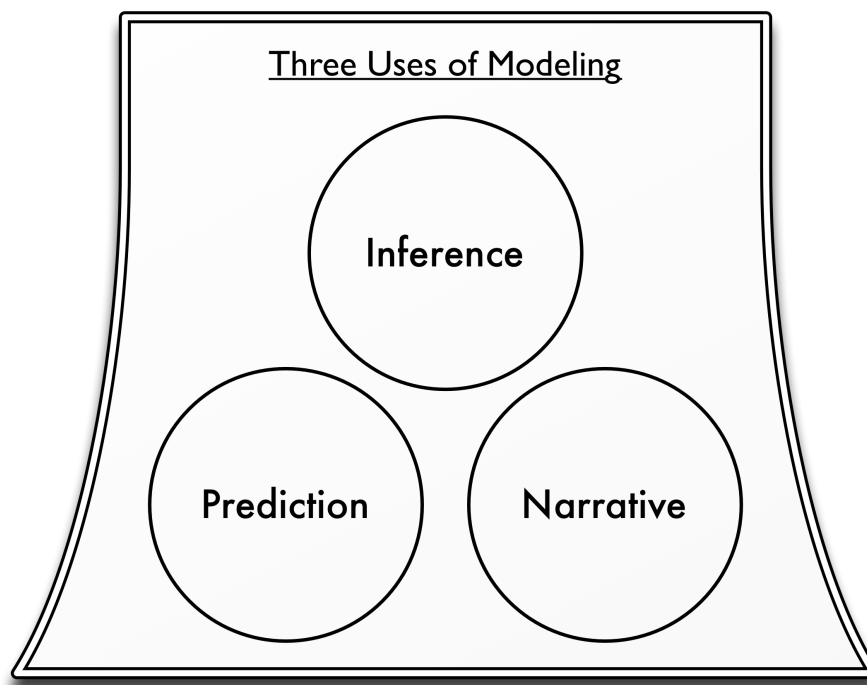


Figure 1. Three Usages of Models

classification system we can use, however, that once we complete it will really clarify the core dichotomy that is at the heart of modeling.

The Strange Case of Inference

To get at this reduced classification scheme, let's talk for a moment about the process of inference. Take our example of finding whether wealth results in increased tests scores in high-school. We phrased this hypothesis in a specific way: that increased wealth will always increase test scores. This was an illustrative statement, however, that actually differs from what is often done in practice. In general, academic research simply asks the question "Does X affect Y ?" Using this terminology, we would ask the question "Does wealth affect tests scores?"

The gold-standard to answering questions like this is the controlled experiment.

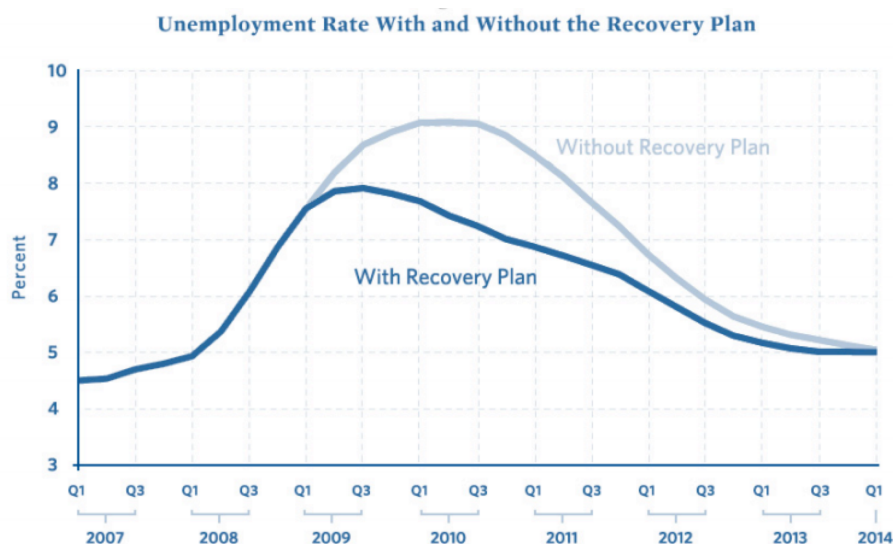


Figure 2. Obama’s Team’s Predictions for the Effects of the Stimulus

For our wealth case, we could take a sample of a thousand families from a school district. When the families’ children enter high-school we randomly select half to be in a “poor” category and the other half to be in a “rich” category. Families in the rich category are given grants of \$500,000 a year to spend how they wish while the parents in the poor category are fired from their jobs and their savings are frozen for the duration of the experiment. Once the students have graduated from high-school, we simply compare the average scores for the students in the poor and rich categories.

This is the ideal approach to answering inferential questions like these. For many types of questions controlled experiments can be done (for instance does treatment with a novel drug help control a disease), but in general complex social questions are simply impossible to answer with them. We can construct the procedure like we have just done as a thought experiment, but it would be impossible (and deeply unethical) to implement in society.

Given this inability to do controlled experiments in many cases, how do we approach inferential questions? The standard way is to collect data and then construct a model and measure the statistical significance of the hypothesis given the data. The model of choice is the linear model:

$$Y = \beta_0 + \beta_1 \times X_1 + \beta_2 \times X_2 \dots$$

For the education example we could simply collect data on a number of students. We could measure their families’ wealth (X_1 in the equation above) and on their test scores (Y). We could then run the linear regression to determine the

coefficient values (β_0 – the intercept – and β_1 – the effect of wealth on test scores). If we thought there were other factors that affected test scores, we could measure them and include them as additional X 's in the regression.

In addition to obtaining the coefficients, we also obtain as an output from the regression the statistical significances or “ p values” of these coefficients. Although p values are commonly used in statistics, they are ubiquitously misunderstood² so it would be useful to briefly review them.

In short a p value measures the probability of seeing the data (or more extreme data) assuming the null hypothesis is true. When looking at the significance of coefficients, this means the probability of seeing that magnitude of coefficient (or one even further from 0), given that the (unknown) truth is that the coefficient actually has a value of 0. In other words, it is the probability of seeing the non-zero value, assuming that the true value is in fact 0. Generally probabilities of 10%, 5% or 1% are taken as indicating statistical significance. Basically meaning that the coefficient value is so far from 0, and the probability of this occurring by chance so low, that we can accept the fact that the coefficient is not 0.

p values do not represent the following commonly used interpretations:

- The probability that the null hypothesis is true (that the coefficient is 0)
- One minus the probability that the alternative hypothesis is true (that the coefficient is not 0)
- Any sort of “proof” that the null or alternative hypothesis is correct or incorrect
- The probability that you are making the correct or incorrect decision if you accept or reject the null or alternative hypothesis

Using the p we can do inference by using the statistical significance of the coefficients. If the probability of the β_1 (the coefficient for wealth) occurring due to chance (given it is false in reality) is less than, say 5%, we can say with reasonable strength that wealth does in fact affect test scores. This is the standard approach taken to model-based inference.

However, let's stop for a second and consider what we have done here. In carrying out these logical steps, we had to make one very large assumption: that the relationship between test scores and wealth is linear. Our equation shown above assumes that for every increase in one unit of wealth (X_1), test scores (Y) will increase on average by the amount of the coefficient (β_1). What if this is not in fact true? For instance, we could easily imagine the case where wealth initially helped test scores by providing students more resources and opportunity to learn. However, after a certain point, wealth could theoretically

²These misunderstandings are not only made by on-the-ground practitioners and analysts, they are frequently shared, and propagated, by university-level statistics instructors; see, for instance, Haller and Krauss (2002).

negatively impact scores as very wealthy students might lack the pressure or motivation to score highly.

If we thought this was the case we could change our regression formula to contain quadratic terms which could replicate this type of relationship:

$$Y = \beta_0 + \beta_1 \times X_1 + \beta_2 \times X_1^2$$

If our assumptions about the quadratic relationship are correct, then these models will yield accurate inferences. However, if they are wrong, our inferences will also be wrong.

What are we really doing when we assume regression forms like this? Now it might not be immediately obvious, but what we really doing is telling a story. Using our first equation, we are telling the story that as wealth increases, test scores will always increase. Bill Gate's children will preform amazingly well! Using the second equation we are telling a different story: that as wealth increases test scores initially do as well but after a certain point increased wealth will hurt test scores. The picture isn't so rosy for Bill Gates's family this time around!

And so we arrive at a very interesting conclusion. By telling these stories, our inferences are in fact based on narrative modeling approaches. Yes, these inferences are based on numerous calculations and very advanced theoretical underpinnings. But ultimately what governs our conclusions and inferences are the stories or narratives we tell about them. These are choices that we as the story teller make, not any objective truth or reality.

Is there an alternative approach to inference? Can we accomplish it without using narrative techniques? The answer is yes. An alternative prediction based approach to inference is available. In which rather than calculating statistical significances as a function of an assumed model, we calculate them as a function of the simple question: "Does knowing X help us to predict Y ?" If the answer to this question is true, then we can say that there is a relationship between X and Y .

The techniques to accomplish prediction-based inference are much newer as they rely upon ample computing power to work. One of them is the A3 method (XXX Citation) which uses resampling based techniques to obtain estimates of predictive accuracy and statistical significance. What they do however is remove model form question from us and instead focus purely on predictive accuracy to determine whether a variable is significant.

Thus we can see that inferential techniques can be split into two categories: narrative modeling methods and predictive modeling methods. So from our original three categories of model purposes – prediction, inference, and narrative – we are left with just two fundamental types of modeling: predictive modeling and narrative modeling.

Predictive versus Narrative Modeling

Predictive Models

How do we define a predictive model? The obvious answer is that predictive model is one that make predictions. If a model makes generates predictions for the future or a given scenario, than it must be a predictive model. By this definition, a weather forecast is a predictive model as were Obama's unemployment predictions we saw earlier.

Unfortunately, this straightforward definition is useless. Worse than being useless, it is actually quite dangerous.

Let us propose a model for next year's unemployment figures in the United States:

Generate a random number from 0 to 1. If the number is less than 0.1, unemployment will be 20%. If the number is greater than or equal to 0.1, unemployment will be 0%.

There, we have just constructed a model of unemployment. Furthermore, our model creates predictions. With just a minute of work we can forecast unemployment for the coming year. Isn't that convenient?

Of course this model is a joke. It is clearly a horrible tool to use to predict unemployment. However, using the naive definition of what it means to a predictive model, it could be classified as one. What makes this simple model, such a poor model for prediction purposes?

There are several answers to this. We might start by saying it is too *simple*. Clearly if we are trying to predict unemployment we should include the current economic state and trends into our model. If the economy is improving, unemployment will probably drop and vice versa. That is probably true, this model could be probably be improved by including data about the economy and future trends. Let's propose a different model:

Generate a random number from 0 to 1. If the number is less than the percentage change in GDP over the past year, unemployment will be 20% plus the current unemployment rate. If the number is greater than or equal to 0.1, unemployment will be the net change in the consumer price index over the past 8 years.

Is this a better model? Clearly, it is more complex and now it is incorporating some relevant economic data and indicators. However, equally as clear it is also a joke no one should treat this as a real predictive model.

So what defines a predictive model? Well the answer is straightforward if a bit more complex than our first proposal. A predictive model is one that:

1. creates predictions,
2. has an accurate estimate of prediction error,
3. and has the lowest prediction error of a reasonable set of other models.

If these three criteria are true, then you have a predictive model. Our two proposed models to estimate unemployment are clearly not predictive as we do not have any estimate of predictive error. We can also apply this definition to Obama's employment predictions we saw earlier. At the point, we called it a narrative model which was probably a tad confusing as the model did generate predictions. However, using these three criteria we can see also that it is not a predictive model. The model contains no estimate of prediction error (and one is not available in the original report containing it) so it simply cannot be considered to be predictive.

If accurate estimate of prediction error are available, you can proceed to criteria 3 and directly compare the prediction errors between different models to find the one with the lowest error. For instance, we could estimate prediction errors for the two simple models proposed here along with Obama's model to find the one with the lowest error. We would hope that the one Obama presented to congress would be the most accurate, however, unless this is explicitly done no statements about predictive accuracy can be made.

Why do we so rarely hear about the predictive accuracy of models. There are a number of reasons but they all boil down to three basic issues:

1. Assessing prediction error accurately is quite difficult.
2. Sharing prediction error will likely decrease subjective belief in a model.
3. Most models people use for prediction are actually narrative models and discussing their predictive error is futile.

Let's look at each of these points in detail. First the issue of the difficulty of assessing prediction error. In general, obtaining an accurate assessment of prediction error is much more difficult than developing the predictions themselves. Most commonly used approaches (for instance the standard R^2 from linear regression) have significant flaws. There are both theoretical and numerical methods that can be used to more accurately predict prediction errors in many cases (will discuss this further in the section the Cost of Complexity). When dealing with time series models, however, like most of those explored in this book, it is almost impossible to truly manage to accurately assess model prediction error. In the past ten years, theoretical techniques to approach these issues have just begun to be developed (e.g. He, Ionides, and King (2009) or A. A. King et al. (2008)) but they are still impractical to apply in most cases.

If the challenge of measuring prediction error is overcome, there is an even more insidious barrier to its being made available. There is a perverse phenomena, that the act of reporting prediction error will often *decrease* the credence an audience gives a model. (XXX model)

Finally, probably the most likely reason supposedly predictive models do not include prediction error is that they simply aren't predictive. Generally models developed for a purportedly predictive purpose are actually narrative models in disguise. This often happens as the modelers tasked with building the model do not understand predictive techniques or the client demands features and capabilities in the model that are incompatible with predictive accuracy. There is nothing inherently wrong with the models and they can be very useful, but it is important to recognize them for what they are: narrative storytelling.

Narrative Models

In practice many modeling projects will try to fulfill multiple usages. For instance, it is common to for clients of modelers to want to build a model for prediction

Chapter 4

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