[DRAFT] Beyond Connecting the Dots: Mastering the Hidden Connections in Everything that Matters

Gene Bellinger Scott Fortmann-Roe

2013-07-24

Contents

Preface

Ludwig von Bertalanffy(1) first proposed, in 1937, that the same basic structures operated across all disciplines, and if one learned how these structures operated one could transfer much of their learning from one discipline to another. When moving from one discipline to another, one would simply have to learn the structures that were operating, and the labels on the elements of the structures. On first reading this may seem most profound, or maybe even preposterous.

However, if you think about it, maybe there is some truth to it after all. What follows is the introduction to a live Systems Thinking book presented from a cross discipline models perspective. Live in the sense that the models are presented in a form that allows you to actually interact with them.

von Bertalanffy wrote "Allegemein Systemlehre" which was translated into English as "General Systems Theory"(2) and I expect we've still not recovered from the translation error. What he intended was a "General Theory of Systems" or "General Systems Teaching," a way to support learning about the structures which operated across all disciplines. Today there are a set of structures referred to as Systems Archetypes which I believe are just what Bertalanffy had in mind.

In the words of von Bertalanffy, "The student in 'system science' receives a technical training which makes systems theory – originally intended to overcome current overspecialization – into another of the hundreds of academic specialties"(1)

Systems Thinking is not a method though more of a way of looking at the world around us and understanding based not from understanding things though more from understanding relations and interactions between things. And while there are many who believe that Systems Thinking or a Systems Perspective provides the best foundation for creating effective approaches of dealing with challenges and shaping a better tomorrow. Yet even with that view, over the past 75 years it has not become widely adopted, even though during that period dozens of approaches have been developed with claim to embrace the Systems Thinking world view. I believe Pogo had it right when he said, "We have met the enemy and he is us." I have repeatedly commented to people that the greatest impediment to the adoption of Systems Thinking is Systems Thinkers.

This should provide you with a sense of why this book has to be different. Now let me offer you a view of how it will be different.

It is our intent to provide a basis for recovering from this overspecialization by offering an extensive series of models from everyday life that will show the value of looking at things though a different lense. We will then build on this to develop an understanding without all the terminology and complexity that typically drives people away from Systems Thinking.

References

- Davidson, Mark. 1983. Uncommon Sense: The Life and Thought of Ludwig von Bertalanffy http://www.amazon.com/Uncommon-Sense-Thought-Bertalanffy-1901-1972/dp/087477165X/

Chapter 1

It's The Pattern That Connects

What you learn, and your capacity to learn, serves as a basis for everything you do in life. Yet, have you ever really thought about how you learn about the world around you? There are some things you memorize early in life, like the times tables. While you memorize these is that really learning? Do you remember that if you put your hand on something very hot it will burn you, or is that something you learned? And if you learned that, how was it that the learning happened? In this chapter we will investigate how you actually learn. We will also present a introduction to how you can improve your learning and actually test whether what you have learned is actually correct.

Patterns

Consider the following

- I have a box that's about 3 feet wide, 3 feet deep and 6 feet high
- It's a rather heavy box
- The box has a couple of doors on it
- When you open the doors it's cooler inside the box than outside
- One compartment is much colder than the other
- When you open the door a light comes on
- There's food inside the box
- The box is in the kitchen
- There are sticky notes all over the front of the box
- There's a collection of papers and stuff on top of the box
- If you move the box you'll probably find a lot of dust under it
- The box is plugged into an electrical outlet
- From time to time you can hear the box running

At some point in this sequence you probably became convinced that what was being described was a refrigerator. Now stop for a moment and ask yourself just how you realized what was being described was a refrigerator? Yes it would have been easier if you had just been shown you a picture of a refrigerator, though that would have spoiled it, wouldn't it.

Figure 1. It's a refrigerator, but how do you know?

As long as you knew beforehand what a refrigerator was, the statements could have been given to you in any order, and at some point you would have finally realized what was being described. If you had never seen, nor heard about, a refrigerator before, you would still be wondering what was being described and what to call it.

You have also most likely come to understand that all refrigerators are not identical. Some have one door with a separate compartment inside. Some have two doors and a drawer. Some are much smaller than others. Some can fit under a counter and some even fit on top of a counter. Some can be so large you can walk into them.

Figure 2. Many kinds of refrigerators, or freezers, but how do you know?

If you see any of the items in Figure 2 you quickly decide it's a refrigerator. How does that happen? Gregory Bateson, one of the great thinkers of our time, said, "It's the pattern that connects." If you reflect on this statement you should come to realize there are actually different ways to interpret what it means. In this particular case the pattern connects you to the following purpose

- The box keeps food from readily spoiling by keeping it cold
- Part of the box is a freezer which keeps food from spoiling for even longer

and you understand it to be a refrigerator. Though now that we've arrived at this point we still haven't addressed the question of how you know. You were probably not actually taught that it's the above purpose that defines the essence of a refrigerator. Most people were not, though they have essentially learned it over time.

Models

Models are the way we look at, and understand the world around us. All we have are our models. They are the way we understand everything. This is so because we build our understanding based on what we already understand.

The world around us simply has too much detail for us to pay attention to everything. A refrigerator has many pieces though how many do you really pay attention to? Probably not many unless you build or repair refrigerators. We choose what to pay attention to in the world around us and we filter out much of the detail we don't become overloaded. Sometimes we do this consciously and sometimes we do it subconsciously though experience. In the midst of what we choose to pay attention to there are patterns. Whether we realize it or not it is these patterns that we pay attention to and attempt to make sense of. We understand these patterns by linking them to extend patterns we already understand while we ignore much of the detail around us.

Model

A model is a simplified version of some aspect of the world around us to help us understand something.

Learning

When we experience something that experience falls somewhere between complete novelty, meaning that we can't connect it with anything in our past experience, and complete confirmation, meaning that it represents something we already completely understand. Experiences which lie somewhere between complete novelty and complete confirmation provide a basis for learning. They represent a basis for connecting to understood patterns, extending those patterns and our understanding, and what results is learning. {Cite: Jantach, Eric. 1980. The Self-Organizing Universe: Scientific and Human Implications. Pergamon Press. http://www.amazon.com/The-Self-Organizing-Universe-Implications-Innovations/dp/0080243118/}

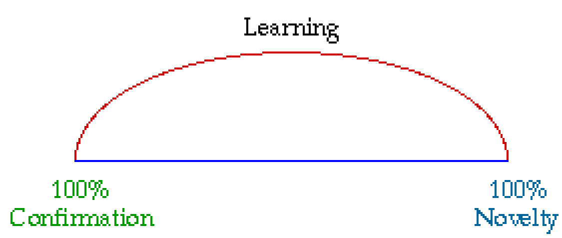


Figure 3. Experience as a Basis for Learning

Consider running into a refrigerator that looks like no refrigerator you've ever seen before. From an initial view you are unlikely to perceive it as a refrigerator.

As you inspect it and find it serves the purpose you've come to understand for refrigerators, or if someone tells you it's a refrigerator, you then expand or extend your awareness of the range of patterns that constitute a refrigerator. And as Bateson said, "It's the pattern that connects."

A Basis for Flawed Learning

While reading the previous paragraphs did it dawn on you that much of this pattern recognition/connection/extension learning doesn't happen consciously? We connect with patterns and extend our knowledge at times without even being consciously aware that it is happening. And when this happens subconsciously there isn't really any critical validation that happens along with the learning. Because this ongoing learning happens without critical validation there are things we learn and come to believe which are actually incorrect. We have perceived patterns and extended our learning in a flawed manner. The really annoying thing is that we then act on these beliefs, and when we produce results that don't go the way we planned we wonder why. Or even worse, we don't actually learn from the results and correct the flawed models which served as the basis for our flawed actions.

When we act on flawed beliefs attempting to solve problems we typically create more problems. It has been said repeatedly that the majority of today's problems are the direct result of yesterday's solutions. Wouldn't this provide a sense that we might really benefit from a better way to think about the world around us, develop better understanding, and develop solutions that don't come back to haunt us in the future?

A Better Way

Based on what has been presented to this point it should be obvious that we could benefit from a better way to develop models of what we believe. Models that are more likely to be correct, as well as surface flaws in many of our current beliefs.

Ludwig von Bertalanffy first proposed, in 1937, that the same basic structures operated across all disciplines, and if one learned how these structures operated one could transfer much of their learning from one discipline to another.{Davidson, Mark. 1983. Uncommon Sense: The Life and Thought of Ludwig von Bertalanffy. J.P. Tarcher, Inc. http://www.amazon.com/Uncommon-Sense-Thought-Bertalanffy-1901-1972/dp/087477165X/} When moving from one discipline to another, one would simply have to learn the structures that were operating, and the labels on the elements of the structures. On first reading this may seem most profound, or maybe even preposterous. However, if you think about it, maybe there is some truth to it after all.

We're not asking you to believe the previous statement because it was provided here. Through the experience presented shortly it is hoped that you will arrive at a sensibility of the statement from your own perspective.

Consider the following images and ask yourself what is it that all these different images actually have in common.

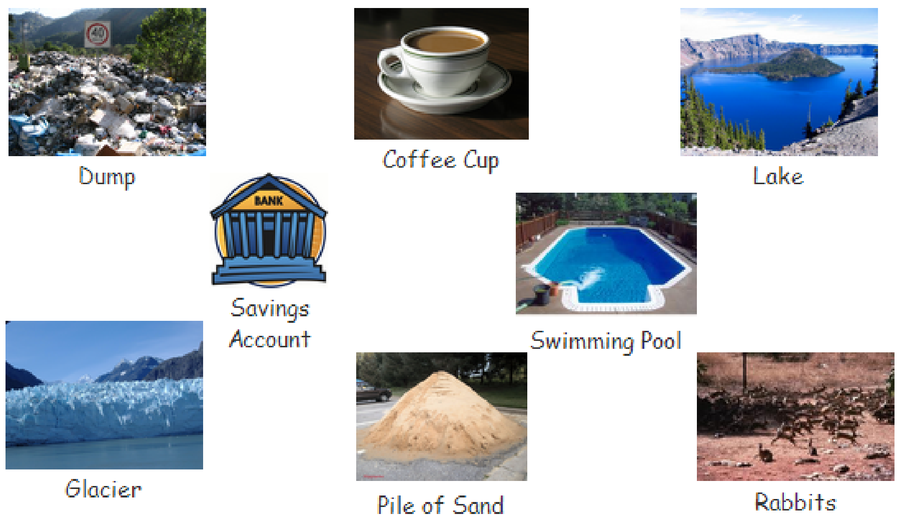


Figure 4. What do these images have in common?

Each of these items represents a collection of stuff. Admittedly each image represents different stuff, though stuff just the same. Because in each case the collection of stuff collected over time it's really more appropriate to refer to the collections of stuff as accumulations. As you will come to realize it is extremely important to remember that accumulations take time to accumulate, and often even longer to get rid of when you find out you don't want them.

The shorter term often used to refer to an accumulation is "stock." Just where this term originated is uncertain. What you call an accumulation of stuff isn't nearly as important as remembering it's a bunch of stuff that collects, or goes away, over time. How much time is different for each one of the accumulations. Now is probably a good time to talk about how accumulations happen over time.

How each of the accumulations in Figure 4 changes is a bit different, as are the time frames of concern. Time frame being the time it takes for some noticeable change in the accumulation. The following segments describe each accumulation in some detail.

Coffee Cup

You usually fill a coffee cup from a coffee pot and it takes a few seconds. Then you take a few minutes to drink the coffee as it's usually too hot to drink when you initially pour it.

Dump

A dump generally accumulates by the truckload after the garbage is picked up at houses or businesses in your community. If the dump were just getting started you'd probably notice it grow with each additional truck load. As it gets bigger and bigger it's gets more difficult to notice that it's growing, even though it is. While the dump is likely to grow almost every day we are probably more likely to think about the growth of the dump in months or years. And does it ever really go away? Usually when it gets to be too much a new dump is started somewhere else and the current dump is buried. Though when the dump is buried it doesn't really go away does it? It's still there and we'll probably talk more about dumps later on.

Glacier

A glacier is a long term accumulation of snow which packs down and turns to ice. Glaciers generally get bigger in winter when it's colder and snow falls, then they get smaller in summer when some portion of the glacier melts. The time frame one usually uses to think about glaciers is years, or even decades.

Lake

Lakes are bigger than a pond and smaller than an ocean and usually filled with fresh water, not salty, unless it's The Great Salt Lake or course. The lake is filled by rivers and streams that flow into it as well as rain water. One might think of this filling in terms of gallons per hour, or gallons per minute in the case of a large inflow such as at Niagara Falls where the water flows into Lake Ontario in the USA. Water leaves the lake through rivers and streams as well as evaporation into the air. For a lake one might think about the water flowing into or out of the lake in hours though when considering the level of the lake itself the change might be considered over days or weeks. It sort of depends on what you're interested in.

Pile of Sand

The pile of sand probably showed up in a truck that dumped it right where it is. While it may have taken a while for the truck to drive from wherever it was filled it probably only took a few minutes to dump the sand once the truck arrived. The sand is probably referred to in cubic yards, which is how much sand it takes to fill a box that's 1 yard wide, 1 yard deep, and 1 yard high. How long it takes for the sand to go away depends on how it's taken away. If you use a wheel barrow then you have to shovel the sand into the wheel barrow and take it to wherever you're going to use it. At this rate it may take days to move it. If you move it with a small piece of machinery, a Bobcat or a Backhoe, then will will probably only take a few minutes to an hour to get it moved.

Rabbits

A population of rabbits gets larger with new rabbit births and gets smaller with rabbit deaths. Have you ever heard the phrase "multiply like rabbits?" What it means is that it doesn't take very long for a few rabbits to become many rabbits, as long as there is a good food supply and not to many predators like wolves and coyotes. The time frame for considering a rabbit population is probably months to years.

Savings Account

A savings account is an account where if you put money in, and keep it there, you will periodically be given money just for keeping your money there. You won't get very much, though some. If you keep putting money in your savings account every so often and never take it out one day you'll be rich. Yet, for some reason that doesn't seem to happen for too many people. We'll have to talk about that sometime later. One generally thinks about the money associated with a savings account in dollars, the interest rate as a percentage, and the time frame in months to years.

Swimming Pool

Swimming pools usually hold thousands of gallons of water and you usually have a couple of options to fill one. You might use a garden hose, which will take days, or a hose from a fire hydrant, which will take a few hours, or from a tanker truck, which probably takes a few loads. In each case the water to fill the pool is probably measured in gallons. Once you fill the pool you loose a little water when people in the pool get out, thought not too much. Most of the water loss from a pool is though evaporation due to the sun and when you backwash the filter used to keep the pool clean. The change in the amount of water is usually measured in gallons per hour or per day.

Exercise 1-1

Take a few minutes and identify half a dozen situations you're familiar with where there are stocks that increase, or decrease, over time. What are the quantities for those stocks, e.g., gallons, pounds, kilograms, etc? What are the flows that increase or decrease those stocks? What are the time frames over which you think about the increase, or decrease of the stock?

At this point you may be wondering why so much time was spent walking though all these examples for increasing and decreasing accumulations of stuff. Since we said this was an interactive book you're probably wondering where the interaction is.

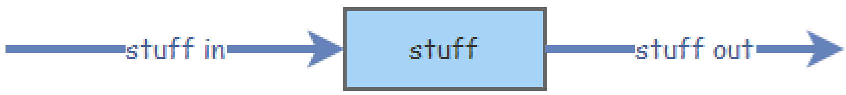


Figure 5. The Accumulation of Stuff

All the accumulations depicted in Figure 4 can be represented in a general form by the model in Figure 5. Remember we defined a model as a simplified version of some aspect of the world around us that we use to help us understand something. It doesn't get much simpler than this does it?

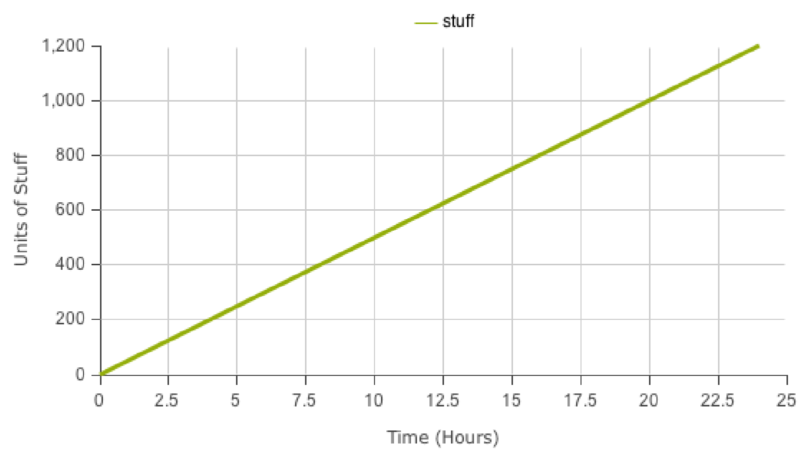
Some amount of [**stuff in**] flowing causes [**stuff**] to increase over time and the amount of [**stuff out**] flowing causes stuff to decrease over time. With both of these happening at the same time [**stuff**] increases if [**stuff in**] is larger than [**stuff out**]. And if [**stuff out**] is greater than [**stuff in**] then the accumulation of [**stuff**] gets smaller. The most critical aspect of this to remember is that it takes time for [**stuff**] to increase or decrease. How fast the change happens depends on the amount of stuff in the flows.

Swimming Pool

The following model investigates a swimming pool as a stock.

1. The model diagram should now look something like this:

2. The diagram represents a swimming pool being filled with a hose at a rate of 50 gallons an hour.
3. The drain is closed so there is no water draining out of the pool.
4. If we let the hose run for 24 hours how much water will be in the pool? Admittedly the math is pretty straight forward though the idea here is to show how you can use a model, a simulation of a model actually, to show changes over time.
5. Run the model. Here are sample results:



6. This graph indicates that after 24 hours the swimming pool will have 1,200 gallons of water in it. Yes, it's about as interesting as watching paint dry. Actually, as we hope you will come to find out, that's a good thing because this is really easy. A more interesting question might be, if the swimming pool holds 20,000 gallons of water how long with it take to fill it with water at 50 gallons per hour? We'll get to this shortly.

The next chapter will present an introduction to Insight Maker so you can build models and run simulations. Building and working with simulations will allow you to extend your understanding of the world around you.

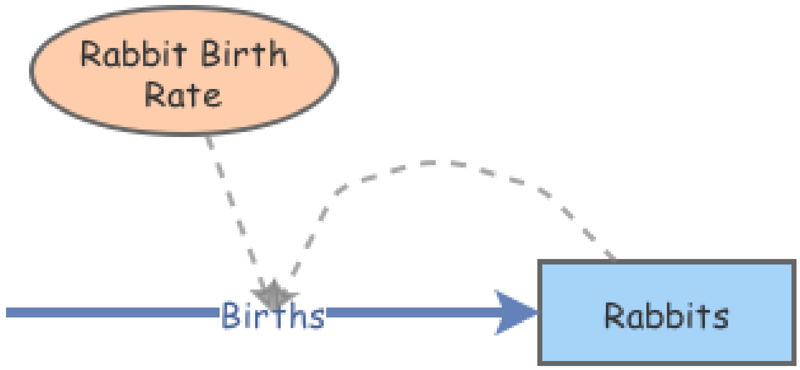
Rabbit Population Growth

If you considered the accumulation of rabbits in Figure 4 you may have already realized that the model of Figure 5 is missing something. Yes, if you add rabbits to rabbits you get more rabbits. Though if you have more rabbits don't they create even more rabbits?

Rabbit Population Growth

This model that reflects the the notion that more rabbits create even more rabbits.

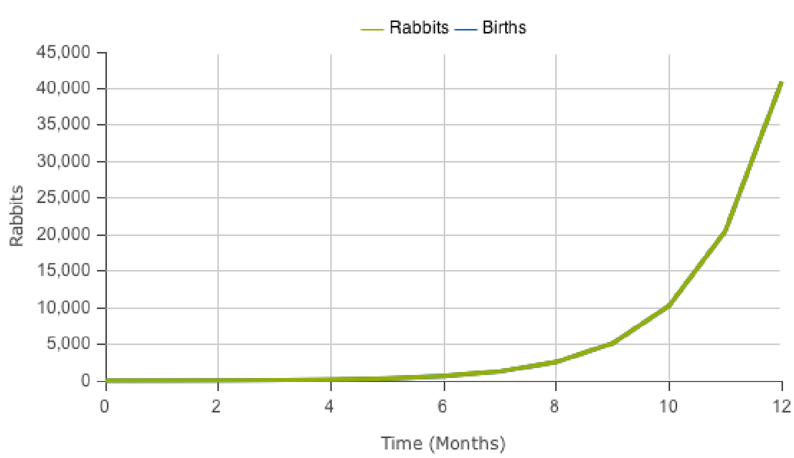
1. The model diagram should now look something like this:



2. This model indicates that if you start with some population of [**Rabbits**] and each time period the current number of [**Rabbits**] times the [**Rabbit Birth Rate**] will result in a number of [**Births**]. This number of [**Births**] will then be added to the accumulation of [**Rabbits**] and figure into the calculation for the next period.

3. Suppose we start with 10 rabbits, half of which are male and half of which are female. Research indicates that a female rabbit can give birth to between 18 and 26 Rabbits a year. The average for this, (18 + 24) / 2 = 22, though we'll round this up to 24 just because it will make the math easier. If a female rabbit can produce 24 Rabbits a year, that's 2 per months, though it actually takes two Rabbits, one male and one female. With all these assumptions we get about 1 new Rabbit per month for each Rabbit.

4. Run the model. Here are sample results:



5. Forty thousand Rabbits in a year? That seems a bit bizarre doesn't it? This result actually points out the real value of modeling, which is

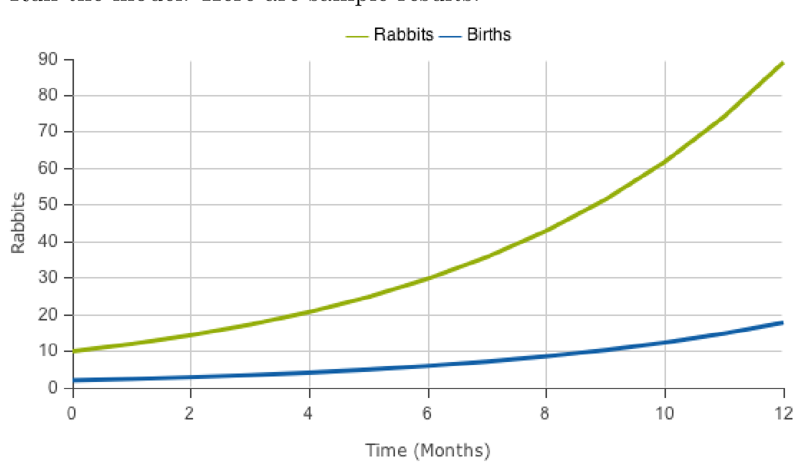
learning. You build a model based on what you think you understand. You then populate it with assumptions about the relations and values, and then you run it. The result then either seems to make sense or seems really bizarre. When the results are really bizarre what the model is telling you is that either the structure is wrong, the assumptions are wrong, or both, because the world can't possibly be this bizarre. As a result you investigate the model and your assumptions. As your understanding improves the model gets better. At some point the model finally serves its purpose, to be a simplification of some aspect of the world which leads to a better understanding. Hopefully you come to find that going round and round with a model can be a delightful learning process.

6. Also, did you notice the choppy nature of the graph? This is a result of some incorrect settings and we'll talk about why this is happens and how to address it in Chapter 2.

7. After that sidetrack lets get back to our 40,000 Rabbits that can't possibly exist after a year. You can be pretty sure how many Rabbits you started with at the beginning. And when you check the formula for Births it seems to be in order. This sort of means the assumption for Rabbit Birth Rate must be too big. If you think about what the model is doing it's probably not too difficult to figure out that the model assumes that a Rabbit can be born this month and then give birth to another Rabbit next month. If a Rabbit has to mature for six months before it gives birth to Rabbits then the Rabbit Birth Rate might be something more like 20%.

8. Change the **Equation** property of the primitive [**Rabbit Birth Rate**] to `.2`.

9. Run the model. Here are sample results:



10. Is this right? A good thing to remember at this point is that "Is it right?" is actually the wrong question. A better question might be, "What have I learned, and is there more I can learn?" The graph sure seems more reasonable than what the model presented in the previous run, though do you have a high degree of confidence in the current Rabbit Birth Rate. Are there a number of other questions we could ask about our Rabbits. What is the Rabbit Death Rate? Do they have enough food to eat? Are they living out in the open where Coyotes and Foxes can get at them? Does their owner have a passion for Rabbit Stew? These might each be a basis for building a better model, though at this point we're going to leave the Rabbits alone and move on to something else.

The most important learning you should take away from this model is that when what flows into the accumulation increases as the accumulation increases the accumulation can get real big in a hurry. This is actually called exponential grown and we'll talk in more detail about this in Chapter 2.

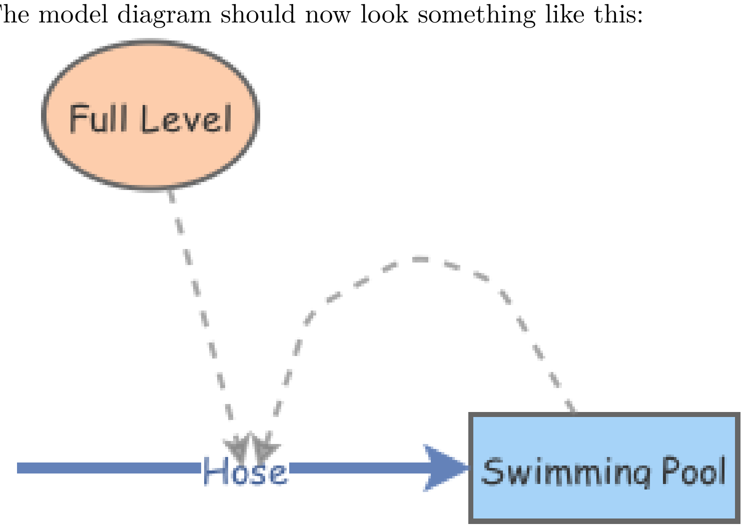
Filling A Swimming Pool

The filling a swimming pool model previous presented was not not very detailed. A more useful question might be, if the pool holds 20,000 gallons of water and the hose fills the pool at 50 gallons per hour, how long will it take to fill the pool. Yes, you can do the math faster than it will take to build the model. Please bear with as there's another aspect of models right around the corner you will find very useful on an ongoing basis.

Filling a Swimming Pool Revisited

This version of the model adds a goal representing a full swimming pool.

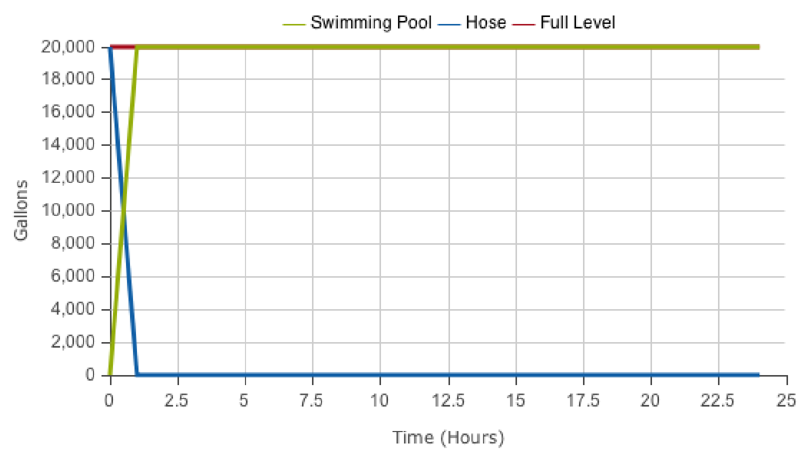
1. The model diagram should now look something like this:



2. We begin with a Swimming Pool that needs to be filled with a hose. We know how many gallows of water it takes to fill the pool and we don't want to put too much water in the pool. The model is created to compare the amount of the water in the Swimming Pool with the the Full Level and use that to decide whether water is flowing in the hose or not.

3. **Hose = IfThenElse([Swimming Pool] < [Full Level], [Full Level]-[Swimming Pool], 0) }**

4. Run the model. Here are sample results:



5. This is really great. We can fill the Swimming Pool in just 1 day, or can we? Either it's a really really big hose or we've done something wrong because it's probably not possible to fill the Swimming Pool with a Hose in one day if it takes 20,000 gallons or water.

Isn't it curious that the structure of this model looks just like the one for the Rabbit Population growth? How a model behaves depends on more than how it looks. We'll come back to this after we figure out how long it's going to take to fill the Swimming Pool.

Hopefully you will come to understand that when your models don't do what you expect them to do it's not a problem – it's an opportunity for learning. This is the real reason why we do modeling - to understand and learn. Just think of it as, the more things don't go the way you expect them too, the more opportunities you have to learn.

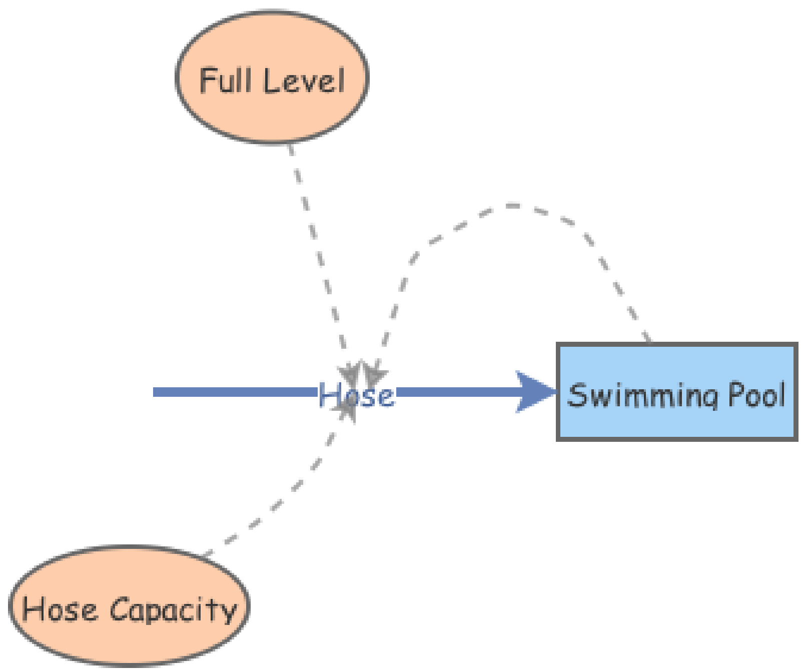
If you look back at the formula for the Hose, notice it didn't take into account the initial statement that the Hose could only deliver 50 gallons per hour. And, might it be useful if we could see what happened with different Hose capacities?

Lets us a revised version of the model with Hose Capacity as a variable so you can set the capacity of the hose before you run the model.

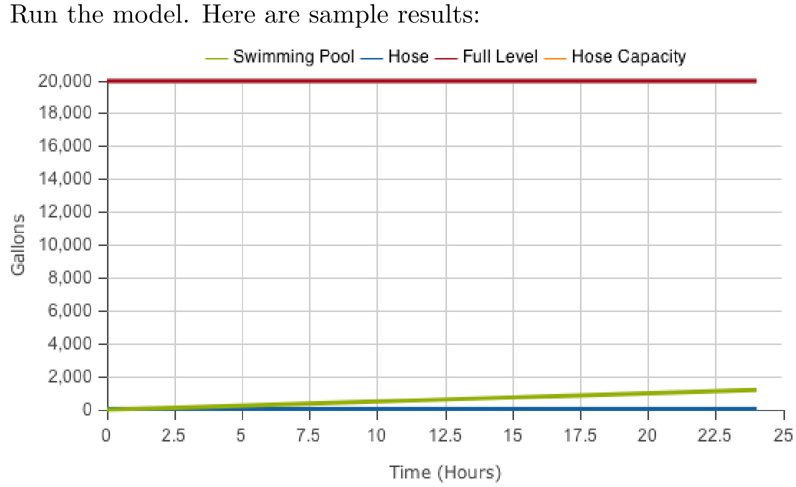
Filling a Swimming Pool One More Time

This version of the model uses a an explicit Hose Capacity.

1. The model diagram should now look something like this:



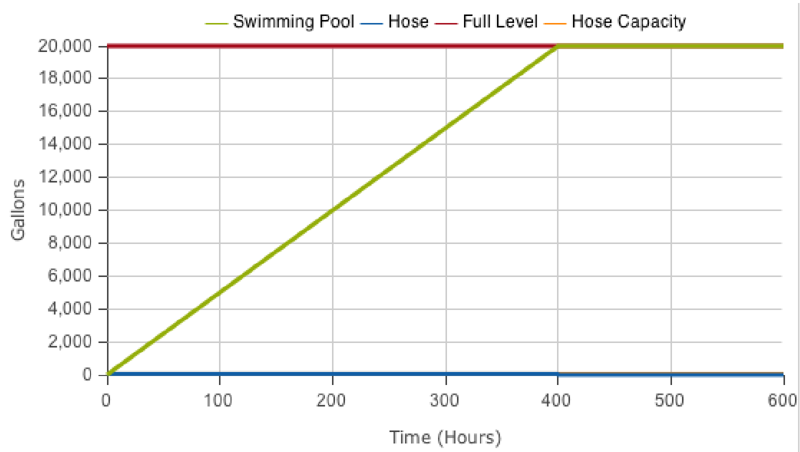
2. The new formula for Hose takes into account both the current amount of water in the [**Swimming Pool**], [**Full Level**] and [**Hose Capacity**]
3. [**Hose**] = **IfThenElse([Swimming Pool] < [Full Level], min([Full Level]-[Swimming Pool],[Hose Capacity]), 0) }**
4. Run the model. Here are sample results:



5. With [**Hose Capacity**] = 50 over a period of 24 hours we've not even come close to filling the [**Swimming Pool**].

6. Change the **Simulation Length** property of the Time Settings to `600`.

7. Run the model. Here are sample results:



8. This graph indicates we need to wait 400 hours to fill the pool. That's a little over 16.5 days. Do we need a bigger hose?

While there are a number of things we could do to improve the model at this point we've gone far enough. We'll begin to get far more sophisticated in the next chapter.

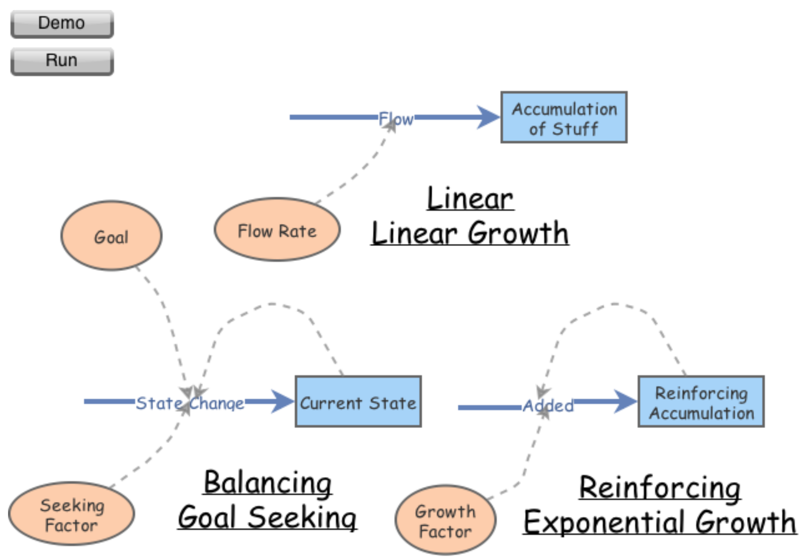
Similar Structures / Different Behavior

If you compare the models presented to this point you should find them quite similar. And yet, the behavior of the models are distinctly different.

Similar Structures / Different Behavior

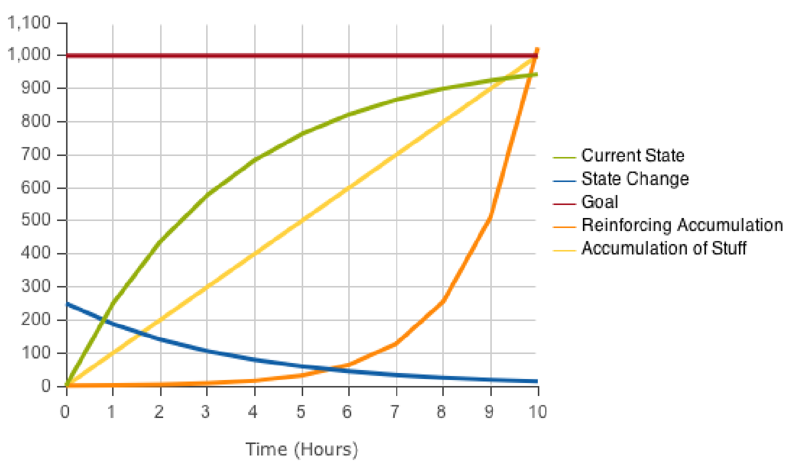
The behavior of a model depends on the structure as well as the formulas that define the nature of the relationships.

1. The model diagram should now look something like this:



2. These three models are in a general form so you can compare the different behavior of the structures that are very similar. [**Flow Rate**], [**Seeking Factor**] and [**Growth Factor**] are each factors which govern he rate of flow. [**Goal**] is a target value which the Growth model doesn't have.

3. Run the model. Here are sample results:



4. The difference that makes a difference is what happens in the connection between the accumulation, or stock, and the flow.

The link between the stock and the flow provides information from the stock to the flow and is generally referred to as feedback, mostly likely because the information travels in the opposite direction as the flow. The nature of feedback results in the three types of models.

Linear

In the Linear/Linear Growth model the Flow simply depends on the [**Flow Rate**] variable, which is expected to be some constant value. This model is referred to as linear because the [**Accumulation of Stuff**] is a straight line as you can see in the graph. Note that if the [**Flow Rate**] isn't a constant, or linear, the [**Accumulation of Stuff**] won't be linear.

Balancing

In the Balancing/Goal Seeking model the [**State Change**] depends on the difference between the [**Goal**] and the [**Current State**]. This difference influences the [**State Change**] to increase the [**Current State**] until it reaches the [**Goal**]. The structure tries to bring about a balance between the [**Current State**] and the [**Goal**] so the difference is zero, and then there's no more [**State Change**].

Reinforcing

In the Reinforcing/Exponential Growth model [**Added**] depends on the value of [**Reinforcing Accumulation**]. This influences [**Added**] to increase the [**Reinforcing Accumulation**] which increases [**Added**]. One might consider a Reinforcing structure to be a Balancing structure that's out of control.

Would you believe that no matter how complicated a model may look it's really only some number of these structures connected together? In the next chapter you will learn about the Insight Maker modeling and simulation environment so you can begin building models and investigating their implications.

Exercise 1-2

The values in the previous model were contrived so when you click the Demo button the model will produce the graph in displayed graph.

- Can you figure out why the values assigned are responsible for the curves produced?
- Alter the slider values for the parameters in the **Configuration Panel** and run the model to get a sense of the impact initial values have on the behavior of these structures.
- Can you explain to someone else the difference between the Linear, Balancing and Reinforcing models in terms of why the structures produce the behavior they do?

Summary

- Models are simplified versions of the world around us.
- We build models to help us understand and learn.
- We build simple models and add to them as we learn with them.
- Building models and learning is an iterative process.
- We learn as we go and seldom do we get models right the first time.
- Linear, Reinforcing and Balancing structures are the basic building blocks for all models.
- These building blocks can aid in understanding aspects of our interactions with the world around us.

Please continue to the next chapter where you will learn about the Insight Maker environment so you can actually build the models that were presented in this chapter.

References

- Kauffman, Draper L. 1980. Systems One: An Introduction to Systems Thinking. http://www.amazon.com/Systems-One-An-Introduction-Thinking/dp/9996280519/
- Meadows, Donella. 2013. Thinking in Systems: A Primer. http://www.amazon.com/Thinking-Systems-A-Primer-ebook/dp/B005VSRFEA/
- McDermott, Ian & O'Connor, Joseph. Unk. The Art of Systems Thinking. http://www.amazon.com/The-Art-Systems-Thinking-ebook/dp/B0091XFU70/
- Sherwood, Dennis. 2002. Seeing the Forest for the Trees: A Manager's Guide to Applying Systems Thinking. http://www.amazon.com/Seeing-Forest-Trees-Managers-ebook/dp/B004GCK63Y/

Chapter 2

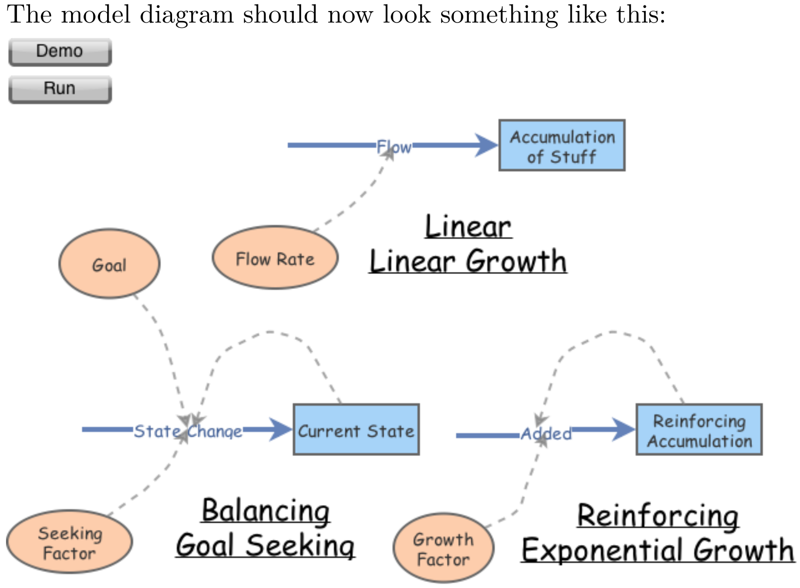
Tools for Understanding

As we work with tools to build things we develop skills using the tools and these skills have a very definite impact on things we create. In this chapter you will learn to use Insight Maker. You will learn to construct the models below and develop an understanding of why these structures form the building blocks of everything you will create in Insight Maker to further your understanding. As such it is strongly recommended that you actually use the features described and build the models. You can't learn to ride a bicycle by reading a book. You develop skills by actually riding the bicycle.

Similar Structures/Different Behaviors

These are the structures from the previous chapter which you will investigate in detail as you learn various aspects of Insight Maker.

1. The model diagram should now look something like this:



2. In the next few segments you will learn how to create these three basic structures from which all models are created.

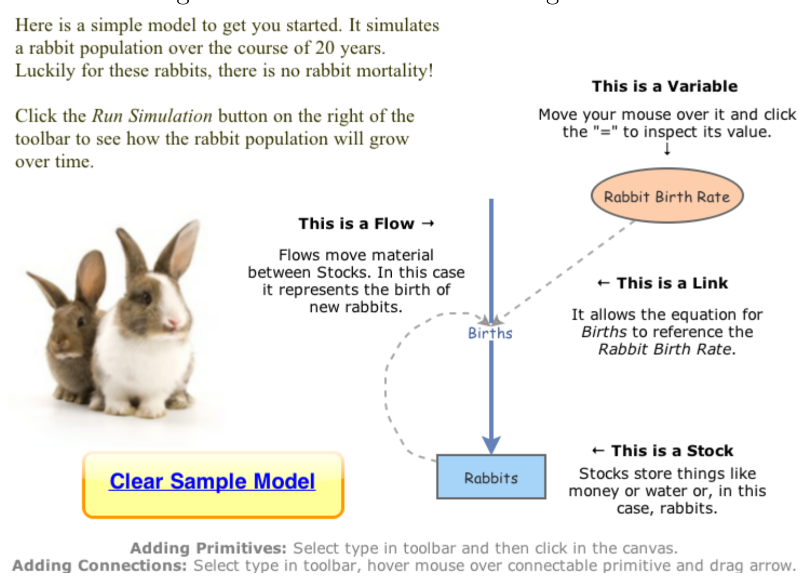
New Insight

When you create a New Insight in Insight Maker you don't actually have to start with a blank canvas. Insight Maker presents you with a very simple working Rabbits Population model so there's something there to interact with and get you started.

Sample Model

The Sample model is something to start with so you don't have to face a blank page.

1. The model diagram should now look something like this:



2. If you click the [**Clear Sample Model**] button you will then have a blank canvas on which to create a new model.

In the next few segments you will learn how to create the three basic structures from which all models are constructed. Various aspects of the modeling environment will be explained when they are initially used in a model, or needed to do the exercises. We won't spend a lot of time on pieces you're not going to use immediately so please don't let any of the displays overwhelm you. It's far easier to remember things when you actually use them.

Canvas

The center area is the work area where you create a model. This area may be scrolled if necessary. All the modeling and simulation is done on the canvas.

Toolbar

The **Toolbar** at the top of the window and the **Configuration Panel** at the right provide all the tools you will use to create and simulate models. The **Toolbar** is depicted in Figure 1. For certain functions small windows will open on top of the canvas though you'll never be taken away from the canvas while working with a model.

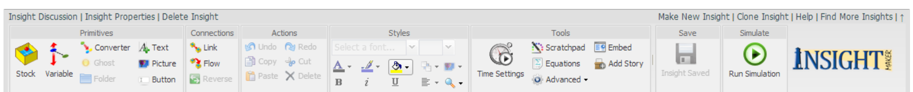


Figure 1. Toolbar

If the Toolbar isn't visible is may simply be closed, in which case there will be a small down arrow as depicted in Figure 2. If you click on this arrow it will open the toolbar displayed in Figure 1. The alternative is also true. If the Toolbar is visible you can click on this arrow to hide it.

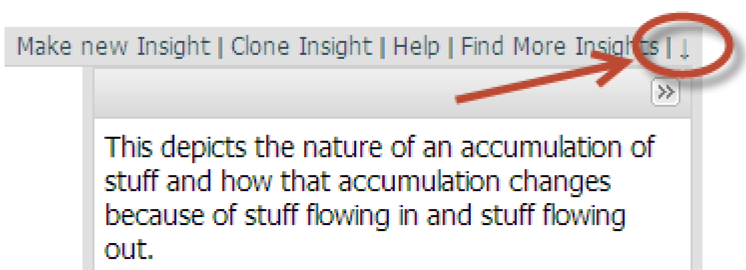


Figure 2. Open Toolbar

Models are created with **Primitives** and then relations established between them with **Connectors**. In this chapter we're only going to use the [**Stock**] and [**Variable**] primitives.

Stock

A rectangle indicates a quantity of something that changes over time. Remember the over time part as stocks don't change in the blink of an eye, well unless there is a flow and you blink very slowly.

Flow

A directed arrow represents the flow of something into or out of a stock. Remember that a stock can only be changed by a flow. Hand waving, magic or smoke and mirrors don't work. The flow has to be explicit to cause a stock to change, which is why it takes time.

Creating Stocks and Flows

To use a [**Primitive**] click on the icon on the **Toolbar** to select it, then click on the canvas where you want the item located. For each **Primitive** there are a set of allowed uses in a simulation model, and we'll cover these later. Once you place the item on the canvas it is named for what it is, with that name selected so you can type in the name you want. Names can contain any characters except braces "{}", brackets "[]", parentheses (), and quotes '. If the label is not selected you can double-click the label to select it and then enter a new one, or you can enter the label in the **Configuration Panel**, though we'll address that in a bit more detail later.

Exercise 2-1

Clear the sample model presented previously and practice placing [**Stock**] and [**Variable**] **Primitives** on a blank canvas and naming them. You can remove a **Primitive** by clicking on it to select it and then pressing the **Delete** key or clicking the **Delete** function in the **Actions** section of the **Toolbar**.

Stocks, Flows, Variables and Links

[**Stocks**] and [**Variables**] are connected to other [**Stocks**] and [**Variables**] using **Link** and **Flow Connectors**. The rules for connections are very explicit because Insight Maker has to figure out how to simulate the model. The allowed connections are constructed in the following model. The next chapter will present several types of models where the rules for connections aren't nearly as rigid.

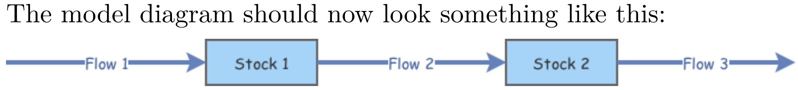
If you select **Link** from the **Connections** segment of the **Toolbar** then hover over a model **Primitive** on the canvas a small arrow pointing to the right shows at the center of the **Primitive**. If you select a **Flow** the small arrow will only show up over a [**Stock**] as a **Flow** can only connect to a [**Stock**].

Center the **cursor crossing double arrows** over the right arrow, which should then change to a **pointing finger hand**. Drag the mouse over to a second model element and the arrow tags along while the connection is drawn. If neither the **Link** or **Flow** is selected in the **Toolbar** then there will be no right pointing arrow when you mouse over the **Primitive**. We'll go into more detail about the validity of connections shortly.

Valid Stock & Variable Connections

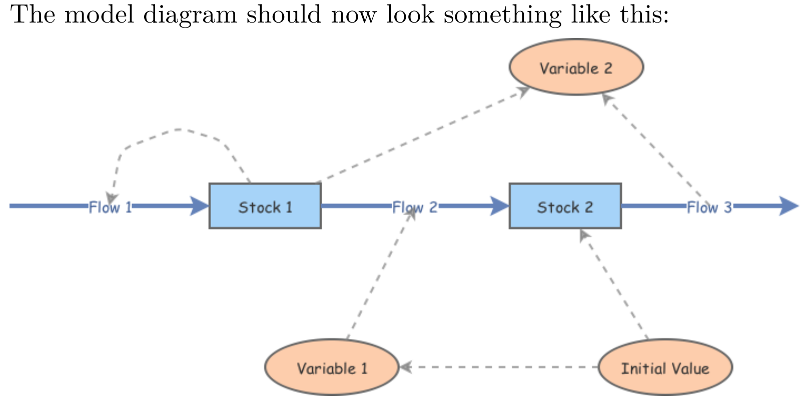
The following sequence presents, and explains, the valid connections for elements of a simulation model.

1. Create a new **Stock** named [**Stock 1**].
2. A [**Stock**] represents a quantity of something that will increase or decrease during the simulation.
3. Create a new **Flow** going from empty space to the primitive [**Stock 1**]. Name that flow [**Flow 1**].
4. A [**Stock**] can only change via a [**Flow**]. To create an inflow first create an outflow and then select [**Reverse**] to change the direction of the flow. This is just a quirk of the environment. A flow that doesn't have a specific origin simply means we're not concerned with where the flow comes from. When you fill the bathtub do you think about where the water comes from?
5. Create a new **Stock** named [**Stock 2**].
6. Create a new **Flow** going from the primitive [**Stock 1**] to the primitive [**Stock 2**]. Name that flow [**Flow 2**].
7. A [**Flow**] can move stuff from one [**Stock**] to another. [**Stock 1**] decreases by the same amount that [**Stock 2**] increases. Flows need to be conserved because you can't create something form nothing in a model.
8. Create a new **Flow** going from the primitive [**Stock 2**] to empty space. Name that flow [**Flow 3**].
9. A [**Flow**] that doesn't have a specific destination simply means we're not concerned with where the [**Flow**] goes.
10. The model diagram should now look something like this:



11. The diagram presents the valid [**Flow**] connections which can only be to, from, or between [**Stocks**]
12. You can connect a [**Stock**] to a [**Flow**] with a [**Link**] to indicate that somehow the [**Flow**] depends on the value of the [**Stock**]. The [**Link**] does not change the value of the [**Stock**].

13. Create a new **Link** going from the primitive [**Stock 1**] to the primitive [**Flow 1**].
14. If you select a link then hold the {shift key} while you click on the link you create {handles} that you can move separately to create a segmented curve.
15. Create a new **Variable** named [**Variable 1**].
16. Create a new **Link** going from the primitive [**Variable 1**] to the primitive [**Flow 2**].
17. A [**Link**] can be used to have a [**Variable**] influence a [**Flow**] and the [**Link**] does not change the value of the [**Variable**].
18. Create a new **Variable** named [**Variable 2**].
19. Create a new **Link** going from the primitive [**Stock 1**] to the primitive [**Variable 2**].
20. Create a new **Link** going from the primitive [**Flow 3**] to the primitive [**Variable 2**].
21. You can use a [**Link**] from a [**Stock**] or a [**Flow**] to influence a [**Variable**]. The [**Link**] does not change the value of the [**Stock**] or the [**Flow**].
22. Create a new **Variable** named [**Initial Value**].
23. Create a new **Link** going from the primitive [**Initial Value**] to the primitive [**Stock 2**].
24. You can use a [**Link**] from a [**Variable**] to a [**Stock**] to assign an initial value to the [**Stock**] when the simulation starts. The [**Link**] does not change the value of the [**Variable**].
25. Create a new **Link** going from the primitive [**Initial Value**] to the primitive [**Variable 1**].
26. You can use a [**Link**] from a [**Variable**] to influence another [**Variable**]. The [**Link**] does not change the value of the source [**Variable**].
27. The model diagram should now look something like this:



28. You have now completed a model that represents the valid connections for a simulation model.

Exercise 2-2

Now that you understand the valid connections for a simulation model recreate this model for yourself. And as you create each element think about what the particular element is for. Actually making the connections helps develop a level of skill and comfort which will serve you well in the future. You can't learn to ride a bicycle by reading a book. You have to actually get on the bicycle. Learning to build models is the same way. To learn to build models you have to build models. Think Nike, "Just do it!"

Configuration Panel

Just below the arrow you clicked to open or close the **Toolbar** is a right pointing double caret as depicted in Figure 3.

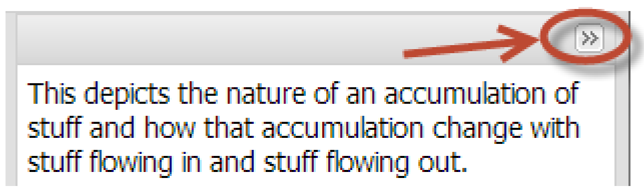


Figure 3. Configuration Panel

If you click this the **Configuration Panel** will close and the right pointing double caret will now point left and can be used to open the configuration panel.

The **Configuration Panel** serves two different purposes. If there are no elements of the model selected on the canvas the {Configuration Panel} will be similar to Figure 4 and contain the model description, tags, and parameter sliders used to set parameter values just before running the model.

If there is a single element selected on the canvas then the **Configuration Panel** will present the list of parameters that can be set for that element. Figure 5 shows the parameters for the stuff element of the model. Please don't be overwhelmed by this long list of parameters. We'll cover them one at a time as they are actually used in a model.

You should note in Figure 5 under the User Interface section it indicates that there should be a slider for stuff and it can be set for values from 0 to 100.

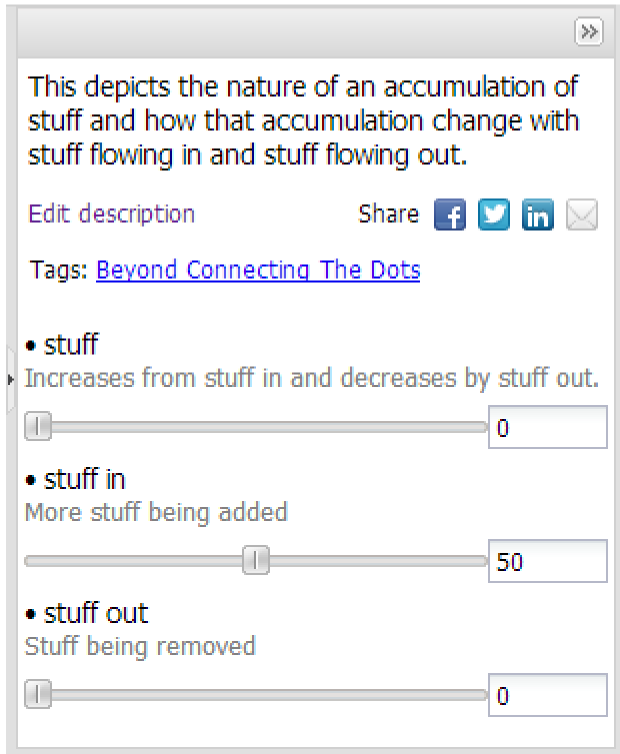


Figure 4. Configuration Panel

Each element has some of the same parameters and some unique to it. Click one of the flows and see what its parameters are.

The Configuration Panels for Stock 1, Flow 1 and Variable 1 from the previous model are displayed in Figure 6 and are described below.

General

This section is where you can assign the (name) and Note for an item.

- **name**. This is the label that you see on the item. You can double-click the item on the canvas and edit the label on the item itself or change it here in the configuration panel.
- **Note**. Here you can enter a description of the item. You can enter short descriptions directly into the field. If you click the down arrow in right of the field it will open the **Note Editor** dialogue window which allows some formatting options. The note that you enter here will pop up when you mouse over an item and click on the little **i** that appears on the item. If the element of the model is selected you can also open the **Note Editor**

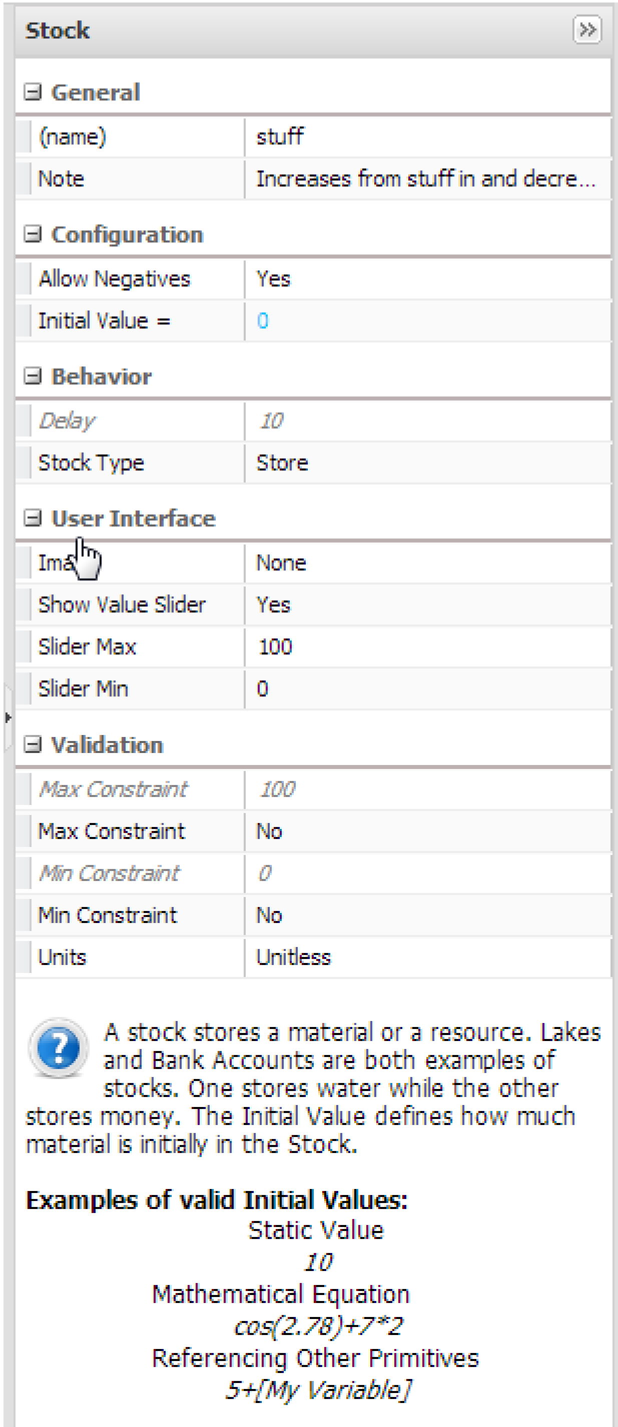


Figure 5. Element Parameters

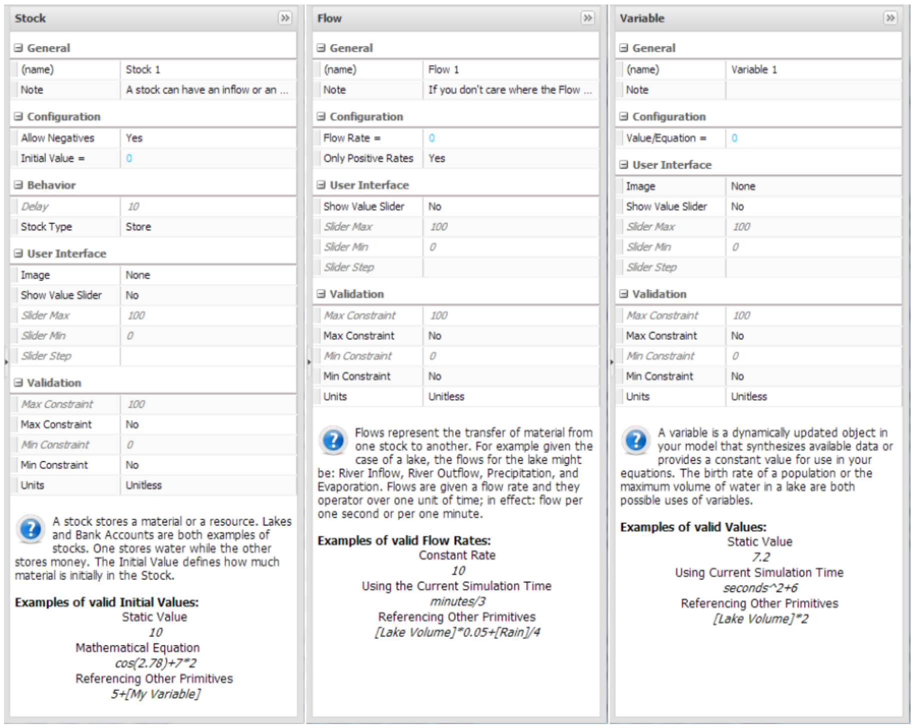


Figure 6. Stock, Flow & Variable Configuration Panels

window by **CTRL+‘(Control+Backquote)**. Adding comments to a model helps others to understand what you were thinking when you constructed the model. And, when you go back to the model in the future the comments will help you understand what you were thinking when you constructed the model together. Yes, you completely understand now, though will you remember next week, or a year from now?

Configuration

This section is used to define how the primitive behaves during the simulation and is a little different for Stocks, Flows and Variables. The behavior is essentially controlled by an equation which is defined in terms of the variables connected to it. This is an initial value for a Stock. You may enter a short value into the field though if you click the down arrow in the right of the field the **Equation Editor** window will open. In this window you can define the formula that defines the behavior of the element. You can also open the **Equation Editor** for an element by mousing over the element and clicking on the **equals (=)** sign that appears. All the built in functions on the tabs at the bottom of the window have descriptions associated descriptions and examples.

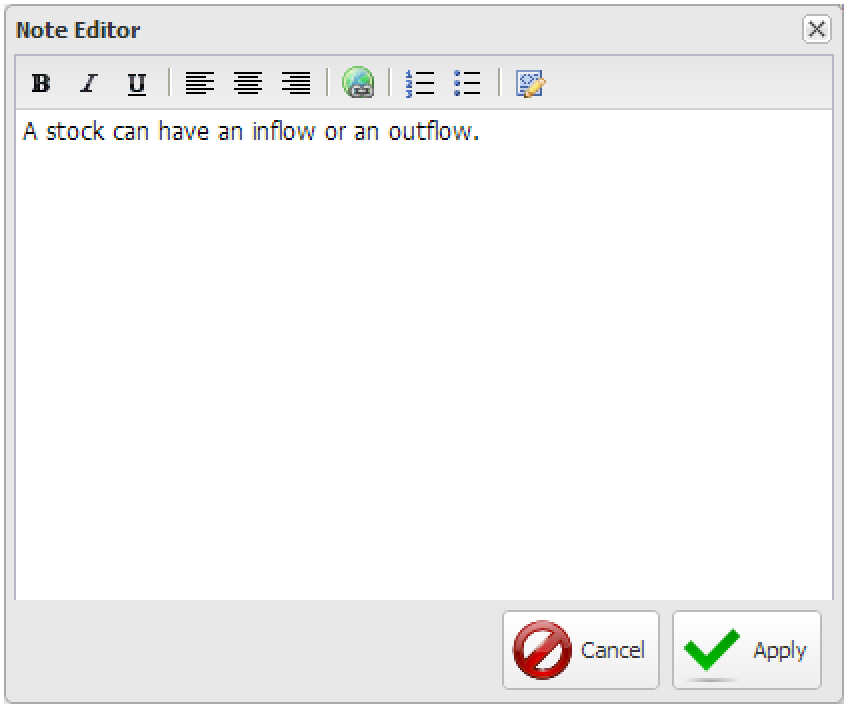


Figure 7. Note Editor

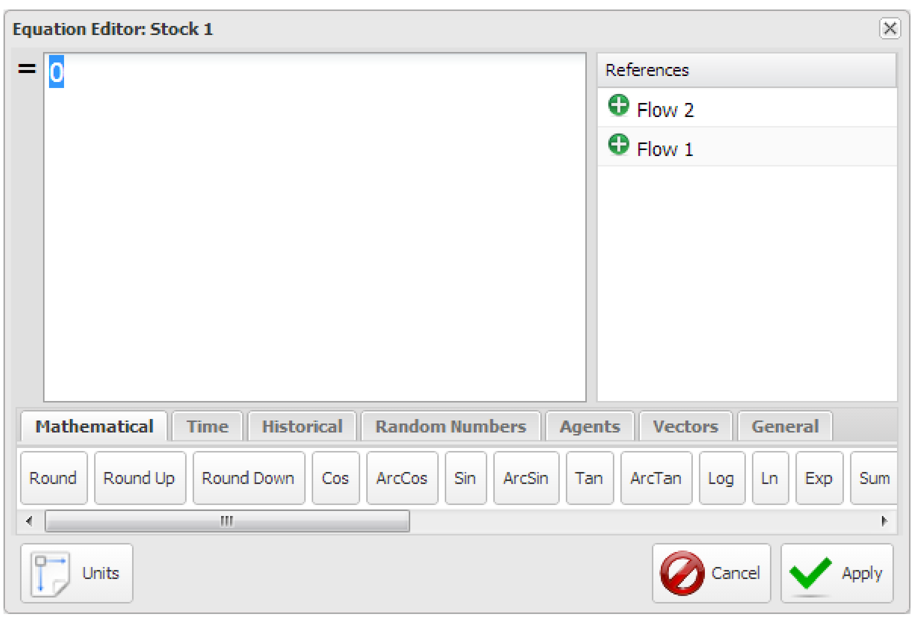


Figure 8. Equation Editor

Additionally in this section you define whether stocks can have negative values and whether flows can flow in one direction or both directions. We'll talk more about these options the first time we use them.

User Interface

It is in this segment of the configuration panel that you define a slider for an element, if there is to be one. You can define a sliders for Stocks, Flows or Variables and use them to establish their value at the start of the simulation. Once you indicate there is to be a slider you then define the maximum and minimum values it may have, as well as the step size, how small are the variations allowed. If you leave the step size field blank then the slider can vary continuously.

An element may have a slider or a formula though not both. Sliders override equations. If you enter an equation and it disappears check to see if there was a slider defined and it hasn't been turned off.

Validation

This section allows you to indicate if there are Maximum and/or Minimum constraints on a Stock, Flow or Variable.

Additionally it is in this section that you assign the Units for an Stock, Flow, and Variable. Units are very useful in helping to ensure the soundness of a model. Units will be covered extensively in Chapter 4.

Time Settings

Time Settings as depicted in Figure 9 are used to indicate when the simulation starts, how long it runs, and the time step it increments during the simulation. Just how to determine the step size will be addressed shortly.

Simulation Results

When you click the **Run** button the model is stepped through the defined time period and produces a display of the results as depicted in Figure 10. There are various options for the type of display and which elements are displayed as shown in Figure 11.

Configure Simulation Results

A default configuration is put together when the model is constructed on the canvas. If you click the **Configure** button in the upper right corner of the {Simulation Results} window the **Chart/Table Configuration** window as depicted in Figure 11 will open. It is in this window you indicate what type of display you want and which items of the model are to be displayed. The only part you need to be concerned about at the moment is the Y-Axis Label field.

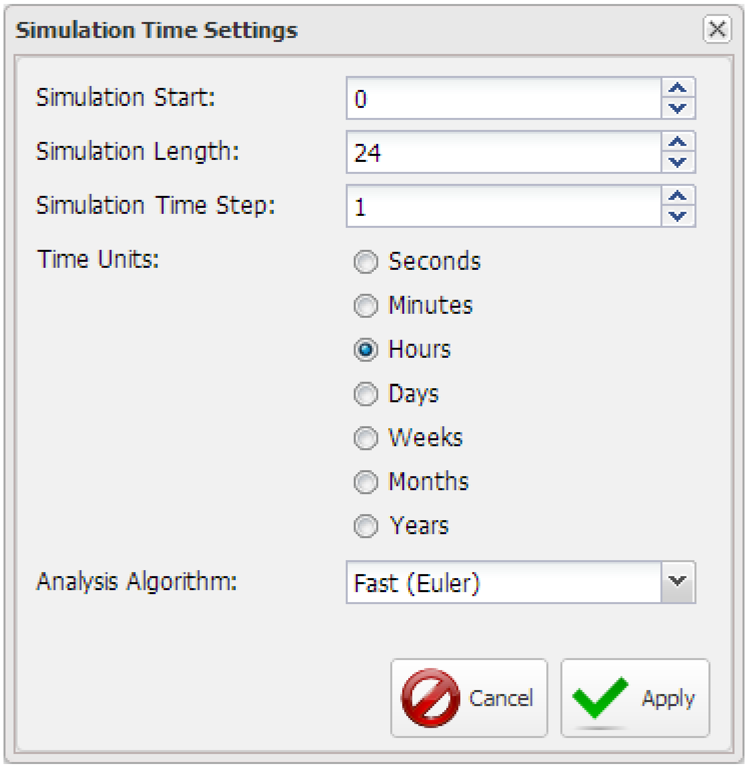


Figure 9. Time Settings

This where it was indicated that the items displayed were in Gallons. You will need to change this shortly in the next exercise.

Note that if you change items in the configuration they will be immediately reflected in the **Simulation Results** window when you click **Apply**. You don't need to run the model again to see a different configuration of the data. This makes it very convenient when when you decide you need another display for some items of the model.

Hopefully you haven't found this short introduction to the modeling environment too overwhelming. Different parts of the environment will be presented just as you need them to interact with the models presented.

Too much explaining and not enough hands on interaction may get real boring in a hurry. As such you are encouraged to actually do the exercises. By interacting with the various aspects of the modeling environment you will develop a level of comfort and expertise which will serve you well throughout the rest of the book.

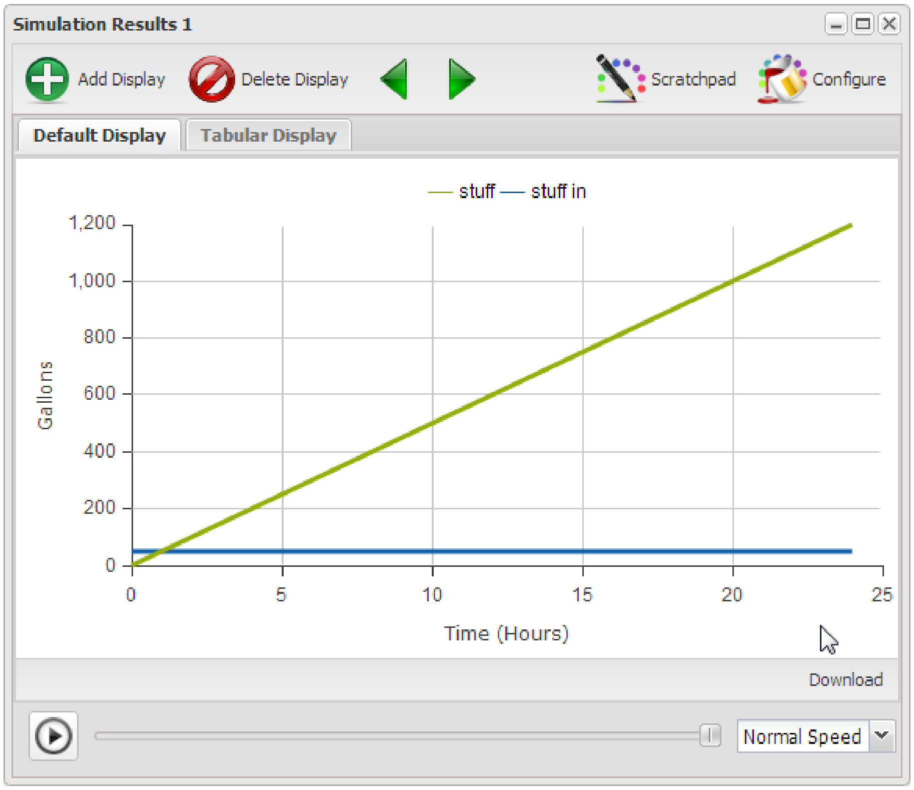


Figure 10. Simulation Results

Common Property # 1

To this point you've learned how to develop a static picture of a model. It is actually a model and provides a sense of the relationships between the various elements. What it doesn't give you a sense of is the dynamic nature of these over time. What are the implications of the relationships? In the next few sections you'll learn how to bring your model to life.

Look at the images in Figure 12 and ask yourself what these images have in common. The images all represent very different kinds of things, some living, some not, though there is a characteristic they all have in common. Have you figured it out?

Maybe you notice the rabbits from the previous chapter? All the images represent things that grow over time in one way or another, and some faster than others.

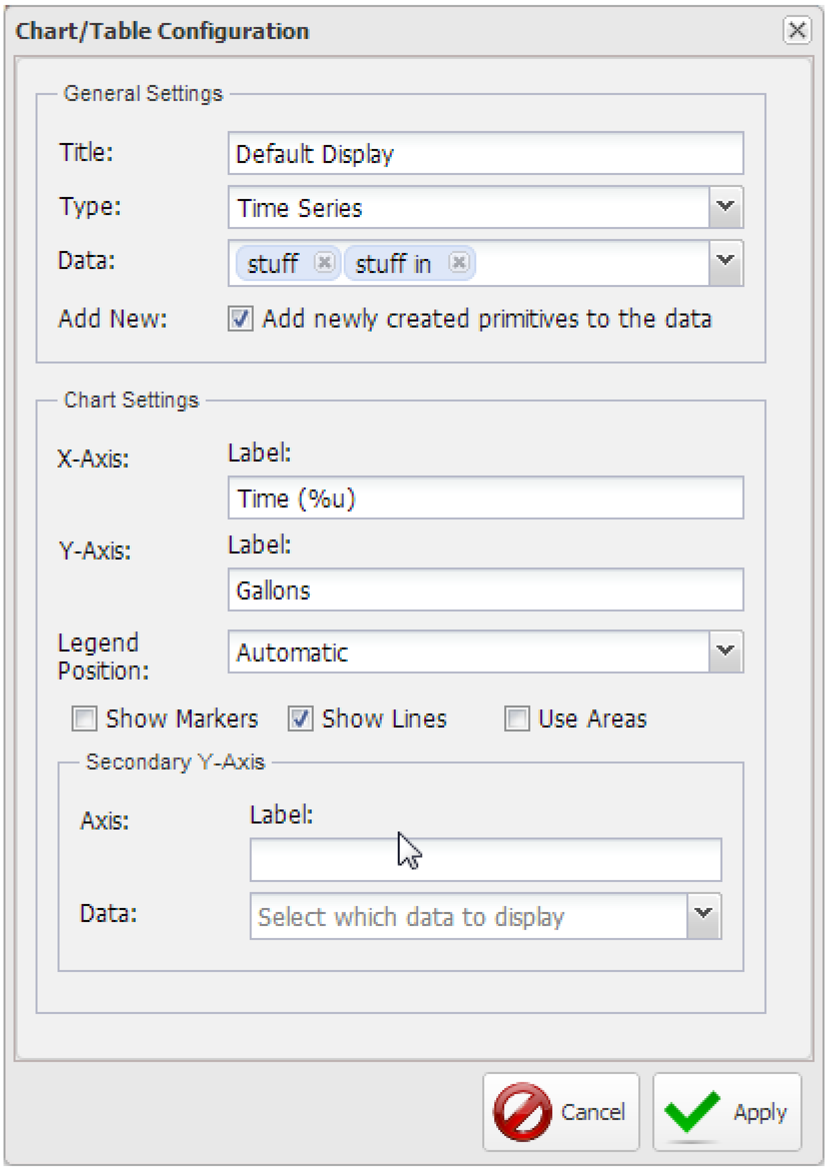


Figure 11. Chart/Table Configuration

Figure 12. Common Property # 1

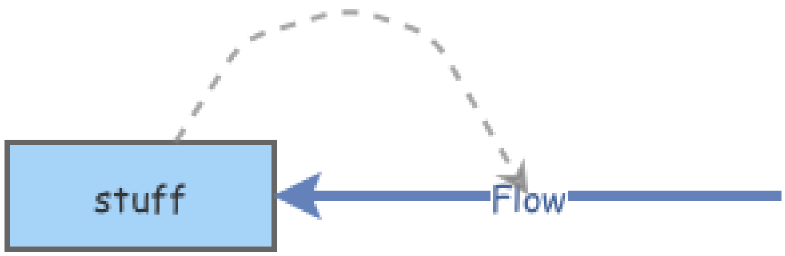
Reinforcing Growth Structure

A reinforcing growth structure is one where growth produces a result which promotes even more growth.

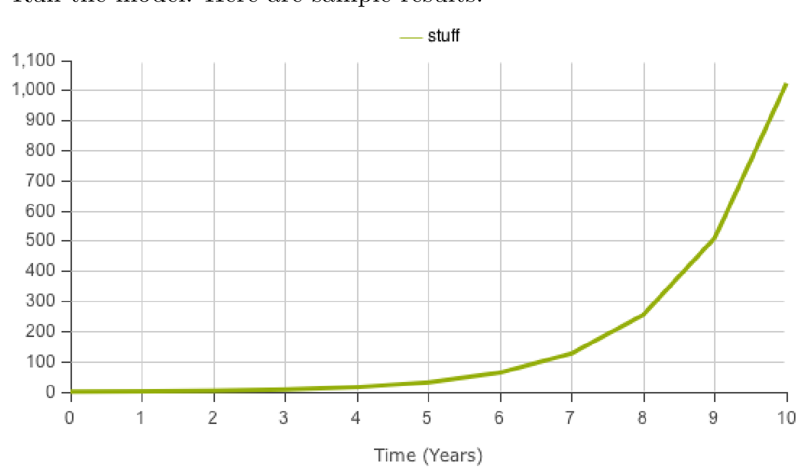
Reinforcing Growth Model

We're going to create a generic exponential growth structure for study and reuse.

1. First we build the linear accumulator model.
2. Create a new **Stock** named [**stuff**].
3. Create a new **Flow** going from empty space to the primitive [**stuff**]. Name that flow [**Flow**].
4. Then we add the reinforcing feedback loop.
5. Create a new **Link** going from the primitive [**stuff**] to the primitive [**Flow**].
6. The model diagram should now look something like this:



7. Now we assign values to the relationships.
8. Change the **Initial Value** property of the primitive [**stuff**] to `1`.
9. Change the **Flow Rate** property of the primitive [**Flow**] to `[stuff]`.
10. And set up the Time Settings for the simulation.
11. Change the **Simulation Length** property of the Time Settings to `10`.
12. Change the **Simulation Time Step** property of the Time Settings to `1`.
13. Run the model. Here are sample results:



14. The graph shows the value of **[stuff]** after 10 Years is 1,024 which you should realize is just 2^10 as expected because we started with a value of 1 and doubled it every year. This curve is referred to as an exponential growth curve.

Exercise 2-3

Notice that the curve in the previous model is a bit choppy where it turns up. Run the model with a Time Step of .5, .25, .125, .0625 and compare the results. What questions are raised by the the results?

Time Units and Time Step Selection

The **Time Units** and **Time Step** selected for a model should be consistent with the time frame and level of detail of the model. You probably wouldn't develop a model about filling a bathtub with water and use **Time Units** of months. Minutes are probably more appropriate for this model. The **Time Step** is then selected to ensure none of the relevant transitions associated with the dynamic nature of the model are missed. A **Time Step** of .25, meaning 15 seconds for a model with **Time Units** in minutes, is probably sufficiently small to ensure there are no transitions missed.

Trial is actually the most appropriate approach to determine if you have an appropriate value for **Time Step**. If you think .5 is appropriate then run the model with 1, .5, and .25 and if the results for 1 and .25 don't differ from .5 then you're probably OK. If .25 produced a different result then compare the .25 result with the .125 result. Once you get two runs where the values don't change then use the larger value.

Exercise 2-4

Given this guidance how would you interpret the results you experienced in the previous exercise?

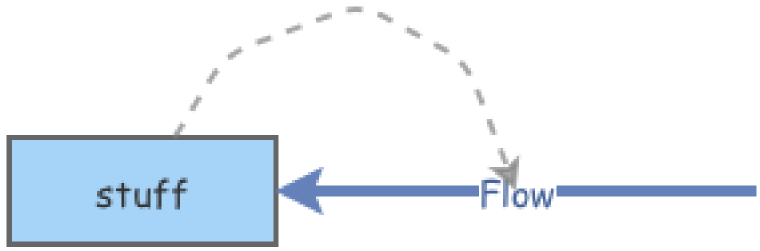
Answer Available

One aspect of trying to model the contexts of Figure 12 that should have become apparent is that there is a piece of the model that's missing.

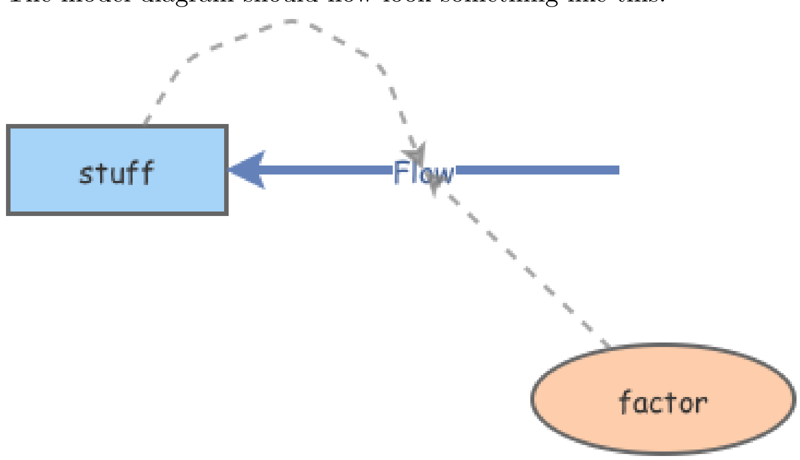
Feedback Dependent Growth with Control

We're now going to add a factor to the previous model so you can control the extent to which the stock influences the flow.

1. The model diagram should now look something like this:

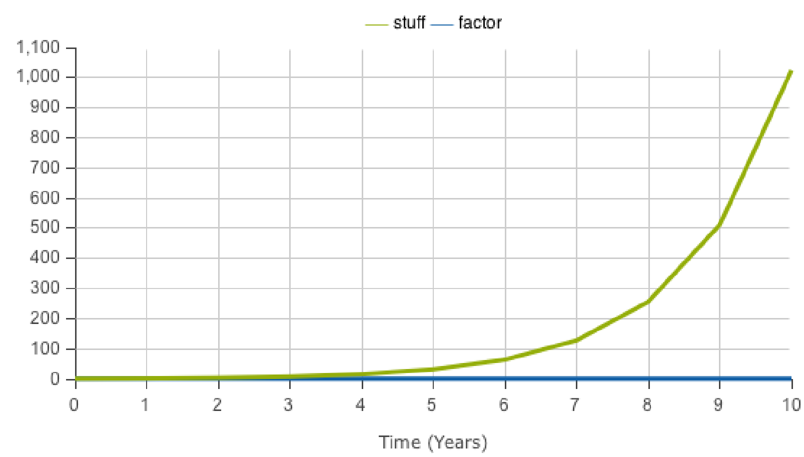


2. We begin with the previous model and we add a factor to control the flow.
3. Create a new **Variable** named [**factor**].
4. Create a new **Link** going from the primitive [**factor**] to the primitive [**Flow**].
5. Change the **Flow Rate** property of the primitive [**Flow**] to `[stuff]*[factor]`.
6. And we'll add a slider to the [**factor**] to make it easy to adjust its values.
7. Change the **ShowSlider** property of the primitive [**factor**] to `true`.
8. Change the **SliderMax** property of the primitive [**factor**] to `1`.
9. Change the **SliderStep** property of the primitive [**factor**] to `0.1`.
10. Change the **Equation** property of the primitive [**factor**] to `1`.
11. The model diagram should now look something like this:



12. With this version of the model you can vary the values of [**factor**] to get a sense of the impact the [**factor**] has on the growth of [**stuff**]].

13. Run the model. Here are sample results:



Exercise 2-5

Using the Feedback Dependent Growth model to implement the models does this structure allow you to construct more realistic representations of the growth situations presented in Figure 12? Why?

Answer Available

Exercise 2-6

Imagine that the feedback dependent growth model is actually a Savings Account that is defined as compounding annually, i.e. calculating and adding interest once a year. This means that the most appropriate **Time Units** would be years with a **Time Step** of 1. There are no other transitions in this model that need to be accommodated. If you run this model with any **Time Step** other than 1 it will result in a less accurate result. Why does this happen?

Answer Available

This model just developed is the standard reinforcing growth model depicted at the beginning of this chapter. In the process of arriving at this model the linear growth was developed first, and then evolved. Hopefully through the exercises to this point you have gained a deeper understanding of how this structure works and the extent to which it may be applied to various situations.

Common Property # 2

Look at the activities depicted by the images in Figure 13 and ask yourself what it is that these activities have in common. The images represent very different kinds of activities though there is a characteristics they all have in common. Have you figured it out?

Figure 13. Common Property # 2

Each activity depicted in Figure 13 represents the pursuit of some goal or objective. Admittedly the goals are very different and each is pursued in a very different manner.

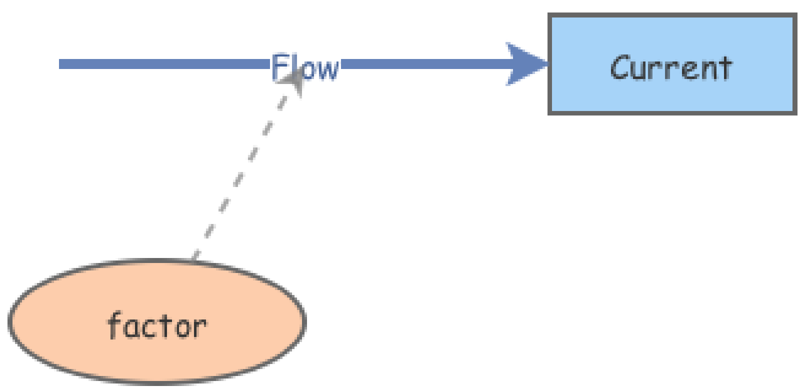
Constructing a Balancing/Goal Seeking Structure

A Balancing/Goal Seeking structure is one where there is a difference between two values and the activity of relationships works to develop a balance between the two values. Essentially what the structure does is move the [**Current**] value to the value of the [**Goal**].

Balancing/Goal Seeking Model

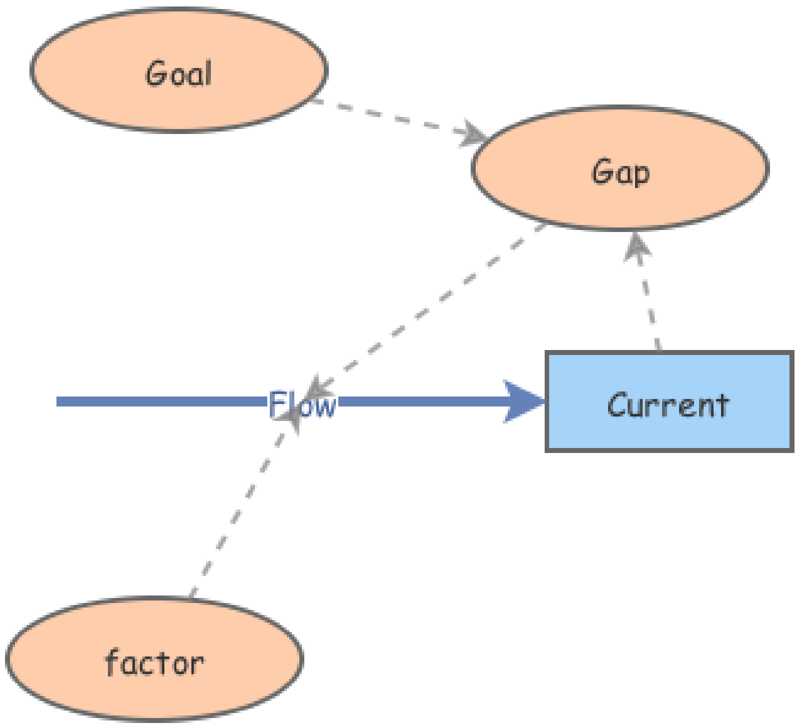
As we have done repeatedly to this point we begin with a linear model consisting of a flow and a stock, along with a flow rate variable. To this we simply have to add a goal and the appropriate feedback and we end up with a goal seeking model.

1. First we build the linear accumulator model with a control factor.
2. Create a new **Stock** named [**Current**].
3. Create a new **Flow** going from empty space to the primitive [**Current**]. Name that flow [**Flow**].
4. Create a new **Variable** named [**factor**].
5. Create a new **Link** going from the primitive [**factor**] to the primitive [**Flow**].
6. The model diagram should now look something like this:



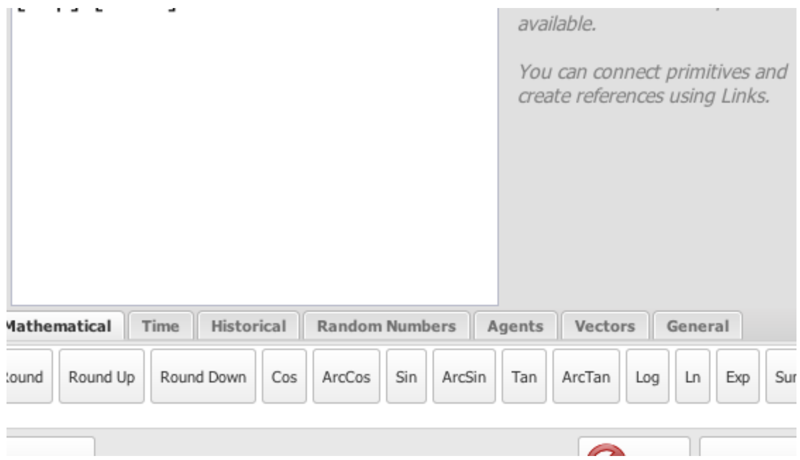
7. Now we add a goal and compare the goal to the current value.
8. Create a new **Variable** named [**Goal**].
9. Create a new **Variable** named [**Gap**].
10. Create a new **Link** going from the primitive [**Goal**] to the primitive [**Gap**].
11. Create a new **Link** going from the primitive [**Current**] to the primitive [**Gap**].
12. Create a new **Link** going from the primitive [**Gap**] to the primitive [**Flow**].

13. The model diagram should now look something like this:



14. When you look at the model admittedly we added [**Gap**] which we haven't addressed before. This was done so we could explicitly plot the difference between the value of [**Current**] and [**Goal**].
15. Now assign values to the various primitives.
16. Change the **Initial Value** property of the primitive [**Current**] to `0`.
17. Change the **Equation** property of the primitive [**Goal**] to `1`.
18. Change the **Equation** property of the primitive [**Goal**] to `[Goal]-[Current]`.
19. Change the **Equation** property of the primitive [**Goal**] to `[Gap]*[factor]`.
20. Create the slider to adjust the [**factor**] value and set it to 0.5.
21. Change the **ShowSlider** property of the primitive [**factor**] to `true`.
22. Change the **SliderMax** property of the primitive [**factor**] to `1`.
23. Change the **SliderStep** property of the primitive [**factor**] to `0.1`.

24. Change the **Equation** property of the primitive [**factor**] to `0.5`.
25. Now set the Time Settings for the simulation run.
26. Change the **Simulation Length** property of the Time Settings to `10`.
27. Change the **Simulation Time Step** property of the Time Settings to `0.5`.
28. Change the **Time Units** property of the Time Settings to `Hours`.
29. Run the model. Here are sample results:



30. Take a look at the Time Settings for the model and you'll see that the model was set up to run from 0 to 10 with a [**Time Step**] of 0.5 and a [**Units**] of hours. These were just selected to create a generic model where you could consider the Goal to be 100% and the other values as having values between 0 and 100%. This way we can consider the implications of the interactions without getting hung up on the actual values.
31. The graph shows that as [**Current**] moves toward the [**Goal**] the [**Gap**] decreases as does [**Change**] which is moving [**Current**] in the direction of [**Goal**]. Once [**Current**] reaches [**Goal**] the [**Gap**] is zero is [**Change**]. This structure endeavors to remove the tension between [**Current**] and [**Goal**], the [**Gap**], to bring a balance to the situation.

Exercise 2-7

Run the above model with various values for factor. What do you notice about the relation between [**Current**] and [**Gap**]? And what do you notice about the curves as the factor gets larger and larger?

Under Time Units and Step Selection we talked about it being essential that the the **Time Units** were selected appropriate to what was being modeled. In this case since it's a generic model one Time Unit is pretty much as appropriate as any other. The Time Step is another matter though, or is it? We said one chooses a Time Step such that none of the relevant interactions are missed and the change from one Time Step to another doesn't change the result.

Exercise 2-8

Set up the previous model to run with Current = 0, Goal = 1, and factor = .75. Now run the model with a Time Step of 1, .5. 25. .125. Does the result actually change? Look at the Tabular Display associated with the Simulation Result. As you make the Time Step smaller and smaller are the results more correct?

Considering that we don't know anything about a real environment being modeled it's a bit difficult to determine if the result is actually more correct as the Time Step used is smaller and smaller.

You might have also realized by this point that it would be quite difficult if we attempted to use this model to model any of the situations depicted in Figure 13. While progress toward the goal in the situations depicted is promoted by the Gap between the Goal and Current the change in those situations isn't likely to be proportional to that Gap.

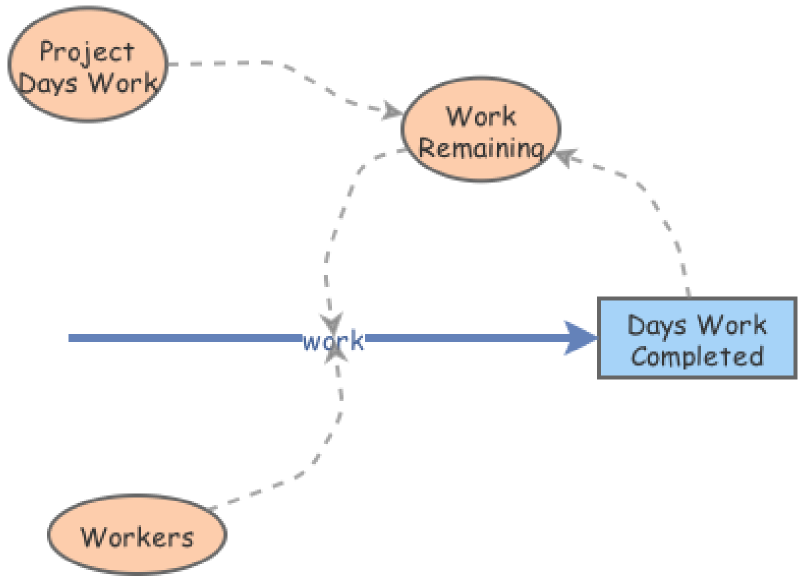
Work Completion Model

The following model presents a modification to the previous model where the factor has been replaced by a constraint. It looks like there have been lots of changes though they all cosmetic except the way Workers influence work on a daily basis.

Work Completion Model

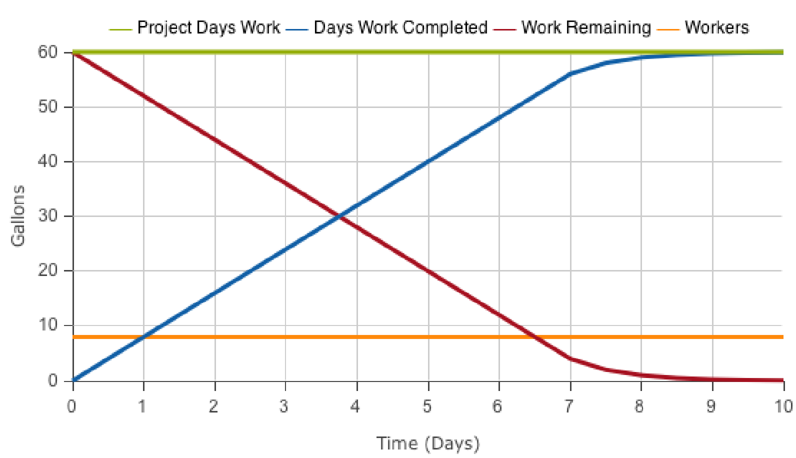
In this model [**Workers**] is not a factor but a limit on the amount of [**work**] that can be performed in a time period.

1. The model diagram should now look something like this:



2. Settings for the model are: Project Days Work = 60, Workers = 8 and Days Work Completed set at the default of zero and Time Step = 1.

3. Run the model. Here are sample results:



4. The reason the graph looks like this is because of the constraint placed on the the work by the number of Workers available. This is accomplished by the formula embedded in the flow.

5. [**Work**] = IfThenElse([**Work Remaining**] > [**Workers**],[**Workers**],[**Work Remaining**])

6. This says that if there is more [**Work Remaining**] than there are [**Workers**] available to do the work then the amount of work that day equals the number of [**Workers**]. This goes on for the first 7 days then on the 8th day there are only four days work required to finish the project which is represented by the different slope on the line on the 8th day. You can see this in detail if you look at the Tabular Display.

Note that in this model you might have considered the [**Workers**] as a [**Stock**] as they are actually a collection. The reason they're not considered as a [**Stock**] in this model is that the number remains constant in the context of this particular model. In a different model [**Workers**] might actually be a [**Stock**] with an inflow and and outflow.

Exercise 2-9

Set up the above model to run with Time Step of 0.5. Compare the results of this run with the results of the previous run above. By making the time step smaller have we improved the accuracy of result? Why?

Again the appropriate Time Step is one that captures the activity occurring within the model. In this case the Workers are in integers and Project Work in days, both of which are in integers, and with the Time Units in days the appropriate Time Step is 1. If there were events which happened in the model on the order of hours then you would have to decide whether to alter the model to run in hours or reduce the Time Step to ensure it was small enough so no interactions in the model were missed.

Exercise 2-10

Use the previous model and reconfigure it for a couple of the activities depicted in Figure 13. Note that for this exercise you will have to relabel the stock, flow, and variables accordingly. You will also have to decide on the most appropriate Time Units and Time Step to use.

Summary

Hopefully this chapter has helped you become more familiar with the modeling environment and the four model elements you will use most often.

- **Stock**. An accumulation of something that can only be changed by something flowing into or out of it.
- **Flow**. Something moving over time which adds to a stock or subtracts from a stock.
- **Variable**. Constant or equation computed each time the simulation steps.
- **Link**. Used to communicate a value from a Stock, Flow, or Variable, to a Stock, Flow or Variable. The source is not changed and a link to a stock can only be used to set it's initial value.

Because of the nature of the building blocks themselves there are only a small number of valid connections as depicted in valid connections model.

These valid connections are used to create only three different types of structures, linear growth, goal seeking and reinforcing growth. If you are comfortable with these you should be relieved to know that's all there are. Just three simple structures will be used for all the models you will ever build. Of course at times there may be quite a few of these connected together though you should be confident that you know about the pieces.

The models that you have experienced in Chapter 1 and Chapter 2 are referred to as Stock & Flow Simulation Models. They are also referred to as quantitative models because of the values associated with the simulation of these models. In the next chapter we'll investigate a number qualitative models which are also used in developing understanding. These are referred to as qualitative models because there are no numerical values associated with them, though there are times when they can be quite useful.

Chapter 3

A Model Is A Model Is A Model

In previous chapters you worked with what is referred to Stock & Flow Simulation Models. These are also referred to as quantitative models because of the numerical values associated with them. While there are other types of quantitative models in this text we're focusing on Stock & Flow Simulation Models. There is also a class of models referred to as qualitative models because they have no numeric values associated with them. There is a level of understanding well served by qualitative models. This chapter will serve to acquaint you with a couple types you can create with Insight Maker and are likely to be useful in your study of systems.

The models presented in this chapter are presented as images because they're really just pictures and not simulations. Links are provided if you want to access the sample models directly in Insight Maker.

As we've said repeatedly, models are simplifications of some aspect of the world around us intended to help us understand something. The Stock & Flow Simulation Models are very explicit as they the relations are represented by numerical formulas and the model may be iterated over time. The models presented in the sections of this chapter may also assist in understanding situations though only from a relationship perspective. There are generally no numerical values associated with these model and they are not iterated over time.

Rich Pictures

A Rich Pictures is a pictorial representation of a set of relationships intended to convey a level of understanding about those relationships and the implication of those relationships. The strength of Rich Pictures is that there are no rules associated with creating a Rich Picture. This means you create the picture that helps you understand the relationships.

The model in Figure 1 is intended to represent the relationships for a Savings

Account. It is created using the Picture Primitive connected together with Links.

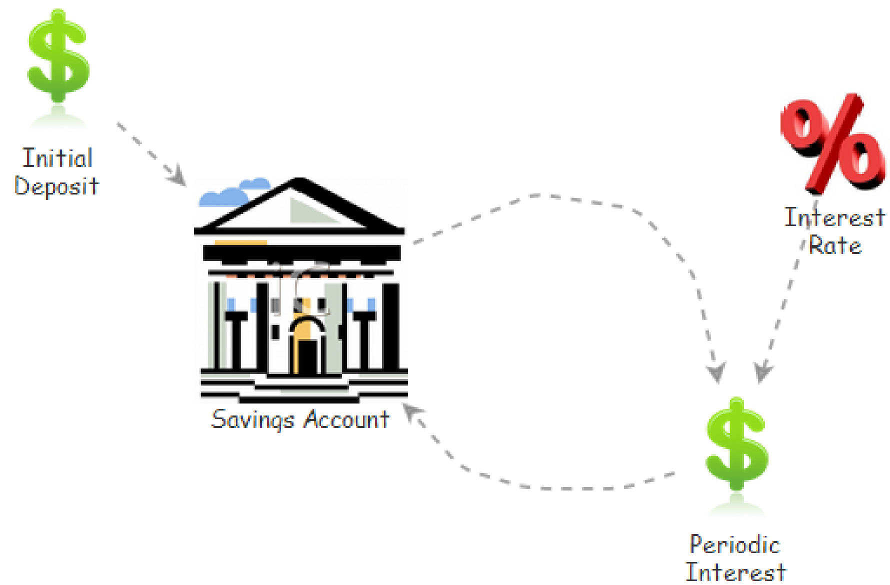


Figure 1. Rich Pictures/Part 1

Picture Primitives are added to the canvas like any of the other Primitives. Just click on the Primitive to select it then click on the canvas where you want the Picture to be. You don't have to be too specific about the placement as items are very easy to move around on the canvas.

Once the Picture Primitive is placed on the canvas there are several options available on the **Configuration Panel**.

The General section of the Picture Primitive is just like the Primitives you've used to this point with a **(name)** and **Note** parameters. The Image option in the User Interface section is new. Here you may select one of the built in pictures from the drop down menu, no picture, the first option on the menu, or paste a URL into the field which points to some graphic on the Internet. Note that the Internet option only works if you're connected to the Internet.

Once you've created the Picture simply use the Link element to connect them together as you did in the Stock & Flow models. After you select the Link tool and when you mouse over a Picture Primitive the small right arrow will appear in the center as the connection point. Simply click on the selection point and drag to the Primitive you want to connect to. You can't use Flows to connect Picture Primitives.

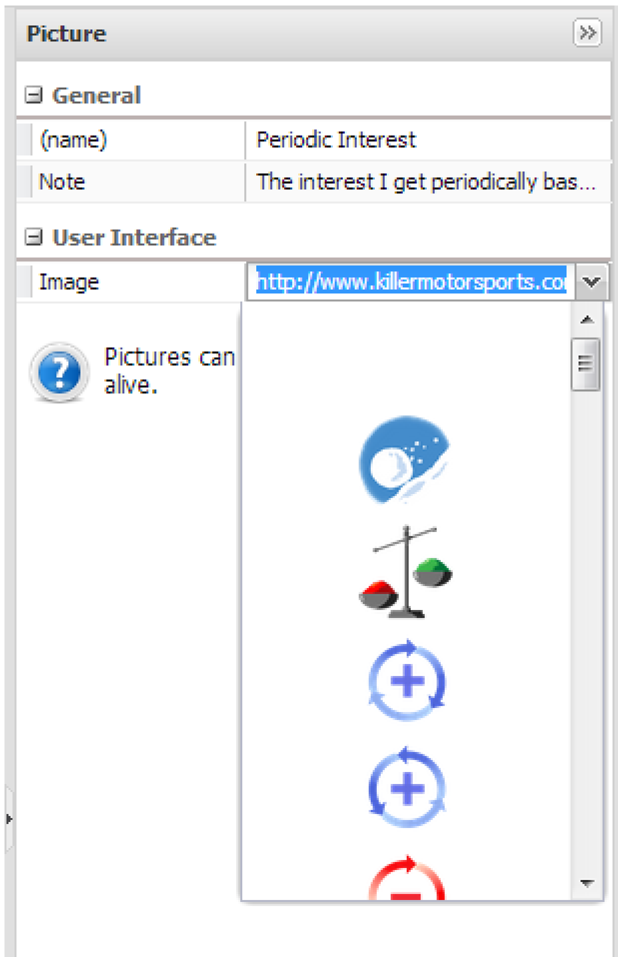


Figure 2. Picture Primitive Configuration Panel

Note that you can use [**Stocks**], [**Variables**], [**Converters**] and [**Text**] in Rich Pictures. You can add images to all of them except [**Text**]. Even though this is the case most of the time using Picture Primitives will serve just fine. We'll talk about Buttons somewhere further along the way.

Exercise 3-1

Use the Picture Primitive and create a rich picture similar to Figure 1 and connect the Pictures together using the Link tool. Note that in the Configuration segment of the Configuration Panel for the Link there is an option to indicate whether a Link is bi-directional. Use some of these in your exercise.

The Text primitive is similar to any other primitive in that you select it then click on the canvas where you want the Text to appear. Once you create the text item you can edit the text, assign notes to it, connect to or from it with links, and style it with commands form the Style segment of the Toolbar.

In this version of the model we add a [Folder] and a [Link Label] to Rich Picture/Part 1.

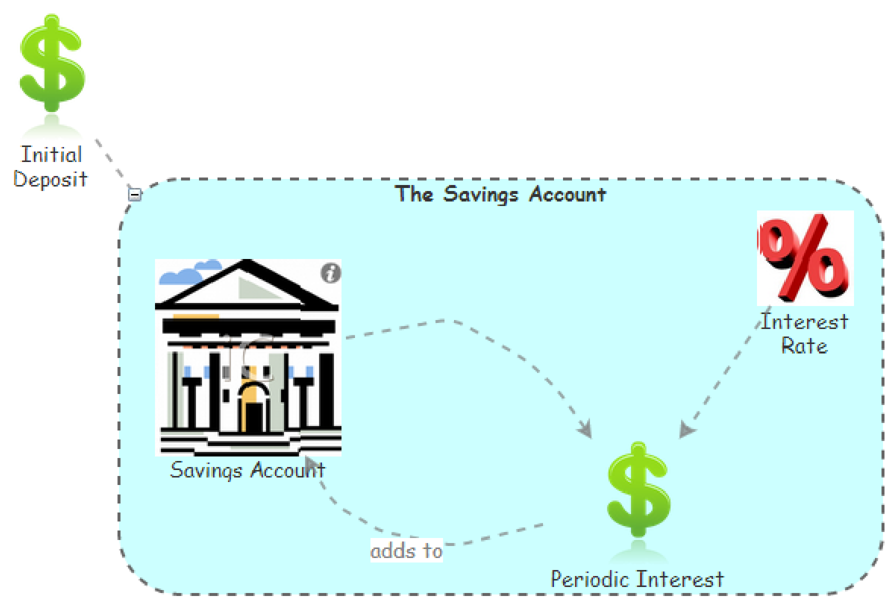


Figure 3. Rich Pictures/Part 2

If you select the Link that has the **adds to** label on it you'll notice that the **(name)** field in the Configuration Panel contains the displayed **adds to** label and there's text in the **Note** field. These two attributes are like other Primitives. If the **(name)** field is blank or named **Link** it won't display. Any other text in the field will display on the Link, or near it if the link is segmented and curved. If you segment the Link the program doesn't really know where you want the label to be. As such if you select the link you'll see a small yellow node on the text. You can click on this node and move the label to wherever you would like it to be. Be warned that at times the positioning is a bit artful, though you'll readily get the knack of it.

Folders are provided so you can enclose a number of items you want to explicitly draw attention to as a concept and have the ability to hide the detail if you choose. To hide the detail click on the little minus (-) sign in the upper left corner of the folder. when you do this the folder closes, displays it's name and the minus (-) sign changes to a plus (+) sign. And you probably also surmised when you closed the folder that you can select an image for a folder or provide an image URL.

Remember the intent of a model is to be some simplification of the world around you to promote understanding. And while there are really no rules for creating a Rich Picture, if you want others to understand one you might want to ensure it is easily understandable rather than confusing. How is the best way to ensure this? Present it to others and have them let you know where they think it is clear and where they think it is confusing. Then go work on it some more.

Causal Loop Diagrams

A Causal Loop Diagram is more structured than a Rich Picture and less structured than a Stock & Flow Diagram. The Causal Loop Diagram was initially invented as a way to express the findings of a Stock & Flow simulation model without having to show the entire simulation model as it was expected it might overwhelm the audience.

While a Causal Loop Diagram is a qualitative model there is still much one can come to understand from one because of the information presented about the relations in one.

We begin with a model of the most basic relations between two elements of a model.

The good news is there are no new aspects of Insight Maker you need to learn to create a Causal Loop Diagram. The diagram is created with [**Stocks**], [**Variables**], [**Pictures**] and [**Links**] with [**Text**] used to indicate the relationship between influencing variables and the influenced variables. After you create the elements, which you may represent with the [Stock}, [**Variable**], [**Picture**] or [**Text**] **Primitives** you use **Links** to connect them together and identify the relationship represented by the **Link**.

There are two widely used notations associated with Causal Loop Diagrams, both of which are presented below.

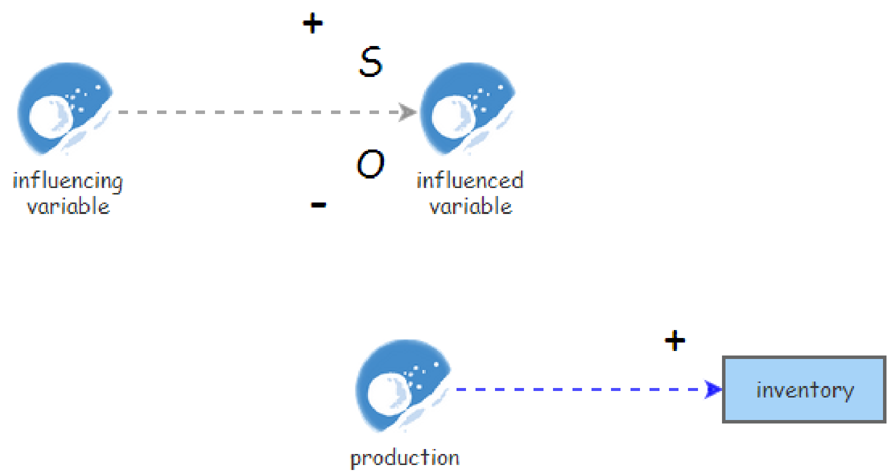


Figure 4. Causal Loop Diagrams/Part 1

The first notation popularized by Senge in The Fifth Discipline{Cite:Senge, P. 1980. The Fifth Discipline: The Art and Practice of the Learning Organization. http://www.amazon.com/Fifth-Discipline-Practice-Organization-ebook/dp/B000SEIFKK/} uses an **S** and **O** notation as depicted in Figure 4.

- **S** implies that the influenced variable changes in the same direction as the influencing variable. If the influencing variable increases the influenced variable increases. If the influencing variable decreases the influenced variable decreases.
- **O** implies that the influenced variable changes in the opposite direction as the influencing variable. If the influencing variable increases then the influenced variable decreases. If the influencing variable decreases then the influenced variable increases.

The influence indicators are created as Text elements and positioned as you deem appropriate to represent the influence representative of the Link.

The + and **-** notation is an older notation and each has two possible meanings which are deduced from the context of the diagram.

- + implies that the influenced variable changes in the same direction as the influencing variable or the influencing variable adds to the influenced variable.
- **-** implies that the influenced variable changes in the opposite direction as the influencing variable or the influencing variable subtracts from the influenced variable.

The + and **-** notation are considered less likely to generate inconsistencies in a model when there are elements such as production and inventory are depicted. In this relationship the + interpreted as **adds to** for production adds to inventory. The situation is such that as production increases inventory increases and as production decreases inventory still increases, just not as rapidly. This should allow you to easily see that the **S** notation might result in a misinterpretation of the diagram as production decreases inventory decreases. To aid in avoiding the confusion some people use [**Stocks**] in their Causal Loop Diagrams to indicate those elements which are actually accumulations.

Because creating the **Link** indicators can get tedious in a hurry after you create the first one it's much easier to hold down the **Control Key** then click on the indicator and drag to a new location. This quickly creates you a copy of the indicator. You can actually use this sequence to duplicate any element, or selection of elements, of a model.

As this section is about Causal Loop Diagrams there probably should be some loops defined here somewhere shouldn't there? Figure 6, Causal Loop Diagrams/Part 2, presents a Causal Loop of the Balancing and Reinforcing loops presented in Chapter 1 and Chapter 2.

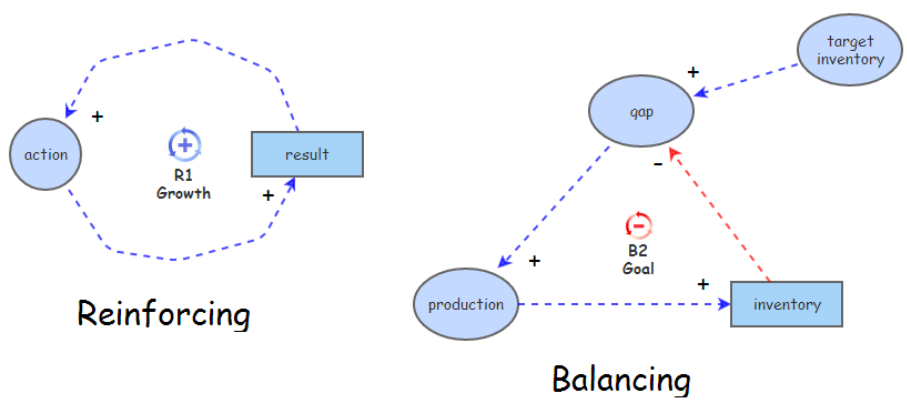


Figure 5. Causal Loop Diagram/Part 2

The Balancing and Reinforcing models presented vary slightly from typical causal loop diagrams in the following ways.

- **Variables**. Variables in a Causal Loop Diagram are typically just Text though here we've used Stocks for those variable which represent accumulations and Pictures, with no images, to represent the variables. The advantage in using Picture elements for variables is that there are no restrictions on labels because Picture elements are never used in equations.
- **Link Colors**. Because the Link identifiers have to be created separately and are not attached to the Link it's much easier to make + indicators

Blue and **-** indicators Red. This seems to be an evolving convention and when used you could just leave the Link indicators out altogether.

While models are a simplification of the world around you intended to promote understanding they also represent an unfolding story. It is appropriate to sequence and label the loops. This gives others a guide to what order to read the story and the intent of the loop itself. The images are created with Picture elements and the images for reinforcing + and balancing **-** loops, both clockwise and counterclockwise are defined images available from the pull images pull down on the [**Picture**] **Configuration Panel**.

The easiest way to identify whether a loop is a reinforcing or balancing loop is to count the number of minus (-) influences around the loop. If the number is zero or even then the loop is a reinforcing loop. If the number is odd then it's a balancing loop.

Hybrid Models

There are times when the message you want to get across may be best represented by a combination of Rich Picture and Causal Loop forms as depicted in Figure 6.

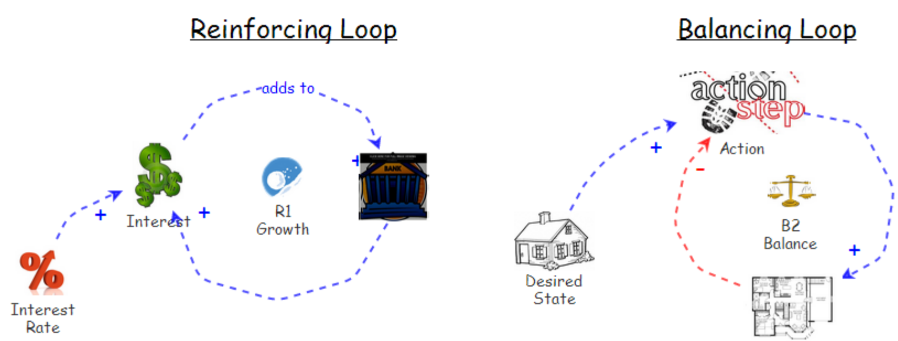


Figure 6. Hybrid Model

If the images make it easier for the model to develop and convey understanding then it helps them achieve their purpose. The model in Figure 7 presents a Hybrid Causal Loop Diagram for The Boy Who Cried Wolf Aesop's Fable.

In the next section you will be introduced to the Unfolding feature of Insight Maker which you can use to build a script so the model will explain itself to someone else when you're not there.

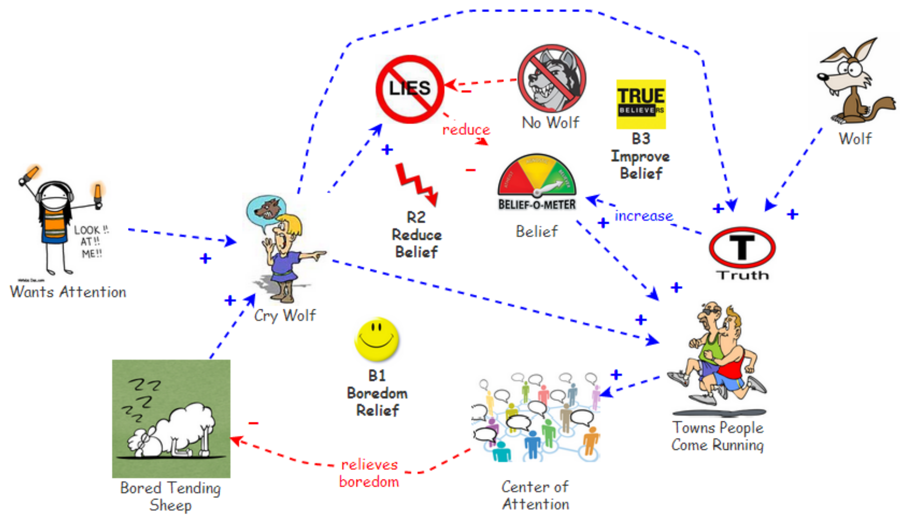


Figure 7. The Boy Who Cried Wolf

Storytelling

Even though you may conscientiously develop your model and add comments often it turns out that people initially look at the whole model and are immediately overwhelmed. It might be somewhat like trying to eat a whole Elephant in a single bite. Storytelling provides a way to overcome this difficulty.

Storytelling a model is intended to reveal a model little by little and explain it along the way. Click on the View Story button in the lower left corner of the screen, read the text, and then click the Step Forward arrow on the right repeatedly to have the model presented as a story.

Adding a story to a model is very straight forward and is initiated by clicking **Add Story** in the **Tools** section of the **Toolbar**. This opens the **Story Designer** which is described as follows.

The main elements of this window are...

- **Enabled**. This check box allows you to actually enable Storytelling. If this box is checked the green plus sign and View Story will show up in the lower left corner of the window.
- **Automatically View**. There are four options on this drop down allowing you to indicate the conditions under which View Story should execute automatically when the model is opened. The options are Never, For Editors, For Non-Editors, Always.
- **Story Steps**. Lists the steps that you have defined as part of the story. You may reorder steps by clicking and dragging them to the new location.

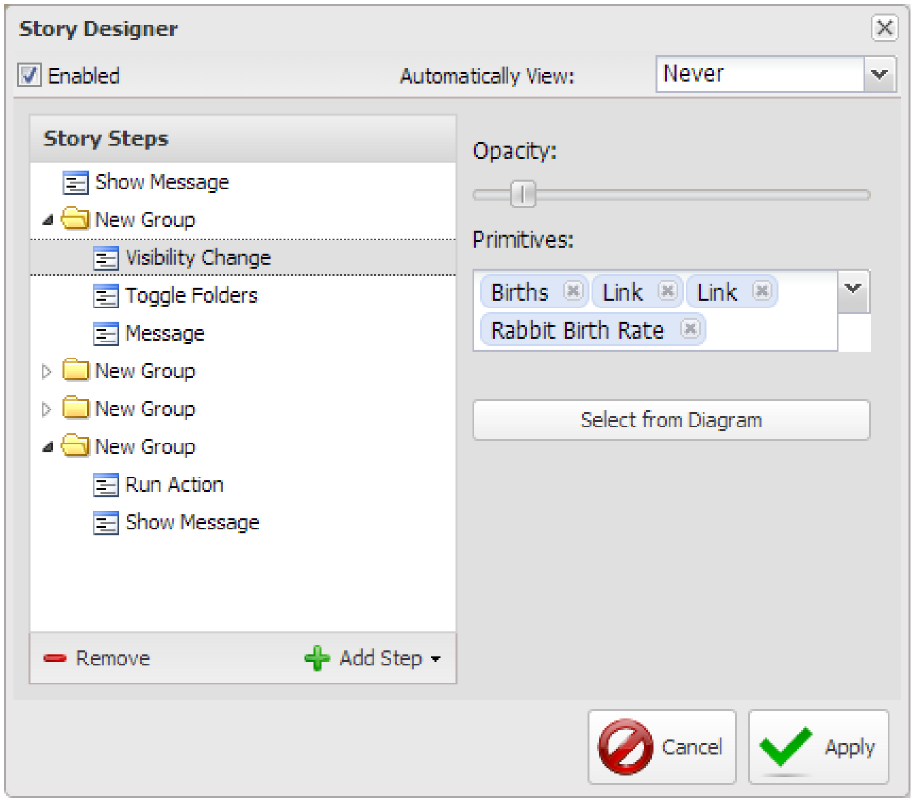


Figure 8. Story Designer

If you click on one of the steps it's definition will be displayed on the right side of the Story Designer window. Currently the definition of the first Visibility Change is displayed.

- **- Remove**. To remove an Story Step first click on it to select it then click the **- Remove** button.
- **+ Add Step**. A drop down which allows you to select which type of step you want to add. New steps are added after the currently selected step. If the new step is not created where you want it just select it and drag it to the location in the sequence where you want it. The various steps will be described shortly.
- **Cancel**. Allows you to exit the Story Designer and not save any changes.
- **Apply**. Applies all the changes you have made in the Story Designer and exits.

There are five different types of steps you can include in a story.

When you select any one of these steps it will be added after the currently

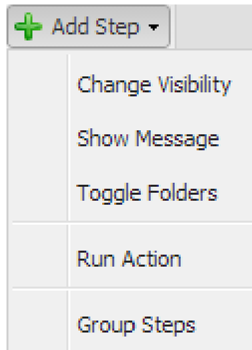


Figure 9. Steps of a Story

selected step in the list. As the following step types are described you might want open the Story Designer and click on different Unfolding Steps to visualize how they're defined.

- **Change Visibility**. Allows you to specify the Opacity, with the slider, for one or more elements of the model when the step is executed. The steps may be selected from the drop down or you may select one or more items from the model and then click the **Select from Diagram** button to put those items in the list. Note that clicking the **Select from Diagram** button will replace the already selected elements with whatever you've selected in the model. If you want to add or remove one more more Primitives it's probably better to use the drop down to add or click the existing item to remove it.
- **Show Message**. Allows you to enter a text message you want to be displayed. There are some formatting options available in the Message edit window.
- **Toggle Folders**. Use this option to Expand or Collapse one or more folders. This is really useful if you want to expand a folder, walk through the items in the folder and then close the folder.
- **Run Action**. Provides you with a window in which you can enter Javascript commands to control various aspects of the model. There are a large number of functions in the Insight Maker API that you can employ in this step.
- **Group Steps**. This step creates a folder in the sequence in which you can place multiple steps. This allows you to indicate there are several steps you want to execute with a single Next Step click. You can see how this was used in the Figure 8 definition of an unfolding. You can open and close a New Group folder by toggling the little triangle to the left of it. Also if you click on a folder you can rename it in the **Name** field on the right.

By creating a story of your model you significantly increase the likelihood that others will understand the insights your model is endeavoring to surface. You might even find that in the process of creating the story you uncover ways to improve or clarify the model for yourself.

The following storytelling examples may be viewed by clicking the following links.

- Rabbit Population
- The Boy Who Cried Wolf
- Systems Thinking: The Essence of And?

Summary

Always remember that a model is a simplification of the world around you intended to help you understand something. You should use a modeling form that adequately serves your quest for understanding. Stock & Flow, Rich Picture, and Causal Loop models are only three of a very large array of possible modeling forms that exist. Use them to the extent that they serve you and when they don't find a form that does. The Modeling and Simulation references provide access to several additional types of diagrams that have been created with Insight Maker, though there are probably more that haven't yet been identified. The Model Thinking Course by Scott E. Page presents a very extensive exposure to many types of models useful for understanding different aspects of the world around us.

Rich Pictures

- Often easier for others to relate to and understand because of the images in the model.
- No rules for creating them though remember that you need to tell a story.
- Because you need to tell a story make the model as understandable as possible.

Causal Loop Diagrams

- Specific Guidelines for how to depict the relationships between the elements.
- Two conventions for expressing relations.
- S and O notation can produce misinterpretations when it comes to stocks.
- The older + and - notation is considered more appropriate.
- Color coding relations is an alternative to both notations, though be consistent.
- Employing explicit stock representations often reduces misinterpretations.

- Hybrid Rich Pictures/Causal Loop Diagrams are often the very meaningful.

Storytelling

- Makes it far easier for others to understand the insights you're trying to surface with your model.

References

- Senge, P. 1990. The Fifth Discipline: The Art & Practice of the Learning Organization
- Modeling & Simulation with Insight Maker
- Model Thinking Course by Scott E. Page, University of Michigan
- Rich Pictures from Open University Course T552

Chapter 4

Building a Model

```
"Would you tell me, please, which way I ought to go from here?"
"That depends a good deal on where you want to get to," said the Cat.
"I don't much care where-" said Alice.
"Then it doesn't matter which way you go," said the Cat.
"-so long as I get SOMEWHERE," Alice added as an explanation.
"Oh, you're sure to do that," said the Cat, "if you only walk long enough."

Lewis Carroll - Alice in Wonderland
```

Now that the most relevant aspects of Insight Maker have been introduced it is appropriate to provide you with a meaningful process and guidelines to use when you set out to build a model to promote an understanding of an area of interest. An aspect of this essential for the development of sound models is the topic of units. While units don't ensure a model is sound, if the units don't match up one can be certain the model is not sound.

Model Construction Process

We develop models to help us understand the implications of interactions, and sometimes guidance. As such, as with Alice above, it is essential that before you begin to build a model you know what it is that you want to understand otherwise how will you know if the model does what you needs to do.

There are a number of guidelines or rules of thumb that you will find helpful when developing a model. These will be presented as Modeling Tips throughout this chapter. The idea is to ensure that the model serves the purpose you started building it for.

The difference between [**Real Events**] and [**Conclusions and Behaviors**] result in the creation of an [**Abstract Version of Real Events**]. The abstraction is then used to develop a [**Model**] which promotes a revision to

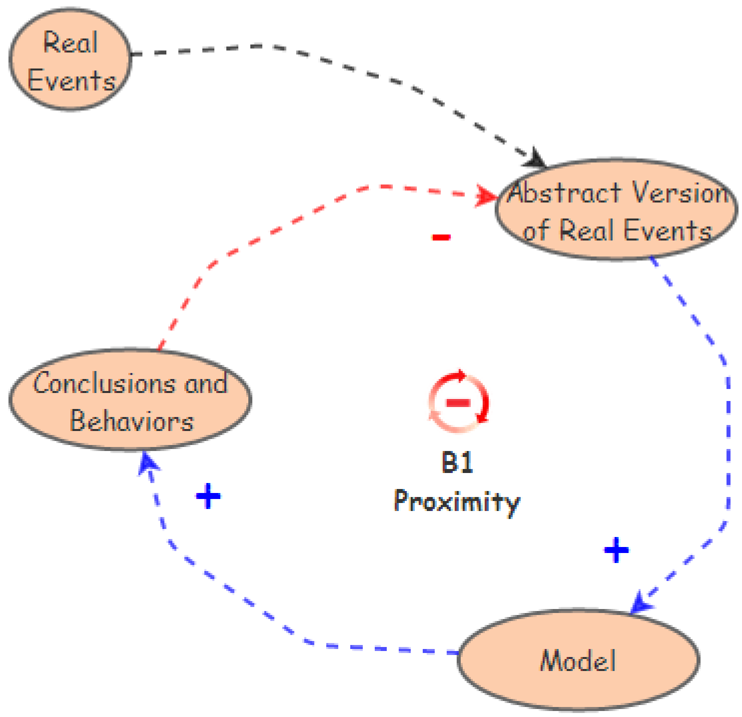


Figure 1. Model Construction Process

[**Conclusions and Behaviors**]. This cycle continues until the [**Model**] produces a set of [**Conclusions and Behaviors**] which are congruent with [**Real Events**]. At this point there's no longer a need to create an [**Abstract Version of Real Events**], meaning you have achieved the understanding you were seeking.

The model construction process of Figure 1 is very conceptual. As you continue to develop models you will arrive at a sequence that you are comfortable with an enables you to achieve the understanding your seeking.

The following two figures present the two model formulation processes presented by Andrew Ford in Modeling the Environment{cite: Ford, A. 2009. Modeling the Environment. http://www.amazon.com/Modeling-Environment-Second-Edition-Andrew/dp/1597264733/}

In approach presented in Figure 2 one focuses on the understanding the qualitative dynamics, i.e., problem familiarization, problem definition and model formulation. Not until such time as there is a level of comfort in the understanding of these dimensions, which may employ Rich Pictures or Causal Loop Diagrams, does one progress to the quantitative aspect of model building, i.e., estimating parameters, simulating to explain the problem and sensitivity and

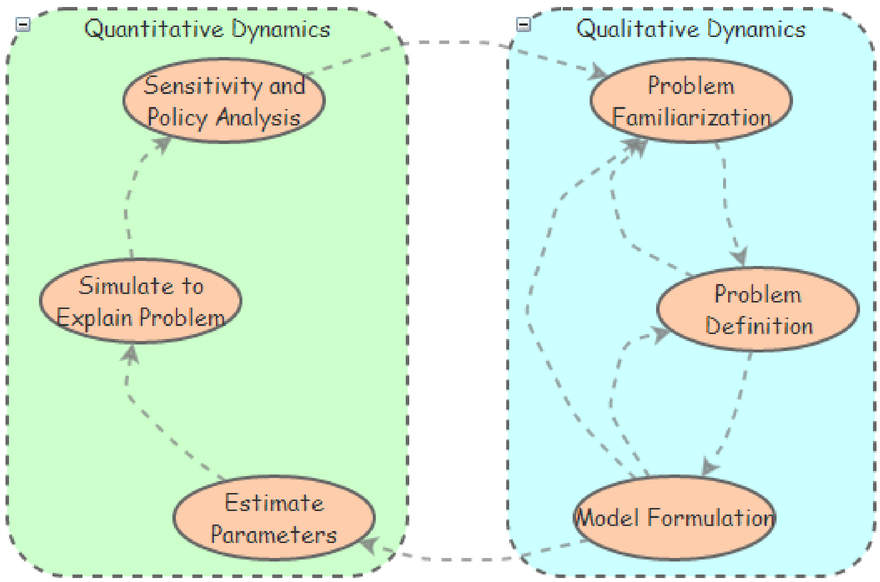


Figure 2. Emphasis on Model Formulation

policy analysis, which is where the Stock & Flow simulation model is employed. The quantitative dynamics may produce sufficient understanding or the process may continue back into the qualitative dynamics area. Model development is an iterative process.

Figure 3, which may look like complete chaos, emphasizes simulation to provide feedback to provide a better understanding of all other aspects of the modeling process.

Here the belief is that actually simulating all stages of the model are the best way to ground one's understanding of all other aspects of the model development process.

As you develop models you will develop an approach which is probably somewhere between Figure 2 and Figure 3 that you are comfortable with. That is probably the most critical aspect, i.e., that you be comfortable with your process and it make sense to you and helps you understand.

Modeling Guidelines

Figure 4 presents a number of guidelines or rules of thumb it would be good for you to keep in mind as you develop models. Some of the following are only relevant to Stock & Flow simulations, and which they are should be quite obvious.

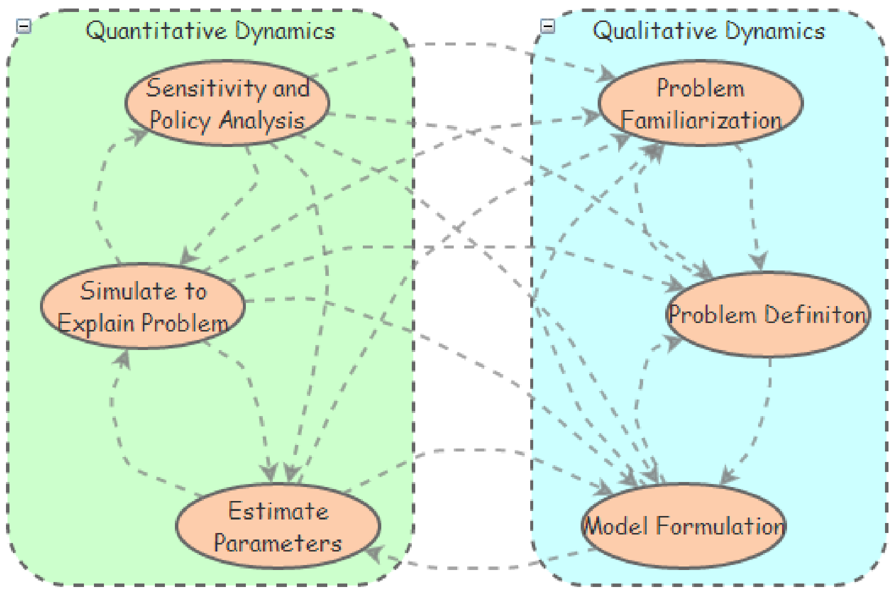


Figure 3. Emphasis on Simulation Early & Simulate Often

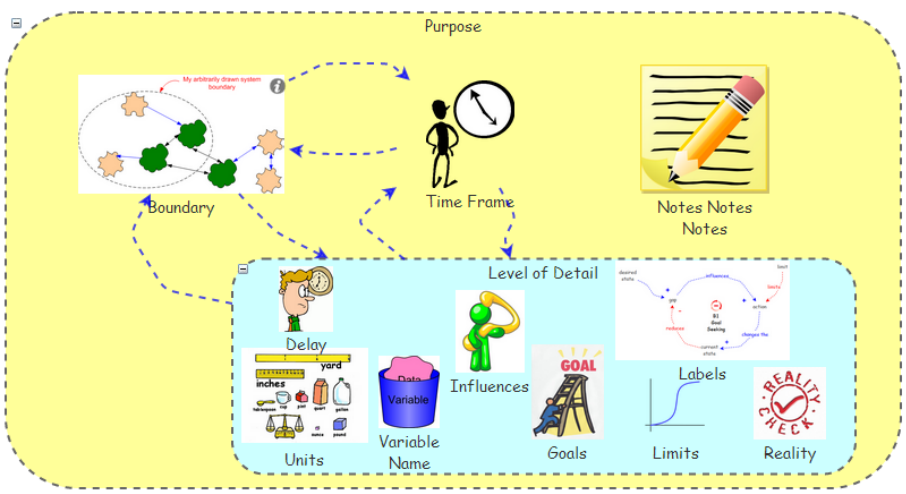


Figure 4. Modeling Guidelines

Remember that model development and the understanding that comes with it is an iterative process. It's almost impossible to create all the pieces as they should be the first time around. Look at it as do a little, learn a little, and repeat.

- **Purpose**. Have a sense of what you want the model to accomplish and expect the thought to evolve as you develop the model.
- **Boundary**. A boundary allows you to explicitly define what's part of the model and what's not part of the model. If you're unclear on the purpose of the model and unable to establish a boundary how will you ever know when to stop adding things to the model?
- **Time Units**. Will the interactions in this model be depicted over Years, Months, Days, etc. And you should realize that you initial thought may have to be revised once you begin developing the model.
- **Simulation Length**. How long might the interactions have to be modeled for. Here again the answer may be obvious, or you may have to start with an estimate and revise it after working with the model.
- **Time Step**. Here again you have to estimate a value based on the smallest time of transition you expect in the interactions and then test it to see if you you're close enough.
- **Notes Notes Notes**. As you build your model add notes to the elements so you can refer back to them later to get a sense of what you were thinking when you created them. Yes, you tell yourself you know what you're doing at the moment, though you'll be surprised at what you won't remember a week, a month or even a year from now. Notes also make it much easier for others to understand what you intended when you created elements.
- **Variable Names**. A stock represents a quantity and should be labeled with a directionless noun our noun phrase, you know, a person, place or thing. Avoid directional modifiers such as increasing, decreasing, growing, slowing, etc. as they tend to make a model very difficult to understand. A flow represents something moving over time so it's label should be something one would easily think of as moving over time as walk, speed, flow, etc.
- **Loop Labels**. If you're developing a Causal Loop Diagram or Stock & Flow Diagram be sure to label and sequence your loops so others have a sense in what order to read your story.
- **Goals**. Balancing loops always have goals. Make sure they're explicitly identified.
- **Influences**. Make sure you include all relevant influences and only the relevant influences. Sometimes you include items because you can't figure out if they're relevant or not. That's OK as long as you remember to later take out the ones that aren't relevant. If you leave influenced which aren't relevant they are likely to result in confusion later.

The following items are most likely relevant only for Stock & Flow simulations.

- **Stocks**. Identify which items are the the stocks, or accumulations, in the model that will change over time. Stocks are often easy to identify if you think about stopping time. When time stops a stock still has a quantity. In this case it's the distance from Grandma's house as Red walks toward it.
- **Flows**. Identify the flows which are responsible for changing the stocks over time. If time stops a flow has no value. In this case it's walking.
- **Delay**. Delays can have very unexpected impacts on the behavior of a model. Where there are delays make sure they're explicitly identified.
- **Units**. Units can be very instrumental in assuring model validity. While consistency of units doesn't guarantee model validity if the units are inconsistent you can be sure the model is not valid.
- **Limits**. If there are limits on Stocks, Variables or Flows be sure they are explicitly stated so Insight Maker can inform you if the model generates out of limit value. This will signal you that there is a problem with assumptions or initial values.
- **Reality Check**. Ensure the model is producing results which are consistent with reality. If it is not then it's an opportunity for learning.

The guidelines are far too much to memorize though if you refer to them as a check list over time they will actually become second nature and you'll find yourself checking them as you're adding elements to a model.

Can Red Get to Grandma's House

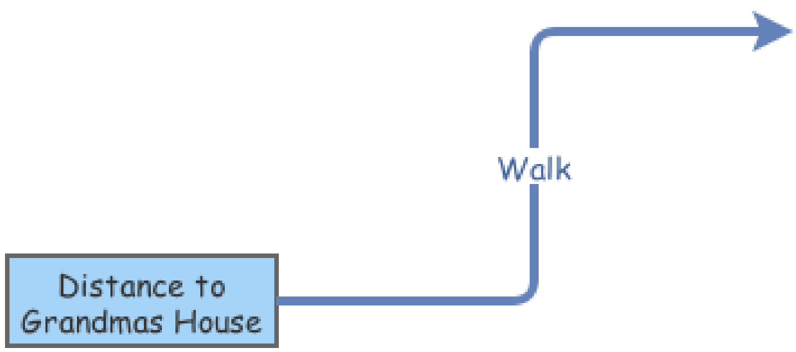
Here's a simple example of a question that might be answered with a model. And yes, it is quite obvious you could just do the math though would you get any better at building models if you did?

Going to Grandma's House

Little Red Riding Hood want's to know how long it will take her to get to Grandma's house if she walks at 2 miles per hour and Grandma's house is 4.5 miles away through the woods.

1. In this statement what is to be figured out is very easy to identify. Sometimes it's not so easy and you have to dig a little.
2. Create a new **Stock** named [**Distance to Grandmas House**].
3. Create a new **Flow** going from the primitive [**Distance to Grandmas House**] to empty space. Name that flow [**Walk**].

4. Now lets add a slider for each [**Primitive**] so we can adjust the values later.
5. Change the **ShowSlider** property of the primitive [**Distance to Grandmas House**] to `true`.
6. Change the **SliderMax** property of the primitive [**Distance to Grandmas House**] to `10`.
7. Change the **SliderStep** property of the primitive [**Distance to Grandmas House**] to `0.5`.
8. Change the **ShowSlider** property of the primitive [**Walk**] to `true`.
9. Change the **SliderMax** property of the primitive [**Walk**] to `5`.
10. Change the **SliderStep** property of the primitive [**Walk**] to `0.1`.
11. The model diagram should now look something like this:



12. This figure represents a simple model of Little Red Riding Hood walking to Grandma's house. While this may look like a rather trivial model there are several aspects of this model that warrant a few notes, and some of them we've not considered before.

Setting Units

If you click on the stock and look at the configuration panel you'll notice that the last item in the list, **Units**, has a value of unitless. Units were not addressed in the first three chapters as they are so important we wanted to ensure we could provide them the focus they deserve. You use units to help ensure that your models are sound. Not that units will guarantee that your model is sound, though if the units don't work out right you can be sure there's a problem, so Insight Maker checks them for you.

Figure 5 shows the Configuration Panel for the stock where Units is assigned a value of miles. For this particular model miles makes sense as we're trying to figure out how long it takes Red to get to Grandma's House and we know it's 4.5 miles away.

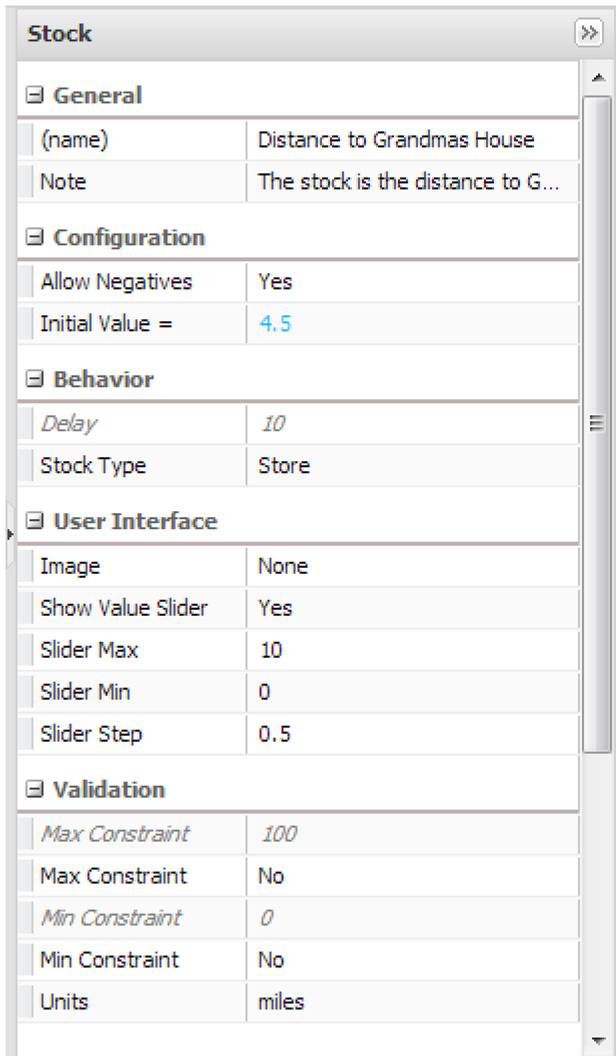


Figure 5. Units for Distance to Grandma's House in miles

If you click on the flow and look at the configuration panel you'll notice that the Units for walk is miles/hour as depicted in Figure 6. A flow represents the movement of something during a time period which is why this is 1/hours.

The flow has a units of hours as that's what will be set up in **Time Settings**

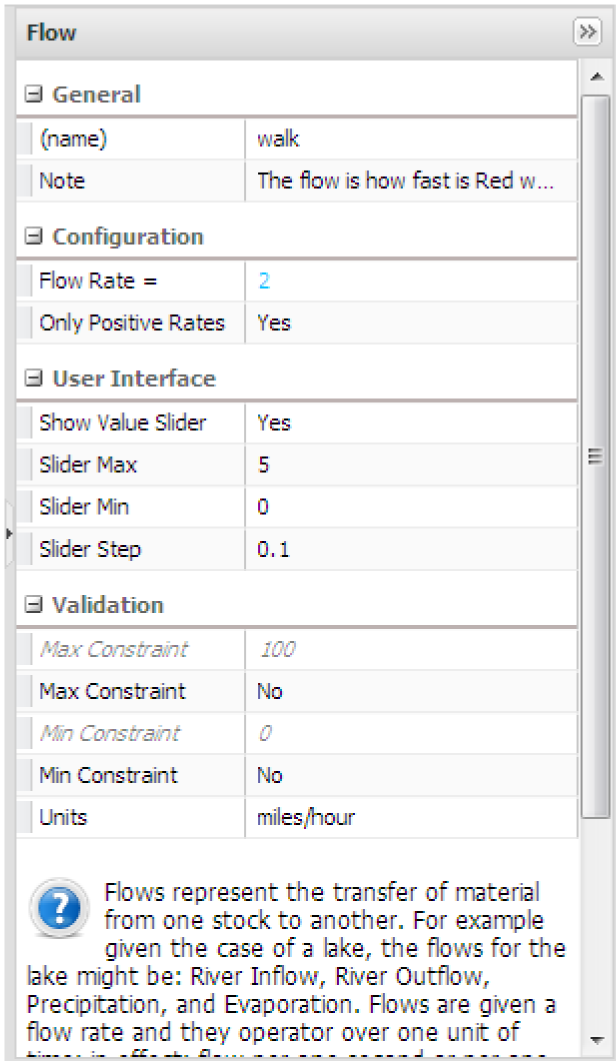


Figure 6. Units for Walk is in miles/hour

as the **Time Units** for the model. All the time settings are shown in Figure 7.

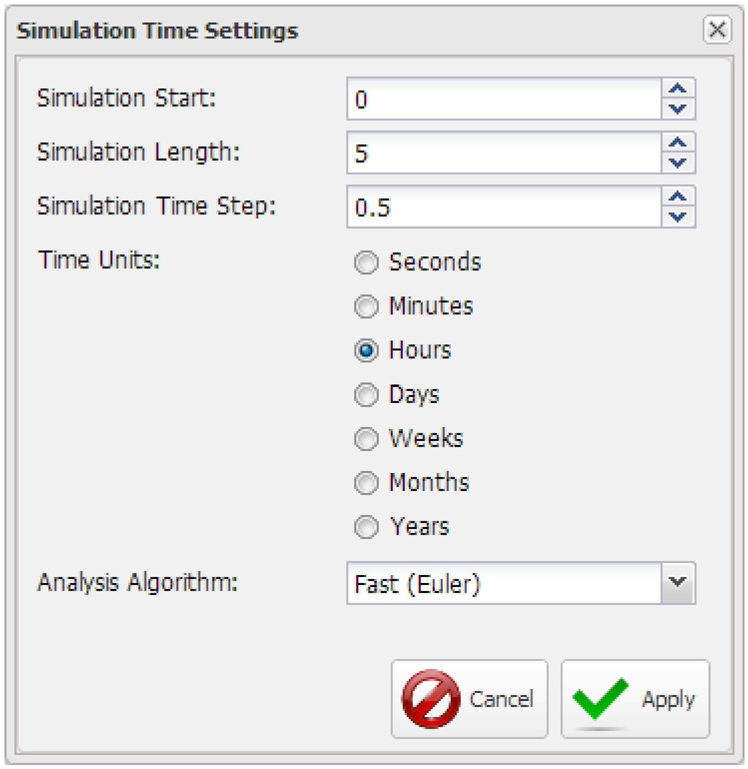


Figure 7. Time Settings for Walk to Grandma's House

You might now be asking, how the [**Walk**] in miles/hour gets turned in to [**Distance to Grandma's**] in miles? Because we've selected a **Time Step** of 0.5 each simulation step multiplies 0.5 hours x 2 miles/hour to get 1 mile traveled each time step. And the units are consistent. Later you can try changing the **Time Units** and running the model to see how that affects the answers. It's not actually this simple though with a constant flow rate this description is close enough.

Modeling Tip

There are a large number of units predefined in Insight Maker. If you click in the Units field and then click on the drop down on the right the **Units Selection** window will open as depicted in Figure 8. Here you can select from predefined units, though it's usually easier to just enter the appropriate units into the **Units** field. There is also a way to define Custom Units thought we'll cover this option in a later chapter.

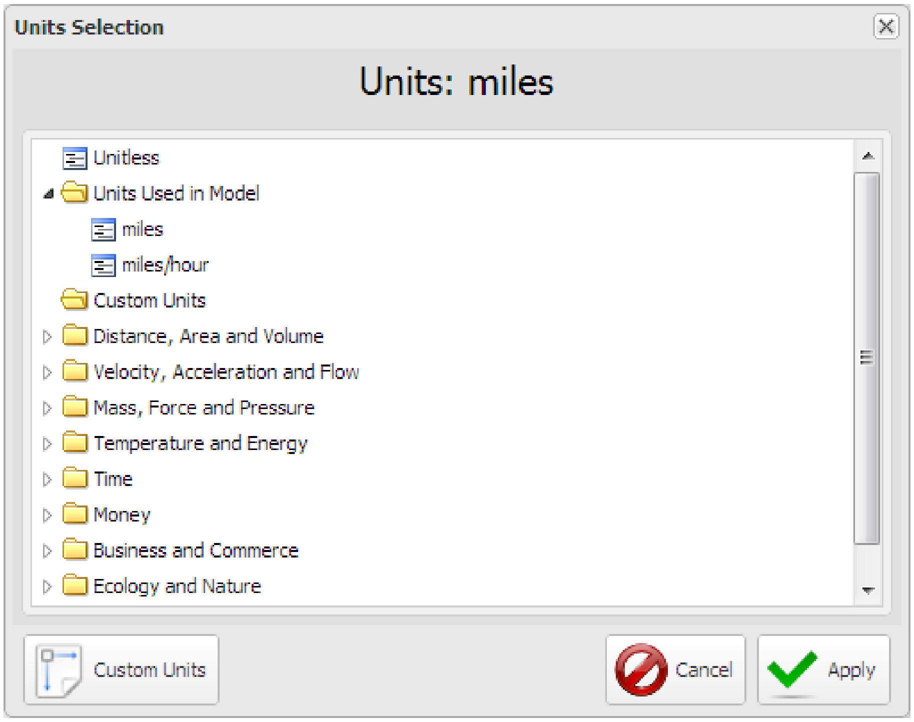


Figure 8. Units Selection

Going to Grandma's House Revisited

Now that we've talked about Units we can finish up this model and run it.

1. The model diagram should now look something like this:

2. Now we can assign the units for the stock and the flow.

3. Change the **Units** property of the primitive [**Distance to Grandmas House**] to `miles`.

4. Change the **Units** property of the primitive [**Walk**] to `miles/hour`.
5. And assign the Time Settings.
6. Change the **Simulation Length** property of the Time Settings to `5`.
7. Change the **Simulation Time Step** property of the Time Settings to `0.5`.
8. Change the **Time Units** property of the Time Settings to `Hours`.
9. Run the model. Here are sample results:

10. From the graphic it should be evident that there are some enhancements that need to be made to our Going to Grandma's House model.

It's evident that Red didn't stop when she got to Grandma's house, and one might wonder where she ended up after 5 hours of walking. It appears that at 2 hours Red was 0.5 miles from Grandma's House and at 2.5 hours she was 0.5 miles past Grandma's House. That there is no time with the Distance to Grandma's House equal to zero indicates that the time step is to large for the relationships in the model.

Exercise 4-1

Run this model with a Time Step of 0.25 and 0.125 and from the Tabular Display which Time Step do you think is most appropriate and why?

Answer Available

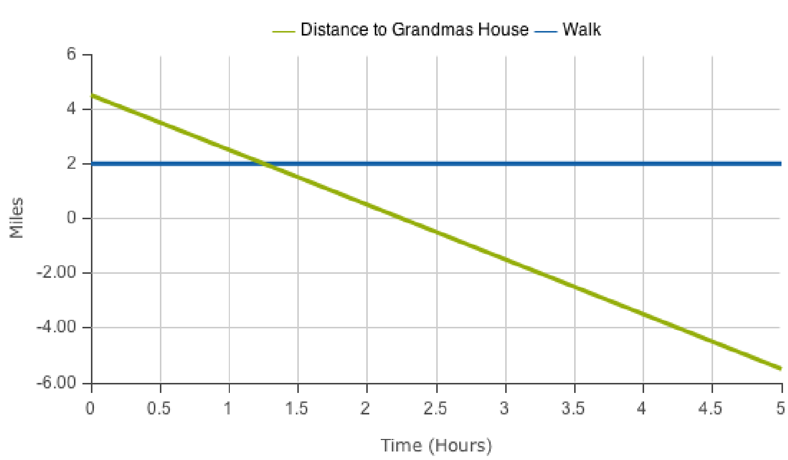
Stopping At Grandma's

We'll begin with the previous model and add an option that tells the model to stop when Red actually gets to Grandma's.

1. The model diagram should now look something like this:

2. The time step in the this version has been set to 0.25.

3. Run the model. Here are sample results:



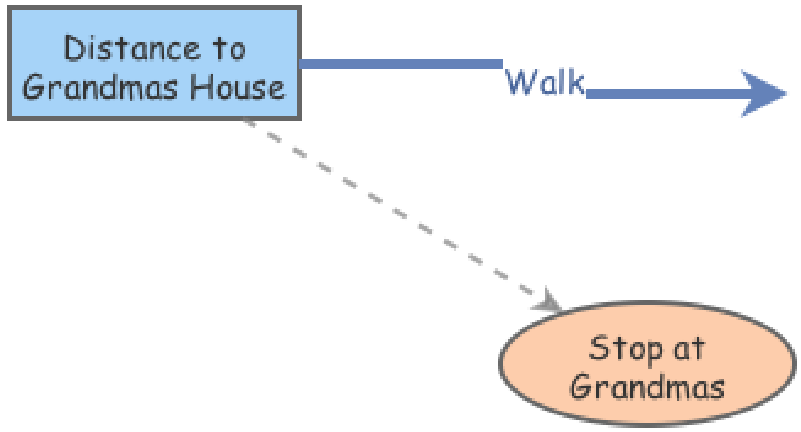
4. Now we'll add a variable to check whether Red is at Grandma's or not, and if she is the simulation should stop.

5. Create a new **Variable** named [**Stop at Grandmas**].

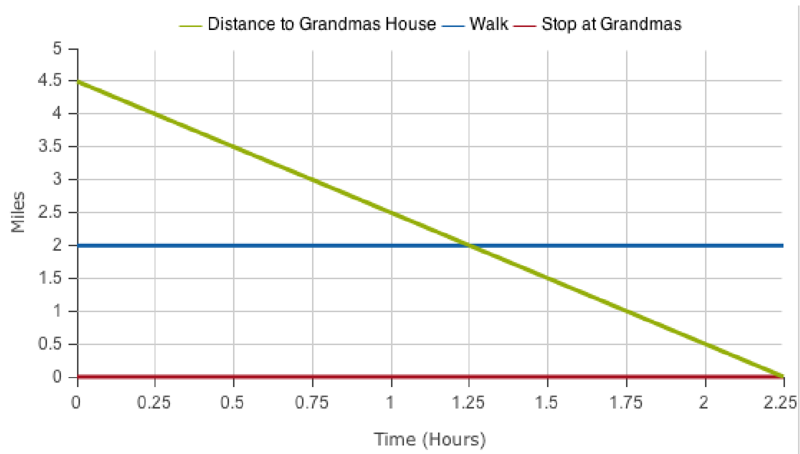
6. Create a new **Link** going from the primitive [**Distance to Grandmas House**] to the primitive [**Stop at Grandmas**].

7. Change the **Equation** property of the primitive [**Stop at Grandmas**] to `IfThenElse([Distance to Grandmas House] < 0 miles, STOP, 0)`.

8. The model diagram should now look something like this:



9. Run the model. Here are sample results:



10. Now you can see that Red stops walking at Grandma's House after 2.25 hours of walking.

If you look at the **Configuration Panel** for [**Stop at Grandmas**] you'll notice that the **Units** are unitless. The variable itself doesn't need a definition of units because it's not participating in any calculations. It's just a test.

As for the formula what you should remember is that when you start using units in a model, which you should do, all formulas have to be consistent from a units perspective otherwise Insight Maker will raise an error message. Just as a test change {0 miles} to 0 and run the model. Because [Distance to Grandmas House] has units what it is compared to has to have units.

Exercise 4-2

In the [Stop at Grandmas] variable change {0 miles} to {0 kilometers}. Does the model still work? Why?

Answer Available

Exercise 4-3

Seldom is there ever just one right way to build a model. You build the model to help you understand something and you might do that in different ways. Even for a model as simple as Going to Grandma's can be structured in several different ways other than starting with a stock of 2.5 and reducing it by walking. Try to build one or two alternatives to this model.

Answer Available

Hopefully the Going to Grandma's model has given you a sense of an approach for developing models along with some useful tips and an introduction to using units and why they can be so useful to you. Oh, and don't forget about putting notes in your models. Wiring diagrams without knowing what the pieces mean are generally not very useful.

Why Aren't We All Rich

If one can put money in an investment account and it grows over time, and it grows even faster with regular deposits, why aren't more people rich and ready for retirement? I've started numerous retirement programs through the years though for one reason or another they've all evaporated in time. What is the basis of this sad state of affairs?

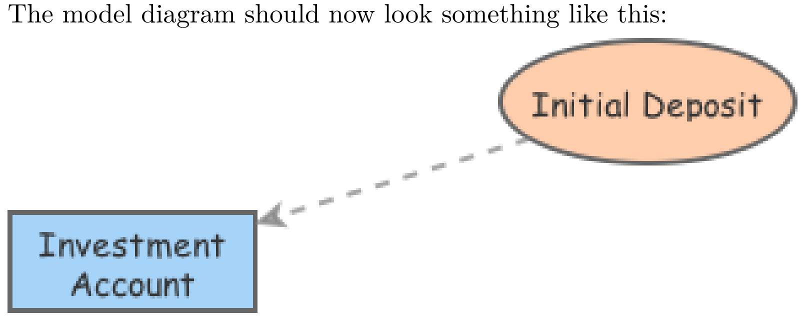
The following is intended to be another example of he development of a model, though somewhat more involved than the previous one. Here's the initial question describing what I'd like to understand.

Why Aren't We All Rich/Initial Setup

We begin with the simplest model possible relevant to the question.

1. First we create the Investment Account and the Initial Deposit and assign the appropriate units.

2. Create a new **Stock** named [**Investment Account**].
3. Create a new **Variable** named [**Initial Deposit**].
4. Create a new **Link** going from the primitive [**Initial Deposit**] to the primitive [**Investment Account**].
5. Now we set the values for the Investment Account and create a slider and set the value for the Initial Deposit.
6. Change the **Initial Value** property of the primitive [**Investment Account**] to `[Initial Deposit]`.
7. Change the **Units** property of the primitive [**Investment Account**] to `Dollars`.
8. Change the **Units** property of the primitive [**Initial Deposit**] to `Dollars`.
9. Change the **ShowSlider** property of the primitive [**Initial Deposit**] to `true`.
10. Change the **SliderMax** property of the primitive [**Initial Deposit**] to `500`.
11. Change the **SliderStep** property of the primitive [**Initial Deposit**] to `100`.
12. Change the **Equation** property of the primitive [**Initial Deposit**] to `100`.
13. Change the **Units** property of the primitive [**Initial Deposit**] to `Dollars`.
14. And we set the initial Time Settings.
15. Change the **Simulation Length** property of the Time Settings to `36`.
16. Change the **Simulation Time Step** property of the Time Settings to `1`.
17. Change the **Time Units** property of the Time Settings to `Months`.
18. The model diagram should now look something like this:



19. All the model does at the moment is assign the Initial Deposit to the Investment Account at the beginning of the simulation.

20. Run the model. Here are sample results:

This figure presents the initial set up for this model.

- **Investment Account**. represents the amount of money, in Dollars, in the account. If you look at the Configuration Panel you'll notice that Units are set to Dollars.
- **Initial Deposit**. is a variable used to specify the amount of money that is initially put into the [Investment Account] when it is opened. Remember we said only a flow can increase or decrease a stock, though you can use a external variable to set the initial value for a stock. This is done done to make the [Initial Deposit] explicit with a slider for testing. The Units for [Initial Deposit] is also set to Dollars.
- **Time Setting**. We've assumed that this is an investment account that will compute and add interest on a monthly basis so the time settings are set up to run for 36 months with an initial Time Step = 1 knowing that we will have to test this later on.

Modeling Tips

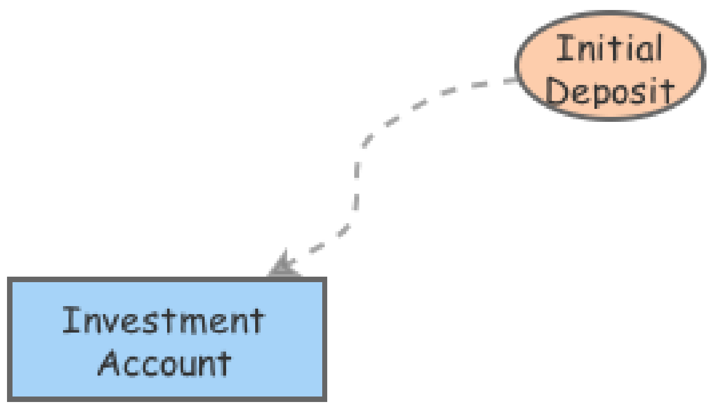
Before you run a model you should develop a sense of the result you expect from the model at this point in its development. Once you run the model you should be certain that is it performing as expected. When the result is not what you expect then either the structure is wrong, your assumptions are wrong, or you simply have an opportunity to further develop your understanding.

You should never be more than a single concept change away from a running model that produces a result that you understand. You may think this a bit strict though after you add several elements to a model and it doesn't work and you spend hours trying to figure out why you may have a better appreciation for this guideline.

Why Aren't We All Rich/Interest

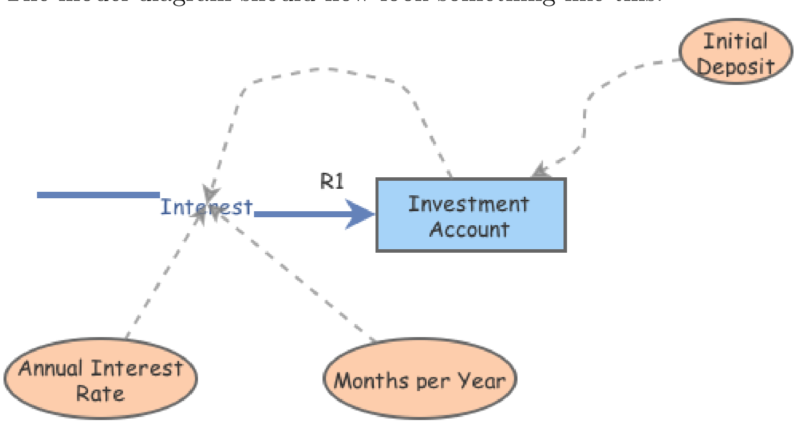
We begin with the previous model and add components to calculate monthly interest.

1. The model diagram should now look something like this:



2. Interest is a product of the Investment Account and the Monthly Interest Rate.
3. Create a new **Flow** going from empty space to the primitive [**Investment Account**]. Name that flow [**Interest**].
4. Change the **Units** property of the primitive [**Interest**] to `Dollars/Month`.
5. Create a new **Link** going from the primitive [**Investment Account**] to the primitive [**Interest**].
6. Create a new **Variable** named [**Annual Interest Rate**].
7. Change the **ShowSlider** property of the primitive [**Annual Interest Rate**] to `true`.

8. Change the **SliderMax** property of the primitive [**Annual Interest Rate**] to `0.1`.
9. Change the **SliderStep** property of the primitive [**Annual Interest Rate**] to `0.01`.
10. Change the **Units** property of the primitive [**Annual Interest Rate**] to `1/Year`.
11. Change the **Equation** property of the primitive [**Annual Interest Rate**] to `0.02`.
12. Create a new **Variable** named [**Months per Year**].
13. Change the **Equation** property of the primitive [**Months per Year**] to `12`.
14. Change the **Units** property of the primitive [**Months per Year**] to `Months/Year`.
15. Create a new **Link** going from the primitive [**Annual Interest Rate**] to the primitive [**Interest**].
16. Create a new **Link** going from the primitive [**Months per Year**] to the primitive [**Interest**].
17. Change the **Flow Rate** property of the primitive [**Interest**] to `[Investment Account] * ([Annual Interest Rate]/[Months per Year])`.
18. Create a new **Picture** named [**R1**].
19. Change the **Image** property of the primitive [**R1**] to `Positive Feedback Counterclockwise`.
20. The model diagram should now look something like this:



21. [**Annual Interest Rate**], as depicted above, is the rate that will be used to compute the interest on the account on a yearly basis. Note the a slider has been included with a .01 step size to make it easy to test different values. Units is 1/year as this is the per year interest rate.

22. [**Months Per Year**], as depicted in the figure, is just the number of months per year, a fixed constant of 12, to be used to convert the Annual Interest Rate to a monthly interest rate. The Units for this variable are Months/Year.

23. [**Interest**] contains the calculation for the Interest at each step of the simulation. The Units for interest are Dollars/Month which is derived from the formula.

24. [**Interest**] = [**Investment Account**] * ([**Annual Interest Rate**]/[**Months per Year**])

25. In Units: Dollars * (1/Year) / (Months/Year) = Dollars/Month

26. And as the simulation sums Dollars/Month over months the result added to the Investment Account is in Dollars which is consistent with the units specified for the Investment Account stock.RESULTS

27. [**R1**] makes use of the Picture primitive used to indicate that the relationship between Investment Account and Interest created a Reinforcing structure, with the 1 simply meaning it's the first one in the model.

28. Run the model. Here are sample results:

29. The run of this model over the three years with a 2% annual interest

rate still isn't very interesting though it does show a growth in the Investment Account as expected.

30. Now change the display so only the [**Investment Account**] value is shown.

31. Admittedly $6 dollars in interest wouldn't seem like much of an incentive to invest in a investment account for three years. Though there are several additional aspects of the Investment Account that we might take into consideration.

Modeling Tips

Making all the elements of a model visible makes it much easier for others to understand it. This is why Months per Year and Initial Deposit were created as explicit variables rather than embedding the valued inside other elements.

And what's definitely worth repeating is that providing comments for all the elements of a model will also make it much easier for others to understand. All one need do is mouse over an element and click on the "i" that appears to read the comment.

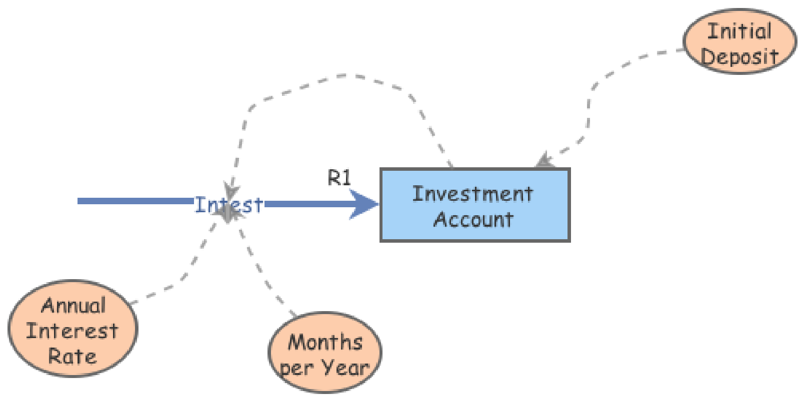
You have the option of adding notes to the Picture element and there are a number of predefined images that you can select from the pull down that can be assigned to the element. There are images for balancing and reinforcing loops, both clockwise and counter clockwise. These pictures can be assigned to Variables and Stocks also.

The other option is that you can put a URL in this field for an image somewhere on the web and that image will be displayed and may be resized.

Why Aren't We All Rich/Monthly Deposits

One typically adds to an investment account on a regular basis.

1. The model diagram should now look something like this:



2. Beginning with the previous model we now add [**Monthly Deposit**] with a slider so we can adjust the value for different runs.

3. Create a new **Flow** going from empty space to the primitive [**Investment Account**]. Name that flow [**Monthly Deposit**].

4. Change the **Units** property of the primitive [**Monthly Deposit**] to `Dollars/Month`.

5. Change the **ShowSlider** property of the primitive [**Monthly Deposit**] to `true`.

6. Change the **SliderStep** property of the primitive [**Monthly Deposit**] to `5`.

7. Change the **Flow Rate** property of the primitive [**Monthly Deposit**] to `75`.

8. Now we adjust the simulation lenght to 30 years, or 360 months.

9. Change the **Simulation Length** property of the Time Settings to `360`.

10. [**Annual Interest Rate**] has been changed to 10% because one is likely to find an investment account that will average 10% over a period of 30 years, or so it would seem based on Whitfield & Co[**1**].

11. Change the **Equation** property of the primitive [**Annual Interest Rate**] to `0.1`.

12. Run the model. Here are sample results:

13. This result is significantly different than the previous version of the model though is it enough to retire on? Not likely.

14. Suppose we change to 40 years and with $100 Dollars/Month recurring deposits.

15. Change the **Simulation Length** property of the Time Settings to `480`.

16. Change the **Flow Rate** property of the primitive [**Monthly Deposit**] to `100`.

17. Run the model. Here are sample results:

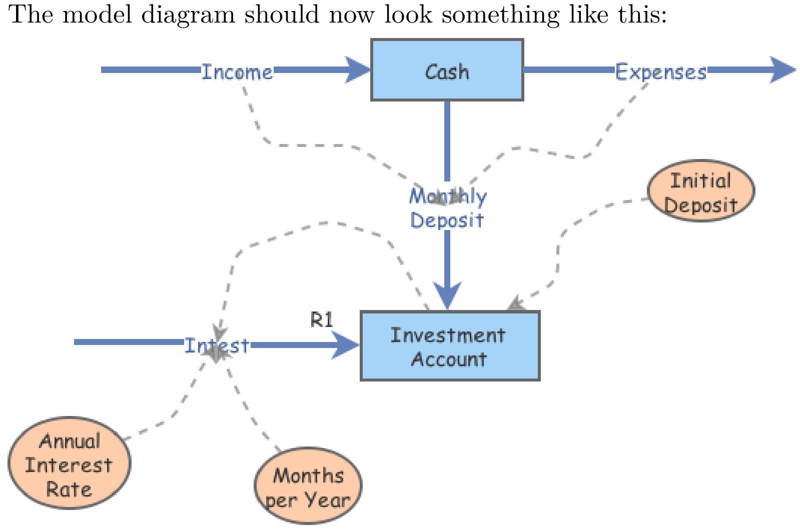
18. This shows a significant difference between $160 thousand dollars and $640 thousand dollars. The difference being based on what you are willing to invest and for how long.

It's best if we don't lose sight of the initial question, that being why more people employ this model and become rich. Part of the difference between the previous model and this one is the extra $30 dollars/month in periodic deposits. One of the difficulties is finding the money to deposit on a monthly basis.

Why Aren't We All Rich/Income & Expenses

In this version of the model we add elements to show where the Monthly Deposit comes from.

1. The model diagram should now look something like this:



2. The Monthly Deposit is just the difference between Income and Expenses.
3. Run the model. Here are sample results:

4. The results from this model are the same as the previous model though with the added definition of where the Monthly Deposit comes from.

Based on this model if one wants to increase the monthly deposits then it is necessary to increase Income or decrease Expenses as the Monthly Deposits are what's left over. Part of the difficulty is that when one has money the tendency seems for most to spend it rather than save it.

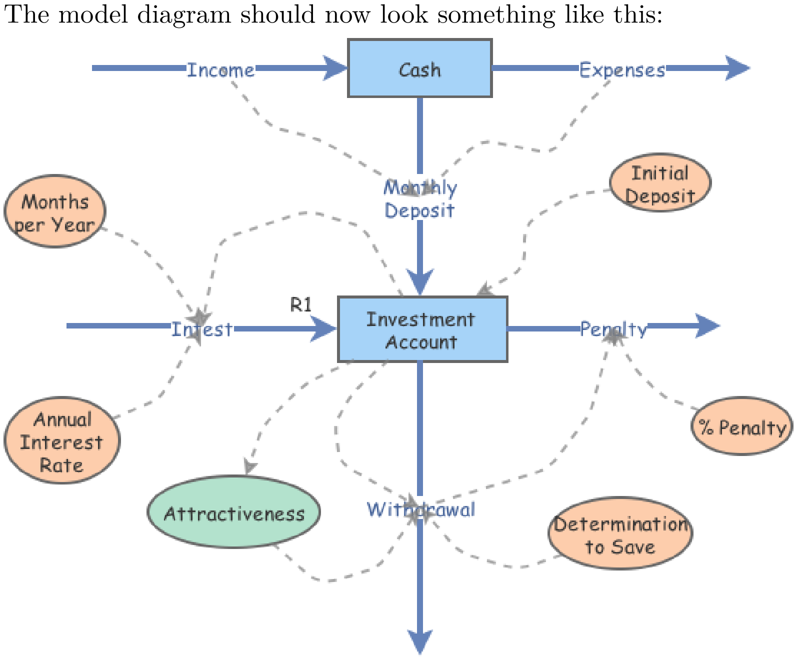
There are a couple additional aspects related to deposits that should be mentioned though won't actually be added to the model. Many companies allow employees to have payroll deductions directly deposited into a retirement account. This helps take care of the problem of having the money and spending it rather than depositing it in the investment account. Also, at least in the US their are tax laws that allow for the investment of some amount of pretax fund, money that you don't have to pay taxes on, to be placed in an investment account. The idea being that you would withdraw the money sometime in the future when you're in a lower tax bracket. Some companies will even match a portion of your investment account deposits up to a certain amount each year. These options, which you could add to the model, would increase the resultant funds available at the end of the simulation.

The question we started with was that is this approach can be used to amass a sizable amount of money then why aren't more people using it to become well off. Part of the answer had to do with the idea that with money in their pocket people are more likely to spend it than save it even though there are incentives to save it.

Why Aren't We All Rich/Attractiveness

The next version of the model provides an enhancement to add Withdrawal and Penalty flows with some associated variables which are described below.

1. The model diagram should now look something like this:



2. With a 50% Determination to Save and an 8% Penalty you notice a very distinct decrease in the dollars actually saved.

3. Run the model. Here are sample results:

Penalty is levied by the Government if the funds are withdrawn before you reach 59 1/2 and is meant to be an encouragement to save. The % Penalty is a variable with a slider defined to you can test the value during runs. The Units for Penalty are Dollars/Month.

Withdrawal represents money taken out of the account to purchase things with. As the amount of money in the Investment Account grows it becomes more and more attractive for use to purchase other things and there develops a tug of war between the Attractiveness of the money in the Investment Account and one's Determination to Save. Attractiveness and Determination to Save both represented by percentages between 0 and 100%. Attractiveness is represented with a Converter, a modeling element not previously described.

Modeling Tip

It is often the case that a variable to be used in a model can not be represented as a constant or some well defined formula. The variable is actually a function of Time or some other variable. In the case of this model Attractiveness is a function of Investment Account and is defined as a set of data points.

The next figure shows the Configuration Panel for Attractiveness Principle. Note that many of the configuration options are the same as other modeling elements. The ones that are different are in Configuration and Input/Output Table.

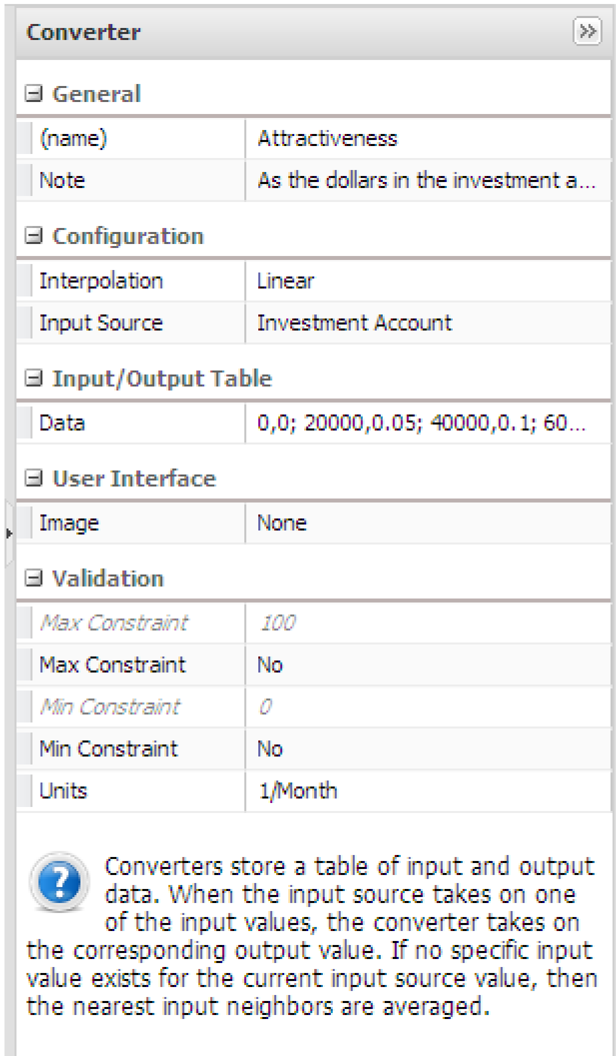


Figure 9. Attractiveness Control Panel

Because the variable is defines as a set of XY coordinates the Data has to be defined point by point as depicted below, or the table may be imported.

Also notice on the Converter Configuration Panel there is an option for Interpolation. This option defines how Insight Maker figures out the Y values in between the defined X points. The graph displayed in Figure 21 depicts the Linear Interpolation meaning that Insight Maker treats the line between two

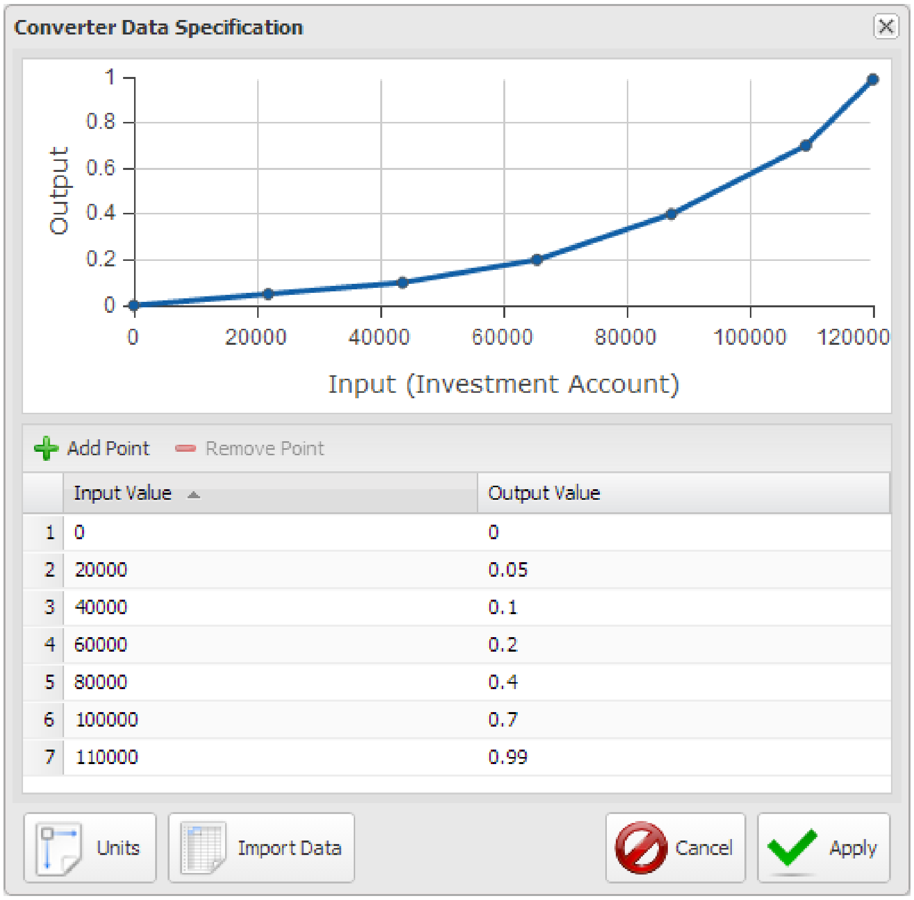


Figure 10. Attractiveness Data Specification

points as a straight line and if computes the Y value from the XY values at the two points on either side of the X value.

Figure 11 shows the curve for the Interpolation option of None meaning that it treats all the Y values between point X1Y1 and X2Y2 as Y1.

The following figures show the various display tabs for a run of this model with a Determination to save of 50%.

When the Investment Account reaches $87,000 dollars after 255 months it is sufficiently attractive to overcome the Determination to Save so money is withdrawn from the account every month and the account no longer grows. Is this a bad thing? That depends on the intent.

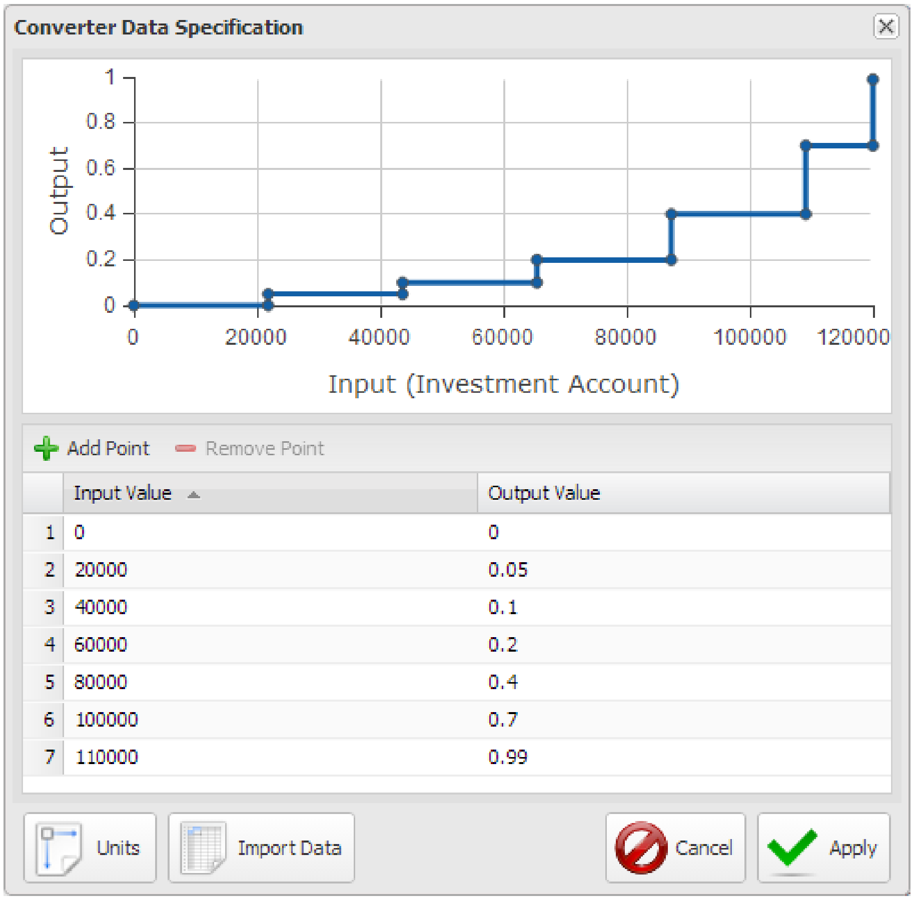


Figure 11. Attractiveness Data Specification with Interpolation = None

Figure 24 just shows that the Attractiveness has reached the Determination to Save level so withdrawals begin happening every month.

The above figure shows that there is almost $800 dollars a month being withdrawn from the account monthly and the account doesn't decrease. Maybe it's accomplishing what it needs to if $800 a month is sufficient to augment other income.

Note the large overshoot on the Withdrawal curve and a small one on the Penalty curve. This is most likely because the Time Step is too large. The next figure is the same display tab for the model run with a Time Step of 0.5. Notice how the curve cleans up.

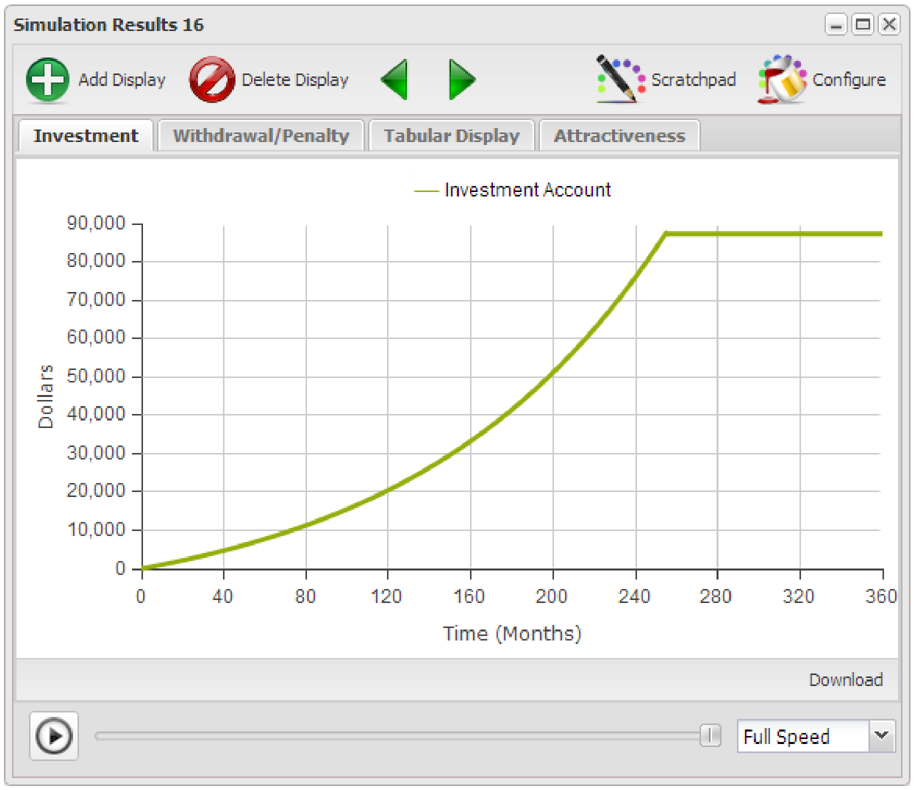


Figure 12. Investment Account Limited by Attractiveness

Exercise 3-4

There is a logic flaw in this model which you might try to repair. The Penalty is not actually taken from the Investment Account but from the Withdrawal itself so it reduces the amount you actually get from the Withdrawal. Be warned that is might be a tricky fix.

We now have a model which provides some incentives to start and continue to deposit in an Investment Account, and some disincentives toward the withdrawal of funds, though have we really addressed the initial situation posed? Not really. As far as starting the Investment Account and regularly depositing money, there are incentives, and for many these incentives were enough to get them to invest. For many the incentive, for one reason or another, has not been sufficient. And, any more strict incentives would likely be looked on unfavorably. People do not like to be manipulated, even when it is for their own benefit. The penalty for

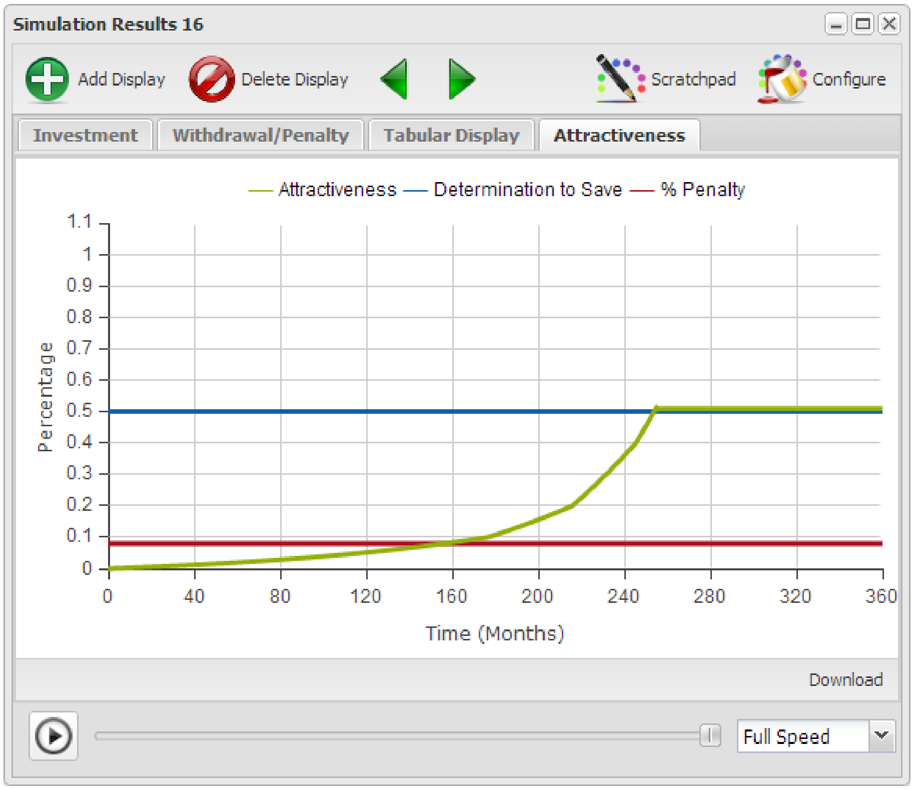


Figure 13. Investment Account Attractiveness and Determination

withdrawal is a deterrent in some respects though as the Investment Account continues to grow its attractiveness in terms of what it can purchase continues to entice. The best answer for this situation is to legally tie up the withdrawal process so it's only an option in the case of dire emergencies. Though as much as people find being manipulated by others distasteful, being controlled by themselves is just as distasteful.

Is the model done? As usual, the answer is; "It Depends!" If it has provided sufficient understanding to address the situation posed then it is sufficient. If not then it should be taken further, though once it is sufficient you should STOP!

Where Have All The Trees Gone?

We had a forest reserve of over a million trees and the logging company guaranteed us they would plant a new tree for every one they cut down, yet all of a sudden there are no trees left to harvest. What happened?

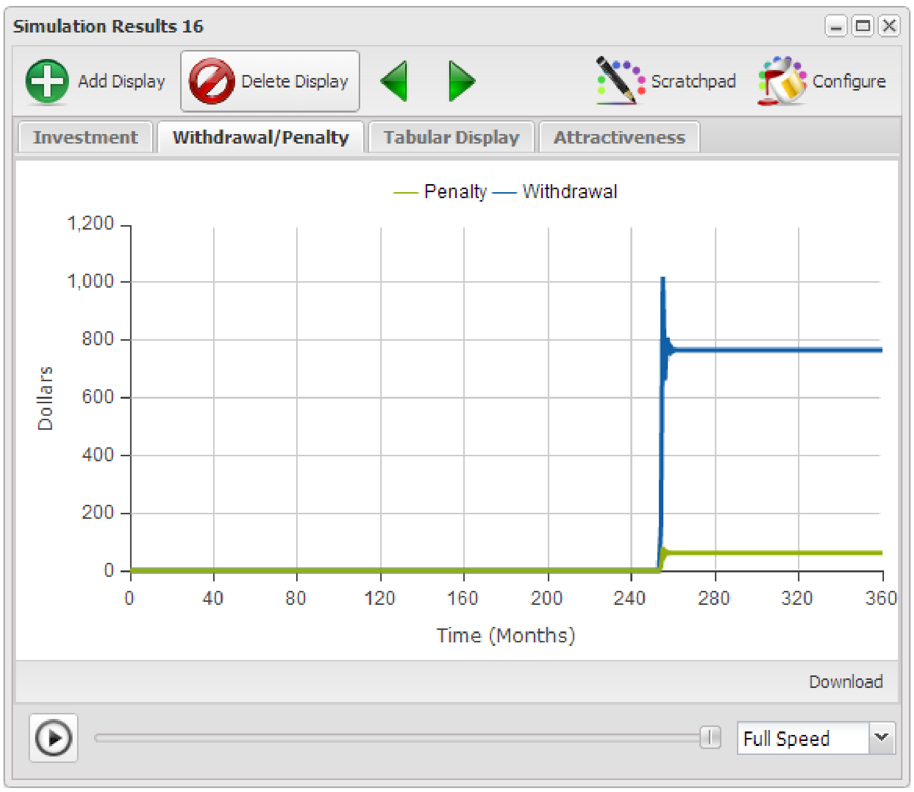


Figure 14. Investment Account Withdrawal and Penalty

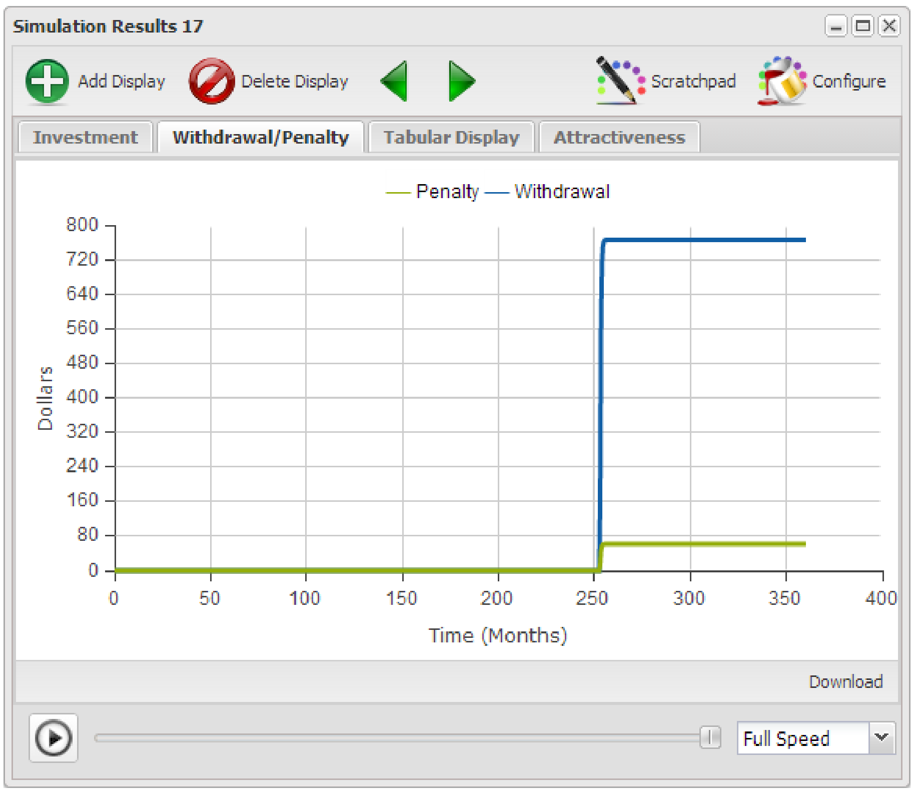
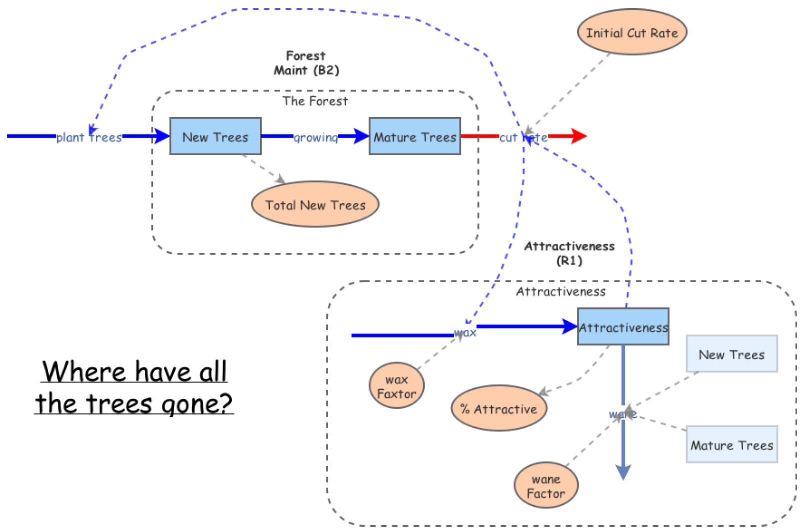


Figure 15. Investment Account Withdrawal and Penalty with Time Step = 0.5

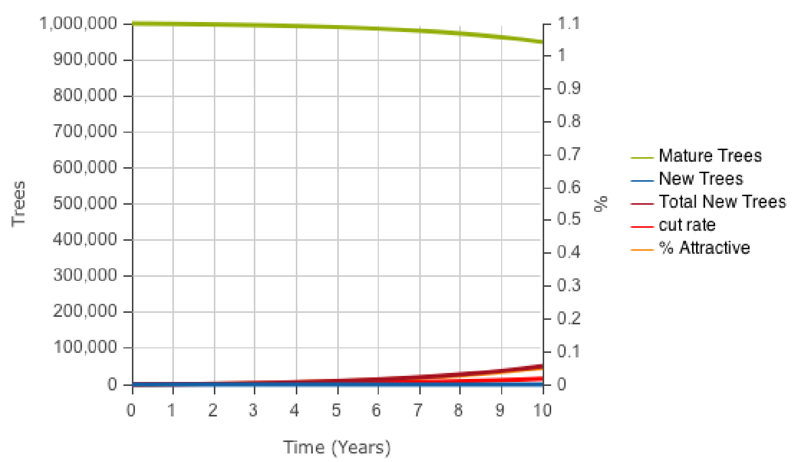
Where Have All The Trees Gone

Investigating the impications of different time horizons.

1. The model diagram should now look something like this:



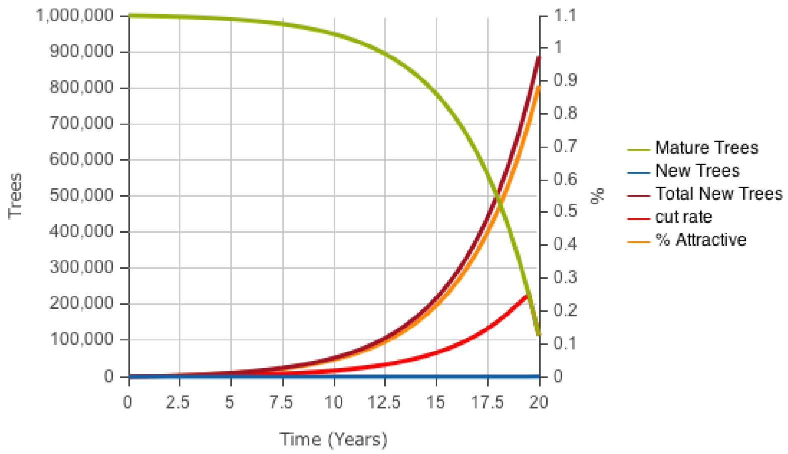
2. Run the model. Here are sample results:



3. When we run this model for 10 years things seem to be fine. The number of mature trees is declining a little thought it doesn't seem to be much to worry about.

4. Change the **Simulation Length** property of the Time Settings to 20.

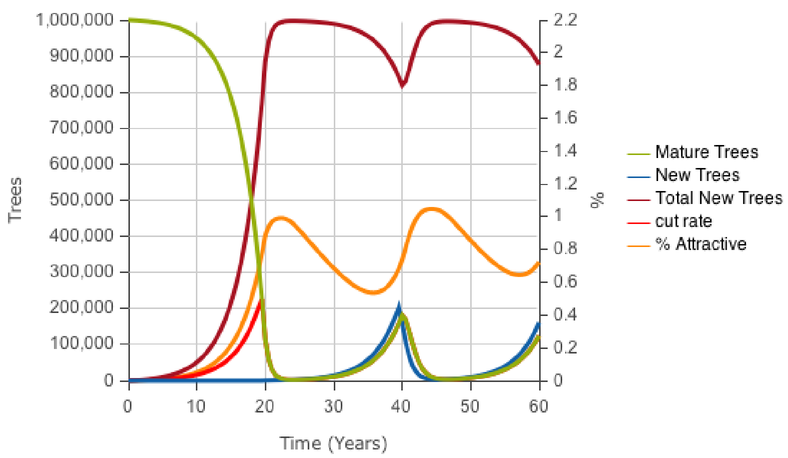
5. Run the model. Here are sample results:



6. When we look at 20 years the situation is very different and should generate major concern.

7. Change the **Simulation Length** property of the Time Settings to 60.

8. Run the model. Here are sample results:



9. And when we look at 60 years there seems to be a very major boom and bust cycle acting here.

** Where have all the trees gone?/story

Exercise 4-4

What have you come to understand about the difference between short term and long term perspectives and how do delays figure into surprises?

Building a Model Summary

- **Intent**. Be sure you have a good idea of what you want the model to help you understand. This may evolve as you develop the model.
- **Time Frame**. Ensure you have a sense of the time frame over which you intend to simulation the model. As you build the mode you may find you need to adjust your initial thought on this.
- **Stocks & Flows**. Identify the Stocks & Flows first as they are key elements of the model.
- **Use Units**. Units help to ensure your model is sound and Insight Maker will test for consistency of units. If the units are consistent it doesn't guarantee the model is sound though it does add a level of confidence.
- **Variables & Links**. Add Variables & Links to influence the flows.
- **Test Often**. Each time you make a logical addition to the model think about how you expect the model to behave then run the model and see if there is agreement with your expectation. If it isn't then it's an opportunity to learn and improve the model. And if it does agree you should still consider the output. It may be that your expectation and the model are both wrong.
- **Time Step**. Test the Time Step to ensure it's small enough to capture all relevant transitions in the model.
- **Stop at the End**. When the model serves the purpose for which you are developing it, STOP! There is always more you can add to a model. You should only include what is relevant to satisfy the initial intent.

References

- Catalina Foothills School District. 2003. Tips for Using System Dynamics Tools. http://www.clexchange.org/ftp/documents/Implementation/IM2003-12TipsUsingSDTools.pdf
- Ford, Andrew. 2009. Modeling the Environment. http://www.amazon.com/Modeling-Environment-Second-Andrew-Ford/dp/1597264733/
- Keeting, Elizabeth K. Internet 2013. Everything You Ever Wanted to Know about How to Develop A System Dynamics Model, But Were Afraid to Ask. http://www.systemdynamics.org/conferences/1998/PROCEED/00024.PDF

- Newell, Barry & Proust, Katrina. 2012. Introduction to Collaborative Conceptual Modelling. https://digitalcollections.anu.edu.au/bitstream/1885/9386/3/Newell_IntroductionCollaborative_2012.pdf
- Whiftield, Caraig. 2012. What Returns Should We Expect from the Stock Market. http://www.whitfieldco.com/blog/?p=39

Chapter 5

Implications of Reality

Chapter 1 presented Bertalanffy's premise that that the same basic structures operated across all disciplines, and if one learned how these structures operated one could transfer much of their learning from one discipline to another. In the previous chapters there has been a focus on three basic structures in support of Bertalanffy's premise. In this chapter will will build on those three basic structures in such a way as to demonstrate that there exists a set of more complex structures composed of combinations of the basic three which also repeatedly occur across all disciplines of science.

Basic Structures

The three basic structures are depicted in Figure 1 along with their characteristics behavior curves in Figure 2.

In the Chapter 1 and Chapter 2 we covered these three basic structures in some detail and it was claimed that all the models you will ever create will simply be a combination of some number of these basic structures. We don't expect that you take this on faith, and while we can't prove it, though in this chapter we will provide you an opportunity to experience some of the more common structures which repeatedly occur across all disciplines of science.

Typical Evolving Relationships

When you undertake something it is either to fix a problem, represented by the Balancing/Goal Seeking structure, or promote growth represented by the Reinforcing/Exponential growth structure. Seldom will you encounter either of these structures in their elementary form. Typically there are multiple structures interacting and even if you create an elementary structure is it likely to readily evolve into a more complex form. Figure 3 depicts the manner in which the Balancing/Goal Seeking and Reinforcing/Exponential Growth structures tend to found as part of, or evolve into, more complex structures. And each structure

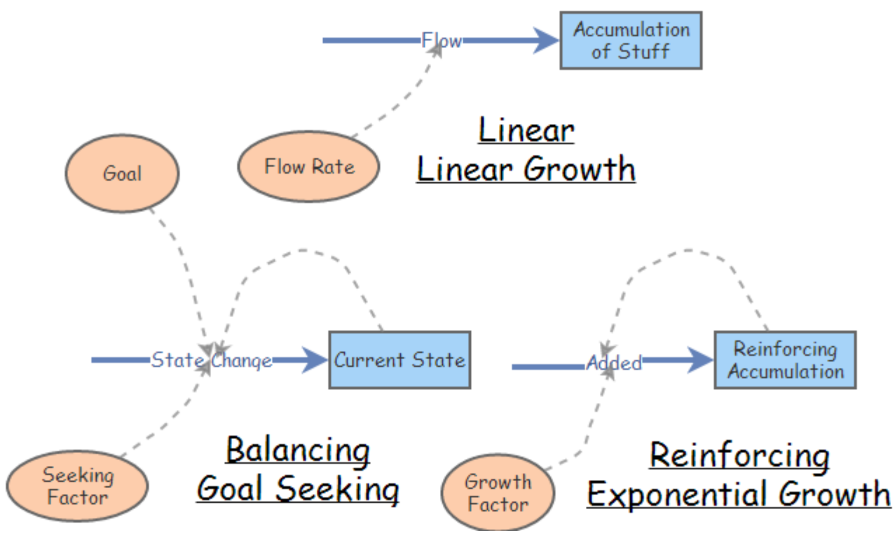


Figure 1. Three Basic Structures

has a characteristics pattern of behavior, which in conjunction with its structure, help to identify the recurring structure.

The sections of this chapter will present an investigation of the more frequently experienced structures and their characteristic patterns of behavior. Links will be provided at the end of the chapter to allow you to continue investigation of those structures not presented here.

Each structure will be presented in a generic form so you can focus on the implications of the relationships rather than what the actual elements are. Each section will also provide appropriate strategies for dealing with the structure as well as a number of explicit example of this structure in different areas.

Because the Balancing/Goal Seeking and Reinforcing/Exponential Growth structures have already been presented we'll simply begin with more complex structures.

Linear Progress Slows Over Time

A Limits to Results structure represents a situation where a Balancing Loop moving toward its goal is slowed in its progress due to some limiting factor. This is generally due to some resource restriction or constraint.

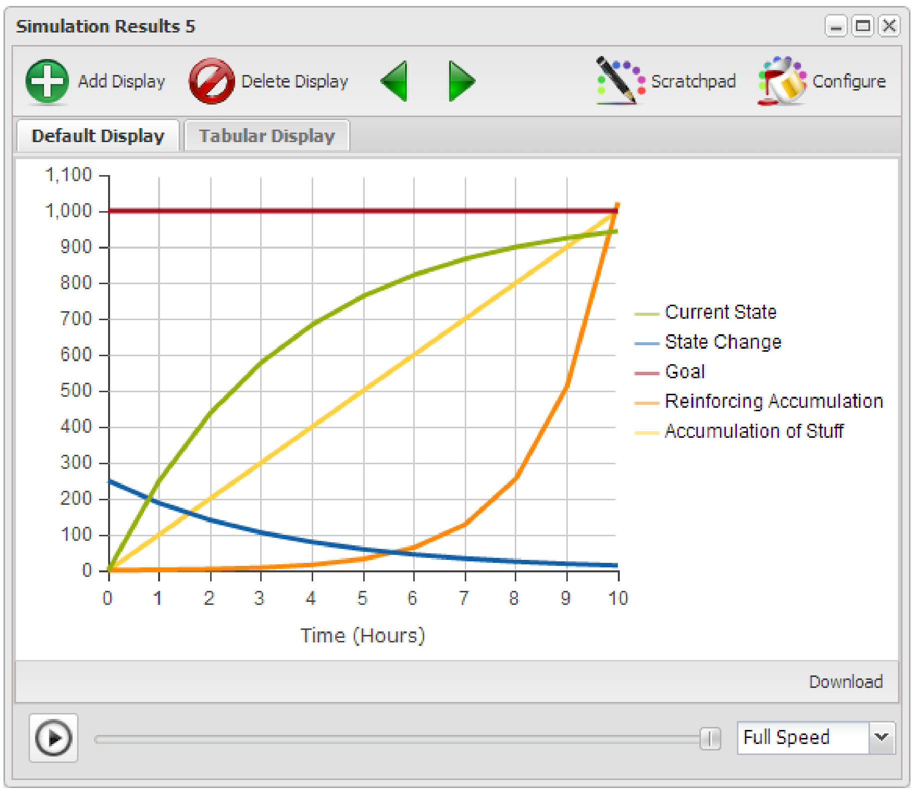


Figure 2. Three Basic Structures Behavior

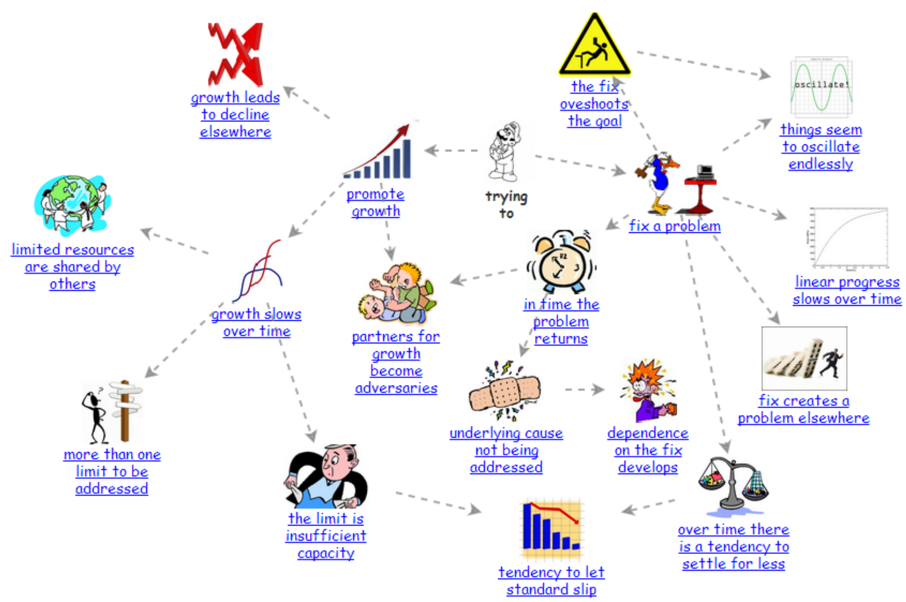
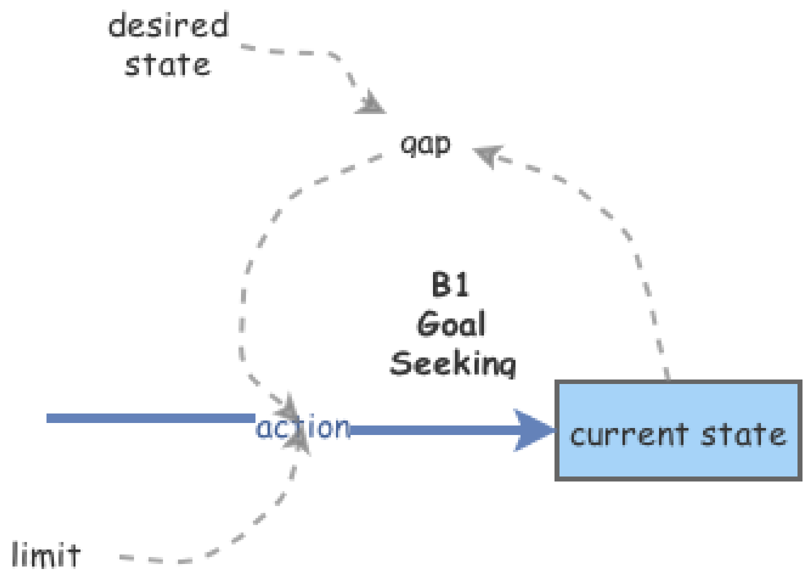


Figure 3. Typical Evolving Relationships

Limits to Results

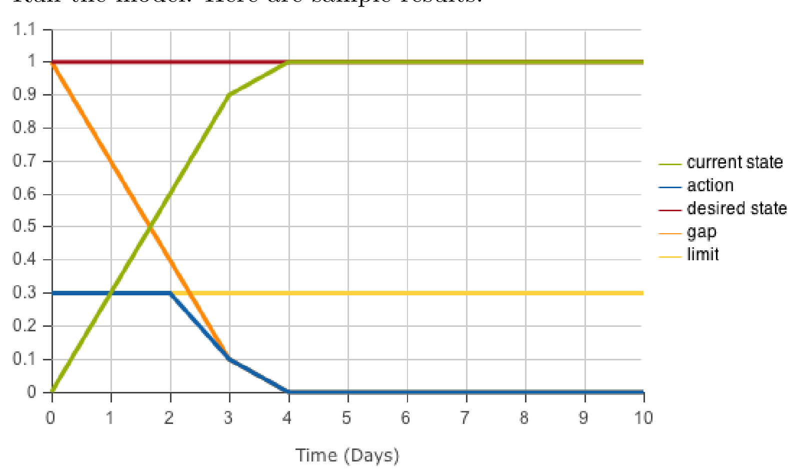
A balancing loop seldom progresses simply based on the difference between the [**current state**] and the [**desired state**].

1. The model diagram should now look something like this:



2. Initial settings are: [**current state**] = 0, [**desired state**] = 1, [**limit**] = 0.3 and [**action**] = IfThenElse([**gap**] > [**limit**], [**limit**], [**gap**])

3. Run the model. Here are sample results:



4. Action is a constant value until the gap < limit. The choppy nature of the diagram will be addressed in the exercise below.

Exercise 5-1

- Run the model with different values for limit. What difference do you see in the curve when the limit is evenly divisible into 1.
- What happens if you change the Time Step to 0.5, 0.25 and 0.125. What is the most appropriate value to use for Time Step?

Answer Available

Examples

Numerous example for this structure should readily come to mind.

- Any undertaking to complete a project is restricted by the availability of resources.
- The flow of anything to fill or empty a stock is restricted by the capacity of what the liquid must flow through.
- The rate of production of a process is limited by the capacity of the process.

Effective Strategy

- The effective way to avoid a Limits to Results scenario is simply to plan ahead to ensure there are sufficient resources available so progress toward results is not limited to a greater extent than acceptable. That said, remember that more of a good thing is not always the best answer. There is often a trade off and more resources may cost more than one gains by reducing the time by using more resources. There's always more than one thing that should be considered.

The Fix Overshoots the Goal

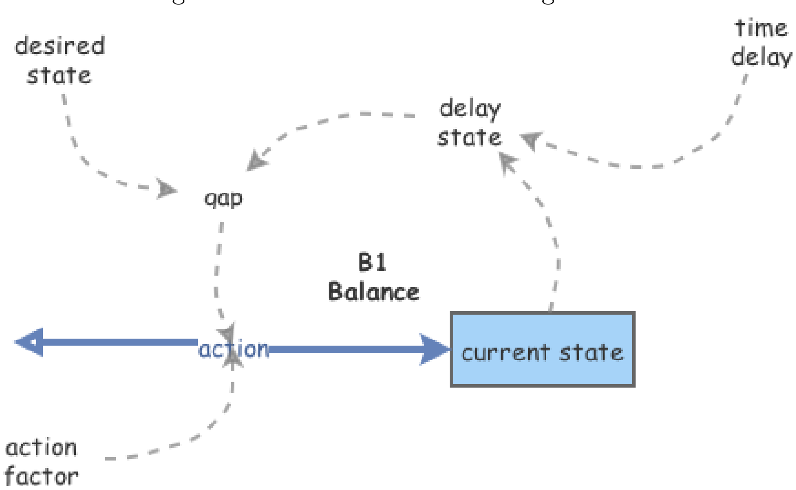
Have you ever pursued a goal and later found that you actually overshot the goal and had to back up to get back to the goal? The Balancing Loop with Delay structure is a variation of the standard Balancing Loop. The variation being that there are one or more delays in the structure which are responsible for producing, as will be demonstrated, a very different behavior pattern than the standard Balancing Loop.

If you look at the Balancing Loop with Delay structure it looks identical to the standard balancing loop with the exception of the delay near the reduces link. The implication is that it takes some amount of time after the current state changes before it is actually realized and figures into the calculation of the gap which influences the subsequent action. Essentially what's happening is that action is being based on old data and therefore is probably not the appropriate action. The implications of this will become evident when we look at the simulation for this structure.

The Fix Overshoots The Goal

Lets take a look at the implications of varying delays on the effect of a balancing loop.

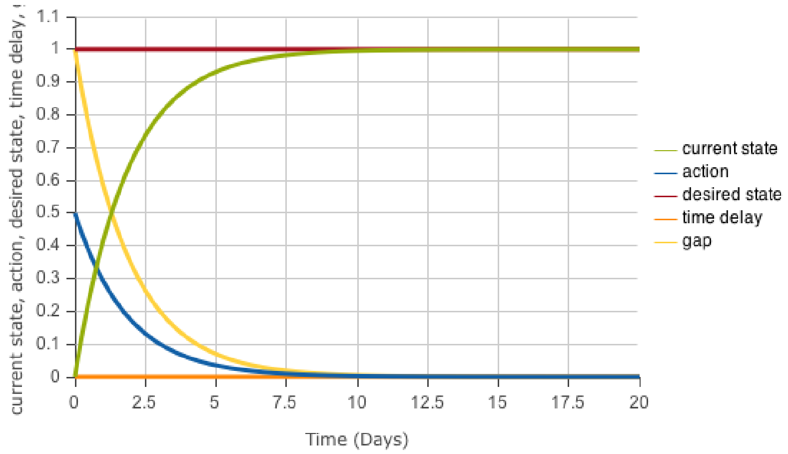
1. The model diagram should now look something like this:



2. Notice in the stock & flow structure the delay has been placed between the current state and the gap. The delay could have just as well been between the gap and action or there could have been a delay between the action and the actual change of the current state though this one is a bit more difficult to structure.
3. Notice the action flow in the diagram actually has an arrow at both ends. Click on the flow and notice the Configuration section of the configuration panel indicates Only Positive Rates is set to No. This means that the flow can flow in either direction based on whether the results of the equation are positive or negative.
4. Initial parameters are [**currents state**] = 0, [**action factor**] = 0.5, [**desired state**] = 1 and [**Time Step**] = 0.25

5. Change the **Equation** property of the primitive [**time delay**] to `0`.

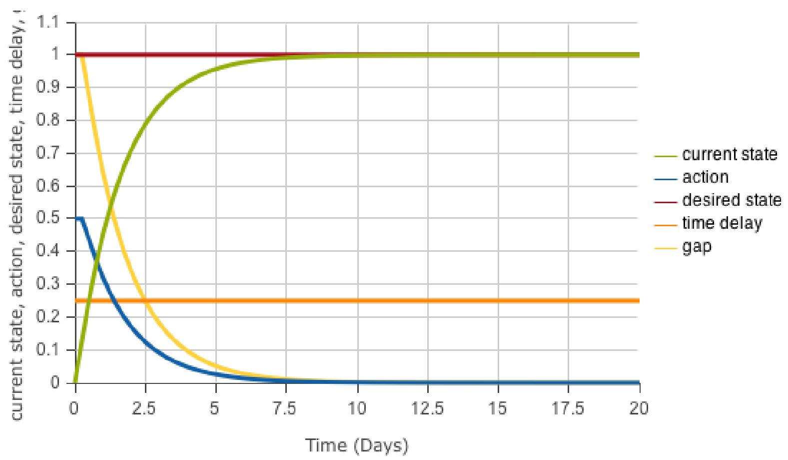
6. Run the model. Here are sample results:



7. With [**Time Delay**] = 0 the results are simply those of the standard balancing loop.

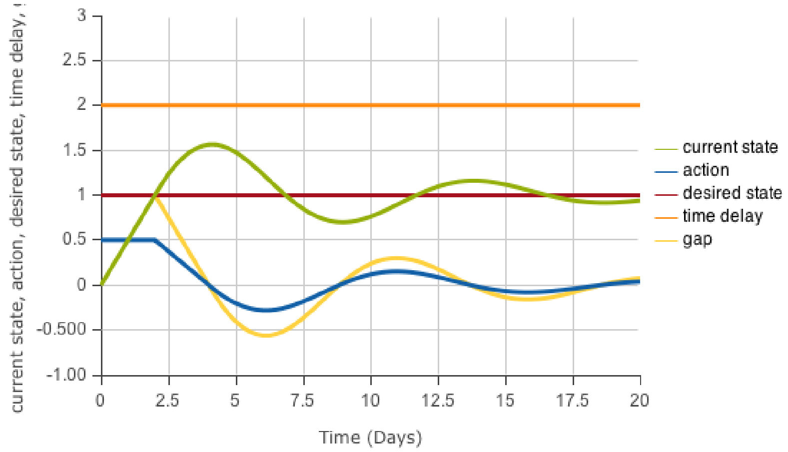
8. Change the **Equation** property of the primitive [**time delay**] to `0.25`.

9. Run the model. Here are sample results:

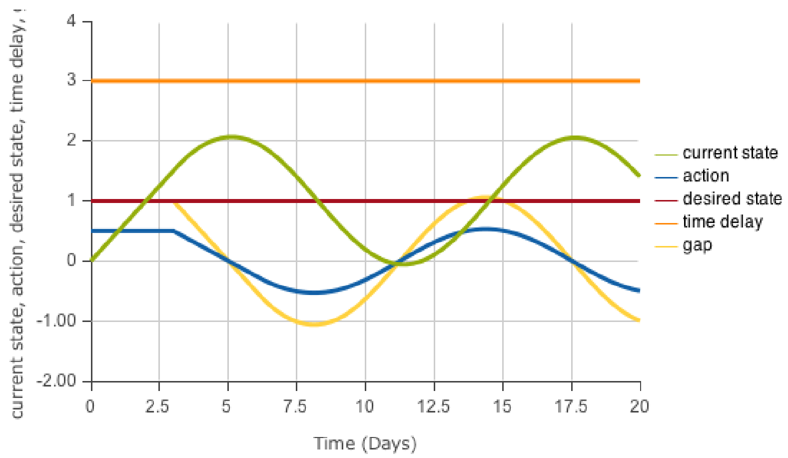


10. Notice that now with a delay the change in the gap and action are delayed for one time period and then the current state actually overshoots the goal and a negative action is required to bring the current state back to the goal.

11. Change the **Equation** property of the primitive [**time delay**] to `2`.
12. Run the model. Here are sample results:

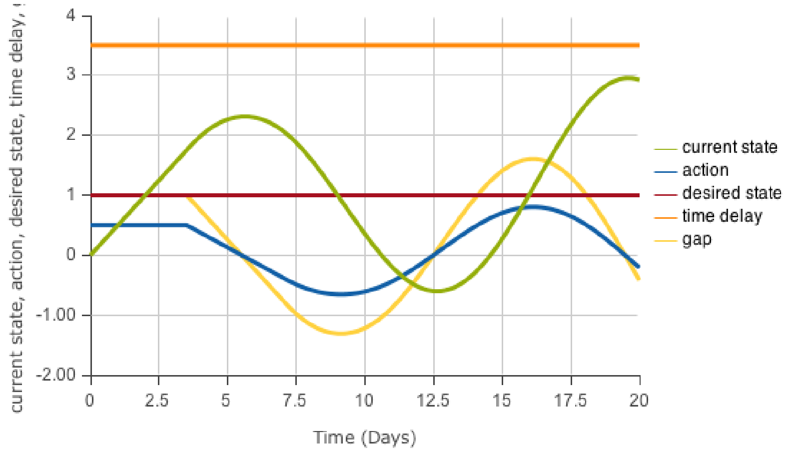


13. With a longer time delay the overshoot is even more severe though after a few time periods the current state actually will reach the goal.
14. Change the **Equation** property of the primitive [**time delay**] to `3`.
15. Run the model. Here are sample results:



16. We've now reached a delay where the action is so out of sync with an awareness of the results that the goal is never reached and the current state continually oscillates around the goal.
17. Change the **Equation** property of the primitive [**time delay**] to `3.5`.

18. Run the model. Here are sample results:



19. Now the situation is described as being out of control because rather oscillations continue to get worse because of the length of the time delay.

[Balancing Loop with Delay Stock & Flow Simulation]

You might ask how could it be that it might take 3.5 days for someone to get a sense of what the results of the previous actions were, which would be a good question. It's probably difficult to find a situation where this is realistic in days though what's important to realize is this structure could operate in this manner if the time units were hours, minutes, seconds or microseconds.

Exercise 5-2

Run this model varying the values of action factor, time delay and time step to develop a sense of how each of these variables influences the behavior of the model.

Examples

- **Adjusting the Shower**. When you adjust the hot and cold water for the shower it takes time for the new mixing ratio to actually be felt in the water temperature to it's easy to over compensate due to impatience.

Effective Strategies

- Advice for dealing with this structure is quite simple. Patience is a virtue. If you know you're dealing with a balancing structure and things are not going as expected then study the structure to see if there could be one or more delays that your impatience is simply having difficulty dealing with. This structure proves that there are times when taking additional action is worse than not taking additional action. More is not always better. If things are waffling back and forth endlessly or out of control a little less effort might be appropriate.
- An alternative is to monitor the Current State on a more frequent basis and ensure the result of the monitoring impacts the action appropriately in a more timely manner. In short, take the delay out of the structure.

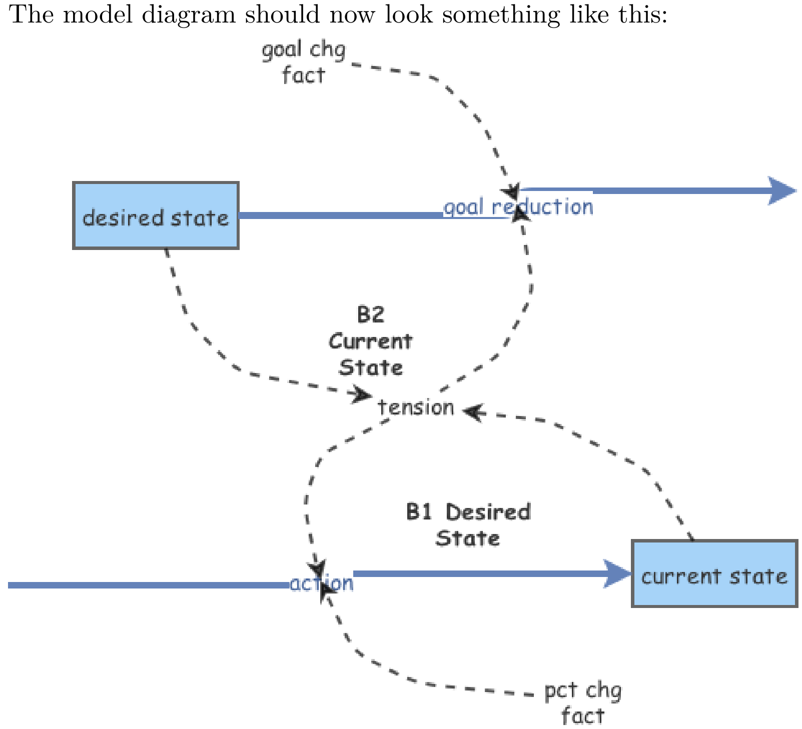
Over Time There Is A Tendency To Settle For Less

Have you ever noticed how difficult it is to bring the best of intentions to fruition? How so many people's New Years resolutions only last a few days? Our inability to achieve the things we set out to achieve is very much a result of the operation of a Drifting Goals structure we generally have little awareness of.

Drifting Goals

If it takes an extended period of time to achieve a goal there is a tendency to settle for achieving a lesser goal.

1. The model diagram should now look something like this:

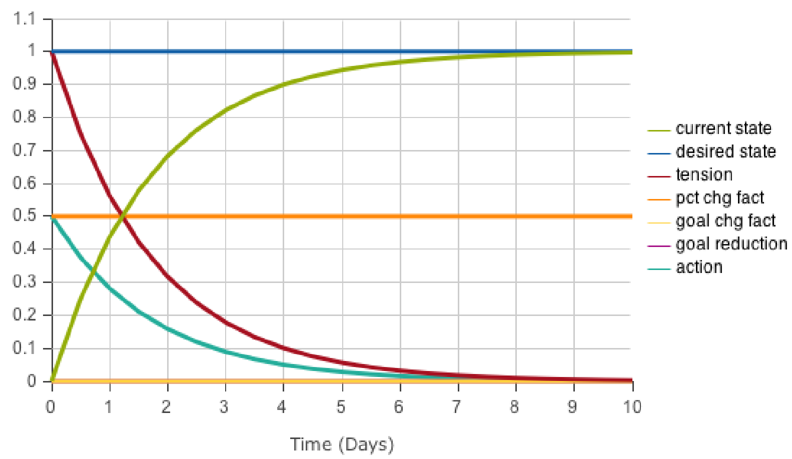


2. Again we have two balancing loops which each provide a goal for the other and because of the delays in the time it takes for the action to produce results one goal overrides the other. What I Want server as the goal for B1 and Pressure to Settle for Less serves as the goal for B2.

3. The initial settings are: [**desried state**] = 1, [**current state**] = 0, [**pct chg fact**] = 0.2, [**Time Step**] = 0.5.

4. Change the **Equation** property of the primitive [**goal chg fact**] to `0`.

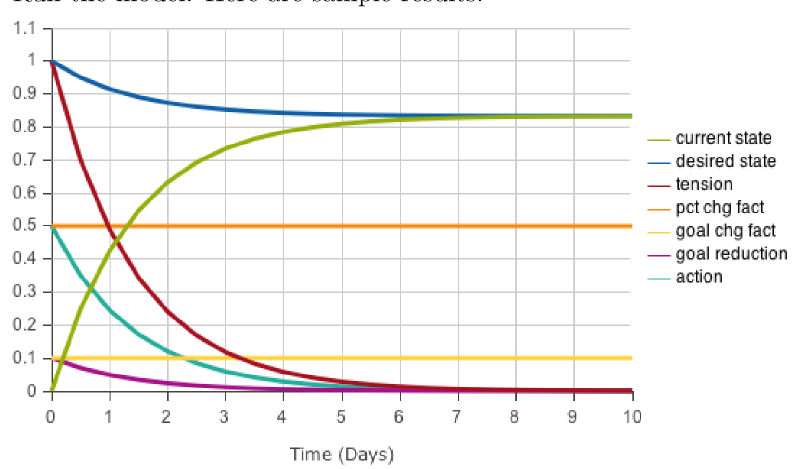
5. Run the model. Here are sample results:



6. With [**goal chg fact**] = 0 the results are those of the simple balancing loop.

7. Change the **Equation** property of the primitive [**goal chg fact**] to `0.1`.

8. Run the model. Here are sample results:



9. In this example with the extent to which the goal drifted is about 30% which is very significant. The extent to which the goal drifts is very dependent on the [**pct chg fact**] and the [**goal chg fact**] variable.

Exercise 5-3

Vary the [**pct chg fact**], [**goal chg fact**] and [**Time Step**] values to get a sense of the impact on the extent to which the goal for the structure is degraded over time.

Examples

- New years resolutions. Need I say more?
- Weight loss programs.

Effective Strategies

- There is only one real effective way to deal with this structure, which is to disconnect the feedback from tension to goal reduction to Desired State so it can no longer subtract from Desired State.
- An alternative strategy is to further increase the action toward the Current State so it reduces the time delay such that there is no time for the tension to reduce the Desired State. This is fine if there are sufficient resources to increase the action.

Areas of Concern

- The action toward the Desired State requires resources, which may have to be developed. Consideration needs to be given as to whether or not there really are sufficient resources to achieve the Desired State. For further insights into this see Growth and Underinvestment with a Drifting Standard.

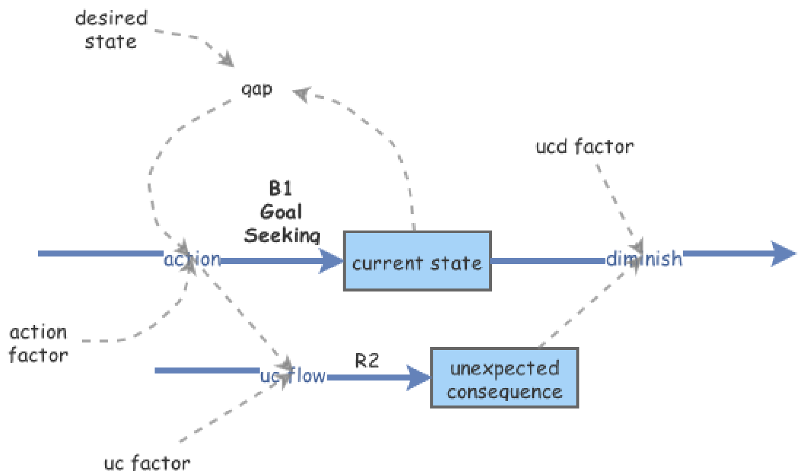
In Time The Problem Returns

Have you noticed how often your best intentions go awry? You set out to fix a problem and shortly thereafter you find yourself fixing the same problem again, and again. This generally results from some unexpected consequences, things that come into play because of your action, or the results of your action, that you never expected, which is why they're called unexpected consequences.

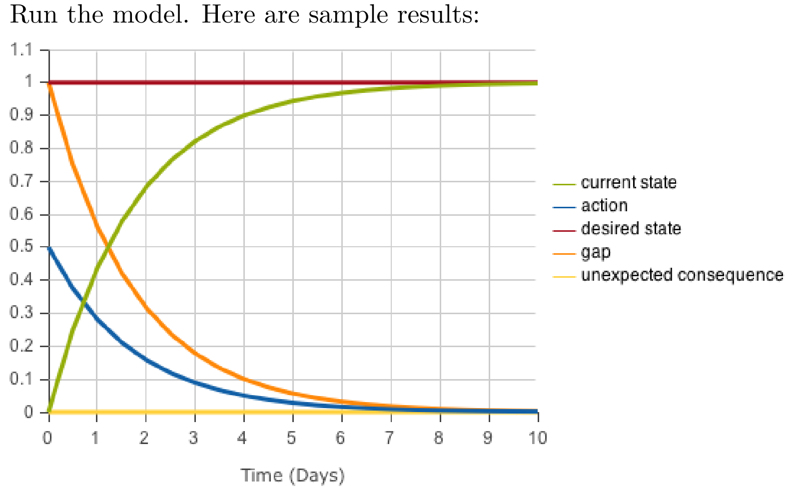
Fixes that Fail

This structure consists of a balancing loop intended to achieve a particular result which is foiled by an insidious reinforcing loop.

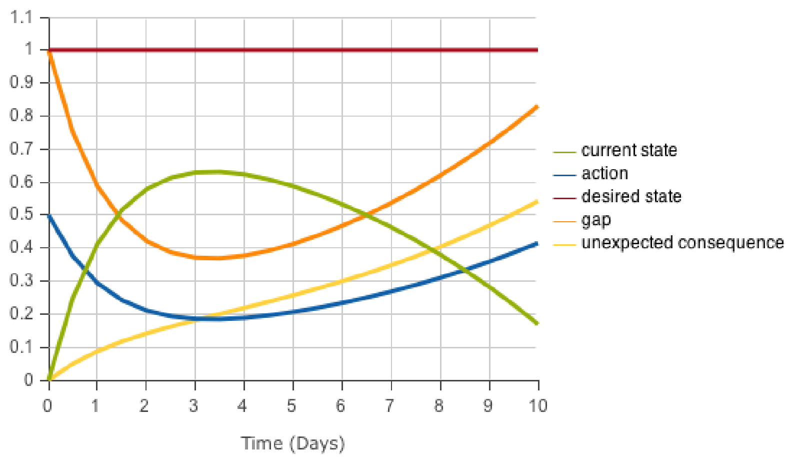
1. The model diagram should now look something like this:



2. The action of the Goal Seeking loop (B1) also influences, after some delay, unexpected consequences which diminishes the migration of the current sate in the direction of the desired state and creates the reinforcing loop (R2).
3. We begin with initial settings of [**desired state**] = 1, [**current state**] = 0, [**action factor**] = 0.5, [**ucd factor**] = 1 and [**Time Step**] = .5
4. Change the **Equation** property of the primitive [**uc factor**] to `0`.
5. Run the model. Here are sample results:



6. With [**uc factor**] = 0 there is no unexpected consequences and therefor the value of [**ucd factor**] is irrelevant and the structure is essentially a standard goal seeking balancing loop (B1).
7. Change the **Equation** property of the primitive [**uc factor**] to `0.2`.
8. Run the model. Here are sample results:



9. As the value of uc factor increases unexpected consequences will increase more rapidly and have a more marked impact on the current state as long as the value of ucd factor remains constant.

Exercise 5-4

Run the Fixes that Fail structure with various values of action factor, uc factor, ucd factor, and Time Step to get a sense of how these four factors influence the behavior of the structure.

Examples

- Your soccer ball is soft so you put air in it though in a few hours you have to put more air in it. And after a few weeks it seems like you spend all your time pumping up your soccer ball.
- Often times what appears to be the most appropriate way to deal with the situation doesn't really solve the problem and in time actually makes the situation worse.

- To deal with a cash shortage one often borrows money ensuring there will be more cash problems in the future.
- In response to cash flow problems companies often choose to layoff employees essentially ensuring they will have more cash flow problems in the future.

Effective Strategies

- The most effective strategy for dealing with this structure is advance planning. Since you can never do just one thing, as everything affects everything else, before taking action to change the current state, think about what else that action, or change in the current state, is likely to affect. And, what effect the effect will have. Sometimes the unexpected consequence may be several effects away, so don't stop at just one. Essentially what one seeks to do is close the loop and identify the unexpected, which means it's no longer unexpected then, is it?
- A less effective strategy would be to figure out how to disconnect the unexpected consequence from influencing the action or the current state. Of course then it wouldn't be a consequence, would it?

Areas of Concern

- There are times when attempts to deal with a situation in a particular way makes it even more difficult to deal with the situation in an appropriate manner later on which is often an indication of a Shifting the Burden Systems Archetype.

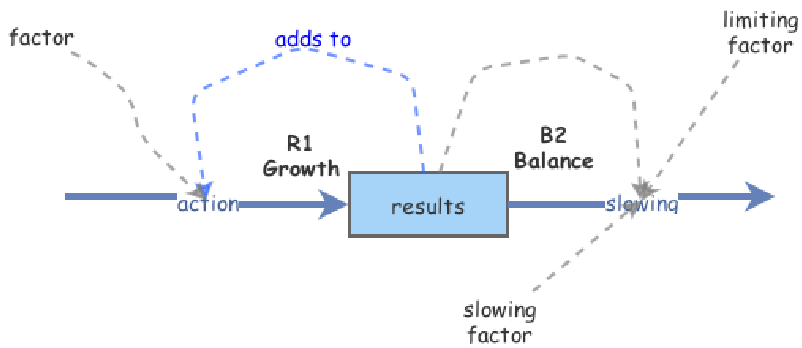
Growth Slows Over Time

A Limits to Growth Systems Archetype consists of a Reinforcing Loop, the growth of which, after some success, is offset by the action of a Balancing Loop. As such it may produce exponential growth for a period of time before the growth slows.

Limits to Growth

The natural exponential growth of the reinforcing structure is restricted by a balancing loop.

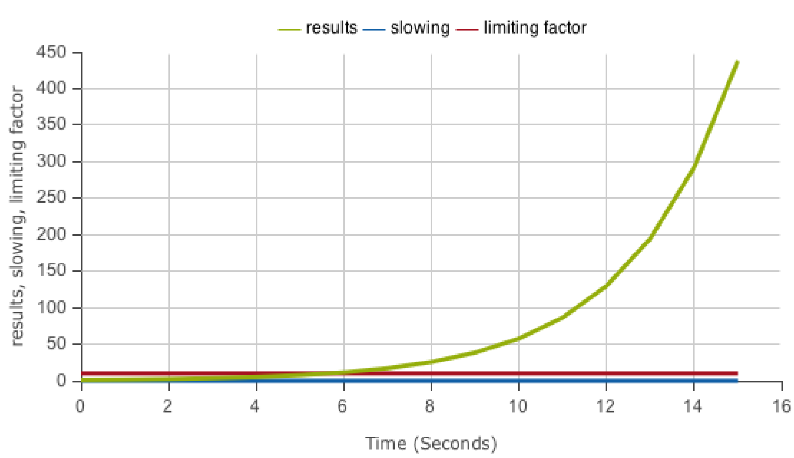
1. The model diagram should now look something like this:



2. The structure is such that the action produces results which add to the action, the typical reinforcing structure. factor is just a value between 0 and 1 to allowing the variance of the rate of interaction. The balancing structure B2 is such that the slowing action drains from the results once the results actually reach the level of the limiting factor. The slowing factor is similar to factor in that allows for control of the rate of interaction.

3. [**slowing**] = IfThenElse([**results**] > [**limiting factor**], ([**results**] - [**limiting factor**]) * [**slowing factor**], 0)

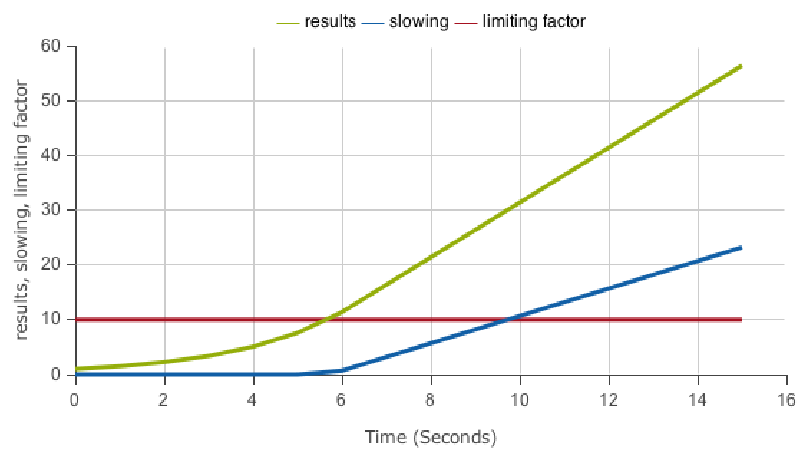
4. Run the model. Here are sample results:



5. With a [**slowing factor**] = 0 the balancing loop is disabled and the typical exponential growth curve is displayed.

6. Change the **Equation** property of the primitive [**slowing factor**] to `0.5`.

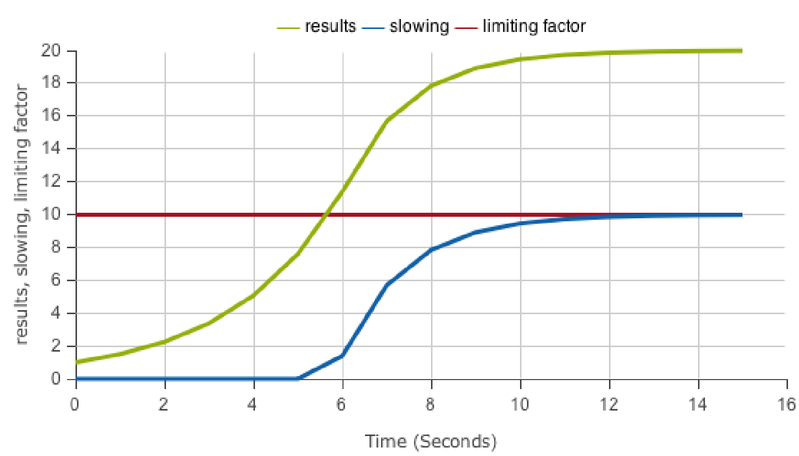
7. Run the model. Here are sample results:



8. With a [**slowing factor**] = 0.5 and a [**limiting factor**] = 10 it is evident that the [**results**] are significantly diminished from the previous results.

9. Change the **Equation** property of the primitive [**slowing factor**] to `1.0`.

10. Run the model. Here are sample results:



11. Now it's quite evident that the slowing action has actually affected the result to such an extent that it's no longer growing.

Exercise 5-5

The structure provided for Limits to Growth limits growth via a balancing loop that attempts to move [results] in the direction of the [limiting factor]. How else might you construct a limits to growth structure.

Answer Available

Examples

- Rabbits tend to multiply very rapidly so why is it we're not completely overrun by rabbits, well maybe everywhere except Australia?
- Keep playing instead of cleaning up the mess in the room, which makes further play difficult, AND the increased mess repels one from cleaning up.
- An epidemic begins with a infected person

Effective Strategies

- The best defense is a good offense. As defined in the effective strategies for the Reinforcing Loop, if there is a Reinforcing Loop operating start looking for what is going to become a limiting factor, and remove it before it even has a chance to create a substantial impact on the result.
- If the structure is already at a stage where the limiting factor is interacting with results to limit results the options are:
- Alter the limiting factor in such a way that it no longer interacts with the results to create a slowing action.
- Find a way to disconnect the results from the slowing action so it can no longer add to it.
- Disconnect the slowing action from the results so it can no longer have a negative impact.

Areas of Concern

- There are often multiple limits to deal with which leads to an Attractiveness Principle.
- It is possible that limited shared resources are the source of the limiting factor leading to a Tragedy of the Commons.
- The limit may be insufficient capacity which leads to Growth and Underinvestment.

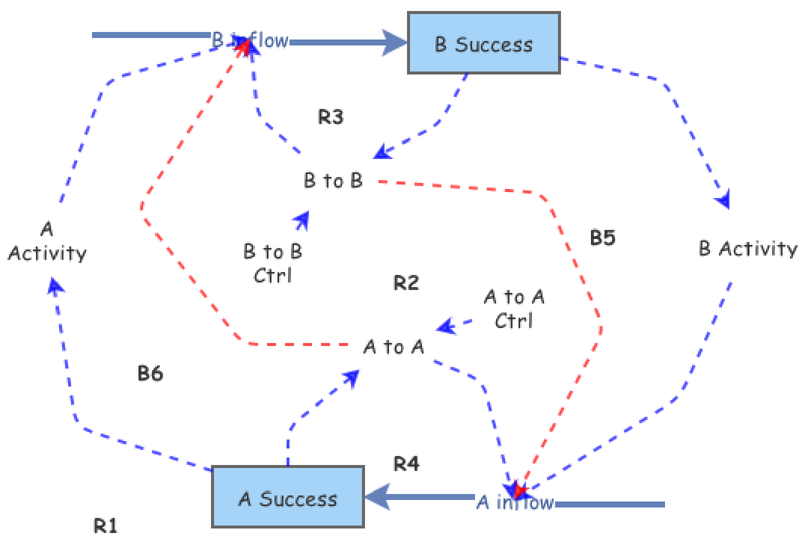
Partners for Growth Become Adversaries

The Accidental Adversaries structure represents a situation where two entities which would be in a synergistic growth relationship end up limiting each others results because of their own activities.

. The problem arises when either A or B does something that promotes their own success which also acts to inhibit the success of the other. The other responds with an attempt to compensate for whatever is inhibiting their success though their action tends to inhibit the success of the other. As such the two entities that should be cooperating for their mutual success become adversaries.

Accidental Adversaries

1. The model diagram should now look something like this:



2. A Activity contributes to B Success and B Activity contributes to A Success and we have a reinforcing structure R1

3. Initial settings of [**A Success**] = 1, [**B Success**] = 1, [**A to A Ctrl**] = 0, and [**B t B Ctrl**] = 0 and [**Time Step**] = 1.

4. Run the model. Here are sample results:

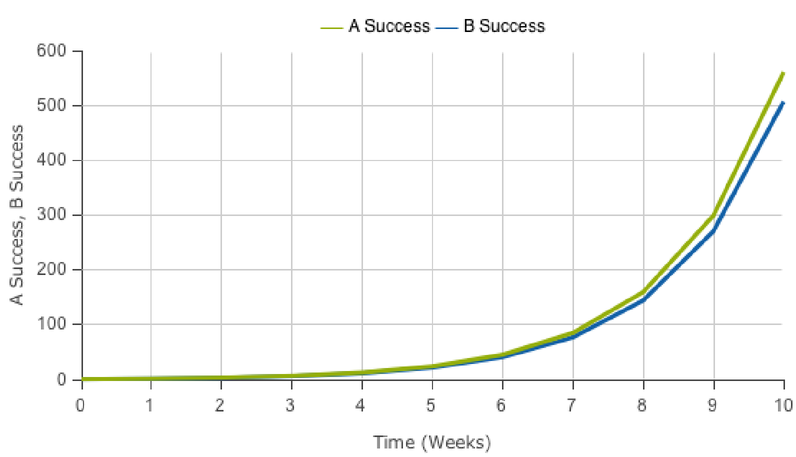
5. With the initial setting the inner loops are essentially turned off so the results are simply the result of the outer reinforcing loop R1.

6. Now because A likes it's success it begins acting to promote it's own success. The difficulty is that these activities by A detract from B's success so B responds to make corrections by promoting it's own success.

7. Change the **Equation** property of the primitive [**A to A Ctrl**] to `0.3`.

8. Change the **Equation** property of the primitive [**B to B Ctrl**] to `0.2`.

9. Run the model. Here are sample results:



10. Here we can see that the total results of A and B are actually less

than they would have been were they cooperating. With A acting to promote it own success its actions diminish B Success and B will then expend energy attempting to compensate for the action by A. And then A will act to compensate for the actions by B which tend to inhibit the success of B. Thus we have a vicious cycle where the two were far better off cooperating.

Exercise 5-6

Why is it that the results balance whenever B to B Ctrl = A to A Ctrl?

Answer Available

Examples

- Parents can be interesting. Both Mums and Dads want to act as good role models and create a positive family environment, but sometimes, each one of them gets caught up in being seen as ‘’the good parent" and gives in to our whims and desires. We all know how this works: If one parent says ’no’ to something, we just ask the other parent. The problem is that when one or both of our parents wants to be seen as the ‘’good parent,’‘the other one ends up being seen as the ’bad’ or ‘tough’ parent, and the whole ‘good role model’ or ‘good family environment’ disintegrates pretty quickly.
- Sales and service acting promoting their own success over promoting each others success.
- Any two individuals who could cooperatively support each others success though who choose to compete with each other actually to the detriments of each other and themselves.

Effective Strategies

This structure points out how myopic local activity, with the best of intentions, can lead to an overall limiting development of the global system, and actually inhibit local development as well. - A and B need to determine whether it is really better to be partners in creating the future or competitors, and do one or the other, not both. At present A and B are neither as they undermine each others success to promote their own success. Sounds like enemies to me. - Alternatively, some higher authority could alter the structure in such a way that A to A and B to B didn’t promote the individual result of A and B. - Another alternative would be to alter the structure in such a way that the result is not measured in terms of A and B individually but in terms of the total result of the two of them together. In this way it should be quite quickly evident that each is undermining their own success through their self-serving actions.

Areas of Concern

- There are currently no known Systems Archetypes that are derivatives of the Accidental Adversaries Systems Archetype.

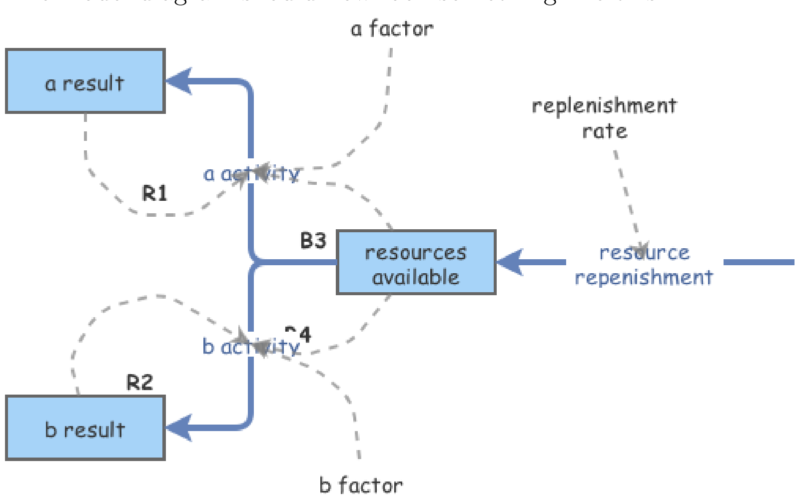
Limited Resources are Shared by Others

A Tragedy of the Commons situation exists whenever two or more activities, each, which in order to produce results, rely on a shared limited resource. Results for these activities continue to develop as long as their use of the limited resource doesn't exceed the resource limit. Once this limit is reached the results produced by each activity are limited to the level at which the resource is replenished. As an example, consider multiple departments with an organization using IT resources, until they've exhausted IT capacity.

Tragedy of the Commons

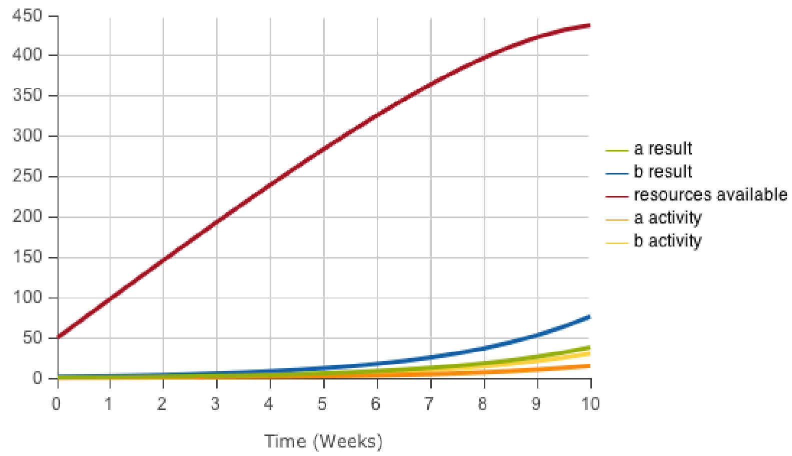
The following model will compare two levels of resource usage for a given set of available resources.

1. The model diagram should now look something like this:



2. Initial settings are [**a result**] = 1, [**b result**] = 2, [**resources available**] = 50, [**a factor**] = 0.4, [**b factor**] = 0.4, [**replenishment rater**] = 50

3. Run the model. Here are sample results:



4. With these settings growth in [**a result**] and [**b result**] ensures that the total activity will exceed the carrying capacity of the resources.

5. Let's see what happens with an increased resource utilization, which is likely as both the [**a result**] and [**b result**] promote more of the same.

6. Change the **Equation** property of the primitive [**b factor**] to `0.8`.

7. Change the **Equation** property of the primitive [**b factor**] to `0.8`.

8. RESULT

9. Notice that both A and B demonstrate exponential behavior until they diminish resources to the replenishment rate. At that time they them begin to demonstrate linear growth as they are limited by the replenishment rate. B result begins greater than A result and grows even faster than A because it consumes more resources.

Exercise 5-7

- What happens if either A or B decides at some point to reduce their consumption of resources?

Examples

- In an organization whenever different departments use resources that support their success though which they don't have to pay for guarantees their resource demands will exceed the supply.
- Over fishing has ruined many of the offshore fishing areas simply because doing more of the same served those doing it until such time as they exceeded the carrying capacity of the system.

Effective Strategies

This structure repeatedly appears in organizational contexts where a service organization supports the success of multiple departments who fail to support the service organization in return. There are two strategies for dealing with this structure, one more effective than the other. - The most effective strategy for dealing with this structure is to wire in feedback paths from A's result and B's result to the resource replenishment so as A and B use resources their results promotes the availability of additional resources. - The alternate, and less effective, strategy for dealing with this structure is to add an additional resource to control the use of resources by A and B. This strategy limits the overall potential results of the structure to the predefined resource limit. It also adds additional resources to the equation, and probably results in endless disputes as to the fairness associated with the allocation of resources. While not really the most appropriate strategy this is the one most often used — out of ignorance I would suspect. - More ecological approach would be for A and B to realize what they're doing and collaboratively manage their activity and the resource such that it isn't depleted. Yes, it could happen.

Summary

The chapter should explicitly depict the relationships between the structures presented in the previous sections and explain the natural evolution paths for the structures.

Systems Archetypes Relationships

References

- Bellinger, Gene. 2013. Systems Archetypes. http://www.systemswiki.org/index.php?title=Systems_Archetypes
- Braun, Bill. 2002. The Systems Archetypes. http://wwwu.uni-klu.ac.at/gossimit/pap/sd/wb_sysarch.pdf

Chapter 6

Applied Understanding

To be filled in.

Chapter 7

Models and Truth

All models are wrong, but some are useful – George E.P. Box

A model is a tool designed to reflect reality. A model is never a perfect mirror of reality, but often models can still be useful even given their imperfections. In this chapter, we will take a journey exploring different types of models and distinctions that are commonly used to classify and understand them. We will consider several approaches to modeling that are quite different from the ones we have introduced throughout this book. These will help to understand the richer ecosystem of modeling tools and techniques that exist and how the ones we have learned fit within this ecosystem.

The ultimate destination of this exploration will be a clear understanding of the fundamental principles and approaches used to construct models. There will be many detours that we must make to arrive at this destination, but in the end we will be able to divide models into two overarching categories based on their purposes and the techniques used to construct them. By mastery this divide, and how the work we and others do fits into it, we will obtain a rich perspective and understanding of the relationship between models and truth. We will also have a renewed appreciation for the strength and power of the techniques introduced in this book for tackling a wide swath of modeling problems.

Before we get there, however, let's introduce some of the terminology that is commonly used to describe models. It is useful to take a step back and first discuss different kinds of models. Modeling is a wide-ranging field with many distinctions made by modelers and mathematicians. Three of these distinctions are presented below:

Deterministic versus Stochastic Models

There are two polar opposite views of the world. One view says the fate of the universe is governed by strictly predictable laws of physics. In this view, the universe acts as if it were a giant machine, where if its current state is known (down to each individual atomic particle), its future states through the rest of time are predetermined. The opposite view is that the universe is ruled by chance and randomness. Random quantum mechanical fluctuations merge and amplify leading to an infinite range of diverging possibilities.

Which of these two views holds more of the truth? We certainly do not know and it is possible that this will be a question that physicists will never cease exploring. Albert Einstein had a viewpoint of special interest, however. He was a strong partisan of the more deterministic view, famously remarking, "God does not play dice with the world."

When creating a model of a system, we must choose how we treat chance. Do we build our model deterministically, such that each time we run it we obtain the same results? Or do we instead incorporate elements of uncertainty so that each time the model is run we may see a different trajectory of outcomes?

Mechanistic versus Statistical Models[1]

When beginning to build a model of a system, there are many questions you should ask, two of which are:

1. Do I know (or have a hypothesis of) the mechanisms that drive the system?
2. Do I have data that describe the observed behavior of the system?

If the first question is answered in the affirmative, you can build a mechanistic model that replicates your understanding (or hypothesis of) the true mechanisms in the system. If, on the other hand, the second question is answered in the affirmative, you can use statistical algorithms such as regressions to create a model of the system based purely on the data.

If neither question is answered affirmatively… well in that case there isn't much of anything you can build.

Exercise 7-1

A credit card company has hired you to build a model to predict defaults of new applicants. They give you a data set containing information on one million of their customers along with whether or not the customer defaulted.

[1] This relates, more broadly, to the contrasting research approaches of induction and deduction. Induction starts with data and observations which are analyzed to create a broader theory (similar to a statistical approach to modeling). Deduction starts with a theory and finishes with the collection of data to confirm the theory (similar to a more mechanistic approach to modeling). It is easy to become mixed up with the meanings of induction and deduction and even great minds have confused them. While Sir Arthur Conan Doyle's character Sherlock Holmes attributes his impressive powers to "deduction", he is actually using induction. Treating what we are calling "statistical" models here as a form of induction, we can also refer to them as "phenomenological" or "empirical" models.

Would it be better to build a mechanistic or statistical model for this data?

Answer Available

Exercise 7-2

You have been commissioned to build a model of population growth for a herd of Zebra in Namibia. You have some data on the historical size of the population of Zebras but this data is limited. You also have access to over a dozen experts who have studied Zebras their whole life and have an intimate understanding of the behavior of the Zebras.

Answer Available

Aggregated versus Disaggregated

When building a model, the issue of scale becomes very important. Imagine we are concerned about the effects of Global Climate Change on water resources. We may wish to examine the question of whether there will be sufficient water supplies given a rise in future temperatures.

At what scale do we build this model? The range of possible scales is wide:

- At the most aggregate, we could estimate total worldwide water demands and supplies into the future.
- Maybe that is too coarse a scale, however, as clearly having excess water in Norway has little direct impact on the situation in Egypt. We could instead create a finer resolution model that separately looked at water demand and consumption in each country.
- Even that may still be too coarse, maybe we should make our model more granular to look at specific cities or population clusters around the globe.
- At the extreme disaggregated level, we might even want to model individual people – all 7 billion of them – and their needs and movements around the world.

There is no simple answer to this question of optimal scale. The best choice is highly context-sensitive and depends on the needs of the specific modeler and application.

Exercise 7-3

You have been hired to build a model of world population growth. What is an appropriate level of aggregation/disaggregation for this model? Does your answer change if you very the time scale? What would be the differences between a model designed to work 10 years in the future, one designed to work for 100 years, and one designed to work for a 1,000 years?

Exercise 7-4

Your company builds rulers. You have been asked to develop a model of global demand for rulers. What is an appropriate level of aggregation/disaggregation for this model?

Prediction, Inference and Narrative

The three distinctions just presented – deterministic vs. stochastic, mechanistic vs. statistical, aggregated vs. disaggregated – can be used to classify models. We can even use them to classify the models we have discussed in this book. Most of our models would be classified as deterministic (random chance is generally not explicitly incorporated in these models), mechanistic (we generally assume mechanisms rather than estimating dependencies from data), and highly aggregated (the agent based models are an exception).

There are many nuances to these broad distinctions that can also be made (e.g. the type of statistical techniques used for a statistical model) and there are also many other distinctions that can be made between model implementations such as the programming language or software that was used to implement the model. These distinctions and technical choices are important when constructing a model, however what is of key importance is the utility of the model for fulfilling a specific goal.

Technical details matter – they can affect maintainability and other factors – but they are of secondary interest to the adequacy of a model in fulfilling it main purpose. It would make as little sense to say a model was fundamentally bad because it was written in a relatively ancient programming language – like Pascal – as it would be to say a model was fundamentally bad because it was, for instance, deterministic. Let's look back at Box's quote at the beginning of this chapter. We know all models are wrong, what we should really care about is their utility in meeting a specific task.

So instead of using the aforementioned technical classifications to discuss models, we think it is more useful to base our discussions of models on the model's driving purpose. This allows us to leave behind relatively mundane technical and implementation details to focus on what we really care about. Among the many different reasons for building models, they all boil down basically to three broad purposes: prediction, inference and narrative.

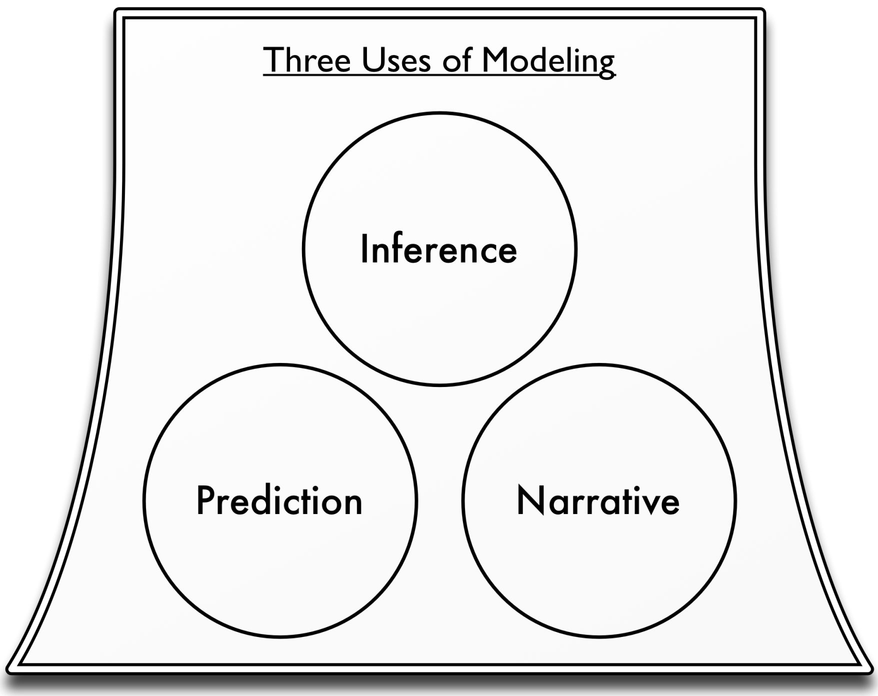


Figure 1. Three Usages of Models

Prediction : Models used for prediction are the most straightforward. They attempt to forecast some outcome given information about variables that may influence that outcome. A weather forecast is an example of a model used for prediction. Likewise, when you apply for a credit card at a bank, they run a predictive model to determine your risk of not paying them back what you owe and defaulting. When you apply for life insurance, the company has a model that predicts how long they think you will live in order to determine how much they should charge you. All these models take in data (the current temperature for the weather forecast, the amount of money in your bank account for your risk of default, your age for the life insurance application) and apply various forms of analysis to generate a prediction of the outcome.

Inference : Models used for inference are most common in academic research. Often, academic research questions distills down to this simple template: "Does X affect Y?" These are inferential questions[2]. As an example, a researcher may make a hypothesis statement such as, "The wealthier a high-school student's family is, then the higher the student's test scores will be." The researcher may then build a model to test the validity of this hypothesis and the model's results will generally be phrased in terms of a p value indicating the statistical significance of the evidence in support of the hypothesis.

Narrative : Models are often used to tell a persuasive story. When the Obama administration wanted to persuade lawmakers and the public to support their economic stimulus, they famously published the graph shown in Figure 2. A great deal of complex modeling and mathematics surely went into constructing this figure. However its core purpose was to tell the nation a story: Things are going to be bad, but the recovery plan will make them less so. Such stories are at the heart of narrative models and we will return to this figure later on and why it is not really a predictive model despite it generating predictions.

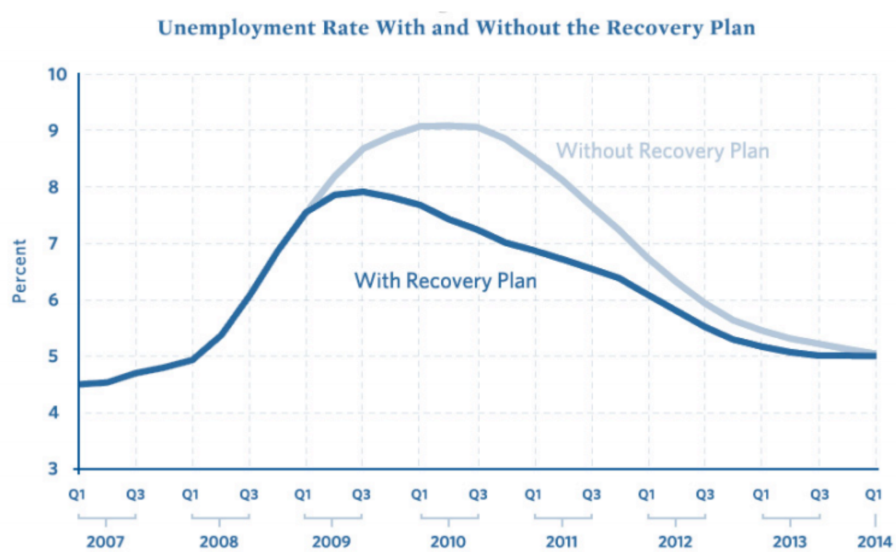


Figure 2. The Obama Administration's Predictions for the Effects of the Recovery Plan (Romer and Bernstein 2009)

All models can be classified in terms of these three primary purposes and we will see how useful it is to discuss modeling projects in terms of them[3].

[2]Predictions are also inferential results, but we prefer to discuss prediction and more hypothesis-testing types of inference separately. This distinction makes our understanding of modeling clearer.

[3]And we strongly recommend doing so. It is important to clearly define the purpose at the start of a project. The techniques used and data required depend significantly on the

Exercise 7-5

Classify each of these modeling tasks as either primary prediction, inference, or narrative tasks:

1. A model to determine the average ocean temperature in 2020.
2. A model to determine whether deforestation affects temperatures.
3. A model to determine whether a company should supply a credit card to a specific applicant.
4. A model to help students understand the risks of global climate change.
5. A model to convince your manager to green-light your new initiative.
6. A model to assess whether nutrition has an effect of infant mortality.

Answer Available

The Strange Case of Inference

To help us get at this fundamental classification scheme, let's first talk for a moment about the process of inference. Take the earlier example of determining whether wealth results in increased high-school test scores. We phrased this hypothesis in a specific way: that increased wealth will always increase test scores. This illustrative statement, however, actually differs from what is often done in practice. In general, researchers simply asks the question "Does X affect Y?" rather than "Does X increase Y?" It's just a slight difference, but it is a more flexible question that allows for many forms of relationships. For our example, we would ask the question "Does wealth affect tests scores?"

The gold standard to answering questions like this is the controlled experiment. Controlled experiences allow you to develop strong inferences as you can see how a system responds when you hold all variables constant except for the single one you are interested in. For our example, we could imagine an experiment where we took a sample of a thousand families from a school district. When these families' children enter high school we would randomly select them to be in a "poor" category and the other half to be in a "rich" category. Families in the rich category are given grants of $500,000 a year to spend how they wish while the parents in the poor category are fired from their jobs and have their savings frozen for the duration of the experiment. Once the students graduate from high school, we would compare the scores for the students in the poor and rich categories.

---

model's overall purpose. To be very clear, it is important to clarify at the outset whether your primary goal is to use a model for prediction or for narrative. Many modeling projects may attempt to do both only to find themselves with a model that does neither.

These controlled randomized experiments are considered the ideal approach to answering inferential questions like these as they allow you to truly determine the effect of your variables, in this case wealth. For many types of questions, such experiments can be implemented (for instance does consumption of a new drug help treat a disease). Unfortunately, in general complex social questions are simply impossible to answer with them. We can consider the testing procedure we just imagined to assess the effect of wealth on scores, but it would be impossible and unethical to undertake in a real community. Furthermore, even if you were to implement the experiment as described, the behavior of a family that was poor or wealthy to begin with might very well differ from a family that experiences a sudden change in income.

Traditional Model Based Inference

Given our general inability to undertake the ideal controlled experiment, how do we answer inferential questions? The standard way is to collect data and then construct a model enabling us to measure the statistical significance of our hypothesis given the data. Due to history and simplicity, linear regression models are by far the most commonly used type of model today. A linear regression predicts an outcome (Y) based on the multiplication of variables (X's) by a set of coefficients determining the effect of the variables on the outcome (β's):

$$Y = \beta_0 + \beta_1 \times X_1 + \beta_2 \times X_2 ...$$

For the education example we could collect data on a number of students, measuring their families' wealth (X_1 in the equation above) and the student's test scores (Y). We would then run the linear regression to determine the coefficient values (β_0 – the intercept – and β_1 – the effect of wealth on test scores). If we thought there were other factors that affected test scores, we could measure them and include them as addition X's in the regression.

In addition to obtaining the values of these coefficients, we also obtain as a result from the regression the statistical significances or "p values" of these coefficients. Although p values are commonly used in statistics, they are ubiquitously misunderstood[4] so it is useful to briefly review them.

In short a p value measures the probability of seeing the measured data (or more extreme data) assuming the null hypothesis is true. Generally the null hypothesis will be that there is no relationship between the variables and the outcomes.

When assessing the significance of coefficients, a p value means the probability of seeing that value of a coefficient (or one even further from 0), assuming that

[4] These misunderstandings are not only made by on-the-ground practitioners and analysts, they are frequently shared, and propagated, by university-level statistics instructors; see, for instance, Haller and Krauss (2002).

the (unknown) truth is that the coefficient actually has a value of 0. In other words, it is the probability of seeing the observed non-zero value, assuming that the true value is in fact 0. Frequently, probabilities of 10%, 5% or 1% or smaller are taken as indicating statistical significance. These low values indicate that the coefficient value is so far from 0, and the probability of this occurring by chance so small, that we can reject the null hypothesis and accept the fact that the coefficient is not 0.

Using the p values enables inference by relying on the statistical significance of the coefficients. If the probability of β_1 (the coefficient for the effect of wealth) occurring due to chance (given it is 0 in reality) is less than, say 5%, we can claim with reasonable strength that wealth does in fact affect test scores. This is the standard approach researchers take to model-based inference and is used ubiquitously.

A Troubled Sea of Assumptions

Let's stop for a second and consider what we have done here. In carrying out these logical steps to apply model based inference to determine whether wealth affects test scores, we have had to make one very large assumption: that the relationship between test scores and wealth is linear.

Our linear regression equation assumes that for every increase in one unit of wealth (X_1), test scores (Y) will increase on average by the amount of the coefficient (β_1). What if this were not in fact the truth? For instance, we could easily imagine the case where wealth initially helped test scores by providing students more resources and opportunities to learn. After a certain point, however, wealth might negatively impact scores as very wealthy students might lack the pressure or motivation to study hard.

If we believed this were the case, then our linear regression model from earlier would be wrong as would the inferences we obtained from the model. We could correct our model and inferences by changing our regression formula to contain a squared term that could replicate this type of relationship:

$$Score = \beta_0 + \beta_1 \times \text{Wealth} + \beta_2 \times \text{Wealth}^2$$

Using this equation, at low values of wealth the $\beta_1 \times \text{Wealth}$ term will have the most effect on scores. Conversely, at high levels of wealth, the $\beta_2 \times \text{Wealth}^2$ term will have the most effect on scores. Thus by having a positive β_1 and a negative β_2 we can model wealth as having an initially beneficial and then detrimental effect. If our assumptions about the quadratic relationship are correct, then this model will yield accurate inferences. If they are wrong, our inferences will be wrong again.

What are we really doing when we assume regression forms like this? Now it might not be immediately obvious, but what we are in fact doing is telling a story.

Using our first equation, we are telling the story that as wealth increases test scores will almost always increase. Bill Gate's children will preform amazingly well here! Using the second equation we are telling a different story: As wealth increases test scores initially do as well but after a certain point increased wealth will hurt test scores. That picture isn't so rosy for the Bill Gates of the world!

And so we arrive at a key insight. By choosing our equations to tell a story, our inferences are in fact based on narrative modeling approaches. True, these inferences build upon numerous calculations and very advanced theoretical underpinnings, but ultimately what governs our conclusions and inferences are the stories or narratives we tell about our system. These are choices that we as narrators make and they not determined by an objective truth or reality.

Exercise 7-6

You are given the following linear regression model that predicts the growth rate of a tree (in meters per year):

$$\text{Growth Rate} = 3.2 + 0.013 \times \text{Mean Annual Temperature} + 0.021 \times \text{Annual Precipitation} - 2.3 \times \text{Moose Dens}$$

Take this mathematical model and convert it to a textual narrative.

Exercise 7-7

You are given the following linear regression model that predicts the demand for hats (in thousands of hats sold per day):

$$\text{Hat Demand} = 23.4 + 3.4 * (\text{Temperature in Celsius} - 22) - 1.2 \times \text{Wind Speed} - 0.21 \times \text{Unemployment Rate}$$

Take this mathematical model and convert it to a textual narrative.

Predictive Inference

Is there an alternative approach to inference that does not rely so heavily on narrative? Can we accomplish it without assuming the relationships between variables? The answer is yes. Although they are not often used, alternative prediction-based approaches to inference are available. In these approaches, rather than calculating statistical significances as a function of an assumed model, we calculate significances as a function of the simple question: "Does knowing X help us to predict Y?" This question is effectively identical to our earlier question – "Does X affect Y?" – but it is structured in an explicitly predictive manner. If the answer to the question is true, then we can say that there is a relationship between X and Y.

The techniques to accomplish prediction-based inference are much newer than classic techniques as linear regression as they rely upon extensive computing power and would not be possible without modern technology. One of these approaches is the *A3* method (XXX Citation) which uses resampling based algorithms to obtain estimates of predictive accuracy and statistical significance. *A3* focuses purely on predictive accuracy of a model to determine whether a variable is significant and often requires the automatic exploration of hundreds or thousands of competing models to find the one that best describes the data. The results of these analyses are inferences that are founded in the data of model fits only, not on subjective assumptions.

Predictive versus Narrative Modeling

As we can see, inferential techniques can be split into two categories: those based on narrative modeling methods and those based on predictive modeling methods. So – and this is a key advance – although there three categories of model purposes – prediction, inference, and narrative – there are only two fundamental approaches to constructing models: predictive modeling and narrative modeling.

This divide is not traditionally used in the modeling field, but it is truly at the heart of modeling. Understanding the distinction between these two types of modeling proves below to be much more valuable than mastering fine technical details. The choice of whether to build a predictive or a narrative model is a fundamental one that shapes every aspect of a model and determines its ultimate utility for a specific purpose. The following sections will describe these two types of models in more detail.

Predictive Models

How do we define a predictive model? The naive answer is that a predictive model is one that makes predictions. If a model generates predictions for a future outcome or a given scenario, than it must be a predictive model. By this definition, a weather forecast is a predictive model as were the Obama administration's unemployment predictions we saw earlier.

Unfortunately, this straightforward definition is useless. Worse than being useless, it is actually quite dangerous.

Let us propose a model for next year's unemployment figures in the United States:

> Generate a random number from 0 to 1. If the number is less than 0.1, unemployment will be 20%. If the number is greater than or equal to 0.1, unemployment will be 0%.

There, we have just constructed a model of unemployment. Furthermore, our model creates predictions. With just a few calculations we can forecast unemployment for the coming year. Isn't that convenient?

Of course, this model is a joke. It is useless in predicting unemployment. However, using the naive definition of what it means to a predictive model, it would be classified as one.

What makes this simple model, such a poor model for prediction purposes?

There are several answers. We might start by saying it is too *simple*. If we are really trying to predict unemployment we should incorporate the current economic state and trends into our model. If the economy is improving, unemployment will probably drop and vice versa. This is a valid point. Let's address it by proposing an "improved" model:

> Generate a random number from 0 to 1. If the number is less than the percentage change in GDP over the past year, unemployment will be 20% plus the current unemployment rate. If the number is greater than or equal to 0.1, unemployment will be the net change in the consumer price index over the past 8 years.

Is this a better model? Clearly, it is more complex than the previous one and it incorporates some relevant economic data and indicators. Equally as clear, however, is that it is also a joke far from being a useful model.

These toy economic models show that just generating predictions is not a helpful criterion to define a predictive model. They also show that complexity and the use of relevant data is not a valid criterion. So how do we specify a predictive model? The answer is straightforward:

> A predictive model is a model that not only creates predictions but also must contain an *accurate assessment of prediction error*.

Read that statement again. The key point is that the assessment of prediction error must be accurate, which is different from the accuracy of the predictions themselves. Of course, ideally the predictions will be accurate; however this is often not possible. Many systems are governed to a significant extent by chance and no model, no matter how good it is, will be able to create accurate predictions for the systems.

If you know the level of prediction error, you can instead contextualize poorly fitting models. You can determine how much to discount their predictions in your decision-making and analysis. Furthermore, and this is crucial, you can compare different predictive models. If your current model is insufficiently accurate, you can develop another one and objectively test it to determine whether it is better than the current model.

Without measures of predictive accuracy, discussing predictions or comparing models that create predictions is an almost nonsensical endeavor. Such discussions will be governed by political concerns and partisanship as there is no objective foundation on which to base them.

Our two proposed models to estimate unemployment are thus clearly not predictive as no estimate of predictive error has been established. We can apply same this requirement to Obama's employment predictions we saw earlier. When we first presented the model, we called it a narrative model, which might have been slightly perplexing as the model did generate predictions. However, using our above definition of a predictive model we can see also that it is in fact not a predictive model. The model contains no estimate of prediction error (and one is not available in the original report) so it simply cannot be considered to be predictive.

If accurate estimates of prediction error are available, you can directly compare the prediction errors between different models to select the one with the lowest error. We could estimate prediction errors for the two joke models we proposed here along with the Obama administration's model to find the one with the lowest error. We would hope that the one the Obama administration presented to Congress would be the most accurate. Before we test it however, we must not make the error of fallaciously accepting a model to be good based on who presented it to us or its complexity.

Why do we so rarely hear about the predictive accuracy of models? There are numerous reasons but they boil down to three basic ones:

1. Assessing prediction error accurately is quite difficult.
2. Sharing prediction error may perversely decrease an audience's belief in a model.
3. Most models people use for prediction are in reality narrative models and their predictive error is either abysmal or irrelevant.

Let's look at each point in detail. First consider the issue of the difficulty of assessing prediction error. In general, obtaining an accurate assessment of prediction error is much more difficult than developing the predictions themselves. Most commonly used approaches (for instance the standard R^2

from linear regression) have significant flaws. There are both theoretical and numerical methods that can be used to make more accurate prediction errors in many cases (this will be discussed further in the section the Cost of Complexity; see also Fortmann-Roe (2012)). When dealing with time series data, however, like most of the models explored in this book, it is often almost impossible to accurately assess model prediction error. Recently, theoretical technique to approach these issues have just begun to be developed (e.g. He, Ionides, and King (2009) or A. A. King et al. (2008)) but they are still impractical to apply in many cases so far.

If the challenge of measuring prediction error is surmounted, there is an even more formidable barrier to its being published with the model. There is a perverse phenomena that the act of reporting prediction error can *decrease* the confidence an audience gives a model. An anecdote was relayed to us by a member of a team working on a model of disease spread. His team shared the predictions from the model with a group of policy-makers. Everything was going fine until the audience saw the error bars around the predictions. Where his audience had been content with the raw predictions, they were quite unhappy with the predictions when accompanied by their accurately estimated uncertainties. Why was this? Was the team's model particularly bad or did these policy-makers have a better model at their disposal? No. In a world where policy-makers and clients are constantly shown models (like Obama's unemployment figures) with no measure of uncertainty (or even worse, poorly calculated, artificially low uncertainty), they come to have unrealistic expectations and often turn away good science in favor of magical thinking.

Finally, the most likely reason supposedly predictive models do not include prediction error is that they simply are not predictive. We have seen how models developed for a purportedly predictive purpose can actually be narrative models in disguise. Just why is this too often the case? You need only look at the reason for most modeling projects. It is very rare that models are commissioned solely for the purpose of generating an accurate prediction. Frequently, models are part of some political process within an organization or across them (whether an organization be a for-profit company or a non-profit such as a university). Ultimately, the people funding the model expect it to prove a point to their benefit. In environments like these, it is to be expected that some predictive modeling efforts will be sidetracked by political concerns or compromised in the process.

We can see the results of such influences in the predictions generated for unemployment presented earlier. Figure 3 shows the projections for the unemployment rates with and without the stimulus plan just as in Figure 2. Overlaid on this are now the true values of unemployment the occurred after the predictions were made. As is readily evident, the original modeling and predictions were well off the mark. Not only was reality worse than the projections assuming the stimulus was enacted (which it was) it is much worse than the projections for the economy assuming the stimulus had never been enacted at all! This is just a small example – one that is sadly replicated over and over again in business and policy-making – of mistakenly treating a narrative model as a predictive one.

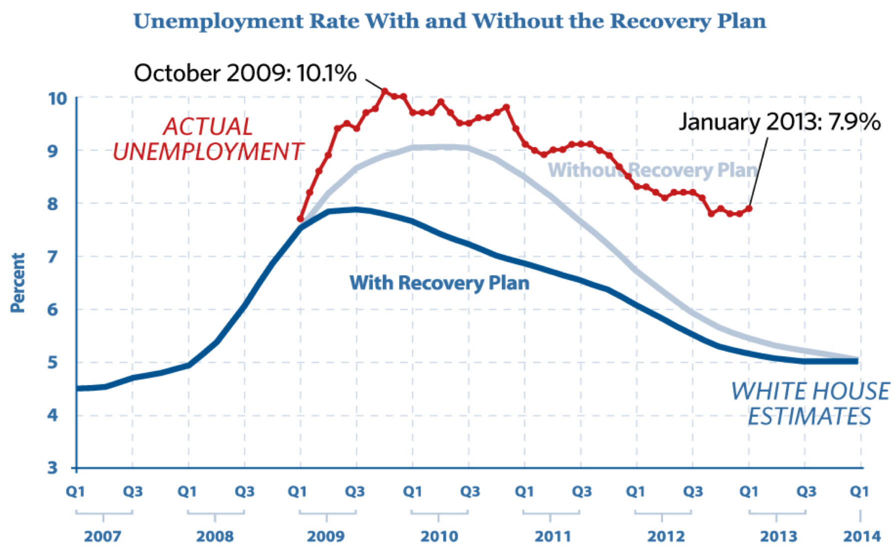


Figure 3. Unemployment predictions versus reality (The Heritage Foundation 2013)

Narrative Models

In contrast to predictive models, a narrative model is one built to persuade and transform an audience's mental models by telling a story. When many people first hear the "narrative" terminology, they respond negatively. "It's just a story." We find this strange, as narratives are the fundamental human form of communication. We tell narratives to our friends and relatives. Politicians communicate their policies to us using narratives. Of course the vast majority of our entertainment is focused on narratives[5]. Business leaders and managers attempt to describe their strategies to us using story lines; and business books are in general dominated by anecdotes plotted along the way to making their points.

We as a species do not view the world as a collection of numbers and probabilities; instead we see consequence and meaning. In short, narratives are how we see the world.

[5]Even sports, a form of entertainment that innately contains no narrative, becomes wrapped in narrative as the announcers and commentators attempt to create stories to engage us.

One critique of the term narrative is that it lacks numbers, quantified data, or mathematics. This could not be further off the mark. There are many ways to construct narratives. Words are one, pictures are another, and music is a third. Numbers and mathematics are just another way of telling a story.

In fact, most statistical and mathematical models are infused with narrative models. We looked earlier at the case of linear regression as a tool to predict test scores as a function of wealth. Again the mathematical equation for this simple model was:

$$Score = \beta_0 + \beta_1 \times Wealth$$

This equation defines a narrative. Translating this narrative into words, we would say:

> Test scores are only determined by the wealth of a student's family. A child whose family is broke will have a test score, on average, of β_0. For every dollar of wealth a child's family accumulates, the child will score, on average, better on tests by β_1.

You might or might not agree with this storyline (in our view it is a nonsensical and reductionist view of child achievement) but it shows the strict equivalence between this mathematical narrative and narrative prose. This process can be applied to all mathematical models. The mathematical definition of the model can be converted directly, with more or less lucidity, into a story describing how the system operates. The same can also be done in the reverse: we can take a descriptive narrative of a system and convert it into a mathematical description. As we have seen (will see? XXX) this is what tools like reference modes and pattern matching are designed to do efficiently: elicit a narrative from a subject in a way which can be reformulated quantitatively.

The question of how to assess the quality of a narrative model is an important one. With predictive models, we can compare competing models based primarily on predictive accuracy[6]. But how do we evaluate and compare the quality of narrative models?

The key criterion in assessing a narrative model is its ability to be *persuasive*. Although persuasion is not an objective measure in the same sense prediction accuracy is, we can decompose persuasiveness into two components for our purposes: believability and clarity. A persuasive model is one that is both believable and effectively communicates its message.

When building a narrative it is very important to use tools that are well suited to meeting these components. Unfortunately, many statistical models, including

[6]Other criteria include ease of use, cost of filling data requirements, and computational requirements. But all those are generally secondary to prediction accuracy.

regressions, are poorly suited to this two-fold task in many ways. Most statistical models depend on unrealistic and highly technical assumptions about the data. If these assumptions were enumerated in plain English, they would often conflict with people's understanding and in fact end up discrediting the model. The "alternative" has been to leave these assumptions hidden creating a black box model opaque to outside inspection.

This is a shame in our view. Such a stratagem can be successful if the authority presenting the model is prestigious enough. But the misdirection will quickly fail if any kind of rigorous scrutiny is applied to the model. Narrative models should never be given any real credence if the operation of the model is not transparent. Most statistical models are built on assumptions that are never made transparent to the audience.

The modeling techniques presented in this book, on the other hand, are well suited for narrative modeling. The techniques we present are "clear box" modeling where the workings of the model are transparently evident and accessible. Our models have their structure explicitly described using an accessible modeling diagram showing the interactions between the different components in the model. The equations governing the model's evolution are clear and readily available for each part of the model[7]. Furthermore, these modeling techniques used here make it straightforward to generate animated illustrations and displays to clearly communicate model results.

Exercise 7-8

Summarize the distinction between predictive and narrative models.

Synthesis

Now that we have thoroughly described the concepts of narrative and predictive models we can conclude this chapter by taking a step back and reemphasizing that these two categories do to represent specific modeling techniques. You can build a stock and flow model to tell a story about a system resulting in a narrative model. If your story of the system accurately represents how the system operates in reality, then you will also have a model that generates accurate predictions.

Similarly, you can apply a linear regression to a dataset. If the relationship in the data is truly a completely linear one, then the result of this regression will be the most accurate predictive model you could build. On the other hand, if you do not assess the predictive accuracy of the model and just use

[7] Admittedly, for complex models it may still require a significant investment on the part of an audience to fully understand the logic and equations in the model. But the opportunity is available.

a linear regression because it is easy to interpret or because it matches you understanding of reality, then you have a narrative model.

The key criteria to remember when building your own models or assessing other peoples models is that a predictive model is one for which you have an accurate assessment of the errors of the predictions. A good predictive model is one that has low relative errors when compared to other predictive models for the same system. A narrative model is one that tells a story about the system. A good narrative model is one that persuades an audience and by persuading, the model transforms the mental models of its audience.

Chapter 8

Building Confidence in Models

When used correctly, the transparency of the modeling techniques presented in this book results in models that are powerful persuasive tools. As with any model, however, there are concerns and questions will invariably be raised which could cause users to doubt the result of the modeling work. There are a number of techniques that you can use to help preemptively address these concerns and increase an audience's confidence in your model.

The idea of building confidence in a model is closely tied to the standard concept of model verification and validation. We dislike this conceptual approach to assessing models as it seems to imply that a model can go through a process to get a big fat "VALID" or "VERIFIED" stamp on it. Returning to Box's quote that "all models are wrong, but some are useful", in reality all models are wrong and none of them are completely valid, period. Models can however be useful, especially narrative models in which the audience has confidence.

We favor the conceptual approach put forth by Forrester and Senge (1978), that there is not any single test or suite of tests that will verify or validate a model and that validity should instead be thought of as a function of confidence. This is a view that differs from that held by some modelers and laypeople. As Forrester and Senge note, "the notion of validity as equivalent to confidence conflicts with the view many seem to hold which equates validity with absolute truth." We share their belief that model confidence is built up piece by piece from a variety of tests that, though they cannot prove anything, together comprise a persuasive case for the quality of a model.

There are three distinct areas where confidence needs to be developed:

1. That the model itself is well designed.
2. Given a design of the model, this design is implemented correctly.
3. The conclusions drawn from the model are accurate.

In the remaining sections of this chapter we will look at each of these different areas in detail. We will explore the different tests and tools that can be used to build confidence for each area.

Model Design

Fundamentally the design of a narrative model is of utmost importance and needs to be justified to an audience[1]. There are two primary aspects to a model's design design: the structure of the model and the data used to parameterize the model.

Structure

The structure of the model should mirror the structure of the system being simulated. Depending on the system complexity, the model structure may need to carry out more or less aggregation and simplification of this reality. Nevertheless, all the primitives in the model should map on to reality in a way that is understandable and relatable to the audience. Thus if there is some object in the real system that behaves as a stock, a stock should exist in the model mirroring the object's position within the system. The same should hold true with the other primitives in the model. Each primitive would ideally be directly mappable onto a counterpart in the real system and any key component in the real system should be mappable onto primitives in the model. Furthermore, feedback loops that exist in the system should exist within the model. These feedback loops should be explicitly identifiable in the model and would ideally be called out or marked in a way that highlighted their presence to an audience.

Furthermore, the model structure should include components that an audience thinks are important drivers of the system. Missing a factor that the audience considers to be a key driver can fatally discredit a model in an audience's mind irrespective of the performance or other qualities of a the model. This is true even if the factor has in fact a negligible effect. Generally speaking, it is much easier to include a factor an audience views as important than it is to later on convince the audience that the factor does not in actuality matter.

Data

The more a model uses real-world data, the more confidence an audience will have in the model. Ideally, you have empirical data to justify the value of every primitive in your model. In practice, such a goal may be a pipe dream. Indeed, for a complex model, obtaining data to parameterize every aspect of it is usually

[1] This is different from predictive models where the results of the model are much more important than the design and the "proof is in the pudding"" so to speak.

impossible[2]. When faced with model primitives that do not have empirical data to parameterize them, an approach must be taken to ensure that it does not appear that their values were chosen without justification or to arrive at a predetermined modeling conclusion. Sensitivity testing, as discussed later on, is one way to achieve this. Another is to carry out a survey of experts in the field in order to solicit a set of recommended parameter values that can then be aggregated or used to justify the ultimate parameterization.

Peer-Review

Going through a peer-review process can be extremely useful in establishing confidence in a model. Two general types of peer-review are available. In one, the model may be incorporated into an academic journal article and submitted for publication. The article will then peer-reviewed by generally two or three anonymous academics in the field who critique it and judge whether or not it is a worthy contribution to the literature, thus meriting publication. In the second type of peer-review, a peer-review committee may be assembled (hired) to review a specific model and provide conclusions and recommendations to clients.

Peer-review can be very useful in establishing the credibility of a model. A credible model is a model one can be more confident in, other things being equal. By engaging an independent group of exports to assess the model, their conclusions about its quality have the appearance of greater validity than those of the self-interested modelers[3]. This can be especially useful when trying to meet some abstract standard such as that the model represents the "best available technology" or the "best available science".

A key risk of a peer-review is, of course, that the peer-review members will find a model deficient in important respects. Good criticism can be very useful and help improve a model. However, some criticism received in practice may be nitpicking details or detrimental advice that would make the model worse if followed.

Model Implementation

Although it is not as much a lightning rod as is model design, the implementation of a model specification is a point where significant error can occur. Programming mistakes or mistyped equations can introduce bugs into a model that can be hard to identify later on. This is a particular problem in black-box models but it is still an important point to consider for all types of models

[2] Leading to the clichéd conclusion of many modeling studies: "We are unable to draw strong conclusions from this modeling work. Instead, our contribution has been to show where additional data needs to be collected."

[3] When the peer review panel is hired by the client there is some conflict of interests, but the panel members should not be swayed by this.

including those presented in this book. Fortunately, a number of steps can be taken to ensure the model is implemented correctly.

Primitive Constraints

For many of the primitives in the model, there will be natural constraints. For instance, a stock representing the volume of water in a lake can never fall below 0. Similarly, if a variable represents the probability of an event occurring, it must be between 0 and 1.

Often these constraints are implicit without being formally specified in the model. A modeler may think, water volume can never become negative so why would I need to specify it? However, the existence of these constraints provides an opportunity to implement a level of automatic model checking. By specifying that a primitive can never go above or below a value (using the **Max Value** and **Min Value** properties in Insight Maker), you can create in effect a canary in the coal mine that warns if something is wrong in the model. If these constraints are violated an error message can be given letting you know that you need to correct some aspect of your model.

This concept of constraints in models is similar to the concept of "contracts" which are support in some programming languages. These contracts define and constrain the interaction between different parts of the program causing an error to be generated if the contract is violated. The Eiffel programming language probably has the best support for this approach to development.

Unit Specification

Since we introduced units in Chapter 3, we showed that they could be a useful tool in constructing models. Units can also be used to ensure that equations are entered correctly. If you fully specify the units in a model, many types of equation errors will result in invalid units, which will create an immediate error. By employing units in your model you can automatically catch a whole class of errors and mistyped equations.

Regression Tests

Other tests than those specified above can be developed. For instance, the proper behavior of one part of the model may be determined and automated tests created to periodically confirm that the model continues to exhibit the correct behavior. Development of such tests are a common part of software engineering that we wish would see more use in model development. Insight Maker itself has a suite of over 1,000 individual regression tests that automatically test every aspect of its simulation engine.

In regards to regression testing, it is important to ensure these tests are automated. It is not enough to examine a portion of the model, determine it is currently working correctly, and leave it at that. The problem is that future changes may break the existing functionality (i.e. a "regression", the introduction of an error or reduced quality compared to an earlier version of the model). Especially for complex models, a change in one part of the model may have an unexpected effect in another part. By implementing a set of automatic checks, you can protect your model against unintended changes and regressions.

George Oster and his class XXX

Exercise 8-1

You have a variable representing the total population size of a small city. What constraints might you place on this variable?

Answer Available

A Second Pair of Eyes

That is not to say, however, that spot and point-in-time checks are not worthwhile. It can be very useful to have a second modeler review your models and cross-check the equations. This helps not only to check simple mistakes but also to question and critique the fundamental structure and choices of the model.

The gold standard in verifying that a model is implemented correctly according to specification is to have a second modeler completely reimplement the model according to that specification. Such reimplementation should ideally occur without access to the original model's code base to ensure that the second modeler does not simply copy bugs from the original model into the reimplementation. If the results from the two implementations concur, that is strong evidence that the model has been implemented correctly. Although potentially an expensive exercise, it will also most likely identify numerous ambiguities in the specification, which could be valuable in and of itself.

Model Results

Given that the design of the model and its implementation are assumed to be correct, the burden still falls upon the modeler to transfer her confidence in the model's results to her audience. There are several different ways this can be done.

Expected Results

The first way is to demonstrate that the model generates expected results for normal inputs. For instance, if you had a model a reservoir, you would expect the volume of the reservoir to decline over time during the summer due to evaporation if no more water flowed into it. You can additionally test extreme scenarios and show that they generate the expected results. If, for example, your reservoir were empty, you would expect the amount of water evaporating from it to be zero. By enumerating these standard cases and showing the model results match the expected results you can help build confidence in the model.

Often these expected results can be described in terms of a curve showing how the values of one of the stocks or variables in the system is expected to change over time. This curve can be taken from historical data (a reference behavior pattern), or simply drawn on a piece of paper by experts familiar with the system (an excepted behavior pattern).

Counterintuitive Results

Another attempt to increase confidence in a model is to show unexpected results that are justifiable. Imagine a model that for a certain set of inputs would create what, at first glance, appeared to be the "wrong" behavior. Some lever in the model could lead to unexpected results. When first shown these results, they could decrease an audience's confidence in the model. If the audience was then walked through the model step by step to show how those results proved to be correct and mirrored reality, then that could well increase their confidence in the model results.

Forecasting

Possibly the most persuasive action to convince an audience of the effectiveness of a model is to forecast the future and then to show this forecast to be correct. This, of course, is difficult to do in practice for multiple reasons including the fact that the scale of a model is often such that it could take several years or decades to generate data to test the model. Additionally, it must be remembered that most narrative models are poor predictors and should not be used for predictive purposes solely.

Sensitivity Testing

Sensitivity testing is a broad field that has the potential to address many questions and doubts that may arise about a model. In general, the variables and numeric configuration values in a model will never be known with complete certainty. When the results from an election poll are published, the pollsters publish not only their predictions but also the uncertainty in the prediction (e.g., "the Democratic candidate will obtain $52\% \pm 3\%$ of the vote"). Similarly when a building is constructed, the materials used will have certain properties – such as strength – that again are only known up to some errors or tolerance. It is the engineer's and contractor's responsibilities to ensure that the materials are sufficient even given the uncertainty of their exact strengths.

The same occurs when modeling. Most primitive values in the model will have to be estimated by the modeler and there will be an error associated with these values. Of course the error will also be propagated through the model when it is simulated and affect the results generated by the model. This error is one factor that can create doubt about a model and reduce an audience's confidence.

As a modeler, one approach to address this doubt would be to try to measure all the model's variables with great accuracy. You could search the available literature, undertake a meta-analysis of current results, carry out new experiments, and survey experts to get as precise a set of parameter values as possible. If you were able to say with strong certainty that these values were so accurate and the errors so small that their effect on the results in negligible, then that would be one way of addressing the issue of uncertainty.

However, all of this is often impossible to do. When dealing with complex systems it is almost always the case that at least a couple variable values will never be known fully with certainty. In this case, no matter how much research or experiments you do, you will never be able to pin down the precise values of these variables. How do we handle these cases?

The answer is straightforward: Rather than trying to eliminate the uncertainty, we embrace it by explicitly including it in the model. If you can then show that the results of your model do not significantly change even given the uncertainty, you have a persuasive case for the validity of your results. Of course the results will always change when the uncertainty is introduced, but if the model conclusions persist even in the face of this uncertainty it will greatly increase your audience's confidence in the results.

Uncertainty can be explicitly integrated into a model by replacing constant primitive values with a construct that represents the uncertainty in that value. Imagine you had a simple population model of rabbits in a cage. You want to know how many rabbits you will have after two years. However, you don't know how many rabbits there initially are in the cage. You have been told that there are probably 12 rabbits, but the true number could range anywhere from 6 to 18.

If you model your population as a single stock, what should the initial value be? A naive model could be built where you the initial value of the rabbit stock was specified as 12. However, that does not incorporate the uncertainty and could be a source of criticism or doubt for the model. An alternative would be to specify that the initial value of the stock is a random number with a minimum value of 6 and a maximum value of 18. So each time you run the model you will get a different result. If you ran the model once, the initial value might be chosen to be 7 and you would obtain one result. If you ran the model again, the initial value might be 13 and you would get another result.

If you run this stochastic model many times, you obtain a range of results. These results can be automatically aggregated to show the range of outputs. For instance if you ran the model 100 times you could see what the maximum and minimum final populations were. This would give you a good feeling for how many rabbits you needed to prepare for after two years. In addition to the maximum and minimum you might be interested in the average of these 100 runs: the expected number of rabbits you would see. You could also plot the distribution of the final population sizes using a histogram to see how the results are distributed. This distribution would show how sensitive the outputs are to the uncertainty in the inputs: a form of sensitivity testing.

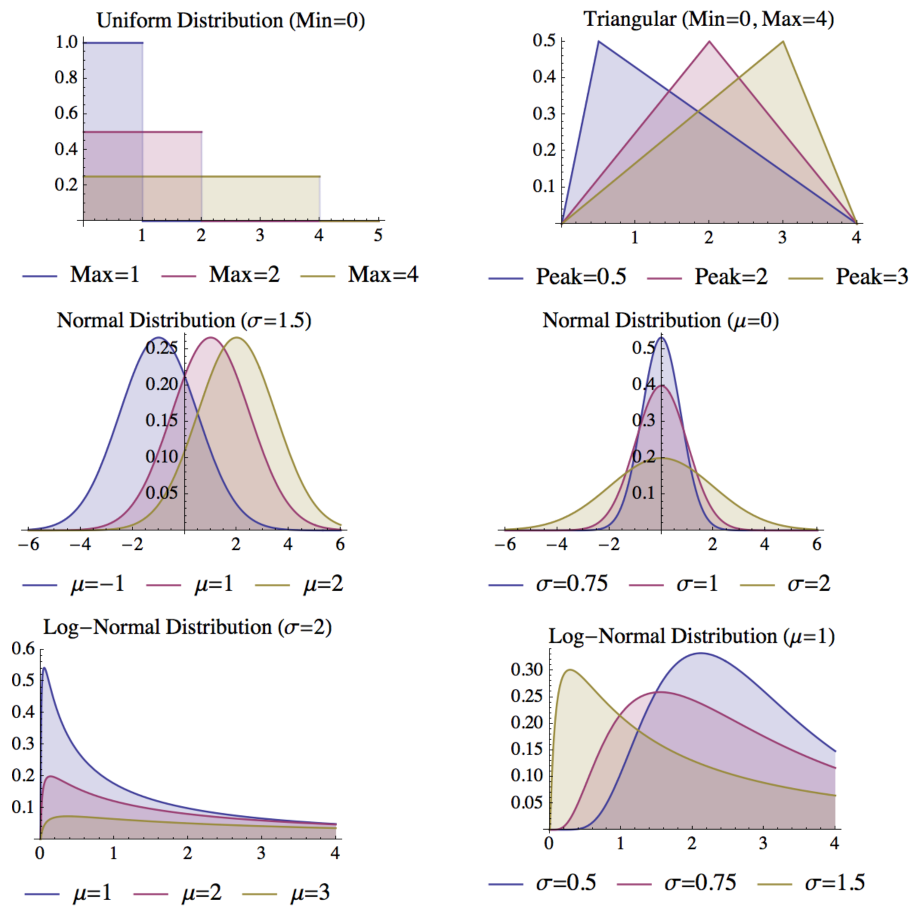


Figure 1. Common Distributions for Sensitivity Testing with Sample Parameter Values

There are four key distributions that are useful for specifying the uncertainty in a variable:

Uniform Distribution : The uniform distribution is defined by two parameters: a minimum and a maximum. Each number within these two boundaries has an equal probability of being sampled. The uniform distribution is useful when you know the boundaries on the values a variable can take on, but you do not have any information on the likelihood of the different values within this region.

The uniform distribution can be used in Insight Maker using the function `Rand(Minimum, Maximum)`, the two parameters are optional and will default to 0 and 1 if `Rand()` is called without them.

Triangular Distribution : The triangular distribution is defined by three parameters: the minimum, the maximum, and the peak. Like the uniform distribution, the triangular distribution will only generate numbers between the minimum and maximum. Unlike the uniform distribution, the triangular distribution will not sample all numbers between these boundaries with equal likelihood. The value specified by the peak will have the most likelihood of being sampled with the likelihood falling off as you move away from the peak towards either the minimum or maximum boundary. The triangular distribution is useful when you know the both the most likely value for a variable and you also know boundaries for the values a variable can take on. The triangular distribution can be used in Insight Maker using the function `RandTriangular(Minimum, Maximum, Peak)`.

Normal Distribution : The normal distribution is defined by two parameters: the mean of the distribution (generally denoted μ) and the standard deviation of the distribution (generally denoted σ). The most likely value to be sampled from the normal distribution is the mean. As you move away from the mean (in either a positive or negative direction), the likelihood of a number being sampled decreases. The standard deviation controls how fast this likelihood falls as you move away from the mean. Small standard deviations result in steep declines in the likelihood while large standard deviations result in more gradual declines. The normal distribution is useful when you do not have boundaries on the values for a variable but you do know what the most likely value for the variable should be (the mean). The normal distribution can be used in Insight Maker using the function `RandNormal(Mean, Standard Deviation)`.

Log-normal Distribution : The log-normal distribution is closely related to the normal distribution. In fact the logarithm of the values samples from a normal distribution will be log-normally distributed. Like the normal distribution, the log-normal distribution is defined by two parameters: the mean and standard deviation. Where the log-normal distribution differs from the normal distribution, is that negative values will never be generated by the log-normal distribution. Thus it is useful when you have a variable which you know cannot be negative but for which you do not have an upper bound. The log-normal distribution can be used in Insight Maker using the function `RandLogNormal(Mean, Standard Deviation)`. The log-normal distribution can also be used to represent other types of one-sided boundaries. For instance, the following equation could be used to represent a variable whose number was always less than 5: `5-RandLogNormal(2, 1)`

There are many other forms of probability distributions. Some notable ones are the Binomial Distribution (`RandBinomial(Count, Probability)`), the Negative Binomial Distribution (`RandNegativeBinomial(Successes, Probability)`), the Poisson Distribution (`RandPoisson(Lambda)`), the Exponential Distribution (`RandExp(Lambda)`) and the Gamma Distribution (`RandGamma(Alpha, Beta)`).

These distributions can be used to address very specific modeling usage cases and needs (for instance, the Poisson distribution can be used to model the number of arrivals over time), however, the four distributions described in detail above should generally be sufficient for most sensitivity testing needs.

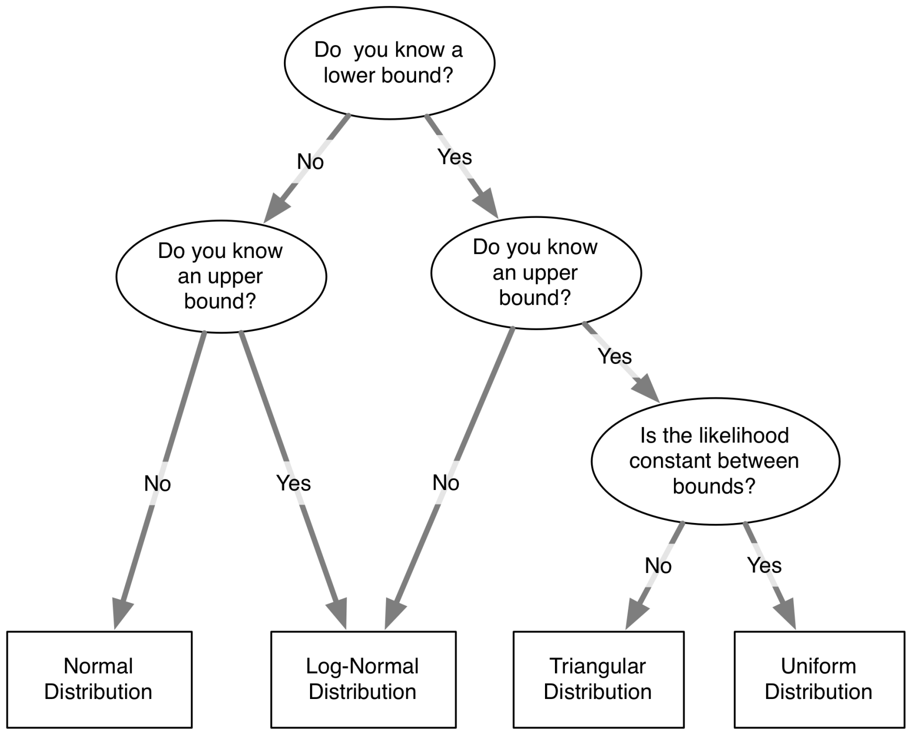


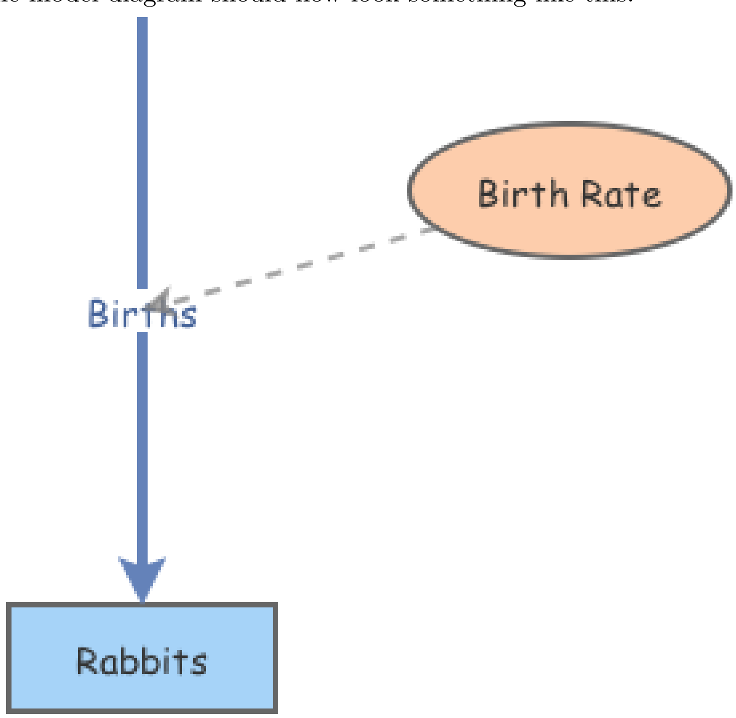
Figure 2. Choices in Selecting a Distribution for a Variable's Value

When important practical tip when using sensitivity testing within the System Dynamics context is to be careful about specifying random numbers within variables. The value of a variable is recalculated each time step. This means that if you have a random number function in the variable, a new random value will be chosen each time step. This can create a problem if the random value is supposed to be fixed across the course of the simulation. For instance, we may not know the birth rate coefficient for our rabbit population, but, whatever it is, we assume it is fixed over the simulation.

A simple way to handle these fixed variable values would be to replace the variables with stocks. The stocks initial value could be set to the random value and it would only be evaluated once at the beginning of the simulation and kept fixed thereafter. This approach, though very workable, however violates the fundamental metaphors at the hard of System Dynamics. In Insight Maker, another approach is to use the `Fix()` function. When used with one argument, this function evaluates whatever argument is passed to it a single time and then returns the results of that initial calculation for subsequent time steps. So instead of having the simple equation `Rand(0, 10)` in a variable to generate a random number between 0 and 10, you could place `Fix(Rand(0, 10))` in the variable. The first equation would generate a new random number each time step, the second equation will generate one random number and keep it constant throughout the simulation.

Sensitivity Testing

1. Let's illustrate the usage of sensitivity testing using our rabbit example. First we will construct a simple exponential growth model.
2. Create a new **Stock** named [**Rabbits**].
3. Change the **Initial Value** property of the primitive [**Rabbits**] to `12`.
4. Create a new **Flow** going from empty space to the primitive [**Rabbits**]. Name that flow [**Births**].
5. Create a new **Variable** named [**Birth Rate**].
6. Change the **Equation** property of the primitive [**Birth Rate**] to `0.05`.
7. Create a new **Link** going from the primitive [**Birth Rate**] to the primitive [**Births**].
8. Change the **Flow Rate** property of the primitive [**Births**] to `[Birth Rate]*[Rabbits]`.
9. The model diagram should now look something like this:



10. This is the basic outline for the model. We assume a fixed value of 12 rabbits and a fixed birth rate of 0.05.

11. Run the model. Here are sample results:

12. When we simulate we obtain the same results each time.

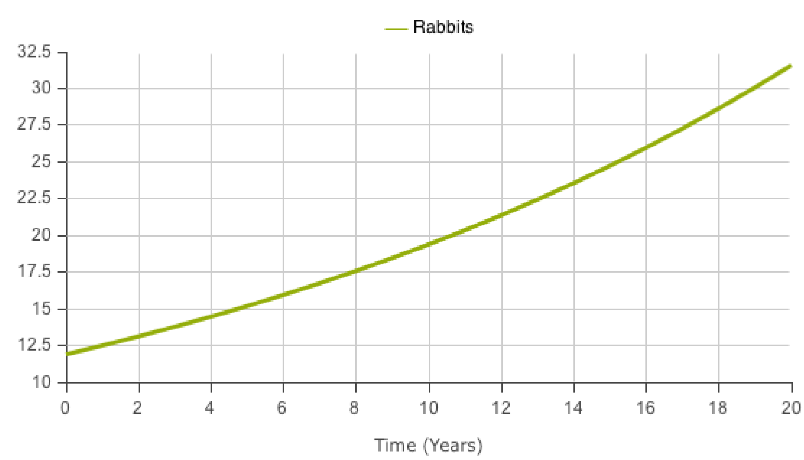
13. Change the **Initial Value** property of the primitive [**Rabbits**] to `RandTriangular(6, 18, 12)`.

14. Now, let's try to incorporate uncertainty. Given that we know that there can be between 6 and 18 rabbits initially with an expected value of 12, we can use the **RandTriangular()** function to model this.

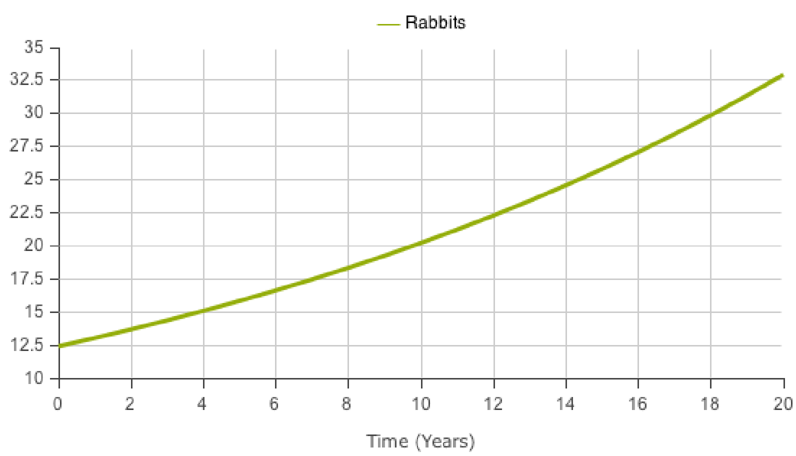
15. Change the **value** property of the primitive [**Birth Rate**] to `RandLogNormal(0.05, 0.03)`.

16. We also do not know the birth rate with certainty. We know, however, that the rate must be greater than 0, and lets say we can assume the expected value is 0.05. We can use the **RandLogNormal()** function to model this type of uncertainty.

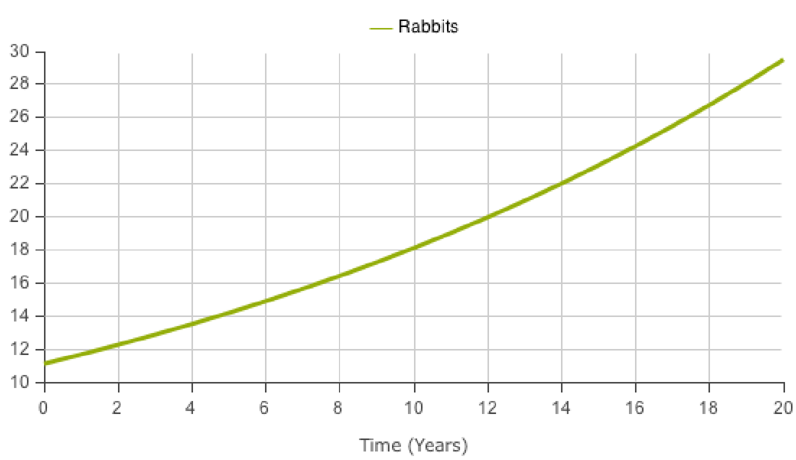
17. Run the model. Here are sample results:



18. Run the model. Here are sample results:

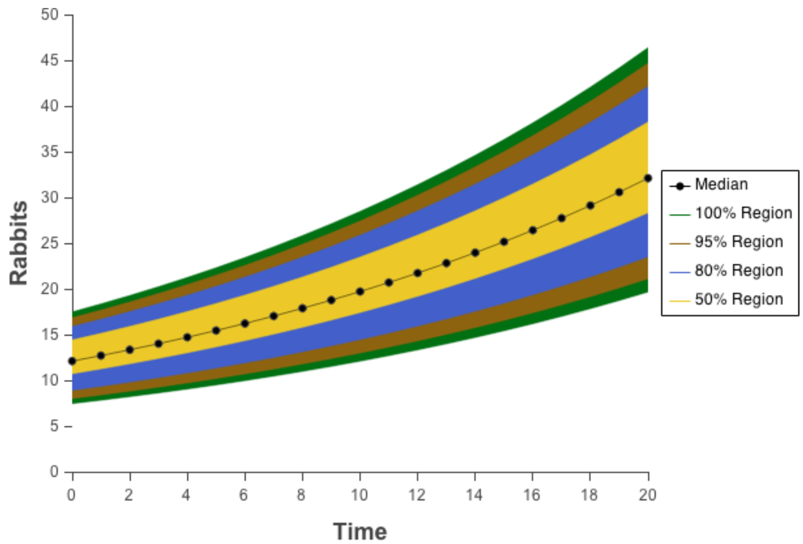


19. Run the model. Here are sample results:



20. Now, we can simulate this mode a few times and see that each time we run the simulation we get a different result.

21. We can now use sensitivity testing to see the range of results given this specified uncertainty. We'll do 100 runs of the model and aggregate the results to see the expected distribution

22. 

23. We can readily see the range of results allowing us to make decisions incorporating our known uncertainty about parameter values.

Exercise 8-2

Create an equation to represent the uncertainty of how many red marbles there are in a bag. You know there are at least 5 red marbles and no more that 14. You do not have any other information.

Answer Available

Exercise 8-3

Create an equation to represent the uncertainty of how many red marbles there are in a bag. You know there are probably about 20 red marbles and you know there are no more than 100 marbles in the bag.

Answer Available

Exercise 8-4

Create an equation to represent the uncertainty of how many red marbles there are in a bag. You know there are probably about 20 red marbles and you do not know how many marbles the bag can hold total.

Answer Available

The astute reader will notice that our discussion up to this has failed to address an important point: how do we determine the uncertainty of a variable? It is very easy to say that we do not know the precise value of a variable, but it is much more difficult to define the uncertainty of it. One case where we can precisely define uncertainty is when you take a random sample of measurements. For instance, suppose our model included the height of the average American man as a variable. We could randomly select a hundred men and measure their heights. In this case our uncertainty would be normally distributed with a mean equal to the mean of our sample of one hundred men and a standard deviation equal to the standard error of our sample of one hundred men[4]. For any random sample of n values from a population, the same should hold true: you will be able to model your uncertainty using a normal distribution with:

$$\mu = \frac{Value_1 + Value_2 + Value_3 + ... + Value_n}{n}$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Value_i - \mu)^2}$$

However, in most applied cases you will not be able to apply this normality assumption. Generally you will not have a nice random sample, or you might have no data at all and instead have some abstract variable you need to specify a value for. In these cases, it is up to you to make a judgment call on the uncertainty. Choose one of the four distributions detailed above and use whatever expert knowledge available to you to place an estimate on the parameterization of uncertainty. One rule of thumb, however, is that it is better to overestimate uncertainty than underestimate it. It is better to err on the side of overestimating your lack of knowledge than it is to obtain undue confidence in model results due to an underestimation of uncertainty.

Exercise 8-5

You have tested the diameter of 15 widgets coming out of a factory and obtained the following values: 2.3, 2.5, 1.9, 1.4, 2.0, 2.7, 1.9, 2.1, 2.1, 2.2, 1.6, 2.4, 2.0, 1.8, 2.6.

[4]Please note that this contradicts slightly what we said earlier. Clearly, a person cannot have a negative height while the normal distribution will sometimes generate negative values. So wouldn't a log-normal distribution be better than a normal distribution? Mechanistically, it would, however statistically we can show that due to the Central Limit Theorem the normal distribution does asymptotically precisely model our uncertainty. Given a large enough sample size (100 is more than enough in this case), the standard deviations for uncertainty will be so small that the chances of seeing a negative number (or even one far from the mean) are effectively none.

Create an equation to generate a new widget size with the same distribution as the widgets arriving from the factory.

Answer Available

Exercise 8-6

You have taken 12 sheep from a population and weighed the amount of wool on each sheep to obtain the following weights in kilograms: 1.005, 0.817, 0.756, 0.821, 0.9, 0.962, 0.692, 0.976, 0.721, 0.828, 0.718, 0.852.

Create an equation to generate how much a random variable for how much wool you will obtain from a sheep.

Answer Available

Confidence and Philosophy

The quality of a model in the eye of an evaluator is to a significant extent influenced by the worldview (more or less coherent and the philosophical orientation (if any) of the evaluator. Broad world-views or epistemologies[5] exist. One key divide in different epistemological theories continues to be between those theories that contain a strong belief in a concrete true reality that our knowledge can accurately capture and those theories that believe our knowledge is partially or wholly independent from reality.

Epistemological theories that are primarily in the first camp are those such as positivism or empiricism. Theories in the latter camp include constructivism and idealism. Constructivism is a popular theory of knowledge that claims knowledge is constructed with social context and historical time. Our presentation of confidence building for narrative models in this chapter is implicitly in line with a constructivist theory of knowledge.

In our discussion of confidence building we repeatedly refer to matching the beliefs of the audience. We recommend creating simulations and behavior in our models that match an audience's expectations for the behavior of the system. This is distinct from saying that you should match the reality of true system. Ideally, true behavior of the system and an audience's mental models of the system should be equivalent, but in practice they may well differ. Although confidence in a model will be boosted by strictly matching the mental models

[5]From the Greek word "epistēmē" meaning "knowledge" or "understanding", epistemology is the branch of philosophy describing how we understand or come to know the world around us.

of an audience, a truly effective narrative model should be persuasive enough to change the mental models of an audience.

Our discussion of predictive models from the previous chapter does not fall within a constructivist world-view as we are claiming that there are objective "outside-ourselves" measures of predictive accuracy we can obtain. It should go without saying that predictive models may not be accurate reflections of reality, even in their own terms. The mathematics of a predictive model may be unrelated to the true system that is being modeled, yet is may still create accurate predictions. As such, our discussion of predictive models is not really a positivist or empiricist one. Instead this discussion would fall under the epistemological theories of pragmatism or instrumentalism which claim a theory or model should be assessed on how well it predicts which may be independent of the truth of the theory itself.

Exercise 8-7

You are asked to evaluate a model simulating the growth of an endangered species in its habitat. What tests and demonstrations would you like to see in order to trust the model and recommend its use in practice?

Exercise 8-8

You are asked to evaluate a model simulating the potential adoption for a new product at your company. The basic results of the model are very encouraging for the product suggesting it would make a significant return on investment.

What tests and demonstrations of the model would you like to see in order to recommend the product be produced based on the model results?

Chapter 9

The Process of Modeling

Now that you are well on your way to being a modeling expert, you may begin to receive requests for assistance with various modeling projects. As a motivating example, a friend – it could also be a colleague or client – comes to you and asks for help. This friend has been involved with the effort to protect the rare Aquatic Hamster.

The Aquatic Hamster is an endangered species that spends most of its life living in lakes and rivers. Unfortunately, development and human encroachment has steadily reduced the available habitat for these hamsters and their population has plummeted. Indeed, now there is just one last population of them left located on a lake just south of the Canada/United States border.

Your friend asks you to build a model of this hamster population in order to help prioritize protection efforts and to rally support from governmental agencies and non-profits to protect this last hamster colony. You want to be of real assistance to your friend, and the hamsters are admittedly cute, so you agree to take on this modeling project.

You are at your desk to start building the model, but then realize something: You aren't sure what to do next. There are so many candidates for first steps. Do you start sketching diagrams? Do you talk to hamster experts? Do you start coding up a model? You are paralyzed by the sheer number of different choices. You know your friend is counting on you, so what do you do now?

In this chapter, we answer that question. We explore the modeling process from start to finish, introducing the tools and techniques for getting from "I need a model" to a final product that works. As you will see, our experience is that the best approach to tackling tough modeling problems is to start deceivingly small: build the simplest model possible (what we call the "Minimum Viable Model") to get going and then iterate aggressively on this initial version.

Why Model?

The first step to building a model is answering the simple question: *Why am I building this model?*

This question seems obvious, but it is often hard to answer in practice. Let's try answering it for our hamster population model: Why are we building this model? The truth is that so far we do not have a real understanding of this.

Oftentimes, the lack of focus begins with the friend/client/colleague who commissioned the model. Laypeople frequently do not have a strong understanding of what modeling is,including what modeling can accomplish and what it cannot. Instead, your friend might have a simplistic view of a model, almost as if were a magic wand. He feels he just needs a model and then, *abracadabra*, it will solve his problem. His thought process on what to do with a model might be as bareboned as:

1. Build Model.
2. ...
3. Hamsters Saved.

Of course this is not the case. You build a model with a specific purpose in mind otherwise it will most likely accomplish nothing. Worse yet, when it comes to the hamsters, it will be too little too late. Your first action should be to work with your friend to make sure you have filled in the "..." step. The best way to do this is generally working backwards from the final step rather than working upwards from the first one. For us that would be to first figure out how the hamster population is to be protected.

Paradoxically, in order to answer the question of why we are building a model, we are going to need to ask many questions of our own. Why should we protect the hamsters? What risks do the hamsters face? What do the hamsters need to be protected? What avenues to obtaining these protections are there? What techniques to protecting the hamsters are most effective? Cheapest? Most expedient? And so on. We need to obtain a good understanding of the root cause of the problem you friend wants to tackle with this model and force out the concrete steps to getting there.

After discussing this with your friend and the two of you come to the conclusion that you will need two things in order to reliably protect the hamster population. First, government regulatory agencies need to pass (stronger) rules protecting the hamster habitat. Second, non-governmental organizations (NGO's) need to provide funds for hamster conservation and protection efforts.

Using this, we can expand our plan with more details:

1. Build Model.

2. ...
3. Agencies enact rules to reliably protect hamsters. NGO's provide money for conservation efforts.
4. Hamsters Saved.

This focuses things for us. Rather than "Building a model to save the hamsters" (which is too vague and completely unactionable leading to our quandary about what to model), we are building a model designed to persuade governmental regulators and NGO's that they should devote resources to protecting the hamsters.

So how do we do that? Let's simplify the complex issue into two specific goals for our model:

- Show that given the *status quo* (business as usual) the hamster population will go extinct.
- Show that alternatives to the *status quo* exist (which require regulatory action or investments) that enable the hamster population not only to survive, but also to thrive.

If our model demonstrates both these things it could be a highly persuasive tool to shape decisions and policies. By building a model that does these two things[1] we will have given our friend a powerful tool to push for regulatory action and financial support.

When building your own models you'll want to go through a similar thought process to get at the core goal or question the model should address. Going into a modeling project with the attitude "First we'll build a great model, then we'll figure out how to apply it" is a prescription for failure. Of course, as you go through the process you might discover insights you never expected or you might in fact determine that your original hypothesis was wrong. Such discovery is always a great outcome, but you can never count on it happening in the course of building your model. It's best to start very focused in your modeling efforts and treat any discoveries or broadening of scope later on as a lucky bonus.

Model Project Management

When tackling modeling projects such as our hamster-population model, there are two basic overarching project management approaches. The first is founded on detailed planning and preparation. Tackling the hamster model using this approach might look something like the following sequential phases:

[1] The model of course must also inspire confidence in its audience. They must believe its results are reliable otherwise the results will have no persuasive power. Review the previous chapter for tools for building confidence in models.

Research : Find and obtain relevant literature on Aquatic Hamsters. Read peer-reviewed publications. Locate hamster experts and interview them. Identify key mechanisms affecting hamster population growth. Some mechanisms may require further study. For example, if human expansion and urbanization affect the hamster habitat area, for example, you may need to study the forces influencing urbanization. These may require additional literature searches and expert interviews.

Design : Once you have completed your background research on the hamsters, start to design the model. Create causal loop diagrams and develop stock and flow diagrams. Break your hamster population model into different sectors. You will have the hamster-specific sector, which includes sub-sectors for each of the life-stages these endangered hamsters go through. You will also need sectors for other parts of the model that affect the hamster population growth: an urbanization sector with its own model, a climate sector with a climate model, and so on. Write out equations for all these sectors and resurvey experts you have contacted to review the overall model design and the specific equations. There will probably be several cycles of iteration and model expansion during this stage as additional key areas to include are identified.

Construction : Now that you have completed a model design and received a seal of approval from experts in the field, you are ready to start building the model itself. Decide what modeling software package (or programming environment) you will use. Implement the equations as they were specified in the design phase.

Wrapping Things Up : Go through the confidence building steps from the previous chapter. Develop tests for your model to ensure it works correctly. Create model documentation. Show the model demonstrates expected behavior and obtain final approval from experts.

This approach to building a model is a very linear process where you go sequentially from stage to stage. In the project management field, this is the classic "waterfall" project where you proceed phase by phase through the project. You plan out the whole thing ahead of time estimating how long each phase would take and identifying dependencies between phases. This form of project management can work well if done expertly and it is well suited for certain kinds of projects such as constructing a building.

In our opinion, however, this approach to tackling a project is quite poorly suited to the task of building a model. There are several reasons for this.

First, each model is inherently unique[2]. You may have developed a dozen different population models in your career, but when it comes to developing a model for a new species or location, you will inevitably run into situations and problems you have never encountered before. The quantity and quality of

[2] Lots of "cookie cutter" models out there are designed to model a certain class of problems. Without custom work, however, these models are of dubious validity and may serve more to "check a box" that a model has been built rather than to be a useful decision-making tool.

data will differ from the cases before. If not, the biology of the animal you are modeling will be different. If not, the model goals and constraints will be different, and so on. Given these differences, rigid project management techniques such as the waterfall approach do not generally provide the predictability that is needed.

Secondly, when building a model you will find that many of your assumptions may simply be wrong. This can happen with every aspect of model construction: the data you thought you had will turn out to be non-existent, the equations provided to you by experts end up not working, and the model code you write will invariably have a bug or two that needs to be identified and squashed. Because of this you will continually need to be adjusting and adapting your model as you learn more about the system and what pieces of information you can rely upon and what you cannot.

Such a high likelihood of error and need for readjustment are not well suited towards techniques based on sequential, long-term planning formats. What good is a great plan if the assumptions it is based on are substantially wrong?

Take, for instance, the data you use to build your model. It is not uncommon for a collaborator to come to you and say we have X, Y and Z data series for you to use in your model (where these might represent environmental conditions or other important model inputs). When you go to check the data however you may find that in fact X does not exist (the collaborator was confused), Y actually has large gaps in the data set which make it effectively useless for your needs, and Z was collected in such a way that they were actually measuring something completely different than they thought they were.

Take, as another instance, the equations in a model. Imagine you consult an expert on Aquatic Hamsters and she provides an equation governing the survival of hamsters during their first year of life. This equation was developed as part of a scientific study where the hamsters were grown in indoor swimming pools at her university's Aquatic Hamster Research Facility. When you go to apply this equation in your model, however, you find out that how the hamsters behave when living within an indoor swimming pool is very different from how they survive in the wild. Because of this, the equation you have is simply not accurate for the hamsters living in the wild.

Errors like these two examples are *very* common. If you had proceeded with the classic waterfall approach to modeling you might not realize that you cannot rely on the data or equations you were planning to use until the very end of the modeling process. At this point it is much too late to go back and correct your model.

Iteration: Failing Fast and Failing Often

Because of this, we advocate an alternative approach to building models. We support jumping right into the model construction process as early as possible.

As we showed you in the *Red* example from Chapter 4, we think it is important to get a simulation model up and running as quickly as possible. You should never want to be more than a few steps away from a simulating model[3].

When beginning a modeling project we recommend building the simplest model possible to get going. We call this the *Minimum Viable Model*[4] and it is the model that contains just enough to minimally represent the system and nothing more. For the hamster model, this Minimum Viable Model might contain just a single stock representing the hamster population and a couple of flows modifying the population. Nothing more.

You don't have to worry about your equations being right or your model being an accurate predictor in the Minimum Viable Model; you just want to get something up and running as soon as possible.

Once you have the Minimum Viable Model you can start to run it by people and begin to incorporate their feedback. So get your friend's thoughts on the minimal hamster model, talk to experts, study the model's forecasts and see what works and does not. Then iterate on the model: make a change here, add a new component there. If the feedback you are getting is no one trusts the model because it does not contain some key mechanism, add that mechanism to the model[5]. Steadily adjust and refine the model based on the actual results of the model and the feedback you receive.

This feedback will be more useful to you when you have a concrete model that is simulating than it would be if you were just running abstract ideas by people. By putting your stake in the ground with a model that simulates, you allow others to critique and engage with the model providing you with valuable information about what works and what does not. If you do not come with a concrete model, you run the risk of receiving very vague, unactionable feedback.

What is best about this approach of rapid iteration we advocate is that it allows you to identify failures quickly. If a data source is no good, you find that out immediately as you try to integrate it rather than spending days, weeks or

[3] This is a common theme in agile approaches to project management. You never want to be far from a working product. For instance, in the popular Scrum approach to managing software projects, the key unit of collective work is "the sprint". A sprint is a relatively brief amount of time (in the scope of the entire project) to complete a set group of product features. At the end of the sprint, the features must be completed and the software working or they are cut. The goal is always to be close to a working program just like you should always be close to a working model.

[4] This idea is adapted from Eric Ries's excellent book *The Lean Startup* (Ries (2011)). In it he advocates an approach to developing start-up companies and businesses focus on rapid development and innovation. Ries supports developing a "Minimal Viable Product" for the company as quickly as possible and iterating on the feedback received for this initial product.

[5] But the key is to wait until you get this feedback. It's easy on your own or with a group of people to make a list of dozens mechanisms that a model *must* contain to be realistic. Once you have implemented those mechanisms in your model you might find out that no one actually cared about them. It is best to start small and then augment the model when there is a demand for some additional mechanism, than it is to spend a large amount of time implementing a very complex model and then to find out much of that work was unnecessary.

months planning your model with the assumption it's really there or you can really use it. Rapid iteration – failing fast and failing often – is a key goal in the model development process. It can be argued that your successes in life are directly proportional to the number of failures and wrong turns you take: the more things you try, the more times you will both succeed and fail. We believe the same is often true in modeling. By speeding up the process of identifying and iterating past failures, this agile approach to modeling will often result in higher quality models completed more quickly than approaches that rely on extensive planning.

Model Boundaries

There are many different mechanisms and entities we could include in our model of the hamster population[6]. Of course there are the hamsters themselves but there are also hamsters predators, the hamsters' food, climatic conditions that affect the growth and survival of the hamsters, urbanization, eutrophication that affects the hamsters' lake, and so on. Given that it would be impossible to include every single element and mechanism in our model, we must define the boundaries of the system.

We can illustrate model system boundaries using a boundary diagram as illustrated in the excellent book *The Electronic Oracle* (Meadows and Robinson (1985))[7]. When using a model boundary diagram, we classify items of interest into one of three categories:

Endogenous : Endogenous items are at the core of the model. They are things that the model itself determines. For instance, the size of the hamster population is endogenous to the model. The model itself simulates this population.

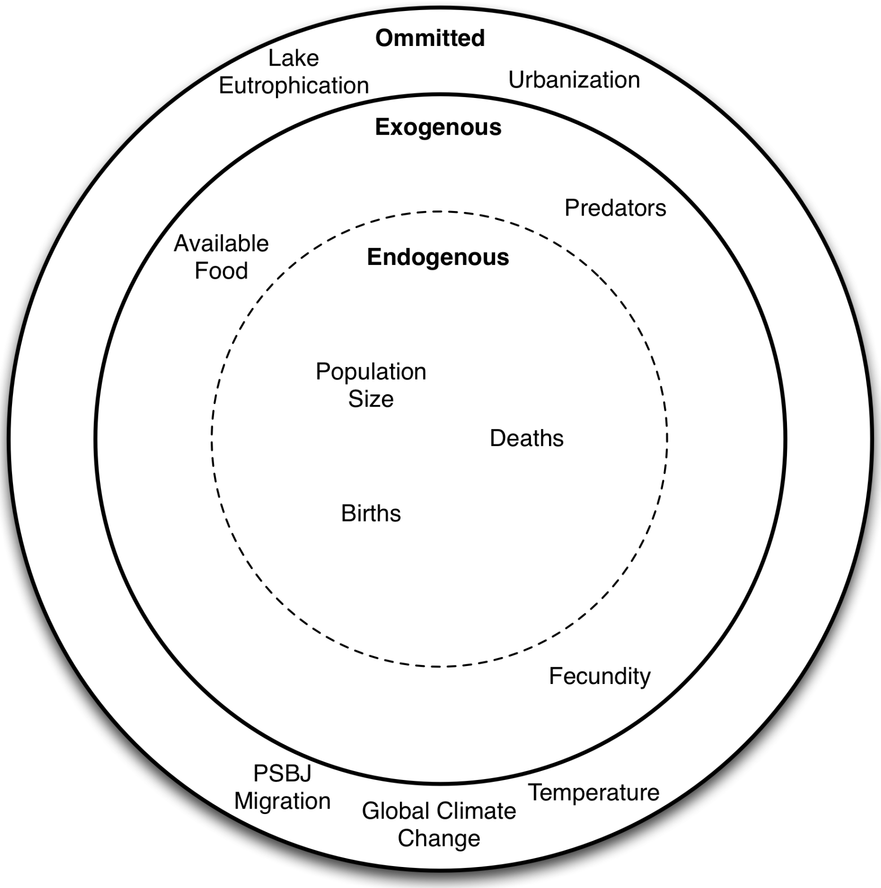
Exogenous : Exogenous items are those that you include in the model but which you do not directly simulate. For instance, if we thought temperature had a significant effect on hamster survival, we might want to include historical temperature data in the model. We do not want to simulate this data though, we just want to use it as an exogenous input into the model.

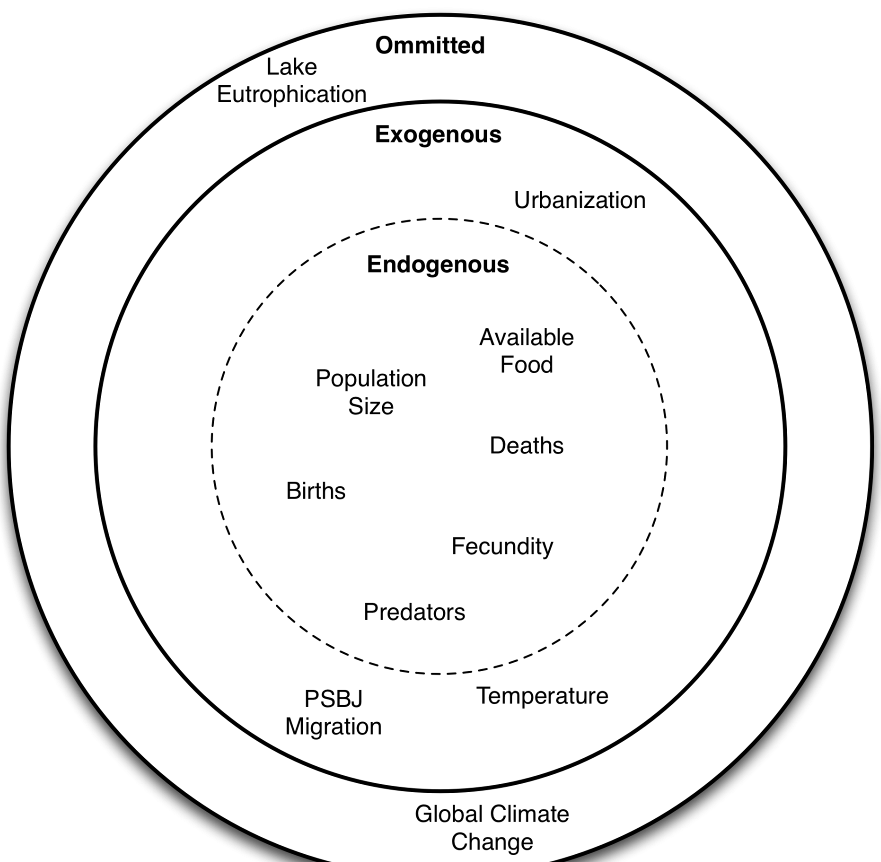
Omitted : Omitted items are those that, though we may acknowledge they do impact the hamsters either directly or indirectly, we choose not to include in the model. Even the most ambitious and comprehensive model will need to draw the line somewhere.

Figure 1 illustrates two different model boundaries for the hamster model. The top diagram depicts a small, conservative model with many features excluded

[6] The idea of the "butterfly effect" is that the flapping of a butterfly's wings in Europe, can initiate slight air disturbances that interact and magnify until they create a hurricane in Florida. If we believe in such avalanche effects to small events, the number of potential items we should include in the model is literally endless.

[7] This book provides an excellent overview of a number of different models and, very interestingly, it tracks the ultimate reception and the success or failure of these models.


Ommitted
Lake Eutrophication
Urbanization
Exogenous
Predators
Available Food
Endogenous
Population Size
Deaths
Births
Fecundity
PSBJ Migration
Global Climate Change
Temperature


Ommitted
Lake Eutrophication
Exogenous
Urbanization
Endogenous
Available Food
Population Size
Deaths
Births
Fecundity
Predators
PSBJ Migration
Temperature
Global Climate Change

from the model. The bottom figure illustrates a much more ambitious model where many additional items are made endogenous to the model and there are much fewer omitted items.

When developing a model, we recommend starting with the boundaries as narrow as possible. In the minimum viable model, you will want to omit as many different mechanisms as possible. As you receive feedback and people push for the inclusion of different mechanisms, you can slowly expand the boundaries of the model. We recommend starting small and expanding as necessary.

Exercise 9-1

Create a boundary diagram for a model of human population growth in the next 100 years. What would be the endogenous, exogenous and omitted items in this model.

Exercise 9-2

Create a boundary diagram for a model forecasting the total quantity of pencils sold within the united states for the next 50 years. What would be the endogenous, exogenous and omitted items in this model.

From Mental Models to Simulation Models

Generally speaking, a single individual should ultimately be responsible for the design and implementation of a model. Models "designed-by-committee" are understandably suffused with compromise and a greater lack of focus. That said, even though one person is ultimately calling the shots, many voices and perspectives are there to be heard in the modeling process. The more input there is into a model, the better the resulting model will most likely be.

The people you are working with generally will not be experts in modeling. Because of this, even if they are intimately familiar with the system you are attempting to model, it will sometimes be difficult to take their freeform insights and transform them into a formal model structure and accompanying numerical equations. In fact people often have great difficulty communicating and describing their own mental models of a system. A number of useful tools and techniques can be used to help elicit information on people's mental models. We discuss three of these tools in the following sections.

Reference Mode Diagrams

A reference mode is a graph that plots how the key stocks and variables in the system change over time. The x-axis of the graph is time, and the y-axis shows the values of the variables as they change. Sometimes reference modes are based on historical data, but you can also create them by asking those involved with the system to sketch out how they think the system will behave in different scenarios.

For our hamster model we could start simply by asking our friend to sketch out what he thinks will happen with the hamster population in the future assuming business as usual (remember that the status quo does not mean no-action). When we do this, he sketches out the top graph in Figure 2.

While your friend probably would use different terminology, to us the curve he sketched immediately looks like an exponential decay model. The instant we see this sketch we should start mapping out a stock and flow diagram in our mind to implement this type of model. Your friend does not need to understand any modeling concepts though, he just needs to be able to draw a picture of what he thinks will happen in the future. This is something that is easy to ask most people to do.

Let's go beyond the simple business as usual scenario. We can also use reference mode diagrams to elicit information on different scenarios. For instance, we have previously been told that development and encroachment on the hamster habitat are key factors reducing the hamster population size. Not only does the development consume key hamster habitat, the construction creates disturbances that have a further negative impact on the hamsters.

We can ask our friend to create a second sketch that shows how the hamster population would respond if development were suspended indefinitely. He responds by drawing the bottom graph in Figure 2. This graph shows the hamster population starting to recover after development stops, initially growing and then leveling off at a certain point.

Again, your friend never said this, but looking at this second drawing we should immediately start thinking of logistic growth models. The leveling off implies that there is some carrying capacity limit for the hamsters. This carrying capacity is probably a function of the available hamster habitat and the disturbances that are going on around the hamsters. We can start to sketch out stock and flow and causal loop diagrams to implement these types of dynamics and reproduce the behaviors our friend has drawn.

These are just two of the reference modes we might ask our friend to think about. We could go on to explore other scenarios and see how he thinks the changes in the scenarios would affect the hamster population. We could also ask him to sketch out other key variables in the system – such as the quantity of food available to the hamsters – to understand how he thinks these key variables interact. We could go on to interview other people familiar with the system and

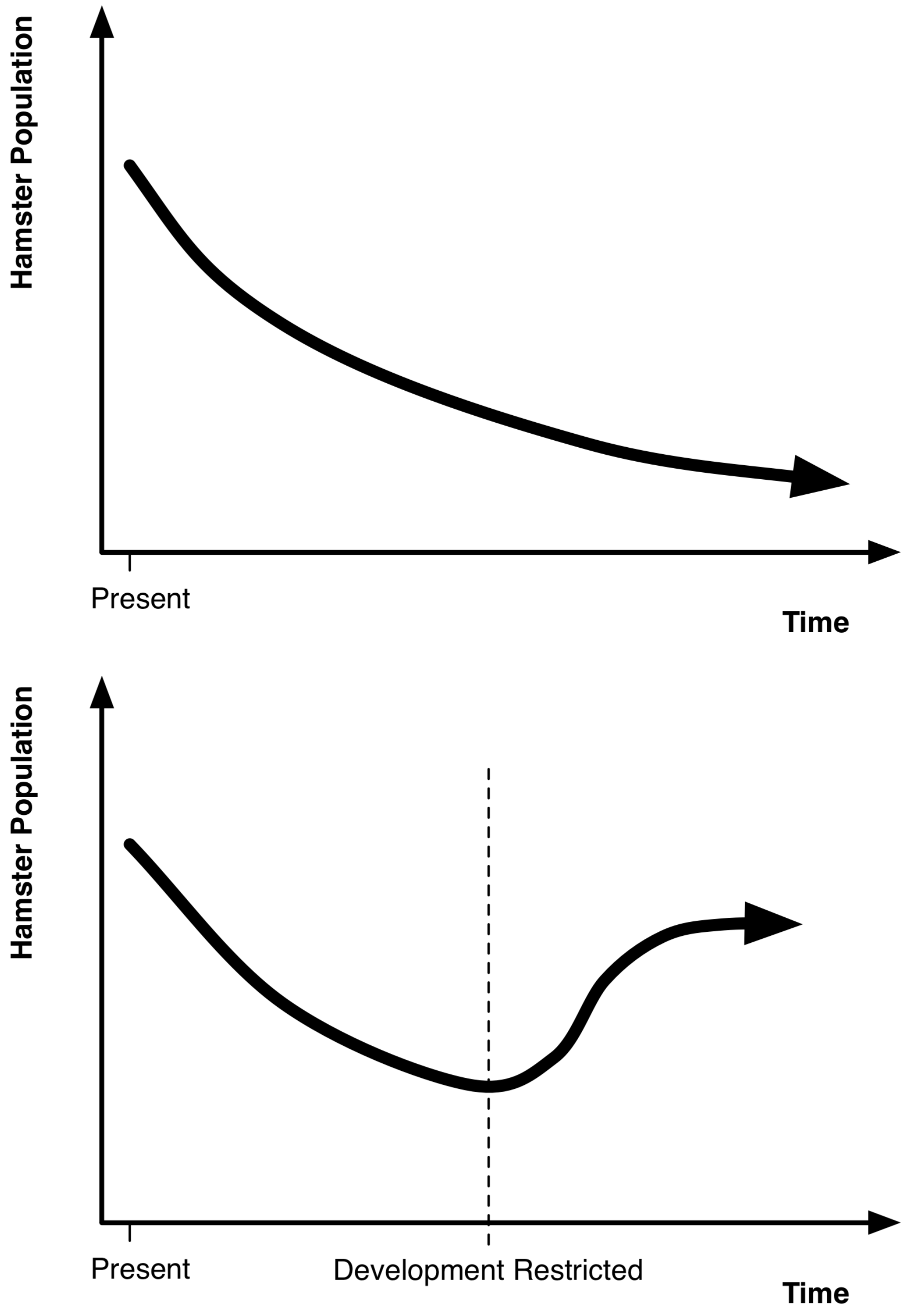


Figure 2. Sample reference modes for our hamster model.

take them through the same process. Ideally, all the reference modes between individual people will agree, but differences are in themselves also useful in revealing different mental models between our interviewees. Bridging differences will be a key interest of ours as we attempt to develop a persuasive model that will bring everyone on board and gain wide support.

Asking non-modelers to sketch out reference modes is a great technique for several reasons. Reference modes are accessible to laypeople, force your interviewees to be concrete, and provide you with very useful and actionable material. Really, a reference mode is a projection of an individual's mental model of the system. They may be unable (or unwilling) to explain their mental model to you in equations or even words, but they generally will be able to describe how they perceive the world using these reference mode diagrams – one small slice of their mental model at a time. Once you have the diagrams, you can proceed to translate them into model structure and equations.

Exercise 9-3

Draw a reference mode diagram for what you will think will happen to the total human population in the next 100 years. Draw additional reference modes for the following scenarios:

1. Cold fusion is invented in 2050. Limitless energy is available for free to everyone.
2. A plague wipes out 1/2 the human population in 2035. Each country is affected equally by the plague.
3. A process for cheaply converting a drop of oil directly into a kilogram of nutritious and delicious food stuffs is invented in 2030. This can replace the need for arable land, but oil become in even greater demand.

Exercise 9-4

You are hired by a paper company to create a model of paper consumption in the next fifty years. Draw reference mode diagrams of world paper demand for the most highly likely future scenarios as you see them. Consider the adoption of digital technologies and the decline of print media.

Pattern-Oriented Modeling

Pattern-oriented modeling focuses on identifying key patterns in the system to be modeled. For example, we may observe a boom-and-bust pattern in our hamster population that is triggered by unusually warm weather. When we develop our model, we formulate relationships and equations that will replicate this boom-and-bust pattern in the simulation.

Developed to help guide the creation of agent-based models, pattern-oriented modeling is very similar in concept to reference modes and system archetypes. Rather than building models around expected dynamic trajectories however, pattern-oriented modeling builds models to recreate patterns. Sometimes a pattern may be the same as a reference mode, but especially when dealing with agent based modeling you may not be able to define a pattern in terms of the dynamic trajectory of a reference mode. For a good overview of pattern-oriented modeling, see Grimm (2005).

Exercise 9-5

What patterns might you see in the how cities are located?

Exercise 9-6

What patterns might you see in the movement of a carnivore like a wolf? In an herbivore like a moose?

Exercise 9-7

What patterns might you see in the movement of a competition between companies in an expanding market? In a contracting market?

Group Model Building

Group modeling sessions are a powerful tool to capitalize on the collective thoughts of a group to inform model structure and design. Instead of individually surveying experts and those involved in a system, a group session with many interested parties can be conducted. The term “group model building” is a bit of a misnomer as generally the model itself will be built away from the group by the facilitator or modeler and the group work will be focused on identifying and ranking key variables and mechanisms and developing high-level causal loop or stock and flow diagrams. See Andersena and Richardsona (1997) for a very practical overview of running and facilitating group model building sessions.

Group modeling sessions can also benefit an organization independently of the success or failure of the model itself. You might expect that the mental models of individuals within an organization would be aligned and the members of the organization would share a common objective and understanding of the challenges and needs required to achieve this objective. However, this is often not the case as different organization members may hold distinct mental models of the organization's purpose and operation within the world. Additionally, it is quite possible that these differences may never be realized as people may fail to adequately communicate their mental models assumptions and beliefs during the course of regular interactions.

The group modeling process can force the concrete discussion of and revealing of these mental models and the stakes involved in having these differences. Once they are revealed, they can be discussed and reconciled, potentially leading to a greater congruity of viewpoints within the group and a greater shared purpose. Vennix, Scheper, and Willems (1993) carried out a survey of participants in group model building sessions and found that this process led to insights and a shared vision more quickly than occurred in standard meetings.

Wrapping it Up

Completing a model is in some ways just the first step in a modeler's work. Once the model is finished you need to make sure to develop adequate tests to ensure it is operating as designed. Moreover, a model by itself is often of little use. You will need to develop extensive sets of documentation, manuals and tutorials if you want the model to be used in practice by people other than yourself. Such efforts take time. Writing clear and useful documentation is a skill in itself and, if done right, may take as long as developing the model in the first place!

In general, it is important to remember the 80/20 rule which also applies to modeling. The first 80% of modeling work generally only takes 20% of the time while the last 20% of the work can take four times as long. Getting the small details right in a model can take much longer than implementing the bulk of the model structure.

Exercise 9-8

You have been asked to model crime trends in a major city. Write out a general overview of stages you might take to developing this model from start to finish.

Chapter 10

The Mathematics of Modeling

This chapter takes the modeling techniques introduced earlier in this book and places them within a firm mathematical framework. The contents of this chapter are quite technical in parts and to fully understand them requires knowledge of basic calculus and linear algebra. We present the material because it is important for both readers who want a deep understanding how their models operate and also those who wish to understand how System Dynamics fits within the larger field of mathematical modeling. For users who approach systems thinking and modeling from a more qualitative angle, this material may be browsed or safely skipped.

Differential Equations and System Dynamics

Differential equations are a common mathematical tool used to study rates of change. Some basic terminology needs to be learned in order to discuss differential equations. After introducing this new terminology, we will then tie it back to the modeling techniques you've already learned.

State Variable : A state variable is an object that represents part of the state of a system. For instance, in a population model you could have a state variable representing the current number of individuals in that population. In a model of a lake, you could have a state variable representing the current volume of water in the lake. In equations, state variables are often represented using Roman letters such as X, Y or Z.

Derivative : Derivatives define rates of change in state variables. For instance, if we had a state variable representing the size of a population, a derivative would specify how this population grows or shrinks over time. The population's derivative would aggregate all changes such as births, deaths and immigration or emigration to show the net change in the state variable over time. Similarly, in the case of a model of a lake, the lake volume state variable would have a derivative showing how much net water flows into or out of the lake over time.

Given a state variable X, the derivative of X with respect to time is generally written as dX/dt but can also be written as X' or $\dot{X}$.

Let's put this new terminology to work to define a simple model. We start by creating an exponential growth population model. We only need one state variable in this model to represent the size of the population. We denote this state variable as P. We need to define one parameter to control the growth rate in the population. We will denote this growth rate parameter α.

The resulting differential equation exponential growth model can be written simply as:

$$\frac{dP}{dt} = \alpha \times P$$

This indicates that the rate of change for the population for one unit of time is $\alpha \times P$. Our model is not quite fully specified yet as we do not know what the initial value of the population is. Differential equation models are often additionally specified by providing the values of the state variables at a specific point in time. Below we indicate that the population size at time 0 is 100.

$$P(0) = 100$$
$$\frac{dP}{dt} = \alpha \times P$$

You may have already noted that this model is easy to construct using the techniques we have already introduced in the book. In fact we have discussed this type of model several times. We could construct it with System Dynamics tools using a stock to represent the population (P), a flow to represent the change of population (dP/dt) and a variable to represent birth rate (α). We could specify our initial condition of a population size of 100, by setting the initial value for the stock for 100.

This is an important point. Many differential equation models[1] can be directly represented using the System Dynamics modeling techniques described in this book. Similarly, a System Dynamics model can be rewritten as a differential equation model.

From this perspective, System Dynamics models and differential equation modeling are one and the same. A System Dynamics model can be expressed using differential equation notation and vice versa. To see this in more detail, we can look at the mapping between System Dynamics and differential equation models. There is a one a one-to-one direct correspondence between the key System Dynamics primitives and components of a differential equation model.

[1]Specifically those where the denominator in the derivative dX/dt is always dt: a very wide class of commonly used models.

System Dynamics Primitive	Differential Equation Equivalent
Stock	State Variable (X, Y, etc…)
Flow	Derivative (dX/dt, dY/dt, etc…)
Variable	Constants/Parameters (α, β, etc…)

Since they do not differ significantly from a mathematical standpoint, what separates these two approaches to modeling? Where System Dynamics and differential equation modeling differ is in their focus and philosophy. The primary goal for differential equation modelers is analytic tractability (in other words, how easy is it to mathematically manipulate and understand the model's equations). This analytic tractability allows these modelers to derive definite results and conclusions from the model's equations. System Dynamics modelers generally are less concerned about analytic tractability and are more comfortable with simulating the model and drawing conclusions from observed trajectories and numerical results.

System Dynamics modelers, to go further, care greatly about communicating their models, deliberately mirroring reality to some extent and exploring the consequences of feedback. The differing focuses on communication between System Dynamics modelers and differential equation modelers can be seen in the method of naming variables. Differential equation models are generally dominated by abstract Greek symbols (e.g. α) while System Dynamics models generally clearly spell out variable names (e.g. "Birth Rate") and additionally use a model diagram to illustrate and communicate the relationships between different parts of the model.

Exercise 10-1

You have a System Dynamics model simulating water leaking out of a hole in a jar. You have a stock [**Jar**] with an initial value of 40. Roughly 10% of the water leaks out of the jar every time period and there is a single flow leading out of the jar with the rate `0.10*[Jar]`. Express this model using differential equations.

Answer Available

Exercise 10-2

You have a System Dynamics model simulating people becoming sick. You have two stocks in the model [**Healthy**] and [**Infected**]. There is a single flow, [**Infection**], going from the healthy to infected stock with a flow rate of

`0.05*[Infected]*[Healthy]`. Initially there are 100 infected people, and 1 infected person. Express this model using differential equations.

Answer Available

Exercise 10-3

You have a differential equation model of an animal population's growth (denoted P). The animals growth is parameterized by the parameter r and a maximum population size or carrying capacity of K. The following differential equations define this model:

$$P(0) = 500$$
$$r = 0.05$$
$$K = 10000$$
$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

Implement a System Dynamics version of this model. What is the size of the population after 100 years?

Answer Available

Solving Differential Equations

Given a differential equation or System Dynamics model specification, how do you go about determining the results of the model? This is typically referred to as "solving" the model. Since differential equation models and system dynamics models are essentially one and the same, the techniques used to solve differential equations can be directly applied to System Dynamics models and they are the techniques used by Insight Maker when you simulate any of the models in this book.

For most of the rest of this chapter, we use the differential equation terminology instead of the System Dynamics one. We do so first because it is more concise and more elegantly addresses the issues discussed in this chapter, but also because we want to familiarize you with its terminology and concepts. If you ever get lost, just refer to the System Dynamics to differential equation translation table we showed above.

Let's start our discussion of solving differential equations using our simple population model. As you recall, this model was:

$$P(0) = 100$$
$$\frac{dP}{dt} = \alpha \times P$$

What is the size of the population, at, let's say $t = 10$ given an α of 0.1? Calculus can be used to solve the model and answer this question. First we separate the terms of the derivative and integrate both sides of the equation. Thereafter it is a simple matter of algebra to solve for P:

$$\begin{aligned}
\frac{dP}{dt} &= \alpha \times P \\
dP &= \alpha \times P \ dt \\
\frac{1}{P} \ dP &= \alpha \ dt \\
\log(P) &= \alpha \times t + A \\
P &= e^{\alpha \times t + A} \\
P &= B \times e^{\alpha \times t}
\end{aligned}$$

In this equation two new variables A and B appeared (where we arbitrarily set $B = e^A$). These are unknown integration constants[2]. We can determine the values of the integration constants based on the initial conditions of the model, as we specified earlier that $P(0) = 100$. We evaluate the solution of the model at this initial condition to determine the value of B.

$$\begin{aligned}
P &= B \times e^{\alpha \times t} \\
100 &= B \times e^{\alpha \times 0} \\
100 &= B
\end{aligned}$$

Thus our generic equation for P at any time and for any α is:

$$P = 100 \times e^{\alpha \times t}$$

Plugging in $\alpha = 0.1$ and $t = 10$, we obtain:

$$\begin{aligned}
P &= 100 \times e^{0.1 \times 10} \\
&= 271.828...
\end{aligned}$$

[2]Recall from calculus that if A is a constant, then $x^2 + A \ dx = 2 \times x$. When we integrate $2 \times x$ we need to add back in the constant term. We don't know the value of this constant term immediately and we have to determine it later on.

For this simple population model we have shown that we can obtain the precise population value at any point in the future. It took a fair amount of algebra even for such a simple model, but we did it!

Unfortunately, many differential equation models cannot be solved using these techniques. For most complex models in practice, it is impossible to analytically determine the values of the state variables in the future. This inability to solve a model can be true for even very simple models. Take for example the following growth model similar to our original one:

$$P(0) = 100$$
$$\frac{dP}{dt} = \alpha \times P \times \log(P)$$

We have simply added a logarithm of P into our growth rate. Despite the smallness of this change, this model is now impossible to solve analytically. There is no analytic solution possible, but feel free to give it a try yourself (but please don't try too hard; we promise there is no solution). When developing complex models it should generally to be assumed that in practice no analytical solution will be available. In cases like these, how can we go about developing solutions to the equations and determining the trajectory of the state variables in the system?

Exercise 10-4

Solve the differential equation:

$$P(0) = 10$$
$$\frac{dP}{dt} = -\alpha$$

Answer Available

Exercise 10-5

Solve the differential equation:

$$P(0) = 10$$
$$\frac{dP}{dt} = 0.05 \times P$$

Answer Available

Exercise 10-6

Solve the differential equation:

$$P(0) = 20$$
$$\frac{dP}{dt} = \beta \times P^2$$

Answer Available

The answer is numerical approximation. Even if we can't solve the model equations analytically, we will always be able to approximate their results numerically. A number of different algorithms exist that allow us to approximate the solution to differential equations by repeatedly plugging values into them. To discuss these methods, it is useful to introduce some additional mathematical notation.

In our previous models, we have only looked at systems with a single state variable at a time. However, we can also consider systems containing multiple state variables. The Lotka-Volterra predator prey system we looked at earlier in the book is an example of this. Given two populations of animals – let's say a population of wolves (W) and a population of moose (M) – where the first population preys upon the second, we obtain a paired set of differential equations representing this predator prey relationship:

$$\frac{dM}{dt} = \alpha \times M - \beta \times M \times W$$
$$\frac{dW}{dt} = \gamma \times M \times W - \delta \times W$$

When looking at algorithms to solve sets of equations like these numerically, it can be useful to denote $\mathbf{y}$ as a vector of all the state variables in the model. So for the case of the exponential growth model $\mathbf{y} = [P]$ while for the Lotka-Volterra model $\mathbf{y} = [M, W]$. When using this notation, $\mathbf{y_t}$ indicates the vector of state variable values at a specific point in time, so $\mathbf{y_0}$ are the initial conditions for this model.

Additionally, we can denote $\mathbf{y'}$ as the vector of derivatives for the different state variables. We treat these derivatives as functions of the current time and the values of the other state variables. So, for instance, to determine the rate of change of the state variables in a model at $t = 10$, we would write $\mathbf{y'}(\mathbf{y_{10}}, 10)$ where $\mathbf{y_{10}}$ are the values of the state variables at $t = 10$.

The use of this notation might seem cumbersome, but it allows us to elegantly describe the mathematics of numerical solution algorithms without getting tied up in the details of a specific model.

Euler's Method

Leonhard Euler

The most basic numerical solution algorithm for differential equations is Euler's method[3]. Simply put, assuming we know the state of the system at time t and we wish to estimate the state of the system at time $t + \Delta t$ (where Δt is pronounced "delta-t" and represents the change in time) we can use the following equation:

$$\mathbf{y_{t+\Delta t}} = \mathbf{y_t} + \Delta t \times \mathbf{y}'(\mathbf{y_t}, t)$$

Let's walk through what this equation is doing. It first takes the derivatives for the state variables at the current point in time. It multiplies these rates of change by the Δt (how far in the future we want to know the results) and adds this change to the values of the state variables at the starting point in time. The result is an estimate of what the values in the future should be.

Let's now apply this to a concrete example. Start with our population scenario, but instead of exponential growth we have a fixed inflow of people at a rate of 20 per year. At $t = 0$ we have 100 people and we want to know the population in 10 years, using Euler's method we obtain the following:

$$\begin{aligned} P_{10} &= P_0 + \Delta t \times \frac{dP}{dt} \\ &= P_0 + 10 \times 20 \\ &= 100 + 200 \\ &= 300 \end{aligned}$$

[3]Leonhard Euler was a brilliant 18th century Swiss mathematician who made many great advances in the theoretical and applied mathematics.

Thus the population size in 10 years will be 300. In this simple example, Euler's method works perfectly and generates the exact same answer as we would have found using analytic solutions.

In general, however, we won't be so lucky. For most problems Euler's method will generate results that contain some level of error compared to what the true value should be. To see this let's explore our exponential growth model again with an α of 0.1. As a reminder, this model is:

$$
\begin{aligned}
P(0) &= 100 \\
\frac{dP}{dt} &= 0.1 \times P
\end{aligned}
$$

As we showed earlier, the precise solution to this model for $t = 10$ (to three decimal places) is 271.828. Let's see what we get using Euler's method with $\Delta t = 10$. Carrying out similar calculations as before we get:

$$
\begin{aligned}
P_{10} &= P_0 + \Delta t \times \frac{dP}{dt} \\
&= P_0 + 10 \times (0.1 \times P_0) \\
&= 100 + 10 \times (0.1 \times 100) \\
&= 100 + 10 \times 10 \\
&= 100 + 100 \\
&= 200
\end{aligned}
$$

So using Euler's method we obtain an estimate 200 for the population size at $t = 10$ when we know the true value should be around 272. That's a pretty large error! Why does this error come about? Why do we so significantly underestimate the final population size?

The reason is that we calculate the population's rate of change only at $t = 0$. For each of the ten years we are simulating, we assume the population grows at the rate it would if there were exactly 100 people. However, the population size is constantly increasing during these ten years, so the rate at which it grows should also be increasing. Imagine, the case of a bank account with an interest rate of 10% yearly. The bank account grows over time so the interest earned should also grow from year to year. It's the same principle of compounding here.

How do we address this issue? Using Euler's method, we can do it simply by changing how often we calculate the rates of change. In our previous calculation, we went straight from $t = 0$ to $t = 10$ all in one step, we used a Δt in Euler's equation of 10. However, we could employ an alternate calculation strategy where, for instance, we stepped from $t = 0$ to $t = 5$, recalculated the derivative based on the new population size and then stepped from $t = 5$ to $t = 10$. This would be equivalent to used a Δt of 5 and iterating the algorithm twice. Here is what we get doing this:

$$\begin{aligned}
P_5 &= P_0 + \Delta t \times \frac{dP}{dt} \\
&= P_0 + 5 \times (0.1 \times P_0) \\
&= 100 + 50 \\
&= 150 \\
P_{10} &= P_5 + \Delta t \times \frac{dP}{dt} \\
&= P_5 + 5 \times (0.1 \times P_5) \\
&= 150 + 5 \times 15 \\
&= 150 + 75 \\
&= 225
\end{aligned}$$

That result is certainly better, and we cut our error by over 33%. However, the error is still too large for most practical purposes. To improve the numerical estimation even more, we can apply smaller and smaller Δt's. You probably have a good grasp of the calculations now, so let's just show the results for each step of the simulation. We'll look at $\Delta t = 2$ and $\Delta t = 1$.

t	P
0	100
2	120
4	144
6	172.8
8	207.4
10	248.8

t	P
0	100
1	110
2	121
3	133.1
4	146.4

5	161.1
6	177.2
7	194.9
8	214.4
9	235.8
10	259.4

We see that as Δt gets smaller and smaller our results become more and more accurate. However, they are never perfect. There is always some error. Even if we made Δt as small as 0.1 (requiring 100 simulation steps), our final population size would be calculated to be 270, an error just under 1%.

Figure 1 illustrates the application of Euler's method to numerically estimate the trajectory for an example function. The smaller the Δt's in the estimation are, the better the results will be. Other terms that can be used in place of Δt are "Step Size", "Time Step" or just "DT". We prefer not to use the notation DT as it is easily confusable with the dt from differential equations. The latter indicates an infinitesimally small change, while step sizes are never infinitesimally small.

As you decrease the step size for the simulation, the results of the simulation become more and more accurate[4]. The cost of this increased accuracy, however, is increased computation time. The computation time required by your model is directly proportional to 1 over the step size. Thus, if you cut the step size in half, your model will take twice as long to complete simulating.

In general, you want a step size small enough that your results are "accurate enough," but one that isn't so small that the simulation takes too long to complete. A rule of thumb for choosing the step size is to choose a starting step size that results in a fast simulation. Then cut the value of the step size in half and simulate the model again. If the results have not change materially between these two simulations, keep the larger step size. If the results have changed, cut the step size in half again and repeat until the results cease to change.

Exercise 10-7

Take the differential equation:

[4] It is important to note at this point that when we discuss accuracies in this context we are specifically referring to models composed of continuous differential equations. If you are using agent based modeling or have discontinuities in your models – which could occur if you use If-Then-Else logic – then a smaller step size may not provide additional accuracy when there is some fundamental time step logic to the model.

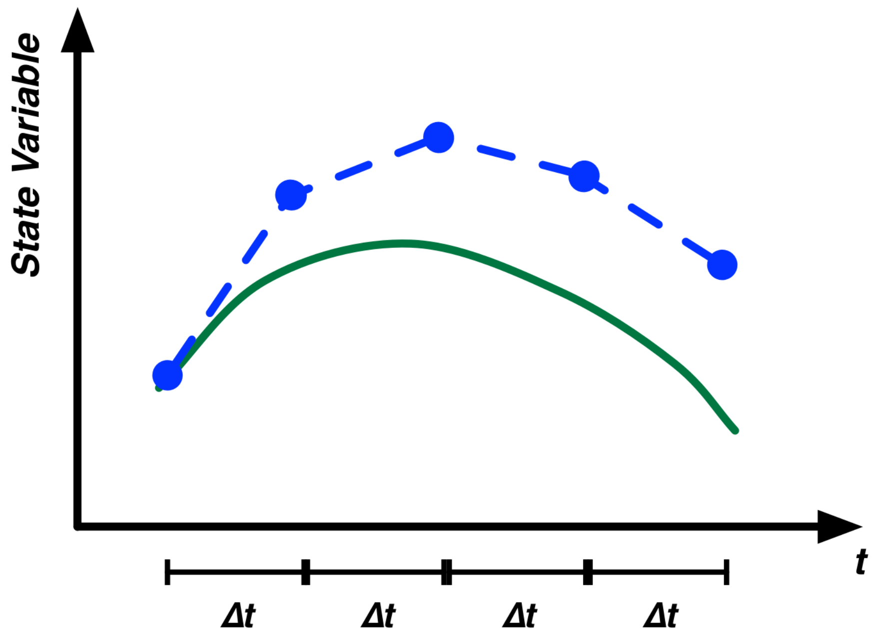


Figure 1. Euler's method at work. The true trajectory for the illustrative state variable is shown in green. Euler's method estimate of this trajectory is shown in blue.

$$P(0) = 20$$

$$\frac{dP}{dt} = \frac{100}{P}$$

Given a step size of 1, find the values of P at $t = 0, 1, 2, 3, 4, 5$ to one decimal place using Euler's method.

Answer Available

Exercise 10-8

Take the differential equation:

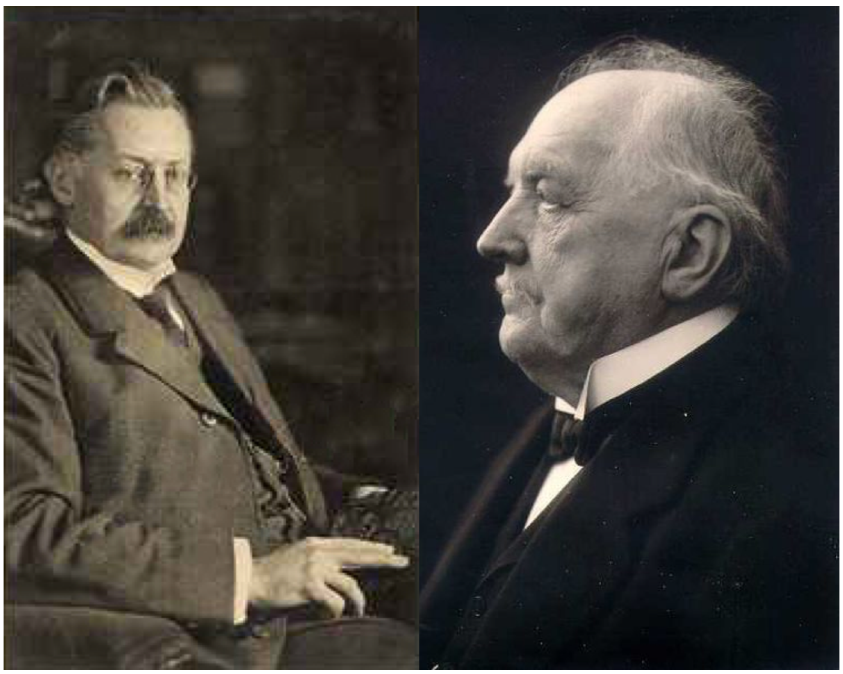
$$P(0) = 20$$

$$\frac{dP}{dt} = P^2 - P$$

Given a step size of 1, find the values of P at $t = 0, 1, 2, 3, 4, 5$ to one decimal place using Euler's method.

Answer Available

Runge-Kutta Methods



Carl Runge and Martin Kutta

Euler's method is not the only technique that can be used to numerically solve differential equations. Another popular set of techniques are called Runge-Kutta methods. Runge-Kutta methods are a family of numerical differential equation solvers. In fact Euler's method itself can be classified as a simple Runge-Kutta method.

One particular member of the Runge-Kutta family of methods that is widely used is a 4th-order Runge-Kutta method. This method differs from Euler's method in that for each step, it evaluates the model multiple times and averages the resulting derivatives. Briefly, the driving set of equations for this method is as follows:

$$\mathbf{y_{t+\Delta t}} = \mathbf{y_t} + \Delta t \frac{\mathbf{a} + 2 \times \mathbf{b} + 2 \times \mathbf{c} + \mathbf{d}}{6}$$

Where:

$$\mathbf{a} = \mathbf{y}'(\mathbf{y_t}, t)$$
$$\mathbf{b} = \mathbf{y}'(\mathbf{y_t} + \frac{\Delta t}{2} \times \mathbf{a}, t + \frac{\Delta t}{2})$$
$$\mathbf{c} = \mathbf{y}'(\mathbf{y_t} + \frac{\Delta t}{2} \times \mathbf{b}, t + \frac{\Delta t}{2})$$
$$\mathbf{d} = \mathbf{y}'(\mathbf{y_t} + \Delta t \times \mathbf{c}, t + \Delta t)$$

What this algorithm does is first compute the derivatives of the system at the current time (**a**) and use them to move the system forward to $t + \Delta t/2$. The derivatives are evaluated at $t + \Delta t/2$ (**b**) and this new set of derivatives is used to again move the system from t to $t + \Delta t/2$. A third set of derivatives are evaluated again at this mid-point (**c**) and they are the used to move the system from t to $t + \Delta t$. A fourth set of derivatives are evaluated at this point (**d**). The system is then returned to its starting point and a weighted average of derivatives are used to move the system the full time step. This weighting puts most of the weight on the middle two derivatives instead of the derivatives from the end points.

This 4th-order Runge-Kutta method is generally much more accurate than Euler's method for a given step size. Using a step size of 10 for our earlier population model, the Runge-Kutta method generates a value of 270.8. A step size of 5 yields a results of 271.7, just a smidgeon away from the precise value of 271.8. Recall that for Euler's method, even with a step size of 0.1 we still were not as accurate as the Runge-Kutta method with a step size of 5. Now it is true that this 4th-Order Runge-Kutta method does a lot more work than Euler's method for each step. It evaluates the model for times and has to do some averaging of derivatives. However, it is still much more accurate than Euler's method for an equivalent level of computational effort.

Exercise 10-9

Take the differential equation:

$$P(0) = 20$$
$$\frac{dP}{dt} = \frac{100}{P}$$

Given a step size of 1, find the values of P at $t = 0, 1, 2, 3, 4, 5$ to one decimal place using the 4th-Order Runge-Kutta method.

Answer Available

Exercise 10-10

Take the differential equation:

$$P(0) = 20$$
$$\frac{dP}{dt} = P^2 - P$$

Given a step size of 1, find the values of P at $t = 0, 1, 2, 3, 4, 5$ to one decimal place using the 4th-Order Runge-Kutta method.

Answer Available

Exercise 10-11

Discuss the differences between the 4th-Order Runge Kutta solutions and the Euler solutions. What causes these differences? Which method is most accurate? Why?

Exercise 10-12

Describe a model where Euler's method would be best suited as a numerical solver. Describe a model where the 4th-Order Runge-Kutta method would be best suited.

Numerical Solution Algorithms

This model explores the selection of the simulation step size and differential equation solution algorithm.

1. Create a new **Stock** named [**Population**].
2. Change the **Initial Value** property of the primitive [**Population**] to `100`.
3. Create a new **Flow** going from empty space to the primitive [**Population**]. Name that flow [**Net Growth**].
4. Create a new **Variable** named [**Growth Rate**].

5. Change the **Equation** property of the primitive [**Growth Rate**] to `0.1`.

6. Create a new **Link** going from the primitive [**Growth Rate**] to the primitive [**Net Growth**].

7. Change the **Flow Rate** property of the primitive [**Net Growth**] to `[Growth Rate]*[Population]`.

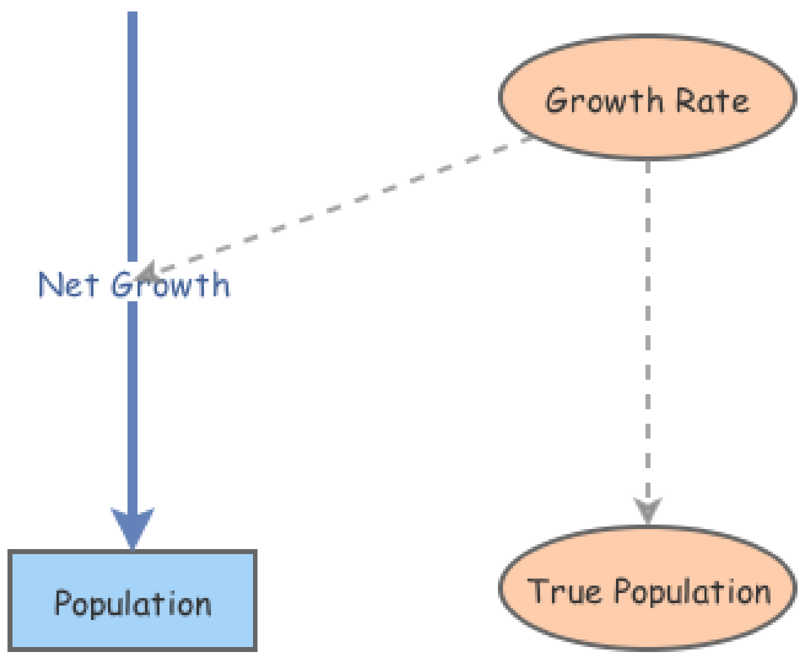
8. Create a new **Variable** named [**True Population**].

9. Create a new **Link** going from the primitive [**Growth Rate**] to the primitive [**True Population**].

10. Change the **Equation** property of the primitive [**True Population**] to `100*Exp([Growth Rate]*Years)`.

11. Change the **Simulation Length** property of the Time Settings to `10`.

12. The model diagram should now look something like this:

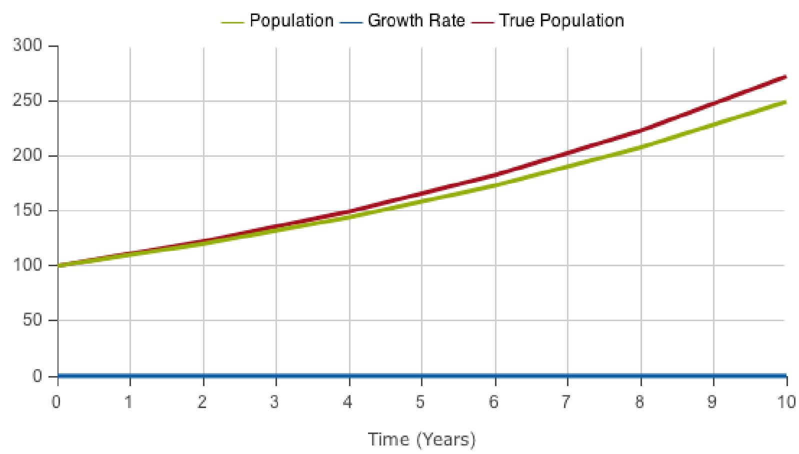


13. Let's now implement the simple exponential growth model we have discussed in this chapter. We have a population that starts with 100 people and increases at a rate of 10% per year. In addition to creating the stock and flow model, we have also created a variable, [**True Population**], that contains the analytical solution to the model.

14. First, we'll use Euler's method with a step size of 2 years and simulate the model.

15. Change the **Simulation Time Step** property of the Time Settings to 2.

16. Run the model. Here are sample results:



17. As we can see these results aren't very accurate. The value of the numerical estimated [**Population**] is quite different from the analytically determined value in [**True Population**]. Let's reduce the step size to 1 year and try again.

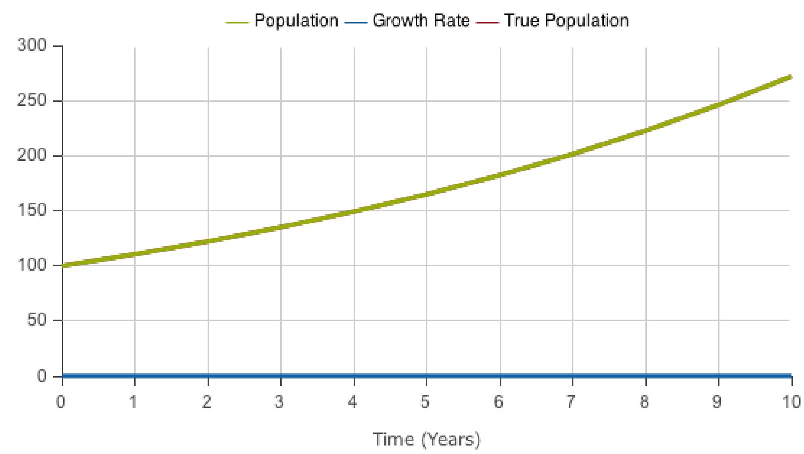
18. Change the **Simulation Time Step** property of the Time Settings to 1.

19. Run the model. Here are sample results:

20. This is better, but we're still off by a fair amount. We could experiment with continuing to reduce the step size, but let's instead switch now to the more accurate Runge-Kutta method. Will simulate the model again with a step size of 1 using the 4th-Order Runge-Kutta solution algorithm.

21. Change the **Analysis Algorithm** property of the Time Settings to `RK4`.

22. Run the model. Here are sample results:



23. That's a lot better! It's so close to being perfect that we can't even see the difference between the two lines in the figure. Just to be clear, let's see how quickly the results degrade when we increase the step size. Let's set the step size to 10 and simulate the model again.

24. Change the **Simulation Time Step** property of the Time Settings to `10`.

25. Run the model. Here are sample results:

26. That's still very good and much better than Euler's Method with a step size of 1. Why don't you go ahead now and experiment with different step sizes and the two solution methods to get a feel for their accuracies.

Other Solution Techniques

While being a brief introduction into numerical solution methods for differential equations, this should provide you with the background you need to intelligently make decisions about controlling the simulation of your models. It should help you identify potential sources of errors in your model and help you to adjust you simulation configuration to account for them.

The two methods we have looked at for solving differential equation models – Euler's method and a 4th-Order Runge-Kutta method – are widely used and they are what are built into Insight Maker. In addition to these two techniques, however, there are many other methods that are used in practice and you should be aware of this richer ecosystem of solution techniques.

Although we do not have space here to delve into the full ecosystem of numerical differential equation algorithms, it is useful to discuss one variant briefly: the adaptive step size algorithm. The methods we have looked at here use a fixed step size specified at the beginning of a simulation. Many models, however, might be characterized by highly variable trajectories. Part of the trajectory might be very smooth and unchanging while other parts might experience numerous rapid changes.

When using a fixed step size algorithm like the ones illustrated above, the step size must be set for the worse case scenario. The step size must be set to a small enough value to account for the rapidly changing areas. That said, the precision of this small step size is unnecessary on the smooth regions of the trajectory where the algorithm must do extra work for minimal gain in precision. Ideally, we would want to have a small step size for the rapidly changing areas and a large one for the smooth regions. This would result in the best of both worlds: high accuracy and quick computation.

Adaptive step size algorithms do just that. They adjust the step size dynamically based on the behavior of the model's derivatives. If the derivatives change rapidly, then the step size will be automatically shrunk; if the derivatives are constant or change very slowly the step size will automatically grow. Figure 2 illustrates the location of steps for an illustrative model using an adaptive step size algorithm. The steps are clustered around changes in the trajectory's derivatives in an attempt to maximize predictive accuracy while minimizing computation effort.

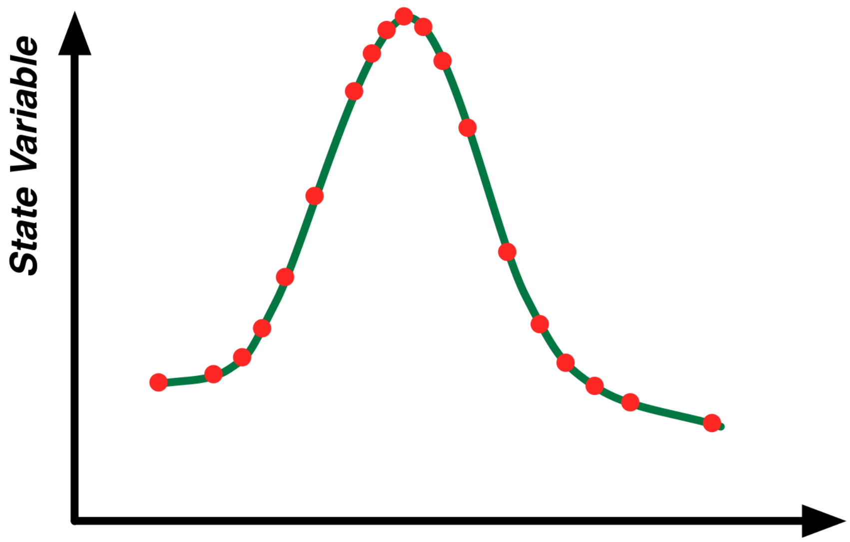


Figure 2. Illustration of an adaptive step size algorithm. Dots show the location of model evaluations. Evaluations are clustered around changes in the derivatives.

Chapter 11

Equilibria and Stability Analysis

This chapter extends on our mathematical analysis of models by introducing the concepts of points of equilibria and stability analysis. These types of analyses allow you to determine many behaviors of a system without needing to fully solving its differential equation model.

Although the trajectory for the state variables in differential equation models generally cannot be determined analytically, several key properties of models can often still be determined. These properties include:

- The location of equilibrium points
- The stability of the equilibrium points

An equilibrium point is defined as a set of state variable values that will cause the system to cease to change. Once the system enters an equilibrium configuration, it will not leave that configuration without an external stimulus. For instance, in our exponential growth model a single equilibrium point exists: that of zero people. If the population is empty, then the population will not grow and instead remain at 0 indefinitely.

In the exponential growth population model there is only one equilibrium point ($P = 0$). In other models you may have multiple equilibrium points. In a model of a highly infectious, incurable disease you can imagine a system where two equilibrium points exist: one where no one is infected and a second point where everyone is infected. As long as there were no infectious individuals, the population would remain healthy. If just a single infected individual were introduced into the population, the infection would, however, spread until everyone was infected and the population would then remain at that point (remember this hypothetical disease is incurable).

Multiple types of equilibria exist. Figure 1 illustrates what is known as the *stability* of equilibrium points. Each of the three panes in this figure show a different form of equilibrium for the ball. In all three the balls are in equilibrium:

if the no external forces come into play, the balls will not move. What differs in each of the three is what occurs if the balls are displaced a small amount.

Stable Equilibrium : In this type of equilibrium the ball will return to its original position if it is displaced. The structure of the system is such that the system is naturally attracted to the point of equilibrium. To use the physical metaphor, the equilibrium is at the bottom of a dip and the system naturally rolls into it.

Unstable Equilibrium : Here the ball will move further and further away from the point of equilibrium if it is displaced even a small amount. The equilibrium is unstable in that if we are just a small distance away from it, we move further away from it. To use the physical metaphor, the equilibrium is at the top of the hill and the system will move away from it unless it is placed at the exact point of equilibrium.

"Neutrally Stable Equilibrium" : This is a less common form of equilibrium and goes by several different names. In this case if the ball is moved it will stay fixed at its new location. It will not move closer to or further from the original equilibrium. Of the three types of equilibrium, this one is less interest or relevance in practice.

In the case of the highly infectious disease model, an equilibrium of everyone being healthy would be classified as an unstable equilibrium. The equilibrium would persist as long as no one brought the disease into the population (someone would not just spontaneously become ill), but if as little as a single sick person entered the population, the population would move further and further away from the equilibrium point of everyone being healthy and would never naturally return to it.

The equilibrium point of everyone being sick is, on the other hand, a stable equilibrium as no one recovers from the disease on their own. Even if you introduced healthy people into a population of sick individuals – moving the population away from the equilibrium – they too will eventually become sick restoring the population to the equilibrium of everyone being sick.

Exercise 11-1

Provide two examples each of situations where stable and unstable equilibria occur in nature. Describe these equilibria.

Answer Available

Equilibrium Points

Often, we can determine the equilibrium points for a system without fully needing to solve the trajectory for the state variables. Let's implement the

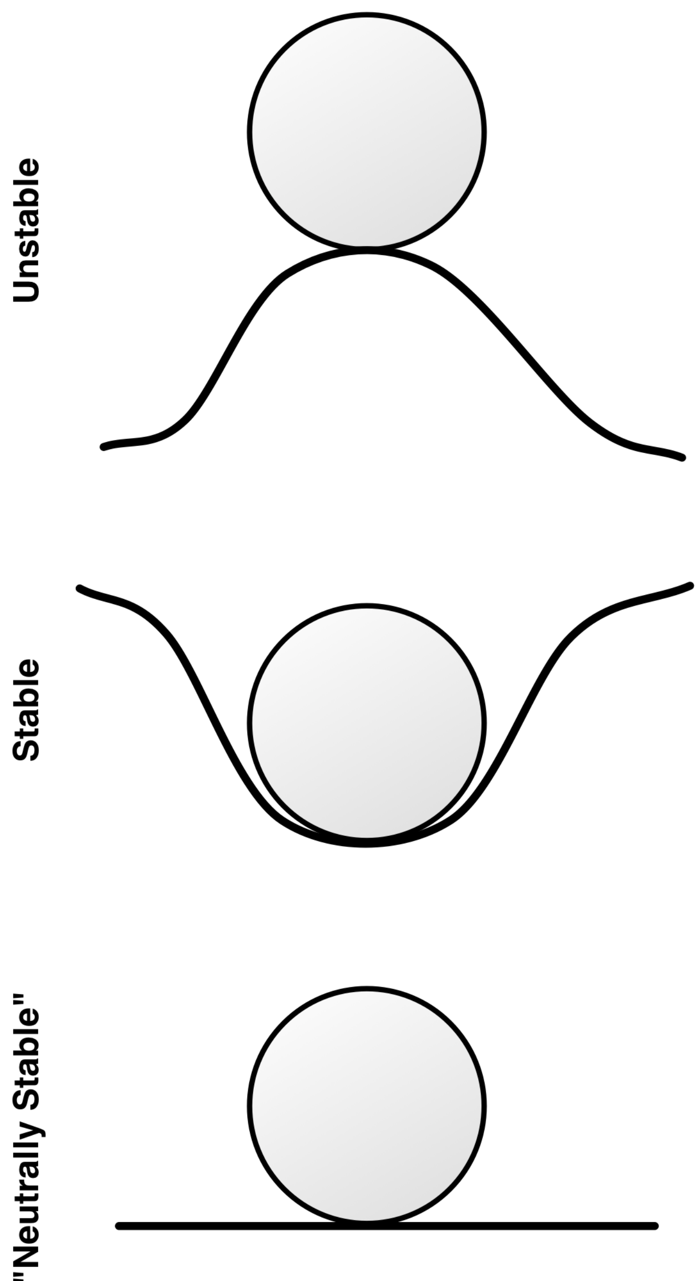


Figure 1. Three different types of stability.

simple disease model we've been discussing. We'll do so for both a differential equation model and a System Dynamics model, but we'll rely on differential equation version to do our analytic analysis.

One way to express the differential version of the model is to define two state variables: the number of healthy people (H) and the number of sick people (S). The rate of infection between sick and healthy people can be made a function of the number of people in each category. Clearly, if there are no sick people the infection rate is 0; but, just as clearly, if everyone is already sick then the infection rate will also be zero. One workable differential equation model to implement this behavior is shown below:

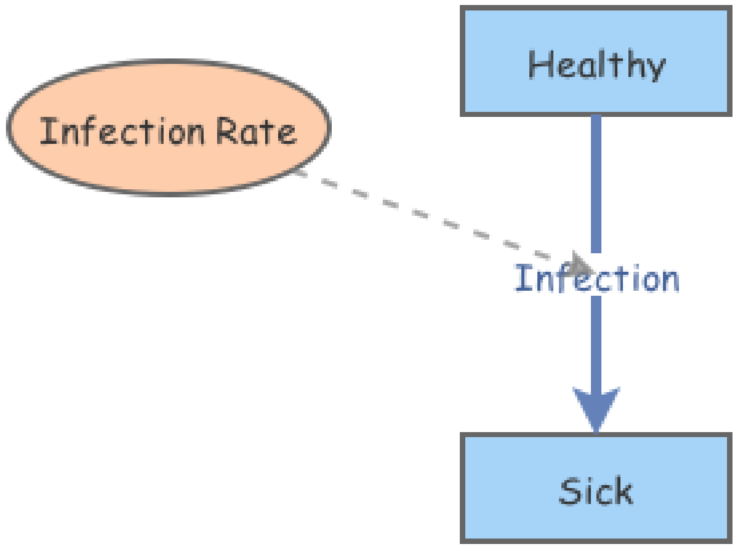
$$
\begin{aligned}
H(0) &= 100 \\
S(0) &= 1 \\
\frac{dH}{dt} &= -\alpha \times H \times S \\
\frac{dS}{dt} &= \alpha \times H \times S
\end{aligned}
$$

This model uses a single parameter (α) to control the infection rate. *alpha* is a non-zero positive value; the smaller α is, the slower the infection will progress and vice versa. This notation illustrates one of the clumsier aspects of implementing stock and flow models using differential equations. The flow values between two stocks have to be repeated twice once for each of the two connected state variable's derivatives.

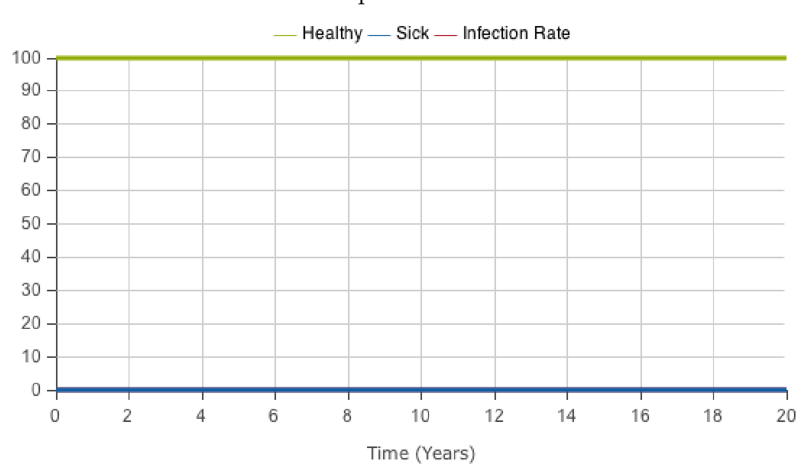
Incurable Disease

This model illustrates stable and unstable equilibria using the scenario of an incurable disease in a population.

1. Create a new **Stock** named [**Healthy**].
2. Create a new **Stock** named [**Sick**].
3. Create a new **Flow** going from the primitive [**Healthy**] to the primitive [**Sick**]. Name that flow [**Infection**].
4. Create a new **Variable** named [**Infection Rate**].
5. Create a new **Link** going from the primitive [**Infection Rate**] to the primitive [**Infection**].
6. The model diagram should now look something like this:



7. This is the structure of our model. We have two stocks of people with people moving from the healthy stock to the sick stock as they become infected. Let's add the values and equations now.
8. Change the **Initial Value** property of the primitive [**Healthy**] to `100`.
9. Change the **Initial Value** property of the primitive [**Sick**] to `0`.
10. Change the **Equation** property of the primitive [**Infection Rate**] to `0.01`.
11. Change the **Flow Rate** property of the primitive [**Infection**] to `[Infection Rate]*[Healthy]*[Sick]`.
12. There our model is fully setup. We've set it to start with everyone being healthy.
13. Run the model. Here are sample results:



14. These results are quite stable. Everyone is healthy and no one gets sick. That indicates we have an equilibrium here. Let's now experiment by making a single person in the population sick.

15. Change the **Initial Value** property of the primitive [**Sick**] to `1`.

16. Change the **Initial Value** property of the primitive [**Healthy**] to `99`.

17. Run the model. Here are sample results:

18. That's more interesting! We can see that everyone being healthy is an unstable equilibrium as the system moves away from it if we deviate from it by even a small amount. We can also see that the second equilibrium (everyone being sick) is stable as the system moves towards it naturally.

Finding the equilibria for differential equation models is by-and-large straightforward analytically. We simply need to harness the definition of an equilibrium point: an equilibrium point is one where the state variables are constant and unchanging. Since the derivatives represent changes in the state variables, this statement is equivalent to saying the derivatives for the model are 0 at equilibrium points.

Based on this, in order to find the equilibrium points we simply need to set the derivatives in our model to 0 and solve the resulting equations. For the disease model we get:

$$
\begin{aligned}
H(0) &= 99 \\
S(0) &= 1 \\
0 &= -\alpha \times H \times S \\
0 &= \alpha \times H \times S
\end{aligned}
$$

The initial conditions will determine what equilibrium is arrived at but they do not affect the existence of the equilibria. Furthermore, the two equations we have set to 0 are equivalent[1] so we can simplify these equations to simply be:

$$0 = \alpha \times H \times S$$

Simple inspection reveals that this equation is true if and only if either $H = 0$, $S = 0$, or $\alpha = 0$. Thus we have mathematically shown that our equilibria are either when everyone is sick or everyone is healthy (or there is no infection whatsoever). Granted this is a trivial conclusion for this model and we stated it earlier. However, for more complex models this type of analysis can be very useful and will often reveal that equilibria are functions of the different parameter values in the model and they may enable you to explicitly determine how the equilibria changes as the model configuration changes.

Let's try a more complex example. Remember the predator prey model from earlier? We had the following set of equations to simulate the relationship between a moose and wolf population:

$$
\begin{aligned}
\frac{dM}{dt} &= \alpha \times M - \beta \times M \times W \\
\frac{dW}{dt} &= \gamma \times M \times W - \delta \times W
\end{aligned}
$$

Let's determine what the equilibrium values are for this model. As before, we start by setting the derivatives to 0:

$$
\begin{aligned}
0 &= \alpha \times M - \beta \times M \times W \\
0 &= \gamma \times M \times W - \delta \times W
\end{aligned}
$$

Solving this set of equations is more difficult than for the disease model. However a little bit of algebra reveals two solutions. One when $M = 0$ and $W = 0$ (there are no animals at all), and the second when $M = \delta/\gamma$ and $W = \alpha/\beta$. This is an example of where the equilibrium location depends on the values of the model parameters.

[1] Although we expressed this model as a function of two state variables H and S, it only has one independent state variable. Given the fixed population size, you know the value of H given S and vice versa.

Exercise 11-2

Find the equilibrium points for the system:

$$\frac{dX}{dt} = X^2 + X - 3$$

Answer Available

Exercise 11-3

Find the equilibrium points for the system:

$$\frac{dX}{dt} = \sin(X)$$

Answer Available

Exercise 11-4

Find the equilibrium points for the system:

$$\frac{dX}{dt} = 2 \times X + Y + 5$$
$$\frac{dY}{dt} = 3 \times X - 4 \times Y$$

Answer Available

Exercise 11-5

Find the equilibrium points for the system:

$$\frac{dX}{dt} = X^2 - Y$$
$$\frac{dY}{dt} = -2 \times X^2 - Y^2$$

Answer Available

Exercise 11-6

Do the locations of equilibria depend on the starting conditions? Does the system arriving at an equilibrium depend on the starting conditions?

Why or why not?

The Phase Plane

Up until now, when looking at model results we have been focused on time series plots and we have mainly been interested in the trajectory of the variables and stocks over time. For the mathematical analysis of differential equations, however, the primary graphical tool is not this time series plot; instead it is what is known as a phase plane plot.

Phase planes are almost like scatterplots. They show one of the state variables plotted against another of the state variables. A scatterplot could be used to show the path for these two variables over the course of a simulation. In the predator prey model the results of a scatterplot of the wolf and moose population will be an ellipsoid. The two populations will cycle continuously. A phase plane plot is similar to this, but rather than just showing one of these cycles for a given simulation run, the phase plane shows the trajectories for *all* combinations of moose and wolf population sizes.

Figure 2 illustrates a phase plane plot for the predator prey system. The trajectory for one set of parameter and state variable values is highlighted in red and, as expected, we see a continual oscillation. We can also see the trajectories for all the other combinations of state variables. We see that the system will always oscillate and the size of this oscillation depends on the initial conditions for the state variables. This illustration provides us with a good deal of information in a single graphic and the phase plane plot is a great way to summarize the behavior of a system with two state variables.

Let's quickly explore the phase plane plots for a simpler system than our predator prey model. Take a system consisting of two state variables[2] both of which grow (or decay) exponentially. These state variables will be assumed to be independent from each other so the value of one does not affect the value of the other:

[2]Just a helpful reminder if you are starting to get lost in some of this differential equation jargon. A "state variable" is just a stock. Return to the table at the beginning of this chapter to see how these terms relate to the system dynamics modeling terminology we have already learned.

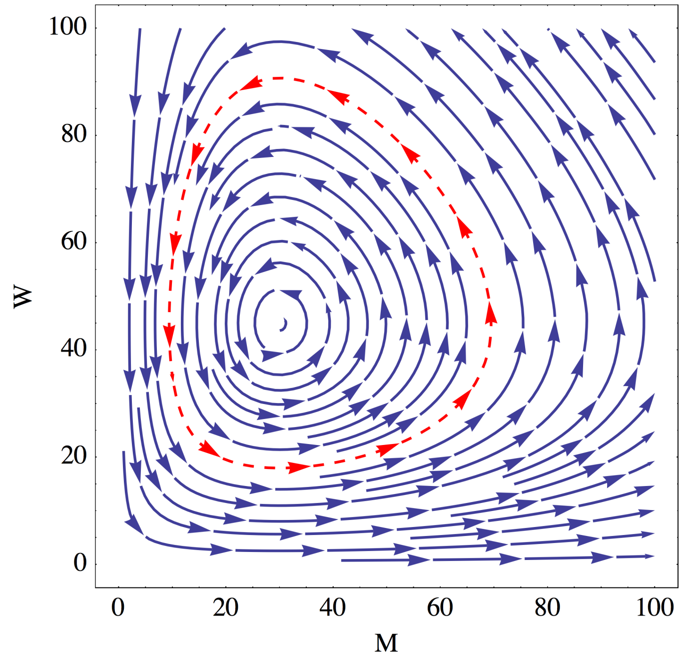


Figure 2. Predator prey phase plane plot. The trajectory for a single set of initial conditions is highlighted in red.

$$\frac{dX}{dt} = \alpha \times X$$
$$\frac{dY}{dt} = \beta \times Y$$

Clearly, there is an equilibrium point for this model at $X = 0$ and $Y = 0$. There are four general types of behavior around this equilibrium. One when $\alpha > 0$ and $\beta > 0$, one when $\alpha < 0$ and $\beta > 0$, one when $\alpha > 0$ and $\beta < 0$, and one when $\alpha < 0$ and $\beta < 0$. The phase planes for each of the four cases are shown in Figure 3.

From these plots we can visually determine how the stability of the equilibrium point at $X = 0, Y = 0$ changes as we change α and β. When $\alpha < 0$ and $\beta < 0$, we have a stable equilibrium; in all other cases we have an unstable equilibrium.

Exercise 11-7

Sketch out the phase plane for the differential equation model:

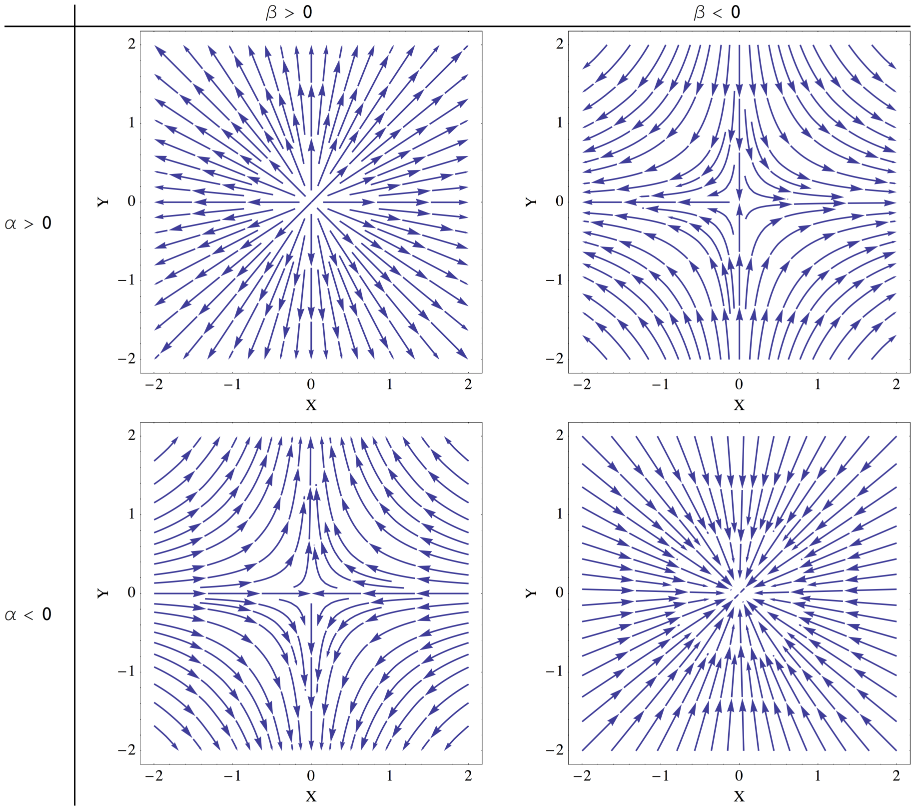


Figure 3. Phase planes for a simple two state variable exponential growth model.

$$\frac{dX}{dt} = -1$$
$$\frac{dY}{dt} = Y$$

Exercise 11-8

Sketch out the phase plane for the differential equation model:

$$\frac{dX}{dt} = X$$
$$\frac{dY}{dt} = Y^2$$

Stability Analysis

Now that we have learned how to analytically determine the location of equilibrium points, we may want to determine what type of stability occurs at these equilibria. As we stated earlier, for the incurable disease model it is trivial to arrive at the conclusion that the state of everyone being healthy is unstable while the state of everyone being sick is stable. In more complex models, it may be harder to draw conclusions or the stability of an equilibrium point may change as a function of the model's parameter values. Fortunately, there is a general way to determine the precise stability nature of the equilibrium points analytically.

The procedure to do this is relatively straightforward, but the theory behind it can be difficult to understand. The first key principle that must be understood is that of "linearization". To get a feel for linearization, let's take the curve in Figure 4. Clearly this curve is not linear. It has lots of bends and does not look at all like a line.

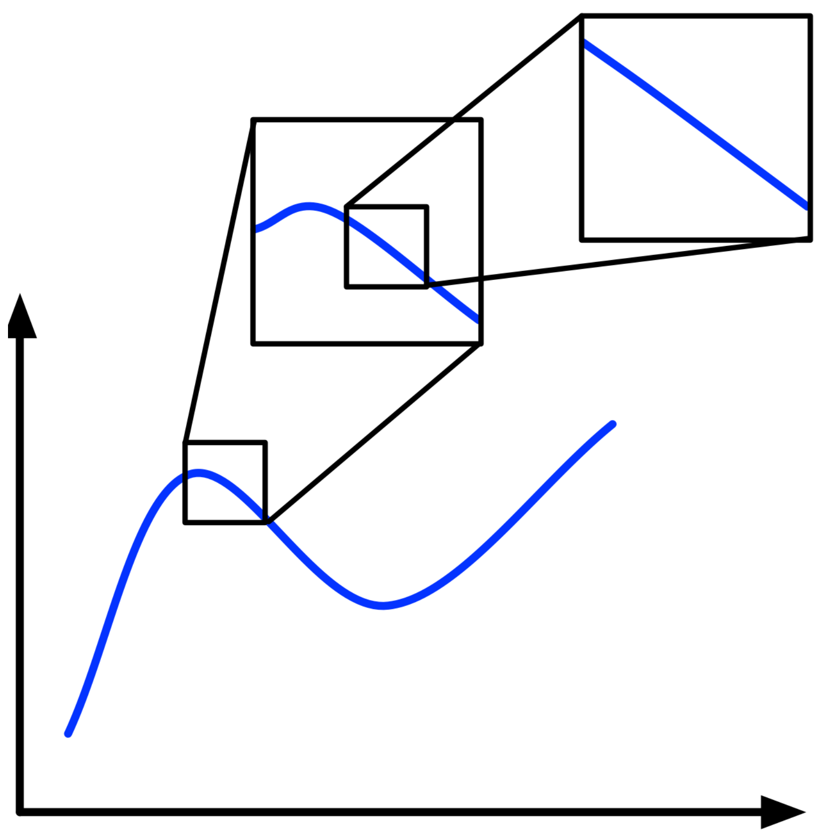


Figure 4. As we zoom in on a function it becomes more and more linear.

If we zoom in on any one part of the curve, however, the section we are zoomed in on starts to straighten out. If we keep zooming in, we will eventually reach a point where the section we are zoomed in on is effectively linear: basically a straight line. This is true for whatever part of the curve we zoom in on[3]. The more bendy parts of the curve will just take more zooming to convert them to a line.

We can conceptually do the same process for the equilibrium points in our phase planes. Even if the trajectories of the state variables in the phase planes are very curvy, if we zoom in enough on the equilibrium points, the trajectories at a point will eventually become effectively linear. The simple, two-state variable exponential growth model we illustrated with phase planes above are examples of a fully linear model. If we zoom in sufficiently on the equilibrium points for most models, the phase planes for the zoomed-in version of the model will eventually start to look like one of these linear cases.

Mathematically, we apply linearization to an arbitrary model by first calculating what is called the Jacobian matrix of the model. The Jacobian matrix is the matrix of partial derivatives of each of derivatives in the model with respect to each of the state variables:

$$\text{Jacobian} = \begin{bmatrix} \dfrac{\partial}{\partial X} X' & \cdots & \dfrac{\partial}{\partial Z} X' \\ \vdots & \ddots & \vdots \\ \dfrac{\partial}{\partial X} Z' & \cdots & \dfrac{\partial}{\partial Z} Z' \end{bmatrix}$$

The Jacobian is a linear approximation of our (potentially) non-linear model derivatives. Let's take the Jacobian matrix for the simple exponential growth model:

$$\frac{dX}{dt} = \alpha \times X$$
$$\frac{dY}{dt} = \beta \times Y$$

$$\text{Jacobian} = \begin{bmatrix} \dfrac{\partial}{\partial X} \alpha \times X & \dfrac{\partial}{\partial Y} \alpha \times X \\ \dfrac{\partial}{\partial X} \beta \times Y & \dfrac{\partial}{\partial Y} \beta \times Y \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

Exercise 11-9

Calculate the Jacobian matrix of the system:

[3]The one exception to this rule is if your curve is some sort of fractal. In this case no matter how much you zoom in on it, it will never become straight. In practice, however, this caveat is a non-issue.

$$\frac{dX}{dt} = X$$
$$\frac{dY}{dt} = Y^2$$

Answer Available

Exercise 11-10

Calculate the Jacobian matrix of the system:

$$\frac{dX}{dt} = X^2 - Y$$
$$\frac{dY}{dt} = -2 \times X^2 - Y^2$$

Answer Available

Exercise 11-11

Calculate the Jacobian matrix of the system:

$$\frac{dX}{dt} = X \times Y + \beta \times Y^2$$
$$\frac{dY}{dt} = \alpha \times X^3 + X^2 \times Y$$

Answer Available

This is complicated so don't worry if you don't completely understand it! Once you have the Jacobian, you calculate what are known as the eigenvalues of the Jacobian at the equilibrium points. This is also a bit complicated, so if your head is starting to spin, just skip forward in this chapter!

Nonetheless, eigenvalues and their sibling eigenvectors are an interesting subject. Given a square matrix (a matrix where the number of rows in equals the number of columns), an eigenvector is a vector which, when multiplied by the matrix, results is the original vector multiplied by some factor. This factor is known as an eigenvalue as is usually denoted λ. Given a matrix $\mathbf{A}$, an eigenvalue λ with associated eigenvector $\mathbf{V}$; the following equation will be true:

$$\mathbf{A} \times \mathbf{V} = \lambda \times \mathbf{V}$$

Let's look at an example for a 2×2 matrix:

$$\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \times \mathbf{V} = \lambda \times \mathbf{V}$$

What eigenvector and eigenvalue combinations satisfy this equation? It turns out there are two key ones:

$$\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 2 \times \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \end{bmatrix} = -1 \times \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Naturally, any multiple of an eigenvector will also be an eigenvector. For instance, in the case above, $[1, 0.5]$ and $[-2, 2]$ are also eigenvectors of the matrix.

We can interpret eigenvectors geometrically. Looking at the 2×2 matrix case, we can think of a vector as representing a coordinate in a two-dimensional plane: $[x, y]$. When we multiply our 2×2 matrix by the point, we transform the point into another point also in the two-dimensional plane. Due to the properties of eigenvectors, we know that when we transform an eigenvector, the transformed point will just be a multiple of the original point. Thus when a point that is on a matrix's eigenvector is transformed by that matrix, it will move inwards or outwards from the origin along the line defined by the matrix's eigenvector.

We can now relate the concept of eigenvalues and eigenvectors to our differential equation models. Take a look back at the phase planes for the exponential model example. For each of the phase planes, there are at least two straight lines of trajectories. In these cases the x- axis and the y-axis are the locations of these trajectories. If you have a system on the x- or y-axis in this example it will remain on that axis as it changes. This indicates that for this model, the eigenvectors are the two axises as a system on either of them does not change direction as it develops. That's the definition of of an eigenvector.

For our purposes though, we do not really care about the actual direction or angle for these eigenvectors. We instead care about whether the state variables move inwards or outwards along these vectors. We can determine this from the eigenvalues of the Jacobian matrix. If the eigenvalue for an eigenvector is negative, then the values move inwards along that eigenvector; while if the eigenvalue is positive, they move outward along the eigenvector.

These eigenvalues tell us all we need to know about the stability of the system. Returning to our illustration of stability as a ball on a hill, we can think of these eigenvalues as being the slopes of the hill around the equilibrium point. If the eigenvalues are negative, the ground slopes down towards the equilibrium point forming a cup (leading to a stable equilibrium). If the eigenvalues are positive, the ground slopes away from the equilibrium point creating a hill (leading to an unstable equilibrium).

Eigenvalues can be calculated straightforwardly for a given Jacobian matrix. Briefly, for the Jacobian matrix J, the eigenvalues λ are the values that satisfy the following equation where det is the matrix determinant and I is the identity matrix.

$$0 = det(J - \lambda \times I)$$

We can do a quick example of calculating the eigenvalues for the Jacobian matrix we derived for our two-state variable exponential growth model.

$$\begin{aligned}
0 &= det\left(\begin{bmatrix}\alpha & 0\\ 0 & \beta\end{bmatrix} - \lambda \times \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)\\
&= det\left(\begin{bmatrix}\alpha-\lambda & 0\\ 0 & \beta-\lambda\end{bmatrix}\right)\\
&= (\alpha-\lambda)\times(\beta-\lambda) - 0\times 0\\
\lambda = \alpha, \lambda = \beta &
\end{aligned}$$

That is a fair amount of work to do. It's even more complicated if you have more than two state variables. However, once you have gone through the calculations and determined the linearized eigenvalues for your equilibrium points, you know everything you might want to know about the stability of the system.

Exercise 11-12

Find the eigenvalues of the following matrix:

$$\begin{bmatrix}2 & 4\\ 4 & 2\end{bmatrix}$$

(Bonus: Determine the associated eigenvectors.)

Answer Available

Exercise 11-13

Find the eigenvalues of the following matrix:

$$\begin{bmatrix} 2 & 0 \\ 5 & 1 \end{bmatrix}$$

(Bonus: Determine the associated eigenvectors.)

Answer Available

Exercise 11-14

Find the eigenvalues of the following matrix:

$$\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$$

(Bonus: Determine the associated eigenvectors.)

Answer Available

Exercise 11-15

$$\begin{bmatrix} \alpha & \beta \\ 0 & \beta \end{bmatrix}$$

(Bonus: Determine the associated eigenvectors.)

Answer Available

In the exponential growth model we can see that when the eigenvalues are both negative we have a stable equilibrium (refer to the graphs we developed earlier), while if either one is positive (or they both are) we have an unstable equilibrium. This makes a lot of sense as if either one is positive it pushes the system away from the equilibrium making it unstable. While if they are both negative then they both push the system towards the equilibrium point. Visualize the ball sitting in the cup or on the hill.

Looking at it this way, we realize that *all we need in order to understand the stability of an equilibrium point are the eigenvalues of the Jacobian at* *the equilibrium point.* This is an incredibly powerful tool. It reduces the complex concept of stability, into an analytical procedure that can be applied straightforwardly.

Let's now look at some more examples.

First let's take our simple disease model from earlier. If you recall that model was:

$$\frac{dH}{dt} = -\alpha \times H \times S$$
$$\frac{dS}{dt} = \alpha \times H \times S$$

First let's calculate the Jacobian for this model. We take the partial derivatives of each of the two derivatives with respect to each of the two state variables to create a two-by-two matrix:

$$\text{Jacobian} = \begin{bmatrix} \frac{\partial}{\partial H} - \alpha \times H \times S & \frac{\partial}{\partial S} - \alpha \times H \times S \\ \frac{\partial}{\partial H} \alpha \times H \times S & \frac{\partial}{\partial S} \alpha \times H \times S \end{bmatrix} = \begin{bmatrix} -\alpha \times S & -\alpha \times H \\ \alpha \times S & \alpha \times H \end{bmatrix}$$

Next, we evaluate this Jacobian at one of our equilibrium points. Let's choose the one where the $S = 0$ (no one is sick) and $H = P$ (where P is the population size) so everyone is healthy:

$$\begin{bmatrix} 0 & -\alpha \times P \\ 0 & \alpha \times P \end{bmatrix}$$

We can now find the eigenvalues for this matrix. Once we go through the math we get two eigenvalues: 0 and $\alpha \times P$. What do these mean? Well, since one of the eigenvalues is positive, this indicates we have movement away from the equilibrium point along at least one of the eigenvectors. The other vector has no movement (0 as the eigenvalue), but this one positive value will ensure we have an unstable equilibrium. Again, think of the ball, the positive eigenvalue indicates the ground slopes downwards from the equilibrium point so a ball balanced on top of this hill will be very unstable.

Now let's do the second equilibrium. The one where $S = P$ and $H = 0$ (everyone is sick). Let's evaluate the Jacobian at this equilibrium:

$$\begin{bmatrix} -\alpha \times P & 0 \\ \alpha \times P & 0 \end{bmatrix}$$

Now let's find the eigenvalues for this matrix. Once we go through the math we get two eigenvalues: this time 0 and $-\alpha \times P$. Again, the 0 eigenvalue can be ignored as it does not cause growth or change. The second eigenvalue however is negative, indicating the system moves toward the equilibrium point again. Look back at our exponential growth phase planes. Negative coefficients indicate trajectories towards the equilibrium (create a cup for the ball). Thus this second equilibrium is a stable one.

It's time to look at a more complex example, we'll consider our predator prey model. First we calculate the Jacobian matrix for this model:

$$\text{Jacobian} = \begin{bmatrix} \frac{\partial}{\partial M}\alpha \times M - \beta \times M \times W & \frac{\partial}{\partial W}\alpha \times M - \beta \times M \times W \\ \frac{\partial}{\partial M}\gamma \times M \times W - \delta \times W & \frac{\partial}{\partial W}\gamma \times M \times W - \delta \times W \end{bmatrix} = \begin{bmatrix} \alpha - \beta \times W & -\beta \times M \\ \gamma \times W & \gamma \times M - \delta \end{bmatrix}$$

Now that we have the Jacobian, we'll evaluate it at the trivial equilibrium of $M = 0$ and $W = 0$. The resulting matrix is:

$$\begin{bmatrix} \alpha & 0 \\ 0 & -\delta \end{bmatrix}$$

The eigenvalues of this matrix are α and $-\delta$. Thus one of the eigenvectors approach the equilibrium and the other moves away from it. This means we have an unstable equilibrium, which is actually good news as it indicates that the two animal populations will not spontaneously go extinct.

Let's now evaluate the more complex equilibrium point we identified earlier of $M = \delta/\gamma$ and $W = \alpha/\beta$. First we calculate the Jacobian at this point:

$$\begin{bmatrix} 0 & \frac{-\beta \times \delta}{\gamma} \\ \frac{\gamma \times \alpha}{\beta} & 0 \end{bmatrix}$$

When we calculate the eigenvalues for this point we obtain $i\sqrt{\alpha \times \delta}$ and $-i\sqrt{\alpha \times \delta}$. Here the i indicates the imaginary number $\sqrt{-1}$. That's a little strange, so how do we interpret this? Well, it turns out that imaginary numbers in the eigenvalues indicate oscillations in the phase planes, thus this results means we have oscillations around the point of equilibrium. Since we have no real component in the eigenvalues, there is neither attraction towards the point of equilibrium or repulsion away from it so we have a stable oscillation around the equilibrium.

Of course we already knew that from our simulations, but this stability analysis allows us to mathematically determine this relationship, a capability that is a very powerful tool. The following table summarizes the different types of eigenvalues that can be found for a system with two state variables and their associated stabilities.

Real Parts	Imaginary Part?	Stability
Both Equal to 0	No	Neutrally Stable
Both Equal to 0	Yes	Stable Oscillations
Both greater than or equal to 0	No	Unstable
Both greater than or equal to 0	Yes	Unstable Oscillations
Both less than or equal to 0	No	Stable
Both less than or equal to 0	Yes	Damped Oscillations (Stable)

Exercise 11-16

A system's Jacobian matrix has two eigenvalues at an equilibrium point. Determine the stability of the system at this point for the following pairs of eigenvalues:

1. 0.5 and 4
2. -3 and 0.2
3. -3 and -1

Answer Available

Exercise 11-17

A system's Jacobian matrix has two eigenvalues at an equilibrium point. Determine the stability of the system at this point for the following pairs of eigenvalues:

1. $1 + 2i$ and $1 - 2i$
2. $-3 + 0.2i$ and $-3 - 0.2i$
3. $0.2i$ and $0.2i$

Answer Available

Exercise 11-18

A system's Jacobian matrix has a single eigenvalue at an equilibrium point. Determine the stability of the system at this point for the following eigenvalues:

1. 2.5
2. -1.2
3. 0.5

Answer Available

Analytical vs. Numerical Analysis

The majority of this book has been focused on the numerical analysis of models and the qualitative conclusions that can be drawn from these results. This chapter has introduced a set of analytical tools that can be used – for the most part – to analyze the same models we have presented elsewhere in the book. Now take a moment to reflect on these different forms of analysis and what each one can offer.

The great benefit of the analytical techniques we present here is that they can provide precise answers to the general behavior of the system. Most of these same answers can also be determined numerically (e.g. running the simulation many times and exploring the results), but those answers will be less precise and definite. If you manually attempt to explore the parameter space of your model, it is possible that you could miss some set of parameter values that will give you unexpected behavior. An analytical analysis may be fully comprehensive and can guarantee the completeness of your conclusions.

A weakness of analytical methods is that your model must be solvable analytically. This means that you will probably need to keep your model from growing too complex in order to keep it analytically tractable. Also, some common functions such as `IfThenElse` logic can make analytical work much more difficult. Further, some models may simply be impossible to analyze analytically and these insolvable models may in fact be very simple in practice. For example, any model containing both X and $\log(X)$ in the same equation will be intractable to many forms of analysis.

We think both analytical and numerical work has a lot of applicability in practice. We do worry, though, about some of the analytical models and work we see presented or published. Sometimes these models seem to us to be much too simple to adequately represent the system they are supposed to be modeling. True, analytically the results of the models appear elegant and clear, but if the model is too simple to be relevant these results have little use and may actually be very misleading in practice. We worry sometimes that a focus on analytical work[4] leads to modelers prioritizing analytical tractability over model utility in their decisions. We believe a focus on analytical results can lead to reductionist models with reduced practical utility and we caution modelers against becoming too focused on elegant solutions and the expense of relevance. Where available, more realistic models are preferable, even if they require numerical solutions than overly simplistic analytically solvable ones.

Exercise 11-19

What are the equilibrium points of the following system and their associated stabilities?

$$\frac{dX}{dt} = X \times Y + X^2$$
$$\frac{dY}{dt} = Y + 2$$

Answer Available

Exercise 11-20

What are the equilibrium points of the following system and their associated stabilities? α is a scalar number that may be positive or negative.

$$\frac{dQ}{dt} = -XQ \times R + R$$
$$\frac{dR}{dt} = \alpha - \alpha \times R^2$$

Answer Available

Exercise 11-21

You have a system dynamics model of a population of wolves. This model consists of a single stock [**Wolves**] (initial value 100), a single flow going into the stock [**Net Growth**], a parameter [**Growth Rate**] (value of 0.05), and a parameter [**Carrying Capacity**] (value of 6,000). The flow has the equation `[Growth Rate]*[Wolves]*(1-[Wolves]*[Carrying Capacity])`.

[4] And, rightly or wrongly, analytical work is generally considered more prestigious and "serious" than numerical work.

Build this model determine the location of the equilibria and their stability. Then prove these conclusions analytically.

Answer Available

Chapter 12

Going Global

Engaging people to cause positive action and change is one of the key goals of systems thinking and modeling. The growth of the Internet has created amazing opportunities to reach out and connect with people in ways that have never before been possible.

The Internet makes it easy to share models with other people. Not only can you email a specific person the tables and graphs of a model's results, but you can also build webpages publishing these results to share with the world. What is more, these results do not have to be limited to static data. Using Insight Maker, you can include an interactive version of your model allowing others to experiment with it directly on your webpage. This can be done on any page you have rights to edit including your personal website, a blog, and a company's information page.

Furthermore, the information flow doesn't have to be one-way from you to others. In a webpage you can include a feedback or comment form that allows anyone to comment and share his or her thoughts on the model right next to the model itself. These comments can be saved directly on the page allowing other people to read them and enabling a discussion to form around the model. This creates many avenues for collaboration and learning that would simply be impossible without the Internet.

In this chapter, we will show you how to develop webpages to showcase your insights and models to the world. We'll also show how to include tools to engage viewers and start a dialogue about your models. Before jumping into the models themselves, we will lay the groundwork by introducing the basic principles of web development. Once we have introduced these key principles, we'll walk through two examples of developing interactive models.

The Web in a Nutshell

The World Wide Web is based on a collection of many different technologies that work together. When developing a webpage there are three major technologies that you need to be familiar with: HTML, CSS and JavaScript. Each of these technologies or languages plays a different role in the development of a webpage.

Technology	Commonly Called	Usage
Hypertext Markup Language	HTML	Webpage Structure
Cascading Style Sheets	CSS	Webpage Style
ECMAScript	JavaScript	Webpage Interactivity

The web is interesting in that each of these technologies is based on old-fashioned, simple text files. You write HTML text files, you write CSS files, and you write JavaScript files[1]. You do not need any fancy tools to create these files. Any simple text editor will do. A web browser takes the simple instructions and code in these files and converts them to the rich interactive webpages you see when you browse the Internet.

When teaching web development many books and sources will recommend that you use some kind of interactive web site builder (like Adobe Dreamweaver http://www.adobe.com/products/dreamweaver.html). That is certainly a great way to get up and running, but ultimately you will find the approach very limiting. To truly harness the different tools offered to you by the Internet, you will need to have some understanding of the underlying technologies and be able to work with them directly. So, rather than using a website builder as a crutch, we recommend jumping right into learning HTML, CSS and JavaScript.

In the following sections we'll give you a brief introductions to each of these fundamental web technologies. This introduction will be rapid so please do not worry if you do not fully understand everything. But the please do your best to engage with this material as it will provide you with everything you need to know in order to be able to get the maximum out of our later examples of interactive modeling webpages.

HTML Basics

HTML defines the structure of a webpage or document. An HTML document is made up out of a set of *tags*. Each tag is enclosed in triangular brackets. For instance, there is a tag called "<hr>" that will create a horizontal division line in your document ("hr" is an abbreviation of "horizontal rule").

Many types of tags will consist of an opening and closing tag paired together. A closing tag is written the same as an opening tag except there is a also backslash

[1]Please note that when you write CSS and JavaScript, your text is case-sensitive. This means that "ABC", "abc", and "Abc" will all be understood differently. HTML, on the other hand, is case-insensitive. In HTML, "ABC", "abc", and "Abc" will all be understood to mean the same thing.

immediately after the first triangular bracket. For instance, you could use a pair of “. . . ” tags to make some text bold:

```
This is some text. <b>This text is bold.</b> This text is not bold.
```

Some tags may also have “attributes” which modify the behavior of the tag. Attributes are included within the opening brackets of the tag after the tag name. For instance, the “<a>” tag is used to make links between webpages. The “” tag has an attribute “href”[2] which is the URL the link should connect to. The following HTML creates a link to Google:

```
If you ever need to search something, just go
to <a href="http://Google.com">Google.</a>.
```

Every HTML page contains some general boilerplate that structures the document. This boilerplate will look almost identical from webpage to webpage and it contains several unique tags which split the document into two sections: a “head” section to store the page title and page keywords for search engines, and a “body” section which contains the page content that is what is actually shown to the user. You will spend most of your time editing the body section. The standard template for a webpage is as follows:

```
<html>
<head>
    <title>A Sample webpage</title>
</head>
<body>
    Document contents goes here...
</body>
</html>
```

There are dozens of different tags you can use in your document to structure it. We can’t cover them all here, but the following table summarizes a few of the most useful ones:

Tag	Usage	Example

[2] The tag name “a” comes from “anchor” and “href” is an abbreviation of “hyperlink reference”. Many of the conventions with web development may seem strange and so you should understand the long history of these technologies and the resulting historical baggage that comes with them.

a	Creates a link	Google.
b	Makes text bold	This text is bold.
i	Makes text italic	This text is <i>italic</i>.
u	Makes text underlined	This text is <u>underlined</u>.
center	Centers a paragraph	<center>In the middle.</center>
p	Creates a paragraph of text	This is a paragraph.
hr	Creates a dividing line	Something <hr> Something Else
h1	Creates a heading	This is a Heading
img	Embeds an image	<img src="http://example.com/image.png">

We can combine these tags together to form more complex documents. The following is an example of a full-featured webpage.

```
<html>
<head>
    <title>A Sample webpage</title>
</head>
<body>
    <h1>Introduction</h1>
        <p>Here is some information about my page.</p>
    <h1>The Content</h1>
        <p>Here we have the meat of the page.</p>
    <hr>
    <h2>For Further Information</h2>
        <p>Here we have links to other sites about this content:<p>
        <p>We could check out <a href="http://BeyondConnectingTheDots.com">
            this book's site</a> for instance.</p>
</body>
</html>
```

Open whatever word processor you use on your computer and save this to *MyPage.html* as a plain text file[3]. You can then open this file in your web

[3]Webpages are always stored as plain text. This differs from, for instance a Microsoft Word document (".doc" or ".docx" extension) or a Rich Text Format document (.rtf extension). You need to ensure you save your document as a plain text document with the extension ".html" or ".htm". You can use any text editor you want, but if you get an editor designed for writing webpages it will have helpful features such as coloring your tags differently from the standard text as you edit the webpage. We recommend Sublime Text (http://www.sublimetext.com/) as a high quality editor for serious work.

browser (Internet Explore, Firefox, Chrome, Safari, etc.). Experiment by adding some more paragraphs and formatting to see how the document changes.

For more information and tutorials on HTML, we recommend the Mozilla Developer Network's guides (https://developer.mozilla.org/en-US/docs/Web/HTML).

Exercise 12-1

Replicate the following formatting in an HTML document:

This **text is *italic* and bold.**

Answer Available

Exercise 12-2

Research HTML on-line, learn how to make a list of items. Create both an ordered and unordered list of the top three countries you wish to visit.

Answer Available

Exercise 12-3

Create an HTML document containing your resume. Use heading tags to separate sections. Include a picture of yourself in the document.

CSS Basics

Where HTML is used to define the structure of a document, CSS is responsible for styling this structure. This styling includes aspects like font and color choices in addition to the general layout. A CSS document is a list of rules where each rule has two parts: a selector that tells the browser what elements of the page the rule applies to, and a set of styles that tells browser how to style those elements. For example, take the following CSS code.

```
p {
    margin: 20px;
}

h1, h2 {
```

```
    font-size: 72px;
    color: red;
}
```

This code has two rules. In the first rule the selector is “p” meaning the rule will apply to all “<p>” tags in the document. The styling for this rule says to apply a 20-pixel margin around each of these paragraph tags. The second rule has the selector “h1, h2”. This means apply the rule to both “<h1>” and “<h2>” tags and to set the contents of those tags to have an extra large font and to be colored red.

There are numerous different aspects of an element’s style you can set with CSS. For a full and detailed reference we recommend the Mozilla Developer Network’s coverage of CSS (https://developer.mozilla.org/en-US/docs/Web/CSS/Reference).

CSS for a webpage can be placed in a standalone file which is referenced by the webpage or it can be included directly within the webpage. Both these can be accomplished by placing a CSS rules within a special tag in the *head* section of the document. For example, taking the *head* section from our earlier document, we could either embed the CSS directly:

```
<head>
    <title>A Sample webpage</title>
    <style>
        p {
            margin: 20px;
        }
        h1, h2 {
            font-size: 72px;
            color: red;
        }
    </style>
</head>
```

Alternatively we could save the CSS to an external text file (such as *MyStyles.css*) and link to it in the *head* of our document:

```
<head>
    <title>A Sample webpage</title>
    <link rel="stylesheet" type="text/css" href="MyStyles.css">
</head>
```

Exercise 12-4

Create a CSS rule to make <u> tags set their text color to green in addition to adding underlining.

Answer Available

Exercise 12-5

Read up about CSS online and create a tag that creates a red box round every link on the web page.

Answer Available

JavaScript Basics

JavaScript[4] provides interactivity for webpages. JavaScript is a powerful programming language that you can use to respond to user actions, run calculations, or modify a webpage. An example of using JavaScript code to calculate a Fibonacci number[5] is below:

```
function fib(n){
    if(n==1 || n==0){
        return 1;
    }
    return fib(n-1) + fib(n-2);
}

alert("The tenth Fibonacci number is: "+fib(10));
```

Like CSS, there are two ways to embed JavaScript into an HTML document. The first is to include the JavaScript directly in the document like we did for the CSS:

[4]The name "JavaScript" is a source of perpetual confusion. What we know colloquially as JavaScript is officially called ECMAScript. Due to trademark issues Microsoft refers to it as JScript when you are using Internet Explorer. It is important to note that *JavaScript* and *Java* are different technologies. They share part of a name due to historic branding purposes but they are completely different languages.

[5]Where the first two Fibonacci numbers are 1 and the Fibonacci numbers thereafter are the sum of the two preceding numbers. The Fibonacci sequence begins: 1, 1, 2, 3, 5, 8, 13, 21, 44. . . .

```
<head>
    <title>A Sample webpage</title>
    <script>
        function fib(n){
            if(n==1 || n==0){
                return 1;
            }
            return fib(n-1) + fib(n-2);
        }

        alert("The tenth Fibonacci number is: "+fib(10));
    </script>
</head>
```

The second method to include the code is to save the JavaScript into a text file (such as *MyScript.js*) and link to it in the document:

```
<head>
    <title>A Sample webpage</title>
    <script src="MyScript.js"></script>
</head>
```

JavaScript is a very powerful tool but also a very complex one. This chapter will illustrate usages of JavaScript but we cannot hope to teach you how to write new JavaScript yourself in this single chapter. Again, we refer you to the Mozilla Developer Network to learn more about JavaScript (https://developer.mozilla.org/en-US/docs/Web/JavaScript).

Exercise 12-6

Learn about JavaScript online. Create a script that prompts the user for two numbers and then adds them.

Answer Available

Creating a Webpage for Engagement

Now that we have made it through some of the technical details, let's jump into building a webpage for an interactive model that users can comment on. There are three basic things we want this webpage to have:

1. A description of the challenge we are tackling, why we built the model, and what the model contains.

2. An interactive version of the model that the user can explore and run simulations with.
3. A discussion forum about the model where users can post comments and see what others have posted.

This might seem ambitious, and it is! But using freely available technologies and services we will be able to put this webpage together very quickly. Let us split the process of developing the webpage into three steps: first we'll create the general page framework, then we will add the interactive model, and lastly we will add the discussion forum.

Creating the Page and Description

Assume we decide we want to create a webpage exploring population growth and whether the Earth can sustain humanity into the future. We start building our webpage by creating an HTML file and putting the following text in it.

```
<html>
<head>
    <title>A Fragile Future</title>
</head>
<body>
    <h1>Introduction</h1>
        <p>This is a model of world population
            changes into the future.</p>

    <h1>The Model</h1>
        [Model goes here]

    <h1>Discussion</h1>
        [Discussion forum goes here]
</body>
</html>
```

This creates a page with three sections: Introduction, The Model, and Discussion. We can fill in the Introduction section with text describing the problem we face and our approach to understanding it in our model. In this example page, we have just written a single sentence but you could extend it with more details on the model to fully explain to the viewer why this is important and how we have modeled it.

The placeholders [Model goes here] and [Discussion forum goes here] are where we will insert our model and discussion forum later on. For now though, we just want to layout the structure of the page.

Adding an Interactive Model

Now that we have created the structure for our webpage, we can add the interactive model. There are several ways to do this. One way would be to write the model in JavaScript and include it directly in the webpage. JavaScript is a full-featured programming language and could be used to implement any of the models described in this book. Although implementing a model in JavaScript is definitely possible, it would require a lot of work. Writing a model in JavaScript would take a large amount of time and would not be possible without extensive programming experience.

Fortunately, using Insight Maker there is a much easier approach. Insight Maker models can be easily embedded in a webpage without any special effort on your part. So rather than writing our world population model in JavaScript, we can simply build the model in Insight Maker and then embed the resulting model in our webpage. So build your model in Insight Maker just as you would build one normally. You can also take an existing model that you have already built and use that one. For this example, we will use the World3 model (http://InsightMaker.com/insight/1954) which has a detailed worldwide model of population change[6].

Once you have finished constructing your model, click the **Embed** button in the **Tools** section of the Insight Maker toolbar. A window will open containing HTML code that you should paste into your webpage. This code will embed a version of the insight when it is placed in a webpage document. For the World3 model this code is something like:

```
<IFRAME SRC="http://InsightMaker.com/insight/1954/embed?topBar=1&sideBar=1&zoom=1"
TITLE="Embedded Insight" width=600 height=420></IFRAME>
```

Take this code and use it to replace the [Model goes here] placeholder in your webpage. Save the webpage and open it in a browser and you will now have a rich interactive version of your model embedded directly in your webpage!

There are several features of the embedding that you can control by editing the "<IFRAME>" tag. For instance the "width" and "height" attributes control the size of the embedded model. They are specified in pixels and you may change them to make the embedded model smaller or larger. The "topBar" and "sideBar" parts of the URL control whether the toolbar and the sidebar will be shown in the embedded model's interface. By default, they are set to 1 indicating these elements will be shown. Set them to 0 to hide the bars when the model is displayed. The "zoom" part determines whether the model diagram is shown at its full size or if it is zoomed to fit the window (the default). Set this to 0 to prevent the model diagram from automatically being resized to fit the window.

[6] This model was described and discussed in detail in the book *The Limits to Growth*.

Adding a Discussion Section

Now we have one last piece to add before we have completed our webpage. We want people to be able to carry on a discussion about the model directly within the page. To make this possible, we need to add some sort of forum or discussion software.

We could program our own custom discussion system; but, like the case of the model itself, this is a place where it is easier to leverage existing free software than it is to develop our own. A number of free commenting and discussion systems are available. One of these is called Disqus (http://disqus.com). If you read a number of different news sites or blogs you have probably already used Disqus as many sites utilize their software.

You will need to sign up for a Disqus account to be able to embed their discussion software, but fortunately like Insight Maker it should not cost you a thing. Once you have completed signing up at http://disqus.com, follow the site for directions on how to embed Disqus in your own webpage. You should be given code that looks similar to the following to place into your webpage:

```
<div id="disqus_thread">Discussion Here</div>
<script type="text/javascript">
    var disqus_shortname = 'SHORT-NAME-DEMO'; // required: replace example with your forum short
    (function() {
        var dsq = document.createElement('script'); dsq.type = 'text/javascript'; dsq.async = tr
        dsq.src = '//' + disqus_shortname + '.disqus.com/embed.js';
        (document.getElementsByTagName('head')[0] || document.getElementsByTagName('body')[0]).a
    })();
</script>
```

First edit this code as instructed (e.g. replace any usernames or ids with the ones you have been provided by Disqus) and then replace the [Discussion forum goes here] placeholder in your page with this code. Load the page and test to see if it is working. One issue with Disqus is it might not work if the webpage is being opened from a file on your computer. You may need to upload it to the domain name you entered when you signed up for Disqus to ensure it works correctly.

Completed Page

We have just put together a powerful site very quickly. Our site lets us share an interactive model with people anywhere in the world and allows them to comment directly on the model. All that it took to do this was the following completed code:

```
<html>
<head>
```

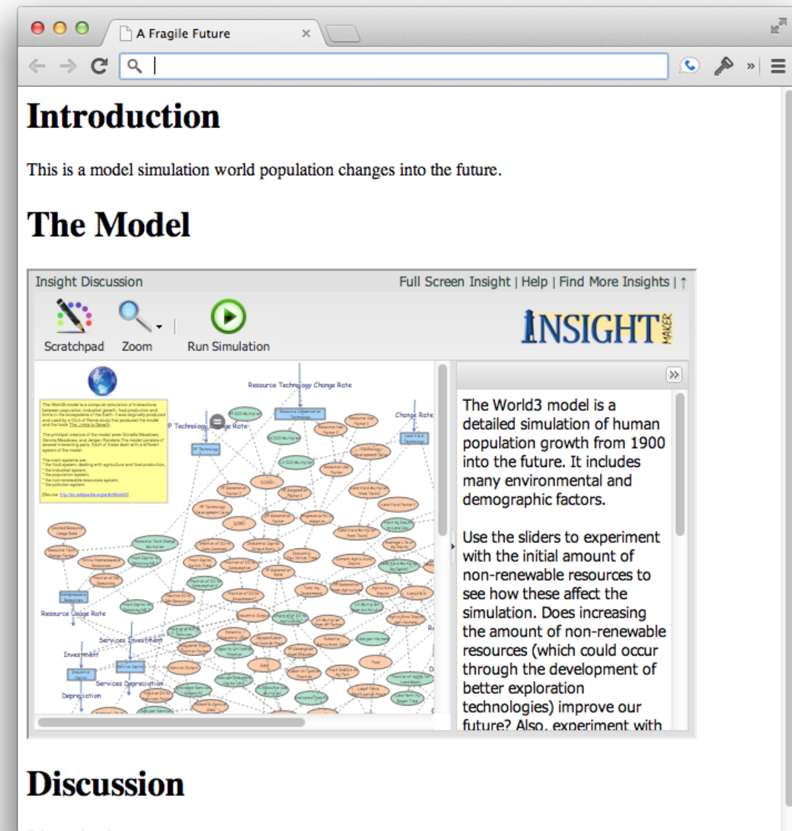


Figure 1. Completed page with embedded model.

```
    <title>A Fragile Future</title>
</head>
<body>
    <h1>Introduction</h1>
        <p>This is a model of world population
            changes into the future.</p>

    <h1>The Model</h1>
        <IFRAME SRC="http://InsightMaker.com/insight/1954/embed?topBar=1&sideBar=1&zoo
        TITLE="Embedded Insight" width=640 height=480></IFRAME>

    <h1>Discussion</h1>
```

```
        <div id="disqus_thread">Discussion here</div>
        <script type="text/javascript">
            var disqus_shortname = ''; // required: replace with your forum shortname
            (function() {
                var dsq = document.createElement('script'); dsq.type = 'text/javascript'; dsq.as
                dsq.src = '//' + disqus_shortname + '.disqus.com/embed.js';
                (document.getElementsByTagName('head')[0] || document.getElementsByTagName('body
            })();
        </script>
</body>
</html>
```

A working version of this site may be viewed at http://BeyondConnectingTheDots.com/book/embedded-model/. There is a lot more we could do with the site. Spend some time now experimenting with it. Add some more descriptive text, maybe add some images, and try to use CSS to adjust the styling.

Exercise 12-7

Use CSS to make a rule to automatically underline the headings in this web page.

Answer Available

Exercise 12-8

Use CSS to add a background color to this page.

Answer Available

Exercise 12-9

Go through these same steps with a model of your choosing. Make your own custom interactive webpage.

Flight Simulators and Serious Games

In the preceding section we described how to rapidly develop a website that contains an interactive model and provides users the ability to comment and discuss the model directly on the page. By leveraging Insight Maker, we were able to get an interactive version of our model embedded in our webpage just by copying a few lines of code. By leveraging Disqus, we were able to include a discussion forum with a similar amount of effort.

In many cases, what we created may be exactly what you are looking for. However, in other cases it is possible that you will wish to provide your users with a unique experience tailored to understanding a specific problem. For instance maybe you would like to develop what is known as a “flight simulator”, a simulation tool that puts the user in the position of trying to manage a problem or achieve an outcome. For example, if you had a model of a business going through a disruptive change, you could place the user in the position of the company’s leader and with instructions to adjust parameters in the model in order to safely shepherd the company through this challenge.

Similarly, “serious games” are tools designed to both engage and educate about a system. You can create a simulation model at the heart of a serious game or a flight simulator. You may give users direct access to this simulation model’s interface, but generally you will want to build a custom interface on top of the model that hides the stock and flow diagram and instead displays a control panel type interface to the user.

Fortunately, web technologies provide a rich environment for developing these flight simulators and serious games. Furthermore, using Insight Maker you can build your model and simulation engine using its model building tools and then build a custom interface on top of the model to provide the exact experience you want to the user. In the following sections we will develop a custom interface to control our world population simulation.

Setting up the Page

We’ll start by stripping down our page from the previous example. Let’s remove the commenting system and the introduction so the page just contains the model (you can add these other items back later on your own as an exercise). After we do this, we will be left with a page just containing the embedded world simulation model.

In this case, however, we do not want the user to actually interact with or even see the embedded model. We will be adding our own custom interface and just using the embedded model to run simulation in the background. To hide the embedded model we can add a CSS rule that makes the <iframe> tag invisible:

```
iframe {
    display: none;
}
```

This rule turns off the display of all <iframe> tags in the page. They are still there and in the page, but they are not shown to the user. The resulting completed template for our page is printed below. When you open this in your browser you should see a completely blank webpage.

```
<html>
<head>
    <title>A Fragile Future</title>
    <style>
        iframe {
            display: none;
        }
    </style>
</head>
<body>
    <IFRAME SRC="http://InsightMaker.com/insight/1954/embed?topBar=1&sideBar=1&zoom=1"
    TITLE="Embedded Insight" width=600 height=420></IFRAME>
</body>
</html>
```

Creating the Control Panel

HTML has a tag called “<input>” that lets you create form elements for users to input data. The <input> tag has an attribute called “type” that determines what the type of the input element will be. There are a wide number of types including “number”, “text”, “color”, “textarea”, “date”, and “button”. For our control panel, we’ll design it to modify two parameters of the model and to have a button users can press to run the simulation. In addition to specifying the type of the inputs, we should also specify their initial values in the control panel. We can do that using the “value” attribute of the <input> tag.

Finally, we will need some method to reference the inputs and to load their values later on. Each tag in an HTML document has an optional “id” attribute. This attribute can be used to obtain a reference to that element from JavaScript. We’ll set the id attribute for our two input fields so we can obtain their values when we are ready to run the simulation.

The resulting control panel will look something like the following code. As you can see we have presented the user with a simple task, to find a combination of settings that results in over 5 billion people in the year 2100 (which is in fact a significant decrease from the current population size so it should not be too hard). You should place this code after the <iframe> tag in your document.

```
<center>
    <p>This is a game to keep the world's population larger than 5 billion in the year 2100.
        We can experiment with the amount of non-renewable resources in the world and the
        start year for a clean energy eco-friendly policy.</p>
    <p> Initial Non-Renewable Resources: <input type="number" value="100" id="resources" /> % </
```

```
    <p> Start Policy Year: <input type="number" value="2013" id="year" /> </p>
    <p> <input type="button" value="Test Scenario" /> </p>
</center>
```

This will create two input fields allowing users to input numeric values. The first, *Initial Non-Renewable Resources* will allow the user to increase or decrease the amount of non-renewable resources assumed in the model at the start of the simulation. The second, *Start Policy Year* allows the user to specify the start date to implement a clean technology policy which will reduce the amount of pollutants being generated in the simulation. A button is also created that lets the user test the scenario in the simulation.

Making it Interactive

We use JavaScript to add interactivity to the webpage. Let's define a JavaScript function *testScenario* that we will use to first read in the user specified options from the control panel, then run the simulation with these parameter values, and finally report to the user whether or not they were successful in keeping the population size above 5 billion.

We will fill out the *testScenario* function with steps later; but for now, just add the following code to the head section of your webpage.

```
<script>
    function testScenario(){
        alert("Scenario tested!");
    }
</script>
```

This creates the function, but we also need a way for the function to be executed when the "Test Scenario" button is pressed. There are several ways to do this. The easiest is to set the "onClick" attribute of the button to call the function. The "onClick" attribute of an input may contain JavaScript code that is executed when the button is clicked. To link up our button with the *testScenario* function, we change our input button in the HTML to:

```
<p> <input type="button" value="Test Scenario" onclick="testScenario()" /> </p>
```

Implement the webpage up to this point and check to make sure that you see a message pop up saying "Scenario tested!" when you press the "Test Scenario" button.

Now that we have implemented basic interactivity, let's flesh out the *testScenario* function.

Load Parameter Values from the Control Panel

To access an input field from JavaScript we use the *document.getElementById* function. This function is built into your browser and allows you to obtain a reference to one of the input elements based on its "id" attribute. Once we have a reference to the input element we can use the element's "value" property to obtain the number the user has entered into the input field.

The following code defines two variables in JavaScript with the same values as the ones the user has entered. Enter this code at the top of your *testScenario* function.

```
var resources = document.getElementById("resources").value;
var year = document.getElementById("year").value;
```

Inject the Parameter Values into the Model

Insight Maker has an extensive JavaScript API[7] that can be used to modify and script models. This is the same API that may be used with Button primitives. Refer to the API reference at http://insightmaker.com/sites/default/files/API/files/API-js.html for full details about the API.

The API instructions provide examples about how to integrate and modify an embedded model. We will adapt those instructions to our own case. First, as the instructions indicate we need to update our <iframe> tag to add an "id" attribute. We adjust our <iframe> tag like so:

```
<IFRAME id="model" SRC="http://InsightMaker.com/insight/1954/embed?topBar=1&sideBar=1&zoom=1"
TITLE="Embedded Insight" width=600 height=420></IFRAME>
```

Now we can obtain a reference to the model using the *document.getElementById* function from before and then we can send API commands to it using its *postMessage* function. Within Insight Maker, we use the *findName* API command to get a reference to a specific primitive and then use the *setValue* API command to set the value of that primitive to the value of the parameter in the control panel. Add the following code to the *testScenario* function.

```
var model = document.getElementById("model").contentWindow;

model.postMessage("setValue(findName('Initial Nonrenewable Resources'), '"+(resources/100)*100000
model.postMessage("setValue(findName('Progressive Policy Adoption'), '"+year+"')", "*");
```

[7] An API, or Application Programming Interface, is a set of commands and functions that can be used to interface programmatically with an application.

This convoluted *postMessage* mechanism to pass JavaScript commands to the embedded model is a constraint necessitated by your browser's security mechanisms. It makes the processing of interacting with embedded models more complex than we would like, but fortunately it is still possible to do everything that we need to do even using it.

Run Simulation and Access Results

To run the model, we use the *runModel* Insight Maker API command. We indicate that the simulation should be run in "silent" mode so the results are returned[8]. We then use the *lastValue* function to obtain the final population size for the simulation in the year 2100. Copy this into your webpage at the end of the *testScenario* function:

```
model.postMessage("runModel({silent: true}).lastValue(findName('Population'))", "*");
```

So far we have just demonstrated one-way communication between the control panel and the embedded model. This is the first point in time when we need to be able to communicate the other way: to receive data back from the embedded model.

Unfortunately, due to the security constraints imposed by your browser, this is slightly complex. In order to receive a message back from the embedded model, we need to register an event handler with your main browser window. Don't worry if you don't fully understand this, just copy the code below into the script tag of your window.

```
function scenarioComplete(event)
{
    if(event.data){
        var pop = Math.round(event.data);
        if(pop > 5000000000){
            alert("You won! The population size of "+pop+" is larger than 5 Billion!")
        }else{
            alert("You failed! The population size of "+pop+" is smaller than 5 Billio
            alert("Please try again.");
        }
    }
}

window.addEventListener("message", scenarioComplete, false);
```

[8] There are two primary ways of running Insight Maker models using the *runModel* API command. One is the regular way where a results diagram will be shown but the results will not automatically be returned in JavaScript. The second way is in silent mode where the results are returned, but results graphs are not shown in the model interface.

Final Result

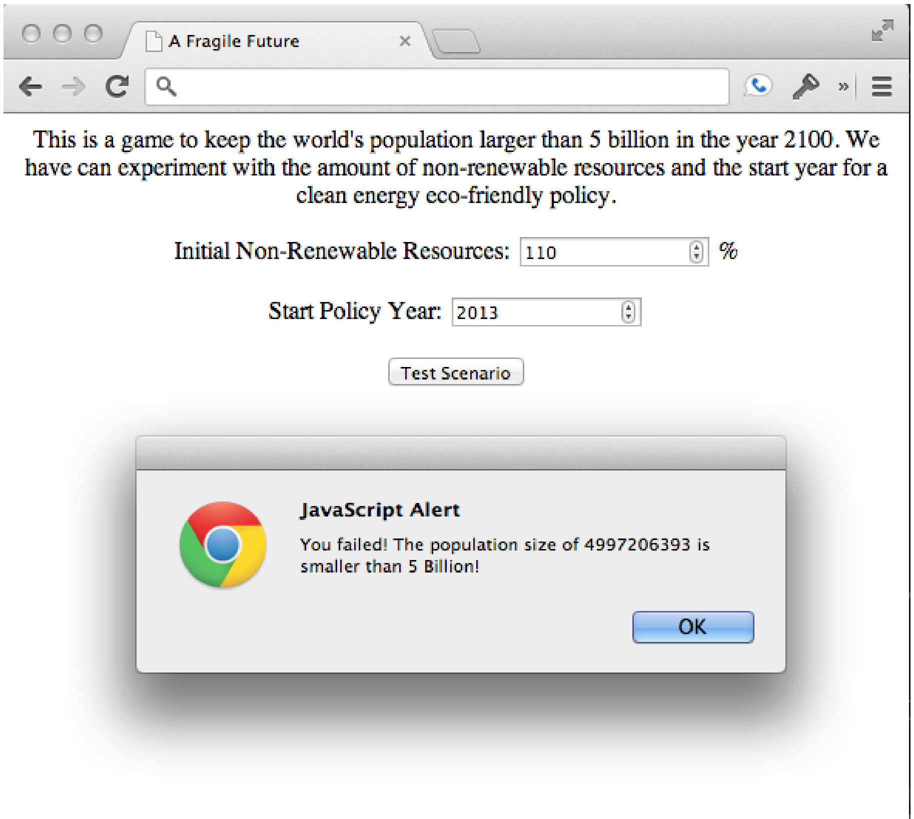


Figure 2. Completed control panel.

The code for the completed webpage is provided below and a working version of the page may viewed at http://BeyondConnectingTheDots.com/book/control-panel/.

```
<html>
<head>
    <title>A Fragile Future</title>
    <style>
        iframe {
            display: none;
        }
    </style>
    <script>
    function testScenario(){
        var resources = document.getElementById("resources").value;
```

```
        var year = document.getElementById("year").value;

        var model = document.getElementById("model").contentWindow;

        model.postMessage("setValue(findName('Initial Nonrenewable Resources'), '"+(re
        model.postMessage("setValue(findName('Progressive Policy Adoption'), '"+year+"

        model.postMessage("runModel({silent: true}).lastValue(findName('Population'))"
    }

    function scenarioComplete(event)
    {
        if(event.data){
            var pop = Math.round(event.data);
            if(pop > 5000000000){
                alert("You won! The population size of "+pop+" is larger than 5 Billio
            }else{
                alert("You failed! The population size of "+pop+" is smaller than 5 Bi
                alert("Please try again.");
            }
        }
    }

    window.addEventListener("message", scenarioComplete, false);
    </script>
</head>
<body>
    <IFRAME id="model" SRC="http://InsightMaker.com/insight/1954?embed=1&topBar=1&side
    TITLE="Embedded Insight" width=600 height=420></IFRAME>

    <center>
        <p>This is a game to keep the world's population larger than 5 billion in the
            We can experiment with the amount of non-renewable resources in the world
            start year for a clean energy eco-friendly policy.</p>
        <p> Initial Non-Renewable Resources: <input type="number" value="100" id="reso
        <p> Start Policy Year: <input type="number" value="2013" id="year" /> </p>
        <p> <input type="button" value="Test Scenario" onclick="testScenario()" /> </p
    </center>

</body>
</html>
```

The key goal of this chapter is not that you completely understand this, but rather that you will be able to adapt it to your own needs. There are a lot of additional changes that could be made to this demonstration. You could clean up the control panel and make it look more attractive by adding some CSS rules. You could add additional inputs to control other parts of the model. You could show the user the trajectory of the population instead of just the final value. Go ahead and experiment with this example to see what you can make it do.

Exercise 12-10

Use CSS to change the style of the inputs. Make inputs have a yellow background and blue text.

Answer Available

Exercise 12-11

Adjust the result message when the users have failed to reach the target population size. Tell them how far away from the target size they are.

Answer Available

Exercise 12-12

Add another input to allow users to adjust the initial amount of potentially arable land in the model.

Additional Tips

Web development is a very complex topic with a lot of nuances. The preceding sections should have given you a brief introduction in how to create interactive models for engaging an audience and encouraging discussion and learning. Although we cannot give you a comprehensive course in web development, there a few additional web development tips that will be very useful when you start to develop your own webpages.

Frameworks and Toolkits

Making an attractive web application is hard. Admittedly, the control panel application we developed does not look very good. We could spend some time improving its appearance by adding additional CSS rules but since we are not professional designers it is quite possible that the results of our efforts would only look amateurish and unattractive. Additionally, writing JavaScript to interact with webpages is also hard. These web technologies were developed over decades and many of the functions and techniques that need to be used are slightly archaic and are difficult to learn.

Fortunately, a number of toolkits and frameworks have been developed that make it easier to develop powerful and attractive web pages and control panels. Below we highlight some important toolkits that you might want to explore and considering adopting for your own usage. These toolkits can be embedded within your webpage extending its functionality. They will help you make more attractive and powerful applications quicker. The ones listed are all also available under open source licenses allowing you to use them for free.

Twitter Bootstrap (http://GetBootstrap.com) : Bootstrap is a framework for developing attractive webpages. It has many tools and rules that can be combined together to create visualizing pleasing webpages with minimal effort. If you don't have a good sense of design, Twitter Bootstrap could be a great help to you.

JQuery (http://jquery.com/) : JQuery is a library designed to improve the JavaScript functions needed to interact with a webpage. It generally greatly simplifies and reduces the number of keystrokes you need to carry out some task. For instance "document.getElementById('item')" becomes in jQuery simply "$('#item')".

JQuery UI (http://jqueryui.com/) : A spin-off from the JQuery project, this toolkit provides control panel elements that are themable and more extensive than the built-in <input> tags. Grids, sliders, and more are all available from this project.

ExtJS (http://www.sencha.com/products/extjs) : ExtJS is a comprehensive library for developing powerful applications. It has extensive tools to develop interface and control panels. It is also what is used to develop Insight Maker's interface.

Exercise 12-13

Install the Twitter Bootstrap toolkit and use it to redesign the control panel web page to make it more attractive.

Debugging Webpages

As you develop your webpage, it is almost certain that you will make many mistakes and typos as you go along. If you make a mistake within the HTML or CSS of the page you will have an immediate visual indication that something is wrong and you can experiment with your code until it is fixed.

JavaScript errors, on the other hand, generally won't provide any visual feedback that an error has occurred. The most likely indication that a JavaScript error has occurred is that nothing happens when you click a button or expect an action to occur. Debugging issues like this can be quite difficult. Fortunately, with just a little bit of additional work you can get access to very rich and informative JavaScript error messages letting you know exactly what went wrong and when.

There are two approaches to obtain access to the JavaScript error messages. The first is to actually edit your webpage and add code to show an error message when an error occurs. Adding the following to a <script> tag in the head section of your document will do that:

```
window.onerror = function(message, url, line) {
  alert("JavaScript Error: \n\n" + message + " (line: " + line + ", url: " + url + ")");
}
```

Now when an error occurs, an alert will pop up with a brief description of the error and information about where it occurred in your code.

The second approach you can take is to use the developer tools that are built into your web browser to study the webpage and observe errors as they occur. Excellent developer tools are built into all modern browsers. These tools let you study the structure of the webpage, profile the performance of your code, and examine how the webpage behaves.

One particular tool is very useful when developing webpages: the JavaScript console. Once you have opened the JavaScript console (search on-line for the exact directions on how to do this for your specific web browser) errors and messages from the webpage will appear in the console as they occur. What's more, the console allows you to evaluate JavaScript commands in the webpage simply by typing the commands into the console.

One approach to debugging code is to put *alert* functions into the code updating you on the progression of the code or displaying values of the JavaScript variables. This works, but can be very clumsy and disruptive. When you have the console open, a better approach is available to you. You can send messages directly to the console providing information on the status of the program. For example:

```
console.log("The value of the variable is: " + myVariable);
console.error("An error has occurred!");
```

Exercise 12-14

Open up a complex web page such as http://nytimes.com. Then use you web browser develope tools to explore how the webpage is structured and designed.

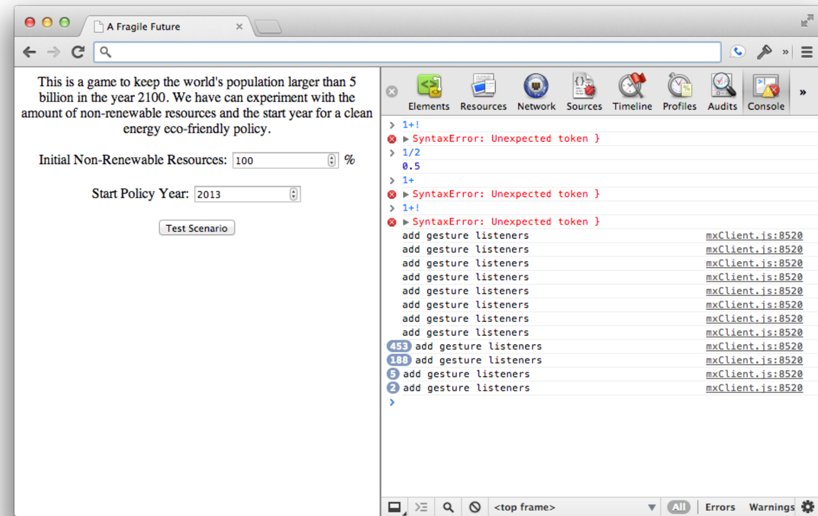


Figure 3. Google Chrome's JavaScript console.

Sending Complex Data Back and Forth

The *postMessage* communication technique to send data back and forth to the embedded model can only be relied upon to send strings and objects that can easily be converted to strings (like numbers)[9]. Oftentimes you will want to pass more complex data from the simulation to the containing window. For instance you might want to pass the whole time series of values one or more primitives took on over the course of the simulation.

To handle these more complex objects you must convert them to strings. JavaScript provides a number of techniques to do so. For instance, if you have an array, you can convert it back and forth from a string using the *join* and *split* functions:

```
var data = [1, 4, 9, 16, 25];
var str = data.join("; "); // "1; 4; 9; 16; 25"
str.split(", "); // [1, 4, 9, 16, 25]
```

By far the most useful and flexible method of converting JavaScript objects to and from strings are the JSON commands. JSON, JavaScript Object Notation, is a general file format for storing data. It is based on the standard method for

[9] The specification for this feature provides that any type of JavaScript object should be supported, however a number of recent browsers only support strings.

declaring JavaScript objects (e.g. {key: value}) but has some differences. What is great about JSON is that your browser already has built-in commands for converting JavaScript objects (a number, array, or other object) into a string and then later back into an object.

You can use this technique to send arbitrarily complex objects back and forth from your simulation to your webpage. Let's see how the JSON commands works:

```
var obj = {title: "I'm a complex object", data: [1, 4, 9]};
var str = JSON.stringify(obj); // '{"title":"I'm a complex object","data":[1,4,9]}'
JSON.parse(str); // {title: "I'm a complex object", data: [1, 4, 9]};
```

Hosting a webpage

In this chapter, we saved the webpages we have created to our personal computers' hard drives and opened them in a browser from there. This works great for development, but it does not allow us to share our creations with others.

Once you are ready to publish your webpages, you must move the HTML, CSS and JavaScript files off your computer and onto a web-server or web-host so that others can access them over the Internet. There are a number of options for web-hosting that range from the simple to the complex and from the free to the expensive.

On the simple and free end of the spectrum there are free blogging sites like Blogger (http://www.blogger.com) or WordPress (http://wordpress.com/). These sites allow you to create free blogs but they also allow you to do much more than that. These types of sites will generally let you edit the source HTML of your pages allowing you to implement the demos in this chapter directly within a blog post.

A step up from simple sites like these blogging platforms are shared hosting providers. Shared hosting providers such as DreamHost (http://dreamhost.com) take a server and allow multiple people to purchase space on the server to run their webpages. There are numerous shared hosting providers available. A more advanced version of shared hosting is Virtual Private Server (VPS) hosting. VPS providers such as RimuHosting (http://rimuhosting.com/) are similar to shared hosting providers in that they fit many customers on a single server. Where they differ is that a VPS host will give each customer a virtualized computer. Each individual customer will feel like they have complete control over their own computer and operating system even though they are sharing the actual hardware with others.

At the high end of the spectrum of complexity, cost and power are dedicated servers. In this case you purchase or rent a machine dedicated solely to the hosting your projects. This gives you complete control of your hosting situation but is expensive and may take a lot of effort to set up and maintain.

In general, we recommend starting small. Sign up for a Blogger account and experiment with these techniques there. If you keep at it and your site grows, at some point you will outgrow this simple solution and at that time you can upgrade to a more advanced hosting solution.

Chapter 13

Modeling with Agents

The modeling techniques we have taught up until this point focused on gaining insights using highly aggregated models of a system. This means that when we looked at models of population growth, we did not explore individual people and instead focused on understanding the population as a whole. This high-level aggregate approach to modeling helps us cut through unnecessary details to understand the core dynamics of a system.

For certain models however, this high-level view may hamstring our ability to explore important questions. For instance in a disease model we may care about the physical relationship between people in the model. Are they near each other? How often do they come into contact? Can we attempt to control the disease by manipulating how people move about and relate to each other? These are all questions that are very hard to answer with a standard System Dynamics model.

Heterogeneity, differences between individuals, is difficult to represent using System Dynamics models. One approach to heterogeneity that is sometimes used is simply to duplicate the model structure for each different class of person or entity in the model. We recall seeing one model that explored education in the United States. The modelers wanted to explore the differences between male and female students. To do so, they simply copy and pasted the entire model structure (consisting of dozens of stocks and flows) and calibrated one of these copies for male students and the other copy for female students.

Granted, this approach can be made to work, but it requires a lot of effort to set up and configure even in the simple two-gender case. When you have more than two cases it can quickly become completely unmanageable. Furthermore, duplicating parts of your model is a recipe for creating unmaintainable models afflicted by hard to track down bugs. The reason for this is that when you later make changes to your model, you are going to need to ensure the changes are made correctly to each one of the model copies. Although simple in principle, in practice this is very easy to mess up and it is a direct route for bugs to be introduced into the model.

Fortunately, an alternative modeling paradigm to System Dynamics exists that is excellent for modeling discrete individuals. It is called Agent Based Modeling and is focused on simulating individual agents and the interactions between these agents[1]. In this chapter we will introduce Agent Based Modeling and show how you can use it to explore questions that cannot be answered with pure System Dynamics.

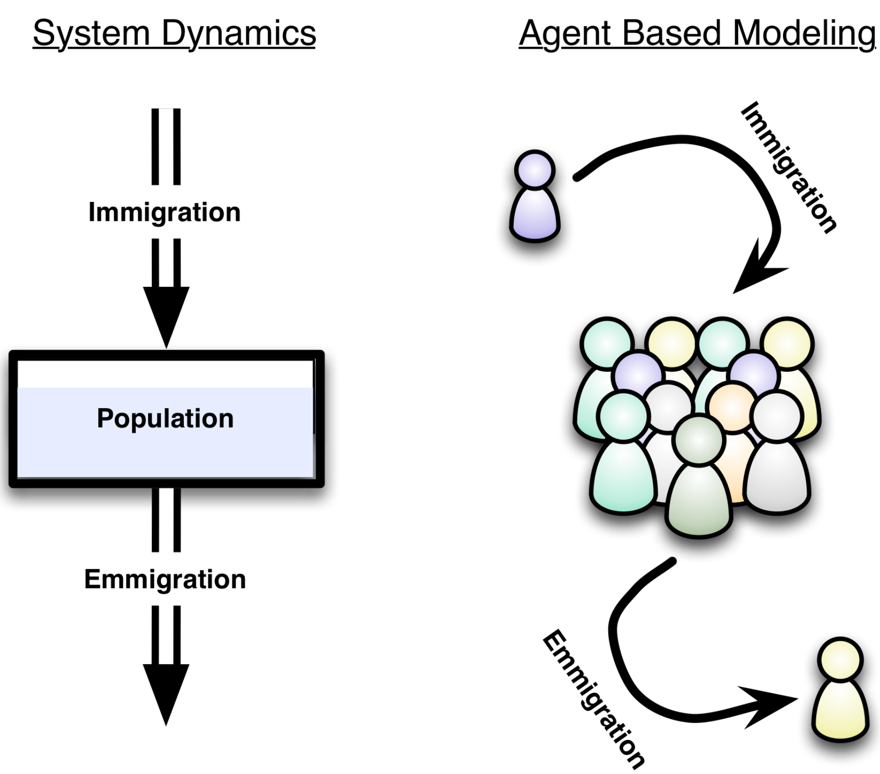


Figure 1. Two paradigms for modeling a population: System Dynamics and Agent Based Modeling.

[1]System Dynamics also has another standard tool for dealing with heterogeneity. This tool is called "vectors", "arrays", "subscripting", or "indexing" and allows you to transparently create multiple copies of your model during simulation to match different classes. Arrays are not as flexible as fully Agent Based Models though. If you consider the continuum of fully aggregate System Dynamics models on one end to fully individualized Agent Based Models on the other, we can think of arrays as existing part way along this continuum.

Exercise 13-1

Discuss the challenges you might face using System Dynamics to model a the adoption of a new product like an improved mousetrap. Identify issues that could be addressed by modeling discrete consumers.

The State Transition Diagram

Up until now our primary modeling tool has been the stock and flow diagram. This type of diagram is useful for summarizing systems from a high-level viewpoint. The stock is a primitive that can model entities that take on a range of values and flows are well suited for specifying the changes in stocks

In addition to representing aggregate systems, stock and flow diagrams are also used to model things on an individual level. For instance, a model of a person's motivations could be represented using a stock and flow diagram. The strength or importance of each type of motivation – money, family, etc... – could be represented as stocks with flows modulating the strength of these motivations over time.

When looking at the individual scale however, we will oftentimes find ourselves wanting to define characteristics of the individual using simple on/off logic. For instance, take the issue of an individual's sex. We can represent this using two categories: Male or Female (leaving aside transgendered individuals for the sake of simplicity). Similarly, when constructing a model of a disease, we might want to say a person is either sick or not sick (with no nuances such as "slightly sick" or "highly sick"). You can attempt to represent these different categories using stocks, but the formulation and equations to do so will be overly complicated.

Where the stock and flow diagram is used to model changing systems with continuous stocks, the state transition diagram is used to model systems with discrete on/off states. Within Insight Maker, state transition diagrams are constructed in almost the same way as stock and flow diagrams. The key difference is all stocks are replaced with *State* primitives and all flows are replaced with *Transition* primitives. State primitives can be added to the model by right-clicking on the model diagram and selecting **Add State**. Transition primitives will automatically be created when you connect two state primitives together using the standard "Flow" connection type.

A state primitive is possibly the simplest primitive available as it can only take on one of two values: true or false. When the state value is true, the state is active. When the state value is false, the state is not active and the agent does not occupy that state. When configuring a state primitive, you only need to specified whether the state is **initially active** or not at the start of the simulation. This initial condition can simply be `true` or `false`, but it can also be a logical equation that depends on the values of other primitives in the agent.

Figure 2. Sample states you might use for agents in a disease model.

For example, if you had a variable in the agent called [**Size**], and you wanted a state to be initially active if the value of [**Size**] was greater than 5, you could use the following as the initially active property for the state: `[Size] > 5`.

A transition primitive moves an agent between states. For instance, if you had two states in your model – *Healthy* and *Sick* – you could have one transition primitive moving agents from the healthy state to the sick state (simulating infection) and another transition primitive moving them the other way (simulating recovery).

There are three different ways a transition from one state to another can be triggered:

Timeout : In this mode the transition will be triggered a specific amount of time after the first state becomes active. For instance, if we had a disease model where the disease lasted 10 days, we could have a transition from the sick to healthy state using a timeout trigger with a period of 10 days.

Probability : In this mode there is a probability of the transition happening each time period. For instance, in the disease model if the disease only lasted 10 days on average but could randomly last longer or shorter, you could use a probability transition with a daily probability of 0.1.

Condition : In this mode you create an equation that will trigger than transition when it becomes true. For instance, if we had a stock, [**Infection Level**] in our agent indicating how sick the agent was, we could have them transition out of the sick state once that stock fell to zero. The trigger condition to enable this could be something like: `[Infection Level] = 0`.

Exercise 13-2

Specify a transition trigger type and value for the following types of transition:

1. Transition after 10 days.
2. 20% chance of transitioning each year.
3. Transitioning when value of the primitive [**Volume**] is greater than 5.

Answer Available

Exercise 13-3

Create a state transition diagram for a model of a person who three states: [**Child**], [**Adult**], [**Retired**]. The person starts in the [**Child**] state, transitions to the [**Adult**] state when they are 18 years old, and has a 2% chance of transitioning to the [**Retired**] state each year.

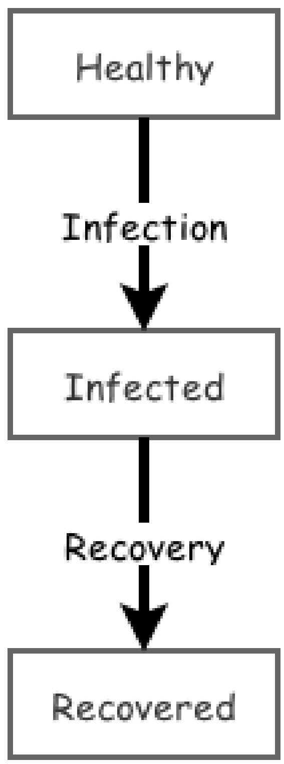
A State Transition Diagram for Disease

This model illustrates the use of state transition diagrams to model a simple disease. This is a disease such as the flu where immunity is obtained once the individual recovers from the disease.

1. Create a new **State** named [**Healthy**].
2. Create a new **State** named [**Infected**].
3. Create a new **State** named [**Recovered**].
4. The model diagram should now look something like this:

5. States can be used to represent a person's condition. In our model a person can either be healthy, infected, or recovered from the infection. Now, let's add transitions that move a person from state to state.
6. Create a new **Transition** going from the primitive [**Healthy**] to the primitive [**Infected**]. Name that transition [**Infection**].

7. Create a new **Transition** going from the primitive [**Infected**] to the primitive [**Recovered**]. Name that transition [**Recovery**].
8. Please note that in this model someone who is recovered cannot become sick again. They have gained immunity to the disease.
9. Now that the model structure has been designed, let's add equations and configure the primitives.
10. Change the **Start Active** property of the primitive [**Healthy**] to `True`.
11. The model diagram should now look something like this:



12. When a state is active, it means a person is in that state. By setting [**Healthy**] to start active, we have the person start in the healthy state.
13. Change the **Trigger Type** property of the primitive [**Infection**] to `Probability`.
14. Change the **Value/Equation** property of the primitive [**Infection**] to `0.3`.

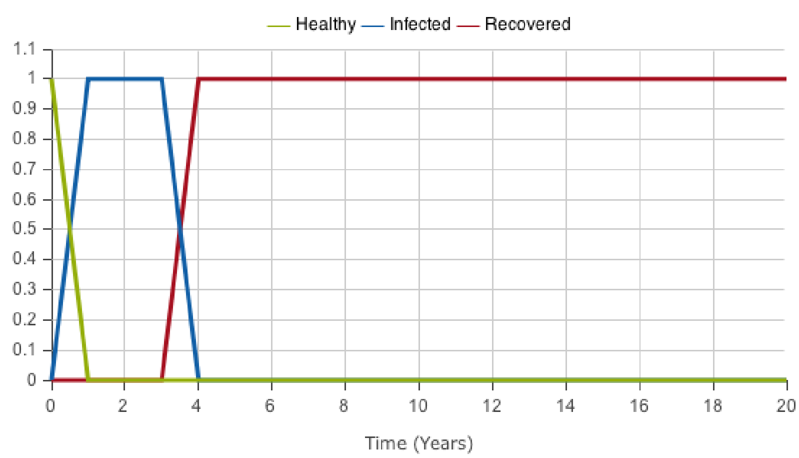
15. Change the **Trigger Type** property of the primitive [**Recovery**] to `Probability`.

16. Change the **Value/Equation** property of the primitive [**Recovery**] to `0.2`.

17. Using the Probability type for the transition trigger means that the person has a fixed probability of transitioning from one state to the next each year. We will assume a 30% probability of the person becoming sick each year and once sick, a 20% chance of recovering each year.

18. Let's run the model now.

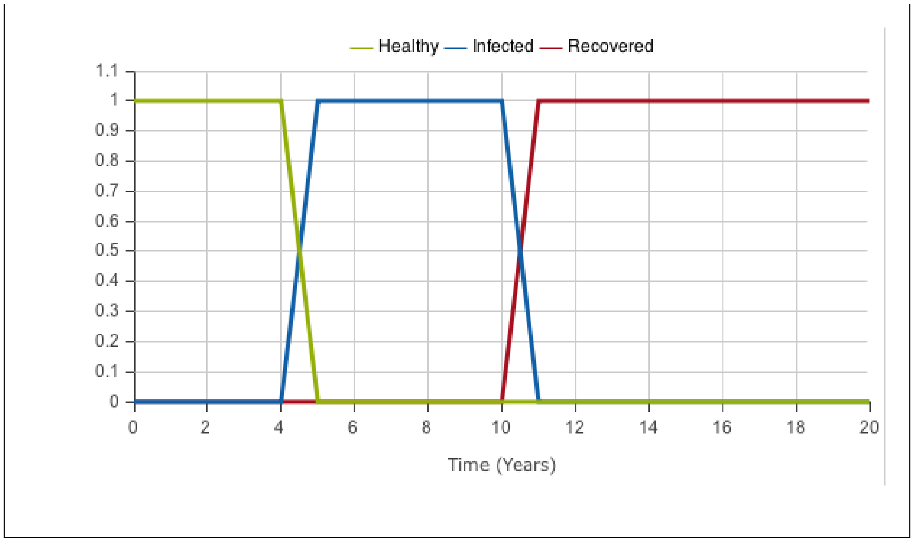
19. Run the model. Here are sample results:



20. A value of 1 for a state primitive means it is active. A value of 0 means it is not active. We can see from this diagram when the individual transitions from the healthy to the infected state and then from the infected state to recovered state.

21. We can run the model again and we will see that we get different results each time we run it. This is because the model is stochastic and the transition triggers are random.

22. Run the model. Here are sample results:



Creating Agents

Now that we have learned about state transition diagrams, we are ready to start creating agents. There are three key parts of creating agents in a model:

1. Defining what an agent is
2. Creating a group of agents
3. Viewing agent results

Defining Agents

We have already introduced the folder primitive as a tool for grouping primitives together and also as a tool for unfolding a model. The folder primitive plays an additional role in Agent Based Modeling as we use folders to define what our agent consists of.

To create an agent construct the state transition diagram for your agent (and also add any stocks, flows or any other primitives you want to this agent). Then create a folder containing all these primitives. Give the folder the name of your agent such as “Person” or “Individual” or even just “Agent”. This is all similar to what we have done with folders before, but now there is one extra step. Edit the folder configuration and set the folder **Behavior** to “Agent”. You have now created the definition of your first agent!

You can have as many different types of agents in your model as you would like. Just create a new agent model and use a new folder to define each of the different types of agents. For instance if you had a predator prey model you could have one agent definition describing the behavior of the prey, and a second agent definition describing the behavior of the predators.

Creating a Population of Agents

After you have defined an agent in your model, you are ready to create a collection or population of agents. This is done by adding an *Agent Population* primitive to your model. The agent population primitive takes the definition of an agent from an agent folder and creates many copies of that agent from the definition. The agent population primitive keeps track of these copies and allows them to operate and to interact with one another.

There are a number of different settings for the agent population primitive but two of them are of key importance. The first is to select what type of agent will be in the population. Each population primitive can only have one type of agent in it. You can have multiple populations though and the agents in one population can interact with the agents in another population.

After specifying what type of agent is in the population, you need to specify how many agents are in the population at the start of the simulation. This is done by setting the **Size** property for the agent population. Later on you can add or remove agents to a population by using the `Add()` and `Remove()` functions.

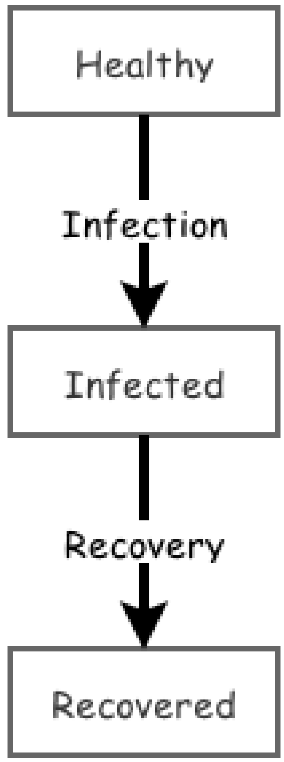
Viewing Agent Results

Many of the standard Insight Maker display types can be used to show the results of an agent based simulation. If you add an agent population to a time series or tabular display, the results for the number of agents in each of the various agent states will automatically be shown. You can also use the **Map** display type to illustrate agents within a geographic region.

An Agent Based Model of Disease

Here we convert a state transition diagram into a model containing multiple agents.

1. The model diagram should now look something like this:



2. We start with our state transition diagram from our previous modeling example.

3. Create a new **Folder** named [**Person**]. The folder should surround the primitives [**Healthy**], [**Infected**], [**Recovered**], [**Infection**] and [**Recovery**].

4. Change the **Type** property of the primitive [**Person**] to `Agent`.

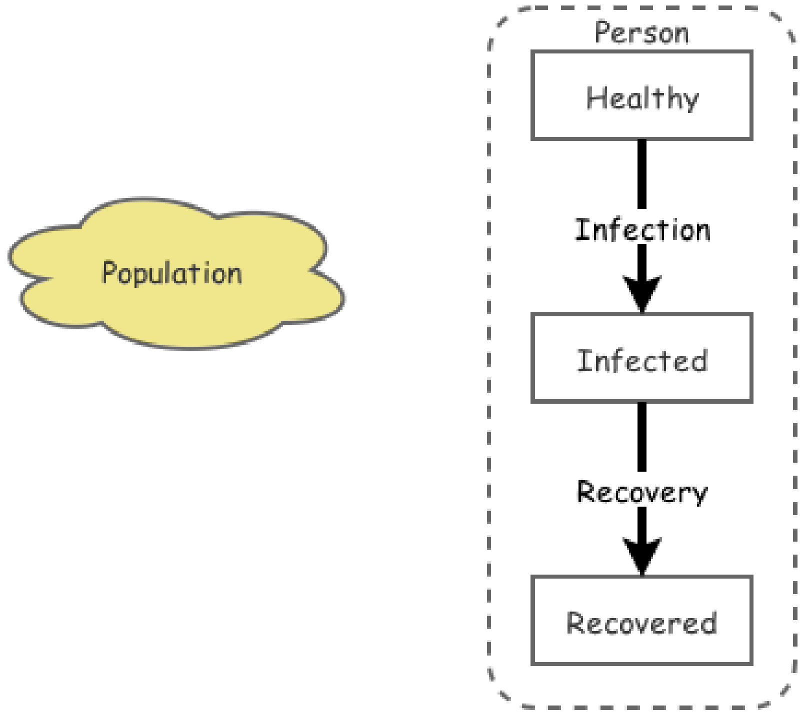
5. First we create an agent folder to encapsulate our state transition diagram. This is a definition of what an agent in our model will be and we make sure the folder behavior is set to "Agent".

6. Create a new **Agent Population** named [**Population**].

7. Change the **Agent Base** property of the primitive [**Population**] to `Person`.

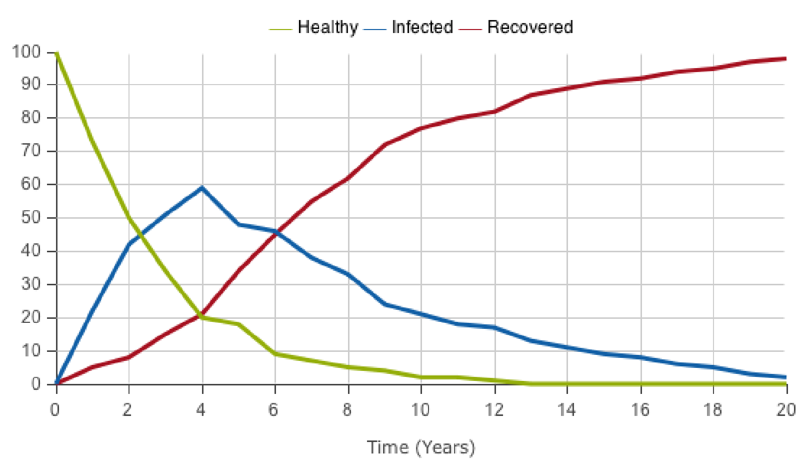
8. Next we create an agent population [**Population**] and set it to contain instances of our **Person** agent. We'll start with a population size of 100 agents.

9. The model diagram should now look something like this:



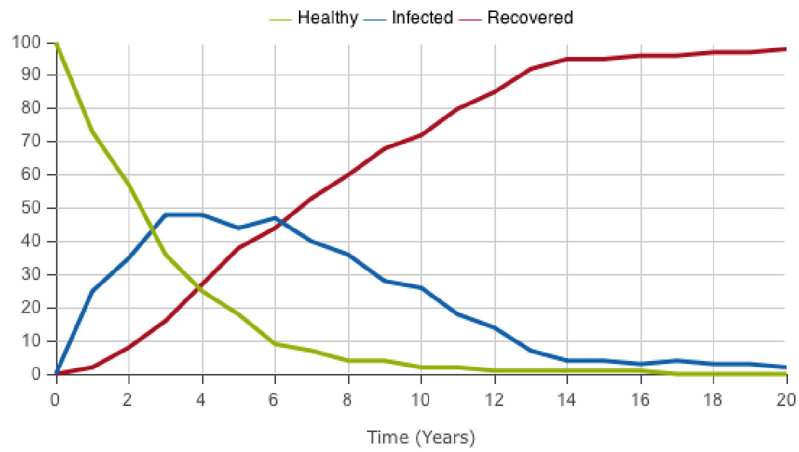
10. We can now run the model to see how the disease affects the 100 people in our population.

11. Run the model. Here are sample results:



12. Each time we run the model we will get different results due to the stochasticity in the model.

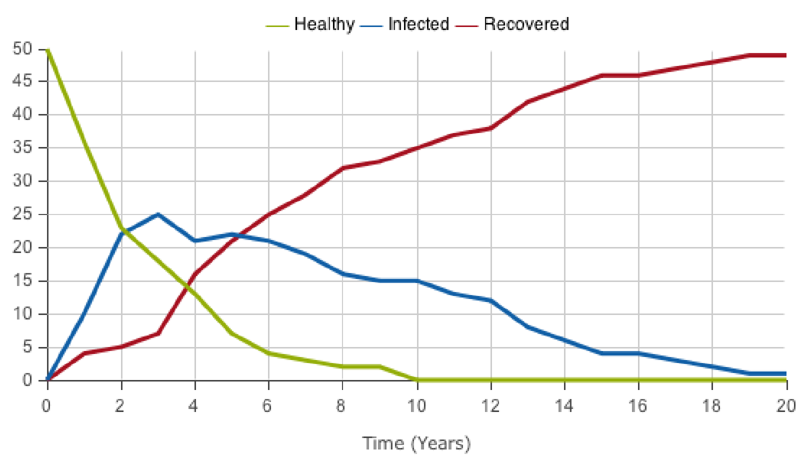
13. Run the model. Here are sample results:



14. Change the **Size** property of the primitive [**Population**] to `50`.

15. We can easily change the number of people in the population. Let's set it to 50 then run the model again.

16. Run the model. Here are sample results:



17. When we have a smaller number of people, the overall population changes are more variable. As we add more and more people, the randomness in the model has less of an effect and trends become smoother approaching the average results we would see if we had used a System Dynamics model.

Working with Agents

Working with agents is fundamentally different from working with primitives in a pure System Dynamics model. For instance, if you have a regular model and you refer to the value of a variable or stock you get a single value back. With agents, however, when you refer to the value of a primitive you might get a separate value for each individual agent in your model.

So for instance, if you have 100 agents and you refer to the primitive [**Height**], you will get 100 different heights one for each of the agents in the model. Similarly, in the case of our disease model, if you you request the value of the state [**Infected**], you will get a different infected value for each of the agents in the model.

You will need to extend your modeling toolkit in order to be able to effectively manage agents and accomplish your goals in your model. The key building block of this extended toolkit is the vector[2]. In the following sections we will first introduce the general concept of vectors and then show how you can use them to interact with agents.

Working with Vectors

A vector is an ordered list of items. In Insight Maker vectors can be written using the '«' sign (or '<<') followed by the '»' sign (or '>>'). For instance imagine we had a small population of only four people. If we asked the model for the heights of those four people[3] in meters we might get something like this:

```
«2, 1.8, 1.9, 1.5»
```

This indicates that our population has four people with heights of 2, 1.8, 1.9 and 1.5 meters. Insight Maker has an extensive set of capabilities and functions for manipulating and summarizing vectors such as this. For instance, if we wanted to know the height of the tallest person in our population, we could use the `Max()` function:

```
Max(«2, 1.8, 1.9, 1.5») # = 2
```

If we wanted to know the height of the smallest person in the population we could use the `Min()` function:

```
Min(«2, 1.8, 1.9, 1.5») # = 1.5
```

Let's say we wanted to know the average height of the people in our our population. We could either use the `Mean()` or `Median()` functions:

```
Mean(«2, 1.8, 1.9, 1.5») # = 1.8
```

[2] In other programing languages and modeling environments vectors are sometimes called "Arrays" or "Lists".

[3] Using an equation like *Value(FindAll([Population]), [Height])*. We'll see later how to construct equations like this.

```
Median(«2, 1.8, 1.9, 1.5») # = 1.85
```

We can also use basic mathematical operations on our vectors. For example, assume we needed to design a room such that the top of the room was at least half a meter above a person's head. We could find the required room height for each person by adding 0.5 to the vector of heights:

```
«2, 1.8, 1.9, 1.5» + 0.5 # = «2.5, 2.3, 2.4, 2»
```

We can also add vectors together. For instance, let's imagine that some of the agents had hats on and we have measured the height of these hats and got the following vector of heights: `«0.05, 0, 0.1, 0»` (two of the people do not wear hats). We could find the height of the agents when they are wearing their hats using:

```
«2, 1.8, 1.9, 1.5» + «0.05, 0, 0.1, 0» # = «2.05, 1.8, 2, 1.5»
```

Another useful vector function is the `Count()` function. Assuming we did not know there were four agents, we could determine how many elements there were in the vector using this function:

```
Count(«2, 1.8, 1.9, 1.5») # = 4
```

You can do a lot with these basic functions but there are also two very powerful vector functions we should mention: `Map()` and `Filter()`. Map takes each element in a vector and applies some transformation to it and returns a vector of the transformations. As an example, let's say we wanted to test whether or not are agents were tall enough to ride an amusement park ride with a cutoff of 1.85 meters. We could get a vector containing whether or not each agent was tall enough using:

```
Map(«2, 1.8, 1.9, 1.5», x >= 1.85) # = «true, false, true, false»
```

Here the function `x >= 2` is applied to each element in the vector (with `x` representing the element value) and the results of this element-by-element evaluation of the function is returned.

Filter takes a function and applies it to each element in a vector. If the function evaluates to true, the element is included in the resulting vector; if the function evaluates to false, the element is not included in the results. For instance, if we just wanted the heights of the people who were tall enough to ride the ride, we could use:

```
Filter(«2, 1.8, 1.9, 1.5», x >= 1.85) # = «2, 1.9»
```

Lastly, there are a couple of very useful functions are available to combine vectors together. `Union()` takes two vectors and combines them together removing duplicated elements.

```
Union(«1, 2 ,3», «2, 3 ,4») # = «1, 2, 3, 4»
```

`Intersection()` takes two vectors and returns a vector containing the elements that are in both of the vectors.

```
Intersection(«1, 2 ,3», «2, 3 ,4») # = «2, 3»
```

`Difference()` takes two vectors and returns a vector containing the elements that are in either one of the vectors but *not* in both of the vectors.

```
Difference(«1, 2 ,3», «2, 3 ,4») # = «1, 4»
```

There are many more vector functions available, but these are some of the key ones. They will prove invaluable when you come to working with vectors of agents.

Exercise 13-4

Given a vector of heights `«2, 1.8, 1.9, 1.5»`, write an equation to find the tallest height under 1.95 meters:

Answer Available

Exercise 13-5

Given a vector named a, write an equation to find the median of the squares of all the elements in a.

Answer Available

Exercise 13-6

Given a vector named a and a vector named b, write an equation to find the smallest element that is in both vectors.

Answer Available

Exercise 13-7

Given the vector named a. Find the mean of the vector without using the `Mean()` function.

Answer Available

Accessing Agents

Insight Maker includes a number of functions to access the individual agents within a population. The simplest of these is the `FindAll()` function. Given an agent population primitive that we'll call [**Population**], the FindAll function returns a vector containing all the agents within that agent population:

```
FindAll([Population])
```

So if your agent population currently had 100 agents in it, this would return a vector with 100 elements where the first element referred to the first agent, the second element referred to the second agent and so on. It is important to note that these elements are agent references, not numbers. So you can use a function like `Reverse()` on the resulting vector, but you cannot directly use functions like `Mean()` as the agent references are not numerical values[4]. We will see how to access the values for agents next.

In addition to the FindAll function, there are other find functions that return a subset of the agents in the model. For instance, the `FindState()` and `FindNotState()` functions return, respectively, agents that either have the given state active or not active. For instance, if we go back to our agent-based disease model, our agents had a state primitive called [**Infected**] that represented if the agent was currently sick, we could get a vector of the agents in our population that were currently sick using the following:

```
FindState([Population], [Infected])
```

And we could obtain a vector of the agents that were not currently infected with:

```
FindNotState([Population], [Infected])
```

Find functions can also be nested. For instance, if we added a [**Male**] state primitive to our agents representing whether or not the agent was a man; we could obtain a vector of all currently infected men with something like the following:

```
FindState(FindState([Population], [Infected]), [Male])
```

Nesting find statements is effectively using Boolean AND logic (like you might use on a search engine: "Infected AND Male"). To do Boolean OR logic (e.g. "Infected OR Male") and return all the agents that are either infected or a man (or both), you can use the Union function to merge two vectors:

```
Union(FindState([Population], [Infected]), FindState([Population], [Male]))
```

If you wanted the agents that were either infected or men (but not both simultaneously), you could use:

```
Difference(FindState([Population], [Infected]), FindState([Population], [Male]))
```

[4] The agents certainly contain many numerical values in their stocks, variables, or states; but an agent reference itself is not numerical and so you cannot do things such as directly taking the average of the agents or sorting them.

Exercise 13-8

Write an equation using the disease example to return a vector of all female infected individuals.

Answer Available

Exercise 13-9

Write an equation using the disease example to return a vector of all female individuals, healthy individuals or healthy females.

Answer Available

Agent Values

Once you have a vector of agents, you can extract the values of the specific primitives in those agents using the `Value()` and `SetValue()` functions.

The Value function takes two arguments: a vector of agents and the primitive for which you want the value. It then returns the value of that primitive in each of the agents. For instance, let us say our agents have a primitive named [**Height**]. We could get a vector of the height of all the people in the model like so:

```
Value(FindAll([Population]), [Height])
```

A vector of heights by itself is generally of not too much use. Often we will want to summarize it, for instance by finding the average height of the people in our population:

```
Mean(Value(FindAll([Population]), [Height]))
```

In addition to determining the value of a primitive in an agent, you can also manually set the agents' primitive values using the SetValue function. It takes the same arguments as the Value function in addition to the value you want to set primitives to. For instance, we could use the following to set the height of all our agents to 2.1:

```
SetValue(FindAll([Population]), [Height], 2.1)
```

Exercise 13-10

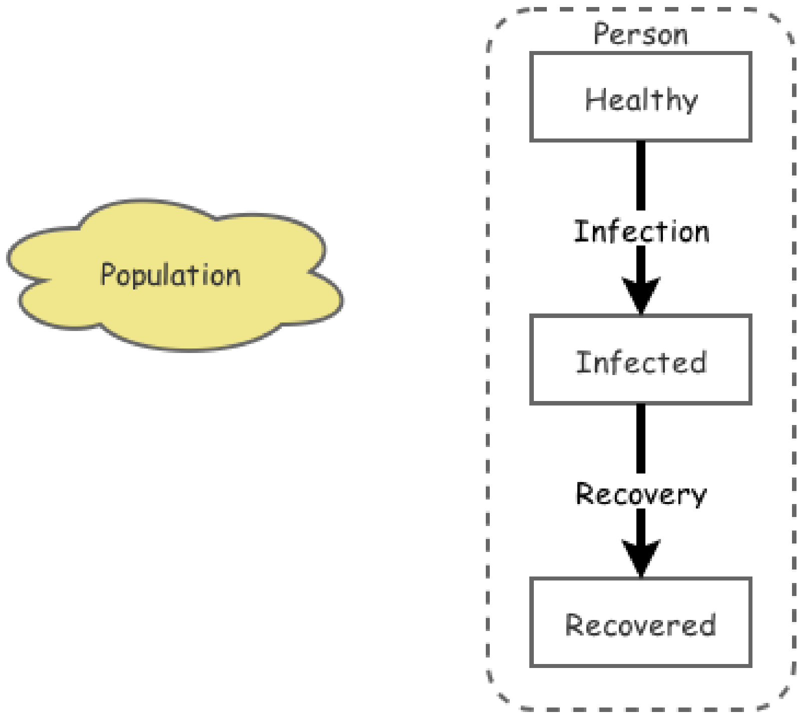
Assume our disease model population had a height stock. Provide an equation to find the average difference in heights between males and females.

Answer Available

Agents Interacting

This example shows how agents can interact with each other using the Find functions.

1. The model diagram should now look something like this:



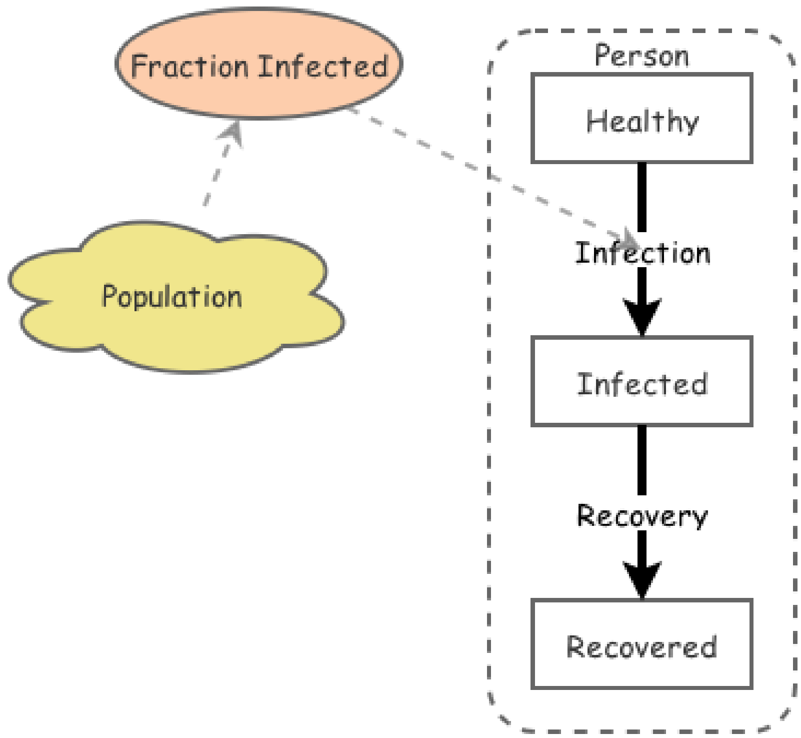
2. Let's make our agent based disease model from earlier more realistic. We will add a variable [**Fraction Infected**] that calculates what fraction of the population is currently infected. We will then use this variable to determine the infection rate so the more people in the population who are infected, the faster the disease will spread.

3. Create a new **Variable** named [**Fraction Infected**].

4. Create a new **Link** going from the primitive [**Population**] to the primitive [**Fraction Infected**].

5. Create a new **Link** going from the primitive [**Fraction Infected**] to the primitive [**Infection**].

6. The model diagram should now look something like this:



7. Now let's configure the value of [**Percent Infected**] and change the [**Infection**]transition to use it.

8. Change the **Equation** property of the primitive [**Fraction Infected**] to `Count(FindState([Population], [Infected]))/PopulationSize([Population])`.

9. This equation uses the FindState function to select all the people in the [**Population**] who are in the Infected state. It then divides the number of those people by the total size of the population.

10. Change the **Value/Equation** property of the primitive [**Infection**] to `[Fraction Infected]`.

11. Now that we have set our infection probability to the value of the [**Fraction Infected**] primitive, we are ready to run the model.

12. Run the model. Here are sample results:

13. That was a bit of a disappointment wasn't it? Nothing happened. Why is this?

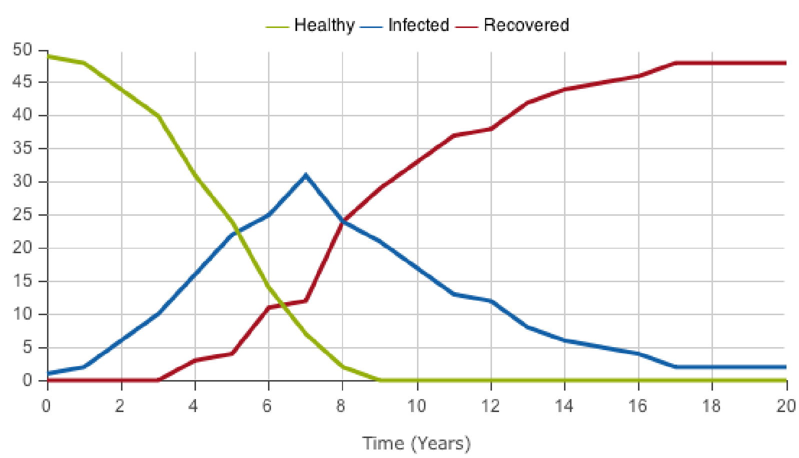
14. Well since our infection rate now depends on the number of people who are infected we have to have at least one person infected to get the epidemic going. Let's change the [**Healthy**] and [**Infected**] states so one person starts in the infected state at the beginning of the simulation.

15. Change the **Start Active** property of the primitive [**Healthy**] to `Index([Self]) <> 1`.

16. Change the **Start Active** property of the primitive [**Infected**] to `Index([Self]) == 1`.

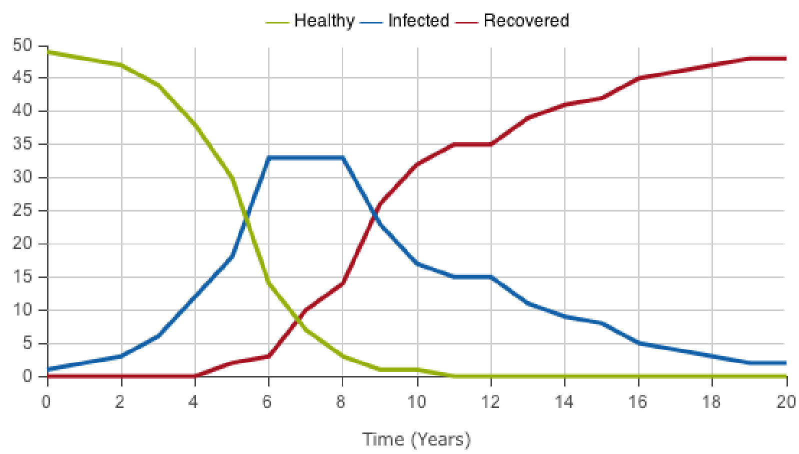
17. Each agent has an index starting with 1, we have set our initially active equations so the first agent in the population will start the simulation in the infected state. Let's run the model to see this working.

18. Run the model. Here are sample results:



19. Each time we run the model we will get a different set of results. Sometimes the infection will die off after the first infected person recovers. Many other times an epidemic spread of the disease will occur.

20. Run the model. Here are sample results:



Agent Geography

One of the key strengths of Agent Based Modeling is that it allows us to study the geographic relationship between our agents. So if we are developing a disease model we do not have to assume that all the agents are perfectly mixed together like atoms in a gas (such as we generally would in System Dynamics). Instead, using Agent Based Modeling we can explicitly define the physical relationship between the different agents and study how this geography affects the spread of the disease.

In general when we talk about geography we mean spatial geography: the locations of people within a region in terms of their latitude and longitude (and sometimes their elevation). Insight Maker supports this kind of geography, but it also supports a second kind of geography: network geography. Insight Maker allows the specification of "connections" between agents. This leads to a new type of geography where you have centrally located agents (ones connected to many other agents) and agents far from the network's center (those that are unconnected or just connected to a very few other agents).

Both these types of geographies can be useful in exploring important features of real-world systems. In the following sections, we will introduce their properties and show you how to utilize them in your own models.

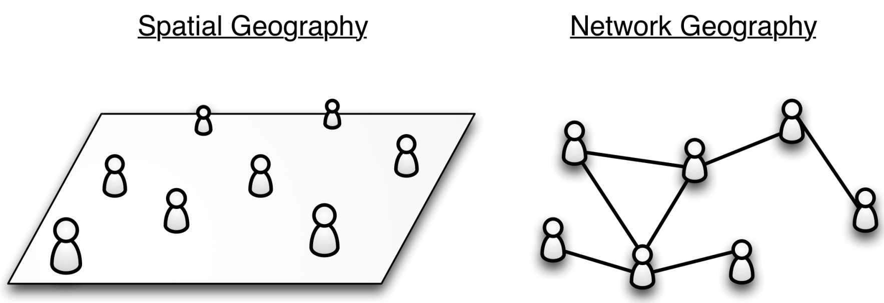


Figure 3. Spatial geography and network geography.

Spatial Geography

In Insight Maker, each Agent Population can be given dimensions in terms of a width and a height. By default, agents are placed at a random location within this region. You can, however, choose a different placement method for the starting position of the agents. The following placement methods are available:

Random : The default. Agents are placed at random positions within the geometry specified for the agent population.

Grid : Agents are aligned in a grid within the population. When using this placement method, you will need to ensure that you have enough agents so that the grid is complete. You might need to experiment with increasing or decreasing the number of agents to make the grid fit perfectly for a given set of region dimensions.

Ellipse : Agents are arranged in a single ellipse within the region. If the region geometry is a square, then the agents will be arranged in a circle.

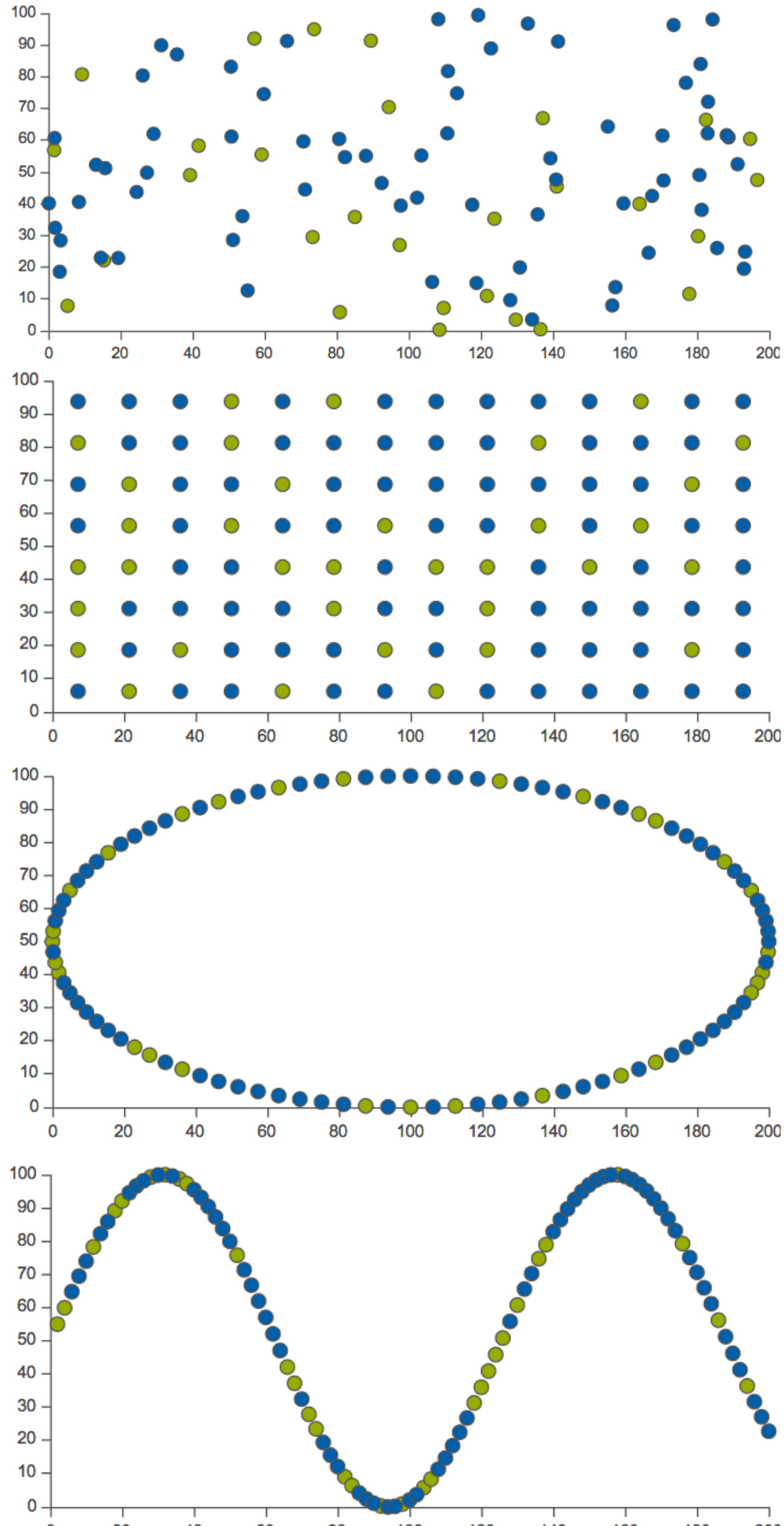
Network : Assuming network connections between agents have been specified, the agents will be arranged in an attempt to create a pleasing layout of the network structure.

Custom Function : Here you can specify a custom function to control the layout of the agents. This function will be called once for each agent in the population and should return a two-element vector where the first element is the x-coordinate of the agent, and the second element is the y-coordinate. The primitive [**Self**] in this function will refer to the agent that is being positioned.

Spatial Find Functions

When working with a spatially explicit model, a number of additional find functions are available for you to obtain references to agents that match a given spatial criteria.

`FindNearby()` is a function that returns a vector of agents that are within a given proximity to a target agent. It takes three arguments: the agent



population primitive, the agent target for which you want nearby neighbors, and a distance. All agents within the specified distance to the target agent will be returned as a vector.

It is useful now to introduce a concept that will be very helpful to you. When used in an Agent, [**Self**] always refers to the agent itself. If you have a primitive within an agent, [**Self**] can be used from that primitive to get a reference to the agent containing the primitive. So the following equation in an agent will return a vector of agents that are within 15 miles of the agent itself:

```
FindNearby([Population], [Self], {15 Miles})
```

Two other useful functions for finding agents in spatial relation to each other are `FindNearest()` and `FindFurthest()`. FindNearest returns the nearest agent to the target while FindFurthest returns the agent that is farthest away from it. Each of them also supports an optional third argument determining how many nearby (or far away) agents to return (this optional argument defaults to one when omitted).

For example, the following equation finds the nearest agent to the current agent:

```
FindNearest([Population], [Self])
```

While this finds the three agents that are furthest from the current agent:

```
FindFurthest([Population], [Self], 3)
```

Movement Functions

You can also move agents to new locations during simulation. To do this, it is helpful to introduce a new primitive we have not yet discussed. This primitive is the *Action* primitive. Action primitives are designed to execute some action that changes the state of your model. For instance, they can be used to move agents or change the values of the primitives within an agent. An action is triggered in the same way a transition is triggered. Like a transition, there are three possible methods of triggering the action: timeout, probability, and condition.

For instance, we can use an action primitive in an agent and the `Move()` function to make agents move during the simulation. The Move function takes two arguments: the agent to be moved, and a vector containing the x- and y-distances to move the agent. Thus, we could place an action primitive in our agent and give it the following action property to make the agent move randomly over time[5]. The equation will move the agent a random distance between -0.5 and 0.5 units in the x-direction and a random distance between -0.5 and 0.5 units in the y-direction.

```
Move([Self], «rand, rand»-0.5)
```

[5]What we are implementing here is known as a “random walk” or Brownian motion. It is a commonly studied pattern of movement with wide applications in science.

Another useful movement function is the `MoveTowards()` function. MoveTowards moves an agent towards (or away from) the location of another agent. MoveTowards takes three arguments: the agent to be moved, the target agent to move towards, and how far to move towards that agent (with negative values indicating movement away). The following command would move an agent one meter closer to its nearest neighbor in the population.

```
MoveTowards([Self], FindNearest([Population], [Self]), {1 Meter})
```

Exercise 13-11

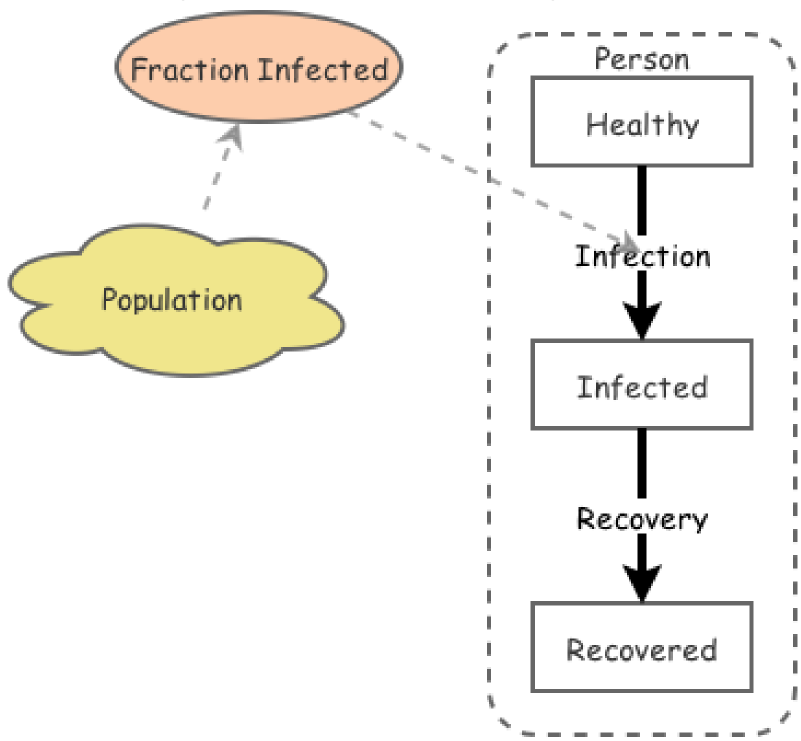
Write an equation to move an agent 2 meters towards the furthest healthy agent.

Answer Available

Agent Movement

This model illustrates the use of movement within agent based models. We adapt the previous disease model so that healthy agents flee from the nearest infected agent.

1. The model diagram should now look something like this:



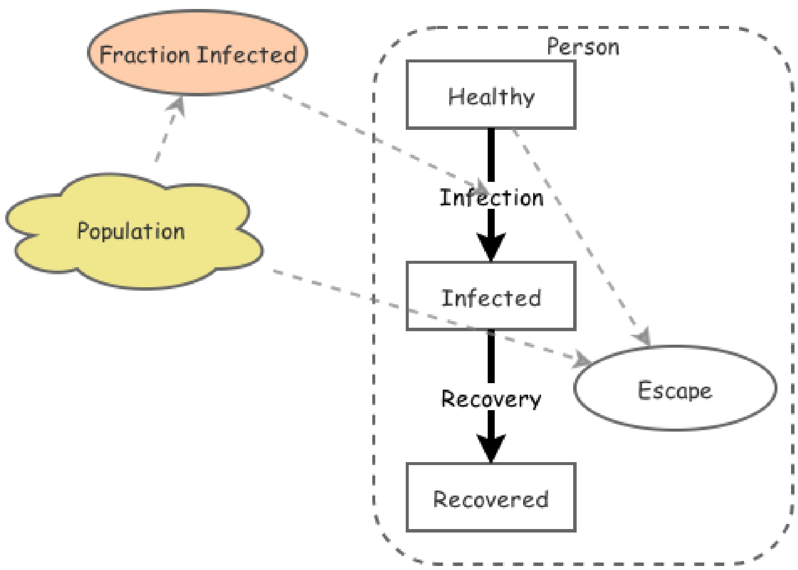
2. We will extend our disease model from earlier by adding movement to the agents. First we need to create an action primitive.

3. Create a new **Action** named [**Escape**].

4. Create a new **Link** going from the primitive [**Healthy**] to the primitive [**Escape**].

5. Create a new **Link** going from the primitive [**Population**] to the primitive [**Escape**].

6. The model diagram should now look something like this:



7. We will have this action be triggered when the agent is healthy and there is at least one infected agent in the simulation.

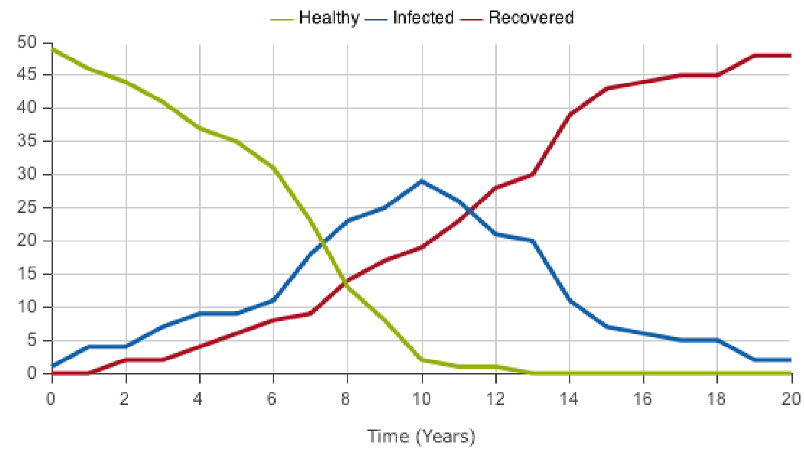
8. Change the **Value/Equation** property of the primitive [**Escape**] to `[Healthy] and Count(FindState([Population], [Infected])) > 0`.

9. The action will cause healthy agents to move away from the nearest infected agent. In effect, fleeing from sick individuals.

10. Change the **Action** property of the primitive [**Escape**] to `MoveTowards([Self], FindNearest(FindState([Population], [Infected]), [Self]), -2)`.

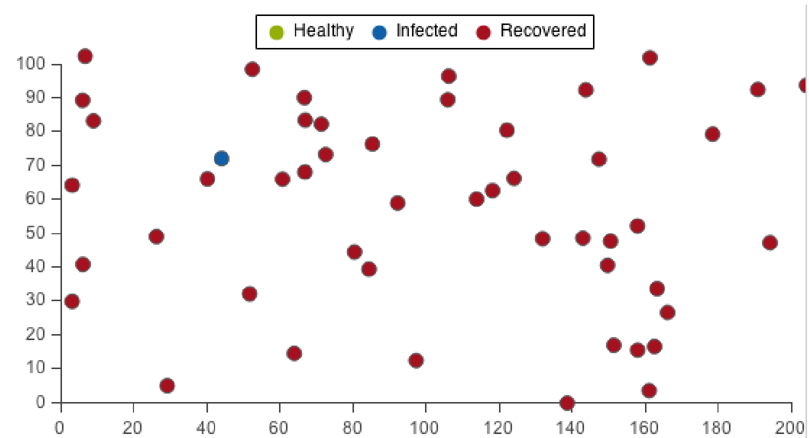
11. We can now run the simulation.

12. Run the model. Here are sample results:



13. These results are not too interesting as they do not show the locations of the agents. To see the agents moving, we need to change the display type to **Map** which gives us a visualization of the location of agents. Then we run the simulation again.

14. Run the model. Here are sample results:



Network Geography

To create connections and remove connections between agents you can use the `Connect()` and `Unconnect()` functions. Both of these functions take two arguments: the agents that should be connected or disconnected. For example, to connect an agent to its nearest neighbor, you could use the following

```
Connect([Self], FindNearest([Population], [Self]))
```

To disconnect an agent from its nearest neighbor (assuming they are connected), you would use:

```
Unconnect([Self], FindNearest([Population], [Self]))
```

To obtain a vector of connections to an agent, use the `Connected()` function:

```
Connected([Self])
```

Connections are not directed so creating a connection from agent A to agent B is the same as creating a connection from agent B to agent A. Also only one connection between a given pair of agents will exist at a time. So creating two connections between a given pair of agents will have the same effect as creating a single connection.

By default, no connections are created when a simulation is initially started. If you change the **Network Structure** configuration property of the agent population primitive, you can specify a function to create connections when the simulation is started. This function is called once for each pair of agents in the model. The agents are available in the function as the variables a and b. If the function evaluates to `true`, then the agents will start connected. If the function evaluates to `false`, the agents will not be initially connected.

You could use this function to, for instance, specify that 40% of agents will be directly connected to each other at the start of the simulation. The following equation would do that by generating a random true/false value with 40% probability of returning `true` each time it is called:

```
RandBoolean(0.4)
```

Multiline Equations

So far in this book, the equations we have looked at have generally been straightforward mathematical formulae. We have introduced some more advanced concepts, such as vectors, but for the most part our equations have been relatively simple one-liners. When doing Agent Based Modeling however, at some point you will find these one-line equations to be limiting. When you begin to run into these limitations with your own models, you may need to start using multiline equations to achieve certain agent behavior.

Almost everyplace in Insight Maker where you can write a mathematical expression, you can also right a multiline equation. It turns out that Insight Maker's language for specifying equations is actually a complete computer programming language and you can exploit the strength of this programming language by writing your equations over several lines instead of using a single line mathematical formula.

We delayed introducing these capabilities until now, as they can sometimes be a distraction from focusing on understanding a system. However, when you build complex Agent Based Models, they can be necessary to express the model logic you wish. Given this need, we will provide a brief introduction to the programming features that can be used as part of Insight Maker equations. You do not need to delve deeply into these capabilities now, but be aware that they are available for when you need them in your own models.

Variables

Variables are temporary slots to store values to be reused within your equations. Variables are created using the '`<-`' symbol meaning assignment. For instance:

```
a <- 2 # The variable 'a' holds the value 2
b <- a + 2 # The variable 'b' holds the value 4
a <- b^2 # a=16, b=4
```

Variable names can contain any number of letters and numbers and must always start with a letter.

If-Then-Else

You should be familiar with the `IfThenElse()` function. A multiline alternative to it exists. The following is equivalent to `IfThenElse([Height] > 2, 1, 2)`.

```
If [Height] > 10 Then
    1
Else
    2
End If
```

One of the benefits of these multiline equations is that they can be more readable than the single line functions. This is especially true if you are trying to do nested *if* statements. Compare `IfThenElse([Height] > 2, 1, IfThenElse([Height] < 1, -1, 2))` to:

```
If [Height] > 2 Then
    1
Else If [Height] < 1 Then
    -1
Else
    2
End If
```

The second one is much more readable. This makes it easier to maintain and more resilient to potential typographical errors.

Loops

Loops are a programming construct that repeat some code multiple times. There are several different types of loops. One important loop is the *for* loop which repeats a command a specified number of times. Here is an example of it being used:

```
sum <- 0
For i From 1 To 3
    sum <- sum + i
End Loop
sum
```

The inner part of the loop is run three times here. The first time the variable *i* is assigned the value of 1, the next time 2, and the last time 3. So this sums up the values of 1, 2, and 3 resulting in 6.

Another variant of the *for* loop is the *for-in* loop. This uses a vector to assign the values of the iterations. The following code sums the numbers 1, 5, and 10 to get 16.

```
sum <- 0
For i In «1, 5, 10»
    sum <- sum + i
End Loop
sum
```

For-in loops can be very useful to iterate through a vector of agents. Another useful loop is called the *while* loop. It does not repeat a predefined number of times and instead repeats until a condition becomes true. Here is an example:

```
total <- 2
While total < 100
    total <- total^2
End Loop
total
```

This code keeps squaring the *total* variable until the total is greater than 100. In this case, this will result in 256.

Functions

Functions allow you to reuse code in multiple places in your model. For instance, imagine you had a model that dealt with temperatures in both Degrees Fahrenheit and Celsius. If you could not use the built in unit conversion functionality, every time you wanted to convert from one form to the other you would have to include the standard conversion formula in your equations. Not only would this be tedious, it would also be error prone as the more times you type an equation, the higher the chance of making a mistake.

You can define functions in two ways. One is a short one-liner:

```
FtoC(f) <- 5/9*(f+32)
```

And another is a multiline form allowing you to incorporate multiline logic in your functions:

```
Function FtoC(f)
    5/9*(f+32)
End Function
```

A great place to include your functions is in the *Macros* section of your model. You can enter macros by clicking the **Macros** button in the **Tools** section of the toolbar. The functions you define here will be accessible in any equation in any part of your model.

Exercise 13-12

Write a function to return the range of a vector. The range is the largest element of the vector minus the smallest element.

Answer Available

Exercise 13-13

Write a function to calculate the nth Fibonacci number. The Fibonacci sequence goes 1, 1, 2, 3, 5, 8, 12, ... After the first two, each number is the sum of the two proceeding numbers in the sequence.

What is the 15th Fibonacci number?

Answer Available

Integrating SD and ABM

System Dynamics modeling and Agent Based Modeling are two different ways of approaching a system. In general, System Dynamics looks at highly aggregated systems and encourages the study of feedback. Agent Based Modeling explores individuals and the interactions between these individuals.

Some software packages only do System Dynamics or Agent Based Modeling leading to the perception that they are somehow incompatible methodologies. Although these techniques can be thought of as quite different, it important to realize that, at the end of the day, both of them are simply applied mathematics. To emphasize this, Insight Maker integrates both these techniques together seamlessly in its modeling environment. There is no such thing as an “Insight Maker Agent Based Model” or an “Insight Maker System Dynamics Model”. There are simply models where you may use agent-based techniques, System Dynamics techniques or a mixture of the two.

Insight Maker (and other modeling packages such as AnyLogic http://www.anylogic.com/) allows you to integrate the two seamlessly together. For instance, in this chapter we have used state transition diagrams within our agents. We could have just as well used stock and flow diagrams within the agents so that each agent in effect contained its own System Dynamics model of its state. Similarly if you have a large System Dynamics model you could create an agent-based sub-model that feeds into the main model dynamics.

When doing modeling, it is important to not get focused on labels or taxonomies of different techniques. Given a modeling task, you want to think about what tools and techniques are best used to approach it. You want to make sure not to approach a modeling task by trying to figure out how to force that task into the constraints of a favorite modeling paradigm.

Exercise 13-14

Compare and contrast the Agent Based Modeling and System Dynamics approach to creating models. Provide three examples of modeling tasks where Agent Based Modeling would be better suited than System Dynamics and three examples where the reverse would be true.

Chapter 14

Optimization and Complexity

We start this chapter by taking up our hamster population model from The Process of Modeling and reconsidering it. As you recall, your friend requested our help in constructing a model to simulate the population of the endangered Aquatic Hamsters. There are many ways to exploit valuable empirical data to improve your models. For instance, if we had data on hamster fecundity, we might be able to plug that information in directly as a parameter in our hamster population model.

One of the most useful kinds of empirical data is historical time series. Some of these time series might represent data and factors that feed into the model, but are not directly modeled. For example, we might have historical temperature data. The temperature could be an important thing to include in the model, as it would affect hamster survival however it is not something we directly model. By this we mean that we do not expect our hamsters to have any effect on the temperature in the region but we do expect the temperature to have an effect on the hamsters. Thus, we can feed the temperature data into the model. We can do this by importing this historical temperature data and including it in the model using a converter primitive.

In other cases, the historical data may represent factors you are directly trying to model. For example, we have a data series of biannual hamster population surveys going back 20 years. This data series lets us know roughly how many hamsters there were over time. Because we are trying to model this data, it is not something plug directly into our model as we could with the temperature, but it is something we can use to calibrate and assess the accuracy of our model.

How do we do this and what will be the results?

Assessing Model Accuracy

We first import our historical data into a converter primitive. We then assess the accuracy of the model in two ways: qualitatively and quantitatively. To assess how well our model fits the historical data qualitatively we plot the simulated and historical data series next to each other. Ideally, they will match up closely but if they do not we should pay close attention to how they differ.

If they have the same general shape (except for a vertical or horizontal displacement) that is good news, as it indicates that you may have gotten the general dynamics of your model correct and that you may just need to fine-tune the relationships and parameter values. If the results look considerably different you may have more work to do in improving the model.

You can also assess the accuracy of models quantitatively. One standard tool people use to assess the accuracy of a model is the R^2 metric[1]. R^2 is the fraction of the squared error explained by the model compared to the "null" model. It ranges from 0 (the model basically provides no predictive power), to 1 (the model predicts perfectly). Mathematically, R^2 is calculated like so:

$$R^2 = \sum_t \frac{(\overline{\text{Truth}} - \text{Truth})^2 - (\text{Model} - \text{Truth})^2}{(\overline{\text{Truth}} - \text{Truth})^2}$$

Naively used, R^2 has a number of issues that we will discuss later in this chapter. However, it is still a useful tool that many people use and with which they are familiar. It is also relatively straightforward to calculate. The following code calculates an R^2 for a model fit. This is code written in JavaScript and can be placed as the **Action** for a button primitive in Insight Maker. The code is written assuming two primitives: a converter [**Historical Hamsters**] containing historical population sizes and a stock [**Hamsters**] containing simulated population sizes. You can edit the code to reference the actual names of the primitives in your model.

```
var simulated = findName("Hamsters"); // Replace with your primitive name
var historical = findName("Historical Hamsters"); // Replace with your primitive name

var results = runModel({silent: true});

var sum = 0;
for(var t = 0; t < results.periods; t++){
    sum += results.value(historical)[t];
}

var average = sum/results.periods;

var nullError = 0;
var simulatedError = 0;
```

[1] Though this metric is not often used in systems dynamics or agent-based models, it is widely used for statistical models such as linear regressions.

```
for(var t = 0; t < results.periods; t++){
    nullError += Math.pow(results.value(historical)[t] - average, 2);
    simulatedError += Math.pow(results.value(historical)[t] - results.value(simulated)[t], 2);
}

showMessage("Pseudo R^2: "+((nullError-simulatedError)/nullError));
```

Calibrating the Model

In addition to using historical data to assess the model fit, you can also use historical data to calibrate model parameters. Depending on the model, you may have many parameters for which you do not have a good way to determine their values. Earlier, we discussed how to use sensitivity testing to assess whether our results are resilient to this uncertainty and to build confidence in the model. Another way to build confidence in your parameter values is, instead of guessing the values of these uncertain parameters, to choose the set of values that results in the best fit between simulated and historical data. This is a semi-objective criterion that helps to remove personal biases you might have from the modeling process.

Goodness of Fit

The first step to using historical data to calibrate the model parameters is to understand what is meant by "the best fit" between historical and simulated data. Conceptually, the idea of a "good fit" seems obvious. A good fit is one where the historical and simulated results are very close together (a *perfect* fit is when they are the same, but that is generally more than we can hope for). However, putting a precise mathematical definition on the concept is not trivial.

Many commonly used goodness of fit measures exist, and below we list some key ones.

Squared Error

Squared error is probably the most widely used of all measures of fit[2]. To calculate the squared error we carry out the following procedure. For each time period we take the difference between the historical data value and the simulated value and then we square that difference. We then sum up all these differences to obtain the total error for the fit. Higher totals indicate worse fits, and lower totals indicate better fits.

The following equation could be placed in a variable to calculate the squared error between a primitive named [**Simulated**] and one named [**Historical**]:

[2]The key reason for this is that regular linear regression (ordinary least squares, the most widely used modeling tool) uses squared error as its measure of goodness of fit. Doing so simplifies the mathematics of the regression problem greatly in the linear case.

```
([Simulated]-[Historical])^2
```

Please note that maximizing the R^2 measure we described earlier is equivalent to minimizing the squared error.

Absolute Value Error

A characteristic of squared error is that outliers have high penalties compared to other data points. Outliers are points in time where the fit is unusually bad. Since the squared error metric squares the differences between simulated and historical data, large differences can cause even larger amounts of error when they are squared. This can sometimes be a negative feature of squared error if you do not want to outliers to have special prominence and weight in the analysis.

An alternative to squared error that treats all types of differences the same is the absolute value error. Here, the absolute value of the difference between the simulated and historical data series is taken. The following equation could be placed in a variable to calculate the absolute value error between a primitive named [**Simulated**] and one named [**Historical**]:

```
Abs([Simulated]-[Historical])
```

Other Approaches

Many other techniques are available for measuring error or assessing goodness of fit. Most statistical approaches function by specifying a full probability model for the data and then taking the goodness of fit not as a measure of error, but rather as the *likelihood*[3] of observing the results we saw given the parameter values. To be clear the issue of optimizing parameter values for models is one that is more complex than what we have presented here. Many sources of error exist in time series and analyzing them is a very complex, statistical challenge. The basic techniques we have presented are, however, useful tools that serve as gateways towards further analytical work.

Exercise 14-1

You have a model simulating the number of widgets produced at a factory. The model contains a stock, [**Widgets**] containing the simulated number of widget produced. You also have a converter, [**Historical Production**] containing historical data on how many widgets were produced in the past.

Write two equations. One to calculated squared error for the models simulation of historical production, and one to calculate the absolute value error of the same.

Answer Available

[3] Likelihood is a technical statistical term. It can be roughly thought of as equivalent to "probability", though it is not precisely that.

Exercise 14-2

You like the idea of penalizing outliers in your optimizations. In fact, you like this idea so much that you would like to penalize outliers even more than squared error does. Create an equation to calculate error that penalizes outliers more than squared error.

Answer Available

Exercise 14-3

Describe why this is not a valid equation to calculate error:

```
[Simulated]-[Historical]
```

Answer Available

Multi-Objective Optimizations

So far our examples have focused on optimizing parameter values for a single population of animals. But what if, instead of one population, we had two or more?

Imagine we were simulation two interacting populations of animals such as the hamsters and their food source, the Hippo Toads. If we had historical data on both the toads and the hamsters we would like our choice of parameter values to result in the best fit between the simulated and historical hamster populations while at the same time resulting in the best fit between the simulated an historical toad populations. This is often quite difficult to achieve, as optimizing the fit for one population will often result in non-optimal fits for the second population.

A straightforward way to try to optimize both populations at once is to make our overall error the sum of the errors for the hamsters and the errors for the toads. For instance, if we had two historical data converters, one for the toads and hamsters, and two stocks, one for each population, the following equation would combine the absolute value errors for both populations.

```
Abs([Simulated Hamsters]-[Historical Hamsters]) + Abs([Simulated Toads]-[Historical Toads])
```

Simply summing the values can sometimes create issues in practice however. Let us imagine that the toad population is generally 10 times as large as the hamster population. If this were the case, the error predicting the toads might be much larger than the error predicting the hamsters and so the optimizer will be forced to focus on optimizing the toad predictions to the detriment of the accuracy of the hamster predictions.

One way to attempt to address this issue is to use the percent error instead of the error magnitude. For example:

```
Abs([Simulated Hamsters]-[Historical Hamsters])/[Historical Hamsters] + Abs([Simulated
```

The percent error metric will be more resilient to differences in scales between the different populations. It will run into issues though if either historical population becomes 0 in size or becomes very small.

Another wrinkle with multi-objective optimizations is that one objective may be more important than the other objectives. For instance, let's imagine our toad and hamster populations were roughly the same size so we do not have to worry about scaling. However, in this case we care much more about predicting the hamsters correctly than we care about the toads. The whole point of the model is to estimate the hamster population so we want to make that as accurate as possible, but we would still like to do well predicting the toads if we are able to.

You can tackle issues like these by "weighting" the different objectives in your aggregate error function. This is most simply done by multiplying the different objectives by a quantity indicating their relative importances. For instance, if we thought getting the hamsters right was about twice as important as getting the toads right, we could use something like:

```
2*Abs([Simulated Hamsters]-[Historical Hamsters]) + Abs([Simulated Toads]-[Historical
```

This makes one unit of error in the hamster population simulation count just as much as two units of error for the toad population simulation[4].

Exercise 14-4

Why does the percent error equation have issues when the historical data become very small? What happens when the historical data becomes 0?

Finding the Best Fit

After choosing how to measure the quality of a fit quantitatively, we need to find the set of parameter values that maximize the fit and minimize the error.

[4] Weighting is a useful technique you can use for other optimization tasks. Imagine you had a model simulating the growth of your business in the next 20 years. You want to use this model to adjust your strategy to achieve three objectives: maximizing revenue, maximizing profit, and maximizing company size. Potentially maximizing profit would be the most important objective with maximizing company size being the least important. You can use weights to combine these three criteria into a single criterion for use by the optimizer.

To do this we use a computer algorithm called an optimizer that automatically experiments with many different combinations of parameter values to find the set of parameters that has the best fit.

Many optimizers basically work by starting with an initial combination of parameter values and measuring the error for that combination. The optimizer then slightly changes the parameter values in order to check the error at nearby combinations of parameter values. For instance, if you are optimizing one parameter, say the hamster birth rate, and your initial starting value is a birth rate of 20% per year; the optimizer will first measure the error at 20% and then measure the errors at 19% and 21%.

If one of the neighbors has a lower error than the initial starting point, the optimizer will keep testing additional values in that direction. It will steadily "move" towards the combination of parameters that results in the lowest error, one step at a time. If, however, the optimizer does not find any nearby combination of parameter values with a lower error than its current combination of parameter values, it will assume it has found the optimal combination of parameter values and stop searching for anything better.

The precise details of optimization algorithms are not important. You need to be aware of one key thing however: these algorithms are not perfect and they sometimes make mistakes. The root cause of these mistakes are so-called "local minimums". An optimizer works by searching through combinations of different parameter values trying to find the combination that minimizes the error of the fit. The combination that has the smallest error out of all possible combinations is known as the true minimum or the "global" minimum.

A local minimum is a combination of parameter values that are not the global minimum, yet whose nearby neighbors all have higher errors. Figure 1 illustrates the problem of local minimum. If the optimizer starts near the first minimum in this figure it might head towards that minimum without ever realizing that another, improved minimum exists. Thus, if you are not careful, you may think you have found the optimal set of parameters when in fact you have only found a local minimum that might have much worse error than the true minimum.

There is no foolproof way to deal with local minimums and no guarantee that you have found the true minimum[5]. The primary method for attempting to prevent an optimization from settling in on a local minimum is to introduce stochasticity into the optimization algorithm. Optimization techniques such as *Simulated Annealing* or *Genetic Algorithms* will sometimes choose combinations of parameter values at random that are actually *worse* than what the optimizer has already found. By occasionally moving in the "wrong" direction, away from the nearest local minimum, these optimization algorithms are more resilient

[5]This is true for the type of optimization problems you will generally be dealing with. Other types of optimization problems are much easier than the ones you may be encountering, as they are what are known as *convex* optimization problems and are guaranteed not to have any local minimums.

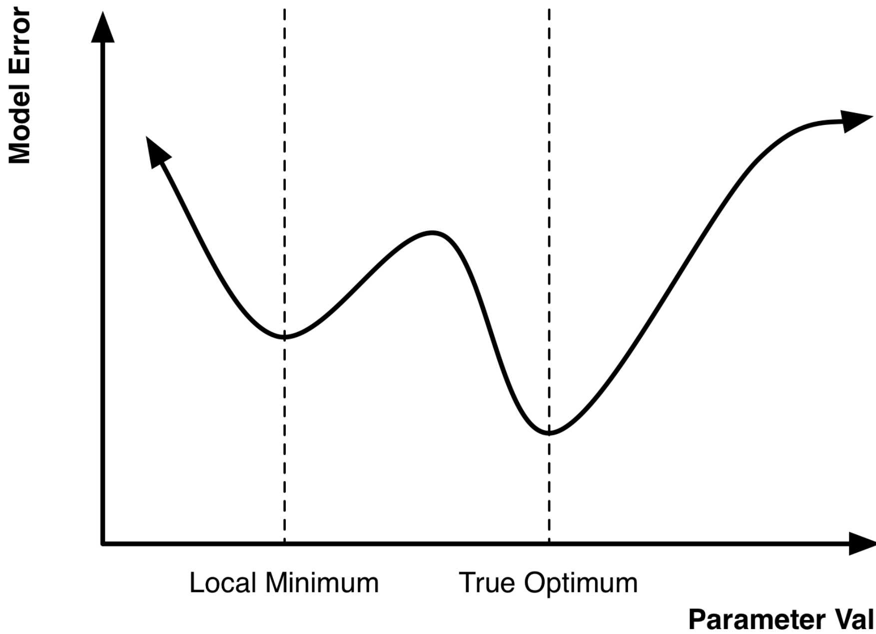


Figure 1. An illustration of local and global minimum for an optimization problem involving a single parameter.

and less likely to become stuck on a local minimum and more likely to keep searching for the global minimum.

Unfortunately, in our experience we have not been satisfied by the performance of these types of stochastic optimization algorithms. They are generally very slow and without fine-tuning by an expert can still easily become stuck in a local minimum. We prefer to use non-stochastic deterministic methods as the core of our optimizations. We then introduce stochasticity into the algorithm by using multiple random starting sets of parameter values. For instance, instead of carrying out a single optimization we will do 10 different optimizations each starting at a different set of parameter values. If all 10 optimizations arrive at the same final minimum that is strong evidence we have found the global minimum. If they all arrive at different minima, then there is a good chance we have not found the global minimum.

Optimizing Parameter Values

This model illustrates the use of optimization and historical data to select the growth rate for a simulated population of hamsters.

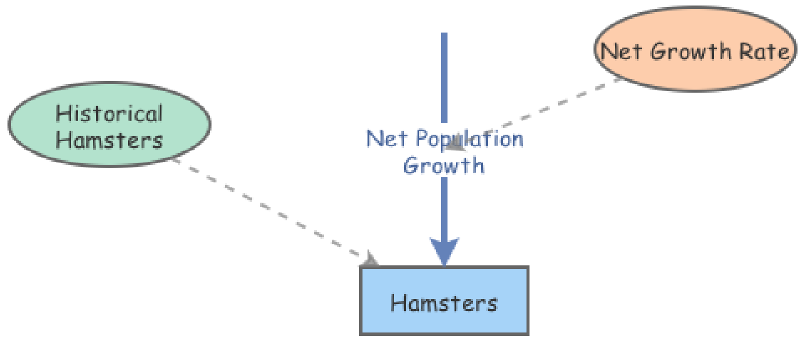
1. Create a new **Converter** named [**Historical Hamsters**].

2. Change the **Data** property of the primitive [**Historical Hamsters**] to `0, 22; 2, 49; 4, 40; 6, 61; 8, 100; 10, 104; 12, 153; 14, 243; 16, 236; 18, 370; 20, 560`.
3. The model diagram should now look something like this:

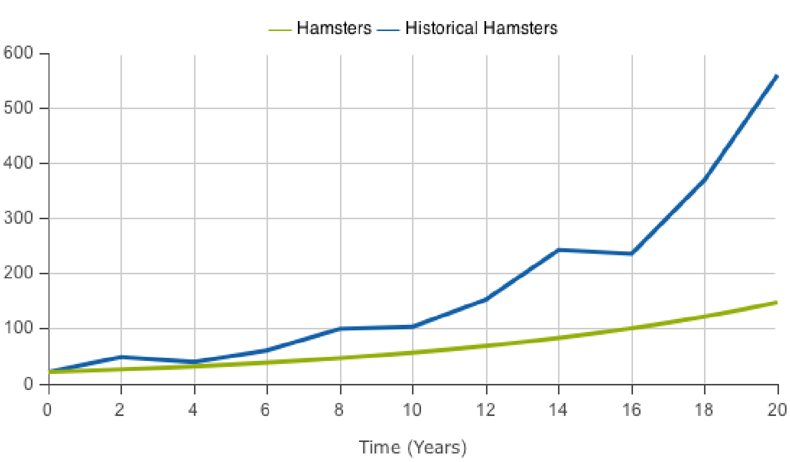
4. We start by importing our historical population data into a converter primitive. In this illustrative example we have twenty years of data with a census of the hamster population being carried out every two years. We run the model to see what this historical data looks like.
5. Run the model. Here are sample results:

6. There is a lot of variability and the population even declines some years. However, it looks like in general the rate of growth increases as the population size increases. This is what we would expect to see with exponential growth. Let's build a simple exponential growth model to attempt to replicate what we see with the historical data.
7. Create a new **Stock** named [**Hamsters**].
8. Create a new **Flow** going from empty space to the primitive [**Hamsters**]. Name that flow [**Net Population Growth**].
9. Create a new **Variable** named [**Net Growth Rate**].
10. Create a new **Link** going from the primitive [**Net Growth Rate**] to the primitive [**Net Population Growth**].

11. Create a new **Link** going from the primitive [**Historical Hamsters**] to the primitive [**Hamsters**].
12. The model diagram should now look something like this:



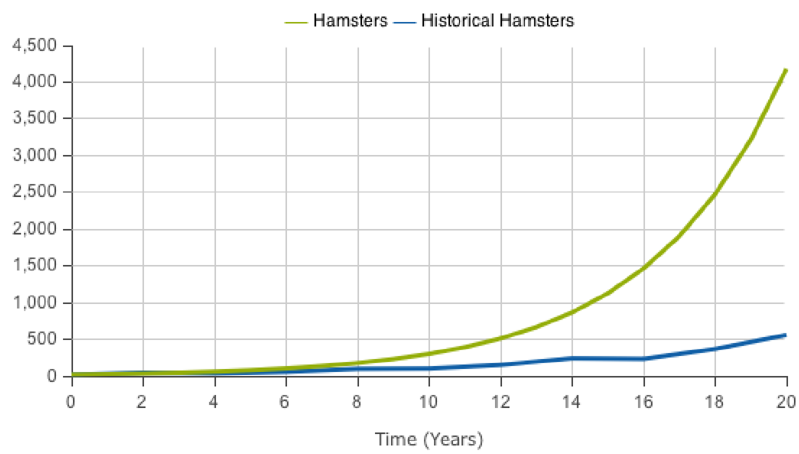
13. That's the structure of our model. Now we can fill in the equations. We'll set the initial population size for our simulated hamster population to be the same as for the historical data.
14. Change the **Initial Value** property of the primitive [**Hamsters**] to `[Historical Hamsters]`.
15. Change the **Flow Rate** property of the primitive [**Net Population Growth**] to `[Hamsters]*[Net Growth Rate]`.
16. What growth rate should we begin with? We do not have any data on this. Let's experiment by starting with 10% per year and see what we end up.
17. Change the **Equation** property of the primitive [**Net Growth Rate**] to `0.1`.
18. Run the model. Here are sample results:



19. That does not look too great. Our simulated population is much smaller than the historical values. Let's try a larger growth rate, say 30%.

20. Change the **Equation** property of the primitive [**Net Growth Rate**] to `0.3`.

21. Run the model. Here are sample results:



22. That's not good either, now our population is too large! We could keep experimenting with different growth rates to find a good one, but that might take a while. Let's just let the optimizer do the work for us. First we need to create a primitive to hold the error. We will use the squared error measure we discussed earlier.

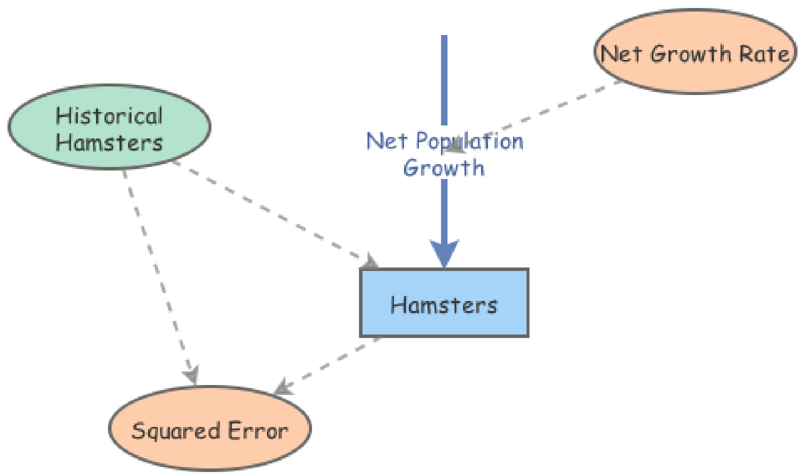
23. Create a new **Variable** named [**Squared Error**].

24. Create a new **Link** going from the primitive [**Hamsters**] to the primitive [**Squared Error**].

25. Create a new **Link** going from the primitive [**Historical Hamsters**] to the primitive [**Squared Error**].

26. Change the **Equation** property of the primitive [**Squared Error**] to `([Hamsters]-[Historical Hamsters])^2`.

27. The model diagram should now look something like this:



28. There, we have set up what we need for the optimizer to work. Now we can run the optimizer. We set the **Goal Primitive** to [**Squared Error**] and the **Primitive to Change** to [**Net Growth Rate**]. We tell the optimizer to minimize the integral of the error and set the optimizer to work.

29. The optimizer gets our results almost instantly: 0.172 or 17.2% is the optimal growth rate. When we run the model with this value the results look great. It's an almost perfect match between the historical and simulated data.

30. Change the **Equation** property of the primitive [**Net Growth Rate**] to `0.172`.

31. Run the model. Here are sample results:

Exercise 14-5

You are building a model to simulated company profits into the future. You have 30 years of historical company profit data you will use to calibrate parameter values using an optimizer.

Choose an error measure to use. Justify this choice and explain why you would use it instead of other measures.

Exercise 14-6

Calculate the pseudo R^2 for [**Growth Rate**] = `0.1`, `0.3`, and `0.172`.

Exercise 14-7

Adjust the JavaScript code to calculate pseudo R^2 to use absolute value error instead of squared error.

Answer Available

Exercise 14-8

Describe local minimum, why they cause issues for optimizers, and strategies for dealing with them.

The Cost of Complexity

After a good deal of work and many sleepless nights you have completed the first draft of your Aquatic Hamster population model. The results are looking great and your friend is really impressed. When he runs it by some colleagues however, they point out that your model does not account for the effects of the annual Pink Spotted Blue Jay migration.

Pink Spotted Blue Jays (PSBJ) are a species of bird that migrates every fall from northern Canada to Florida. In the spring they return from Florida to Canada. Along the way, they usually spend a few days by the lake where the Aquatic Hamsters have their last colony. During this time they eat the same Orange Hippo Toads the hamsters themselves depend upon as food. By reducing the Hippo Toad population, the PSBJ negatively affect the hamsters, at least for this period of time when there is less food available to support them.

The timing of the PSBJ migration can vary by several weeks each year no one knows precisely when the PSBJ's will arrive at the lake or even how long they will stay there. Further, the population of migrating birds can fluctuate significantly with maybe 100 birds arriving one year and 10,000 another year. The amount of toads they eat is proportional to the number of birds. Not much data exist quantifying the birds' effects on the hamsters, but it is a well-established fact that they eat the Hippo Toads the hamsters rely upon for their survival and many conservationists are concerned about the migration.

Your friend's colleagues wonder why you have decided to not include the PSBJ migration in your model. They want to know how they can trust a model that does not include this factor that clearly has an effect on the hamster population.

In response, you may point out that though the migration clearly has an impact, it appears to be a small one that is not as important as the other factors in the model. You add that there are no scientific studies or theoretical basis to define exactly how the migration functions or how it affects the hamster population. Given this, you think it is probably best to leave it out.

You say all this, but they remain unconvinced. "If there is a known process that affects the hamster population, it should be included in the model," they persist. "How can you tell us we shouldn't use what we know to be true in the model? We know the migration matters, and so it needs to be in there."

The Argument for Complexity

Your friend's colleagues have a point. If you intentionally leave out known true mechanisms from the model, how can you ask others to have confidence that the model is accurate? Put another way, by leaving out these mechanisms you ensure the model is wrong. Wouldn't the model *have* to be better if you included them?

This argument is, on the surface, quite persuasive. It is an argument that innately makes sense and appeals to our basic understanding of the world: Really it seems to be "common sense".

It is also an argument that is wrong and very dangerous.

Before we take apart this common sense argument piece by piece, let us talk about when complexity is a good thing. As we will show, complexity is not good from a modeling standpoint, but it can sometimes be a very good tool to help build confidence in your model and to gain support for the model.

Take the case of the PSBJ migration. It might be that adding a migration component to the model ends up *not* improving the predictive accuracy of the model. However, if other people view this migration as important, you may want to include the migration in the model if for no other reason than to get them on board. Yes, from a purely "prediction" standpoint it might be a waste of time and resources to augment the model with this component, but this is sometimes the cost of gaining support for a model. A "big tent" type model that brings lots of people on board might not be as objectively good as a tightly focused model, but if it can gain more support and adoption it might be able to effect greater positive change.

The Argument Against Complexity

Generally speaking, the costs of complexity to modeling are threefold. Two of them are self evident: there are computational costs to complex models as they take longer to simulate and there are also cognitive costs to complex models in that they are harder to understand. There is, however, a third cost to complexity that most people do not initially consider: complexity often leads to less accurate models compared to simpler models.

In the following sections we detail each of these three costs.

Computational Performance Costs

As a model becomes more complex, it takes longer to simulate. When you start building a model it may take less than a second to complete a simulation. As the model's complexity grows, the time required to complete a simulation may grow to a few seconds to a few minutes and then to even a few hours or more.

Lengthy simulation times can significantly impede model construction and validation. The agile approach to model development we recommend is predicated on rapid iteration and experimentation. As your simulation times cross beyond even something as small as 30 seconds, model results will no longer be effectively immediate and your ability to rapidly iterate and experiment will be diminished.

Furthermore, when working with an optimizer or sensitivity-testing tool, performance impacts can have an even larger effect. An optimization or sensitivity testing tool may run the model thousands of times or more in its analysis so even a small increase in the computation time for a single simulation may have a dramatic impact when using these tools.

Optimizations themselves are not only affected by the length of a simulation, they are also highly sensitive to the *number* of parameters being optimized. You should be extremely careful about increasing model complexity if this requires the optimizer to adjust additional parameter values. A simplistic, but useful, rule of thumb is that for every parameter you add for an optimizer to optimize, the optimization will take 10 times as long[6].

[6] In practice an optimizer should ideally perform a bit better than this, but this provides us a useful guideline to understand optimizations. Also it should be noted that the optimizations we are talking about here are for non-linear optimization problems for which gradients (derivatives) cannot be directly calculated. For other types of optimization problems, such as linear problems, much faster optimization techniques are available.

Thus if it takes one minute to find the optimal value for one parameter, then it takes 10 minutes to find the optimal values for two parameters and 100 minutes to find the optimal values for three parameters. Imagine we had built a model and optimized five parameters at once. We then increased the model complexity so we now had to optimize ten parameters. Our intuition would be that the optimization would now take twice as long. This is wrong. Using our power of ten rule we know that the time needed will be closer to 10^5 or 100,000 times as long!

That is a huge difference and highlights how important it is to keep model complexity at a manageable level. In practice, a rule of thumb is that you should have no difficulty optimizing one or two parameters at a time. As you add more parameters that optimization task becomes rapidly more difficult. At five or so parameters you have a very difficult but generally tractable optimization challenge. Above five parameters you may be lucky to obtain good results.

Cognitive Costs

In addition to the computational cost of complexity, there is also a cognitive cost. As humans we have a finite ability to understand systems and complexity. This is partly why we model in the first place: to help us simplify and understand a world that is beyond our cognitive capacity.

Returning to our hamster population model, including the bird migration could create a confounding factor in the model that makes it more difficult to interpret the effects of the different components of the model and extract insights from them. If we observe an interesting behavior in the expanded model we will have to do extra work to determine if it is due to the migration or some other part of the model. Furthermore, the migration may obscure interesting dynamics in the model making it more difficult for us to understand the key dynamics in the hamster system and extract insights from the model.

We can describe this phenomenon using a simple conceptual model defined by three equations. The number of available insights in a model is directly proportional to model complexity. As the model complexity increases, the number of insights available in the model also grows.

$$\text{Available Insights} \propto \text{Complexity}$$

Conversely, our ability to understand the model and extract insights from it is inversely proportional to model complexity. α is a constant indicating how much understandability decreases as complexity increases. This relationship is non-linear as each item added to a model can interact with every other item currently in the model. Thus, the cognitive burden increases exponentially as complexity increases.

$$\text{Understandability} \propto \alpha^{-\text{Complexity}}$$

The number of insights we actually obtain from a model is the product of the number of available insights and our ability to understand the model:

$$\text{Insights} = \text{Available Insights} \times \text{Understandability}$$

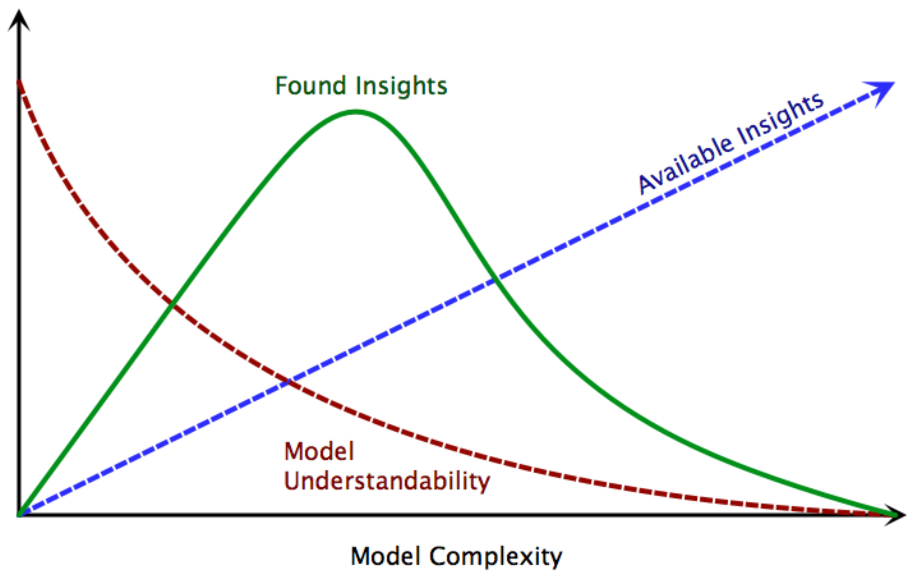


Figure 2. Expected discoveries of insights as model complexity increases.

Thus when the model complexity is 0 – in effect basically no model – we gain no insights from the model. As the model complexity starts to rise, we begin to gain additional insights. After a certain point however, the added model complexity actually inhibits additional understanding. As complexity rises our insights will fall back down towards 0. This phenomenon is illustrated in Figure 2.

Accuracy Costs

The negative effects of complexity on computational performance and our cognitive capacity should not be a surprise. What may be surprising on the other hand, is the fact that complex models are in fact often *less accurate* than simpler alternatives.

To illustrate this phenomenon, let us imagine that for part of our hamster population model we wanted to predict the size of the hamsters after a year[7]. The hamsters go through two distinct life stages in their first year: an infant

[7] Size could affect hamster survival and fecundity so it could be an important variable to model.

life stage that lasts 3 months and a juvenile life stage that lasts 9 months. The hamsters' growth patterns are different during each of these periods.

Say a scientific study was conducted measuring the sizes of 10 hamsters at birth, at 3 months and at 12 months. The measurements at birth and 12 months are known to be very accurate (with just a small amount of error due to the highly accurate scale used to weigh the hamsters). Unfortunately, the accurate scale was broken when the hamsters were weighed at 3 months and a less accurate scale was used instead for that period. The data we obtain from this study are tabulated below and plotted in Figure 3:

Hamster	Birth	3 Months	12 Months
1	9.0	23.2	44.4
2	9.7	19.8	44.0
3	10.2	23.5	44.7
4	8.8	32.2	43.3
5	10.1	31.3	44.5
6	10.0	27.2	44.2
7	10.0	21.4	46.1
8	11.1	24.1	46.0
9	8.7	41.0	44.9
10	11.2	31.7	43.8

Now, unbeknownst to us, there are a pair of very simple equations that govern Aquatic Hamster growth. During the infant stage they gain 200% of their birth weight in that three-month period. Their growth rate slows down once they reach the juvenile stage such that at the end of the juvenile stage their weight is 50% greater than it was when they completed the infant stage. Figure 3 plots this true (albeit unknown) size trajectory compared to the measured values. The higher inaccuracy of the measurements at 3 months compared to 0 and 12 months is readily visible in this figure by the greater spread of measurements around the 3 month period.

We can summarize this relationship mathematically:

$$\text{Size}_{t=3\text{ months}} = 3.00 * \text{Size}_{t=0\text{ months}}$$
$$\text{Size}_{t=12\text{ months}} = 1.50 * \text{Size}_{t=3\text{ months}}$$

Naturally, we can combine these equations to directly calculate the weight of the hamsters at 12 months from their weight at birth:

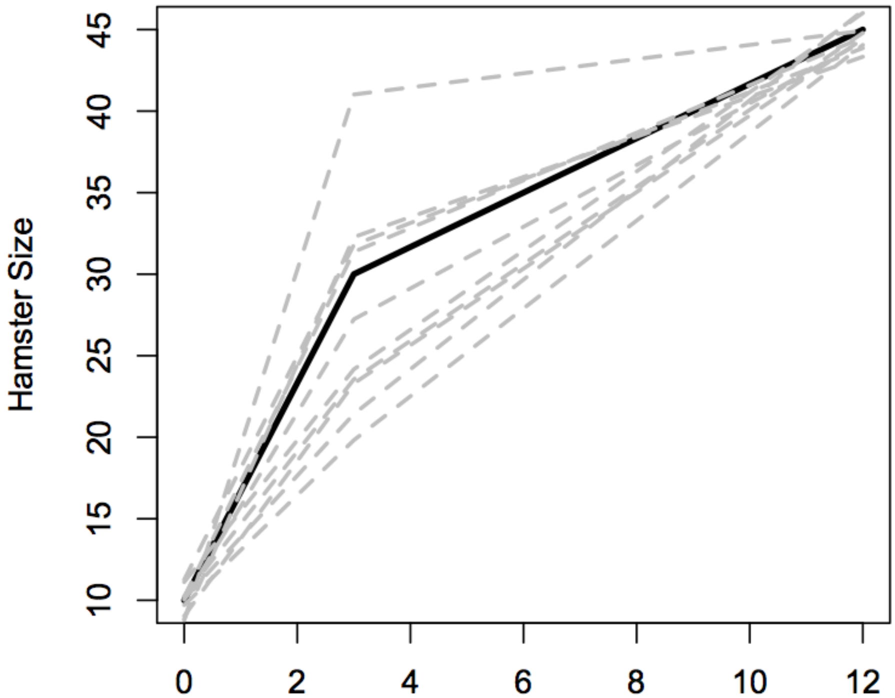


Figure 3. Recorded hamster sizes (dashed grey lines) and the unknown true size trajectory for a hamster starting with size 10 (solid black line).

$$\text{Size}_{t=12\text{ months}} = 4.50 * \text{Size}_{t=0\text{ months}}$$

Again, we don't know this is the relationship, so we need to estimate it from the data. All we care about is the size of hamsters at 12 months given their birth size. The simplest way to estimate this relationship is to do a linear regression estimating the final size as a function of the initial size. This regression would result in the following relationship:

$$\text{Size}_{t=12\text{ months}} = 4.65 * \text{Size}_{t=0\text{ months}}$$

This result is quite good. The linear coefficient we estimate of 4.65 is very close to the true value of 4.50. Our model so far is doing pretty well.

However, like with the bird migration, someone might point out that this model is too crude. "We know that the hamster go through an infant and juvenile stage", they might say, "we should model these stages separately so the model is more accurate."

This viewpoint moreover has actually been held to be the case in the law. For instance, there have been judicial decisions that "life-cycle" models, those that model each stage of an animal's life are the only valid ones[8]. If we were presenting this model in to an audience that believed that, we would have to create two regressions: one for the infant stage and one for the juvenile stage.

Using the data we have, we would obtain these two regressions:

$$\text{Size}_{t=3 \text{ months}} = 2.74 * \text{Size}_{t=0 \text{ months}}$$
$$\text{Size}_{t=12 \text{ months}} = 1.54 * \text{Size}_{t=3 \text{ months}}$$

Combining these regression to get the overall size change for the 12 months we obtain the following:

$$\text{Size}_{t=12 \text{ months}} = 4.22 * \text{Size}_{t=0 \text{ months}}$$

Now, in this illustrative example we are fortunate to know that the true growth multiplier should be 4.50 so we can test how well our regression actually were. The error for this relatively detailed life-cycle model is $(4.50 - 4.22)/4.50$ or 6.2%. For the "cruder" model where we did not attempt to model the individual stages, the overall error is $(4.50 - 4.65)/4.50$ or 3.3%.

So by trying to be more accurate and detailed, we built a more complex model that has almost twice the error of our simpler model! Let's repeat that: The more complex model is significantly worse in accuracy than the simpler model.

Why is that? We can trace the key issue back to the problem that our data for the 3 month period are significantly worse than our data for 0 months or 12 months. By introducing it into the model, we bring down the overall quality of the model by injecting more error into it. When someone comes to you asking you to add a feature to a model you have to consider if this feature may actually introduce more error into the model as it did in this example.

We can think of life-cycle and many other kinds of models as a chain. Each link of the chain is a sub-model that takes data from the previous link, transforms them and feeds them into the next link. Like a chain, models may only be as good as their weakest link. It is often better to build a small model where all the links are strong, than a more complex model with many weak links.

Exercise 14-9

Implement a model tracking the growth of a hamster from birth to 12 months. Create the model for a single hamster and then using sensitivity testing to

[8] Technically the determination is that life-cycle models are the "best available science". These decisions are misguided and frankly wrong, but that is what occurs when judges are put in the position of making highly technical scientific decisions.

obtain a distribution of hamster size. Assume hamster are born with an average size of 10 and a standard deviation of 1. Use the true parameter growth rates and do not incorporate measurement uncertainty in the model;

Exercise 14-10

Define a procedure for fitting a System Dynamics model of hamster growth to the hamster growth data in the table. Assume you know that their are two linear growth rates for the infant and juvenile stages but you do not know the values of these rates.

Answer Available

Exercise 14-11

Apply the optimization procedure to your System Dynamics model to determine the hamster rates of growth from the empirical data.

Overfitting The act of building models that are too complex for the data you have is known as "overfitting" the data[9]. In the model of hamster sizes, the model where we look at each life stage separately is an overfit model; We do not have the data to justify this complex of a model. The simpler model (ignoring the different stages) is superior.

Overfitting is unfortunately too common in model construction. Part of the reason is that the techniques people use to assess the accuracy of a model are often incorrect and inherently biased to cause overfitting. To see this, let's explore a simple example. Say we want to create a model to predict the heights of students in high schools (this is seemingly trivial, but bear with us). To build the model we have data from five hundred students at one high school.

We begin by averaging the heights of all the students in our data set and we find that the average student height is 5 feet 7 inches. That number by itself is a valid model for student height. It is a very simple model[10], but it is a model nonetheless: Simply predict 5 feet 7 inches for the height of any student.

We know we can make this model more accurate. To start, we decide to create a regression for height where gender is a variable. This gives us a new model

[9] The reverse – building models that are too simple – is called "underfitting". In practice, underfitting will be less of a problem as our natural tendency is to overfit.

[10] Statisticians would call this the "null" model, the simplest model possible.

which predicts women high-school students have a height of 5 feet 5 inches on average, while men have a height of 5 feet 9 inches on average. We calculated the R^2 for the model to be 0.21.

That's not bad, but for prediction purposes we can do better. We decide to include students' race as a predictor as we think that on average there might be differences in heights for different ethnicities. We complete this extended model including ethnic status as a predictor alongside gender and the R^2 fit of our model increases to 0.33.

We still think we can do better though, so we add age as a third predictor: We hypothesize that the older the students are, the taller they will be. The model including age as an additional linear variable is significantly improved with an R^2 of 0.56.

Once we have built this model, we realize that maybe we should not just have a linear relationship with age because as students grow older, their rate of growth will probably slow down. To account for this we decide to also include the square of age in our regression. With this added variable our fit improves to an R^2 of 0.59.

This is going pretty well, we might be on to something. But why stop with the square; what happens if we add higher order polynomial terms based on age? Why not go further and use the cube of age. The fit improves slightly again. We think we are on a roll and so we keep going. We add age taken to the fourth power, and then to the fifth power, and then to the sixth, and so on.

We get a little carried away and end up including 100 different powers of age and each time we add and new power our R^2 gets slightly better. We could keep going, but it's time to do a reality check.

Do really we think that including AGE^{100} made our model any better than when we only had 99 terms based on age? According to the R^2 metric it did (if only by a very small amount). However, we know intuitively it did not. Maybe the first few age variables helped, but once we get past a quadratic ($\text{AGE} + \text{AGE}^2$) or cubic ($\text{AGE} + \text{AGE}^2 + \text{AGE}^3$) relationship, we probably are not capturing any more real characteristics of how age affects a person's size.

Variables	R^2
Gender	0.21
Gender, Race	0.33
Gender, Race, Age	0.56
Gender, Race, Age^2	0.59
Gender, Race, Age^2, ..., Age^{100}	0.63
Gender, Race, Age^2, ..., Age^{500}	1.00

So why does our reported model accuracy – R^2 – keep getting better and better as we add these higher order power terms based on age to our regression?

This question is at the heart of overfitting. Let's imagine taking our exploitation of age to its logical conclusion. We could build a model with 500 different terms based on age ($\text{AGE} + \text{AGE}^2 + \text{AGE}^3 + ... + \text{AGE}^{500}$). The result of this regression would go through every single point in our population of five hundred students[11].This model would have a perfect R^2 of one (as it matches each point perfectly) but we know intuitively that it would be a horrible model.

Why is this model so bad? Imagine two students born a day apart one with a height of 6 feet 2 inches the other with a height of 5 feet 5 inches. Our model would indicate that a single day caused a 7-inch difference in height. Even more ridiculous, the model would predict a roller coaster ride for students as they aged. They would gain inches one day (according to the model) and lose them the next. Clearly this model is nonsensical. However, this nonsensical model has a perfect R^2, it is a paradox!

The key to unlocking the solution to the paradox and overcoming overfitting turns out to be surprisingly simple: *assess the accuracy of a model using data that were not used to build the model*.

The reason our overfit model for students looks so good using the R^2 error metric is that we measured the R^2 using the same data that we just used to build the model. This is an issue as we can force an arbitrarily high R^2 simply by continually increasing the complexity of our model. In this context the R^2 we are calculating turns out to be meaningless.

What we need to do is to find new data – new students – to test our model on. That will be a more reliable test of its accuracy. If we first built our model and then took it and applied it to a different high school and calculated the R^2 using this new data, we would obtain a truer measure of how good our model actually was.

Figure 4 illustrates the effect of overfitting using observation from 9 students. The top three graphs show plots of the heights and ages for these nine students. We fit three models to these data: a simple linear one, a quadratic polynomial, and an equation with nine terms so that it goes through each point exactly.

Below the three graphs we show the regular R^2 that most people use when fitting models, and also what the true R^2[12] would be if we applied the resulting model to new data. The regular R^2 always increases so if we used this naive metric we would always end up choosing the most complex model. As we can see, the true accuracy of the model decreases after we reach a certain

[11] Remember a polynomial equation with two terms can perfectly pass through two data points, an equation with three terms can perfectly pass through three points, and so on.

[12] You might have heard of R^2 variants such as the Adjusted R^2. The Adjusted R^2 is better than the regular R^2; however it is important to note that it is not the true R^2. Adjusted R^2 also has some issues with overfitting.

complexity. Therefore the middle model is really the better model in this case. When illustrated like this, this concept of overfitting should make a lot of sense; but, surprisingly, it is often overlooked in practice even by modeling experts.

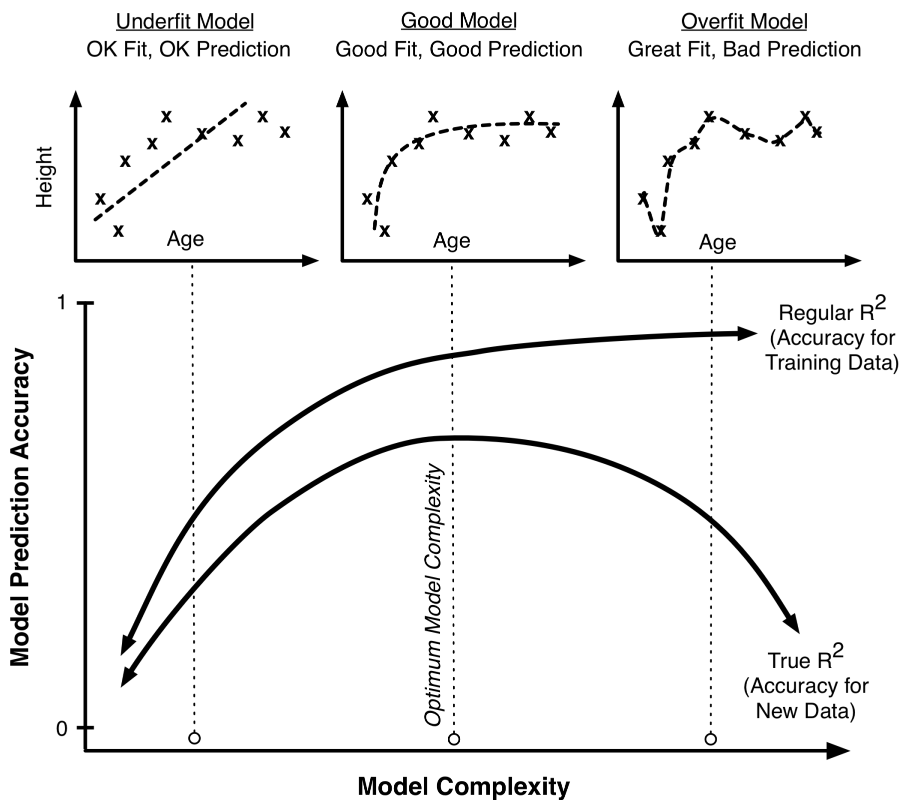


Figure 4. Illustration of overfitting. The best model is not necessarily the one that fits the data the closest.

In general, overfitting should be watched for carefully. If you do not have a good metric of model error, the inclination to add complexity to your model will be validated by misleadingly optimistic measures of error that make you think your model is getting better when it is actually getting worse. The optimization techniques we described earlier in this chapter are also susceptible to these problems as every time you add a new variable to be optimized the optimization error will always go down further (assuming the true optimal parameter configuration can be found). The more parameters you add the worse this effect will be.

How do we estimate the true error of the model fit? The simplest approach is to take your dataset and split it into two parts. Build the model with one half of the data and then measure the accuracy using the other half. So with our high-school students we would randomly assign each one to be used either to build the model or to assess the model's error. Advanced statistical techniques such as *cross-validation* or *bootstrapping* are other approaches and can be more effective given a finite amount of data. Unfortunately, we do not have space to discuss them here, but we would recommend the reader explore them on their own if they are interested in this topic.

No one ever got fired for saying, "Let's make this model more complex." After this chapter, we hope you understand why this advice, though safe to say, is often exactly the wrong advice.

Exercise 14-12

What is overfitting? What is underfitting?

Exercise 14-13

You have been asked to evaluate a model built by a consulting company. The company tells you that their model has an R^2 of 0.96 and is therefore a very accurate model.

Do you agree? What questions or tests do you need to do to determine if the model is good?

Chapter 15

Exercise Answers

This section contains answers to selected exercises.

Chapter 2

Exercise 2-4

You haven't been given enough information about what's being modeled to determine what might be the appropriate **Time Units** or **Time Step**.

References

- How does DT work? from isee Systems
- DT Situations Requiring Special Care from isee Systems

~ Exercise

Consider the images in Figure 12 and think about what **Time Units** and **Time Step** you would use in a model representing the growth in each of these areas.

Exercise 2-5

While each of the situations in Figure 12 represents growth dependent on the current value of the situation each one has a different growth factor.

Exercise 2-6

Other values for **Time Step** produce less correct answers because 1 is the value most appropriately representing what the model was created to model.

Chapter 4

Exercise 4-1

Hopefully you found that both 0.25 and 0.125 produced a step with a distance to Grandma's House of 0 at 2.25 hours. In finding no difference between the results for 0.25 and 0.125 you should have concluded that 0.25 was a small enough for this model. Smaller is not always better. In this case it just makes the model run for more steps.

Exercise 4-2

The simulation engine in Insight Maker is smart enough to convert between the myriad of similar dimensions, e.g., miles, kilometers, feet, etc. Though it's recommended that you make conversions explicit otherwise models become very difficult to understand.

Insight Maker doesn't complain because you're still comparing distance to distance and it doesn't matter that they're in different scales simply because zero = zero in any scale. It's really better to keep your units in the same scale.

Exercise 4-3

One alternative would be to start with Distance to Grandmas House = 0 and add to the stock as one walks toward it. This way you're tracking the distance traveled rather than the distance left to travel.

Chapter 5

Exercise 5-1

- If the limit is evenly divisible into 1 then there is no change in the cure as the goal is reached.
- There isn't a noticeable difference between the graph for 0.25 and 0.125 so 0.25 would be the most appropriate value to use for Time Step.

Exercise 5-5

The [limiting factor] could interact with [results] in such a way that what becomes limited is the [action] that produces the [results].

Exercise 5-6

Simply because the structure is constructed in a balanced way. In a real situation this is not likely to be the case.

Chapter 7

Exercise 7-1

It would be better to build a statistical model in this case.

Exercise 7-2

It would be better to build a mechanistic model in this case.

Exercise 7-5

1. Prediction
2. Inference
3. Prediction
4. Narrative
5. Narrative
6. Inference

Chapter 8

Exercise 8-1

Minimum value: 0

Maximum value: 10,000,000 (this value is somewhat arbitrary but should be larger than the maximum size you expect this city to ever grow to)

Exercise 8-2

We use a standard deviation of 4 as we lack any information on what the dispersion should be.

```
Round(Rand(5, 15))
```

Exercise 8-3

```
Round(RandTriangular(0, 100, 20))
```

Exercise 8-4

```
Round(RandLogNormal(20, 4))
```

We use a standard deviation of 4 as we lack any information on what the dispersion should be.

Exercise 8-5

```
RandNormal(2.1, 0.3625)
```

Exercise 8-6

```
RandNormal(0.837, 0.106)
```

Chapter 10

Exercise 10-1

You can denote volume of water in the jar using the state variable J. Our equations will then be:

$$J(0) = 40$$

$$\frac{dJ}{dt} = -0.10 \times J$$

Exercise 10-2

You can denote the healthy stock using state variable H and the infected stock I. Our equations will then be:

$$H(0) = 100$$

$$I(0) = 1$$

$$\frac{dH}{dt} = -0.05 \times H \times I$$

$$\frac{dI}{dt} = 0.05 \times H \times I$$

Exercise 10-3

Approximately 8,865 animals.

Exercise 10-4

$$P = 10 - \alpha \times t$$

Exercise 10-5

$$P = 10 \times e^{0.05 \times t}$$

Exercise 10-6

$$P = \frac{20}{1 - 20 \times \beta \times t}$$

Exercise 10-7

20.0, 25.0, 29.0, 32.4, 35.5, 38.3

Exercise 10-8

20.0, 27.0, 37.5, 53.9, 78.3, 124.5

Exercise 10-9

20.0, 24.5, 28.3, 31.6, 34.6, 37.4

Exercise 10-10

20.0, 29.1, 44.7, 73.6, 131.5, 260.4

Chapter 11

Exercise 11-1

Stable Equilibria: A piece of rubber that returns to its original shape after pulled, a forest where trees grow back once cut down.

Unstable Equilibria: A ball balanced on top of a sloped roof, a pole balanced perfectly on the floor.

Exercise 11-2

$X = -2.30$ and $X = 1.30$

Exercise 11-3

$X = 0, \pi, 2\pi, 3\pi, 4\pi, ...$

Exercise 11-4

$X = -1.82, Y = -1.36$

Exercise 11-5

$X = 0, Y = 0$ and $X = -1.41, Y = -2$ and $X = 1.41, Y = -2$

Exercise 11-9

$$\begin{bmatrix} 1 & 0\ 0 & 2 \times Y \end{bmatrix}$$

Exercise 11-10

$$\begin{bmatrix} 2 \times X & -1 - 4 \times X & -2 \times Y \end{bmatrix}$$

Exercise 11-11

$$\begin{bmatrix} Y \times X & X + 2 \times \beta \times Y \; 3 \times \alpha \times X^2 + 2 \times X \times Y & X^2 \end{bmatrix}$$

Exercise 11-12

Eigenvalue of 6 with eigenvector of $[1, 1]$. Eigenvalue of -2 with eigenvector of $[-1, 1]$.

Exercise 11-13

Eigenvalue of 2 with eigenvector of $[1, 5]$. Eigenvalue of 1 with eigenvector of $[0, 1]$.

Exercise 11-14

Eigenvalue of $\alpha - \beta$ with eigenvector of $[-1, 1]$. Eigenvalue of $\beta + \alpha$ with eigenvector of $[1, 1]$.

Exercise 11-15

Eigenvalue of α with eigenvector of $[1, 0]$. Eigenvalue of β with eigenvector of $\left[\frac{-\beta}{\alpha-\beta}, 1\right]$.

Exercise 11-16

1. Unstable
2. A saddle (unstable)
3. Stable

Exercise 11-17

1. Unstable oscillations
2. Damped oscillations (stable)
3. Stable oscillations

Exercise 11-18

1. Unstable
2. Stable
3. Unstable

Exercise 11-19

Equilibrium $X = 2, Y = -2$ is unstable.

$X = 0, Y = -2$ is an unstable saddle point.

Exercise 11-20

Equilibrium $Q = 1, R = 1$ is stable if $\alpha \geq 0$. Otherwise it is unstable.

$Q = 1, R = -1$ is unstable.

Exercise 11-21

The first equilibrium has no wolves and is unstable.

The second equilibrium is when the population size is equal to the carrying capacity. This equilibrium is stable.

Chapter 12

Exercise 12-1

```
This <b>text is <i>italic</i> and bold.</b>
```

Exercise 12-2

Ordered list:

```
<ol>
    <li>Croatia</li>
    <li>Greece</li>
    <li>Peru</li>
</ol>
```

Unordered list:

```
<ul>
    <li>Croatia</li>
    <li>Greece</li>
    <li>Peru</li>
</ul>
```

Exercise 12-4

```
u {
    color: green;
}
```

Exercise 12-5

```
a {
    border: solid 2px red;
}
```

Exercise 12-6

```
var a = prompt("Enter the first number:");
var b = prompt("Enter the second number:");
var sum = a+b;

alert("There sum is: "+sum);
```

Exercise 12-7

```
h1 {
    text-decoration: underline;
}
```

Exercise 12-8

```
body {
    background-color: azure;
}
```

Exercise 12-10

```
input {
    background-color: yellow;
    color: navy;
}
```

Exercise 12-11

Change the alert to:

```
alert("Failed! You need "+(5000000000-pop)+" more people!");
```

Chapter 13

Exercise 13-2

1. Timeout trigger with value 10 days.
2. Probability trigger with value 20% (assuming time units of years).
3. Condition trigger. Value: `[Volume] > 5`

Exercise 13-4

```
Max(Filter(«2, 1.8, 1.9, 1.5», x < 1.95))
```

Exercise 13-5

```
Median(a^2)
```

Exercise 13-6

```
Min(Intersection(a, b))
```

Exercise 13-7

```
Sum(a)/Count(a)
```

Exercise 13-8

```
FindState(FindState([Population], [Infected]), [Female])
```

Exercise 13-9

```
Union(FindNotState([Population], [Infected]), FindState([Population], [Female]))
```

Exercise 13-10

```
Mean(Value(FindState([Population], [Male]), [Height]))-Mean(Value(FindState([Population], [Femal
```

Exercise 13-11

```
MoveTowards([Self], FindFurthest(FindState([Population, [Healthy]), [Self]), {2 Meters})
```

Exercise 13-12

```
range(x) <- max(x)-min(x)
```

or

```
Function Range(x)
    Max(x)-Min(x)
End Function
```

Exercise 13-13

```
Function Fib(n)
    If n = 1 or n = 2 Then
        1
    Else
        Fib(n-1) + Fib(n-2)
    End If
End Function
```

The 15th Fibonacci number is 610.

Chapter 14

Exercise 14-1

Squared error:

`([Widgets]-[Historical Production])^2`

Absolute value error:

`Abs([Widgets]-[Historical Production])`

Exercise 14-2

`([Simulated]-[Historical])^4`

Exercise 14-3

The optimizer can always minimize this simply making [**Simulated**] as small as possible. This will not result in a fit to the historical data.

Exercise 14-7

Change:

```
nullError += Math.pow(results.value(historical)[t] - average, 2);
simulatedError += Math.pow(results.value(historical)[t] - results.value(simulated)[t],
```

To:

```
nullError += Math.abs(results.value(historical)[t] - average);
simulatedError += Math.abs(results.value(historical)[t] - results.value(simulated)[t])
```

Exercise 14-10

Example procedure:

1. Find the average hamster size at each time period by taking the mean of observations at that period.
2. Define two variables in the model: [**Infant Rate**] and [**Juvenile Rate**].
3. Define an error primitive [**Error**] the equation taking the absolute value of the difference between the simulated size and the average empirical size.
4. Run the optimizer to minimize this error term by adjusting the two rate variables.

Chapter 16

References

Andersena, D., and G. Richardsona. 1997. "Scripts for group model building." *System Dynamics Review* 13 (2): 107–129.

Forrester, Jay Wright, and Peter M. Senge. 1978. "Tests for building confidence in system dynamics models."

Fortmann-Roe, Scott. 2012. "Accurately Measuring Model Prediction Error" (apr). http://scott.fortmann-roe.com/docs/MeasuringError.html.

Grimm, V. 2005. "Pattern-Oriented Modeling of Agent-Based Complex Systems: Lessons from Ecology." *Science* 310 (5750) (nov): 987–991.

Haller, H., and S. Krauss. 2002. "Misinterpretations of Significance: A Problem Students Share with Their Teachers." *Methods of Psychological Research Online* 7 (1): 1–20.

He, D., E. L. Ionides, and A. A. King. 2009. "Plug-and-play inference for disease dynamics: measles in large and small populations as a case study." *Journal of The Royal Society Interface* 7 (43) (dec): 271–283.

King, Aaron A., Edward L. Ionides, Mercedes Pascual, and Menno J. Bouma. 2008. "Inapparent infections and cholera dynamics." *Nature* 454 (7206) (aug): 877–880.

Meadows, D. H., and J. M. Robinson. 1985. *The Electronic Oracle*. Albany, NY: System Dynamics Society.

Ries, Eric. 2011. *The Lean Startup*. New York: Crown Business.

Romer, Christina, and Jared Bernstein. 2009. "The job impact of the American recovery and reinvestment plan."

The Heritage Foundation. 2013. "Unemployment Rate January 2013" (feb). http://www.heritage.org/multimedia/infographic/2013/02/unemployment-rate-january-2013.

Vennix, Jac AM, Wim Scheper, and Rob Willems. 1993. “Group model-building: what does the client think of it:” 534–543.