[DRAFT] Beyond Connecting the Dots: Mastering the Hidden Connections in Everything that Matters

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Preface

Ludwig von Bertalanffy(1) first proposed, in 1937, that the same basic structures operated across all disciplines, and if one learned how these structures operated one could transfer much of their learning from one discipline to another. When moving from one discipline to another, one would simply have to learn the structures that were operating, and the labels on the elements of the structures. On first reading this may seem most profound, or maybe even preposterous.

However, if you think about it, maybe there is some truth to it after all. What follows is the introduction to a live Systems Thinking book presented from a cross discipline models perspective. Live in the sense that the models are presented in a form that allows you to actually interact with them.

von Bertalanffy wrote "Allegemein Systemlehre" which was translated into English as "General Systems Theory"(2) and I expect we've still not recovered from the translation error. What he intended was a "General Theory of Systems" or "General Systems Teaching," a way to support learning about the structures which operated across all disciplines. Today there are a set of structures referred to as Systems Archetypes which I believe are just what Bertalanffy had in mind.

In the words of von Bertalanffy, “The student in ‘system science’ receives a technical training which makes systems theory – originally intended to overcome current overspecialization – into another of the hundreds of academic specialties”(1)

Systems Thinking is not a method though more of a way of looking at the world around us and understanding based not from understanding things though more from understanding relations and interactions between things. And while there are many who believe that Systems Thinking or a Systems Perspective provides the best foundation for creating effective approaches of dealing with challenges and shaping a better tomorrow. Yet even with that view, over the past 75 years it has not become widely adopted, even though during that period dozens of approaches have been developed with claim to embrace the Systems Thinking world view. I believe Pogo had it right when he said, “We have met the enemy and he is us.” I have repeatedly commented to people that the greatest impediment to the adoption of Systems Thinking is Systems Thinkers.

This should provide you with a sense of why this book has to be different. Now let me offer you a view of how it will be different.

It is our intent to provide a basis for recovering from this overspecialization by offering an extensive series of models from everyday life that will show the value of looking at things though a different lense. We will then build on this to develop an understanding without all the terminology and complexity that typically drives people away from Systems Thinking.

References

- Davidson, Mark. 1983. Uncommon Sense: The Life and Thought of Ludwig von Bertalanffy http://www.amazon.com/Uncommon-Sense-Thought-Bertalanffy-1901-1972/dp/087477165X/

Chapter 1

It's The Pattern That Connects

What you learn, and your capacity to learn, serves as a basis for everything you do in life. Yet, have you ever thought about how you really learn about the world around you? There are some things you memorize early in life, like the times tables. While you memorize these is that really learning? Do you remember that if you put your hand on something very hot it will burn you, or is that something you learned? And if you learned that, how was it that the learning happened? In this chapter we will investigate how you actually learn. And, we will present a introduction to how you can improve your learning and actually test whether what you have learned is actually correct.

Patterns

Consider the following

- I have a box that's about 3' wide, 3' deep and 6' high
- It's a rather heavy box
- The has a couple of doors on it
- When you open the doors it's cooler inside the box than outside
- One compartment is much colder than the other
- When you open the door a light comes on
- There's food inside the box
- The box is in the kitchen
- There are sticky notes all over the front of the box
- There's a collection of papers and stuff on top of the box
- If you move the box you'll probably find a lot of dust under it
- The box is plugged into an electrical outlet
- From time to time you can hear the box running

At some point in this sequence you probably became convinced that what was being described was a refrigerator. Now stop for a moment and ask yourself just how was it that you realized what was being described was a refrigerator? Yes it would have been easier if I had just shown you a picture of a refrigerator, though that would have spoiled it, wouldn't it.

Figure 1. It's A Refrigerator. How do you know?

As long as you knew beforehand what a refrigerator was, the statements could have been given to you in any order, and still at some point you would have finally realized what was being described. If you had never seen, nor heard about, a refrigerator before, you would still be wondering what was being described and what to call it.

You have also most likely come to understand that all refrigerators are not identical. Some have one door with a separate compartment inside. Some have two doors and a drawer. Some are much smaller than others. Some can fit under a counter and some even fit on top of a counter. Some can be so large you can walk into them.

Figure 2. Many Kinds of Refrigerators, or Freezers - How do you know?

If you see any of these you quickly decide it's a refrigerator. How does that happen? Gregory Bateson, one of the great thinkers of our time, said, "It's the pattern that connects." If you reflect on this statement you should come to realize there are actually different ways to interpret what it means. In this particular case the pattern connects you to the following purpose

- The box keeps food from readily spoiling by keeping it cold
- Part of the box is a freezer which keeps food from spoiling for even longer

and you understand it to be a refrigerator. Though now that we've arrived at this point we still haven't addressed the question of how you know. You were probably not actually taught that it's the above purpose that defines the essence of a refrigerator. Most people were not, though they have essentially learned it over time.

Models

Models are the way we look at, and understand the world around us. All we have are our models. They are the way we understand everything. This is so because we build our understanding based on what we already understand.

The world around us simply has too much detail for us to pay attention to everything. A refrigerator has many pieces though how many do you really pay attention to? Probably not many unless you build or repair refrigerators. We filter out much of the detail around us so we don't become overloaded and we choose what to pay attention to. Sometimes we do this consciously and sometimes subconsciously. In the midst of what we choose to pay attention to there are patterns. Whether we realize it or not it is these patterns that we pay attention to and attempt to make sense of. We understand these patterns by linking them to extend patterns we already understand. And much of the world around us we simply ignore for if we didn't we would just be overwhelmed.

Model

A model is a simplified version of some aspect of the world around us to help us understand something.

Learning

When we experience something that experience falls somewhere between complete novelty, meaning that we can't connect it with anything in our past experience, and complete confirmation, meaning that it represents something we perceive as already completely understood. Experiences which lie somewhere between complete novelty and complete confirmation provide a basis for learning. They represent a basis for connecting to understood patterns, extending those patterns and our understanding, and what results is learning. {Cite: Jantach, Eric. 1980. The Self-Organizing Universe: Scientific and Human Implications. Pergamon Press. http://www.amazon.com/The-Self-Organizing-Universe-Implications-Innovations/dp/0080243118/}

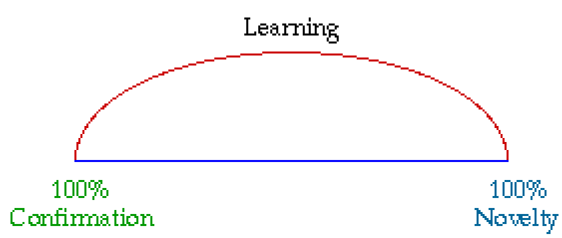


Figure 3. Experience as a Basis for Learning

Consider running into a refrigerator that looks like no refrigerator you've never seen before. From an initial view you are likely not to perceive it as a refrigerator.

As you inspect it to find it serves the purpose you've come to understand for refrigerators or if someone tells you it's a refrigerator you then expand or extend your awareness of the range of patterns that constitute a refrigerator.

A Basis for Flawed Learning

While reading the previous paragraphs did it dawn on you that much of this pattern recognition/connection/extension learning doesn't happen consciously? We connect with patterns and extend our knowledge at times without even being consciously aware that it is happening. And when it happens in a subconscious manner there isn't really any critical validation that happens along with the learning. Because this ongoing learning happens without critical validation there are things we learn and come to believe which are actually incorrect. We have perceived patterns and extended our learning in a flawed manner. The really annoying thing is that we then act on these beliefs, and when we produce results that don't go the way we planned we wonder why. Or even worse, we don't actually learn from the results and correct the flawed models which served as the basis for our actions.

When we act on flawed beliefs attempting to solve problems we typically create more problems than we fix. It has been said repeatedly that the majority of today's problems are the direct result of yesterday's solutions. Wouldn't this provide a sense that we might really benefit from a better way to think about the world around us, develop better understanding, and develop solutions that don't come back to haunt us in the future?

A Better Way

Based on what has been presented to this point it should be obvious that we could benefit from a better way to develop models of what we believe. Models that are more likely to be correct, as well as surface flaws in many of our current beliefs.

Ludwig von Bertalanffy first proposed, in 1937, that the same basic structures operated across all disciplines, and if one learned how these structures operated one could transfer much of their learning from one discipline to another.{Davidson, Mark. 1983. Uncommon Sense: The Life and Thought of Ludwig von Bertalanffy. J.P. Tarcher, Inc. http://www.amazon.com/Uncommon-Sense-Thought-Bertalanffy-1901-1972/dp/087477165X/} When moving from one discipline to another, one would simply have to learn the structures that were operating, and the labels on the elements of the structures. On first reading this may seem most profound, or maybe even preposterous. However, if you think about it, maybe there is some truth to it after all.

We're not asking you to believe the previous statement because it was provided here. Though through the experience presented shortly it is hoped that you will arrive at a sensibility of the statement from your own perspective.

Consider the following images and ask yourself what it is that all these different items actually have in common.

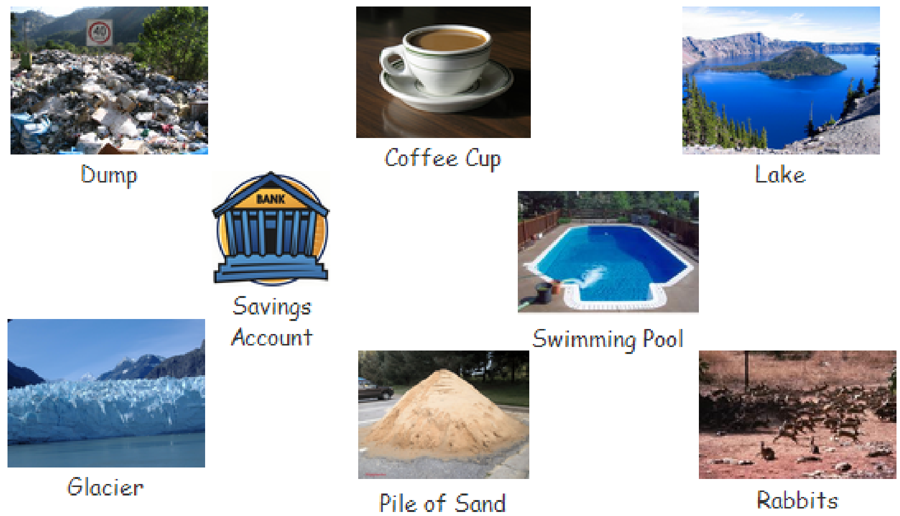


Figure 4. What do these items have in common?

Each of these items represents a collection of stuff. Admittedly each image represents different stuff though stuff just the same. Because in each case this stuff collected over time it's really more appropriate to refer to the the collections of stuff as accumulations. And as you will come to realize it is extremely important to remember that accumulations take time to accumulate, and often even longer to get rid of when you find out you don't want them.

The shorter term often used to refer to an accumulation is "stock." Just where this term originated is uncertain. What you call an accumulation of stuff isn't nearly as important as remembering it's a bunch of stuff that collects, or goes away, over time. How much time is different for each one of the accumulations. Now is probably a good time to talk about how accumulations happen over time.

How each of the accumulations in Figure 4 changes is a bit different, as are the time frames of concern. Time frame being the time it takes for some noticeable change in the accumulation. The following segments describe each accumulation in some detail.

Coffee Cup

You usually fill a coffee cup from a coffee pot and it takes a few seconds. Then you take a few minutes to drink the coffee as it's usually too hot to drink when you initially get it.

Dump

A dump generally accumulates by the truckload after the garbage is picked up at houses or businesses in your community. If the dump were just getting started you'd probably notice it grow with each additional truck load. As it gets bigger and bigger it's gets more difficult to notice that it's growing, even though it is. While the dump is likely to grow almost every day we are probably more likely to think about the growth of the dump in months and years. And does it ever really go away? Usually when it gets to be too much a new dump is started somewhere else and the current dump is buried. Though when it's buried it doesn't really go away does it? It's still there and we'll probably talk more about dumps later on.

Glacier

A glacier is a long term accumulation of snow which packs down and turns to ice. Glaciers get bigger in the winter when it's generally colder and snow falls, then they get smaller in summer when some portion of the glacier melts. The time frame one usually uses to think about glaciers is years or even decades.

Lake

Lakes are bigger than a pond and smaller than an ocean and usually filled with fresh water, not salty, unless it's The Great Salt Lake. The lake is filled by rivers and streams that flow into it as well as rain water. One might think of this in terms of gallons per hour or gallons per minute in the case of a large inflow such as at Niagara Falls where the water flows into Lake Ontario in the USA. Water leaves the lake through rivers and streams as well as evaporation into the air. For a lake one might think about the water flowing into or out of the lake in hours though when considering the level of the lake itself the change might be considered over days or weeks. It sort of depends on what you're interested in.

Pile of Sand

The pile of sand probably showed up in a truck that dumped it right where it is. While it may have taken the truck a while to drive from the wherever it started it probably only took a few minutes to dump the sand once the truck arrived. The sand is probably referred to in cubic yards, which is how much sand it takes to fill a box that's 1 yard wide, 1 yard deep, and 1 yard high. How long it takes for the sand to go away depends on how it's taken away. If you use a wheel barrow then you have to shovel the sand into the wheel barrow and take it to wherever you're going to use it. At this rate it may take days to move it. If you move it with a small piece of machinery, a Bobcat or a Backhoe, then will will probably only take a few minutes to an hour to get it moved.

Rabbits

A population of rabbits gets larger with new rabbit births and gets smaller with rabbit deaths. Have you ever heard the phrase "multiply like rabbits?" What it means is that it doesn't take very long for a few rabbits to become many rabbits, as long as there is a good food supply and not to many predators like wolves and coyotes. The time frame for considering a rabbit population is probably months to years.

Savings Account

A savings account is a bank account where if you put money in, and keep it there, the bank will periodically give you money just for keeping it there. They won't give you very much, though some. If you keep putting money in your savings account every so often and never take it out one day you'll be rich. Yet, for some reason that doesn't seem to happen for too many people. We'll have to talk about that sometime later in the book. One generally thinks about the money associated with a savings account in dollars, the interest rate as a percentage, and the time frame in months and years.

Swimming Pool

Swimming pools usually hold thousands of gallons of water and you usually have a couple of options to fill one. You might use a garden hose, which will take days, or a hose from a fire hydrant, which will take a few hours, or from a tanker truck, which probably takes a few loads. In each case the water filling the pool is probably measured in gallons per hour. Once you fill the pool you loose a little water when people in the pool get out, thought not too much. Most of the water loss from a pool is though evaporation due to the sun and when you backwash the filter used to keep the pool clean. The change in the amount of water is usually measured in gallons per hour.

Exercise 1-1

Take a few minutes and identify half a dozen situations you're familiar with where there are stocks that increase, or decrease, over time. What are the quantities for those stocks, e.g., gallons, pounds, kilograms, etc. What are the flows that increase or decrease those stocks, and what are the time frames over which you think about the increase, or decrease?

At this point you may be wondering why so much time was spent making you walk though all these examples for increasing and decreasing accumulation of stuff. Since we said this was an interactive book you're probably wondering where the interaction is.

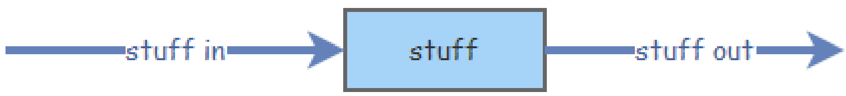


Figure 5. The Accumulation of Stuff

All the accumulations depicted in Figure 4 can be represented in a general form by the model in Figure 5. Remember we defined a model as a simplified version of some aspect of the world around us to help us understand something. It doesn't get much simpler than this does it?

Some amount of stuff flowing in causes stuff to increase over time and the amount of stuff flowing out causes stuff to decrease over time. With both of these happening at the same time stuff increases if stuff in is larger than stuff out. And if stuff out is greater than stuff in then the accumulation of stuff gets smaller. The most critical aspect of this to remember is that it takes time for stuff to increase or decrease. How fast the change happens depends on the amount of stuff in the flows.

Swimming Pool

This model uses a swimming pool as a stock.

1. The model diagram should now look something like this:


2. The diagram represents a swimming pool being filled with a hose at a rate pf 50 gallons an hour.
3. The drain is closed so there is not water draining out of the pool.
4. If we let the hose run for 24 hours how much water will be in the pool? Admittedly the math is pretty straight forward though the idea here is to show how you can use a model, actually a simulation of a model, to show changes over time.
5. Run the model. Here are sample results:

6. This graph indicates that after 24 hours the swimming pool with have 1,200 gallons of water in it. Yes, it's about as interesting as watching paint dry. Actually, as you will come to find out, that's a good thing because this is really easy. A more interesting question might be, if the swimming pool holds 20,000 gallons of water how long with it take to fill with water at 50 gallons per hour? We'll get to this shortly.

In the next chapter you will actually learn to build the models being presented in here.

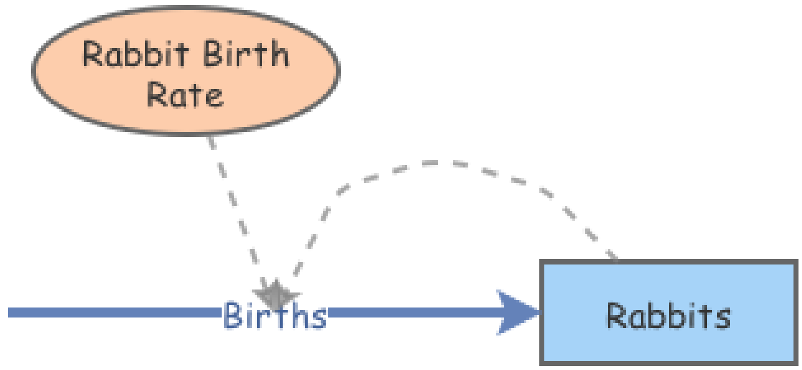
Rabbit Population Growth

If you considered the accumulation of rabbits you may have already realized that the model of Figure 5 is missing something. Yes, if you add rabbits to rabbits you get even more rabbits. Though if you have more rabbits don't they create even more rabbits?

Rabbit Population Growth

This model that reflects the the notion that more rabbits create even more rabbits.

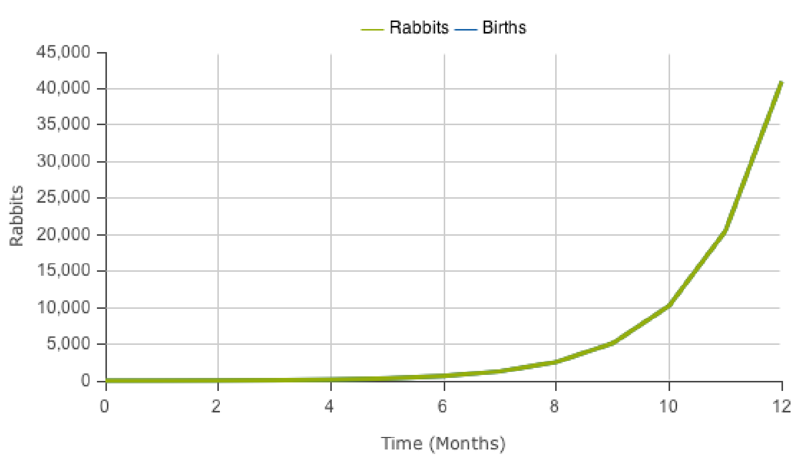
1. The model diagram should now look something like this:



2. This model indicates that if you start with some population of Rabbits and each time period the current number of Rabbits times the Rabbit Birth Rate will result in a number of Births. This number of Births will then be added to the accumulation of Rabbis and figure into the calculation for the next period.

3. Suppose we start with 10 Rabbits, half of which are male and half of which are female. Research indicates that a female rabbit can give birth to between 18 and 26 Rabbits a year. The average for this, (18 + 24) / 2 = 22, though we'll round this up to 24 just because it will make the math easier. If a female Rabbit can produce 24 Rabbits a year, that's 2 per months, though it actually takes two Rabbits, one male and one female. With all these assumptions we get about 1 new Rabbit per month for each Rabbit.

4. Run the model. Here are sample results:



5. Forty thousand Rabbits in a year? That seems a bit bizarre doesn't it? This result actually points out the real value of modeling, which is

learning. You build a model based on what you think you understand. You then populate it with assumptions about the values and you run it. The result then either seems to make sense or seems really bizarre. When the results are really bizarre what the model is telling you is that either the structure is wrong, the assumptions are wrong, or both, because the world can't possibly be this bizarre. As a result you investigate the model and your assumptions. As your understanding improves the model gets better. At some point the model finally serves its purpose, to be a simplification of some aspect of the world which leads to a better understanding. Hopefully you come to find that going round and round with a model can be a delightful learning process.

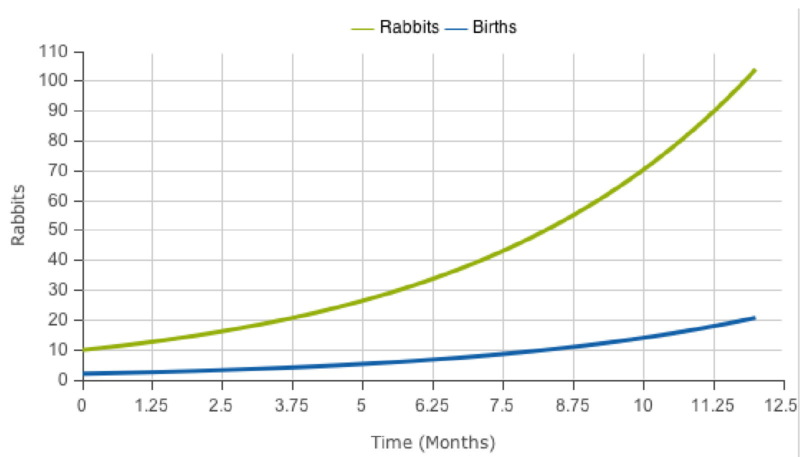
6. Also, did you notice the choppy nature of the graph? This is a result of some incorrect settings and we'll talk about why this is happens and how to address it in Chapter 2.

7. After that sidetrack lets get back to our 40,000 Rabbits that can't possibly exist after a year. You can be pretty sure how many Rabbits you started with at the beginning. And when you check the formula for Births it seems to be in order. This sort of means the assumption for Rabbit Birth Rate must be too big. If you think about what the model is doing it's probably not too difficult to figure out that the model assumes that a Rabbit can be born this month and then give birth to another Rabbit next month. If a Rabbit has to mature for six months before it gives birth to Rabbits then the Rabbit Birth Rate might be something more like 20%.

8. Change the **Equation** property of the primitive [**Rabbit Birth Rate**] to `.2`.

9. Change the **Simulation Time Step** property of the Time Settings to `0.25`.

10. Run the model. Here are sample results:



11. Is this right? A good thing to remember at this point is that "Is it right?" is actually the wrong question. A better question might be, "What have I learned, and is there more I can learn?" The graph sure seems more reasonable than what the model presented in Figure 16 though do you have a high degree of confidence in the current Rabbit Birth Rate. Are there a number of other questions we could ask about our Rabbits. What is the Rabbit Death Rate? Do they have enough food to eat? Are they living out in the open where Coyotes and Foxes can get at them? Does their owner have a passion for Rabbit Stew? These might each be a basis for building a better model, though at this point we're going to leave the Rabbits alone and move on to something else.

The most important learning you should take away from this model is that when what flows into the accumulation increases as the accumulation increases the accumulation can get real big in a hurry. This is actually called exponential grown and we'll talk in more detail about this in Chapter 2.

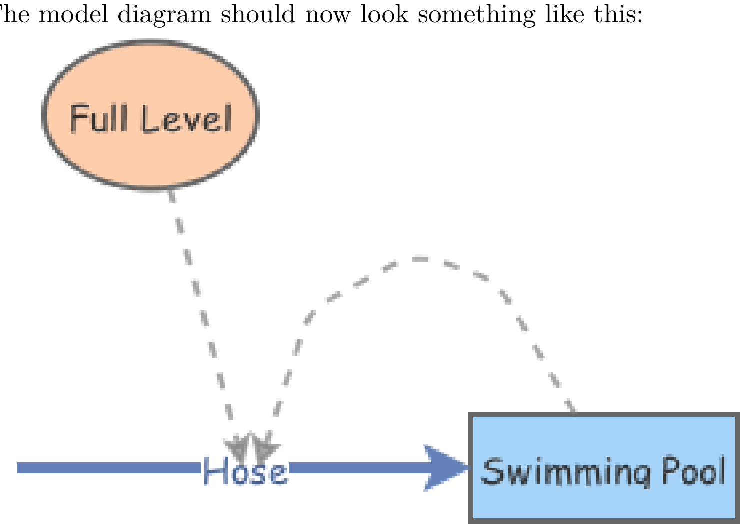
Filling A Swimming Pool

The filling a swimming pool model previous presented was not not very detailed. A more useful question might be, if the pool holds 20,000 gallons of water and the hose fills the pool at 50 gallons per hour, how long will it take to fill the pool. Yes, you can do the math faster than it will take to build the model. Please bear with as there's another aspect of models right around the corner you will find very useful on an ongoing basis.

Filling a Swimming Pool Revisited

This version of the model adds a goal.

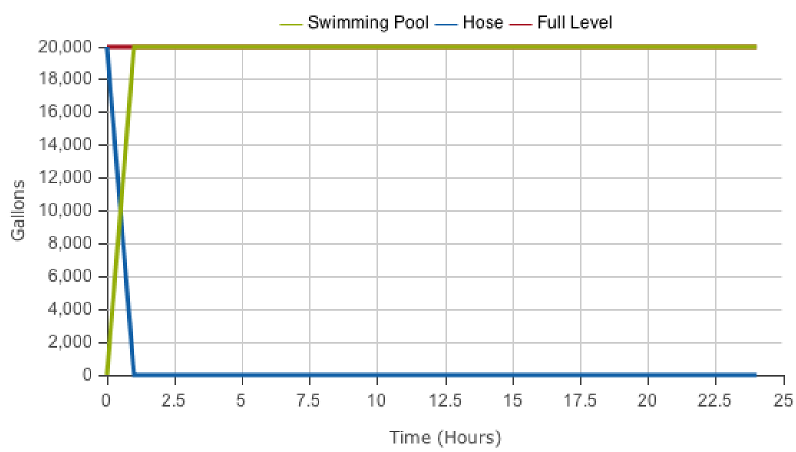
1. The model diagram should now look something like this:



2. We begin with a Swimming Pool that needs to be filled with a hose. We know how many gallows of water it takes to fill the pool and we don't want to put too much water in the pool. The model is created to compare the amount of the water in the Swimming Pool with the the Full Level and use that to decide whether water is flowing in the hose or not.

3. 'Hose = IfThenElse([**Swimming Pool**] < [**Full Level**], [**Full Level**]-[**Swimming Pool**], 0)'{.javascript}

4. Run the model. Here are sample results:



5. This is really great. We can fill the Swimming Pool in just 1 day, or can we? Either it's a really really big hose or we've done something wrong because it's probably not possible to fill the Swimming Pool with a Hose in one day if it takes 20,000 gallons or water.

Isn't it curious that the structure of this model looks just like the one for the Rabbit Population growth? We'll come back to this after we figure out how long it's going to take to fill the Swimming Pool.

And hopefully you will come to understand that when your models don't do what you expect them to do it's not a problem – it's an opportunity for learning. This is the real reason why we do modeling - to understand and learn. Just think of it as, the more things don't go the way you expect them too, the more opportunities you have to learn.

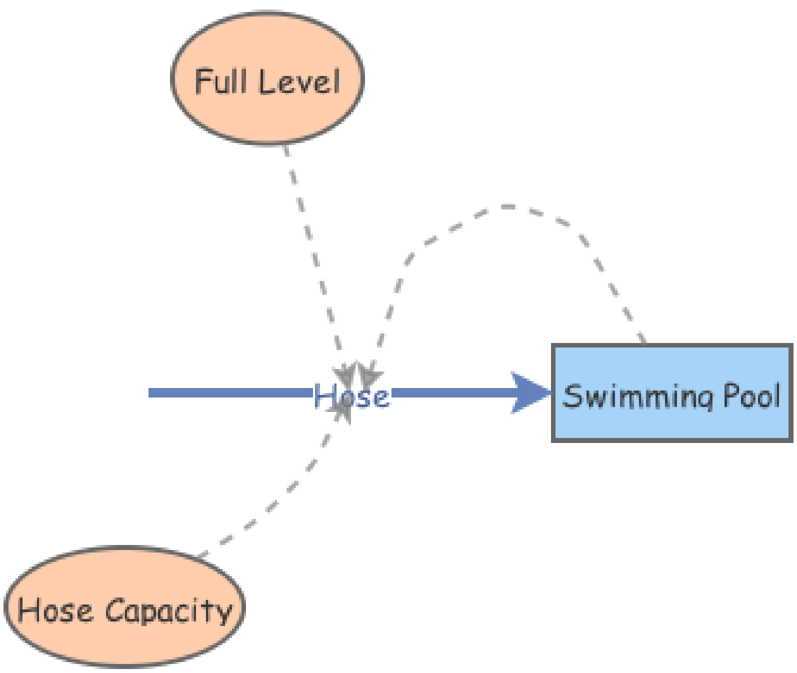
If you look back at the formula for the Hose, notice it didn't take into account the initial statement that the Hose could only deliver 50 gallons per hour. And, might it be useful if we could see what happened with different Hose capacities?

Lets us a revised version of the model with Hose Capacity as a variable so you can set the capacity of the hose before you run the model.

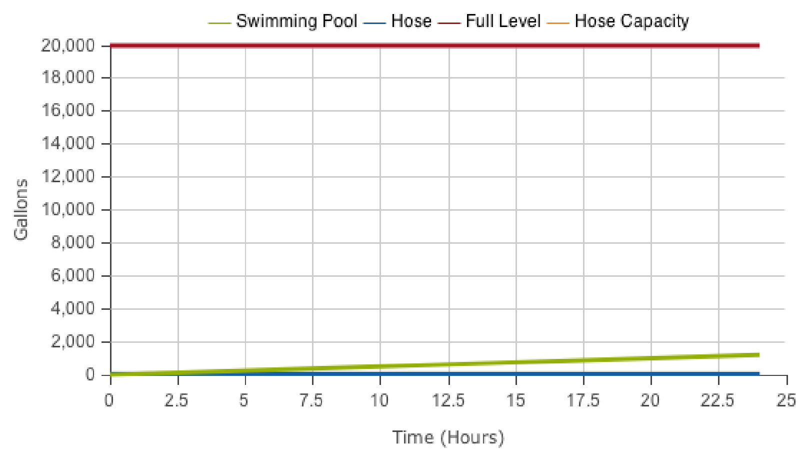
Filling a Swimming Pool One More Time

This version of the model uses a an explicit Hose Capacity.

1. The model diagram should now look something like this:



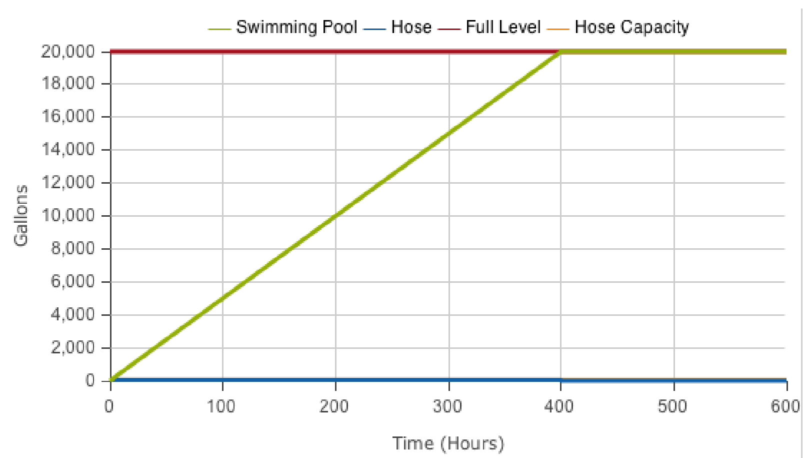
2. The new formula for Hose takes into account both the current amount of water in the Swimming Pool, Full Level and Hose Capacity
3. ‘Hose = IfThenElse([**Swimming Pool**] < [**Full Level**], min([**Full Level**]-[**Swimming Pool**],[**Hose Capacity**]), 0)‘{.javascript}
4. Run the model. Here are sample results:



5. With Hose Capacity = 50 over a period of 24 hours we've not even come close to filling the Swimming Pool.

6. Change the **Simulation Length** property of the Time Settings to `600`.

7. Run the model. Here are sample results:



8. This graph indicates we need to wait 400 hours to fill the pool. That's a little over 16.5 days. Do we need a bigger hose?

While there are a number of things we could do to improve the model at this point we've gone far enough with this one.

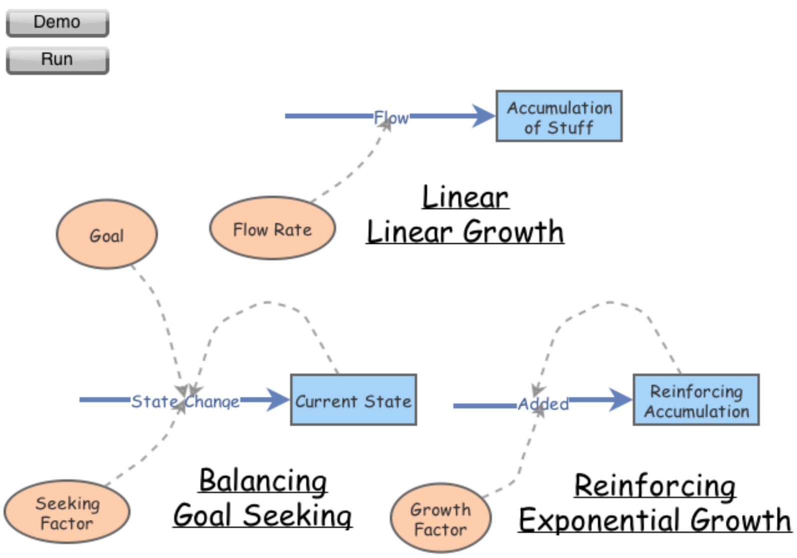
Similar Structures / Different Behavior

If you compare models presented to this point you should find them to be quite similar. And yet the behavior of the models are distinctly different.

Similar Structures / Different Behavior

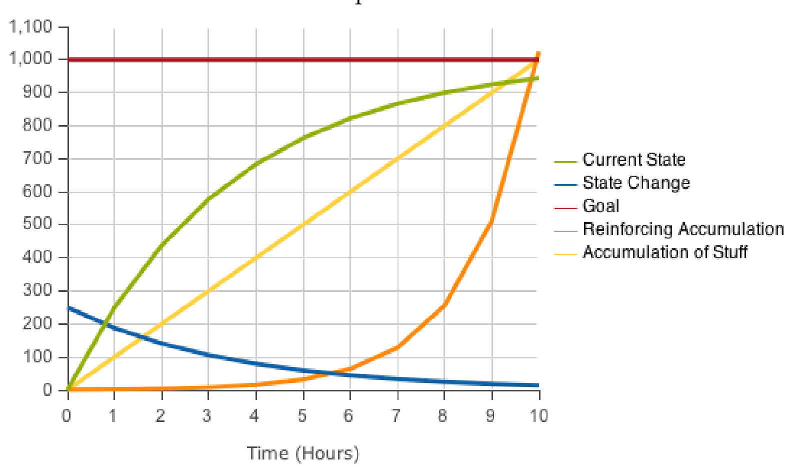
The behavior of a model depends on the structure of the model as well as the formulas that define the nature of the relationships.

1. The model diagram should now look something like this:



2. These three models are in a general form so you can compare the different behavior of the structures that are very similar. Flow Rate, Seeking Factor and Growth Factor are each factors which govern he rate of flow. Goal is a target value which the Growth model doesn't have.

3. Run the model. Here are sample results:



4. The difference that makes a difference is what happens in the connection between the accumulation, or stock, and the flow.

The link between the stock and the flow provides information from the stock to the flow and is generally referred to as feedback, mostly likely because the information travels in the opposite direction as the flow. The nature of feedback results in the three types of models.

Linear

In the Linear model the Flow simply depends on the Flow Rate variable, which is expected to be some constant value. This model is referred to as linear because the Accumulation of Stuff is a straight line as you can see in Figure 24. Not that if the Flow Rate isn't a constant or linear the Accumulation of Stuff won't be linear.

Balancing

In the Balancing model the State Change depends on the difference between the Goal and the Current State. This difference influences the State Change to increase the Current State until it reaches the Goal. The structure tries to bring about a balance between the Current State and the Goal so the difference is zero, and then there's no more State Change.

Reinforcing

In the Reinforcing model Added depends on the value of Reinforcing Accumulation. This influences Added to increase the Reinforcing Accumulation which increases Added. One might consider a Reinforcing structure to be a Balancing structure that's out of control.

Would you believe that no matter how complicated a model may look it's really only some number of these structures connected together? In the next chapter you will learn about the modeling and simulation environment and actually begin building some models and investigating the implications of these structures.

Exercise 1-2

The values in the previous model were contrived so when you click the Demo button the model it will produce the graph in displayed graph.

- Can you figure out why the values assigned are responsible for the curves produced?

- Alter the values for the parameters in the Configuration Panel and run the model to get a sense of the impact initial values have on the behavior of these structures.
- Can you explain to someone else the difference between the Linear, Balancing and Reinforcing models in terms of why the structures produce the behavior they do?

Summary

- Models are simplified versions of the world around us.
- We build models to help us understand and learn.
- We build simple models and add to them as we learn with them.
- Building models and learning is an iterative process.
- We learn as we go and seldom do we get models right the first time.
- Linear, Reinforcing and Balancing structures are the basic building blocks for all models.
- These building blocks can aid in understanding aspects of our interactions with the world around us.

In the next chapter you will learn about the Insight Maker environment and actually build the models that were presented in this chapter.

References

- Kauffman, Draper L. 1980. Systems One: An Introduction to Systems Thinking. http://www.amazon.com/Systems-One-An-Introduction-Thinking/dp/9996280519/
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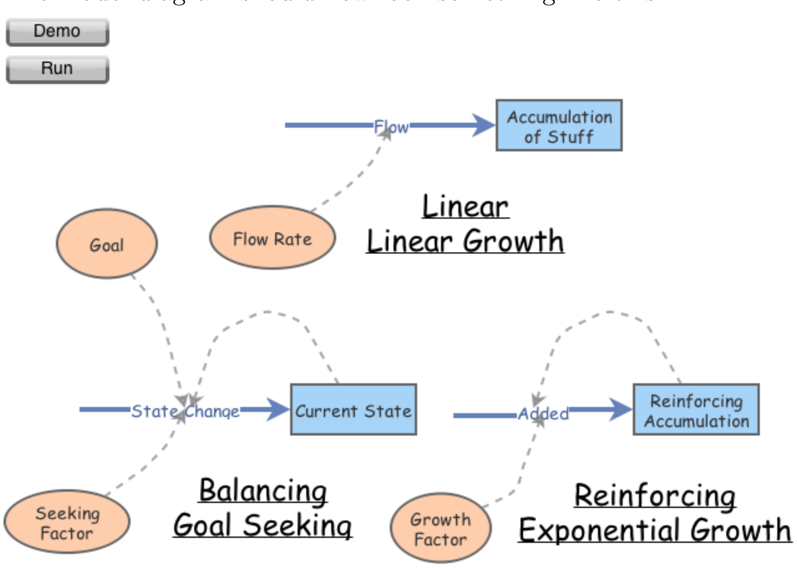
Chapter 2

Dynamic Building Blocks

When we build things our skill in using the tools we work with has a very definite impact on things we create. Figure 1 is the last model from the previous chapter. In this chapter we hope you will become far more familiar with, and comfortable with, the Insight Maker environment, the creation of these structures, as well as a deeper understanding of why these structures for the building blocks of everything else you will create in the Insight Maker environment to further your understanding and learning.

Similar Structures/Different Behaviors

1. The model diagram should now look something like this:

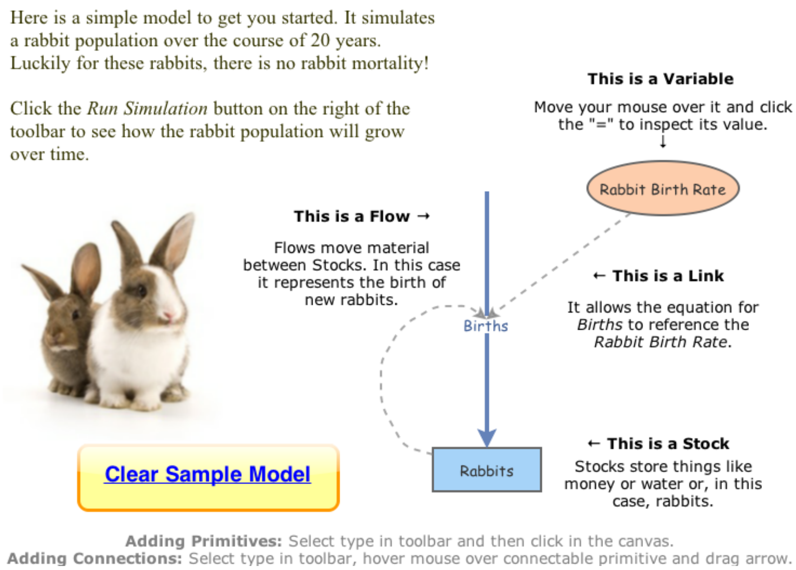


The Blank Canvas

When you create a New Insight you don't actually have to start with a blank canvas. Insight Maker presents you with a very simple working Rabbits Population model. This is just so there's something there to work with when you first start.

Sample Model

1. The model diagram should now look something like this:



2. If you click the Clear Sample Model button you will then have a blank canvas on which to create. In the next few segments you will learn how to create the three basic structures from which all models are constructed.

All the tools you use to work with models are located on the **toolbar** at the top of the screen which is depicted in Fig 1.

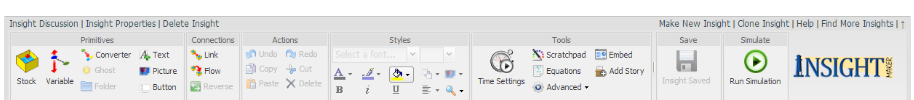


Figure 1. Toolbar

To use any of the **Primitives** click on the icon on the **Toolbar** to select it, then click on the canvas where you want the item located. For each tool there are a set of allowed uses. Once you place the item on the canvas it is named for what it is, with that name selected so you can type in the name you want. Names can contain any characters except braces "{}", brackets "[]", parentheses (), and quotes '. If the label is not selected you can double-click it to select the label and then enter a new one, or you can enter the label in the **Configuration Panel** though we'll address that in a bit more detail later.

Exercise 2-1

Practice placing [**Stock**] and [**Variable**] [**Primitives**] on the blank canvas in Figure 2 and naming them. You can remove a [**Primitive**] by clicking on it to select it and then pressing the **Delete** key or clicking the **Delete** button in the **Actions** section of the **Toolbar**. Note that the **Save** option is disabled so you won't be able to save what you create. **Note**: This is only for the review copy. In the final electronic copy you will be able to save what you create.

Stocks, Flows, Variables and Links

[**Stocks**] and [**Variables**] are connected to other [**Stocks**] and [**Variables**] using [**Link**] and [**Flow**] [**Connections**]. The rules for connections are very explicit because Insight Maker has to figure out how to simulate the model. The allowed connections are depicted in Figure 3. The next chapter will present several types of models where the rules for connections aren't nearly as rigid.

Valid Primitive Connections

1. The model diagram should now look something like this:

2. You have now completed a model that represents

If you select **Link** from the **Connections** segment of the **toolbar** then hover over a model \p{Primitives) on the canvas a small arrow pointing to the right shows at the center of the \p{Primitives). If you select a **Flow** the small arrow will only show up over a [**Stock**] as a **Flow** can only connect to a [**Stock**].

Center the **cursor crossing double arrows** over the right arrow, which should then change to a **pointing finger hand**. Drag the mouse over to a second model element and the arrow tags along while the **Connections** is drawn. If neither **Link** or **Flow** is selected then there will be no right pointing arrow when you mouse over the primitive. We'll go into more detail about connections shortly.

Exercise 2-2

Click on the **Unfold Model** button in the lower left corner of Figure 3, and then repeatedly click the **Step Forward** arrow to walk though an unfolding of the Valid Primitives model while you read the comments on the lower bar.

Note the setup takes a few seconds to please be patient. This will only be the case in the review copy, not the final eBook.

Hopefully the rules associated with the connections were easy to understand. Just remember that Flows represent the movement of stuff while Links only communicate the value of something from one location to another.

Valid Primitive Connections

The valid primitive connections are described as follows.

Flow

A Flow adds stuff to a Stock, subtracts stuff from a Stock, or moves stuff from one Stock to another. The only way to change the quantity of stuff in a Stock is with a Flow.

- A flow out of a stock decreases it. If where the flow goes isn't relevant to the model then it just flows from the stock to the canvas. Select Flow from the toolbar and then click on the arrow that appears on the stock when you mouse over it and drag onto the canvas and release.
- A flow into a stock will increase it. If you don't care where the Flow is coming from then you first have to draw the Flow from the Stock to the canvas and click the Reverse button in the Connections section to get the Flow to come into the Stock from nowhere. It's just a quirk of the web implementation.
- A flow from one stock to another decreases the source and increases the destination. The get a flow between two Stocks draw the Stocks first and then draw the Flow from one Stock to the other.
- Flows can be bidirectional and we'll talk more about that the first time we use one in a model.
- Flows take time! Please remember this.

Link

A Link is used to communicate a value from one element to another. There is no flow of stuff through the link itself. The communication is considered to be instantaneous.

- You can use a Link from a Stock to a Variable to communicate the value of the Stock to be used in an equation. This does not change the Stock.
- You can use a Link to communicate the value of a Stock to a Flow to be used in the equation determining the value of the Flow in the next iteration. The Link does not change the value of the Stock.
- You can use a Link to communicate the value of a Flow to a Variable to be used in an equation. This does not change the value of the flow.
- You can use a Link to communicate the value of a Variable to a Flow to be used in the equation that defines the flow. This does not change the value of the Variable.
- You can use a Link to communicate the value of a Variable to another Variable so that value can be used in an equation in the destination variable. The link does not change the value of the source Variable.
- You can use a Link to communicate the value of a Variable to a Stock to be used as it's Initial Value when the simulation begins. The value of the Variable is computed and assigned to the Stock as the simulation begins and it has no influence on the Stock during the simulation.

When you draw a link from one element to another it is created as a straight line. There are times when you would prefer that the connection be other than a straight line to make the diagram easier to follow. You can turn a straight line into a multiple segment line as follows.

- Click on the link to select it.
- Hold down the shift key and click somewhere in the middle of the link then release. This puts a little node on the line.
- Click on the node and move it as you wish to create a two segment link.
- You can create as many segments as you need, simply repeat the second step above.
- If you wish to remove the segments select the head of the link, move it off the element it's connected to and then reconnect it. It will now be a straight link.

Exercise 2-3

Go back to Figure 2 and recreate Figure 3 for yourself and as you create each element think about what that particular element is for. Actually making the connections helps develop a level of skill and comfort which will serve you well in the future.

Configuration Panel

Each of the four elements used to build a model has some of the same configuration options though because each has a different function there are some unique configuration options for each item. Some of the most frequently used options will be described in the following sections. The ones not described here will be described the first time they are used.

The Configuration Panels for Stock 1, Flow 1 and Variable 1 form Figure 3 are displayed in Figure 3a and are described below.

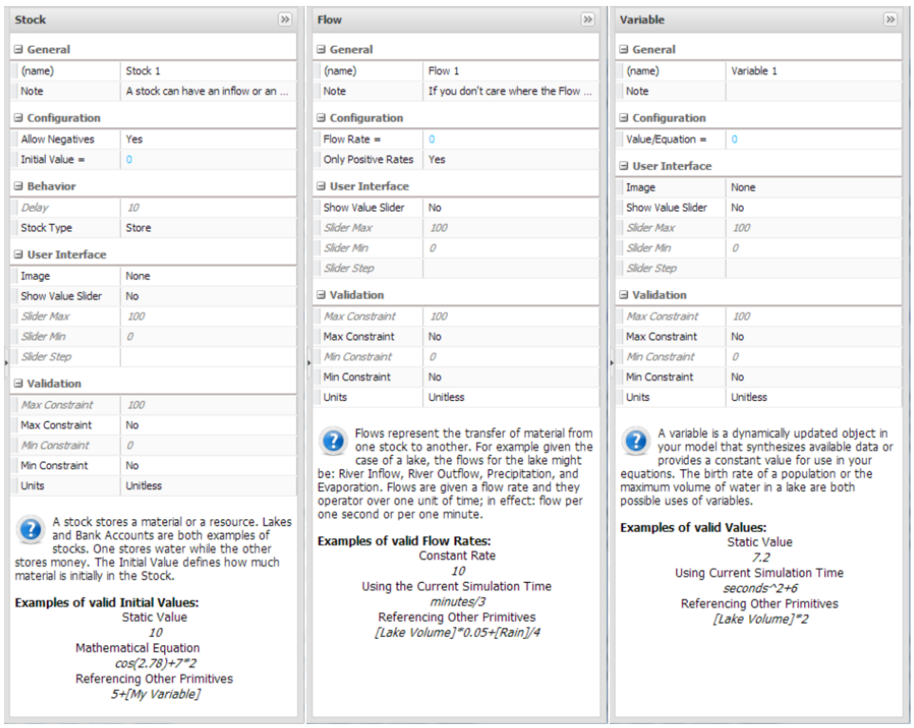


Figure 2. Stock, Flow & Variable Configuration Panels

General

This section is where you can assign the (name) and Note for an item.

- **(name)**. This is the label that you see on the item. You can double-click the item on the canvas and edit the label on the item itself or change it here in the configuration panel.
- **Note**. Here you can enter a description of the item. You can enter short descriptions directly into the field. If you click the down arrow in right

of the field it will open the **Note Editor** dialogue window which allows some formatting options. The note that you enter here will pop up when you mouse over an item and click on the little **i** that appears on the item. If the element of the model is selected you can also open the Note Editor window by CTRL+‘(Control+Backquote). Adding comments to a model helps others to understand what you were thinking and when you go back to the model in the future the comments will help you understand what you were thinking when you put the model together. Yes, you completely understand now, though will you remember next week, or a year from now?

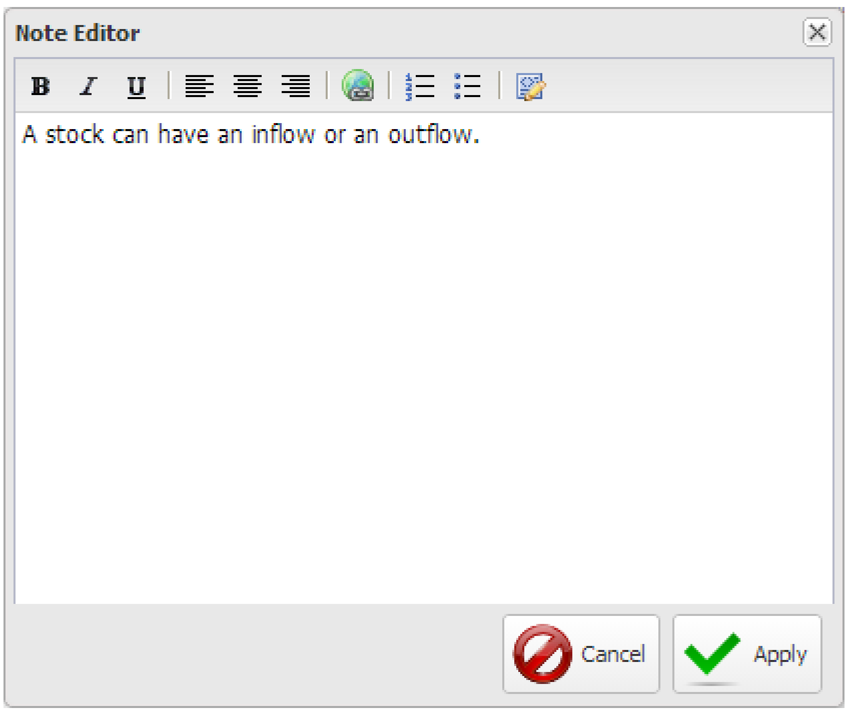


Figure 3. Note Editor

Configuration

This section is used to define how the element behaves during the simulation and is a little different for Stock, Flow and Variables, though quite similar. The behavior is essentially controlled by an equation which is defined in terms of the variables connected to it. This is an initial value for a Stock. You may enter a short value into the field though if you click the down arrow in the right of the field the **Equation Editor** window will open. In this window you can define the formula that defines the behavior of the element. You can also open the

Equation Editor for an element by mousing over the element and clicking on the **equals** (=) sign that appears. All the built in functions on the tabs at the bottom of the window have descriptions associated descriptions and examples.

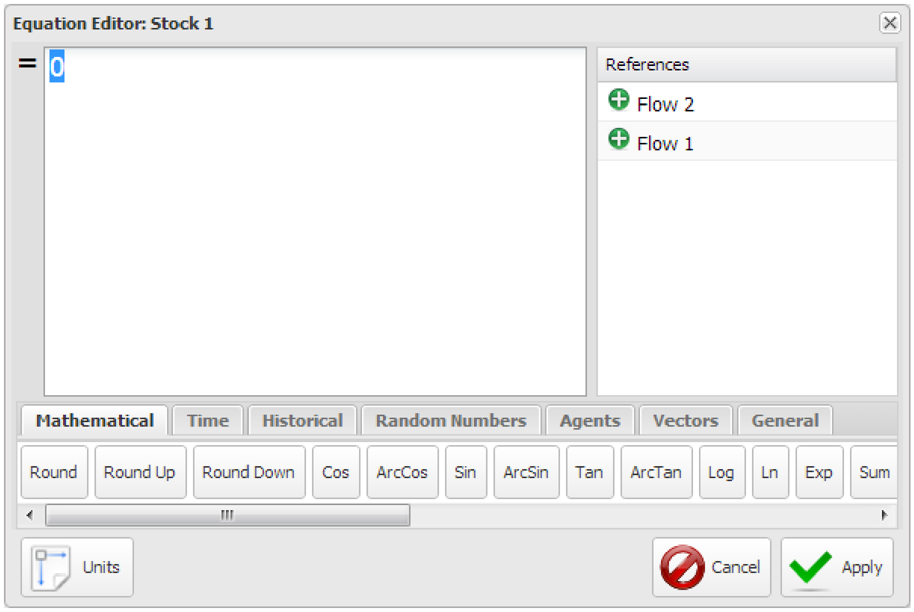


Figure 5. Equation Editor

Additionally in this section you define whether stocks can have negative values and whether flows can flow in both directions. We'll talk more about these options the first time we use them.

User Interface

It is in this segment of the configuration panel that you define a slider for an element, if there is to be one. You can define a sliders for Stocks, to define it's initial value and for Flows and for Variables. Once you indicate there is to be a slider you then define the maximum and minimum values it may have, as well as the step size, how small are the variations allowed. If you leave the step size field blank then the slider can vary continuously.

An element may have a slider or a formula though not both. Sliders override equations. If you enter an equation and it disappears check to see if there was a slider defined and it hasn't been turned off.

Validation

This section allows you to indicate if there are Maximum and/or Minimum constraints on a Stock, Flow or Variable.

Additionally it is in this section that you assign the Units for an Stock, Flow, and Variable. Units are very useful in helping to ensure the soundness of a model. Units will be covered extensively in Chapter 4.

Common Property # 1

To this point you've learned how to develop a static picture of a model. It is actually a model and provides a sense of the relationships between the various elements. What it doesn't give you a sense of is the dynamic nature of these interactions over time. What are the implications of the relationships? In the next few sections you'll learn how to bring your model to life.

Look at the pictures in Figure 4 and ask yourself what it is that these images have in common. The images all represent very different kinds of things, some living, some not, though there is a characteristics they all have in common. Have you figured it out?

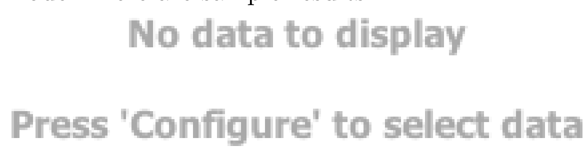
Figure 5. Common Property # 1

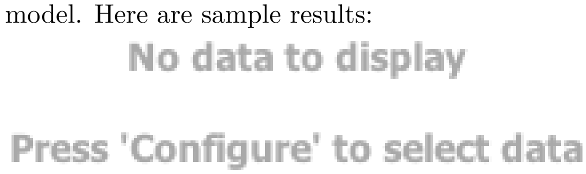
Maybe you notice the rabbits from the previous chapter? The things depicted in the various images all grow in one way or another, and some faster than others.

Constructing a Growth Structure

Lets use Figure 5 to construct a basic growth structure and in the process you'll learn about several of the parameters associated with the different elements of a model.

Growth Structure

1. The model diagram should now look something like this:
2. Change the **Initial Value** property of the primitive [**stuff**] to `0`.
3. Change the **Flow Rate** property of the primitive [**Flow**] to `1`.
4. Run the model. Here are sample results:


5. Notice that the model ran for 20 years. That's because we used the default Time Settings.
6. If we evolve the model of Figure 5 into Figure 8 so the flow is dependent on the amount of stuff we find the resultant growth to be very different.
7. Create a new **Link** going from the primitive [**stuff**] to the primitive [**Flow**].
8. Change the **Flow Rate** property of the primitive [**Flow**] to `[stuff]`.
9. Run the model. Here are sample results:


10. The result of the run from the model in Figure 8 is depicted in Figure 9. The value after 10 Years is 1,024 which you should realize is just 2ˆ{}10 as expected because we started with a value of 1 and doubled it every year. This curve is referred to as an exponential growth curve.

Exercise 2-4

Notice that the curve in Figure 10 is a bit choppy where it turns up. Run the model in Figure 8 with a Time Step of .5, .25, .125, .0625 and compare the results. What questions are raised by the the results?

Time Units and Time Step Selection

The **Time Units** and **Time Step** selected for a model should be consistent with the time frame and level of detail of the model. You probably wouldn't develop a model about filling a bathtub with water and use **Time Units** of months. Minutes are probably more appropriate for this model. The **Time Step** is then selected to ensure none of the relevant transitions associated with the dynamic nature of the model are missed. A **Time Step** of .25, meaning 15 seconds, is probably sufficiently small to ensure there are no transitions missed.

Trial is actually the most appropriate approach to determine if you have an appropriate value for **Time Step**. If you think .5 is appropriate then run the model with 1, .5, and .25 and if the results for 1 and .25 don't differ from .5 then you're probably OK. If .25 produced a different result then compare the .25 result with the .125 result. Once you get two runs where the values don't change then use the larger one.

Given this guidance how would you interpret the results you experienced in Exercise 2-5?

References

- How does DT work? from isee Systems
- DT Situations Requiring Special Care from isee Systems

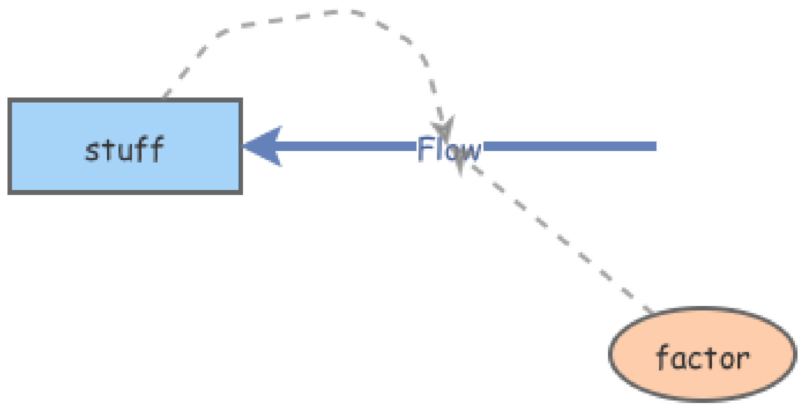
Exercise 2-5

Consider the images in Figure 5 and think about what **Time Units** and **Time Step** you would use in a model representing the growth in each of these areas.

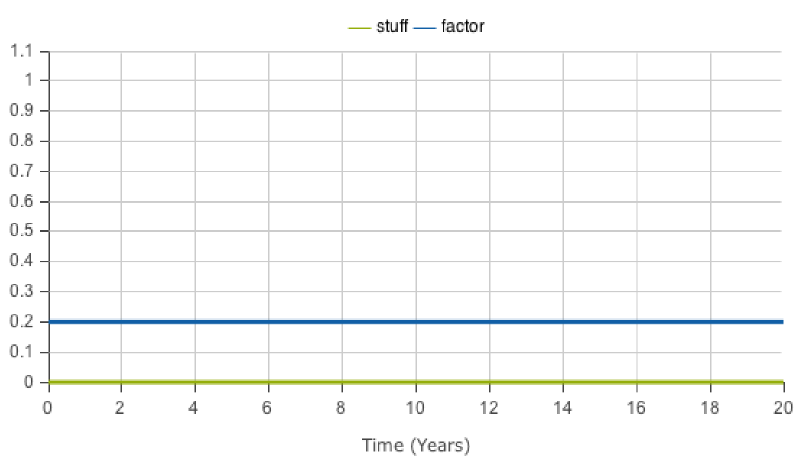
One aspect of trying to model the contexts of Figure 5 that should have become apparent is that there is a piece of the model that's missing.

Feedback Dependent Growth

1. Create a new **Variable** named [**factor**].
2. Create a new **Link** going from the primitive [**factor**] to the primitive [**Flow**].
3. Change the **Equation** property of the primitive [**factor**] to `.2`.
4. Change the **Flow Rate** property of the primitive [**Flow**] to `[stuff] * [factor]`.
5. The model diagram should now look something like this:



6. This version of the model adds a factor, which is allowed to vary between 0 and 1, which is simply used to govern the flow. Mouse over the Flow and click the equal (=) sign to view the formula governing the flow.
7. Run the model. Here are sample results:



Exercise 2-6

Using the Feedback Dependent Growth model to implement the models does this structure allow you to construct more realistic representations of the growth situations presented in Figure 5?

Exercise 2-7

The model in Figure 11 is the model for a Savings Account that is defined as compounding annually, i.e. calculating and adding interest once a year. This means that the most appropriate **Time Units** is years with a **Time Step** of 1. There are no other transitions in this model that need to be accommodated. If you run this model with any **Time Step** other than 1 it will result in a less accurate result. Why does this happen?

This model is the standard reinforcing growth model depicted in Figure 1 at the beginning of this chapter. In the process of arriving this model the linear growth model of Figure 1 was developed first, and then evolved. Hopefully through the exercises to this point you have gained a deeper understanding of how this structure works and the extent to which it may be applied to various situations.

Common Property # 2

Look at the activities depicted by the images in Figure 12 and ask yourself what it is that these activities have in common. The images represent very different kinds of activities though there is a characteristics they all have in common. Have you figured it out?

Each activity depicted in Figure 6 represents the pursuit of some goal or objective. Admittedly the goals are very different and each is pursued in a very different manner.

Constructing a Balancing/Goal Seeking Structure

As we have done repeatedly to this point we begin with a linear model consisting of a flow and a stock, along with a flow rate variable. To this we simply have to add a goal and the appropriate feedback and we end up with the model in Figure 13.

Figure 6. Common Property # 2

Balancing/Goal Seeking Model

1. Change the **Initial Value** property of the primitive [**Current**] to `0`.
2. Change the **Equation** property of the primitive [**factor**] to `0.5`.
3. Change the **Equation** property of the primitive [**Goal**] to `1`.
4. Change the **Equation** property of the primitive [**Goal**] to `[Goal]-[Current]`.
5. Change the **Equation** property of the primitive [**Goal**] to `[Gap]*[factor]`.
6. Change the **Simulation Length** property of the Time Settings to `10`.
7. Change the **Time Units** property of the Time Settings to `Hours`.
8. The model diagram should now look something like this:
9. When you look at the model admittedly we added Gap which we haven't addressed before. This was done so we could explicitly plot the difference between the Current value and the Goal. The factor is

simply a multiplier between 0 and 1 to govern the extent to which the Gap governs the change.

10. Run the model. Here are sample results:

11. Take a look at the Time Settings for the model and you'll see that the model was set up to run from 0 to 10 with a time step of 1 and a units of hours. These were just selected to create a generic model where you could consider the Goal to be 100% and the other values as having values between 0 and 100%. This way we can consider the implications of the interactions without getting hung up on the actual values.

12. The graph shows that as Current moves toward the Goal the Gap decreases as does the change which is moving Current in the direction of Goal. Once Current reaches Goal the Gap is zero is change. This structure endeavors to remove the tension between Current and Goal, the Gap, to bring a balance to the situation.

Exercise 2-8

Run the model in Figure 13 with various values for factor. What do you notice about the relation between Current and Gap? And what do you notice about the curves as the factor gets larger and larger?

Under Time Units and Step Selection we talked about it being essential that the the Time Units were selected appropriate to what was being modeled. In this case since it's a generic model one Time Unit is pretty much as appropriate as any other. The Time Step is another matter though, or is it? We said one chooses a Time Step such that none of the relevant interactions are missed and the change from one Time Step to another doesn't change the result.

Exercise 2-9

Set up the model in Figure 13 to run with Current = 0, Goal = 1, and factor = .75. Now run the model with a Time Step of 1, .5. 25. .125. Does the result actually change? Look at the Tabular Display associated with the Simulation Result. As you make the Time Step smaller and smaller are the results more correct?

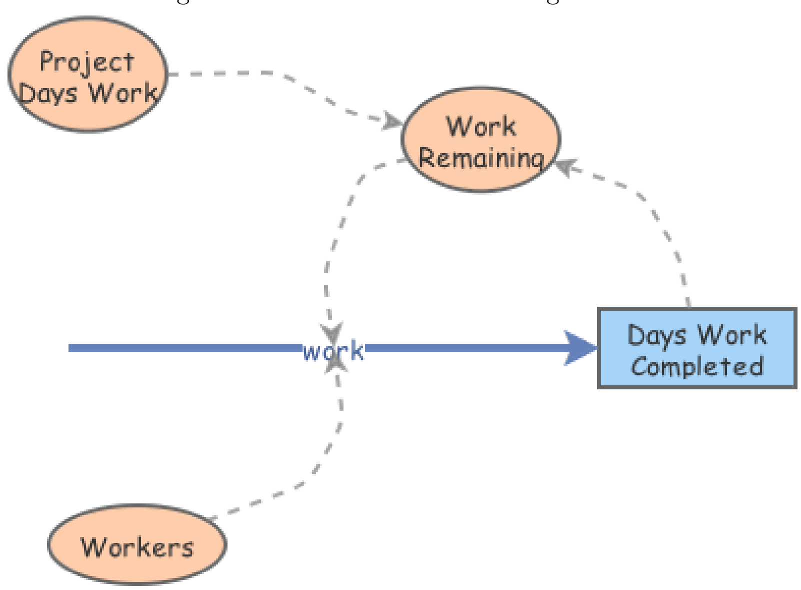
Considering that we don't know anything about a real environment being modeled in Figure 13 it's a bit difficult to determine if the result is actually more correct as the Time Step used is smaller and smaller.

You might have also realized by this point that it would be quite difficult if we attempted to use this model to model any of the situations depicted in Figure 12. While progress toward the goal in the situations depicted is promoted by the Gap between the Goal and Current the change in those situations isn't likely to be proportional to that Gap.

Figure 15 presents a modification to the model of Figure 13 where the factor has been replaced by a constraint. It looks like there have been lots of changes though they all cosmetic except the way Workers influence work on a daily basis.

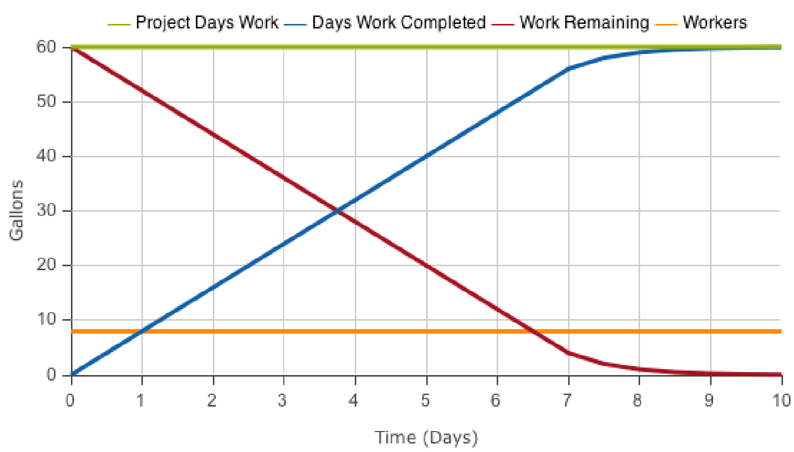
Work Completion Model

1. The model diagram should now look something like this:



2. If you run the model with Project Days Work = 60, Workers = 8 and Days Work Completed set at the default of zero and Time Step = 1 you should see the graph in Figure 16.

3. Run the model. Here are sample results:



4. The reason the graph looks like this is because of the constraint placed on the the work because of the number of Workers available. This is accomplished by the formula embedded in the flow.

5. > work = IfThenElse([**Work Remaining**] > [**Workers**],[**Workers**],[**Work Remaining**])

6. This says that if there is more Work Remaining than there are Workers available to do the work then the amount of work that day equals the number of Workers. This goes on for the first 7 days then on the 8th day there are only four days work required to finish the project which is represented by the different slope on the line on the 8th day. You can see this in detail if you look at the Tabular Display.

Exercise 2-10

Set up the model in Figure 16 to run with Time Step of .5. Compare the Tabular Display of this run with the results of the previous run above. By making the time step smaller have we improved the accuracy of result? Why?

Again the appropriate Time Step is one that captures the activity occurring within the model. In this case the Workers are in integers and Project Work days are in integers, and with the Time Units in days the appropriate Time Step is 1. If there were events which happened in the model on the order of hours then you would have to decide whether to alter the model to run in hours or reduce the Time Step to ensure it was small enough so no interactions in the model were missed.

Exercise 2-11

Use the model in Figure 15 and reconfigure it for a couple of the activities depicted in Figure 12. Note that for this exercise you will have to relabel the stock, flow, and variables accordingly. You will also have to decide on the most appropriate Time Units and Time Step to use.

Summary

Hopefully this chapter has helped you become more familiar with the modeling environment and the four model elements you will use most often.

- **Stock**. An accumulation of something that can only be changed by something flowing into or out of it.
- **Flow**. Something moving over time which adds to a stock or subtracts from a stock.
- **Variable**. Constant or equation computed each time the simulation steps.
- **Link**. Used to communicate a value from a Stock, Flow, or Variable, to a Stock, Flow or Variable. The source is not changed and a link to a stock can only be used to set it's initial value.

Because of the nature of the building blocks themselves there are only a small number of valid connections as depicted in Figure 3.

These valid connections are used to create only three different types of structures, linear growth, goal seeking and reinforcing growth. If you are comfortable with these you should be relieved to know that's all there are. Just three simple structures will be used for all the models you will ever build. Of course at times there may be quite a few of these connected together though you should be confident that you know about the pieces.

The models that you have experienced in Chapter 1 and Chapter 2 are referred to as Stock & Flow Simulation Models. These are also referred to as quantitative models because of the values associated with the simulation of these models. In the next chapter we'll investigate a number qualitative models which are also used in developing understanding. These are referred to as qualitative models because there are no numerical values associated with them, though there are times when they can be quite useful.

Chapter 3

A Model Is A Model Is A Model

Chapter 4

Building a Model

Chapter 5

Implications of Reality

Chapter 6

Applied Understanding

To be filled in.

Chapter 7

Models and Truth

> All models are wrong, but some are useful – George E.P. Box

A model is a tool designed to reflect reality. A model is never a perfect mirror of reality, but often models can still be useful even given their imperfections. In this chapter, we will take a journey exploring different types of models and distinctions that are commonly used to classify and understand them. We will consider several approaches to modeling that are quite different from the ones we have introduced throughout this book. These will help to understand the richer ecosystem of modeling tools and techniques that exist and how the ones we have learned fit within this ecosystem.

The ultimate destination of this exploration will be a clear understanding of the fundamental principles and approaches used to construct models. There will be many detours that we must make to arrive at this destination, but in the end we will be able to divide models into two overarching categories based on their purposes and the techniques used to construct them. By mastery this divide, and how the work we and others do fits into it, we will obtain a rich perspective and understanding of the relationship between models and truth. We will also have a renewed appreciation for the strength and power of the techniques introduced in this book for tackling a wide swath of modeling problems.

Before we get there, however, let's introduce some of the terminology that is commonly used to describe models. It is useful to take a step back and first discuss different kinds of models. Modeling is a wide-ranging field with many distinctions made by modelers and mathematicians. Three of these distinctions are presented below:

Deterministic versus Stochastic Models

There are two polar opposite views of the world. One view says the fate of the universe is governed by strictly predictable laws of physics. In this view, the universe acts as if it were a giant machine, where if its current state is known (down to each individual atomic particle), its future states through the rest of time are predetermined. The opposite view is that the universe is ruled by chance and randomness. Random quantum mechanical fluctuations merge and amplify leading to an infinite range of diverging possibilities.

Which of these two views holds more of the truth? We certainly do not know and it is possible that this will be a question that physicists will never cease exploring. Albert Einstein had a viewpoint of special interest, however. He was a strong partisan of the more deterministic view, famously remarking, "God does not play dice with the world."

When creating a model of a system, we must choose how we treat chance. Do we build our model deterministically, such that each time we run it we obtain the same results? Or do we instead incorporate elements of uncertainty so that each time the model is run we may see a different trajectory of outcomes?

Mechanistic versus Statistical Models[1]

When beginning to build a model of a system, there are many questions you should ask, two of which are:

1. Do I know (or have a hypothesis of) the mechanisms that drive the system?
2. Do I have data that describe the observed behavior of the system?

If the first question is answered in the affirmative, you can build a mechanistic model that replicates your understanding (or hypothesis of) the true mechanisms in the system. If, on the other hand, the second question is answered in the affirmative, you can use statistical algorithms such as regressions to create a model of the system based purely on the data.

If neither question is answered affirmatively… well in that case there isn't much of anything you can build.

Exercise 7-1

A credit card company has hired you to build a model to predict defaults of new applicants. They give you a data set containing information on one million of their customers along with whether or not the customer defaulted.

[1] This relates, more broadly, to the contrasting research approaches of induction and deduction. Induction starts with data and observations which are analyzed to create a broader theory (similar to a statistical approach to modeling). Deduction starts with a theory and finishes with the collection of data to confirm the theory (similar to a more mechanistic approach to modeling). It is easy to become mixed up with the meanings of induction and deduction and even great minds have confused them. While Sir Arthur Conan Doyle's character Sherlock Holmes attributes his impressive powers to "deduction", he is actually using induction. Treating what we are calling "statistical" models here as a form of induction, we can also refer to them as "phenomenological" or "empirical" models.

Would it be better to build a mechanistic or statistical model for this data?

Answer Available

Exercise 7-2

You have been commissioned to build a model of population growth for a herd of Zebra in Namibia. You have some data on the historical size of the population of Zebras but this data is limited. You also have access to over a dozen experts who have studied Zebras their whole life and have an intimate understanding of the behavior of the Zebras.

Answer Available

Aggregated versus Disaggregated

When building a model, the issue of scale becomes very important. Imagine we are concerned about the effects of Global Climate Change on water resources. We may wish to examine the question of whether there will be sufficient water supplies given a rise in future temperatures.

At what scale do we build this model? The range of possible scales is wide:

- At the most aggregate, we could estimate total worldwide water demands and supplies into the future.
- Maybe that is too coarse a scale, however, as clearly having excess water in Norway has little direct impact on the situation in Egypt. We could instead create a finer resolution model that separately looked at water demand and consumption in each country.
- Even that may still be too coarse, maybe we should make our model more granular to look at specific cities or population clusters around the globe.
- At the extreme disaggregated level, we might even want to model individual people – all 7 billion of them – and their needs and movements around the world.

There is no simple answer to this question of optimal scale. The best choice is highly context-sensitive and depends on the needs of the specific modeler and application.

Exercise 7-3

You have been hired to build a model of world population growth. What is an appropriate level of aggregation/disaggregation for this model? Does your answer change if you very the time scale? What would be the differences between a model designed to work 10 years in the future, one designed to work for 100 years, and one designed to work for a 1,000 years?

Exercise 7-4

Your company builds rulers. You have been asked to develop a model of global demand for rulers. What is an appropriate level of aggregation/disaggregation for this model?

Prediction, Inference and Narrative

The three distinctions just presented – deterministic vs. stochastic, mechanistic vs. statistical, aggregated vs. disaggregated – can be used to classify models. We can even use them to classify the models we have discussed in this book. Most of our models would be classified as deterministic (random chance is generally not explicitly incorporated in these models), mechanistic (we generally assume mechanisms rather than estimating dependencies from data), and highly aggregated (the agent based models are an exception).

There are many nuances to these broad distinctions that can also be made (e.g. the type of statistical techniques used for a statistical model) and there are also many other distinctions that can be made between model implementations such as the programming language or software that was used to implement the model. These distinctions and technical choices are important when constructing a model, however what is of key importance is the utility of the model for fulfilling a specific goal.

Technical details matter – they can affect maintainability and other factors – but they are of secondary interest to the adequacy of a model in fulfilling it main purpose. It would make as little sense to say a model was fundamentally bad because it was written in a relatively ancient programming language – like Pascal – as it would be to say a model was fundamentally bad because it was, for instance, deterministic. Let's look back at Box's quote at the beginning of this chapter. We know all models are wrong, what we should really care about is their utility in meeting a specific task.

So instead of using the aforementioned technical classifications to discuss models, we think it is more useful to base our discussions of models on the model's driving purpose. This allows us to leave behind relatively mundane technical and implementation details to focus on what we really care about. Among the many different reasons for building models, they all boil down basically to three broad purposes: prediction, inference and narrative.

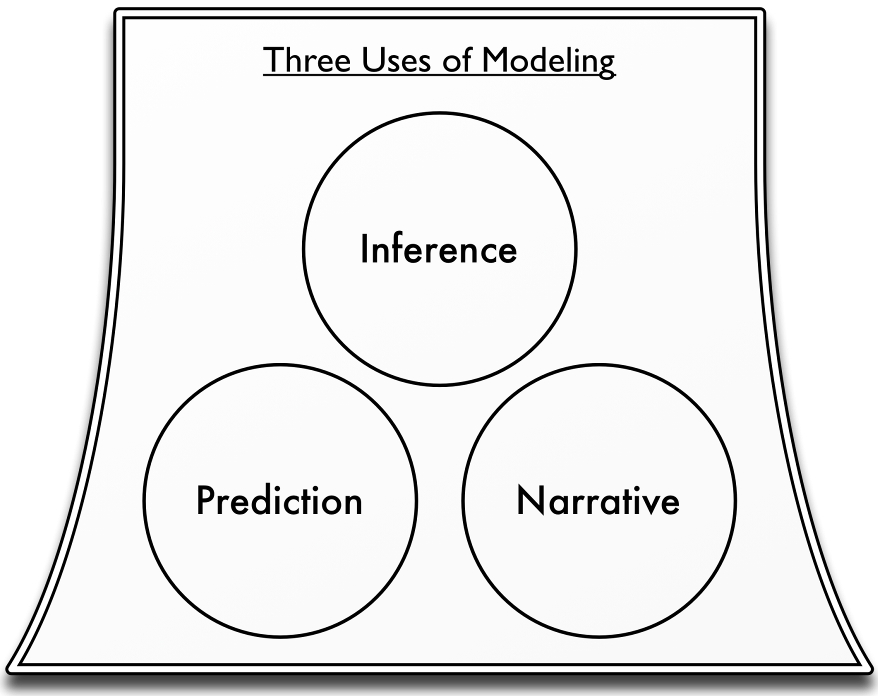


Figure 1. Three Usages of Models

Prediction : Models used for prediction are the most straightforward. They attempt to forecast some outcome given information about variables that may influence that outcome. A weather forecast is an example of a model used for prediction. Likewise, when you apply for a credit card at a bank, they run a predictive model to determine your risk of not paying them back what you owe and defaulting. When you apply for life insurance, the company has a model that predicts how long they think you will live in order to determine how much they should charge you. All these models take in data (the current temperature for the weather forecast, the amount of money in your bank account for your risk of default, your age for the life insurance application) and apply various forms of analysis to generate a prediction of the outcome.

Inference : Models used for inference are most common in academic research. Often, academic research questions distills down to this simple template: “Does X affect Y?” These are inferential questions[2]. As an example, a researcher may make a hypothesis statement such as, “The wealthier a high-school student’s family is, then the higher the student’s test scores will be.” The researcher may then build a model to test the validity of this hypothesis and the model’s results will generally be phrased in terms of a p value indicating the statistical significance of the evidence in support of the hypothesis.

Narrative : Models are often used to tell a persuasive story. When the Obama administration wanted to persuade lawmakers and the public to support their economic stimulus, they famously published the graph shown in Figure 2. A great deal of complex modeling and mathematics surely went into constructing this figure. However its core purpose was to tell the nation a story: Things are going to be bad, but the recovery plan will make them less so. Such stories are at the heart of narrative models and we will return to this figure later on and why it is not really a predictive model despite it generating predictions.

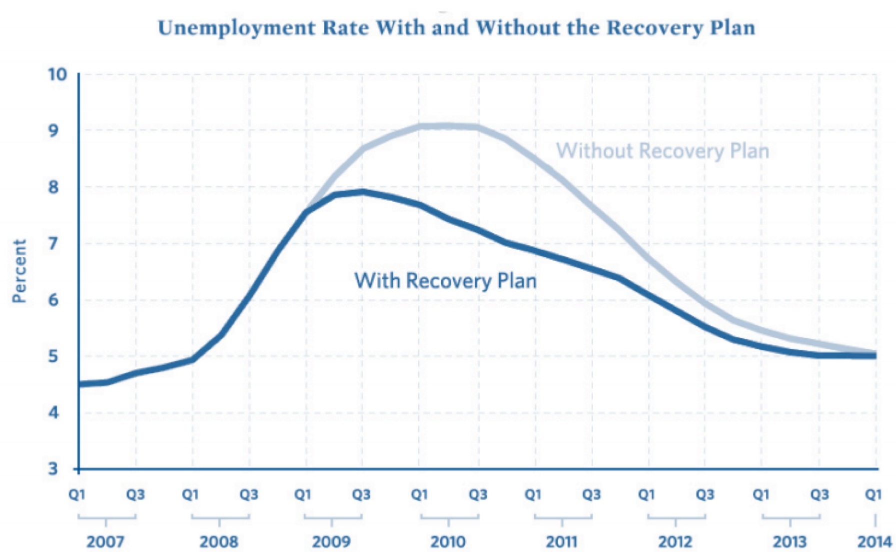


Figure 2. The Obama Administration’s Predictions for the Effects of the Recovery Plan [@Romer:2009tx]

All models can be classified in terms of these three primary purposes and we will see how useful it is to discuss modeling projects in terms of them[3].

[2] Predictions are also inferential results, but we prefer to discuss prediction and more hypothesis-testing types of inference separately. This distinction makes our understanding of modeling clearer.

[3] And we strongly recommend doing so. It is important to clearly define the purpose at the start of a project. The techniques used and data required depend significantly on the

Exercise 7-5

Classify each of these modeling tasks as either primary prediction, inference, or narrative tasks:

1. A model to determine the average ocean temperature in 2020.
2. A model to determine whether deforestation affects temperatures.
3. A model to determine whether a company should supply a credit card to a specific applicant.
4. A model to help students understand the risks of global climate change.
5. A model to convince your manager to green-light your new initiative.
6. A model to assess whether nutrition has an effect of infant mortality.

Answer Available

The Strange Case of Inference

To help us get at this fundamental classification scheme, let's first talk for a moment about the process of inference. Take the earlier example of determining whether wealth results in increased high-school test scores. We phrased this hypothesis in a specific way: that increased wealth will always increase test scores. This illustrative statement, however, actually differs from what is often done in practice. In general, researchers simply asks the question "Does X affect Y?" rather than "Does X increase Y?" It's just a slight difference, but it is a more flexible question that allows for many forms of relationships. For our example, we would ask the question "Does wealth affect tests scores?"

The gold standard to answering questions like this is the controlled experiment. Controlled experiences allow you to develop strong inferences as you can see how a system responds when you hold all variables constant except for the single one you are interested in. For our example, we could imagine an experiment where we took a sample of a thousand families from a school district. When these families' children enter high school we would randomly select them to be in a "poor" category and the other half to be in a "rich" category. Families in the rich category are given grants of $500,000 a year to spend how they wish while the parents in the poor category are fired from their jobs and have their savings frozen for the duration of the experiment. Once the students graduate from high school, we would compare the scores for the students in the poor and rich categories.

model's overall purpose. To be very clear, it is important to clarify at the outset whether your primary goal is to use a model for prediction or for narrative. Many modeling projects may attempt to do both only to find themselves with a model that does neither.

These controlled randomized experiments are considered the ideal approach to answering inferential questions like these as they allow you to truly determine the effect of your variables, in this case wealth. For many types of questions, such experiments can be implemented (for instance does consumption of a new drug help treat a disease). Unfortunately, in general complex social questions are simply impossible to answer with them. We can consider the testing procedure we just imagined to assess the effect of wealth on scores, but it would be impossible and unethical to undertake in a real community. Furthermore, even if you were to implement the experiment as described, the behavior of a family that was poor or wealthy to begin with might very well differ from a family that experiences a sudden change in income.

Traditional Model Based Inference

Given our general inability to undertake the ideal controlled experiment, how do we answer inferential questions? The standard way is to collect data and then construct a model enabling us to measure the statistical significance of our hypothesis given the data. Due to history and simplicity, linear regression models are by far the most commonly used type of model today. A linear regression predicts an outcome (Y) based on the multiplication of variables (X's) by a set of coefficients determining the effect of the variables on the outcome (β's):

$$Y = \beta_0 + \beta_1 \times X_1 + \beta_2 \times X_2 ...$$

For the education example we could collect data on a number of students, measuring their families' wealth (X_1 in the equation above) and the student's test scores (Y). We would then run the linear regression to determine the coefficient values (β_0 – the intercept – and β_1 – the effect of wealth on test scores). If we thought there were other factors that affected test scores, we could measure them and include them as addition X's in the regression.

In addition to obtaining the values of these coefficients, we also obtain as a result from the regression the statistical significances or "p values" of these coefficients. Although p values are commonly used in statistics, they are ubiquitously misunderstood[4] so it is useful to briefly review them.

In short a p value measures the probability of seeing the measured data (or more extreme data) assuming the null hypothesis is true. Generally the null hypothesis will be that there is no relationship between the variables and the outcomes.

When assessing the significance of coefficients, a p value means the probability of seeing that value of a coefficient (or one even further from 0), assuming that

[4] These misunderstandings are not only made by on-the-ground practitioners and analysts, they are frequently shared, and propagated, by university-level statistics instructors; see, for instance, @Haller:2002vo.

the (unknown) truth is that the coefficient actually has a value of 0. In other words, it is the probability of seeing the observed non-zero value, assuming that the true value is in fact 0. Frequently, probabilities of 10%, 5% or 1% or smaller are taken as indicating statistical significance. These low values indicate that the coefficient value is so far from 0, and the probability of this occurring by chance so small, that we can reject the null hypothesis and accept the fact that the coefficient is not 0.

Using the p values enables inference by relying on the statistical significance of the coefficients. If the probability of β_1 (the coefficient for the effect of wealth) occurring due to chance (given it is 0 in reality) is less than, say 5%, we can claim with reasonable strength that wealth does in fact affect test scores. This is the standard approach researchers take to model-based inference and is used ubiquitously.

A Troubled Sea of Assumptions

Let's stop for a second and consider what we have done here. In carrying out these logical steps to apply model based inference to determine whether wealth affects test scores, we have had to make one very large assumption: that the relationship between test scores and wealth is linear.

Our linear regression equation assumes that for every increase in one unit of wealth (X_1), test scores (Y) will increase on average by the amount of the coefficient (β_1). What if this were not in fact the truth? For instance, we could easily imagine the case where wealth initially helped test scores by providing students more resources and opportunities to learn. After a certain point, however, wealth might negatively impact scores as very wealthy students might lack the pressure or motivation to study hard.

If we believed this were the case, then our linear regression model from earlier would be wrong as would the inferences we obtained from the model. We could correct our model and inferences by changing our regression formula to contain a squared term that could replicate this type of relationship:

$$Score = \beta_0 + \beta_1 \times \text{Wealth} + \beta_2 \times \text{Wealth}^2$$

Using this equation, at low values of wealth the $\beta_1 \times \text{Wealth}$ term will have the most effect on scores. Conversely, at high levels of wealth, the $\beta_2 \times \text{Wealth}^2$ term will have the most effect on scores. Thus by having a positive β_1 and a negative β_2 we can model wealth as having an initially beneficial and then detrimental effect. If our assumptions about the quadratic relationship are correct, then this model will yield accurate inferences. If they are wrong, our inferences will be wrong again.

What are we really doing when we assume regression forms like this? Now it might not be immediately obvious, but what we are in fact doing is telling a story.

Using our first equation, we are telling the story that as wealth increases test scores will almost always increase. Bill Gate's children will preform amazingly well here! Using the second equation we are telling a different story: As wealth increases test scores initially do as well but after a certain point increased wealth will hurt test scores. That picture isn't so rosy for the Bill Gates of the world!

And so we arrive at a key insight. By choosing our equations to tell a story, our inferences are in fact based on narrative modeling approaches. True, these inferences build upon numerous calculations and very advanced theoretical underpinnings, but ultimately what governs our conclusions and inferences are the stories or narratives we tell about our system. These are choices that we as narrators make and they not determined by an objective truth or reality.

Exercise 7-6

You are given the following linear regression model that predicts the growth rate of a tree (in meters per year):

$$\text{Growth Rate} = 3.2 + 0.013 \times \text{Mean Annual Temperature} + 0.021 \times \text{Annual Precipitation} - 2.3 \times \text{Moose Dens}$$

Take this mathematical model and convert it to a textual narrative.

Exercise 7-7

You are given the following linear regression model that predicts the demand for hats (in thousands of hats sold per day):

$$\text{Hat Demand} = 23.4 + 3.4 * (\text{Temperature in Celsius} - 22) - 1.2 \times \text{Wind Speed} - 0.21 \times \text{Unemployment Rate}$$

Take this mathematical model and convert it to a textual narrative.

Predictive Inference

Is there an alternative approach to inference that does not rely so heavily on narrative? Can we accomplish it without assuming the relationships between variables? The answer is yes. Although they are not often used, alternative prediction-based approaches to inference are available. In these approaches, rather than calculating statistical significances as a function of an assumed model, we calculate significances as a function of the simple question: "Does knowing X help us to predict Y?" This question is effectively identical to our earlier question – "Does X affect Y?" – but it is structured in an explicitly predictive manner. If the answer to the question is true, then we can say that there is a relationship between X and Y.

The techniques to accomplish prediction-based inference are much newer than classic techniques as linear regression as they rely upon extensive computing power and would not be possible without modern technology. One of these approaches is the *A3* method (XXX Citation) which uses resampling based algorithms to obtain estimates of predictive accuracy and statistical significance. *A3* focuses purely on predictive accuracy of a model to determine whether a variable is significant and often requires the automatic exploration of hundreds or thousands of competing models to find the one that best describes the data. The results of these analyses are inferences that are founded in the data of model fits only, not on subjective assumptions.

Predictive versus Narrative Modeling

As we can see, inferential techniques can be split into two categories: those based on narrative modeling methods and those based on predictive modeling methods. So – and this is a key advance – although there three categories of model purposes – prediction, inference, and narrative – there are only two fundamental approaches to constructing models: predictive modeling and narrative modeling.

This divide is not traditionally used in the modeling field, but it is truly at the heart of modeling. Understanding the distinction between these two types of modeling proves below to be much more valuable than mastering fine technical details. The choice of whether to build a predictive or a narrative model is a fundamental one that shapes every aspect of a model and determines its ultimate utility for a specific purpose. The following sections will describe these two types of models in more detail.

Predictive Models

How do we define a predictive model? The naive answer is that a predictive model is one that makes predictions. If a model generates predictions for a future outcome or a given scenario, than it must be a predictive model. By this definition, a weather forecast is a predictive model as were the Obama administration's unemployment predictions we saw earlier.

Unfortunately, this straightforward definition is useless. Worse than being useless, it is actually quite dangerous.

Let us propose a model for next year's unemployment figures in the United States:

> Generate a random number from 0 to 1. If the number is less than 0.1, unemployment will be 20%. If the number is greater than or equal to 0.1, unemployment will be 0%.

There, we have just constructed a model of unemployment. Furthermore, our model creates predictions. With just a few calculations we can forecast unemployment for the coming year. Isn't that convenient?

Of course, this model is a joke. It is useless in predicting unemployment. However, using the naive definition of what it means to a predictive model, it would be classified as one.

What makes this simple model, such a poor model for prediction purposes?

There are several answers. We might start by saying it is too *simple*. If we are really trying to predict unemployment we should incorporate the current economic state and trends into our model. If the economy is improving, unemployment will probably drop and vice versa. This is a valid point. Let's address it by proposing an "improved" model:

> Generate a random number from 0 to 1. If the number is less than the percentage change in GDP over the past year, unemployment will be 20% plus the current unemployment rate. If the number is greater than or equal to 0.1, unemployment will be the net change in the consumer price index over the past 8 years.

Is this a better model? Clearly, it is more complex than the previous one and it incorporates some relevant economic data and indicators. Equally as clear, however, is that it is also a joke far from being a useful model.

These toy economic models show that just generating predictions is not a helpful criterion to define a predictive model. They also show that complexity and the use of relevant data is not a valid criterion. So how do we specify a predictive model? The answer is straightforward:

> A predictive model is a model that not only creates predictions but also must contain an *accurate assessment of prediction error*.

Read that statement again. The key point is that the assessment of prediction error must be accurate, which is different from the accuracy of the predictions themselves. Of course, ideally the predictions will be accurate; however this is often not possible. Many systems are governed to a significant extent by chance and no model, no matter how good it is, will be able to create accurate predictions for the systems.

If you know the level of prediction error, you can instead contextualize poorly fitting models. You can determine how much to discount their predictions in your decision-making and analysis. Furthermore, and this is crucial, you can compare different predictive models. If your current model is insufficiently accurate, you can develop another one and objectively test it to determine whether it is better than the current model.

Without measures of predictive accuracy, discussing predictions or comparing models that create predictions is an almost nonsensical endeavor. Such discussions will be governed by political concerns and partisanship as there is no objective foundation on which to base them.

Our two proposed models to estimate unemployment are thus clearly not predictive as no estimate of predictive error has been established. We can apply same this requirement to Obama's employment predictions we saw earlier. When we first presented the model, we called it a narrative model, which might have been slightly perplexing as the model did generate predictions. However, using our above definition of a predictive model we can see also that it is in fact not a predictive model. The model contains no estimate of prediction error (and one is not available in the original report) so it simply cannot be considered to be predictive.

If accurate estimates of prediction error are available, you can directly compare the prediction errors between different models to select the one with the lowest error. We could estimate prediction errors for the two joke models we proposed here along with the Obama administration's model to find the one with the lowest error. We would hope that the one the Obama administration presented to Congress would be the most accurate. Before we test it however, we must not make the error of fallaciously accepting a model to be good based on who presented it to us or its complexity.

Why do we so rarely hear about the predictive accuracy of models? There are numerous reasons but they boil down to three basic ones:

1. Assessing prediction error accurately is quite difficult.
2. Sharing prediction error may perversely decrease an audience's belief in a model.
3. Most models people use for prediction are in reality narrative models and their predictive error is either abysmal or irrelevant.

Let's look at each point in detail. First consider the issue of the difficulty of assessing prediction error. In general, obtaining an accurate assessment of prediction error is much more difficult than developing the predictions themselves. Most commonly used approaches (for instance the standard R^2

from linear regression) have significant flaws. There are both theoretical and numerical methods that can be used to make more accurate prediction errors in many cases (this will be discussed further in the section the Cost of Complexity; see also @FortmannRoe:2012tf). When dealing with time series data, however, like most of the models explored in this book, it is often almost impossible to accurately assess model prediction error. Recently, theoretical technique to approach these issues have just begun to be developed (e.g. @He:2009jp or @King:2008jq) but they are still impractical to apply in many cases so far.

If the challenge of measuring prediction error is surmounted, there is an even more formidable barrier to its being published with the model. There is a perverse phenomena that the act of reporting prediction error can *decrease* the confidence an audience gives a model. An anecdote was relayed to us by a member of a team working on a model of disease spread. His team shared the predictions from the model with a group of policy-makers. Everything was going fine until the audience saw the error bars around the predictions. Where his audience had been content with the raw predictions, they were quite unhappy with the predictions when accompanied by their accurately estimated uncertainties. Why was this? Was the team's model particularly bad or did these policy-makers have a better model at their disposal? No. In a world where policy-makers and clients are constantly shown models (like Obama's unemployment figures) with no measure of uncertainty (or even worse, poorly calculated, artificially low uncertainty), they come to have unrealistic expectations and often turn away good science in favor of magical thinking.

Finally, the most likely reason supposedly predictive models do not include prediction error is that they simply are not predictive. We have seen how models developed for a purportedly predictive purpose can actually be narrative models in disguise. Just why is this too often the case? You need only look at the reason for most modeling projects. It is very rare that models are commissioned solely for the purpose of generating an accurate prediction. Frequently, models are part of some political process within an organization or across them (whether an organization be a for-profit company or a non-profit such as a university). Ultimately, the people funding the model expect it to prove a point to their benefit. In environments like these, it is to be expected that some predictive modeling efforts will be sidetracked by political concerns or compromised in the process.

We can see the results of such influences in the predictions generated for unemployment presented earlier. Figure 3 shows the projections for the unemployment rates with and without the stimulus plan just as in Figure 2. Overlaid on this are now the true values of unemployment the occurred after the predictions were made. As is readily evident, the original modeling and predictions were well off the mark. Not only was reality worse than the projections assuming the stimulus was enacted (which it was) it is much worse than the projections for the economy assuming the stimulus had never been enacted at all! This is just a small example – one that is sadly replicated over and over again in business and policy-making – of mistakenly treating a narrative model as a predictive one.

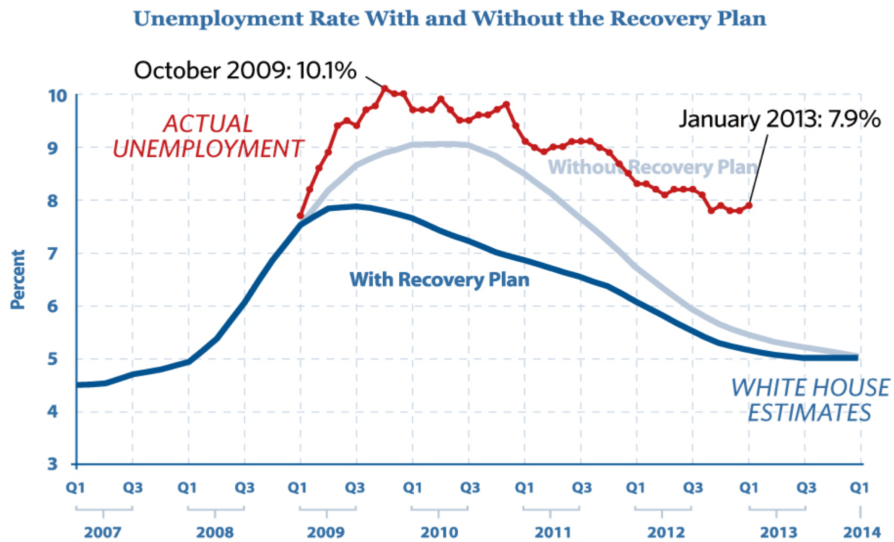


Figure 3. Unemployment predictions versus reality [@TheHeritageFoundation:2013vu]

Narrative Models

In contrast to predictive models, a narrative model is one built to persuade and transform an audience's mental models by telling a story. When many people first hear the "narrative" terminology, they respond negatively. "It's just a story." We find this strange, as narratives are the fundamental human form of communication. We tell narratives to our friends and relatives. Politicians communicate their policies to us using narratives. Of course the vast majority of our entertainment is focused on narratives[5]. Business leaders and managers attempt to describe their strategies to us using story lines; and business books are in general dominated by anecdotes plotted along the way to making their points.

We as a species do not view the world as a collection of numbers and probabilities; instead we see consequence and meaning. In short, narratives are how we see the world.

One critique of the term narrative is that it lacks numbers, quantified data, or mathematics. This could not be further off the mark. There are many ways to

[5]Even sports, a form of entertainment that innately contains no narrative, becomes wrapped in narrative as the announcers and commentators attempt to create stories to engage us.

construct narratives. Words are one, pictures are another, and music is a third. Numbers and mathematics are just another way of telling a story.

In fact, most statistical and mathematical models are infused with narrative models. We looked earlier at the case of linear regression as a tool to predict test scores as a function of wealth. Again the mathematical equation for this simple model was:

$$Score = \beta_0 + \beta_1 \times Wealth$$

This equation defines a narrative. Translating this narrative into words, we would say:

> Test scores are only determined by the wealth of a student's family. A child whose family is broke will have a test score, on average, of β_0. For every dollar of wealth a child's family accumulates, the child will score, on average, better on tests by β_1.

You might or might not agree with this storyline (in our view it is a nonsensical and reductionist view of child achievement) but it shows the strict equivalence between this mathematical narrative and narrative prose. This process can be applied to all mathematical models. The mathematical definition of the model can be converted directly, with more or less lucidity, into a story describing how the system operates. The same can also be done in the reverse: we can take a descriptive narrative of a system and convert it into a mathematical description. As we have seen (will see? XXX) this is what tools like reference modes and pattern matching are designed to do efficiently: elicit a narrative from a subject in a way which can be reformulated quantitatively.

The question of how to assess the quality of a narrative model is an important one. With predictive models, we can compare competing models based primarily on predictive accuracy[6]. But how do we evaluate and compare the quality of narrative models?

The key criterion in assessing a narrative model is its ability to be *persuasive*. Although persuasion is not an objective measure in the same sense prediction accuracy is, we can decompose persuasiveness into two components for our purposes: believability and clarity. A persuasive model is one that is both believable and effectively communicates its message.

When building a narrative it is very important to use tools that are well suited to meeting these components. Unfortunately, many statistical models, including regressions, are poorly suited to this two-fold task in many ways. Most statistical models depend on unrealistic and highly technical assumptions about the data.

[6] Other criteria include ease of use, cost of filling data requirements, and computational requirements. But all those are generally secondary to prediction accuracy.

If these assumptions were enumerated in plain English, they would often conflict with people's understanding and in fact end up discrediting the model. The "alternative" has been to leave these assumptions hidden creating a black box model opaque to outside inspection.

This is a shame in our view. Such a stratagem can be successful if the authority presenting the model is prestigious enough. But the misdirection will quickly fail if any kind of rigorous scrutiny is applied to the model. Narrative models should never be given any real credence if the operation of the model is not transparent. Most statistical models are built on assumptions that are never made transparent to the audience.

The modeling techniques presented in this book, on the other hand, are well suited for narrative modeling. The techniques we present are "clear box" modeling where the workings of the model are transparently evident and accessible. Our models have their structure explicitly described using an accessible modeling diagram showing the interactions between the different components in the model. The equations governing the model's evolution are clear and readily available for each part of the model[7]. Furthermore, these modeling techniques used here make it straightforward to generate animated illustrations and displays to clearly communicate model results.

Exercise 7-8

Summarize the distinction between predictive and narrative models.

Synthesis

Now that we have thoroughly described the concepts of narrative and predictive models we can conclude this chapter by taking a step back and reemphasizing that these two categories do to represent specific modeling techniques. You can build a stock and flow model to tell a story about a system resulting in a narrative model. If your story of the system accurately represents how the system operates in reality, then you will also have a model that generates accurate predictions.

Similarly, you can apply a linear regression to a dataset. If the relationship in the data is truly a completely linear one, then the result of this regression will be the most accurate predictive model you could build. On the other hand, if you do not assess the predictive accuracy of the model and just use a linear regression because it is easy to interpret or because it matches you understanding of reality, then you have a narrative model.

[7] Admittedly, for complex models it may still require a significant investment on the part of an audience to fully understand the logic and equations in the model. But the opportunity is available.

The key criteria to remember when building your own models or assessing other peoples models is that a predictive model is one for which you have an accurate assessment of the errors of the predictions. A good predictive model is one that has low relative errors when compared to other predictive models for the same system. A narrative model is one that tells a story about the system. A good narrative model is one that persuades an audience and by persuading, the model transforms the mental models of its audience.

Chapter 8

Building Confidence in Models

When used correctly, the transparency of the modeling techniques presented in this book results in models that are powerful persuasive tools. As with any model, however, there are concerns and questions will invariably be raised which could cause users to doubt the result of the modeling work. There are a number of techniques that you can use to help preemptively address these concerns and increase an audience's confidence in your model.

The idea of building confidence in a model is closely tied to the standard concept of model verification and validation. We dislike this conceptual approach to assessing models as it seems to imply that a model can go through a process to get a big fat "VALID" or "VERIFIED" stamp on it. Returning to Box's quote that "all models are wrong, but some are useful", in reality all models are wrong and none of them are completely valid, period. Models can however be useful, especially narrative models in which the audience has confidence.

We favor the conceptual approach put forth by @Forrester:1978vy, that there is not any single test or suite of tests that will verify or validate a model and that validity should instead be thought of as a function of confidence. This is a view that differs from that held by some modelers and laypeople. As Forrester and Senge note, "the notion of validity as equivalent to confidence conflicts with the view many seem to hold which equates validity with absolute truth." We share their belief that model confidence is built up piece by piece from a variety of tests that, though they cannot prove anything, together comprise a persuasive case for the quality of a model.

There are three distinct areas where confidence needs to be developed:

1. That the model itself is well designed.
2. Given a design of the model, this design is implemented correctly.
3. The conclusions drawn from the model are accurate.

In the remaining sections of this chapter we will look at each of these different areas in detail. We will explore the different tests and tools that can be used to build confidence for each area.

Model Design

Fundamentally the design of a narrative model is of utmost importance and needs to be justified to an audience[1]. There are two primary aspects to a model's design design: the structure of the model and the data used to parameterize the model.

Structure

The structure of the model should mirror the structure of the system being simulated. Depending on the system complexity, the model structure may need to carry out more or less aggregation and simplification of this reality. Nevertheless, all the primitives in the model should map on to reality in a way that is understandable and relatable to the audience. Thus if there is some object in the real system that behaves as a stock, a stock should exist in the model mirroring the object's position within the system. The same should hold true with the other primitives in the model. Each primitive would ideally be directly mappable onto a counterpart in the real system and any key component in the real system should be mappable onto primitives in the model. Furthermore, feedback loops that exist in the system should exist within the model. These feedback loops should be explicitly identifiable in the model and would ideally be called out or marked in a way that highlighted their presence to an audience.

Furthermore, the model structure should include components that an audience thinks are important drivers of the system. Missing a factor that the audience considers to be a key driver can fatally discredit a model in an audience's mind irrespective of the performance or other qualities of a the model. This is true even if the factor has in fact a negligible effect. Generally speaking, it is much easier to include a factor an audience views as important than it is to later on convince the audience that the factor does not in actuality matter.

Data

The more a model uses real-world data, the more confidence an audience will have in the model. Ideally, you have empirical data to justify the value of every primitive in your model. In practice, such a goal may be a pipe dream. Indeed, for a complex model, obtaining data to parameterize every aspect of it is usually

[1]This is different from predictive models where the results of the model are much more important than the design and the "proof is in the pudding"" so to speak.

impossible[2]. When faced with model primitives that do not have empirical data to parameterize them, an approach must be taken to ensure that it does not appear that their values were chosen without justification or to arrive at a predetermined modeling conclusion. Sensitivity testing, as discussed later on, is one way to achieve this. Another is to carry out a survey of experts in the field in order to solicit a set of recommended parameter values that can then be aggregated or used to justify the ultimate parameterization.

Peer-Review

Going through a peer-review process can be extremely useful in establishing confidence in a model. Two general types of peer-review are available. In one, the model may be incorporated into an academic journal article and submitted for publication. The article will then peer-reviewed by generally two or three anonymous academics in the field who critique it and judge whether or not it is a worthy contribution to the literature, thus meriting publication. In the second type of peer-review, a peer-review committee may be assembled (hired) to review a specific model and provide conclusions and recommendations to clients.

Peer-review can be very useful in establishing the credibility of a model. A credible model is a model one can be more confident in, other things being equal. By engaging an independent group of exports to assess the model, their conclusions about its quality have the appearance of greater validity than those of the self-interested modelers[3]. This can be especially useful when trying to meet some abstract standard such as that the model represents the "best available technology" or the "best available science".

A key risk of a peer-review is, of course, that the peer-review members will find a model deficient in important respects. Good criticism can be very useful and help improve a model. However, some criticism received in practice may be nitpicking details or detrimental advice that would make the model worse if followed.

Model Implementation

Although it is not as much a lightning rod as is model design, the implementation of a model specification is a point where significant error can occur. Programming mistakes or mistyped equations can introduce bugs into a model that can be hard to identify later on. This is a particular problem in black-box models but it is still an important point to consider for all types of models

[2] Leading to the clichéd conclusion of many modeling studies: "We are unable to draw strong conclusions from this modeling work. Instead, our contribution has been to show where additional data needs to be collected."

[3] When the peer review panel is hired by the client there is some conflict of interests, but the panel members should not be swayed by this.

including those presented in this book. Fortunately, a number of steps can be taken to ensure the model is implemented correctly.

Primitive Constraints

For many of the primitives in the model, there will be natural constraints. For instance, a stock representing the volume of water in a lake can never fall below 0. Similarly, if a variable represents the probability of an event occurring, it must be between 0 and 1.

Often these constraints are implicit without being formally specified in the model. A modeler may think, water volume can never become negative so why would I need to specify it? However, the existence of these constraints provides an opportunity to implement a level of automatic model checking. By specifying that a primitive can never go above or below a value (using the **Max Value** and **Min Value** properties in Insight Maker), you can create in effect a canary in the coal mine that warns if something is wrong in the model. If these constraints are violated an error message can be given letting you know that you need to correct some aspect of your model.

This concept of constraints in models is similar to the concept of "contracts" which are support in some programming languages. These contracts define and constrain the interaction between different parts of the program causing an error to be generated if the contract is violated. The Eiffel programming language probably has the best support for this approach to development.

Unit Specification

Since we introduced units in Chapter 3, we showed that they could be a useful tool in constructing models. Units can also be used to ensure that equations are entered correctly. If you fully specify the units in a model, many types of equation errors will result in invalid units, which will create an immediate error. By employing units in your model you can automatically catch a whole class of errors and mistyped equations.

Regression Tests

Other tests than those specified above can be developed. For instance, the proper behavior of one part of the model may be determined and automated tests created to periodically confirm that the model continues to exhibit the correct behavior. Development of such tests are a common part of software engineering that we wish would see more use in model development. Insight Maker itself has a suite of over 1,000 individual regression tests that automatically test every aspect of its simulation engine.

In regards to regression testing, it is important to ensure these tests are automated. It is not enough to examine a portion of the model, determine it is currently working correctly, and leave it at that. The problem is that future changes may break the existing functionality (i.e. a "regression", the introduction of an error or reduced quality compared to an earlier version of the model). Especially for complex models, a change in one part of the model may have an unexpected effect in another part. By implementing a set of automatic checks, you can protect your model against unintended changes and regressions.

George Oster and his class XXX

Exercise 8-1

You have a variable representing the total population size of a small city. What constraints might you place on this variable?

Answer Available

A Second Pair of Eyes

That is not to say, however, that spot and point-in-time checks are not worthwhile. It can be very useful to have a second modeler review your models and cross-check the equations. This helps not only to check simple mistakes but also to question and critique the fundamental structure and choices of the model.

The gold standard in verifying that a model is implemented correctly according to specification is to have a second modeler completely reimplement the model according to that specification. Such reimplementation should ideally occur without access to the original model's code base to ensure that the second modeler does not simply copy bugs from the original model into the reimplementation. If the results from the two implementations concur, that is strong evidence that the model has been implemented correctly. Although potentially an expensive exercise, it will also most likely identify numerous ambiguities in the specification, which could be valuable in and of itself.

Model Results

Given that the design of the model and its implementation are assumed to be correct, the burden still falls upon the modeler to transfer her confidence in the model's results to her audience. There are several different ways this can be done.

Expected Results

The first way is to demonstrate that the model generates expected results for normal inputs. For instance, if you had a model a reservoir, you would expect the volume of the reservoir to decline over time during the summer due to evaporation if no more water flowed into it. You can additionally test extreme scenarios and show that they generate the expected results. If, for example, your reservoir were empty, you would expect the amount of water evaporating from it to be zero. By enumerating these standard cases and showing the model results match the expected results you can help build confidence in the model.

Often these expected results can be described in terms of a curve showing how the values of one of the stocks or variables in the system is expected to change over time. This curve can be taken from historical data (a reference behavior pattern), or simply drawn on a piece of paper by experts familiar with the system (an excepted behavior pattern).

Counterintuitive Results

Another attempt to increase confidence in a model is to show unexpected results that are justifiable. Imagine a model that for a certain set of inputs would create what, at first glance, appeared to be the "wrong" behavior. Some lever in the model could lead to unexpected results. When first shown these results, they could decrease an audience's confidence in the model. If the audience was then walked through the model step by step to show how those results proved to be correct and mirrored reality, then that could well increase their confidence in the model results.

Forecasting

Possibly the most persuasive action to convince an audience of the effectiveness of a model is to forecast the future and then to show this forecast to be correct. This, of course, is difficult to do in practice for multiple reasons including the fact that the scale of a model is often such that it could take several years or decades to generate data to test the model. Additionally, it must be remembered that most narrative models are poor predictors and should not be used for predictive purposes solely.

Sensitivity Testing

Sensitivity testing is a broad field that has the potential to address many questions and doubts that may arise about a model. In general, the variables and numeric configuration values in a model will never be known with complete certainty. When the results from an election poll are published, the pollsters publish not only their predictions but also the uncertainty in the prediction (e.g., "the Democratic candidate will obtain $52\% \pm 3\%$ of the vote"). Similarly when a building is constructed, the materials used will have certain properties – such as strength – that again are only known up to some errors or tolerance. It is the engineer's and contractor's responsibilities to ensure that the materials are sufficient even given the uncertainty of their exact strengths.

The same occurs when modeling. Most primitive values in the model will have to be estimated by the modeler and there will be an error associated with these values. Of course the error will also be propagated through the model when it is simulated and affect the results generated by the model. This error is one factor that can create doubt about a model and reduce an audience's confidence.

As a modeler, one approach to address this doubt would be to try to measure all the model's variables with great accuracy. You could search the available literature, undertake a meta-analysis of current results, carry out new experiments, and survey experts to get as precise a set of parameter values as possible. If you were able to say with strong certainty that these values were so accurate and the errors so small that their effect on the results in negligible, then that would be one way of addressing the issue of uncertainty.

However, all of this is often impossible to do. When dealing with complex systems it is almost always the case that at least a couple variable values will never be known fully with certainty. In this case, no matter how much research or experiments you do, you will never be able to pin down the precise values of these variables. How do we handle these cases?

The answer is straightforward: Rather than trying to eliminate the uncertainty, we embrace it by explicitly including it in the model. If you can then show that the results of your model do not significantly change even given the uncertainty, you have a persuasive case for the validity of your results. Of course the results will always change when the uncertainty is introduced, but if the model conclusions persist even in the face of this uncertainty it will greatly increase your audience's confidence in the results.

Uncertainty can be explicitly integrated into a model by replacing constant primitive values with a construct that represents the uncertainty in that value. Imagine you had a simple population model of rabbits in a cage. You want to know how many rabbits you will have after two years. However, you don't know how many rabbits there initially are in the cage. You have been told that there are probably 12 rabbits, but the true number could range anywhere from 6 to 18.

If you model your population as a single stock, what should the initial value be? A naive model could be built where you the initial value of the rabbit stock was specified as 12. However, that does not incorporate the uncertainty and could be a source of criticism or doubt for the model. An alternative would be to specify that the initial value of the stock is a random number with a minimum value of 6 and a maximum value of 18. So each time you run the model you will get a different result. If you ran the model once, the initial value might be chosen to be 7 and you would obtain one result. If you ran the model again, the initial value might be 13 and you would get another result.

If you run this stochastic model many times, you obtain a range of results. These results can be automatically aggregated to show the range of outputs. For instance if you ran the model 100 times you could see what the maximum and minimum final populations were. This would give you a good feeling for how many rabbits you needed to prepare for after two years. In addition to the maximum and minimum you might be interested in the average of these 100 runs: the expected number of rabbits you would see. You could also plot the distribution of the final population sizes using a histogram to see how the results are distributed. This distribution would show how sensitive the outputs are to the uncertainty in the inputs: a form of sensitivity testing.

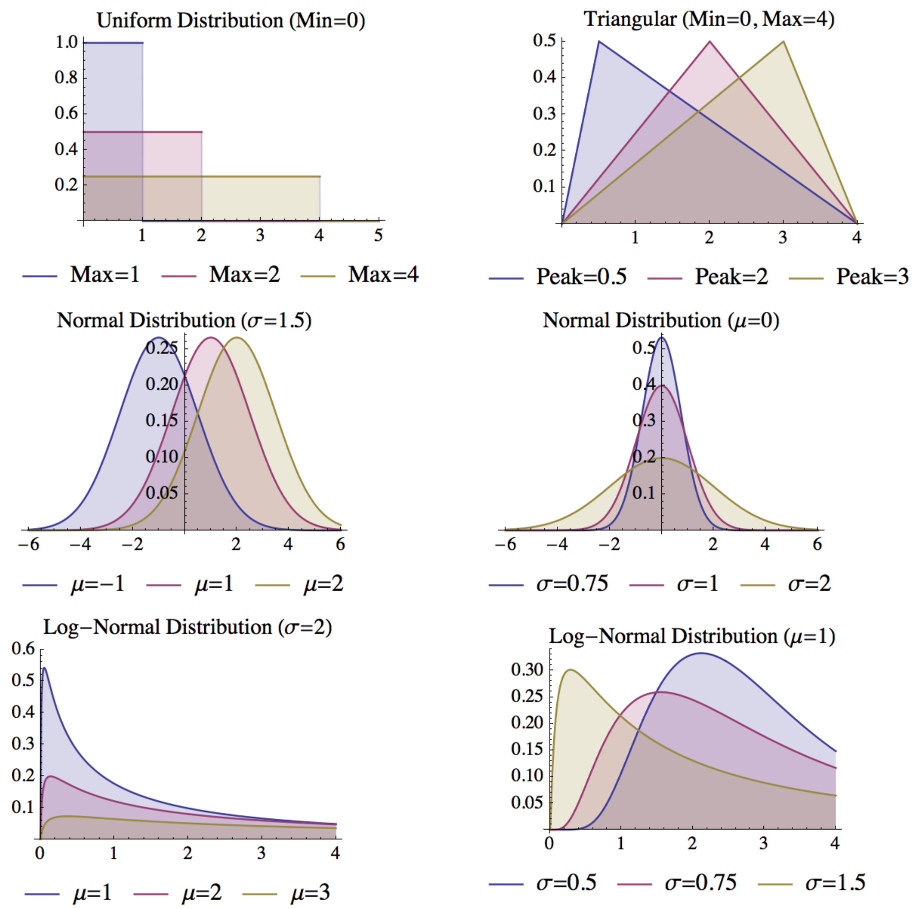


Figure 1. Common Distributions for Sensitivity Testing with Sample Parameter Values

There are four key distributions that are useful for specifying the uncertainty in a variable:

Uniform Distribution : The uniform distribution is defined by two parameters: a minimum and a maximum. Each number within these two boundaries has an equal probability of being sampled. The uniform distribution is useful when you know the boundaries on the values a variable can take on, but you do not have any information on the likelihood of the different values within this region.

The uniform distribution can be used in Insight Maker using the function `Rand(Minimum, Maximum)`, the two parameters are optional and will default to 0 and 1 if `Rand()` is called without them.

Triangular Distribution : The triangular distribution is defined by three parameters: the minimum, the maximum, and the peak. Like the uniform distribution, the triangular distribution will only generate numbers between the minimum and maximum. Unlike the uniform distribution, the triangular distribution will not sample all numbers between these boundaries with equal likelihood. The value specified by the peak will have the most likelihood of being sampled with the likelihood falling off as you move away from the peak towards either the minimum or maximum boundary. The triangular distribution is useful when you know the both the most likely value for a variable and you also know boundaries for the values a variable can take on. The triangular distribution can be used in Insight Maker using the function `RandTriangular(Minimum, Maximum, Peak)`.

Normal Distribution : The normal distribution is defined by two parameters: the mean of the distribution (generally denoted μ) and the standard deviation of the distribution (generally denoted σ). The most likely value to be sampled from the normal distribution is the mean. As you move away from the mean (in either a positive or negative direction), the likelihood of a number being sampled decreases. The standard deviation controls how fast this likelihood falls as you move away from the mean. Small standard deviations result in steep declines in the likelihood while large standard deviations result in more gradual declines. The normal distribution is useful when you do not have boundaries on the values for a variable but you do know what the most likely value for the variable should be (the mean). The normal distribution can be used in Insight Maker using the function `RandNormal(Mean, Standard Deviation)`.

Log-normal Distribution : The log-normal distribution is closely related to the normal distribution. In fact the logarithm of the values samples from a normal distribution will be log-normally distributed. Like the normal distribution, the log-normal distribution is defined by two parameters: the mean and standard deviation. Where the log-normal distribution differs from the normal distribution, is that negative values will never be generated by the log-normal distribution. Thus it is useful when you have a variable which you know cannot be negative but for which you do not have an upper bound. The log-normal distribution can be used in Insight Maker using the function `RandLogNormal(Mean, Standard Deviation)`. The log-normal distribution can also be used to represent other types of one-sided boundaries. For instance, the following equation could be used to represent a variable whose number was always less than 5: `5-RandLogNormal(2, 1)`

There are many other forms of probability distributions. Some notable ones are the Binomial Distribution (`RandBinomial(Count, Probability)`), the Negative Binomial Distribution (`RandNegativeBinomial(Successes, Probability)`), the Poisson Distribution (`RandPoisson(Lambda)`), the Exponential Distribution (`RandExp(Lambda)`) and the Gamma Distribution (`RandGamma(Alpha, Beta)`).

These distributions can be used to address very specific modeling usage cases and needs (for instance, the Poisson distribution can be used to model the number of arrivals over time), however, the four distributions described in detail above should generally be sufficient for most sensitivity testing needs.

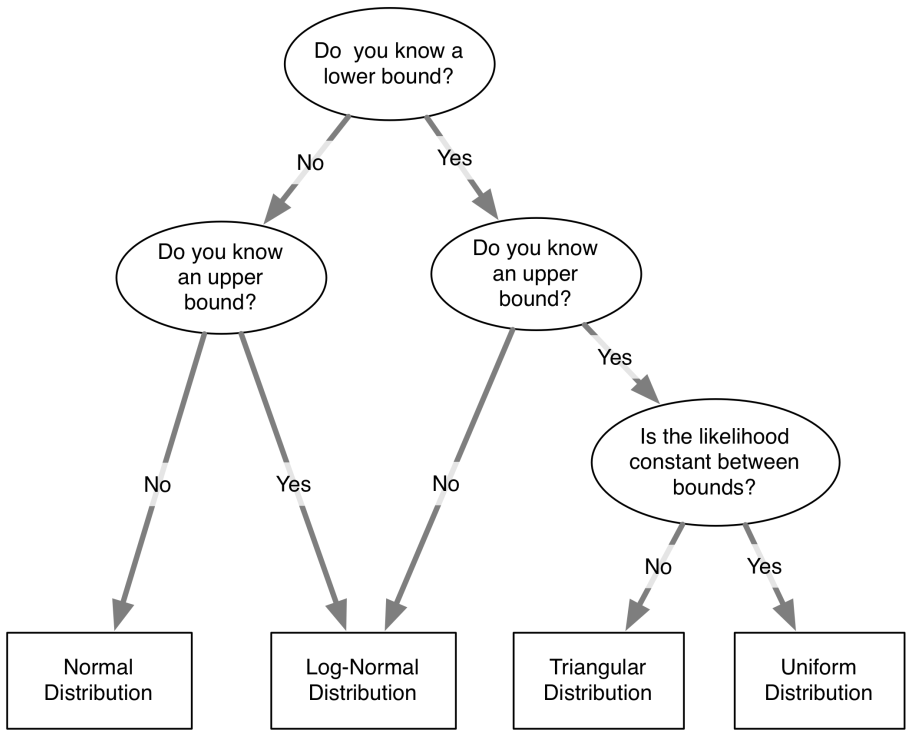


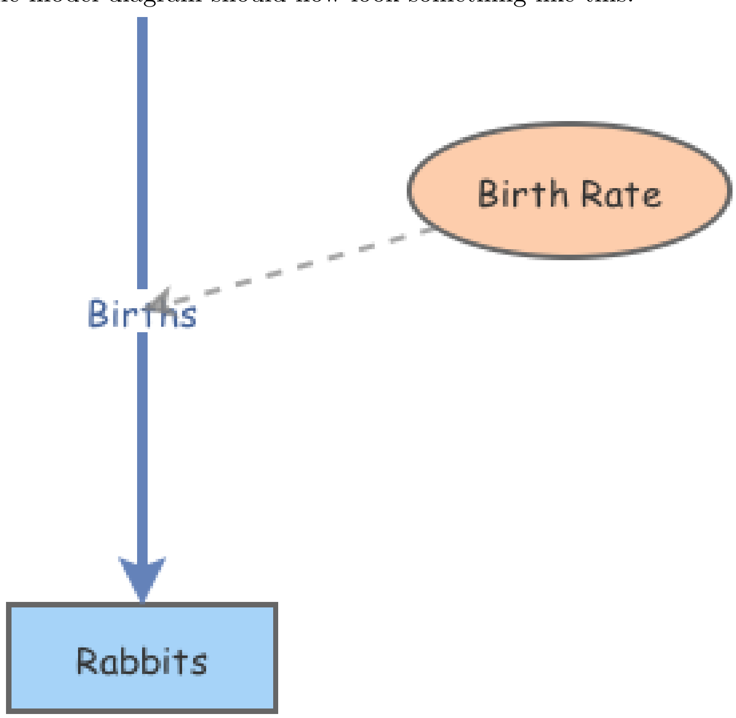
Figure 2. Choices in Selecting a Distribution for a Variable's Value

When important practical tip when using sensitivity testing within the System Dynamics context is to be careful about specifying random numbers within variables. The value of a variable is recalculated each time step. This means that if you have a random number function in the variable, a new random value will be chosen each time step. This can create a problem if the random value is supposed to be fixed across the course of the simulation. For instance, we may not know the birth rate coefficient for our rabbit population, but, whatever it is, we assume it is fixed over the simulation.

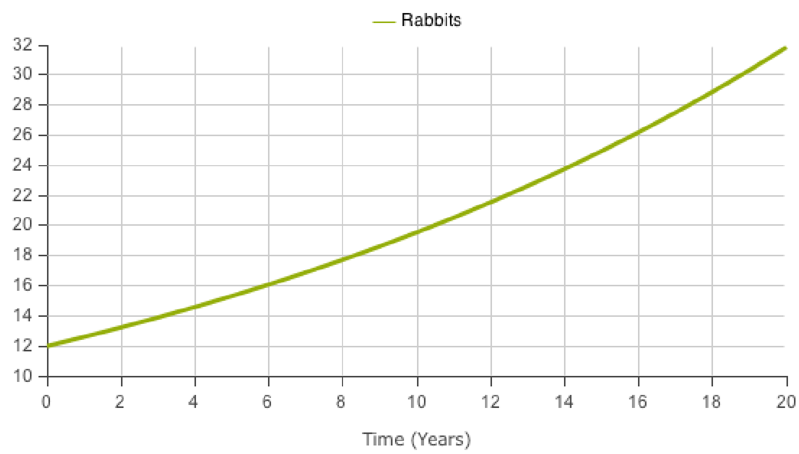
A simple way to handle these fixed variable values would be to replace the variables with stocks. The stocks initial value could be set to the random value and it would only be evaluated once at the beginning of the simulation and kept fixed thereafter. This approach, though very workable, however violates the fundamental metaphors at the hard of System Dynamics. In Insight Maker, another approach is to use the `Fix()` function. When used with one argument, this function evaluates whatever argument is passed to it a single time and then returns the results of that initial calculation for subsequent time steps. So instead of having the simple equation `Rand(0, 10)` in a variable to generate a random number between 0 and 10, you could place `Fix(Rand(0, 10))` in the variable. The first equation would generate a new random number each time step, the second equation will generate one random number and keep it constant throughout the simulation.

Sensitivity Testing

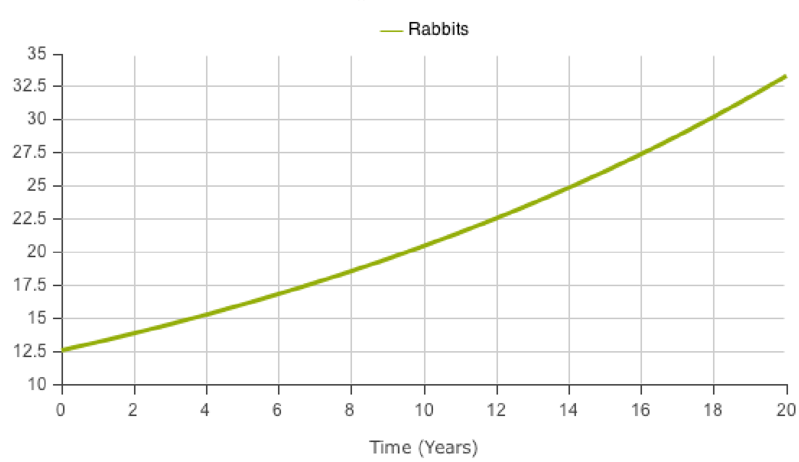
1. Let's illustrate the usage of sensitivity testing using our rabbit example. First we will construct a simple exponential growth model.
2. Create a new **Stock** named [**Rabbits**].
3. Change the **Initial Value** property of the primitive [**Rabbits**] to `12`.
4. Create a new **Flow** going from empty space to the primitive [**Rabbits**]. Name that flow [**Births**].
5. Create a new **Variable** named [**Birth Rate**].
6. Change the **Equation** property of the primitive [**Birth Rate**] to `0.05`.
7. Create a new **Link** going from the primitive [**Birth Rate**] to the primitive [**Births**].
8. Change the **Flow Rate** property of the primitive [**Births**] to `[Birth Rate]*[Rabbits]`.
9. The model diagram should now look something like this:



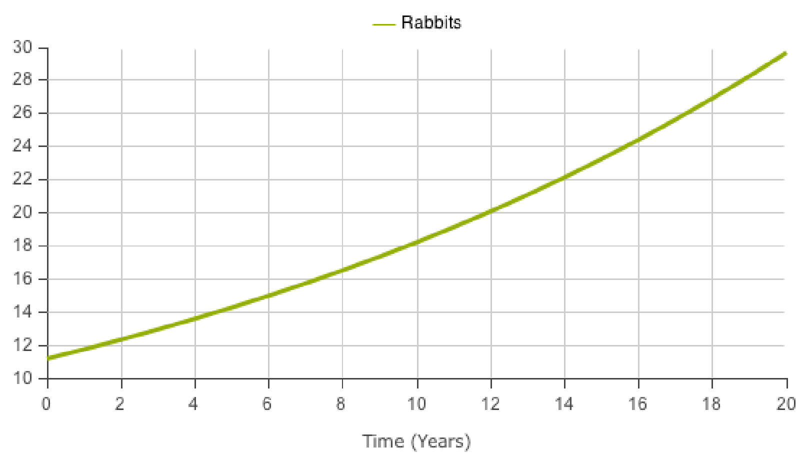
10. This is the basic outline for the model. We assume a fixed value of 12 rabbits and a fixed birth rate of 0.05.
11. Run the model. Here are sample results:



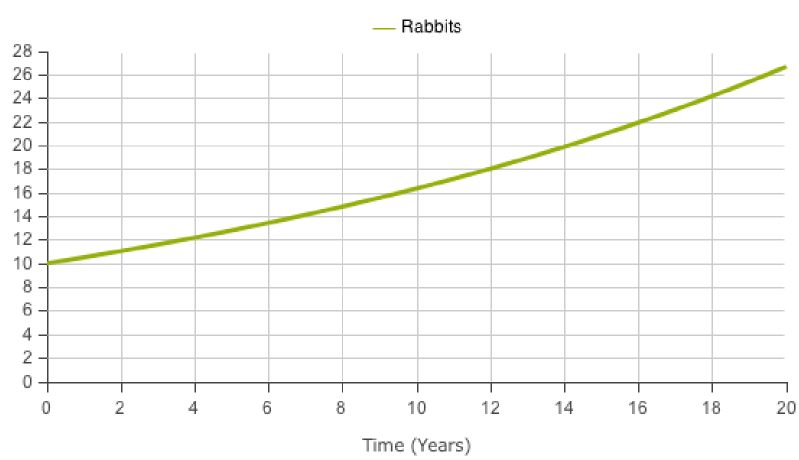
12. When we simulate we obtain the same results each time.
13. Change the **Initial Value** property of the primitive [**Rabbits**] to `RandTriangular(6, 18, 12)`.
14. Now, let's try to incorporate uncertainty. Given that we know that there can be between 6 and 18 rabbits initially with an expected value of 12, we can use the **RandTriangular()** function to model this.
15. Change the **value** property of the primitive [**Birth Rate**] to `RandLogNormal(0.05, 0.03)`.
16. We also do not know the birth rate with certainty. We know, however, that the rate must be greater than 0, and lets say we can assume the expected value is 0.05. We can use the **RandLogNormal()** function to model this type of uncertainty.
17. Run the model. Here are sample results:



18. Run the model. Here are sample results:

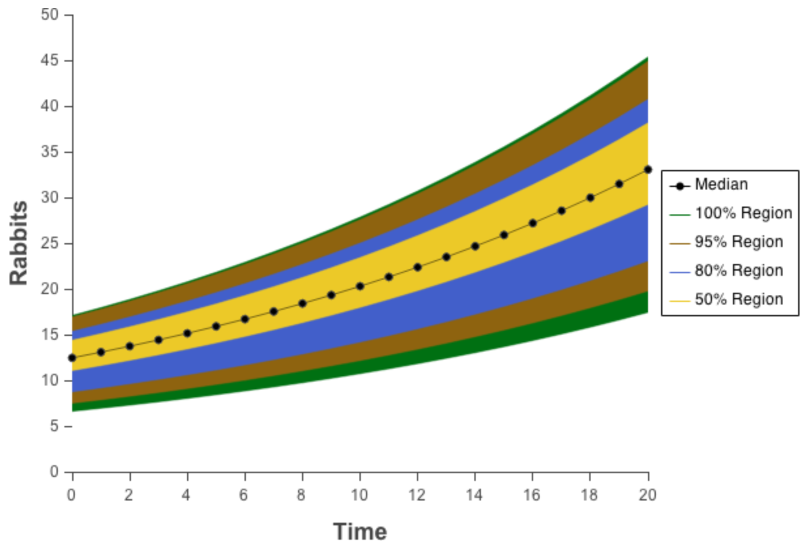


19. Run the model. Here are sample results:



20. Now, we can simulate this mode a few times and see that each time we run the simulation we get a different result.

21. We can now use sensitivity testing to see the range of results given this specified uncertainty. We'll do 100 runs of the model and aggregate the results to see the expected distribution

22. 

23. We can readily see the range of results allowing us to make decisions incorporating our known uncertainty about parameter values.

Exercise 8-2

Create an equation to represent the uncertainty of how many red marbles there are in a bag. You know there are at least 5 red marbles and no more that 14. You do not have any other information.

Answer Available

Exercise 8-3

Create an equation to represent the uncertainty of how many red marbles there are in a bag. You know there are probably about 20 red marbles and you know there are no more than 100 marbles in the bag.

Answer Available

Exercise 8-4

Create an equation to represent the uncertainty of how many red marbles there are in a bag. You know there are probably about 20 red marbles and you do not know how many marbles the bag can hold total.

Answer Available

The astute reader will notice that our discussion up to this has failed to address an important point: how do we determine the uncertainty of a variable? It is very easy to say that we do not know the precise value of a variable, but it is much more difficult to define the uncertainty of it. One case where we can precisely define uncertainty is when you take a random sample of measurements. For instance, suppose our model included the height of the average American man as a variable. We could randomly select a hundred men and measure their heights. In this case our uncertainty would be normally distributed with a mean equal to the mean of our sample of one hundred men and a standard deviation equal to the standard error of our sample of one hundred men[4]. For any random sample of n values from a population, the same should hold true: you will be able to model your uncertainty using a normal distribution with:

$$\mu = \frac{Value_1 + Value_2 + Value_3 + ... + Value_n}{n}$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Value_i - \mu)^2}$$

However, in most applied cases you will not be able to apply this normality assumption. Generally you will not have a nice random sample, or you might have no data at all and instead have some abstract variable you need to specify a value for. In these cases, it is up to you to make a judgment call on the uncertainty. Choose one of the four distributions detailed above and use whatever expert knowledge available to you to place an estimate on the parameterization of uncertainty. One rule of thumb, however, is that it is better to overestimate uncertainty than underestimate it. It is better to err on the side of overestimating your lack of knowledge than it is to obtain undue confidence in model results due to an underestimation of uncertainty.

Exercise 8-5

You have tested the diameter of 15 widgets coming out of a factory and obtained the following values: 2.3, 2.5, 1.9, 1.4, 2.0, 2.7, 1.9, 2.1, 2.1, 2.2, 1.6, 2.4, 2.0, 1.8, 2.6.

[4]Please note that this contradicts slightly what we said earlier. Clearly, a person cannot have a negative height while the normal distribution will sometimes generate negative values. So wouldn't a log-normal distribution be better than a normal distribution? Mechanistically, it would, however statistically we can show that due to the Central Limit Theorem the normal distribution does asymptotically precisely model our uncertainty. Given a large enough sample size (100 is more than enough in this case), the standard deviations for uncertainty will be so small that the chances of seeing a negative number (or even one far from the mean) are effectively none.

Create an equation to generate a new widget size with the same distribution as the widgets arriving from the factory.

Answer Available

Exercise 8-6

You have taken 12 sheep from a population and weighed the amount of wool on each sheep to obtain the following weights in kilograms: 1.005, 0.817, 0.756, 0.821, 0.9, 0.962, 0.692, 0.976, 0.721, 0.828, 0.718, 0.852.

Create an equation to generate how much a random variable for how much wool you will obtain from a sheep.

Answer Available

Confidence and Philosophy

The quality of a model in the eye of an evaluator is to a significant extent influenced by the worldview (more or less coherent and the philosophical orientation (if any) of the evaluator. Broad world-views or epistemologies[5] exist. One key divide in different epistemological theories continues to be between those theories that contain a strong belief in a concrete true reality that our knowledge can accurately capture and those theories that believe our knowledge is partially or wholly independent from reality.

Epistemological theories that are primarily in the first camp are those such as positivism or empiricism. Theories in the latter camp include constructivism and idealism. Constructivism is a popular theory of knowledge that claims knowledge is constructed with social context and historical time. Our presentation of confidence building for narrative models in this chapter is implicitly in line with a constructivist theory of knowledge.

In our discussion of confidence building we repeatedly refer to matching the beliefs of the audience. We recommend creating simulations and behavior in our models that match an audience's expectations for the behavior of the system. This is distinct from saying that you should match the reality of true system. Ideally, true behavior of the system and an audience's mental models of the system should be equivalent, but in practice they may well differ. Although confidence in a model will be boosted by strictly matching the mental models

[5] From the Greek word "epistēmē" meaning "knowledge" or "understanding", epistemology is the branch of philosophy describing how we understand or come to know the world around us.

of an audience, a truly effective narrative model should be persuasive enough to change the mental models of an audience.

Our discussion of predictive models from the previous chapter does not fall within a constructivist world-view as we are claiming that there are objective "outside-ourselves" measures of predictive accuracy we can obtain. It should go without saying that predictive models may not be accurate reflections of reality, even in their own terms. The mathematics of a predictive model may be unrelated to the true system that is being modeled, yet is may still create accurate predictions. As such, our discussion of predictive models is not really a positivist or empiricist one. Instead this discussion would fall under the epistemological theories of pragmatism or instrumentalism which claim a theory or model should be assessed on how well it predicts which may be independent of the truth of the theory itself.

Exercise 8-7

You are asked to evaluate a model simulating the growth of an endangered species in its habitat. What tests and demonstrations would you like to see in order to trust the model and recommend its use in practice?

Exercise 8-8

You are asked to evaluate a model simulating the potential adoption for a new product at your company. The basic results of the model are very encouraging for the product suggesting it would make a significant return on investment.

What tests and demonstrations of the model would you like to see in order to recommend the product be produced based on the model results?

Chapter 9

The Process of Modeling

Now that you are well on your way to being a modeling expert, you may begin to receive requests for assistance with various modeling projects. As a motivating example, a friend – it could also be a colleague or client – comes to you and asks for help. This friend has been involved with the effort to protect the rare Aquatic Hamster.

The Aquatic Hamster is an endangered species that spends most of its life living in lakes and rivers. Unfortunately, development and human encroachment has steadily reduced the available habitat for these hamsters and their population has plummeted. Indeed, now there is just one last population of them left located on a lake just south of the Canada/United States border.

Your friend asks you to build a model of this hamster population in order to help prioritize protection efforts and to rally support from governmental agencies and non-profits to protect this last hamster colony. You want to be of real assistance to your friend, and the hamsters are admittedly cute, so you agree to take on this modeling project.

You are at your desk to start building the model, but then realize something: You aren't sure what to do next. There are so many candidates for first steps. Do you start sketching diagrams? Do you talk to hamster experts? Do you start coding up a model? You are paralyzed by the sheer number of different choices. You know your friend is counting on you, so what do you do now?

In this chapter, we answer that question. We explore the modeling process from start to finish, introducing the tools and techniques for getting from "I need a model" to a final product that works. As you will see, our experience is that the best approach to tackling tough modeling problems is to start deceivingly small: build the simplest model possible (what we call the "Minimum Viable Model") to get going and then iterate aggressively on this initial version.

Why Model?

The first step to building a model is answering the simple question: *Why am I building this model?*

This question seems obvious, but it is often hard to answer in practice. Let's try answering it for our hamster population model: Why are we building this model? The truth is that so far we do not have a real understanding of this.

Oftentimes, the lack of focus begins with the friend/client/colleague who commissioned the model. Laypeople frequently do not have a strong understanding of what modeling is,including what modeling can accomplish and what it cannot. Instead, your friend might have a simplistic view of a model, almost as if were a magic wand. He feels he just needs a model and then, *abracadabra*, it will solve his problem. His thought process on what to do with a model might be as bareboned as:

1. Build Model.
2. . . .
3. Hamsters Saved.

Of course this is not the case. You build a model with a specific purpose in mind otherwise it will most likely accomplish nothing. Worse yet, when it comes to the hamsters, it will be too little too late. Your first action should be to work with your friend to make sure you have filled in the ". . . " step. The best way to do this is generally working backwards from the final step rather than working upwards from the first one. For us that would be to first figure out how the hamster population is to be protected.

Paradoxically, in order to answer the question of why we are building a model, we are going to need to ask many questions of our own. Why should we protect the hamsters? What risks do the hamsters face? What do the hamsters need to be protected? What avenues to obtaining these protections are there? What techniques to protecting the hamsters are most effective? Cheapest? Most expedient? And so on. We need to obtain a good understanding of the root cause of the problem you friend wants to tackle with this model and force out the concrete steps to getting there.

After discussing this with your friend and the two of you come to the conclusion that you will need two things in order to reliably protect the hamster population. First, government regulatory agencies need to pass (stronger) rules protecting the hamster habitat. Second, non-governmental organizations (NGO's) need to provide funds for hamster conservation and protection efforts.

Using this, we can expand our plan with more details:

1. Build Model.

2. ...
3. Agencies enact rules to reliably protect hamsters. NGO's provide money for conservation efforts.
4. Hamsters Saved.

This focuses things for us. Rather than "Building a model to save the hamsters" (which is too vague and completely unactionable leading to our quandary about what to model), we are building a model designed to persuade governmental regulators and NGO's that they should devote resources to protecting the hamsters.

So how do we do that? Let's simplify the complex issue into two specific goals for our model:

- Show that given the *status quo* (business as usual) the hamster population will go extinct.
- Show that alternatives to the *status quo* exist (which require regulatory action or investments) that enable the hamster population not only to survive, but also to thrive.

If our model demonstrates both these things it could be a highly persuasive tool to shape decisions and policies. By building a model that does these two things[1] we will have given our friend a powerful tool to push for regulatory action and financial support.

When building your own models you'll want to go through a similar thought process to get at the core goal or question the model should address. Going into a modeling project with the attitude "First we'll build a great model, then we'll figure out how to apply it" is a prescription for failure. Of course, as you go through the process you might discover insights you never expected or you might in fact determine that your original hypothesis was wrong. Such discovery is always a great outcome, but you can never count on it happening in the course of building your model. It's best to start very focused in your modeling efforts and treat any discoveries or broadening of scope later on as a lucky bonus.

Model Project Management

When tackling modeling projects such as our hamster-population model, there are two basic overarching project management approaches. The first is founded on detailed planning and preparation. Tackling the hamster model using this approach might look something like the following sequential phases:

[1] The model of course must also inspire confidence in its audience. They must believe its results are reliable otherwise the results will have no persuasive power. Review the previous chapter for tools for building confidence in models.

Research : Find and obtain relevant literature on Aquatic Hamsters. Read peer-reviewed publications. Locate hamster experts and interview them. Identify key mechanisms affecting hamster population growth. Some mechanisms may require further study. For example, if human expansion and urbanization affect the hamster habitat area, for example, you may need to study the forces influencing urbanization. These may require additional literature searches and expert interviews.

Design : Once you have completed your background research on the hamsters, start to design the model. Create causal loop diagrams and develop stock and flow diagrams. Break your hamster population model into different sectors. You will have the hamster-specific sector, which includes sub-sectors for each of the life-stages these endangered hamsters go through. You will also need sectors for other parts of the model that affect the hamster population growth: an urbanization sector with its own model, a climate sector with a climate model, and so on. Write out equations for all these sectors and resurvey experts you have contacted to review the overall model design and the specific equations. There will probably be several cycles of iteration and model expansion during this stage as additional key areas to include are identified.

Construction : Now that you have completed a model design and received a seal of approval from experts in the field, you are ready to start building the model itself. Decide what modeling software package (or programming environment) you will use. Implement the equations as they were specified in the design phase.

Wrapping Things Up : Go through the confidence building steps from the previous chapter. Develop tests for your model to ensure it works correctly. Create model documentation. Show the model demonstrates expected behavior and obtain final approval from experts.

This approach to building a model is a very linear process where you go sequentially from stage to stage. In the project management field, this is the classic "waterfall" project where you proceed phase by phase through the project. You plan out the whole thing ahead of time estimating how long each phase would take and identifying dependencies between phases. This form of project management can work well if done expertly and it is well suited for certain kinds of projects such as constructing a building.

In our opinion, however, this approach to tackling a project is quite poorly suited to the task of building a model. There are several reasons for this.

First, each model is inherently unique[2]. You may have developed a dozen different population models in your career, but when it comes to developing a model for a new species or location, you will inevitably run into situations and problems you have never encountered before. The quantity and quality of

[2]Lots of "cookie cutter" models out there are designed to model a certain class of problems. Without custom work, however, these models are of dubious validity and may serve more to "check a box" that a model has been built rather than to be a useful decision-making tool.

data will differ from the cases before. If not, the biology of the animal you are modeling will be different. If not, the model goals and constraints will be different, and so on. Given these differences, rigid project management techniques such as the waterfall approach do not generally provide the predictability that is needed.

Secondly, when building a model you will find that many of your assumptions may simply be wrong. This can happen with every aspect of model construction: the data you thought you had will turn out to be non-existent, the equations provided to you by experts end up not working, and the model code you write will invariably have a bug or two that needs to be identified and squashed. Because of this you will continually need to be adjusting and adapting your model as you learn more about the system and what pieces of information you can rely upon and what you cannot.

Such a high likelihood of error and need for readjustment are not well suited towards techniques based on sequential, long-term planning formats. What good is a great plan if the assumptions it is based on are substantially wrong?

Take, for instance, the data you use to build your model. It is not uncommon for a collaborator to come to you and say we have X, Y and Z data series for you to use in your model (where these might represent environmental conditions or other important model inputs). When you go to check the data however you may find that in fact X does not exist (the collaborator was confused), Y actually has large gaps in the data set which make it effectively useless for your needs, and Z was collected in such a way that they were actually measuring something completely different than they thought they were.

Take, as another instance, the equations in a model. Imagine you consult an expert on Aquatic Hamsters and she provides an equation governing the survival of hamsters during their first year of life. This equation was developed as part of a scientific study where the hamsters were grown in indoor swimming pools at her university's Aquatic Hamster Research Facility. When you go to apply this equation in your model, however, you find out that how the hamsters behave when living within an indoor swimming pool is very different from how they survive in the wild. Because of this, the equation you have is simply not accurate for the hamsters living in the wild.

Errors like these two examples are *very* common. If you had proceeded with the classic waterfall approach to modeling you might not realize that you cannot rely on the data or equations you were planning to use until the very end of the modeling process. At this point it is much too late to go back and correct your model.

Iteration: Failing Fast and Failing Often

Because of this, we advocate an alternative approach to building models. We support jumping right into the model construction process as early as possible.

As we showed you in the *Red* example from Chapter 4, we think it is important to get a simulation model up and running as quickly as possible. You should never want to be more than a few steps away from a simulating model[3].

When beginning a modeling project we recommend building the simplest model possible to get going. We call this the *Minimum Viable Model*[4] and it is the model that contains just enough to minimally represent the system and nothing more. For the hamster model, this Minimum Viable Model might contain just a single stock representing the hamster population and a couple of flows modifying the population. Nothing more.

You don't have to worry about your equations being right or your model being an accurate predictor in the Minimum Viable Model; you just want to get something up and running as soon as possible.

Once you have the Minimum Viable Model you can start to run it by people and begin to incorporate their feedback. So get your friend's thoughts on the minimal hamster model, talk to experts, study the model's forecasts and see what works and does not. Then iterate on the model: make a change here, add a new component there. If the feedback you are getting is no one trusts the model because it does not contain some key mechanism, add that mechanism to the model[5]. Steadily adjust and refine the model based on the actual results of the model and the feedback you receive.

This feedback will be more useful to you when you have a concrete model that is simulating than it would be if you were just running abstract ideas by people. By putting your stake in the ground with a model that simulates, you allow others to critique and engage with the model providing you with valuable information about what works and what does not. If you do not come with a concrete model, you run the risk of receiving very vague, unactionable feedback.

What is best about this approach of rapid iteration we advocate is that it allows you to identify failures quickly. If a data source is no good, you find that out

[3] This is a common theme in agile approaches to project management. You never want to be far from a working product. For instance, in the popular Scrum approach to managing software projects, the key unit of collective work is "the sprint". A sprint is a relatively brief amount of time (in the scope of the entire project) to complete a set group of product features. At the end of the sprint, the features must be completed and the software working or they are cut. The goal is always to be close to a working program just like you should always be close to a working model.

[4] This idea is adapted from Eric Ries's excellent book *The Lean Startup* (@Ries:2011vp). In it he advocates an approach to developing start-up companies and businesses focus on rapid development and innovation. Ries supports developing a "Minimal Viable Product" for the company as quickly as possible and iterating on the feedback received for this initial product.

[5] But the key is to wait until you get this feedback. It's easy on your own or with a group of people to make a list of dozens mechanisms that a model *must* contain to be realistic. Once you have implemented those mechanisms in your model you might find out that no one actually cared about them. It is best to start small and then augment the model when there is a demand for some additional mechanism, than it is to spend a large amount of time implementing a very complex model and then to find out much of that work was unnecessary.

immediately as you try to integrate it rather than spending days, weeks or months planning your model with the assumption it's really there or you can really use it. Rapid iteration – failing fast and failing often – is a key goal in the model development process. It can be argued that your successes in life are directly proportional to the number of failures and wrong turns you take: the more things you try, the more times you will both succeed and fail. We believe the same is often true in modeling. By speeding up the process of identifying and iterating past failures, this agile approach to modeling will often result in higher quality models completed more quickly than approaches that rely on extensive planning.

Model Boundaries

There are many different mechanisms and entities we could include in our model of the hamster population[6]. Of course there are the hamsters themselves but there are also hamsters predators, the hamsters' food, climatic conditions that affect the growth and survival of the hamsters, urbanization, eutrophication that affects the hamsters' lake, and so on. Given that it would be impossible to include every single element and mechanism in our model, we must define the boundaries of the system.

We can illustrate model system boundaries using a boundary diagram as illustrated in the excellent book *The Electronic Oracle* (@Meadows:1985wb)[7]. When using a model boundary diagram, we classify items of interest into one of three categories:

Endogenous : Endogenous items are at the core of the model. They are things that the model itself determines. For instance, the size of the hamster population is endogenous to the model. The model itself simulates this population.

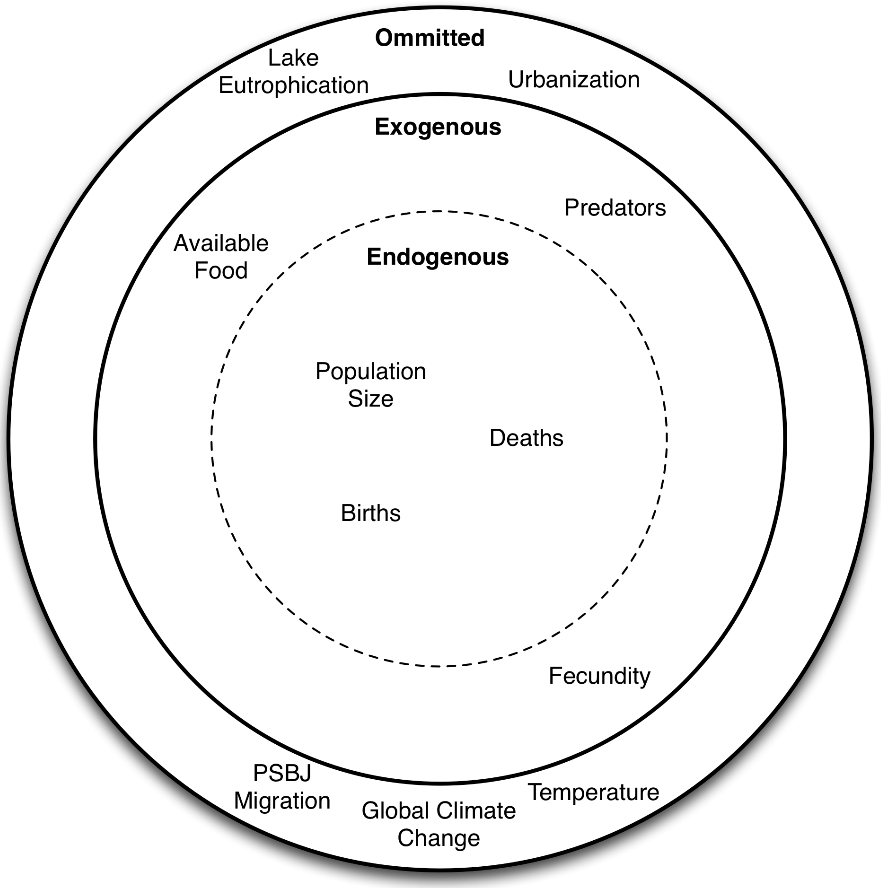
Exogenous : Exogenous items are those that you include in the model but which you do not directly simulate. For instance, if we thought temperature had a significant effect on hamster survival, we might want to include historical temperature data in the model. We do not want to simulate this data though, we just want to use it as an exogenous input into the model.

Omitted : Omitted items are those that, though we may acknowledge they do impact the hamsters either directly or indirectly, we choose not to include in the model. Even the most ambitious and comprehensive model will need to draw the line somewhere.

Figure 1 illustrates two different model boundaries for the hamster model. The top diagram depicts a small, conservative model with many features excluded

[6]The idea of the "butterfly effect" is that the flapping of a butterfly's wings in Europe, can initiate slight air disturbances that interact and magnify until they create a hurricane in Florida. If we believe in such avalanche effects to small events, the number of potential items we should include in the model is literally endless.

[7]This book provides an excellent overview of a number of different models and, very interestingly, it tracks the ultimate reception and the success or failure of these models.


Ommitted
Lake
Eutrophication
Urbanization
Exogenous
Predators
Available
Food
Endogenous
Population
Size
Deaths
Births
Fecundity
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Migration
Global Climate
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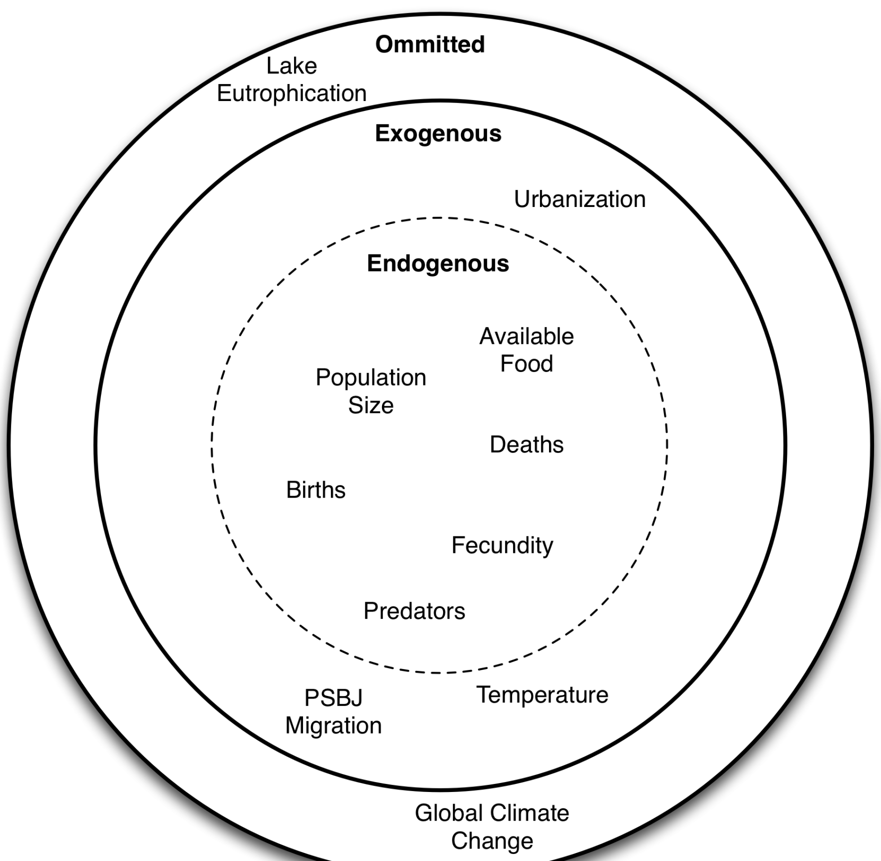
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Predators
PSBJ
Migration
Temperature
Global Climate
Change

from the model. The bottom figure illustrates a much more ambitious model where many additional items are made endogenous to the model and there are much fewer omitted items.

When developing a model, we recommend starting with the boundaries as narrow as possible. In the minimum viable model, you will want to omit as many different mechanisms as possible. As you receive feedback and people push for the inclusion of different mechanisms, you can slowly expand the boundaries of the model. We recommend starting small and expanding as necessary.

Exercise 9-1

Create a boundary diagram for a model of human population growth in the next 100 years. What would be the endogenous, exogenous and omitted items in this model.

Exercise 9-2

Create a boundary diagram for a model forecasting the total quantity of pencils sold within the united states for the next 50 years. What would be the endogenous, exogenous and omitted items in this model.

From Mental Models to Simulation Models

Generally speaking, a single individual should ultimately be responsible for the design and implementation of a model. Models "designed-by-committee" are understandably suffused with compromise and a greater lack of focus. That said, even though one person is ultimately calling the shots, many voices and perspectives are there to be heard in the modeling process. The more input there is into a model, the better the resulting model will most likely be.

The people you are working with generally will not be experts in modeling. Because of this, even if they are intimately familiar with the system you are attempting to model, it will sometimes be difficult to take their freeform insights and transform them into a formal model structure and accompanying numerical equations. In fact people often have great difficulty communicating and describing their own mental models of a system. A number of useful tools and techniques can be used to help elicit information on people's mental models. We discuss three of these tools in the following sections.

Reference Mode Diagrams

A reference mode is a graph that plots how the key stocks and variables in the system change over time. The x-axis of the graph is time, and the y-axis shows the values of the variables as they change. Sometimes reference modes are based on historical data, but you can also create them by asking those involved with the system to sketch out how they think the system will behave in different scenarios.

For our hamster model we could start simply by asking our friend to sketch out what he thinks will happen with the hamster population in the future assuming business as usual (remember that the status quo does not mean no-action). When we do this, he sketches out the top graph in Figure 2.

While your friend probably would use different terminology, to us the curve he sketched immediately looks like an exponential decay model. The instant we see this sketch we should start mapping out a stock and flow diagram in our mind to implement this type of model. Your friend does not need to understand any modeling concepts though, he just needs to be able to draw a picture of what he thinks will happen in the future. This is something that is easy to ask most people to do.

Let's go beyond the simple business as usual scenario. We can also use reference mode diagrams to elicit information on different scenarios. For instance, we have previously been told that development and encroachment on the hamster habitat are key factors reducing the hamster population size. Not only does the development consume key hamster habitat, the construction creates disturbances that have a further negative impact on the hamsters.

We can ask our friend to create a second sketch that shows how the hamster population would respond if development were suspended indefinitely. He responds by drawing the bottom graph in Figure 2. This graph shows the hamster population starting to recover after development stops, initially growing and then leveling off at a certain point.

Again, your friend never said this, but looking at this second drawing we should immediately start thinking of logistic growth models. The leveling off implies that there is some carrying capacity limit for the hamsters. This carrying capacity is probably a function of the available hamster habitat and the disturbances that are going on around the hamsters. We can start to sketch out stock and flow and causal loop diagrams to implement these types of dynamics and reproduce the behaviors our friend has drawn.

These are just two of the reference modes we might ask our friend to think about. We could go on to explore other scenarios and see how he thinks the changes in the scenarios would affect the hamster population. We could also ask him to sketch out other key variables in the system – such as the quantity of food available to the hamsters – to understand how he thinks these key variables interact. We could go on to interview other people familiar with the system and

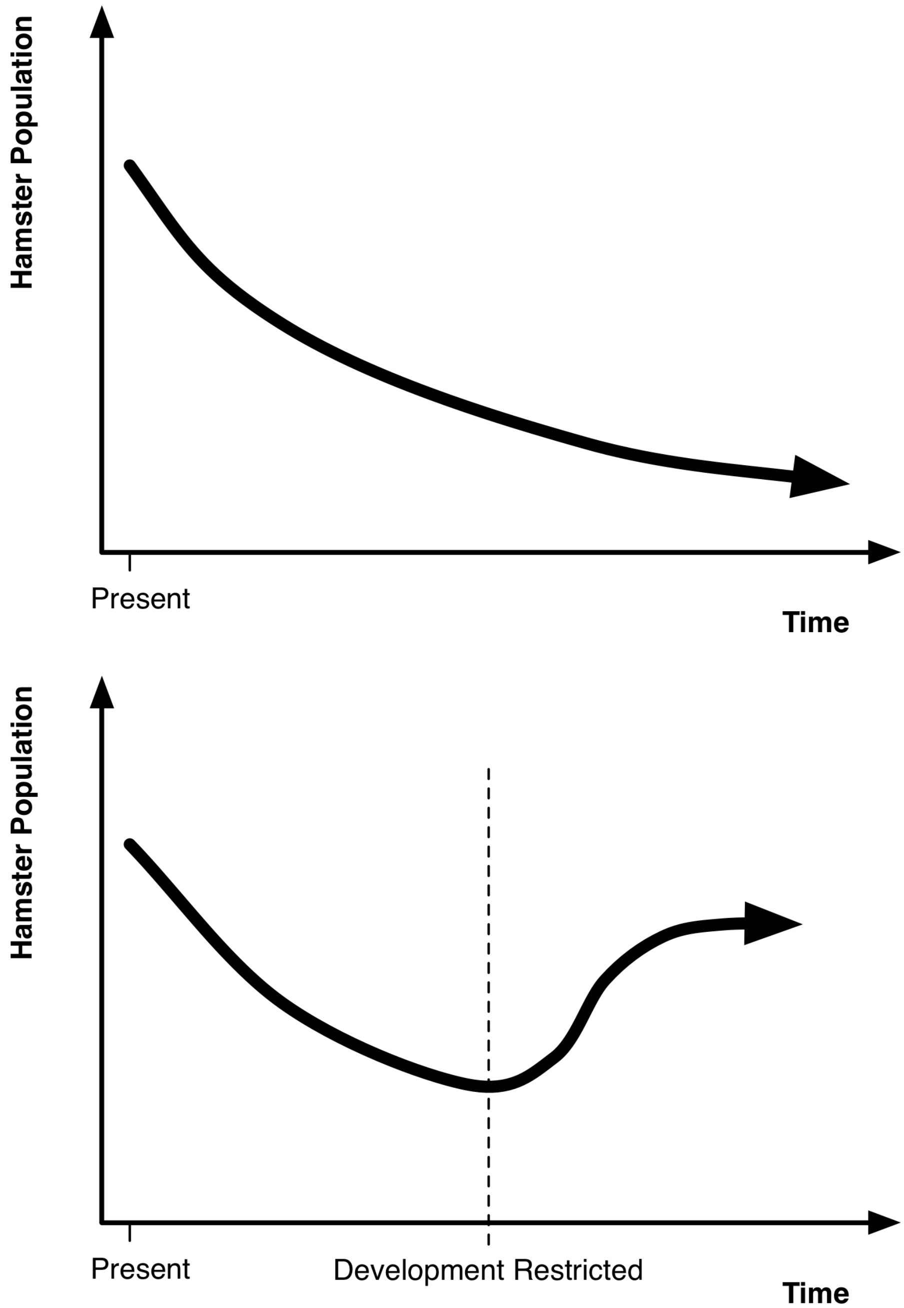


Figure 2. Sample reference modes for our hamster model.

take them through the same process. Ideally, all the reference modes between individual people will agree, but differences are in themselves also useful in revealing different mental models between our interviewees. Bridging differences will be a key interest of ours as we attempt to develop a persuasive model that will bring everyone on board and gain wide support.

Asking non-modelers to sketch out reference modes is a great technique for several reasons. Reference modes are accessible to laypeople, force your interviewees to be concrete, and provide you with very useful and actionable material. Really, a reference mode is a projection of an individual's mental model of the system. They may be unable (or unwilling) to explain their mental model to you in equations or even words, but they generally will be able to describe how they perceive the world using these reference mode diagrams – one small slice of their mental model at a time. Once you have the diagrams, you can proceed to translate them into model structure and equations.

Exercise 9-3

Draw a reference mode diagram for what you will think will happen to the total human population in the next 100 years. Draw additional reference modes for the following scenarios:

1. Cold fusion is invented in 2050. Limitless energy is available for free to everyone.
2. A plague wipes out 1/2 the human population in 2035. Each country is affected equally by the plague.
3. A process for cheaply converting a drop of oil directly into a kilogram of nutritious and delicious food stuffs is invented in 2030. This can replace the need for arable land, but oil become in even greater demand.

Exercise 9-4

You are hired by a paper company to create a model of paper consumption in the next fifty years. Draw reference mode diagrams of world paper demand for the most highly likely future scenarios as you see them. Consider the adoption of digital technologies and the decline of print media.

Pattern-Oriented Modeling

Pattern-oriented modeling focuses on identifying key patterns in the system to be modeled. For example, we may observe a boom-and-bust pattern in our hamster population that is triggered by unusually warm weather. When we develop our model, we formulate relationships and equations that will replicate this boom-and-bust pattern in the simulation.

Developed to help guide the creation of agent-based models, pattern-oriented modeling is very similar in concept to reference modes and system archetypes. Rather than building models around expected dynamic trajectories however, pattern-oriented modeling builds models to recreate patterns. Sometimes a pattern may be the same as a reference mode, but especially when dealing with agent based modeling you may not be able to define a pattern in terms of the dynamic trajectory of a reference mode. For a good overview of pattern-oriented modeling, see @Grimm:2005ei.

Exercise 9-5

What patterns might you see in the how cities are located?

Exercise 9-6

What patterns might you see in the movement of a carnivore like a wolf? In an herbivore like a moose?

Exercise 9-7

What patterns might you see in the movement of a competition between companies in an expanding market? In a contracting market?

Group Model Building

Group modeling sessions are a powerful tool to capitalize on the collective thoughts of a group to inform model structure and design. Instead of individually surveying experts and those involved in a system, a group session with many interested parties can be conducted. The term "group model building" is a bit of a misnomer as generally the model itself will be built away from the group by the facilitator or modeler and the group work will be focused on identifying and ranking key variables and mechanisms and developing high-level causal loop or stock and flow diagrams. See @Andersena:1997tg for a very practical overview of running and facilitating group model building sessions.

Group modeling sessions can also benefit an organization independently of the success or failure of the model itself. You might expect that the mental models of individuals within an organization would be aligned and the members of the organization would share a common objective and understanding of the challenges and needs required to achieve this objective. However, this is often not the case as different organization members may hold distinct mental models of the organization's purpose and operation within the world. Additionally, it is quite possible that these differences may never be realized as people may fail to adequately communicate their mental models assumptions and beliefs during the course of regular interactions.

The group modeling process can force the concrete discussion of and revealing of these mental models and the stakes involved in having these differences. Once they are revealed, they can be discussed and reconciled, potentially leading to a greater congruity of viewpoints within the group and a greater shared purpose. @Vennix:1993wv carried out a survey of participants in group model building sessions and found that this process led to insights and a shared vision more quickly than occurred in standard meetings.

Wrapping it Up

Completing a model is in some ways just the first step in a modeler's work. Once the model is finished you need to make sure to develop adequate tests to ensure it is operating as designed. Moreover, a model by itself is often of little use. You will need to develop extensive sets of documentation, manuals and tutorials if you want the model to be used in practice by people other than yourself. Such efforts take time. Writing clear and useful documentation is a skill in itself and, if done right, may take as long as developing the model in the first place!

In general, it is important to remember the 80/20 rule which also applies to modeling. The first 80% of modeling work generally only takes 20% of the time while the last 20% of the work can take four times as long. Getting the small details right in a model can take much longer than implementing the bulk of the model structure.

Exercise 9-8

You have been asked to model crime trends in a major city. Write out a general overview of stages you might take to developing this model from start to finish.

Chapter 10

The Mathematics of Modeling

This chapter takes the modeling techniques introduced earlier in this book and places them within a firm mathematical framework. The contents of this chapter are quite technical in parts and to fully understand them requires knowledge of basic calculus and linear algebra. We present the material because it is important for both readers who want a deep understanding how their models operate and also those who wish to understand how System Dynamics fits within the larger field of mathematical modeling. For users who approach systems thinking and modeling from a more qualitative angle, this material may be browsed or safely skipped.

Differential Equations and System Dynamics

Differential equations are a common mathematical tool used to study rates of change. Some basic terminology needs to be learned in order to discuss differential equations. After introducing this new terminology, we will then tie it back to the modeling techniques you've already learned.

State Variable : A state variable is an object that represents part of the state of a system. For instance, in a population model you could have a state variable representing the current number of individuals in that population. In a model of a lake, you could have a state variable representing the current volume of water in the lake. In equations, state variables are often represented using Roman letters such as X, Y or Z.

Derivative : Derivatives define rates of change in state variables. For instance, if we had a state variable representing the size of a population, a derivative would specify how this population grows or shrinks over time. The population's derivative would aggregate all changes such as births, deaths and immigration or emigration to show the net change in the state variable over time. Similarly, in the case of a model of a lake, the lake volume state variable would have a derivative showing how much net water flows into or out of the lake over time.

Given a state variable X, the derivative of X with respect to time is generally written as dX/dt but can also be written as X' or $\dot{X}$.

Let's put this new terminology to work to define a simple model. We start by creating an exponential growth population model. We only need one state variable in this model to represent the size of the population. We denote this state variable as P. We need to define one parameter to control the growth rate in the population. We will denote this growth rate parameter α.

The resulting differential equation exponential growth model can be written simply as:

$$\frac{dP}{dt} = \alpha \times P$$

This indicates that the rate of change for the population for one unit of time is $\alpha \times P$. Our model is not quite fully specified yet as we do not know what the initial value of the population is. Differential equation models are often additionally specified by providing the values of the state variables at a specific point in time. Below we indicate that the population size at time 0 is 100.

$$P(0) = 100$$
$$\frac{dP}{dt} = \alpha \times P$$

You may have already noted that this model is easy to construct using the techniques we have already introduced in the book. In fact we have discussed this type of model several times. We could construct it with System Dynamics tools using a stock to represent the population (P), a flow to represent the change of population (dP/dt) and a variable to represent birth rate (α). We could specify our initial condition of a population size of 100, by setting the initial value for the stock for 100.

This is an important point. Many differential equation models[1] can be directly represented using the System Dynamics modeling techniques described in this book. Similarly, a System Dynamics model can be rewritten as a differential equation model.

From this perspective, System Dynamics models and differential equation modeling are one and the same. A System Dynamics model can be expressed using differential equation notation and vice versa. To see this in more detail, we can look at the mapping between System Dynamics and differential equation models. There is a one a one-to-one direct correspondence between the key System Dynamics primitives and components of a differential equation model.

[1]Specifically those where the denominator in the derivative dX/dt is always dt: a very wide class of commonly used models.

System Dynamics Primitive	Differential Equation Equivalent
Stock	State Variable (X, Y, etc...)
Flow	Derivative (dX/dt, dY/dt, etc...)
Variable	Constants/Parameters (α, β, etc...)

Since they do not differ significantly from a mathematical standpoint, what separates these two approaches to modeling? Where System Dynamics and differential equation modeling differ is in their focus and philosophy. The primary goal for differential equation modelers is analytic tractability (in other words, how easy is it to mathematically manipulate and understand the model's equations). This analytic tractability allows these modelers to derive definite results and conclusions from the model's equations. System Dynamics modelers generally are less concerned about analytic tractability and are more comfortable with simulating the model and drawing conclusions from observed trajectories and numerical results.

System Dynamics modelers, to go further, care greatly about communicating their models, deliberately mirroring reality to some extent and exploring the consequences of feedback. The differing focuses on communication between System Dynamics modelers and differential equation modelers can be seen in the method of naming variables. Differential equation models are generally dominated by abstract Greek symbols (e.g. α) while System Dynamics models generally clearly spell out variable names (e.g. "Birth Rate") and additionally use a model diagram to illustrate and communicate the relationships between different parts of the model.

Exercise 10-1

You have a System Dynamics model simulating water leaking out of a hole in a jar. You have a stock [**Jar**] with an initial value of 40. Roughly 10% of the water leaks out of the jar every time period and there is a single flow leading out of the jar with the rate `0.10*[Jar]`. Express this model using differential equations.

Answer Available

Exercise 10-2

You have a System Dynamics model simulating people becoming sick. You have two stocks in the model [**Healthy**] and [**Infected**]. There is a single flow, [**Infection**], going from the healthy to infected stock with a flow rate of

`0.05*[Infected]*[Healthy]`. Initially there are 100 infected people, and 1 infected person. Express this model using differential equations.

Answer Available

Exercise 10-3

You have a differential equation model of an animal population's growth (denoted P). The animals growth is parameterized by the parameter r and a maximum population size or carrying capacity of K. The following differential equations define this model:

$$P(0) = 500$$
$$r = 0.05$$
$$K = 10000$$
$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

Implement a System Dynamics version of this model. What is the size of the population after 100 years?

Answer Available

Solving Differential Equations

Given a differential equation or System Dynamics model specification, how do you go about determining the results of the model? This is typically referred to as "solving" the model. Since differential equation models and system dynamics models are essentially one and the same, the techniques used to solve differential equations can be directly applied to System Dynamics models and they are the techniques used by Insight Maker when you simulate any of the models in this book.

For most of the rest of this chapter, we use the differential equation terminology instead of the System Dynamics one. We do so first because it is more concise and more elegantly addresses the issues discussed in this chapter, but also because we want to familiarize you with its terminology and concepts. If you ever get lost, just refer to the System Dynamics to differential equation translation table we showed above.

Let's start our discussion of solving differential equations using our simple population model. As you recall, this model was:

$$P(0) = 100$$
$$\frac{dP}{dt} = \alpha \times P$$

What is the size of the population, at, let's say $t = 10$ given an α of 0.1? Calculus can be used to solve the model and answer this question. First we separate the terms of the derivative and integrate both sides of the equation. Thereafter it is a simple matter of algebra to solve for P:

$$\begin{aligned}
\frac{dP}{dt} &= \alpha \times P \\
dP &= \alpha \times P \; dt \\
\frac{1}{P} \; dP &= \alpha \; dt \\
\log(P) &= \alpha \times t + A \\
P &= e^{\alpha \times t + A} \\
P &= B \times e^{\alpha \times t}
\end{aligned}$$

In this equation two new variables A and B appeared (where we arbitrarily set $B = e^A$). These are unknown integration constants[2]. We can determine the values of the integration constants based on the initial conditions of the model, as we specified earlier that $P(0) = 100$. We evaluate the solution of the model at this initial condition to determine the value of B.

$$\begin{aligned}
P &= B \times e^{\alpha \times t} \\
100 &= B \times e^{\alpha \times 0} \\
100 &= B
\end{aligned}$$

Thus our generic equation for P at any time and for any α is:

$$P = 100 \times e^{\alpha \times t}$$

Plugging in $\alpha = 0.1$ and $t = 10$, we obtain:

$$\begin{aligned}
P &= 100 \times e^{0.1 \times 10} \\
&= 271.828...
\end{aligned}$$

[2]Recall from calculus that if A is a constant, then $x^2 + A \; dx = 2 \times x$. When we integrate $2 \times x$ we need to add back in the constant term. We don't know the value of this constant term immediately and we have to determine it later on.

For this simple population model we have shown that we can obtain the precise population value at any point in the future. It took a fair amount of algebra even for such a simple model, but we did it!

Unfortunately, many differential equation models cannot be solved using these techniques. For most complex models in practice, it is impossible to analytically determine the values of the state variables in the future. This inability to solve a model can be true for even very simple models. Take for example the following growth model similar to our original one:

$$P(0) = 100$$
$$\frac{dP}{dt} = \alpha \times P \times \log(P)$$

We have simply added a logarithm of P into our growth rate. Despite the smallness of this change, this model is now impossible to solve analytically. There is no analytic solution possible, but feel free to give it a try yourself (but please don't try too hard; we promise there is no solution). When developing complex models it should generally to be assumed that in practice no analytical solution will be available. In cases like these, how can we go about developing solutions to the equations and determining the trajectory of the state variables in the system?

Exercise 10-4

Solve the differential equation:

$$P(0) = 10$$
$$\frac{dP}{dt} = -\alpha$$

Answer Available

Exercise 10-5

Solve the differential equation:

$$P(0) = 10$$
$$\frac{dP}{dt} = 0.05 \times P$$

Answer Available

Exercise 10-6

Solve the differential equation:

$$P(0) = 20$$
$$\frac{dP}{dt} = \beta \times P^2$$

Answer Available

The answer is numerical approximation. Even if we can't solve the model equations analytically, we will always be able to approximate their results numerically. A number of different algorithms exist that allow us to approximate the solution to differential equations by repeatedly plugging values into them. To discuss these methods, it is useful to introduce some additional mathematical notation.

In our previous models, we have only looked at systems with a single state variable at a time. However, we can also consider systems containing multiple state variables. The Lotka-Volterra predator prey system we looked at earlier in the book is an example of this. Given two populations of animals – let's say a population of wolves (W) and a population of moose (M) – where the first population preys upon the second, we obtain a paired set of differential equations representing this predator prey relationship:

$$\frac{dM}{dt} = \alpha \times M - \beta \times M \times W$$
$$\frac{dW}{dt} = \gamma \times M \times W - \delta \times W$$

When looking at algorithms to solve sets of equations like these numerically, it can be useful to denote $\mathbf{y}$ as a vector of all the state variables in the model. So for the case of the exponential growth model $\mathbf{y} = [P]$ while for the Lotka-Volterra model $\mathbf{y} = [M, W]$. When using this notation, $\mathbf{y_t}$ indicates the vector of state variable values at a specific point in time, so $\mathbf{y_0}$ are the initial conditions for this model.

Additionally, we can denote $\mathbf{y'}$ as the vector of derivatives for the different state variables. We treat these derivatives as functions of the current time and the values of the other state variables. So, for instance, to determine the rate of change of the state variables in a model at $t = 10$, we would write $\mathbf{y'}(\mathbf{y_{10}}, 10)$ where $\mathbf{y_{10}}$ are the values of the state variables at $t = 10$.

The use of this notation might seem cumbersome, but it allows us to elegantly describe the mathematics of numerical solution algorithms without getting tied up in the details of a specific model.

Euler's Method

Leonhard Euler

The most basic numerical solution algorithm for differential equations is Euler's method[3]. Simply put, assuming we know the state of the system at time t and we wish to estimate the state of the system at time $t + \Delta t$ (where Δt is pronounced "delta-t" and represents the change in time) we can use the following equation:

$$\mathbf{y_{t+\Delta t}} = \mathbf{y_t} + \Delta t \times \mathbf{y}'(\mathbf{y_t}, t)$$

Let's walk through what this equation is doing. It first takes the derivatives for the state variables at the current point in time. It multiplies these rates of change by the Δt (how far in the future we want to know the results) and adds this change to the values of the state variables at the starting point in time. The result is an estimate of what the values in the future should be.

Let's now apply this to a concrete example. Start with our population scenario, but instead of exponential growth we have a fixed inflow of people at a rate of 20 per year. At $t = 0$ we have 100 people and we want to know the population in 10 years, using Euler's method we obtain the following:

$$\begin{aligned} P_{10} &= P_0 + \Delta t \times \frac{dP}{dt} \\ &= P_0 + 10 \times 20 \\ &= 100 + 200 \\ &= 300 \end{aligned}$$

[3] Leonhard Euler was a brilliant 18th century Swiss mathematician who made many great advances in the theoretical and applied mathematics.

Thus the population size in 10 years will be 300. In this simple example, Euler's method works perfectly and generates the exact same answer as we would have found using analytic solutions.

In general, however, we won't be so lucky. For most problems Euler's method will generate results that contain some level of error compared to what the true value should be. To see this let's explore our exponential growth model again with an α of 0.1. As a reminder, this model is:

$$P(0) = 100$$
$$\frac{dP}{dt} = 0.1 \times P$$

As we showed earlier, the precise solution to this model for $t = 10$ (to three decimal places) is 271.828. Let's see what we get using Euler's method with $\Delta t = 10$. Carrying out similar calculations as before we get:

$$\begin{aligned}
P_{10} &= P_0 + \Delta t \times \frac{dP}{dt} \\
&= P_0 + 10 \times (0.1 \times P_0) \\
&= 100 + 10 \times (0.1 \times 100) \\
&= 100 + 10 \times 10 \\
&= 100 + 100 \\
&= 200
\end{aligned}$$

So using Euler's method we obtain an estimate 200 for the population size at $t = 10$ when we know the true value should be around 272. That's a pretty large error! Why does this error come about? Why do we so significantly underestimate the final population size?

The reason is that we calculate the population's rate of change only at $t = 0$. For each of the ten years we are simulating, we assume the population grows at the rate it would if there were exactly 100 people. However, the population size is constantly increasing during these ten years, so the rate at which it grows should also be increasing. Imagine, the case of a bank account with an interest rate of 10% yearly. The bank account grows over time so the interest earned should also grow from year to year. It's the same principle of compounding here.

How do we address this issue? Using Euler's method, we can do it simply by changing how often we calculate the rates of change. In our previous calculation, we went straight from $t = 0$ to $t = 10$ all in one step, we used a Δt in Euler's equation of 10. However, we could employ an alternate calculation strategy where, for instance, we stepped from $t = 0$ to $t = 5$, recalculated the derivative based on the new population size and then stepped from $t = 5$ to $t = 10$. This would be equivalent to used a Δt of 5 and iterating the algorithm twice. Here is what we get doing this:

$$
\begin{aligned}
P_5 &= P_0 + \Delta t \times \frac{dP}{dt} \\
&= P_0 + 5 \times (0.1 \times P_0) \\
&= 100 + 50 \\
&= 150 \\
P_{10} &= P_5 + \Delta t \times \frac{dP}{dt} \\
&= P_5 + 5 \times (0.1 \times P_5) \\
&= 150 + 5 \times 15 \\
&= 150 + 75 \\
&= 225
\end{aligned}
$$

That result is certainly better, and we cut our error by over 33%. However, the error is still too large for most practical purposes. To improve the numerical estimation even more, we can apply smaller and smaller Δt's. You probably have a good grasp of the calculations now, so let's just show the results for each step of the simulation. We'll look at $\Delta t = 2$ and $\Delta t = 1$.

t	P
0	100
2	120
4	144
6	172.8
8	207.4
10	248.8

t	P
0	100
1	110
2	121
3	133.1
4	146.4

5	161.1
6	177.2
7	194.9
8	214.4
9	235.8
10	259.4

We see that as Δt gets smaller and smaller our results become more and more accurate. However, they are never perfect. There is always some error. Even if we made Δt as small as 0.1 (requiring 100 simulation steps), our final population size would be calculated to be 270, an error just under 1%.

Figure 1 illustrates the application of Euler's method to numerically estimate the trajectory for an example function. The smaller the Δt's in the estimation are, the better the results will be. Other terms that can be used in place of Δt are "Step Size", "Time Step" or just "DT". We prefer not to use the notation DT as it is easily confusable with the dt from differential equations. The latter indicates an infinitesimally small change, while step sizes are never infinitesimally small.

As you decrease the step size for the simulation, the results of the simulation become more and more accurate[4]. The cost of this increased accuracy, however, is increased computation time. The computation time required by your model is directly proportional to 1 over the step size. Thus, if you cut the step size in half, your model will take twice as long to complete simulating.

In general, you want a step size small enough that your results are "accurate enough," but one that isn't so small that the simulation takes too long to complete. A rule of thumb for choosing the step size is to choose a starting step size that results in a fast simulation. Then cut the value of the step size in half and simulate the model again. If the results have not change materially between these two simulations, keep the larger step size. If the results have changed, cut the step size in half again and repeat until the results cease to change.

Exercise 10-7

Take the differential equation:

[4] It is important to note at this point that when we discuss accuracies in this context we are specifically referring to models composed of continuous differential equations. If you are using agent based modeling or have discontinuities in your models – which could occur if you use If-Then-Else logic – then a smaller step size may not provide additional accuracy when there is some fundamental time step logic to the model.

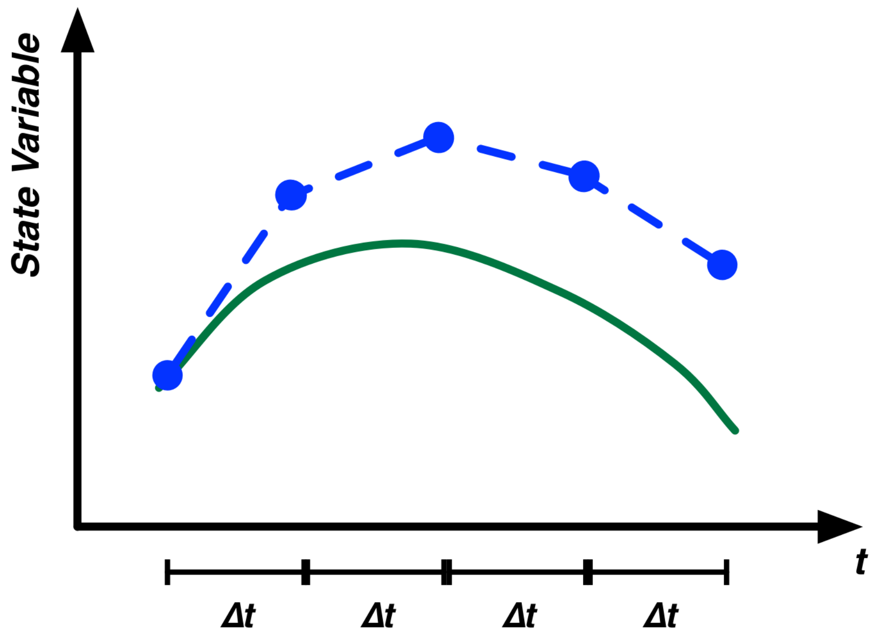


Figure 1. Euler's method at work. The true trajectory for the illustrative state variable is shown in green. Euler's method estimate of this trajectory is shown in blue.

$$P(0) = 20$$
$$\frac{dP}{dt} = \frac{100}{P}$$

Given a step size of 1, find the values of P at $t = 0, 1, 2, 3, 4, 5$ to one decimal place using Euler's method.

Answer Available

Exercise 10-8

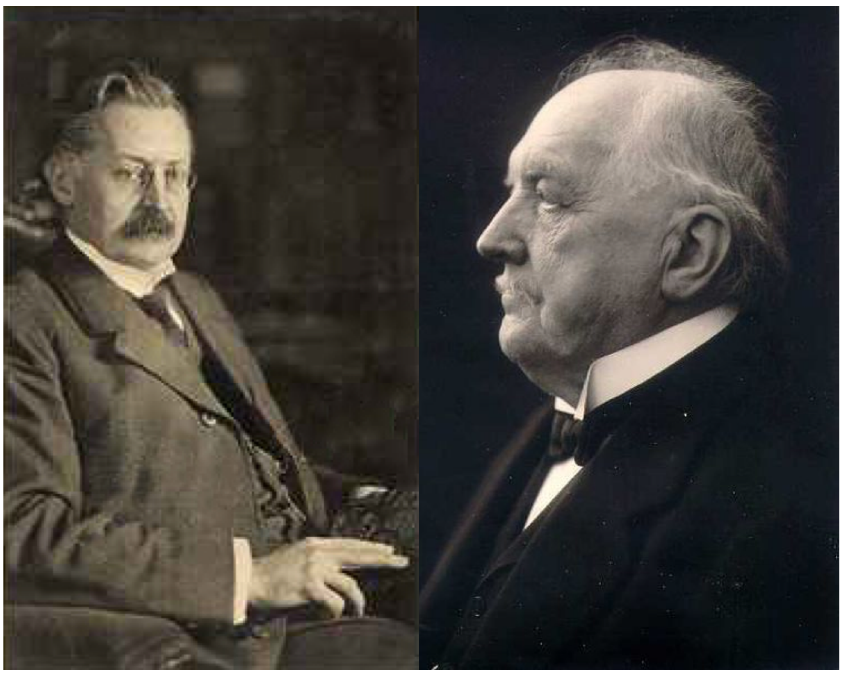
Take the differential equation:

$$P(0) = 20$$
$$\frac{dP}{dt} = P^2 - P$$

Given a step size of 1, find the values of P at $t = 0, 1, 2, 3, 4, 5$ to one decimal place using Euler's method.

Answer Available

Runge-Kutta Methods



Carl Runge and Martin Kutta

Euler's method is not the only technique that can be used to numerically solve differential equations. Another popular set of techniques are called Runge-Kutta methods. Runge-Kutta methods are a family of numerical differential equation solvers. In fact Euler's method itself can be classified as a simple Runge-Kutta method.

One particular member of the Runge-Kutta family of methods that is widely used is a 4th-order Runge-Kutta method. This method differs from Euler's method in that for each step, it evaluates the model multiple times and averages the resulting derivatives. Briefly, the driving set of equations for this method is as follows:

$$\mathbf{y_{t+\Delta t}} = \mathbf{y_t} + \Delta t \frac{\mathbf{a} + 2 \times \mathbf{b} + 2 \times \mathbf{c} + \mathbf{d}}{6}$$

Where:

$$\begin{aligned}
\mathbf{a} &= \mathbf{y}'(\mathbf{y_t}, t) \\
\mathbf{b} &= \mathbf{y}'(\mathbf{y_t} + \frac{\Delta t}{2} \times \mathbf{a}, t + \frac{\Delta t}{2}) \\
\mathbf{c} &= \mathbf{y}'(\mathbf{y_t} + \frac{\Delta t}{2} \times \mathbf{b}, t + \frac{\Delta t}{2}) \\
\mathbf{d} &= \mathbf{y}'(\mathbf{y_t} + \Delta t \times \mathbf{c}, t + \Delta t)
\end{aligned}$$

What this algorithm does is first compute the derivatives of the system at the current time (**a**) and use them to move the system forward to $t + \Delta t/2$. The derivatives are evaluated at $t + \Delta t/2$ (**b**) and this new set of derivatives is used to again move the system from t to $t + \Delta t/2$. A third set of derivatives are evaluated again at this mid-point (**c**) and they are the used to move the system from t to $t + \Delta t$. A fourth set of derivatives are evaluated at this point (**d**). The system is then returned to its starting point and a weighted average of derivatives are used to move the system the full time step. This weighting puts most of the weight on the middle two derivatives instead of the derivatives from the end points.

This 4th-order Runge-Kutta method is generally much more accurate than Euler's method for a given step size. Using a step size of 10 for our earlier population model, the Runge-Kutta method generates a value of 270.8. A step size of 5 yields a results of 271.7, just a smidgeon away from the precise value of 271.8. Recall that for Euler's method, even with a step size of 0.1 we still were not as accurate as the Runge-Kutta method with a step size of 5. Now it is true that this 4th-Order Runge-Kutta method does a lot more work than Euler's method for each step. It evaluates the model for times and has to do some averaging of derivatives. However, it is still much more accurate than Euler's method for an equivalent level of computational effort.

Exercise 10-9

Take the differential equation:

$$\begin{aligned}
P(0) &= 20 \\
\frac{dP}{dt} &= \frac{100}{P}
\end{aligned}$$

Given a step size of 1, find the values of P at $t = 0, 1, 2, 3, 4, 5$ to one decimal place using the 4th-Order Runge-Kutta method.

Answer Available

Exercise 10-10

Take the differential equation:

$$P(0) = 20$$
$$\frac{dP}{dt} = P^2 - P$$

Given a step size of 1, find the values of P at $t = 0, 1, 2, 3, 4, 5$ to one decimal place using the 4th-Order Runge-Kutta method.

Answer Available

Exercise 10-11

Discuss the differences between the 4th-Order Runge Kutta solutions and the Euler solutions. What causes these differences? Which method is most accurate? Why?

Exercise 10-12

Describe a model where Euler's method would be best suited as a numerical solver. Describe a model where the 4th-Order Runge-Kutta method would be best suited.

Numerical Solution Algorithms

This model explores the selection of the simulation step size and differential equation solution algorithm.

1. Create a new **Stock** named [**Population**].
2. Change the **Initial Value** property of the primitive [**Population**] to `100`.
3. Create a new **Flow** going from empty space to the primitive [**Population**]. Name that flow [**Net Growth**].
4. Create a new **Variable** named [**Growth Rate**].

5. Change the **Equation** property of the primitive [**Growth Rate**] to `0.1`.

6. Create a new **Link** going from the primitive [**Growth Rate**] to the primitive [**Net Growth**].

7. Change the **Flow Rate** property of the primitive [**Net Growth**] to `[Growth Rate]*[Population]`.

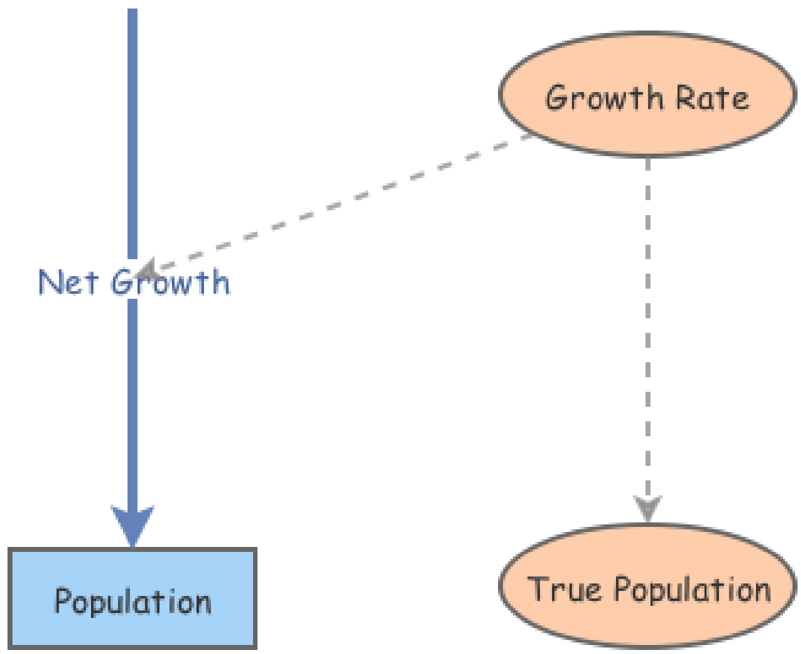
8. Create a new **Variable** named [**True Population**].

9. Create a new **Link** going from the primitive [**Growth Rate**] to the primitive [**True Population**].

10. Change the **Equation** property of the primitive [**True Population**] to `100*Exp([Growth Rate]*Years)`.

11. Change the **Simulation Length** property of the Time Settings to `10`.

12. The model diagram should now look something like this:



13. Let's now implement the simple exponential growth model we have discussed in this chapter. We have a population that starts with 100 people and increases at a rate of 10% per year. In addition to creating the stock and flow model, we have also created a variable, [**True Population**], that contains the analytical solution to the model.

14. First, we'll use Euler's method with a step size of 2 years and simulate the model.

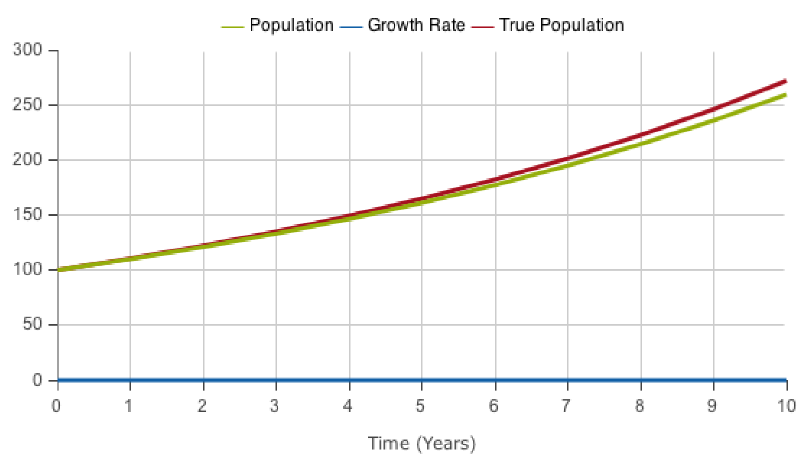
15. Change the **Simulation Time Step** property of the Time Settings to 2.

16. Run the model. Here are sample results:

17. As we can see these results aren't very accurate. The value of the numerical estimated [**Population**] is quite different from the analytically determined value in [**True Population**]. Let's reduce the step size to 1 year and try again.

18. Change the **Simulation Time Step** property of the Time Settings to 1.

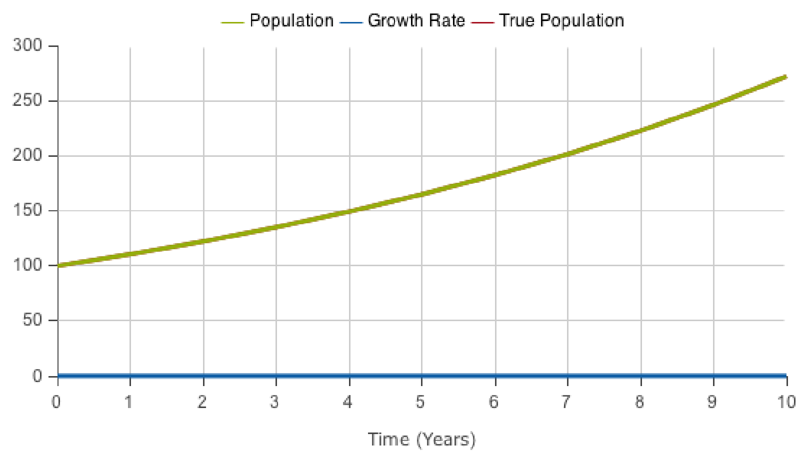
19. Run the model. Here are sample results:



20. This is better, but we're still off by a fair amount. We could experiment with continuing to reduce the step size, but let's instead switch now to the more accurate Runge-Kutta method. Will simulate the model again with a step size of 1 using the 4th-Order Runge-Kutta solution algorithm.

21. Change the **Analysis Algorithm** property of the Time Settings to `RK4`.

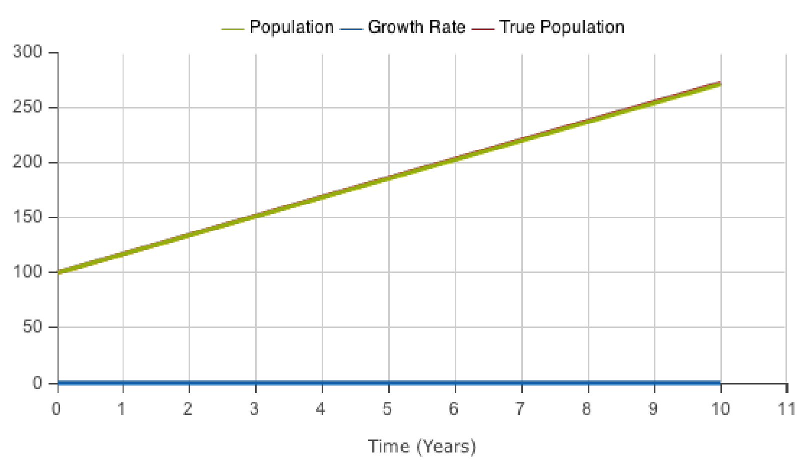
22. Run the model. Here are sample results:



23. That's a lot better! It's so close to being perfect that we can't even see the difference between the two lines in the figure. Just to be clear, let's see how quickly the results degrade when we increase the step size. Let's set the step size to 10 and simulate the model again.

24. Change the **Simulation Time Step** property of the Time Settings to `10`.

25. Run the model. Here are sample results:



26. That's still very good and much better than Euler's Method with a step size of 1. Why don't you go ahead now and experiment with different step sizes and the two solution methods to get a feel for their accuracies.

Other Solution Techniques

While being a brief introduction into numerical solution methods for differential equations, this should provide you with the background you need to intelligently make decisions about controlling the simulation of your models. It should help you identify potential sources of errors in your model and help you to adjust you simulation configuration to account for them.

The two methods we have looked at for solving differential equation models – Euler's method and a 4th-Order Runge-Kutta method – are widely used and they are what are built into Insight Maker. In addition to these two techniques, however, there are many other methods that are used in practice and you should be aware of this richer ecosystem of solution techniques.

Although we do not have space here to delve into the full ecosystem of numerical differential equation algorithms, it is useful to discuss one variant briefly: the adaptive step size algorithm. The methods we have looked at here use a fixed step size specified at the beginning of a simulation. Many models, however, might be characterized by highly variable trajectories. Part of the trajectory might be very smooth and unchanging while other parts might experience numerous rapid changes.

When using a fixed step size algorithm like the ones illustrated above, the step size must be set for the worse case scenario. The step size must be set to a small enough value to account for the rapidly changing areas. That said, the precision of this small step size is unnecessary on the smooth regions of the trajectory where the algorithm must do extra work for minimal gain in precision. Ideally, we would want to have a small step size for the rapidly changing areas and a large one for the smooth regions. This would result in the best of both worlds: high accuracy and quick computation.

Adaptive step size algorithms do just that. They adjust the step size dynamically based on the behavior of the model's derivatives. If the derivatives change rapidly, then the step size will be automatically shrunk; if the derivatives are constant or change very slowly the step size will automatically grow. Figure 2 illustrates the location of steps for an illustrative model using an adaptive step size algorithm. The steps are clustered around changes in the trajectory's derivatives in an attempt to maximize predictive accuracy while minimizing computation effort.

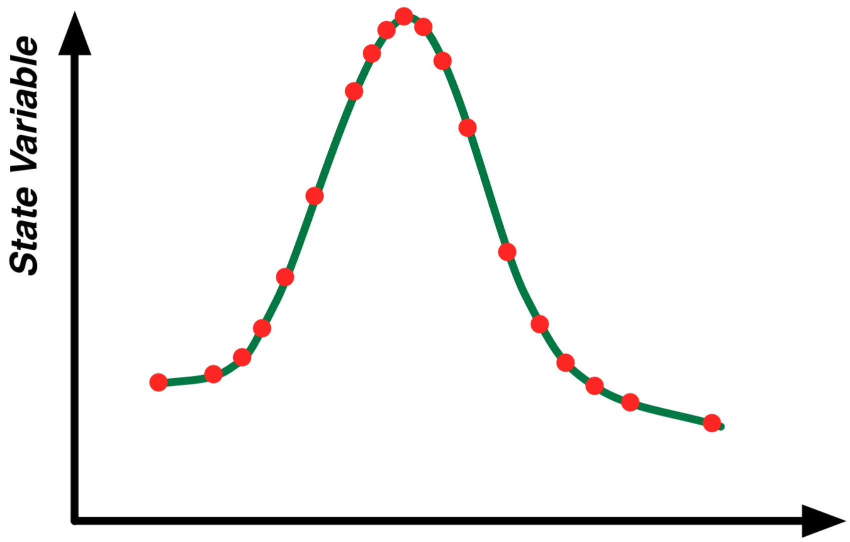


Figure 2. Illustration of an adaptive step size algorithm. Dots show the location of model evaluations. Evaluations are clustered around changes in the derivatives.

Chapter 11

Going Global

Engaging people to cause positive action and change is one of the key goals of systems thinking and modeling. The growth of the Internet has created amazing opportunities to reach out and connect with people in ways that have never before been possible.

The Internet makes it easy to share models with other people. Not only can you email a specific person the tables and graphs of a model's results, but you can also build webpages publishing these results to share with the world. What is more, these results do not have to be limited to static data. Using Insight Maker, you can include an interactive version of your model allowing others to experiment with it directly on your webpage. This can be done on any page you have rights to edit including your personal website, a blog, and a company's information page.

Furthermore, the information flow doesn't have to be one-way from you to others. In a webpage you can include a feedback or comment form that allows anyone to comment and share his or her thoughts on the model right next to the model itself. These comments can be saved directly on the page allowing other people to read them and enabling a discussion to form around the model. This creates many avenues for collaboration and learning that would simply be impossible without the Internet.

In this chapter, we will show you how to develop webpages to showcase your insights and models to the world. We'll also show how to include tools to engage viewers and start a dialogue about your models. Before jumping into the models themselves, we will lay the groundwork by introducing the basic principles of web development. Once we have introduced these key principles, we'll walk through two examples of developing interactive models.

The Web in a Nutshell

The World Wide Web is based on a collection of many different technologies that work together. When developing a webpage there are three major technologies that you need to be familiar with: HTML, CSS and JavaScript. Each of these technologies or languages plays a different role in the development of a webpage.

Technology	Commonly Called	Usage
Hypertext Markup Language	HTML	Webpage Structure
Cascading Style Sheets	CSS	Webpage Style
ECMAScript	JavaScript	Webpage Interactivity

The web is interesting in that each of these technologies is based on old-fashioned, simple text files. You write HTML text files, you write CSS files, and you write JavaScript files[1]. You do not need any fancy tools to create these files. Any simple text editor will do. A web browser takes the simple instructions and code in these files and converts them to the rich interactive webpages you see when you browse the Internet.

When teaching web development many books and sources will recommend that you use some kind of interactive web site builder (like Adobe Dreamweaver http://www.adobe.com/products/dreamweaver.html). That is certainly a great way to get up and running, but ultimately you will find the approach very limiting. To truly harness the different tools offered to you by the Internet, you will need to have some understanding of the underlying technologies and be able to work with them directly. So, rather than using a website builder as a crutch, we recommend jumping right into learning HTML, CSS and JavaScript.

In the following sections we'll give you a brief introductions to each of these fundamental web technologies. This introduction will be rapid so please do not worry if you do not fully understand everything. But the please do your best to engage with this material as it will provide you with everything you need to know in order to be able to get the maximum out of our later examples of interactive modeling webpages.

HTML Basics

HTML defines the structure of a webpage or document. An HTML document is made up out of a set of *tags*. Each tag is enclosed in triangular brackets. For instance, there is a tag called "<hr>" that will create a horizontal division line in your document ("hr" is an abbreviation of "horizontal rule").

Many types of tags will consist of an opening and closing tag paired together. A closing tag is written the same as an opening tag except there is a also backslash

[1] Please note that when you write CSS and JavaScript, your text is case-sensitive. This means that "ABC", "abc", and "Abc" will all be understood differently. HTML, on the other hand, is case-insensitive. In HTML, "ABC", "abc", and "Abc" will all be understood to mean the same thing.

immediately after the first triangular bracket. For instance, you could use a pair of “…” tags to make some text bold:

```
This is some text. <b>This text is bold.</b> This text is not bold.
```

Some tags may also have “attributes” which modify the behavior of the tag. Attributes are included within the opening brackets of the tag after the tag name. For instance, the “<a>” tag is used to make links between webpages. The “” tag has an attribute “href”[2] which is the URL the link should connect to. The following HTML creates a link to Google:

```
If you ever need to search something, just go
to <a href="http://Google.com">Google.</a>.
```

Every HTML page contains some general boilerplate that structures the document. This boilerplate will look almost identical from webpage to webpage and it contains several unique tags which split the document into two sections: a “head” section to store the page title and page keywords for search engines, and a “body” section which contains the page content that is what is actually shown to the user. You will spend most of your time editing the body section. The standard template for a webpage is as follows:

```
<html>
<head>
    <title>A Sample webpage</title>
</head>
<body>
    Document contents goes here...
</body>
</html>
```

There are dozens of different tags you can use in your document to structure it. We can’t cover them all here, but the following table summarizes a few of the most useful ones:

Tag	Usage	Example

[2] The tag name “a” comes from “anchor” and “href” is an abbreviation of “hyperlink reference”. Many of the conventions with web development may seem strange and so you should understand the long history of these technologies and the resulting historical baggage that comes with them.

a	Creates a link	Google.
b	Makes text bold	This text is bold.
i	Makes text italic	This text is <i>italic</i>.
u	Makes text underlined	This text is <u>underlined</u>.
center	Centers a paragraph	<center>In the middle.</center>
p	Creates a paragraph of text	This is a paragraph.
hr	Creates a dividing line	Something <hr> Something Else
h1	Creates a heading	This is a Heading
img	Embeds an image	<img src="http://example.com/image.png">

We can combine these tags together to form more complex documents. The following is an example of a full-featured webpage.

```
<html>
<head>
    <title>A Sample webpage</title>
</head>
<body>
    <h1>Introduction</h1>
        <p>Here is some information about my page.</p>
    <h1>The Content</h1>
        <p>Here we have the meat of the page.</p>
    <hr>
    <h2>For Further Information</h2>
        <p>Here we have links to other sites about this content:<p>
        <p>We could check out <a href="http://BeyondConnectingTheDots.com">
            this book's site</a> for instance.</p>
</body>
</html>
```

Open whatever word processor you use on your computer and save this to *MyPage.html* as a plain text file[3]. You can then open this file in your web

[3]Webpages are always stored as plain text. This differs from, for instance a Microsoft Word document (".doc" or ".docx" extension) or a Rich Text Format document (.rtf extension). You need to ensure you save your document as a plain text document with the extension ".html" or ".htm". You can use any text editor you want, but if you get an editor designed for writing webpages it will have helpful features such as coloring your tags differently from the standard text as you edit the webpage. We recommend Sublime Text (http://www.sublimetext.com/) as a high quality editor for serious work.

browser (Internet Explore, Firefox, Chrome, Safari, etc.). Experiment by adding some more paragraphs and formatting to see how the document changes.

For more information and tutorials on HTML, we recommend the Mozilla Developer Network's guides (https://developer.mozilla.org/en-US/docs/Web/HTML).

Exercise 11-1

Replicate the following formatting in an HTML document:

This **text is *italic* and bold.**

Answer Available

Exercise 11-2

Research HTML on-line, learn how to make a list of items. Create both an ordered and unordered list of the top three countries you wish to visit.

Answer Available

Exercise 11-3

Create an HTML document containing your resume. Use heading tags to separate sections. Include a picture of yourself in the document.

CSS Basics

Where HTML is used to define the structure of a document, CSS is responsible for styling this structure. This styling includes aspects like font and color choices in addition to the general layout. A CSS document is a list of rules where each rule has two parts: a selector that tells the browser what elements of the page the rule applies to, and a set of styles that tells browser how to style those elements. For example, take the following CSS code.

```
p {
    margin: 20px;
}

h1, h2 {
```

```
    font-size: 72px;
    color: red;
}
```

This code has two rules. In the first rule the selector is "p" meaning the rule will apply to all "<p>" tags in the document. The styling for this rule says to apply a 20-pixel margin around each of these paragraph tags. The second rule has the selector "h1, h2". This means apply the rule to both "<h1>" and "<h2>" tags and to set the contents of those tags to have an extra large font and to be colored red.

There are numerous different aspects of an element's style you can set with CSS. For a full and detailed reference we recommend the Mozilla Developer Network's coverage of CSS (https://developer.mozilla.org/en-US/docs/Web/CSS/Reference).

CSS for a webpage can be placed in a standalone file which is referenced by the webpage or it can be included directly within the webpage. Both these can be accomplished by placing a CSS rules within a special tag in the *head* section of the document. For example, taking the *head* section from our earlier document, we could either embed the CSS directly:

```
<head>
    <title>A Sample webpage</title>
    <style>
        p {
            margin: 20px;
        }
        h1, h2 {
            font-size: 72px;
            color: red;
        }
    </style>
</head>
```

Alternatively we could save the CSS to an external text file (such as *MyStyles.css*) and link to it in the *head* of our document:

```
<head>
    <title>A Sample webpage</title>
    <link rel="stylesheet" type="text/css" href="MyStyles.css">
</head>
```

Exercise 11-4

Create a CSS rule to make <u> tags set their text color to green in addition to adding underlining.

Answer Available

Exercise 11-5

Read up about CSS online and create a tag that creates a red box round every link on the web page.

Answer Available

JavaScript Basics

JavaScript[4] provides interactivity for webpages. JavaScript is a powerful programming language that you can use to respond to user actions, run calculations, or modify a webpage. An example of using JavaScript code to calculate a Fibonacci number[5] is below:

```
function fib(n){
    if(n==1 || n==0){
        return 1;
    }
    return fib(n-1) + fib(n-2);
}

alert("The tenth Fibonacci number is: "+fib(10));
```

Like CSS, there are two ways to embed JavaScript into an HTML document. The first is to include the JavaScript directly in the document like we did for the CSS:

[4] The name "JavaScript" is a source of perpetual confusion. What we know colloquially as JavaScript is officially called ECMAScript. Due to trademark issues Microsoft refers to it as JScript when you are using Internet Explorer. It is important to note that *JavaScript* and *Java* are different technologies. They share part of a name due to historic branding purposes but they are completely different languages.

[5] Where the first two Fibonacci numbers are 1 and the Fibonacci numbers thereafter are the sum of the two preceding numbers. The Fibonacci sequence begins: 1, 1, 2, 3, 5, 8, 13, 21, 44...

```
<head>
    <title>A Sample webpage</title>
    <script>
        function fib(n){
            if(n==1 || n==0){
                return 1;
            }
            return fib(n-1) + fib(n-2);
        }

        alert("The tenth Fibonacci number is: "+fib(10));
    </script>
</head>
```

The second method to include the code is to save the JavaScript into a text file (such as *MyScript.js*) and link to it in the document:

```
<head>
    <title>A Sample webpage</title>
    <script src="MyScript.js"></script>
</head>
```

JavaScript is a very powerful tool but also a very complex one. This chapter will illustrate usages of JavaScript but we cannot hope to teach you how to write new JavaScript yourself in this single chapter. Again, we refer you to the Mozilla Developer Network to learn more about JavaScript (https://developer.mozilla.org/en-US/docs/Web/JavaScript).

Exercise 11-6

Learn about JavaScript online. Create a script that prompts the user for two numbers and then adds them.

Answer Available

Creating a Webpage for Engagement

Now that we have made it through some of the technical details, let's jump into building a webpage for an interactive model that users can comment on. There are three basic things we want this webpage to have:

1. A description of the challenge we are tackling, why we built the model, and what the model contains.

2. An interactive version of the model that the user can explore and run simulations with.
3. A discussion forum about the model where users can post comments and see what others have posted.

This might seem ambitious, and it is! But using freely available technologies and services we will be able to put this webpage together very quickly. Let us split the process of developing the webpage into three steps: first we'll create the general page framework, then we will add the interactive model, and lastly we will add the discussion forum.

Creating the Page and Description

Assume we decide we want to create a webpage exploring population growth and whether the Earth can sustain humanity into the future. We start building our webpage by creating an HTML file and putting the following text in it.

```
<html>
<head>
    <title>A Fragile Future</title>
</head>
<body>
    <h1>Introduction</h1>
        <p>This is a model of world population
            changes into the future.</p>

    <h1>The Model</h1>
        [Model goes here]

    <h1>Discussion</h1>
        [Discussion forum goes here]
</body>
</html>
```

This creates a page with three sections: Introduction, The Model, and Discussion. We can fill in the Introduction section with text describing the problem we face and our approach to understanding it in our model. In this example page, we have just written a single sentence but you could extend it with more details on the model to fully explain to the viewer why this is important and how we have modeled it.

The placeholders [Model goes here] and [Discussion forum goes here] are where we will insert our model and discussion forum later on. For now though, we just want to layout the structure of the page.

Adding an Interactive Model

Now that we have created the structure for our webpage, we can add the interactive model. There are several ways to do this. One way would be to write the model in JavaScript and include it directly in the webpage. JavaScript is a full-featured programming language and could be used to implement any of the models described in this book. Although implementing a model in JavaScript is definitely possible, it would require a lot of work. Writing a model in JavaScript would take a large amount of time and would not be possible without extensive programming experience.

Fortunately, using Insight Maker there is a much easier approach. Insight Maker models can be easily embedded in a webpage without any special effort on your part. So rather than writing our world population model in JavaScript, we can simply build the model in Insight Maker and then embed the resulting model in our webpage. So build your model in Insight Maker just as you would build one normally. You can also take an existing model that you have already built and use that one. For this example, we will use the World3 model (http://InsightMaker.com/insight/1954) which has a detailed worldwide model of population change[6].

Once you have finished constructing your model, click the **Embed** button in the **Tools** section of the Insight Maker toolbar. A window will open containing HTML code that you should paste into your webpage. This code will embed a version of the insight when it is placed in a webpage document. For the World3 model this code is something like:

```
<IFRAME SRC="http://InsightMaker.com/insight/1954/embed?topBar=1&sideBar=1&zoom=1"
TITLE="Embedded Insight" width=600 height=420></IFRAME>
```

Take this code and use it to replace the [Model goes here] placeholder in your webpage. Save the webpage and open it in a browser and you will now have a rich interactive version of your model embedded directly in your webpage!

There are several features of the embedding that you can control by editing the "<IFRAME>" tag. For instance the "width" and "height" attributes control the size of the embedded model. They are specified in pixels and you may change them to make the embedded model smaller or larger. The "topBar" and "sideBar" parts of the URL control whether the toolbar and the sidebar will be shown in the embedded model's interface. By default, they are set to 1 indicating these elements will be shown. Set them to 0 to hide the bars when the model is displayed. The "zoom" part determines whether the model diagram is shown at its full size or if it is zoomed to fit the window (the default). Set this to 0 to prevent the model diagram from automatically being resized to fit the window.

[6] This model was described and discussed in detail in the book *The Limits to Growth*

Adding a Discussion Section

Now we have one last piece to add before we have completed our webpage. We want people to be able to carry on a discussion about the model directly within the page. To make this possible, we need to add some sort of forum or discussion software.

We could program our own custom discussion system; but, like the case of the model itself, this is a place where it is easier to leverage existing free software than it is to develop our own. A number of free commenting and discussion systems are available. One of these is called Disqus (http://disqus.com). If you read a number of different news sites or blogs you have probably already used Disqus as many sites utilize their software.

You will need to sign up for a Disqus account to be able to embed their discussion software, but fortunately like Insight Maker it should not cost you a thing. Once you have completed signing up at http://disqus.com, follow the site for directions on how to embed Disqus in your own webpage. You should be given code that looks similar to the following to place into your webpage:

```
<div id="disqus_thread">Discussion Here</div>
<script type="text/javascript">
    var disqus_shortname = 'SHORT-NAME-DEMO'; // required: replace example with your forum short
    (function() {
        var dsq = document.createElement('script'); dsq.type = 'text/javascript'; dsq.async = tr
        dsq.src = '//' + disqus_shortname + '.disqus.com/embed.js';
        (document.getElementsByTagName('head')[0] || document.getElementsByTagName('body')[0]).a
    })();
</script>
```

First edit this code as instructed (e.g. replace any usernames or ids with the ones you have been provided by Disqus) and then replace the [Discussion forum goes here] placeholder in your page with this code. Load the page and test to see if it is working. One issue with Disqus is it might not work if the webpage is being opened from a file on your computer. You may need to upload it to the domain name you entered when you signed up for Disqus to ensure it works correctly.

Completed Page

We have just put together a powerful site very quickly. Our site lets us share an interactive model with people anywhere in the world and allows them to comment directly on the model. All that it took to do this was the following completed code:

```
<html>
<head>
```

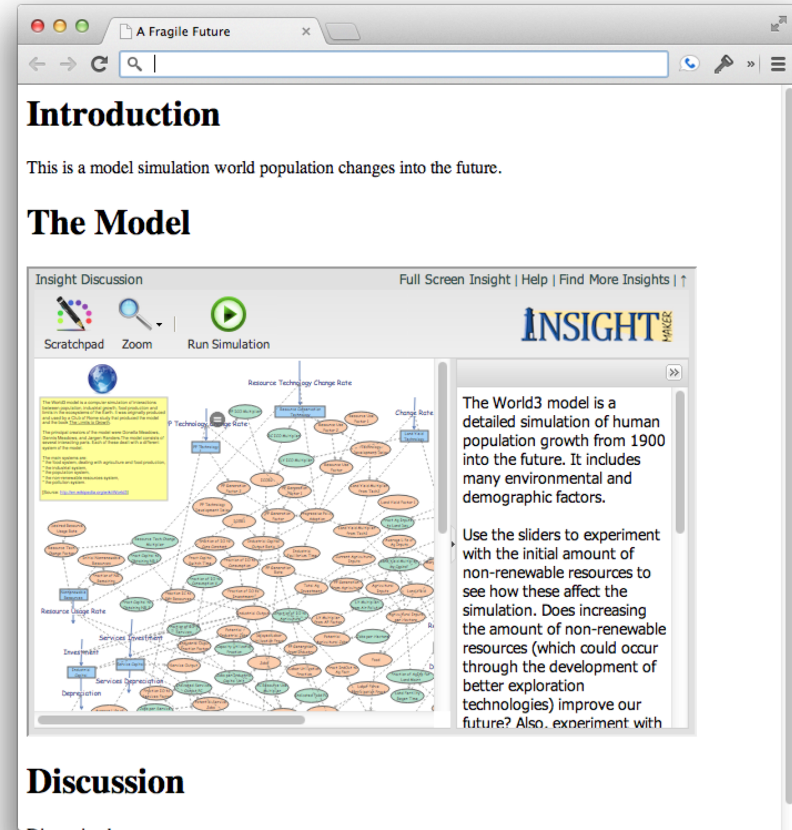


Figure 1. Completed page with embedded model.

```
    <title>A Fragile Future</title>
</head>
<body>
    <h1>Introduction</h1>
        <p>This is a model of world population
            changes into the future.</p>

    <h1>The Model</h1>
        <IFRAME SRC="http://InsightMaker.com/insight/1954/embed?topBar=1&sideBar=1&zoo
        TITLE="Embedded Insight" width=640 height=480></IFRAME>

    <h1>Discussion</h1>
```

```
        <div id="disqus_thread">Discussion here</div>
        <script type="text/javascript">
            var disqus_shortname = ''; // required: replace with your forum shortname
            (function() {
                var dsq = document.createElement('script'); dsq.type = 'text/javascript'; dsq.as
                dsq.src = '//' + disqus_shortname + '.disqus.com/embed.js';
                (document.getElementsByTagName('head')[0] || document.getElementsByTagName('body
            })();
        </script>
</body>
</html>
```

A working version of this site may be viewed at http://BeyondConnectingTheDots.com/book/embedded-model/. There is a lot more we could do with the site. Spend some time now experimenting with it. Add some more descriptive text, maybe add some images, and try to use CSS to adjust the styling.

Exercise 11-7

Use CSS to make a rule to automatically underline the headings in this web page.

Answer Available

Exercise 11-8

Use CSS to add a background color to this page.

Answer Available

Exercise 11-9

Go through these same steps with a model of your choosing. Make your own custom interactive webpage.

Flight Simulators and Serious Games

In the preceding section we described how to rapidly develop a website that contains an interactive model and provides users the ability to comment and discuss the model directly on the page. By leveraging Insight Maker, we were able to get an interactive version of our model embedded in our webpage just by copying a few lines of code. By leveraging Disqus, we were able to include a discussion forum with a similar amount of effort.

In many cases, what we created may be exactly what you are looking for. However, in other cases it is possible that you will wish to provide your users with a unique experience tailored to understanding a specific problem. For instance maybe you would like to develop what is known as a "flight simulator", a simulation tool that puts the user in the position of trying to manage a problem or achieve an outcome. For example, if you had a model of a business going through a disruptive change, you could place the user in the position of the company's leader and with instructions to adjust parameters in the model in order to safely shepherd the company through this challenge.

Similarly, "serious games" are tools designed to both engage and educate about a system. You can create a simulation model at the heart of a serious game or a flight simulator. You may give users direct access to this simulation model's interface, but generally you will want to build a custom interface on top of the model that hides the stock and flow diagram and instead displays a control panel type interface to the user.

Fortunately, web technologies provide a rich environment for developing these flight simulators and serious games. Furthermore, using Insight Maker you can build your model and simulation engine using its model building tools and then build a custom interface on top of the model to provide the exact experience you want to the user. In the following sections we will develop a custom interface to control our world population simulation.

Setting up the Page

We'll start by stripping down our page from the previous example. Let's remove the commenting system and the introduction so the page just contains the model (you can add these other items back later on your own as an exercise). After we do this, we will be left with a page just containing the embedded world simulation model.

In this case, however, we do not want the user to actually interact with or even see the embedded model. We will be adding our own custom interface and just using the embedded model to run simulation in the background. To hide the embedded model we can add a CSS rule that makes the <iframe> tag invisible:

```css
iframe {
    display: none;
}
```

This rule turns off the display of all <iframe> tags in the page. They are still there and in the page, but they are not shown to the user. The resulting completed template for our page is printed below. When you open this in your browser you should see a completely blank webpage.

```
<html>
<head>
    <title>A Fragile Future</title>
    <style>
        iframe {
            display: none;
        }
    </style>
</head>
<body>
    <IFRAME SRC="http://InsightMaker.com/insight/1954/embed?topBar=1&sideBar=1&zoom=1"
    TITLE="Embedded Insight" width=600 height=420></IFRAME>
</body>
</html>
```

Creating the Control Panel

HTML has a tag called “<input>” that lets you create form elements for users to input data. The <input> tag has an attribute called “type” that determines what the type of the input element will be. There are a wide number of types including “number”, “text”, “color”, “textarea”, “date”, and “button”. For our control panel, we’ll design it to modify two parameters of the model and to have a button users can press to run the simulation. In addition to specifying the type of the inputs, we should also specify their initial values in the control panel. We can do that using the “value” attribute of the <input> tag.

Finally, we will need some method to reference the inputs and to load their values later on. Each tag in an HTML document has an optional “id” attribute. This attribute can be used to obtain a reference to that element from JavaScript. We’ll set the id attribute for our two input fields so we can obtain their values when we are ready to run the simulation.

The resulting control panel will look something like the following code. As you can see we have presented the user with a simple task, to find a combination of settings that results in over 5 billion people in the year 2100 (which is in fact a significant decrease from the current population size so it should not be too hard). You should place this code after the <iframe> tag in your document.

```
<center>
    <p>This is a game to keep the world's population larger than 5 billion in the year 2100.
        We can experiment with the amount of non-renewable resources in the world and the
        start year for a clean energy eco-friendly policy.</p>
    <p> Initial Non-Renewable Resources: <input type="number" value="100" id="resources" /> % </
```

```
    <p> Start Policy Year: <input type="number" value="2013" id="year" /> </p>
    <p> <input type="button" value="Test Scenario" /> </p>
</center>
```

This will create two input fields allowing users to input numeric values. The first, *Initial Non-Renewable Resources* will allow the user to increase or decrease the amount of non-renewable resources assumed in the model at the start of the simulation. The second, *Start Policy Year* allows the user to specify the start date to implement a clean technology policy which will reduce the amount of pollutants being generated in the simulation. A button is also created that lets the user test the scenario in the simulation.

Making it Interactive

We use JavaScript to add interactivity to the webpage. Let's define a JavaScript function *testScenario* that we will use to first read in the user specified options from the control panel, then run the simulation with these parameter values, and finally report to the user whether or not they were successful in keeping the population size above 5 billion.

We will fill out the *testScenario* function with steps later; but for now, just add the following code to the head section of your webpage.

```
<script>
    function testScenario(){
        alert("Scenario tested!");
    }
</script>
```

This creates the function, but we also need a way for the function to be executed when the "Test Scenario" button is pressed. There are several ways to do this. The easiest is to set the "onClick" attribute of the button to call the function. The "onClick" attribute of an input may contain JavaScript code that is executed when the button is clicked. To link up our button with the *testScenario* function, we change our input button in the HTML to:

```
<p> <input type="button" value="Test Scenario" onclick="testScenario()" /> </p>
```

Implement the webpage up to this point and check to make sure that you see a message pop up saying "Scenario tested!" when you press the "Test Scenario" button.

Now that we have implemented basic interactivity, let's flesh out the *testScenario* function.

Load Parameter Values from the Control Panel

To access an input field from JavaScript we use the *document.getElementById* function. This function is built into your browser and allows you to obtain a reference to one of the input elements based on its “id” attribute. Once we have a reference to the input element we can use the element’s “value” property to obtain the number the user has entered into the input field.

The following code defines two variables in JavaScript with the same values as the ones the user has entered. Enter this code at the top of your *testScenario* function.

```
var resources = document.getElementById("resources").value;
var year = document.getElementById("year").value;
```

Inject the Parameter Values into the Model

Insight Maker has an extensive JavaScript API[7] that can be used to modify and script models. This is the same API that may be used with Button primitives. Refer to the API reference at http://insightmaker.com/sites/default/files/API/files/API-js.html for full details about the API.

The API instructions provide examples about how to integrate and modify an embedded model. We will adapt those instructions to our own case. First, as the instructions indicate we need to update our <iframe> tag to add an “id” attribute. We adjust our <iframe> tag like so:

```
<IFRAME id="model" SRC="http://InsightMaker.com/insight/1954/embed?topBar=1&sideBar=1&zoom=1"
TITLE="Embedded Insight" width=600 height=420></IFRAME>
```

Now we can obtain a reference to the model using the *document.getElementById* function from before and then we can send API commands to it using its *postMessage* function. Within Insight Maker, we use the *findName* API command to get a reference to a specific primitive and then use the *setValue* API command to set the value of that primitive to the value of the parameter in the control panel. Add the following code to the *testScenario* function.

```
var model = document.getElementById("model").contentWindow;

model.postMessage("setValue(findName('Initial Nonrenewable Resources'), '"+(resources/100)*100000
model.postMessage("setValue(findName('Progressive Policy Adoption'), '"+year+"')", "*");
```

[7]An API, or Application Programming Interface, is a set of commands and functions that can be used to interface programmatically with an application.

This convoluted *postMessage* mechanism to pass JavaScript commands to the embedded model is a constraint necessitated by your browser's security mechanisms. It makes the processing of interacting with embedded models more complex than we would like, but fortunately it is still possible to do everything that we need to do even using it.

Run Simulation and Access Results

To run the model, we use the *runModel* Insight Maker API command. We indicate that the simulation should be run in "silent" mode so the results are returned[8]. We then use the *lastValue* function to obtain the final population size for the simulation in the year 2100. Copy this into your webpage at the end of the *testScenario* function:

```
model.postMessage("runModel({silent: true}).lastValue(findName('Population'))", "*");
```

So far we have just demonstrated one-way communication between the control panel and the embedded model. This is the first point in time when we need to be able to communicate the other way: to receive data back from the embedded model.

Unfortunately, due to the security constraints imposed by your browser, this is slightly complex. In order to receive a message back from the embedded model, we need to register an event handler with your main browser window. Don't worry if you don't fully understand this, just copy the code below into the script tag of your window.

```
function scenarioComplete(event)
{
    if(event.data){
        var pop = Math.round(event.data);
        if(pop > 5000000000){
            alert("You won! The population size of "+pop+" is larger than 5 Billion!")
        }else{
            alert("You failed! The population size of "+pop+" is smaller than 5 Billio
            alert("Please try again.");
        }
    }
}

window.addEventListener("message", scenarioComplete, false);
```

[8]There are two primary ways of running Insight Maker models using the *runModel* API command. One is the regular way where a results diagram will be shown but the results will not automatically be returned in JavaScript. The second way is in silent mode where the results are returned, but results graphs are not shown in the model interface.

Final Result

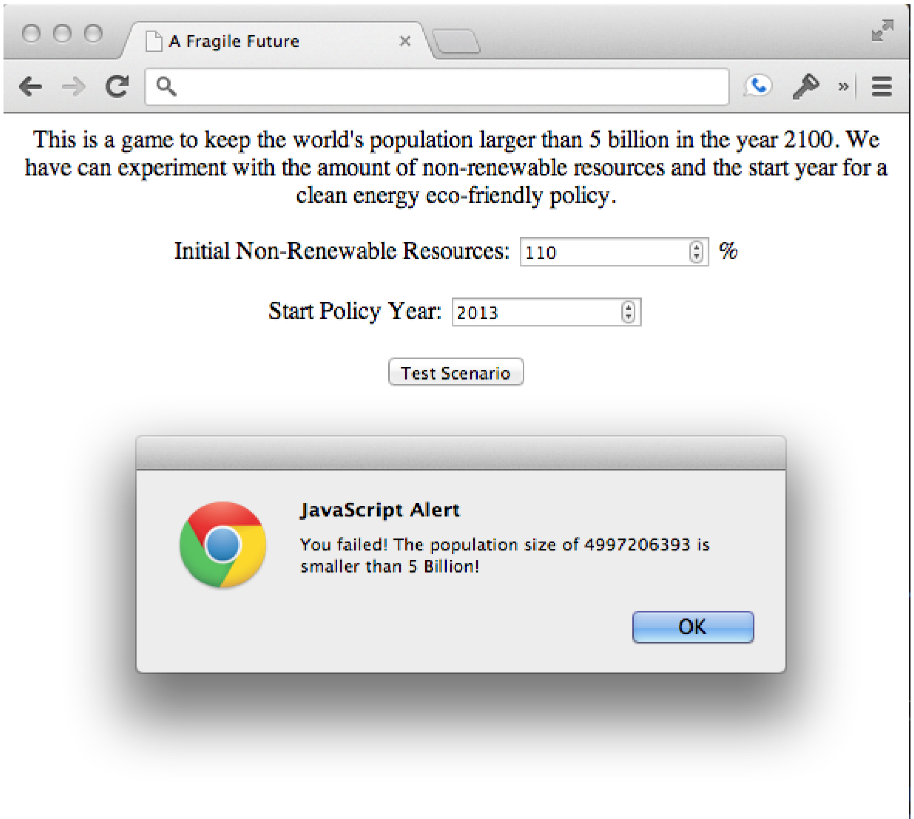


Figure 2. Completed control panel.

The code for the completed webpage is provided below and a working version of the page may viewed at http://BeyondConnectingTheDots.com/book/control-panel/.

```
<html>
<head>
    <title>A Fragile Future</title>
    <style>
        iframe {
            display: none;
        }
    </style>
    <script>
    function testScenario(){
        var resources = document.getElementById("resources").value;
```

```
        var year = document.getElementById("year").value;

        var model = document.getElementById("model").contentWindow;

        model.postMessage("setValue(findName('Initial Nonrenewable Resources'), '"+(re
        model.postMessage("setValue(findName('Progressive Policy Adoption'), '"+year+"

        model.postMessage("runModel({silent: true}).lastValue(findName('Population'))"
    }

    function scenarioComplete(event)
    {
        if(event.data){
            var pop = Math.round(event.data);
            if(pop > 5000000000){
                alert("You won! The population size of "+pop+" is larger than 5 Billio
            }else{
                alert("You failed! The population size of "+pop+" is smaller than 5 Bi
                alert("Please try again.");
            }
        }
    }

    window.addEventListener("message", scenarioComplete, false);
    </script>
</head>
<body>
    <IFRAME id="model" SRC="http://InsightMaker.com/insight/1954?embed=1&topBar=1&side
    TITLE="Embedded Insight" width=600 height=420></IFRAME>

    <center>
        <p>This is a game to keep the world's population larger than 5 billion in the
            We can experiment with the amount of non-renewable resources in the world
            start year for a clean energy eco-friendly policy.</p>
        <p> Initial Non-Renewable Resources: <input type="number" value="100" id="reso
        <p> Start Policy Year: <input type="number" value="2013" id="year" /> </p>
        <p> <input type="button" value="Test Scenario" onclick="testScenario()" /> </p
    </center>

</body>
</html>
```

The key goal of this chapter is not that you completely understand this, but rather that you will be able to adapt it to your own needs. There are a lot of additional changes that could be made to this demonstration. You could clean up the control panel and make it look more attractive by adding some CSS rules. You could add additional inputs to control other parts of the model. You could show the user the trajectory of the population instead of just the final value. Go ahead and experiment with this example to see what you can make it do.

Exercise 11-10

Use CSS to change the style of the inputs. Make inputs have a yellow background and blue text.

Answer Available

Exercise 11-11

Adjust the result message when the users have failed to reach the target population size. Tell them how far away from the target size they are.

Answer Available

Exercise 11-12

Add another input to allow users to adjust the initial amount of potentially arable land in the model.

Additional Tips

Web development is a very complex topic with a lot of nuances. The preceding sections should have given you a brief introduction in how to create interactive models for engaging an audience and encouraging discussion and learning. Although we cannot give you a comprehensive course in web development, there a few additional web development tips that will be very useful when you start to develop your own webpages.

Frameworks and Toolkits

Making an attractive web application is hard. Admittedly, the control panel application we developed does not look very good. We could spend some time improving its appearance by adding additional CSS rules but since we are not professional designers it is quite possible that the results of our efforts would only look amateurish and unattractive. Additionally, writing JavaScript to interact with webpages is also hard. These web technologies were developed over decades and many of the functions and techniques that need to be used are slightly archaic and are difficult to learn.

Fortunately, a number of toolkits and frameworks have been developed that make it easier to develop powerful and attractive web pages and control panels. Below we highlight some important toolkits that you might want to explore and considering adopting for your own usage. These toolkits can be embedded within your webpage extending its functionality. They will help you make more attractive and powerful applications quicker. The ones listed are all also available under open source licenses allowing you to use them for free.

Twitter Bootstrap (http://GetBootstrap.com) : Bootstrap is a framework for developing attractive webpages. It has many tools and rules that can be combined together to create visualizing pleasing webpages with minimal effort. If you don't have a good sense of design, Twitter Bootstrap could be a great help to you.

JQuery (http://jquery.com/) : JQuery is a library designed to improve the JavaScript functions needed to interact with a webpage. It generally greatly simplifies and reduces the number of keystrokes you need to carry out some task. For instance "document.getElementById('item')" becomes in jQuery simply "$('#item')".

JQuery UI (http://jqueryui.com/) : A spin-off from the JQuery project, this toolkit provides control panel elements that are themable and more extensive than the built-in <input> tags. Grids, sliders, and more are all available from this project.

ExtJS (http://www.sencha.com/products/extjs) : ExtJS is a comprehensive library for developing powerful applications. It has extensive tools to develop interface and control panels. It is also what is used to develop Insight Maker's interface.

Exercise 11-13

Install the Twitter Bootstrap toolkit and use it to redesign the control panel web page to make it more attractive.

Debugging Webpages

As you develop your webpage, it is almost certain that you will make many mistakes and typos as you go along. If you make a mistake within the HTML or CSS of the page you will have an immediate visual indication that something is wrong and you can experiment with your code until it is fixed.

JavaScript errors, on the other hand, generally won't provide any visual feedback that an error has occurred. The most likely indication that a JavaScript error has occurred is that nothing happens when you click a button or expect an action to occur. Debugging issues like this can be quite difficult. Fortunately, with just a little bit of additional work you can get access to very rich and informative JavaScript error messages letting you know exactly what went wrong and when.

There are two approaches to obtain access to the JavaScript error messages. The first is to actually edit your webpage and add code to show an error message when an error occurs. Adding the following to a <script> tag in the head section of your document will do that:

```
window.onerror = function(message, url, line) {
  alert("JavaScript Error: \n\n" + message + " (line: " + line + ", url: " + url + ")");
}
```

Now when an error occurs, an alert will pop up with a brief description of the error and information about where it occurred in your code.

The second approach you can take is to use the developer tools that are built into your web browser to study the webpage and observe errors as they occur. Excellent developer tools are built into all modern browsers. These tools let you study the structure of the webpage, profile the performance of your code, and examine how the webpage behaves.

One particular tool is very useful when developing webpages: the JavaScript console. Once you have opened the JavaScript console (search on-line for the exact directions on how to do this for your specific web browser) errors and messages from the webpage will appear in the console as they occur. What's more, the console allows you to evaluate JavaScript commands in the webpage simply by typing the commands into the console.

One approach to debugging code is to put *alert* functions into the code updating you on the progression of the code or displaying values of the JavaScript variables. This works, but can be very clumsy and disruptive. When you have the console open, a better approach is available to you. You can send messages directly to the console providing information on the status of the program. For example:

```
console.log("The value of the variable is: " + myVariable);
console.error("An error has occurred!");
```

Exercise 11-14

Open up a complex web page such as http://nytimes.com. Then use you web browser develope tools to explore how the webpage is structured and designed.

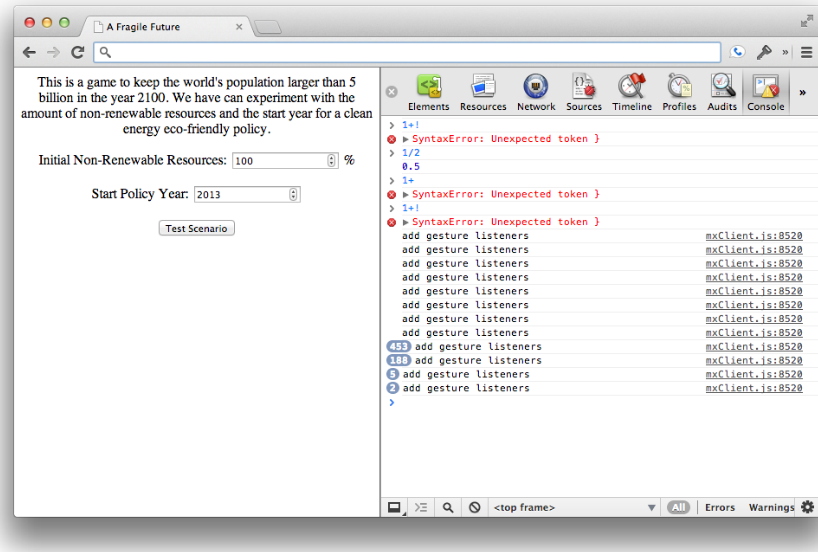


Figure 3. Google Chrome's JavaScript console.

Sending Complex Data Back and Forth

The *postMessage* communication technique to send data back and forth to the embedded model can only be relied upon to send strings and objects that can easily be converted to strings (like numbers)[9]. Oftentimes you will want to pass more complex data from the simulation to the containing window. For instance you might want to pass the whole time series of values one or more primitives took on over the course of the simulation.

To handle these more complex objects you must convert them to strings. JavaScript provides a number of techniques to do so. For instance, if you have an array, you can convert it back and forth from a string using the *join* and *split* functions:

```
var data = [1, 4, 9, 16, 25];
var str = data.join("; "); // "1; 4; 9; 16; 25"
str.split(", "); // [1, 4, 9, 16, 25]
```

By far the most useful and flexible method of converting JavaScript objects to and from strings are the JSON commands. JSON, JavaScript Object Notation, is a general file format for storing data. It is based on the standard method for

[9]The specification for this feature provides that any type of JavaScript object should be supported, however a number of recent browsers only support strings.

declaring JavaScript objects (e.g. {key: value}) but has some differences. What is great about JSON is that your browser already has built-in commands for converting JavaScript objects (a number, array, or other object) into a string and then later back into an object.

You can use this technique to send arbitrarily complex objects back and forth from your simulation to your webpage. Let's see how the JSON commands works:

```
var obj = {title: "I'm a complex object", data: [1, 4, 9]};
var str = JSON.stringify(obj); // '{"title":"I'm a complex object","data":[1,4,9]}'
JSON.parse(str); // {title: "I'm a complex object", data: [1, 4, 9]};
```

Hosting a webpage

In this chapter, we saved the webpages we have created to our personal computers' hard drives and opened them in a browser from there. This works great for development, but it does not allow us to share our creations with others.

Once you are ready to publish your webpages, you must move the HTML, CSS and JavaScript files off your computer and onto a web-server or web-host so that others can access them over the Internet. There are a number of options for web-hosting that range from the simple to the complex and from the free to the expensive.

On the simple and free end of the spectrum there are free blogging sites like Blogger (http://www.blogger.com) or WordPress (http://wordpress.com/). These sites allow you to create free blogs but they also allow you to do much more than that. These types of sites will generally let you edit the source HTML of your pages allowing you to implement the demos in this chapter directly within a blog post.

A step up from simple sites like these blogging platforms are shared hosting providers. Shared hosting providers such as DreamHost (http://dreamhost.com) take a server and allow multiple people to purchase space on the server to run their webpages. There are numerous shared hosting providers available. A more advanced version of shared hosting is Virtual Private Server (VPS) hosting. VPS providers such as RimuHosting (http://rimuhosting.com/) are similar to shared hosting providers in that they fit many customers on a single server. Where they differ is that a VPS host will give each customer a virtualized computer. Each individual customer will feel like they have complete control over their own computer and operating system even though they are sharing the actual hardware with others.

At the high end of the spectrum of complexity, cost and power are dedicated servers. In this case you purchase or rent a machine dedicated solely to the hosting your projects. This gives you complete control of your hosting situation but is expensive and may take a lot of effort to set up and maintain.

In general, we recommend starting small. Sign up for a Blogger account and experiment with these techniques there. If you keep at it and your site grows, at some point you will outgrow this simple solution and at that time you can upgrade to a more advanced hosting solution.

Chapter 12

Modeling with Agents

The modeling techniques we have taught up until this point focused on gaining insights using highly aggregated models of a system. This means that when we looked at models of population growth, we did not explore individual people and instead focused on understanding the population as a whole. This high-level aggregate approach to modeling helps us cut through unnecessary details to understand the core dynamics of a system.

For certain models however, this high-level view may hamstring our ability to explore important questions. For instance in a disease model we may care about the physical relationship between people in the model. Are they near each other? How often do they come into contact? Can we attempt to control the disease by manipulating how people move about and relate to each other? These are all questions that are very hard to answer with a standard System Dynamics model.

Heterogeneity, differences between individuals, is difficult to represent using System Dynamics models. One approach to heterogeneity that is sometimes used is simply to duplicate the model structure for each different class of person or entity in the model. We recall seeing one model that explored education in the United States. The modelers wanted to explore the differences between male and female students. To do so, they simply copy and pasted the entire model structure (consisting of dozens of stocks and flows) and calibrated one of these copies for male students and the other copy for female students.

Granted, this approach can be made to work, but it requires a lot of effort to set up and configure even in the simple two-gender case. When you have more than two cases it can quickly become completely unmanageable. Furthermore, duplicating parts of your model is a recipe for creating unmaintainable models afflicted by hard to track down bugs. The reason for this is that when you later make changes to your model, you are going to need to ensure the changes are made correctly to each one of the model copies. Although simple in principle, in practice this is very easy to mess up and it is a direct route for bugs to be introduced into the model.

Fortunately, an alternative modeling paradigm to System Dynamics exists that is excellent for modeling discrete individuals. It is called Agent Based Modeling and is focused on simulating individual agents and the interactions between these agents[1]. In this chapter we will introduce Agent Based Modeling and show how you can use it to explore questions that cannot be answered with pure System Dynamics.

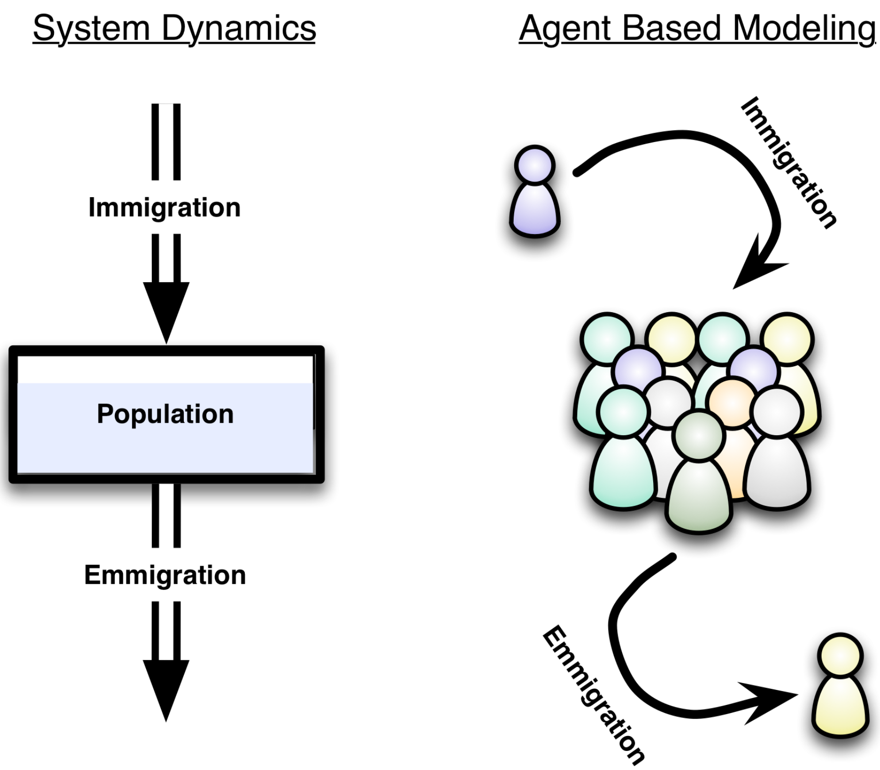


Figure 1. Two paradigms for modeling a population: System Dynamics and Agent Based Modeling.

[1] System Dynamics also has another standard tool for dealing with heterogeneity. This tool is called "vectors", "arrays", "subscripting", or "indexing" and allows you to transparently create multiple copies of your model during simulation to match different classes. Arrays are not as flexible as fully Agent Based Models though. If you consider the continuum of fully aggregate System Dynamics models on one end to fully individualized Agent Based Models on the other, we can think of arrays as existing part way along this continuum.

Exercise 12-1

Discuss the challenges you might face using System Dynamics to model a the adoption of a new product like an improved mousetrap. Identify issues that could be addressed by modeling discrete consumers.

The State Transition Diagram

Up until now our primary modeling tool has been the stock and flow diagram. This type of diagram is useful for summarizing systems from a high-level viewpoint. The stock is a primitive that can model entities that take on a range of values and flows are well suited for specifying the changes in stocks

In addition to representing aggregate systems, stock and flow diagrams are also used to model things on an individual level. For instance, a model of a person's motivations could be represented using a stock and flow diagram. The strength or importance of each type of motivation – money, family, etc... – could be represented as stocks with flows modulating the strength of these motivations over time.

When looking at the individual scale however, we will oftentimes find ourselves wanting to define characteristics of the individual using simple on/off logic. For instance, take the issue of an individual's sex. We can represent this using two categories: Male or Female (leaving aside transgendered individuals for the sake of simplicity). Similarly, when constructing a model of a disease, we might want to say a person is either sick or not sick (with no nuances such as "slightly sick" or "highly sick"). You can attempt to represent these different categories using stocks, but the formulation and equations to do so will be overly complicated.

Where the stock and flow diagram is used to model changing systems with continuous stocks, the state transition diagram is used to model systems with discrete on/off states. Within Insight Maker, state transition diagrams are constructed in almost the same way as stock and flow diagrams. The key difference is all stocks are replaced with *State* primitives and all flows are replaced with *Transition* primitives. State primitives can be added to the model by right-clicking on the model diagram and selecting **Add State**. Transition primitives will automatically be created when you connect two state primitives together using the standard "Flow" connection type.

A state primitive is possibly the simplest primitive available as it can only take on one of two values: true or false. When the state value is true, the state is active. When the state value is false, the state is not active and the agent does not occupy that state. When configuring a state primitive, you only need to specified whether the state is **initially active** or not at the start of the simulation. This initial condition can simply be `true` or `false`, but it can also be a logical equation that depends on the values of other primitives in the agent.

Figure 2. Sample states you might use for agents in a disease model.

For example, if you had a variable in the agent called [**Size**], and you wanted a state to be initially active if the value of [**Size**] was greater than 5, you could use the following as the initially active property for the state: `[Size] > 5`.

A transition primitive moves an agent between states. For instance, if you had two states in your model – *Healthy* and *Sick* – you could have one transition primitive moving agents from the healthy state to the sick state (simulating infection) and another transition primitive moving them the other way (simulating recovery).

There are three different ways a transition from one state to another can be triggered:

Timeout : In this mode the transition will be triggered a specific amount of time after the first state becomes active. For instance, if we had a disease model where the disease lasted 10 days, we could have a transition from the sick to healthy state using a timeout trigger with a period of 10 days.

Probability : In this mode there is a probability of the transition happening each time period. For instance, in the disease model if the disease only lasted 10 days on average but could randomly last longer or shorter, you could use a probability transition with a daily probability of 0.1.

Condition : In this mode you create an equation that will trigger than transition when it becomes true. For instance, if we had a stock, [**Infection Level**] in our agent indicating how sick the agent was, we could have them transition out of the sick state once that stock fell to zero. The trigger condition to enable this could be something like: `[Infection Level] = 0`.

Exercise 12-2

Specify a transition trigger type and value for the following types of transition:

1. Transition after 10 days.
2. 20% chance of transitioning each year.
3. Transitioning when value of the primitive [**Volume**] is greater than 5.

Answer Available

Exercise 12-3

Create a state transition diagram for a model of a person who three states: [**Child**], [**Adult**], [**Retired**]. The person starts in the [**Child**] state, transitions to the [**Adult**] state when they are 18 years old, and has a 2% chance of transitioning to the [**Retired**] state each year.

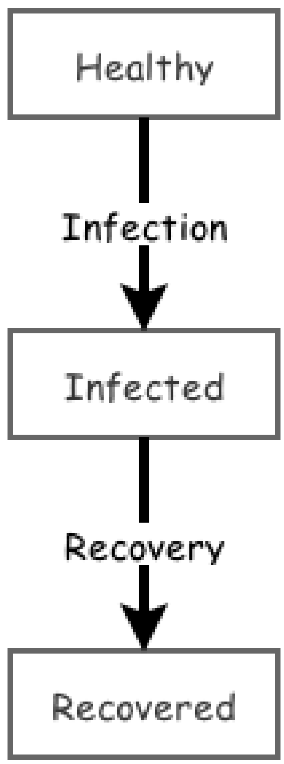
A State Transition Diagram for Disease

This model illustrates the use of state transition diagrams to model a simple disease. This is a disease such as the flu where immunity is obtained once the individual recovers from the disease.

1. Create a new **State** named [**Healthy**].
2. Create a new **State** named [**Infected**].
3. Create a new **State** named [**Recovered**].
4. The model diagram should now look something like this:

5. States can be used to represent a person's condition. In our model a person can either be healthy, infected, or recovered from the infection. Now, let's add transitions that move a person from state to state.
6. Create a new **Transition** going from the primitive [**Healthy**] to the primitive [**Infected**]. Name that transition [**Infection**].

7. Create a new **Transition** going from the primitive [**Infected**] to the primitive [**Recovered**]. Name that transition [**Recovery**].
8. Please note that in this model someone who is recovered cannot become sick again. They have gained immunity to the disease.
9. Now that the model structure has been designed, let's add equations and configure the primitives.
10. Change the **Start Active** property of the primitive [**Healthy**] to `True`.
11. The model diagram should now look something like this:



12. When a state is active, it means a person is in that state. By setting [**Healthy**] to start active, we have the person start in the healthy state.
13. Change the **Trigger Type** property of the primitive [**Infection**] to `Probability`.
14. Change the **Value/Equation** property of the primitive [**Infection**] to `0.3`.

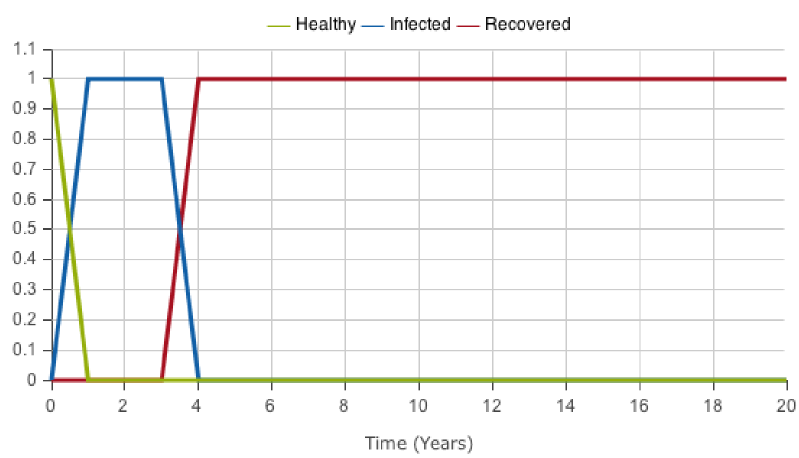
15. Change the **Trigger Type** property of the primitive [**Recovery**] to `Probability`.

16. Change the **Value/Equation** property of the primitive [**Recovery**] to `0.2`.

17. Using the Probability type for the transition trigger means that the person has a fixed probability of transitioning from one state to the next each year. We will assume a 30% probability of the person becoming sick each year and once sick, a 20% chance of recovering each year.

18. Let's run the model now.

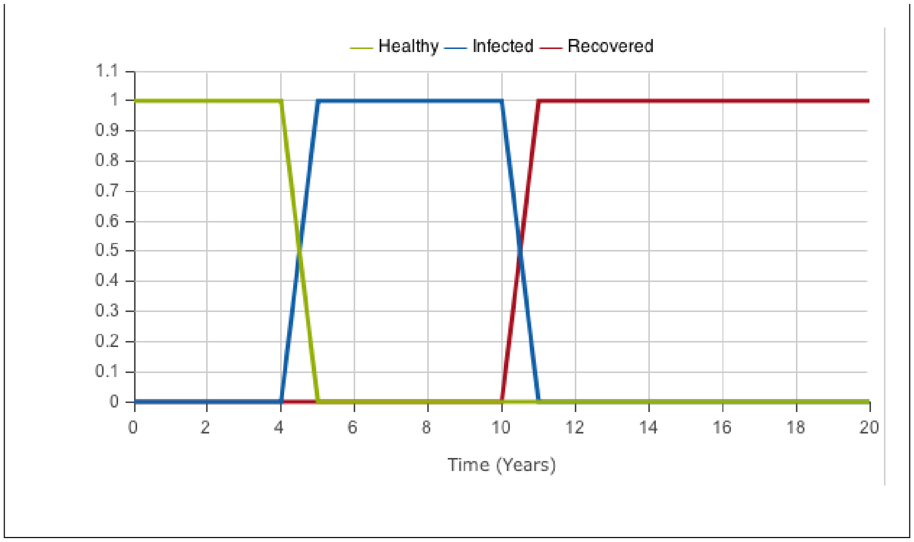
19. Run the model. Here are sample results:



20. A value of 1 for a state primitive means it is active. A value of 0 means it is not active. We can see from this diagram when the individual transitions from the healthy to the infected state and then from the infected state to recovered state.

21. We can run the model again and we will see that we get different results each time we run it. This is because the model is stochastic and the transition triggers are random.

22. Run the model. Here are sample results:



Creating Agents

Now that we have learned about state transition diagrams, we are ready to start creating agents. There are three key parts of creating agents in a model:

1. Defining what an agent is
2. Creating a group of agents
3. Viewing agent results

Defining Agents

We have already introduced the folder primitive as a tool for grouping primitives together and also as a tool for unfolding a model. The folder primitive plays an additional role in Agent Based Modeling as we use folders to define what our agent consists of.

To create an agent construct the state transition diagram for your agent (and also add any stocks, flows or any other primitives you want to this agent). Then create a folder containing all these primitives. Give the folder the name of your agent such as “Person” or “Individual” or even just “Agent”. This is all similar to what we have done with folders before, but now there is one extra step. Edit the folder configuration and set the folder **Behavior** to “Agent”. You have now created the definition of your first agent!

You can have as many different types of agents in your model as you would like. Just create a new agent model and use a new folder to define each of the different types of agents. For instance if you had a predator prey model you could have one agent definition describing the behavior of the prey, and a second agent definition describing the behavior of the predators.

Creating a Population of Agents

After you have defined an agent in your model, you are ready to create a collection or population of agents. This is done by adding an *Agent Population* primitive to your model. The agent population primitive takes the definition of an agent from an agent folder and creates many copies of that agent from the definition. The agent population primitive keeps track of these copies and allows them to operate and to interact with one another.

There are a number of different settings for the agent population primitive but two of them are of key importance. The first is to select what type of agent will be in the population. Each population primitive can only have one type of agent in it. You can have multiple populations though and the agents in one population can interact with the agents in another population.

After specifying what type of agent is in the population, you need to specify how many agents are in the population at the start of the simulation. This is done by setting the **Size** property for the agent population. Later on you can add or remove agents to a population by using the `Add()` and `Remove()` functions.

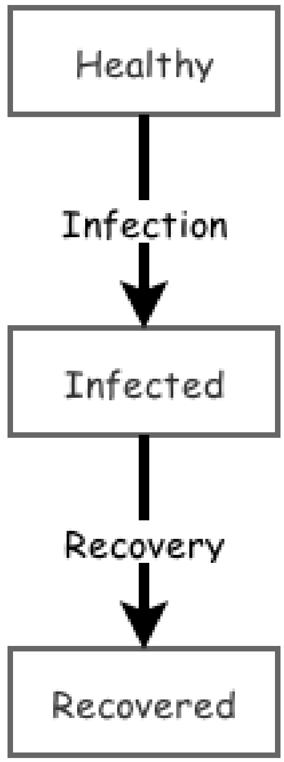
Viewing Agent Results

Many of the standard Insight Maker display types can be used to show the results of an agent based simulation. If you add an agent population to a time series or tabular display, the results for the number of agents in each of the various agent states will automatically be shown. You can also use the **Map** display type to illustrate agents within a geographic region.

An Agent Based Model of Disease

Here we convert a state transition diagram into a model containing multiple agents.

1. The model diagram should now look something like this:



2. We start with our state transition diagram from our previous modeling example.

3. Create a new **Folder** named [**Person**]. The folder should surround the primitives [**Healthy**], [**Infected**], [**Recovered**], [**Infection**] and [**Recovery**].

4. Change the **Type** property of the primitive [**Person**] to `Agent`.

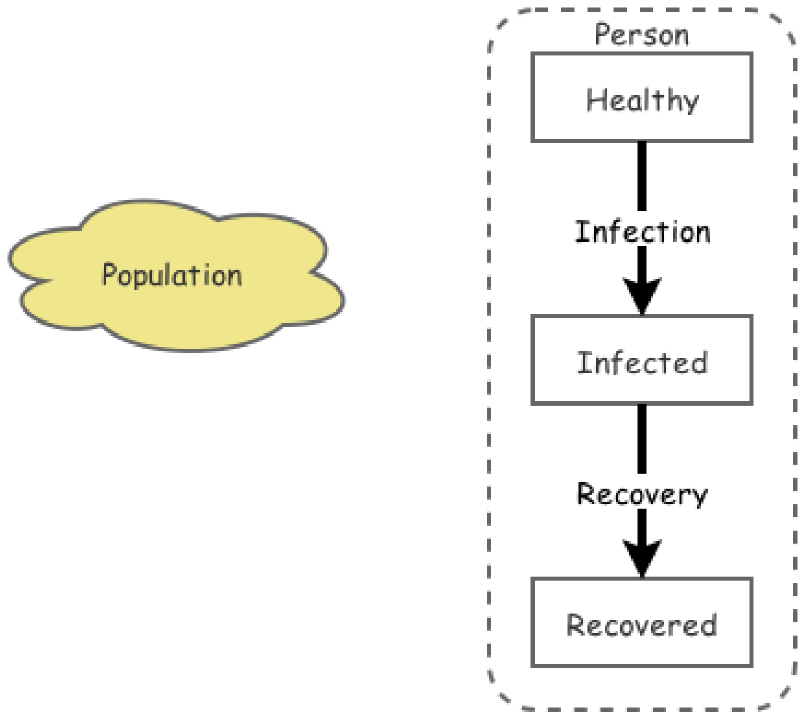
5. First we create an agent folder to encapsulate our state transition diagram. This is a definition of what an agent in our model will be and we make sure the folder behavior is set to "Agent".

6. Create a new **Agent Population** named [**Population**].

7. Change the **Agent Base** property of the primitive [**Population**] to `Person`.

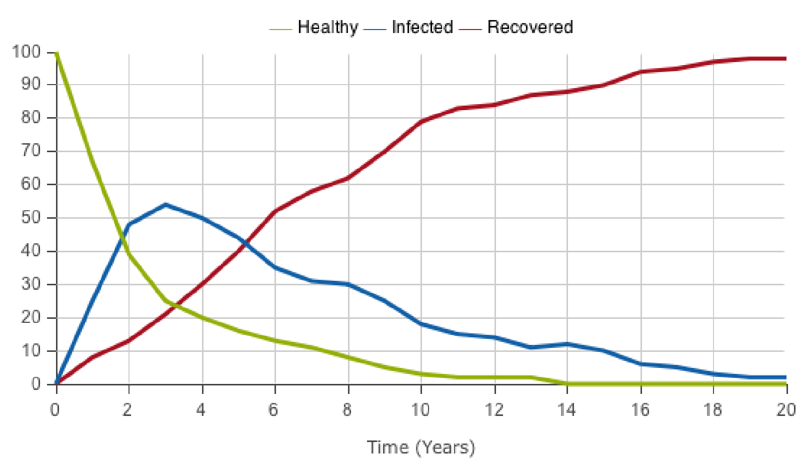
8. Next we create an agent population [**Population**] and set it to contain instances of our **Person** agent. We'll start with a population size of 100 agents.

9. The model diagram should now look something like this:



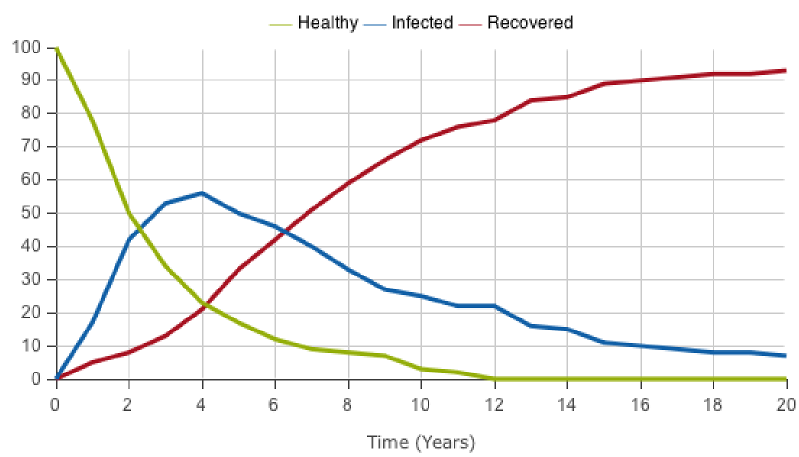
10. We can now run the model to see how the disease affects the 100 people in our population.

11. Run the model. Here are sample results:



12. Each time we run the model we will get different results due to the stochasticity in the model.

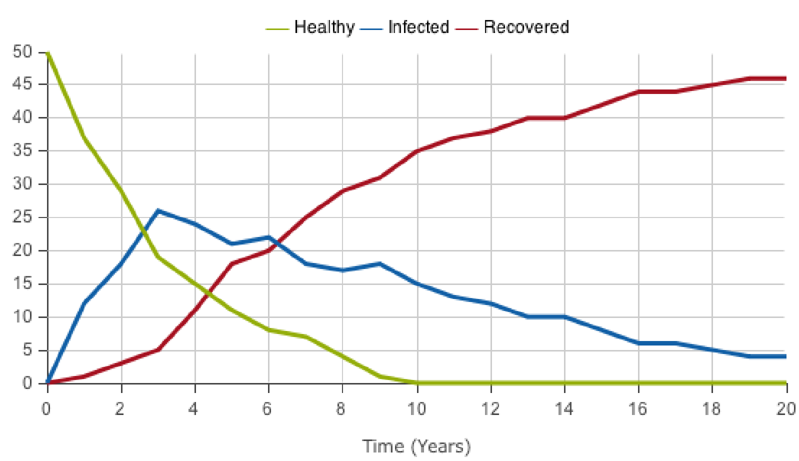
13. Run the model. Here are sample results:



14. Change the **Size** property of the primitive [**Population**] to `50`.

15. We can easily change the number of people in the population. Let's set it to 50 then run the model again.

16. Run the model. Here are sample results:



17. When we have a smaller number of people, the overall population changes are more variable. As we add more and more people, the randomness in the model has less of an effect and trends become smoother approaching the average results we would see if we had used a System Dynamics model.

Working with Agents

Working with agents is fundamentally different from working with primitives in a pure System Dynamics model. For instance, if you have a regular model and you refer to the value of a variable or stock you get a single value back. With agents, however, when you refer to the value of a primitive you might get a separate value for each individual agent in your model.

So for instance, if you have 100 agents and you refer to the primitive [**Height**], you will get 100 different heights one for each of the agents in the model. Similarly, in the case of our disease model, if you you request the value of the state [**Infected**], you will get a different infected value for each of the agents in the model.

You will need to extend your modeling toolkit in order to be able to effectively manage agents and accomplish your goals in your model. The key building block of this extended toolkit is the vector[2]. In the following sections we will first introduce the general concept of vectors and then show how you can use them to interact with agents.

Working with Vectors

A vector is an ordered list of items. In Insight Maker vectors can be written using the '«' sign (or '<<') followed by the '»' sign (or '>>'). For instance imagine we had a small population of only four people. If we asked the model for the heights of those four people[3] in meters we might get something like this:

```
«2, 1.8, 1.9, 1.5»
```

This indicates that our population has four people with heights of 2, 1.8, 1.9 and 1.5 meters. Insight Maker has an extensive set of capabilities and functions for manipulating and summarizing vectors such as this. For instance, if we wanted to know the height of the tallest person in our population, we could use the `Max()` function:

```
Max(«2, 1.8, 1.9, 1.5») # = 2
```

If we wanted to know the height of the smallest person in the population we could use the `Min()` function:

```
Min(«2, 1.8, 1.9, 1.5») # = 1.5
```

Let's say we wanted to know the average height of the people in our our population. We could either use the `Mean()` or `Median()` functions:

```
Mean(«2, 1.8, 1.9, 1.5») # = 1.8
```

[2] In other programing languages and modeling environments vectors are sometimes called "Arrays" or "Lists".

[3] Using an equation like *Value(FindAll([Population]), [Height])*. We'll see later how to construct equations like this.

```
Median(«2, 1.8, 1.9, 1.5») # = 1.85
```

We can also use basic mathematical operations on our vectors. For example, assume we needed to design a room such that the top of the room was at least half a meter above a person's head. We could find the required room height for each person by adding 0.5 to the vector of heights:

```
«2, 1.8, 1.9, 1.5» + 0.5 # = «2.5, 2.3, 2.4, 2»
```

We can also add vectors together. For instance, let's imagine that some of the agents had hats on and we have measured the height of these hats and got the following vector of heights: `«0.05, 0, 0.1, 0»` (two of the people do not wear hats). We could find the height of the agents when they are wearing their hats using:

```
«2, 1.8, 1.9, 1.5» + «0.05, 0, 0.1, 0» # = «2.05, 1.8, 2, 1.5»
```

Another useful vector function is the `Count()` function. Assuming we did not know there were four agents, we could determine how many elements there were in the vector using this function:

```
Count(«2, 1.8, 1.9, 1.5») # = 4
```

You can do a lot with these basic functions but there are also two very powerful vector functions we should mention: `Map()` and `Filter()`. Map takes each element in a vector and applies some transformation to it and returns a vector of the transformations. As an example, let's say we wanted to test whether or not are agents were tall enough to ride an amusement park ride with a cutoff of 1.85 meters. We could get a vector containing whether or not each agent was tall enough using:

```
Map(«2, 1.8, 1.9, 1.5», x >= 1.85) # = «true, false, true, false»
```

Here the function `x >= 2` is applied to each element in the vector (with `x` representing the element value) and the results of this element-by-element evaluation of the function is returned.

Filter takes a function and applies it to each element in a vector. If the function evaluates to true, the element is included in the resulting vector; if the function evaluates to false, the element is not included in the results. For instance, if we just wanted the heights of the people who were tall enough to ride the ride, we could use:

```
Filter(«2, 1.8, 1.9, 1.5», x >= 1.85) # = «2, 1.9»
```

Lastly, there are a couple of very useful functions are available to combine vectors together. `Union()` takes two vectors and combines them together removing duplicated elements.

```
Union(«1, 2 ,3», «2, 3 ,4») # = «1, 2, 3, 4»
```

`Intersection()` takes two vectors and returns a vector containing the elements that are in both of the vectors.

```
Intersection(«1, 2 ,3», «2, 3 ,4») # = «2, 3»
```

`Difference()` takes two vectors and returns a vector containing the elements that are in either one of the vectors but *not* in both of the vectors.

```
Difference(«1, 2 ,3», «2, 3 ,4») # = «1, 4»
```

There are many more vector functions available, but these are some of the key ones. They will prove invaluable when you come to working with vectors of agents.

Exercise 12-4

Given a vector of heights `«2, 1.8, 1.9, 1.5»`, write an equation to find the tallest height under 1.95 meters:

Answer Available

Exercise 12-5

Given a vector named a, write an equation to find the median of the squares of all the elements in a.

Answer Available

Exercise 12-6

Given a vector named a and a vector named b, write an equation to find the smallest element that is in both vectors.

Answer Available

Exercise 12-7

Given the vector named a. Find the mean of the vector without using the `Mean()` function.

Answer Available

Accessing Agents

Insight Maker includes a number of functions to access the individual agents within a population. The simplest of these is the `FindAll()` function. Given an agent population primitive that we'll call [**Population**], the FindAll function returns a vector containing all the agents within that agent population:

```
FindAll([Population])
```

So if your agent population currently had 100 agents in it, this would return a vector with 100 elements where the first element referred to the first agent, the second element referred to the second agent and so on. It is important to note that these elements are agent references, not numbers. So you can use a function like `Reverse()` on the resulting vector, but you cannot directly use functions like `Mean()` as the agent references are not numerical values[4]. We will see how to access the values for agents next.

In addition to the FindAll function, there are other find functions that return a subset of the agents in the model. For instance, the `FindState()` and `FindNotState()` functions return, respectively, agents that either have the given state active or not active. For instance, if we go back to our agent-based disease model, our agents had a state primitive called [**Infected**] that represented if the agent was currently sick, we could get a vector of the agents in our population that were currently sick using the following:

```
FindState([Population], [Infected])
```

And we could obtain a vector of the agents that were not currently infected with:

```
FindNotState([Population], [Infected])
```

Find functions can also be nested. For instance, if we added a [**Male**] state primitive to our agents representing whether or not the agent was a man; we could obtain a vector of all currently infected men with something like the following:

```
FindState(FindState([Population], [Infected]), [Male])
```

Nesting find statements is effectively using Boolean AND logic (like you might use on a search engine: "Infected AND Male"). To do Boolean OR logic (e.g. "Infected OR Male") and return all the agents that are either infected or a man (or both), you can use the Union function to merge two vectors:

```
Union(FindState([Population], [Infected]), FindState([Population], [Male]))
```

If you wanted the agents that were either infected or men (but not both simultaneously), you could use:

```
Difference(FindState([Population], [Infected]), FindState([Population], [Male]))
```

[4]The agents certainly contain many numerical values in their stocks, variables, or states; but an agent reference itself is not numerical and so you cannot do things such as directly taking the average of the agents or sorting them.

Exercise 12-8

Write an equation using the disease example to return a vector of all female infected individuals.

Answer Available

Exercise 12-9

Write an equation using the disease example to return a vector of all female individuals, healthy individuals or healthy females.

Answer Available

Agent Values

Once you have a vector of agents, you can extract the values of the specific primitives in those agents using the `Value()` and `SetValue()` functions.

The Value function takes two arguments: a vector of agents and the primitive for which you want the value. It then returns the value of that primitive in each of the agents. For instance, let us say our agents have a primitive named [**Height**]. We could get a vector of the height of all the people in the model like so:

```
Value(FindAll([Population]), [Height])
```

A vector of heights by itself is generally of not too much use. Often we will want to summarize it, for instance by finding the average height of the people in our population:

```
Mean(Value(FindAll([Population]), [Height]))
```

In addition to determining the value of a primitive in an agent, you can also manually set the agents' primitive values using the SetValue function. It takes the same arguments as the Value function in addition to the value you want to set primitives to. For instance, we could use the following to set the height of all our agents to 2.1:

```
SetValue(FindAll([Population]), [Height], 2.1)
```

Exercise 12-10

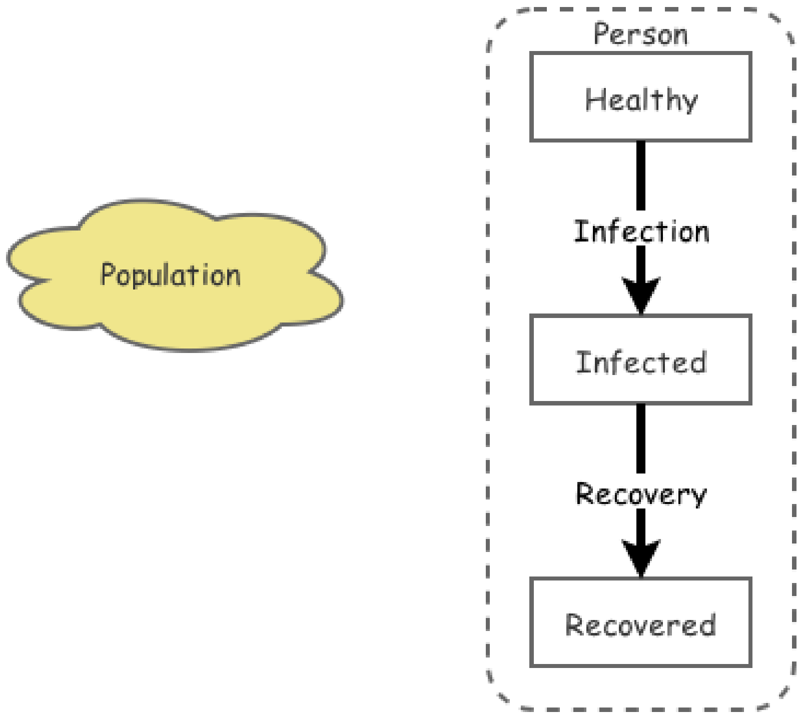
Assume our disease model population had a height stock. Provide an equation to find the average difference in heights between males and females.

Answer Available

Agents Interacting

This example shows how agents can interact with each other using the Find functions.

1. The model diagram should now look something like this:



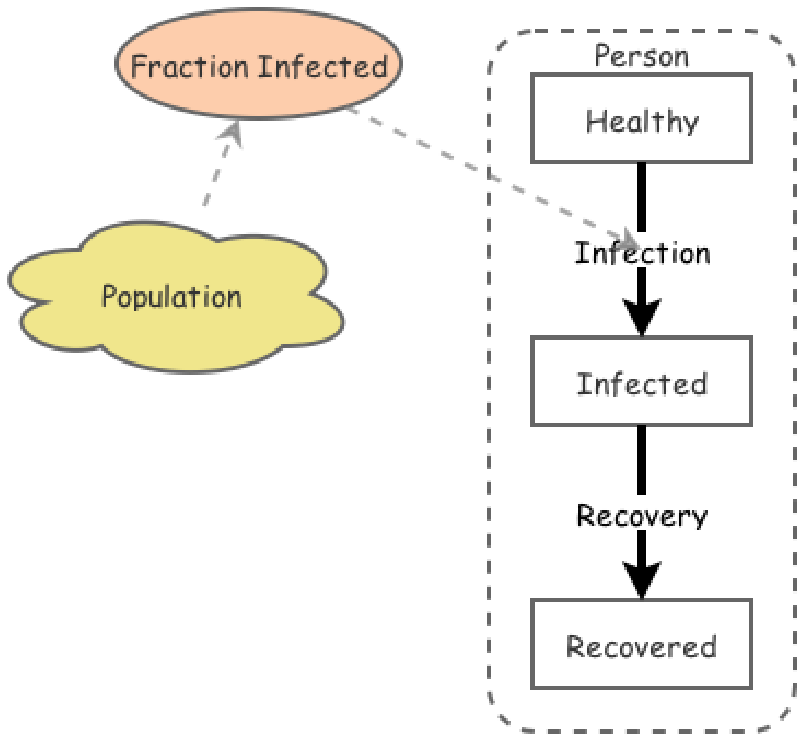
2. Let's make our agent based disease model from earlier more realistic. We will add a variable [**Fraction Infected**] that calculates what fraction of the population is currently infected. We will then use this variable to determine the infection rate so the more people in the population who are infected, the faster the disease will spread.

3. Create a new **Variable** named [**Fraction Infected**].

4. Create a new **Link** going from the primitive [**Population**] to the primitive [**Fraction Infected**].

5. Create a new **Link** going from the primitive [**Fraction Infected**] to the primitive [**Infection**].

6. The model diagram should now look something like this:



7. Now let's configure the value of [**Percent Infected**] and change the [**Infection**]transition to use it.

8. Change the **Equation** property of the primitive [**Fraction Infected**] to `Count(FindState([Population], [Infected]))/PopulationSize([Population])`.

9. This equation uses the FindState function to select all the people in the [**Population**] who are in the Infected state. It then divides the number of those people by the total size of the population.

10. Change the **Value/Equation** property of the primitive [**Infection**] to `[Fraction Infected]`.

11. Now that we have set our infection probability to the value of the [**Fraction Infected**] primitive, we are ready to run the model.

12. Run the model. Here are sample results:

13. That was a bit of a disappointment wasn't it? Nothing happened. Why is this?

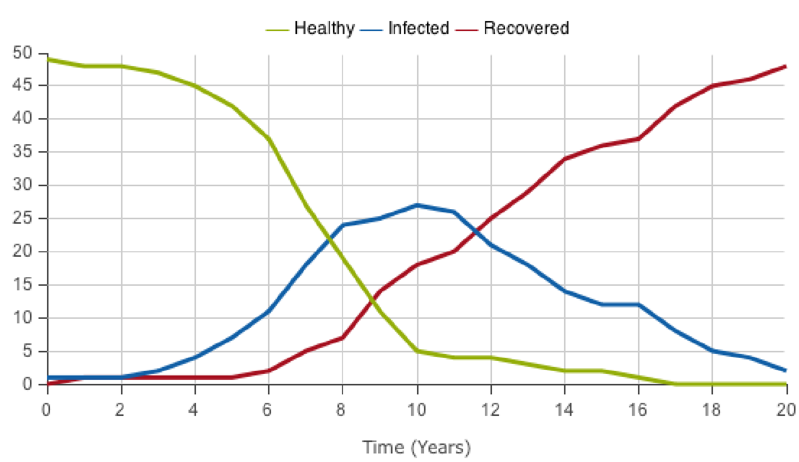
14. Well since our infection rate now depends on the number of people who are infected we have to have at least one person infected to get the epidemic going. Let's change the [**Healthy**] and [**Infected**] states so one person starts in the infected state at the beginning of the simulation.

15. Change the **Start Active** property of the primitive [**Healthy**] to `Index([Self]) <> 1`.

16. Change the **Start Active** property of the primitive [**Infected**] to `Index([Self]) == 1`.

17. Each agent has an index starting with 1, we have set our initially active equations so the first agent in the population will start the simulation in the infected state. Let's run the model to see this working.

18. Run the model. Here are sample results:



19. Each time we run the model we will get a different set of results. Sometimes the infection will die off after the first infected person recovers. Many other times an epidemic spread of the disease will occur.

20. Run the model. Here are sample results:

Agent Geography

One of the key strengths of Agent Based Modeling is that it allows us to study the geographic relationship between our agents. So if we are developing a disease model we do not have to assume that all the agents are perfectly mixed together like atoms in a gas (such as we generally would in System Dynamics). Instead, using Agent Based Modeling we can explicitly define the physical relationship between the different agents and study how this geography affects the spread of the disease.

In general when we talk about geography we mean spatial geography: the locations of people within a region in terms of their latitude and longitude (and sometimes their elevation). Insight Maker supports this kind of geography, but it also supports a second kind of geography: network geography. Insight Maker allows the specification of "connections" between agents. This leads to a new type of geography where you have centrally located agents (ones connected to many other agents) and agents far from the network's center (those that are unconnected or just connected to a very few other agents).

Both these types of geographies can be useful in exploring important features of real-world systems. In the following sections, we will introduce their properties and show you how to utilize them in your own models.

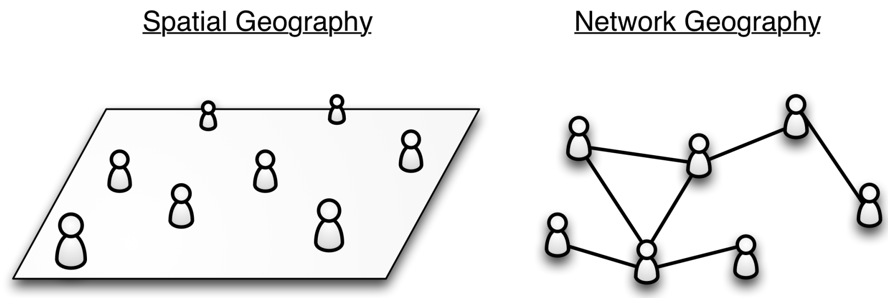


Figure 3. Spatial geography and network geography.

Spatial Geography

In Insight Maker, each Agent Population can be given dimensions in terms of a width and a height. By default, agents are placed at a random location within this region. You can, however, choose a different placement method for the starting position of the agents. The following placement methods are available:

Random : The default. Agents are placed at random positions within the geometry specified for the agent population.

Grid : Agents are aligned in a grid within the population. When using this placement method, you will need to ensure that you have enough agents so that the grid is complete. You might need to experiment with increasing or decreasing the number of agents to make the grid fit perfectly for a given set of region dimensions.

Ellipse : Agents are arranged in a single ellipse within the region. If the region geometry is a square, then the agents will be arranged in a circle.

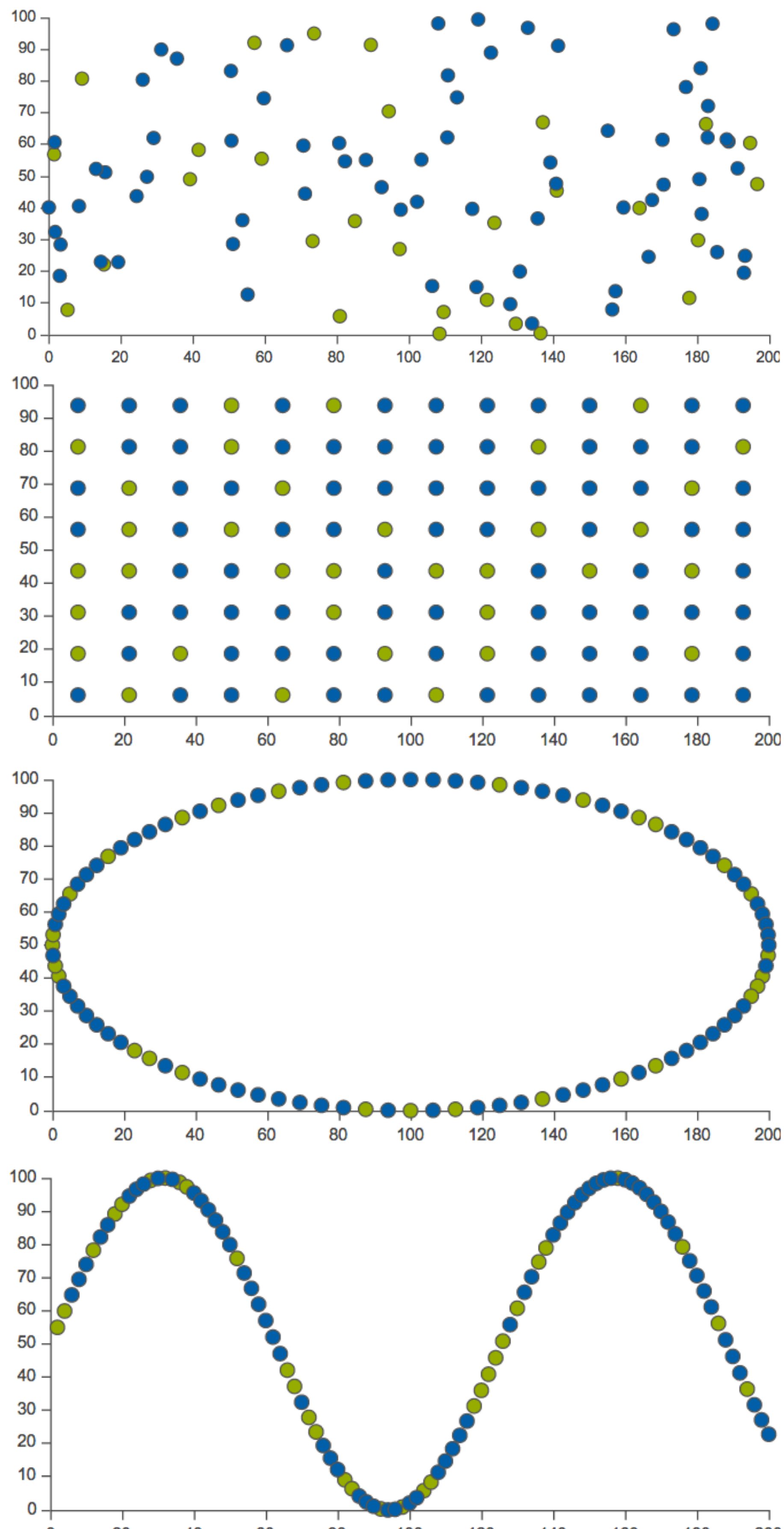
Network : Assuming network connections between agents have been specified, the agents will be arranged in an attempt to create a pleasing layout of the network structure.

Custom Function : Here you can specify a custom function to control the layout of the agents. This function will be called once for each agent in the population and should return a two-element vector where the first element is the x-coordinate of the agent, and the second element is the y-coordinate. The primitive [**Self**] in this function will refer to the agent that is being positioned.

Spatial Find Functions

When working with a spatially explicit model, a number of additional find functions are available for you to obtain references to agents that match a given spatial criteria.

`FindNearby()` is a function that returns a vector of agents that are within a given proximity to a target agent. It takes three arguments: the agent


100
90
80
70
60
50
40
30
20
10
0
0
20
40
60
80
100
120
140
160
180
200

population primitive, the agent target for which you want nearby neighbors, and a distance. All agents within the specified distance to the target agent will be returned as a vector.

It is useful now to introduce a concept that will be very helpful to you. When used in an Agent, [**Self**] always refers to the agent itself. If you have a primitive within an agent, [**Self**] can be used from that primitive to get a reference to the agent containing the primitive. So the following equation in an agent will return a vector of agents that are within 15 miles of the agent itself:

```
FindNearby([Population], [Self], {15 Miles})
```

Two other useful functions for finding agents in spatial relation to each other are `FindNearest()` and `FindFurthest()`. FindNearest returns the nearest agent to the target while FindFurthest returns the agent that is farthest away from it. Each of them also supports an optional third argument determining how many nearby (or far away) agents to return (this optional argument defaults to one when omitted).

For example, the following equation finds the nearest agent to the current agent:

```
FindNearest([Population], [Self])
```

While this finds the three agents that are furthest from the current agent:

```
FindFurthest([Population], [Self], 3)
```

Movement Functions

You can also move agents to new locations during simulation. To do this, it is helpful to introduce a new primitive we have not yet discussed. This primitive is the *Action* primitive. Action primitives are designed to execute some action that changes the state of your model. For instance, they can be used to move agents or change the values of the primitives within an agent. An action is triggered in the same way a transition is triggered. Like a transition, there are three possible methods of triggering the action: timeout, probability, and condition.

For instance, we can use an action primitive in an agent and the `Move()` function to make agents move during the simulation. The Move function takes two arguments: the agent to be moved, and a vector containing the x- and y-distances to move the agent. Thus, we could place an action primitive in our agent and give it the following action property to make the agent move randomly over time[5]. The equation will move the agent a random distance between -0.5 and 0.5 units in the x-direction and a random distance between -0.5 and 0.5 units in the y-direction.

```
Move([Self], «rand, rand»-0.5)
```

[5]What we are implementing here is known as a "random walk" or Brownian motion. It is a commonly studied pattern of movement with wide applications in science.

Another useful movement function is the `MoveTowards()` function. MoveTowards moves an agent towards (or away from) the location of another agent. MoveTowards takes three arguments: the agent to be moved, the target agent to move towards, and how far to move towards that agent (with negative values indicating movement away). The following command would move an agent one meter closer to its nearest neighbor in the population.

```
MoveTowards([Self], FindNearest([Population], [Self]), {1 Meter})
```

Exercise 12-11

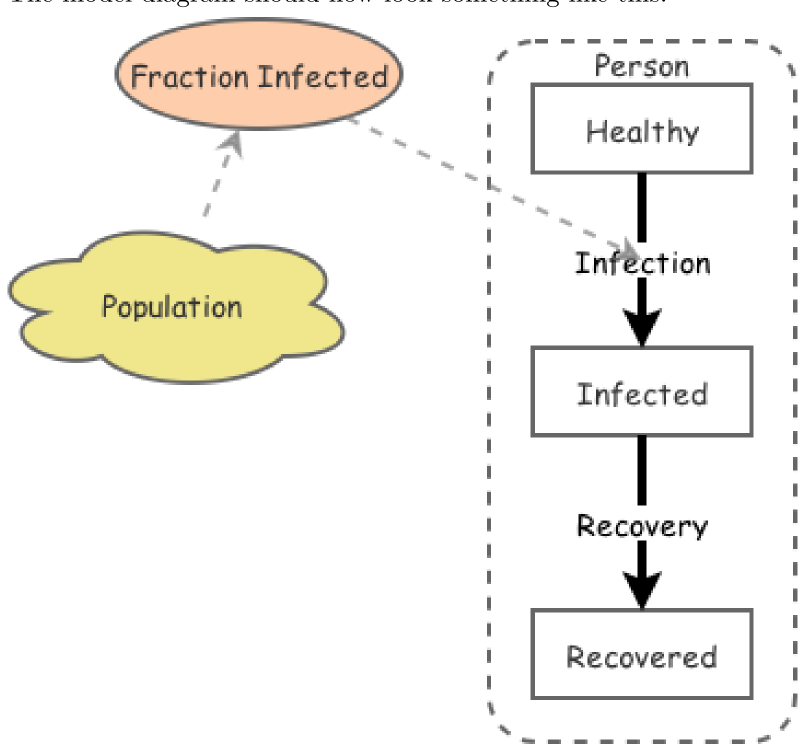
Write an equation to move an agent 1 meters towards the furthest healthy agent.

Answer Available

Agent Movement

This model illustrates the use of movement within agent based models. We adapt the previous disease model so that healthy agents flee from the nearest infected agent.

1. The model diagram should now look something like this:



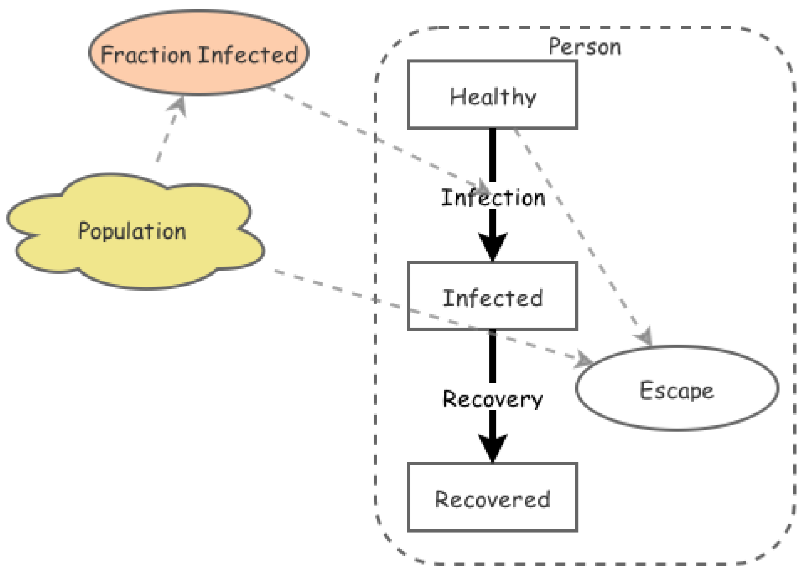
2. We will extend our disease model from earlier by adding movement to the agents. First we need to create an action primitive.

3. Create a new **Action** named [**Escape**].

4. Create a new **Link** going from the primitive [**Healthy**] to the primitive [**Escape**].

5. Create a new **Link** going from the primitive [**Population**] to the primitive [**Escape**].

6. The model diagram should now look something like this:



7. We will have this action be triggered when the agent is healthy and there is at least one infected agent in the simulation.

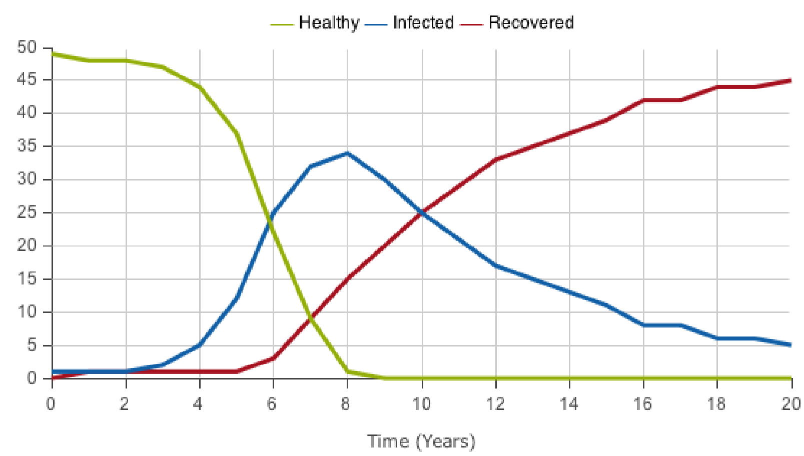
8. Change the **Value/Equation** property of the primitive [**Escape**] to `[Healthy] and Count(FindState([Population], [Infected])) > 0`.

9. The action will cause healthy agents to move away from the nearest infected agent. In effect, fleeing from sick individuals.

10. Change the **Action** property of the primitive [**Escape**] to `MoveTowards([Self], FindNearest(FindState([Population], [Infected]), [Self]), -2)`.

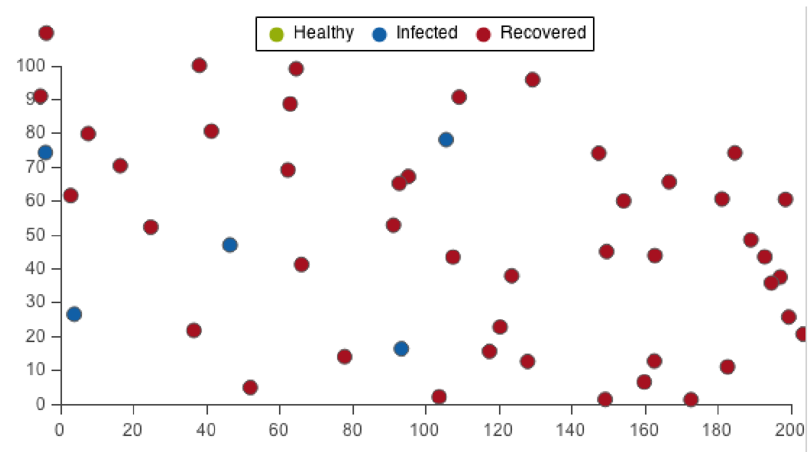
11. We can now run the simulation.

12. Run the model. Here are sample results:



13. These results are not too interesting as they do not show the locations of the agents. To see the agents moving, we need to change the display type to **Map** which gives us a visualization of the location of agents. Then we run the simulation again.

14. Run the model. Here are sample results:



Network Geography

To create connections and remove connections between agents you can use the `Connect()` and `Unconnect()` functions. Both of these functions take two arguments: the agents that should be connected or disconnected. For example, to connect an agent to its nearest neighbor, you could use the following

```
Connect([Self], FindNearest([Population], [Self]))
```

To disconnect an agent from its nearest neighbor (assuming they are connected), you would use:

```
Unconnect([Self], FindNearest([Population], [Self]))
```

To obtain a vector of connections to an agent, use the `Connected()` function:

```
Connected([Self])
```

Connections are not directed so creating a connection from agent A to agent B is the same as creating a connection from agent B to agent A. Also only one connection between a given pair of agents will exist at a time. So creating two connections between a given pair of agents will have the same effect as creating a single connection.

By default, no connections are created when a simulation is initially started. If you change the **Network Structure** configuration property of the agent population primitive, you can specify a function to create connections when the simulation is started. This function is called once for each pair of agents in the model. The agents are available in the function as the variables a and b. If the function evaluates to `true`, then the agents will start connected. If the function evaluates to `false`, the agents will not be initially connected.

You could use this function to, for instance, specify that 40% of agents will be directly connected to each other at the start of the simulation. The following equation would do that by generating a random true/false value with 40% probability of returning `true` each time it is called:

```
RandBoolean(0.4)
```

Multiline Equations

So far in this book, the equations we have looked at have generally been straightforward mathematical formulae. We have introduced some more advanced concepts, such as vectors, but for the most part our equations have been relatively simple one-liners. When doing Agent Based Modeling however, at some point you will find these one-line equations to be limiting. When you begin to run into these limitations with your own models, you may need to start using multiline equations to achieve certain agent behavior.

Almost everyplace in Insight Maker where you can write a mathematical expression, you can also right a multiline equation. It turns out that Insight Maker's language for specifying equations is actually a complete computer programming language and you can exploit the strength of this programming language by writing your equations over several lines instead of using a single line mathematical formula.

We delayed introducing these capabilities until now, as they can sometimes be a distraction from focusing on understanding a system. However, when you build complex Agent Based Models, they can be necessary to express the model logic you wish. Given this need, we will provide a brief introduction to the programming features that can be used as part of Insight Maker equations. You do not need to delve deeply into these capabilities now, but be aware that they are available for when you need them in your own models.

Variables

Variables are temporary slots to store values to be reused within your equations. Variables are created using the '`<-`' symbol meaning assignment. For instance:

```
a <- 2 # The variable 'a' holds the value 2
b <- a + 2 # The variable 'b' holds the value 4
a <- b^2 # a=16, b=4
```

Variable names can contain any number of letters and numbers and must always start with a letter.

If-Then-Else

You should be familiar with the `IfThenElse()` function. A multiline alternative to it exists. The following is equivalent to `IfThenElse([Height] > 2, 1, 2)`.

```
If [Height] > 10 Then
    1
Else
    2
End If
```

One of the benefits of these multiline equations is that they can be more readable than the single line functions. This is especially true if you are trying to do nested *if* statements. Compare `IfThenElse([Height] > 2, 1, IfThenElse([Height] < 1, -1, 2))` to:

```
If [Height] > 2 Then
    1
Else If [Height] < 1 Then
    -1
Else
    2
End If
```

The second one is much more readable. This makes it easier to maintain and more resilient to potential typographical errors.

Loops

Loops are a programming construct that repeat some code multiple times. There are several different types of loops. One important loop is the *for* loop which repeats a command a specified number of times. Here is an example of it being used:

```
sum <- 0
For i From 1 To 3
    sum <- sum + i
End Loop
sum
```

The inner part of the loop is run three times here. The first time the variable i is assigned the value of 1, the next time 2, and the last time 3. So this sums up the values of 1, 2, and 3 resulting in 6.

Another variant of the *for* loop is the *for-in* loop. This uses a vector to assign the values of the iterations. The following code sums the numbers 1, 5, and 10 to get 16.

```
sum <- 0
For i In «1, 5, 10»
    sum <- sum + i
End Loop
sum
```

For-in loops can be very useful to iterate through a vector of agents. Another useful loop is called the *while* loop. It does not repeat a predefined number of times and instead repeats until a condition becomes true. Here is an example:

```
total <- 2
While total < 100
    total <- total^2
End Loop
total
```

This code keeps squaring the *total* variable until the total is greater than 100. In this case, this will result in 256.

Functions

Functions allow you to reuse code in multiple places in your model. For instance, imagine you had a model that dealt with temperatures in both Degrees Fahrenheit and Celsius. If you could not use the built in unit conversion functionality, every time you wanted to convert from one form to the other you would have to include the standard conversion formula in your equations. Not only would this be tedious, it would also be error prone as the more times you type an equation, the higher the chance of making a mistake.

You can define functions in two ways. One is a short one-liner:

```
FtoC(f) <- 5/9*(f+32)
```

And another is a multiline form allowing you to incorporate multiline logic in your functions:

```
Function FtoC(f)
    5/9*(f+32)
End Function
```

A great place to include your functions is in the *Macros* section of your model. You can enter macros by clicking the **Macros** button in the **Tools** section of the toolbar. The functions you define here will be accessible in any equation in any part of your model.

Exercise 12-12

Write a function to return the range of a vector. The range is the largest element of the vector minus the smallest element.

Answer Available

Exercise 12-13

Write a function to calculate the nth Fibonacci number. The Fibonacci sequence goes 1, 1, 2, 3, 5, 8, 12, ... After the first two, each number is the sum of the two proceeding numbers in the sequence.

What is the 15th Fibonacci number?

Answer Available

Integrating SD and ABM

System Dynamics modeling and Agent Based Modeling are two different ways of approaching a system. In general, System Dynamics looks at highly aggregated systems and encourages the study of feedback. Agent Based Modeling explores individuals and the interactions between these individuals.

Some software packages only do System Dynamics or Agent Based Modeling leading to the perception that they are somehow incompatible methodologies. Although these techniques can be thought of as quite different, it important to realize that, at the end of the day, both of them are simply applied mathematics. To emphasize this, Insight Maker integrates both these techniques together seamlessly in its modeling environment. There is no such thing as an "Insight Maker Agent Based Model" or an "Insight Maker System Dynamics Model". There are simply models where you may use agent-based techniques, System Dynamics techniques or a mixture of the two.

Insight Maker (and other modeling packages such as AnyLogic http://www.anylogic.com/) allows you to integrate the two seamlessly together. For instance, in this chapter we have used state transition diagrams within our agents. We could have just as well used stock and flow diagrams within the agents so that each agent in effect contained its own System Dynamics model of its state. Similarly if you have a large System Dynamics model you could create an agent-based sub-model that feeds into the main model dynamics.

When doing modeling, it is important to not get focused on labels or taxonomies of different techniques. Given a modeling task, you want to think about what tools and techniques are best used to approach it. You want to make sure not to approach a modeling task by trying to figure out how to force that task into the constraints of a favorite modeling paradigm.

Exercise 12-14

Compare and contrast the Agent Based Modeling and System Dynamics approach to creating models. Provide three examples of modeling tasks where Agent Based Modeling would be better suited than System Dynamics and three examples where the reverse would be true.

Chapter 13

Optimization and Complexity

We start this chapter by taking up our hamster population model from The Process of Modeling and reconsidering it. As you recall, your friend requested our help in constructing a model to simulate the population of the endangered Aquatic Hamsters. There are many ways to exploit valuable empirical data to improve your models. For instance, if we had data on hamster fecundity, we might be able to plug that information in directly as a parameter in our hamster population model.

One of the most useful kinds of empirical data is historical time series. Some of these time series might represent data and factors that feed into the model, but are not directly modeled. For example, we might have historical temperature data. The temperature could be an important thing to include in the model, as it would affect hamster survival however it is not something we directly model. By this we mean that we do not expect our hamsters to have any effect on the temperature in the region but we do expect the temperature to have an effect on the hamsters. Thus, we can feed the temperature data into the model. We can do this by importing this historical temperature data and including it in the model using a converter primitive.

In other cases, the historical data may represent factors you are directly trying to model. For example, we have a data series of biannual hamster population surveys going back 20 years. This data series lets us know roughly how many hamsters there were over time. Because we are trying to model this data, it is not something plug directly into our model as we could with the temperature, but it is something we can use to calibrate and assess the accuracy of our model.

How do we do this and what will be the results?

Assessing Model Accuracy

We first import our historical data into a converter primitive. We then assess the accuracy of the model in two ways: qualitatively and quantitatively. To assess how well our model fits the historical data qualitatively we plot the simulated and historical data series next to each other. Ideally, they will match up closely but if they do not we should pay close attention to how they differ.

If they have the same general shape (except for a vertical or horizontal displacement) that is good news, as it indicates that you may have gotten the general dynamics of your model correct and that you may just need to fine-tune the relationships and parameter values. If the results look considerably different you may have more work to do in improving the model.

You can also assess the accuracy of models quantitatively. One standard tool people use to assess the accuracy of a model is the R^2 metric[1]. R^2 is the fraction of the squared error explained by the model compared to the "null" model. It ranges from 0 (the model basically provides no predictive power), to 1 (the model predicts perfectly). Mathematically, R^2 is calculated like so:

$$R^2 = \sum_t \frac{(\overline{\text{Truth}} - \text{Truth})^2 - (\text{Model} - \text{Truth})^2}{(\overline{\text{Truth}} - \text{Truth})^2}$$

Naively used, R^2 has a number of issues that we will discuss later in this chapter. However, it is still a useful tool that many people use and with which they are familiar. It is also relatively straightforward to calculate. The following code calculates an R^2 for a model fit. This is code written in JavaScript and can be placed as the **Action** for a button primitive in Insight Maker. The code is written assuming two primitives: a converter [**Historical Hamsters**] containing historical population sizes and a stock [**Hamsters**] containing simulated population sizes. You can edit the code to reference the actual names of the primitives in your model.

```
var simulated = findName("Hamsters"); // Replace with your primitive name
var historical = findName("Historical Hamsters"); // Replace with your primitive name

var results = runModel({silent: true});

var sum = 0;
for(var t = 0; t < results.periods; t++){
    sum += results.value(historical)[t];
}

var average = sum/results.periods;

var nullError = 0;
var simulatedError = 0;
```

[1]Though this metric is not often used in systems dynamics or agent-based models, it is widely used for statistical models such as linear regressions.

```
for(var t = 0; t < results.periods; t++){
    nullError += Math.pow(results.value(historical)[t] - average, 2);
    simulatedError += Math.pow(results.value(historical)[t] - results.value(simulated)[t], 2);
}

showMessage("Pseudo R^2: "+((nullError-simulatedError)/nullError));
```

Calibrating the Model

In addition to using historical data to assess the model fit, you can also use historical data to calibrate model parameters. Depending on the model, you may have many parameters for which you do not have a good way to determine their values. Earlier, we discussed how to use sensitivity testing to assess whether our results are resilient to this uncertainty and to build confidence in the model. Another way to build confidence in your parameter values is, instead of guessing the values of these uncertain parameters, to choose the set of values that results in the best fit between simulated and historical data. This is a semi-objective criterion that helps to remove personal biases you might have from the modeling process.

Goodness of Fit

The first step to using historical data to calibrate the model parameters is to understand what is meant by "the best fit" between historical and simulated data. Conceptually, the idea of a "good fit" seems obvious. A good fit is one where the historical and simulated results are very close together (a *perfect* fit is when they are the same, but that is generally more than we can hope for). However, putting a precise mathematical definition on the concept is not trivial.

Many commonly used goodness of fit measures exist, and below we list some key ones.

Squared Error

Squared error is probably the most widely used of all measures of fit[2]. To calculate the squared error we carry out the following procedure. For each time period we take the difference between the historical data value and the simulated value and then we square that difference. We then sum up all these differences to obtain the total error for the fit. Higher totals indicate worse fits, and lower totals indicate better fits.

The following equation could be placed in a variable to calculate the squared error between a primitive named [**Simulated**] and one named [**Historical**]:

[2] The key reason for this is that regular linear regression (ordinary least squares, the most widely used modeling tool) uses squared error as its measure of goodness of fit. Doing so simplifies the mathematics of the regression problem greatly in the linear case.

```
([Simulated]-[Historical])^2
```

Please note that maximizing the R^2 measure we described earlier is equivalent to minimizing the squared error.

Absolute Value Error

A characteristic of squared error is that outliers have high penalties compared to other data points. Outliers are points in time where the fit is unusually bad. Since the squared error metric squares the differences between simulated and historical data, large differences can cause even larger amounts of error when they are squared. This can sometimes be a negative feature of squared error if you do not want to outliers to have special prominence and weight in the analysis.

An alternative to squared error that treats all types of differences the same is the absolute value error. Here, the absolute value of the difference between the simulated and historical data series is taken. The following equation could be placed in a variable to calculate the absolute value error between a primitive named [**Simulated**] and one named [**Historical**]:

```
Abs([Simulated]-[Historical])
```

Other Approaches

Many other techniques are available for measuring error or assessing goodness of fit. Most statistical approaches function by specifying a full probability model for the data and then taking the goodness of fit not as a measure of error, but rather as the *likelihood*[3] of observing the results we saw given the parameter values. To be clear the issue of optimizing parameter values for models is one that is more complex than what we have presented here. Many sources of error exist in time series and analyzing them is a very complex, statistical challenge. The basic techniques we have presented are, however, useful tools that serve as gateways towards further analytical work.

Exercise 13-1

You have a model simulating the number of widgets produced at a factory. The model contains a stock, [**Widgets**] containing the simulated number of widget produced. You also have a converter, [**Historical Production**] containing historical data on how many widgets were produced in the past.

Write two equations. One to calculated squared error for the models simulation of historical production, and one to calculate the absolute value error of the same.

Answer Available

[3] Likelihood is a technical statistical term. It can be roughly thought of as equivalent to "probability", though it is not precisely that.

Exercise 13-2

You like the idea of penalizing outliers in your optimizations. In fact, you like this idea so much that you would like to penalize outliers even more than squared error does. Create an equation to calculate error that penalizes outliers more than squared error.

Answer Available

Exercise 13-3

Describe why this is not a valid equation to calculate error:

```
[Simulated]-[Historical]
```

Answer Available

Multi-Objective Optimizations

So far our examples have focused on optimizing parameter values for a single population of animals. But what if, instead of one population, we had two or more?

Imagine we were simulation two interacting populations of animals such as the hamsters and their food source, the Hippo Toads. If we had historical data on both the toads and the hamsters we would like our choice of parameter values to result in the best fit between the simulated and historical hamster populations while at the same time resulting in the best fit between the simulated an historical toad populations. This is often quite difficult to achieve, as optimizing the fit for one population will often result in non-optimal fits for the second population.

A straightforward way to try to optimize both populations at once is to make our overall error the sum of the errors for the hamsters and the errors for the toads. For instance, if we had two historical data converters, one for the toads and hamsters, and two stocks, one for each population, the following equation would combine the absolute value errors for both populations.

```
Abs([Simulated Hamsters]-[Historical Hamsters]) + Abs([Simulated Toads]-[Historical Toads])
```

Simply summing the values can sometimes create issues in practice however. Let us imagine that the toad population is generally 10 times as large as the hamster population. If this were the case, the error predicting the toads might be much larger than the error predicting the hamsters and so the optimizer will be forced to focus on optimizing the toad predictions to the detriment of the accuracy of the hamster predictions.

One way to attempt to address this issue is to use the percent error instead of the error magnitude. For example:

```
Abs([Simulated Hamsters]-[Historical Hamsters])/[Historical Hamsters] + Abs([Simulated
```

The percent error metric will be more resilient to differences in scales between the different populations. It will run into issues though if either historical population becomes 0 in size or becomes very small.

Another wrinkle with multi-objective optimizations is that one objective may be more important than the other objectives. For instance, let's imagine our toad and hamster populations were roughly the same size so we do not have to worry about scaling. However, in this case we care much more about predicting the hamsters correctly than we care about the toads. The whole point of the model is to estimate the hamster population so we want to make that as accurate as possible, but we would still like to do well predicting the toads if we are able to.

You can tackle issues like these by "weighting" the different objectives in your aggregate error function. This is most simply done by multiplying the different objectives by a quantity indicating their relative importances. For instance, if we thought getting the hamsters right was about twice as important as getting the toads right, we could use something like:

```
2*Abs([Simulated Hamsters]-[Historical Hamsters]) + Abs([Simulated Toads]-[Historical
```

This makes one unit of error in the hamster population simulation count just as much as two units of error for the toad population simulation[4].

Exercise 13-4

Why does the percent error equation have issues when the historical data become very small? What happens when the historical data becomes 0?

Finding the Best Fit

After choosing how to measure the quality of a fit quantitatively, we need to find the set of parameter values that maximize the fit and minimize the error.

[4] Weighting is a useful technique you can use for other optimization tasks. Imagine you had a model simulating the growth of your business in the next 20 years. You want to use this model to adjust your strategy to achieve three objectives: maximizing revenue, maximizing profit, and maximizing company size. Potentially maximizing profit would be the most important objective with maximizing company size being the least important. You can use weights to combine these three criteria into a single criterion for use by the optimizer.

To do this we use a computer algorithm called an optimizer that automatically experiments with many different combinations of parameter values to find the set of parameters that has the best fit.

Many optimizers basically work by starting with an initial combination of parameter values and measuring the error for that combination. The optimizer then slightly changes the parameter values in order to check the error at nearby combinations of parameter values. For instance, if you are optimizing one parameter, say the hamster birth rate, and your initial starting value is a birth rate of 20% per year; the optimizer will first measure the error at 20% and then measure the errors at 19% and 21%.

If one of the neighbors has a lower error than the initial starting point, the optimizer will keep testing additional values in that direction. It will steadily "move" towards the combination of parameters that results in the lowest error, one step at a time. If, however, the optimizer does not find any nearby combination of parameter values with a lower error than its current combination of parameter values, it will assume it has found the optimal combination of parameter values and stop searching for anything better.

The precise details of optimization algorithms are not important. You need to be aware of one key thing however: these algorithms are not perfect and they sometimes make mistakes. The root cause of these mistakes are so-called "local minimums". An optimizer works by searching through combinations of different parameter values trying to find the combination that minimizes the error of the fit. The combination that has the smallest error out of all possible combinations is known as the true minimum or the "global" minimum.

A local minimum is a combination of parameter values that are not the global minimum, yet whose nearby neighbors all have higher errors. Figure 1 illustrates the problem of local minimum. If the optimizer starts near the first minimum in this figure it might head towards that minimum without ever realizing that another, improved minimum exists. Thus, if you are not careful, you may think you have found the optimal set of parameters when in fact you have only found a local minimum that might have much worse error than the true minimum.

There is no foolproof way to deal with local minimums and no guarantee that you have found the true minimum[5]. The primary method for attempting to prevent an optimization from settling in on a local minimum is to introduce stochasticity into the optimization algorithm. Optimization techniques such as *Simulated Annealing* or *Genetic Algorithms* will sometimes choose combinations of parameter values at random that are actually *worse* than what the optimizer has already found. By occasionally moving in the "wrong" direction, away from the nearest local minimum, these optimization algorithms are more resilient

[5] This is true for the type of optimization problems you will generally be dealing with. Other types of optimization problems are much easier than the ones you may be encountering, as they are what are known as *convex* optimization problems and are guaranteed not to have any local minimums.

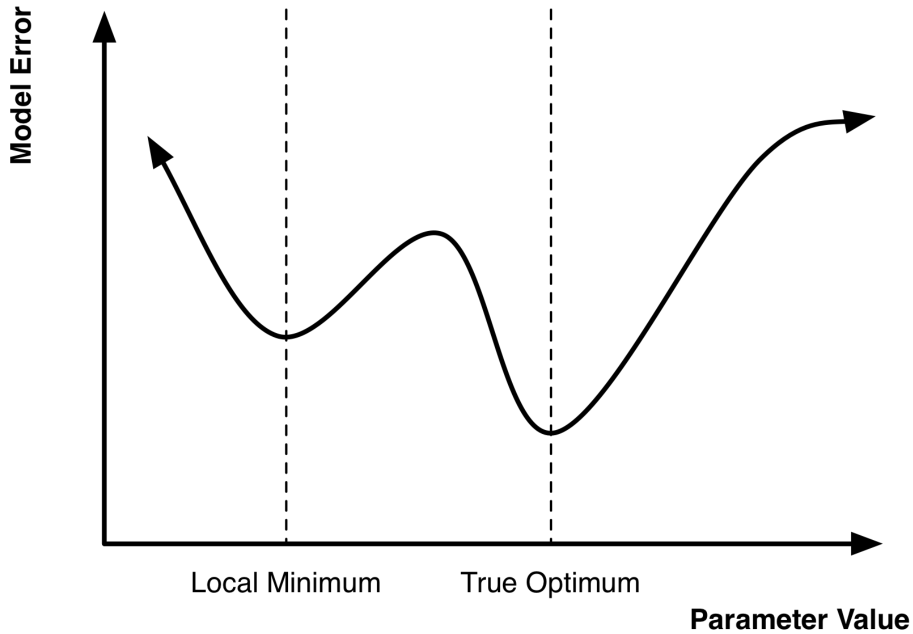


Figure 1. An illustration of local and global minimum for an optimization problem involving a single parameter.

and less likely to become stuck on a local minimum and more likely to keep searching for the global minimum.

Unfortunately, in our experience we have not been satisfied by the performance of these types of stochastic optimization algorithms. They are generally very slow and without fine-tuning by an expert can still easily become stuck in a local minimum. We prefer to use non-stochastic deterministic methods as the core of our optimizations. We then introduce stochasticity into the algorithm by using multiple random starting sets of parameter values. For instance, instead of carrying out a single optimization we will do 10 different optimizations each starting at a different set of parameter values. If all 10 optimizations arrive at the same final minimum that is strong evidence we have found the global minimum. If they all arrive at different minima, then there is a good chance we have not found the global minimum.

Optimizing Parameter Values

This model illustrates the use of optimization and historical data to select the growth rate for a simulated population of hamsters.

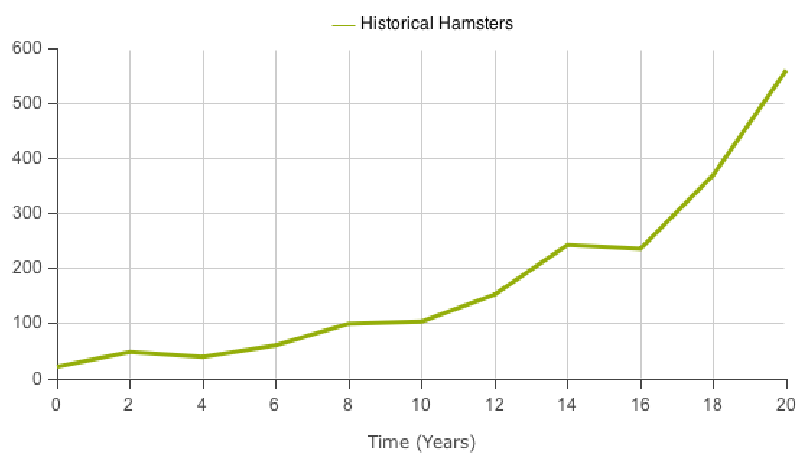
1. Create a new **Converter** named [**Historical Hamsters**].

2. Change the **Data** property of the primitive [**Historical Hamsters**] to `0, 22; 2, 49; 4, 40; 6, 61; 8, 100; 10, 104; 12, 153; 14, 243; 16, 236; 18, 370; 20, 560`.

3. The model diagram should now look something like this:

4. We start by importing our historical population data into a converter primitive. In this illustrative example we have twenty years of data with a census of the hamster population being carried out every two years. We run the model to see what this historical data looks like.

5. Run the model. Here are sample results:



6. There is a lot of variability and the population even declines some years. However, it looks like in general the rate of growth increases as the population size increases. This is what we would expect to see with exponential growth. Let's build a simple exponential growth model to attempt to replicate what we see with the historical data.

7. Create a new **Stock** named [**Hamsters**].

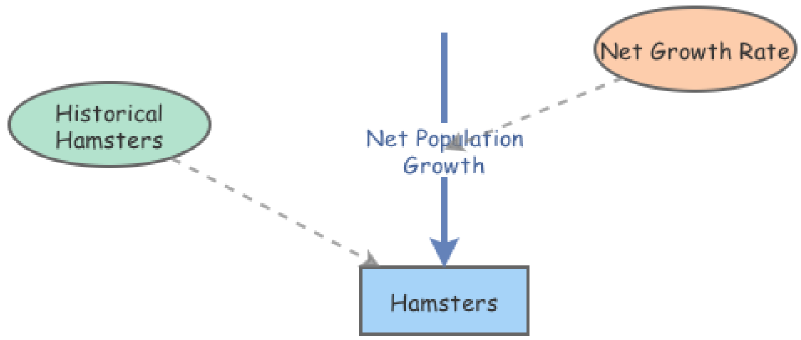
8. Create a new **Flow** going from empty space to the primitive [**Hamsters**]. Name that flow [**Net Population Growth**].

9. Create a new **Variable** named [**Net Growth Rate**].

10. Create a new **Link** going from the primitive [**Net Growth Rate**] to the primitive [**Net Population Growth**].

11. Create a new **Link** going from the primitive [**Historical Hamsters**] to the primitive [**Hamsters**].

12. The model diagram should now look something like this:



13. That's the structure of our model. Now we can fill in the equations. We'll set the initial population size for our simulated hamster population to be the same as for the historical data.

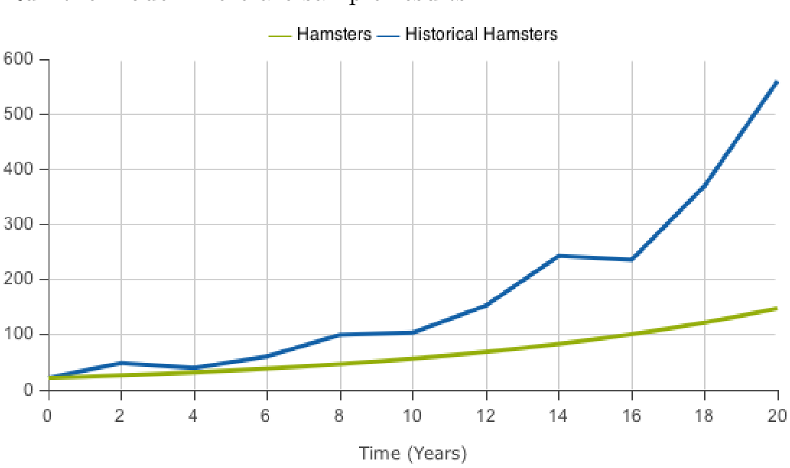
14. Change the **Initial Value** property of the primitive [**Hamsters**] to `[Historical Hamsters]`.

15. Change the **Flow Rate** property of the primitive [**Net Population Growth**] to `[Hamsters]*[Net Growth Rate]`.

16. What growth rate should we begin with? We do not have any data on this. Let's experiment by starting with 10% per year and see what we end up.

17. Change the **Equation** property of the primitive [**Net Growth Rate**] to `0.1`.

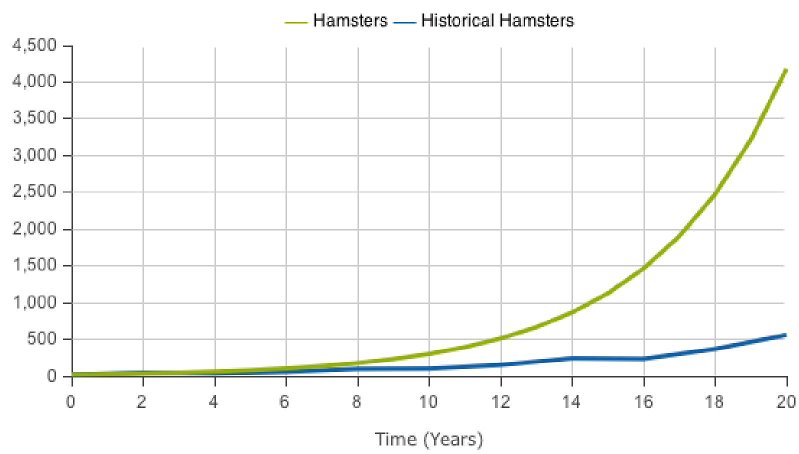
18. Run the model. Here are sample results:



19. That does not look too great. Our simulated population is much smaller than the historical values. Let's try a larger growth rate, say 30%.

20. Change the **Equation** property of the primitive [**Net Growth Rate**] to `0.3`.

21. Run the model. Here are sample results:



22. That's not good either, now our population is too large! We could keep experimenting with different growth rates to find a good one, but that might take a while. Let's just let the optimizer do the work for us. First we need to create a primitive to hold the error. We will use the squared error measure we discussed earlier.

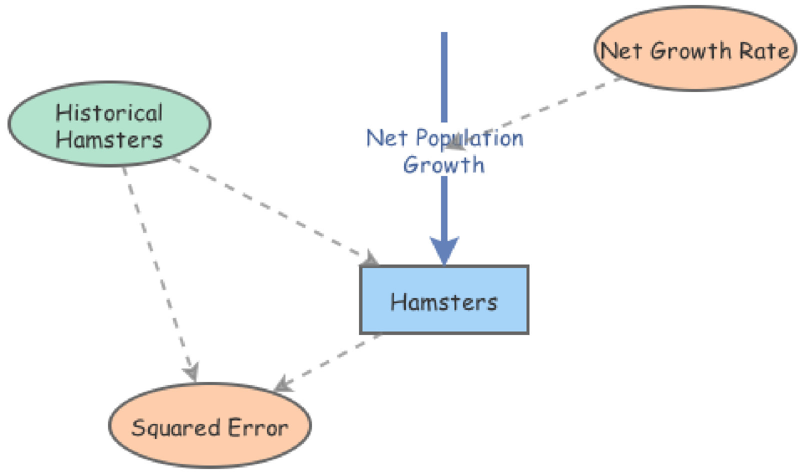
23. Create a new **Variable** named [**Squared Error**].

24. Create a new **Link** going from the primitive [**Hamsters**] to the primitive [**Squared Error**].

25. Create a new **Link** going from the primitive [**Historical Hamsters**] to the primitive [**Squared Error**].

26. Change the **Equation** property of the primitive [**Squared Error**] to `([Hamsters]-[Historical Hamsters])^2`.

27. The model diagram should now look something like this:



28. There, we have set up what we need for the optimizer to work. Now we can run the optimizer. We set the **Goal Primitive** to [**Squared Error**] and the **Primitive to Change** to [**Net Growth Rate**]. We tell the optimizer to minimize the integral of the error and set the optimizer to work.

29. The optimizer gets our results almost instantly: 0.172 or 17.2% is the optimal growth rate. When we run the model with this value the results look great. It's an almost perfect match between the historical and simulated data.

30. Change the **Equation** property of the primitive [**Net Growth Rate**] to `0.172`.

31. Run the model. Here are sample results:

Exercise 13-5

You are building a model to simulated company profits into the future. You have 30 years of historical company profit data you will use to calibrate parameter values using an optimizer.

Choose an error measure to use. Justify this choice and explain why you would use it instead of other measures.

Exercise 13-6

Calculate the pseudo R^2 for [**Growth Rate**] = `0.1`, `0.3`, and `0.172`.

Exercise 13-7

Adjust the JavaScript code to calculate pseudo R^2 to use absolute value error instead of squared error.

Answer Available

Exercise 13-8

Describe local minimum, why they cause issues for optimizers, and strategies for dealing with them.

The Cost of Complexity

After a good deal of work and many sleepless nights you have completed the first draft of your Aquatic Hamster population model. The results are looking great and your friend is really impressed. When he runs it by some colleagues however, they point out that your model does not account for the effects of the annual Pink Spotted Blue Jay migration.

Pink Spotted Blue Jays (PSBJ) are a species of bird that migrates every fall from northern Canada to Florida. In the spring they return from Florida to Canada. Along the way, they usually spend a few days by the lake where the Aquatic Hamsters have their last colony. During this time they eat the same Orange Hippo Toads the hamsters themselves depend upon as food. By reducing the Hippo Toad population, the PSBJ negatively affect the hamsters, at least for this period of time when there is less food available to support them.

The timing of the PSBJ migration can vary by several weeks each year no one knows precisely when the PSBJ's will arrive at the lake or even how long they will stay there. Further, the population of migrating birds can fluctuate significantly with maybe 100 birds arriving one year and 10,000 another year. The amount of toads they eat is proportional to the number of birds. Not much data exist quantifying the birds' effects on the hamsters, but it is a well-established fact that they eat the Hippo Toads the hamsters rely upon for their survival and many conservationists are concerned about the migration.

Your friend's colleagues wonder why you have decided to not include the PSBJ migration in your model. They want to know how they can trust a model that does not include this factor that clearly has an effect on the hamster population.

In response, you may point out that though the migration clearly has an impact, it appears to be a small one that is not as important as the other factors in the model. You add that there are no scientific studies or theoretical basis to define exactly how the migration functions or how it affects the hamster population. Given this, you think it is probably best to leave it out.

You say all this, but they remain unconvinced. "If there is a known process that affects the hamster population, it should be included in the model," they persist. "How can you tell us we shouldn't use what we know to be true in the model? We know the migration matters, and so it needs to be in there."

The Argument for Complexity

Your friend's colleagues have a point. If you intentionally leave out known true mechanisms from the model, how can you ask others to have confidence that the model is accurate? Put another way, by leaving out these mechanisms you ensure the model is wrong. Wouldn't the model *have* to be better if you included them?

This argument is, on the surface, quite persuasive. It is an argument that innately makes sense and appeals to our basic understanding of the world: Really it seems to be "common sense".

It is also an argument that is wrong and very dangerous.

Before we take apart this common sense argument piece by piece, let us talk about when complexity is a good thing. As we will show, complexity is not good from a modeling standpoint, but it can sometimes be a very good tool to help build confidence in your model and to gain support for the model.

Take the case of the PSBJ migration. It might be that adding a migration component to the model ends up *not* improving the predictive accuracy of the model. However, if other people view this migration as important, you may want to include the migration in the model if for no other reason than to get them on board. Yes, from a purely "prediction" standpoint it might be a waste of time and resources to augment the model with this component, but this is sometimes the cost of gaining support for a model. A "big tent" type model that brings lots of people on board might not be as objectively good as a tightly focused model, but if it can gain more support and adoption it might be able to effect greater positive change.

The Argument Against Complexity

Generally speaking, the costs of complexity to modeling are threefold. Two of them are self evident: there are computational costs to complex models as they take longer to simulate and there are also cognitive costs to complex models in that they are harder to understand. There is, however, a third cost to complexity that most people do not initially consider: complexity often leads to less accurate models compared to simpler models.

In the following sections we detail each of these three costs.

Computational Performance Costs

As a model becomes more complex, it takes longer to simulate. When you start building a model it may take less than a second to complete a simulation. As the model's complexity grows, the time required to complete a simulation may grow to a few seconds to a few minutes and then to even a few hours or more.

Lengthy simulation times can significantly impede model construction and validation. The agile approach to model development we recommend is predicated on rapid iteration and experimentation. As your simulation times cross beyond even something as small as 30 seconds, model results will no longer be effectively immediate and your ability to rapidly iterate and experiment will be diminished.

Furthermore, when working with an optimizer or sensitivity-testing tool, performance impacts can have an even larger effect. An optimization or sensitivity testing tool may run the model thousands of times or more in its analysis so even a small increase in the computation time for a single simulation may have a dramatic impact when using these tools.

Optimizations themselves are not only affected by the length of a simulation, they are also highly sensitive to the *number* of parameters being optimized. You should be extremely careful about increasing model complexity if this requires the optimizer to adjust additional parameter values. A simplistic, but useful, rule of thumb is that for every parameter you add for an optimizer to optimize, the optimization will take 10 times as long[6].

[6] In practice an optimizer should ideally perform a bit better than this, but this provides us a useful guideline to understand optimizations. Also it should be noted that the optimizations we are talking about here are for non-linear optimization problems for which gradients (derivatives) cannot be directly calculated. For other types of optimization problems, such as linear problems, much faster optimization techniques are available.

Thus if it takes one minute to find the optimal value for one parameter, then it takes 10 minutes to find the optimal values for two parameters and 100 minutes to find the optimal values for three parameters. Imagine we had built a model and optimized five parameters at once. We then increased the model complexity so we now had to optimize ten parameters. Our intuition would be that the optimization would now take twice as long. This is wrong. Using our power of ten rule we know that the time needed will be closer to 10^5 or 100,000 times as long!

That is a huge difference and highlights how important it is to keep model complexity at a manageable level. In practice, a rule of thumb is that you should have no difficulty optimizing one or two parameters at a time. As you add more parameters that optimization task becomes rapidly more difficult. At five or so parameters you have a very difficult but generally tractable optimization challenge. Above five parameters you may be lucky to obtain good results.

Cognitive Costs

In addition to the computational cost of complexity, there is also a cognitive cost. As humans we have a finite ability to understand systems and complexity. This is partly why we model in the first place: to help us simplify and understand a world that is beyond our cognitive capacity.

Returning to our hamster population model, including the bird migration could create a confounding factor in the model that makes it more difficult to interpret the effects of the different components of the model and extract insights from them. If we observe an interesting behavior in the expanded model we will have to do extra work to determine if it is due to the migration or some other part of the model. Furthermore, the migration may obscure interesting dynamics in the model making it more difficult for us to understand the key dynamics in the hamster system and extract insights from the model.

We can describe this phenomenon using a simple conceptual model defined by three equations. The number of available insights in a model is directly proportional to model complexity. As the model complexity increases, the number of insights available in the model also grows.

$$\text{Available Insights} \propto \text{Complexity}$$

Conversely, our ability to understand the model and extract insights from it is inversely proportional to model complexity. α is a constant indicating how much understandability decreases as complexity increases. This relationship is non-linear as each item added to a model can interact with every other item currently in the model. Thus, the cognitive burden increases exponentially as complexity increases.

$$\text{Understandability} \propto \alpha^{-\text{Complexity}}$$

The number of insights we actually obtain from a model is the product of the number of available insights and our ability to understand the model:

$$\text{Insights} = \text{Available Insights} \times \text{Understandability}$$

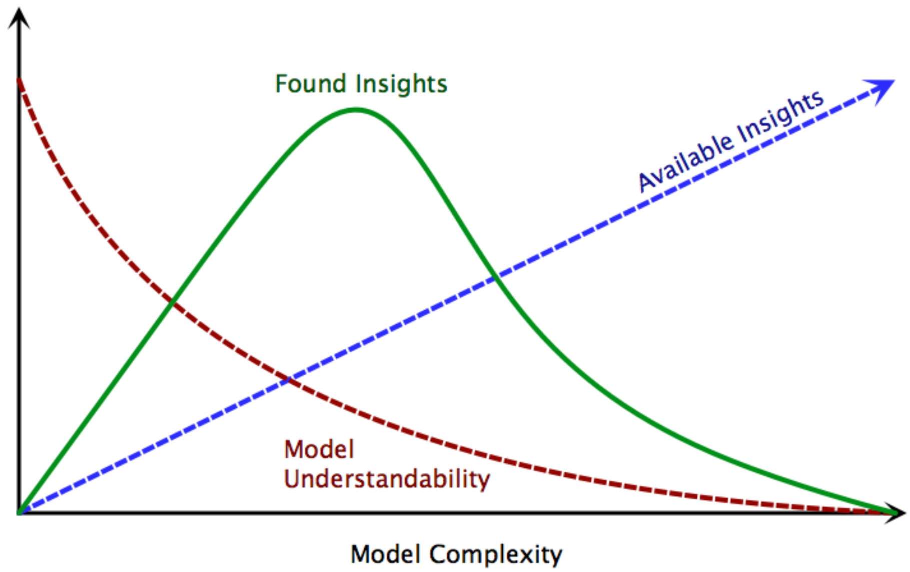


Figure 2. Expected discoveries of insights as model complexity increases.

Thus when the model complexity is 0 – in effect basically no model – we gain no insights from the model. As the model complexity starts to rise, we begin to gain additional insights. After a certain point however, the added model complexity actually inhibits additional understanding. As complexity rises our insights will fall back down towards 0. This phenomenon is illustrated in Figure 2.

Accuracy Costs

The negative effects of complexity on computational performance and our cognitive capacity should not be a surprise. What may be surprising on the other hand, is the fact that complex models are in fact often *less accurate* than simpler alternatives.

To illustrate this phenomenon, let us imagine that for part of our hamster population model we wanted to predict the size of the hamsters after a year[7]. The hamsters go through two distinct life stages in their first year: an infant

[7]Size could affect hamster survival and fecundity so it could be an important variable to model.

life stage that lasts 3 months and a juvenile life stage that lasts 9 months. The hamsters' growth patterns are different during each of these periods.

Say a scientific study was conducted measuring the sizes of 10 hamsters at birth, at 3 months and at 12 months. The measurements at birth and 12 months are known to be very accurate (with just a small amount of error due to the highly accurate scale used to weigh the hamsters). Unfortunately, the accurate scale was broken when the hamsters were weighed at 3 months and a less accurate scale was used instead for that period. The data we obtain from this study are tabulated below and plotted in Figure 3:

Hamster	Birth	3 Months	12 Months
1	9.0	23.2	44.4
2	9.7	19.8	44.0
3	10.2	23.5	44.7
4	8.8	32.2	43.3
5	10.1	31.3	44.5
6	10.0	27.2	44.2
7	10.0	21.4	46.1
8	11.1	24.1	46.0
9	8.7	41.0	44.9
10	11.2	31.7	43.8

Now, unbeknownst to us, there are a pair of very simple equations that govern Aquatic Hamster growth. During the infant stage they gain 200% of their birth weight in that three-month period. Their growth rate slows down once they reach the juvenile stage such that at the end of the juvenile stage their weight is 50% greater than it was when they completed the infant stage. Figure 3 plots this true (albeit unknown) size trajectory compared to the measured values. The higher inaccuracy of the measurements at 3 months compared to 0 and 12 months is readily visible in this figure by the greater spread of measurements around the 3 month period.

We can summarize this relationship mathematically:

$$\text{Size}_{t=3\text{ months}} = 3.00 * \text{Size}_{t=0\text{ months}}$$
$$\text{Size}_{t=12\text{ months}} = 1.50 * \text{Size}_{t=3\text{ months}}$$

Naturally, we can combine these equations to directly calculate the weight of the hamsters at 12 months from their weight at birth:

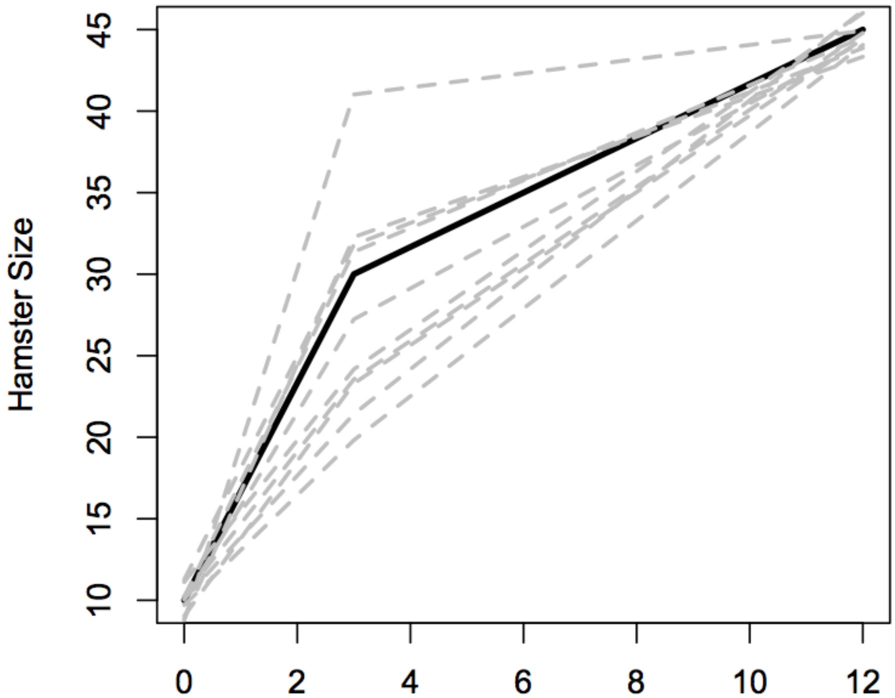


Figure 3. Recorded hamster sizes (dashed grey lines) and the unknown true size trajectory for a hamster starting with size 10 (solid black line).

$$\text{Size}_{t=12\text{ months}} = 4.50 * \text{Size}_{t=0\text{ months}}$$

Again, we don't know this is the relationship, so we need to estimate it from the data. All we care about is the size of hamsters at 12 months given their birth size. The simplest way to estimate this relationship is to do a linear regression estimating the final size as a function of the initial size. This regression would result in the following relationship:

$$\text{Size}_{t=12\text{ months}} = 4.65 * \text{Size}_{t=0\text{ months}}$$

This result is quite good. The linear coefficient we estimate of 4.65 is very close to the true value of 4.50. Our model so far is doing pretty well.

However, like with the bird migration, someone might point out that this model is too crude. "We know that the hamster go through an infant and juvenile stage", they might say, "we should model these stages separately so the model is more accurate."

This viewpoint moreover has actually been held to be the case in the law. For instance, there have been judicial decisions that "life-cycle" models, those that model each stage of an animal's life are the only valid ones[8]. If we were presenting this model in to an audience that believed that, we would have to create two regressions: one for the infant stage and one for the juvenile stage.

Using the data we have, we would obtain these two regressions:

$$\text{Size}_{t=3\text{ months}} = 2.74 * \text{Size}_{t=0\text{ months}}$$
$$\text{Size}_{t=12\text{ months}} = 1.54 * \text{Size}_{t=3\text{ months}}$$

Combining these regression to get the overall size change for the 12 months we obtain the following:

$$\text{Size}_{t=12\text{ months}} = 4.22 * \text{Size}_{t=0\text{ months}}$$

Now, in this illustrative example we are fortunate to know that the true growth multiplier should be 4.50 so we can test how well our regression actually were. The error for this relatively detailed life-cycle model is $(4.50 - 4.22)/4.50$ or 6.2%. For the "cruder" model where we did not attempt to model the individual stages, the overall error is $(4.50 - 4.65)/4.50$ or 3.3%.

So by trying to be more accurate and detailed, we built a more complex model that has almost twice the error of our simpler model! Let's repeat that: The more complex model is significantly worse in accuracy than the simpler model.

Why is that? We can trace the key issue back to the problem that our data for the 3 month period are significantly worse than our data for 0 months or 12 months. By introducing it into the model, we bring down the overall quality of the model by injecting more error into it. When someone comes to you asking you to add a feature to a model you have to consider if this feature may actually introduce more error into the model as it did in this example.

We can think of life-cycle and many other kinds of models as a chain. Each link of the chain is a sub-model that takes data from the previous link, transforms them and feeds them into the next link. Like a chain, models may only be as good as their weakest link. It is often better to build a small model where all the links are strong, than a more complex model with many weak links.

Exercise 13-9

Implement a model tracking the growth of a hamster from birth to 12 months. Create the model for a single hamster and then using sensitivity testing to

[8]Technically the determination is that life-cycle models are the "best available science". These decisions are misguided and frankly wrong, but that is what occurs when judges are put in the position of making highly technical scientific decisions.

obtain a distribution of hamster size. Assume hamster are born with an average size of 10 and a standard deviation of 1. Use the true parameter growth rates and do not incorporate measurement uncertainty in the model;

Exercise 13-10

Define a procedure for fitting a System Dynamics model of hamster growth to the hamster growth data in the table. Assume you know that their are two linear growth rates for the infant and juvenile stages but you do not know the values of these rates.

Answer Available

Exercise 13-11

Apply the optimization procedure to your System Dynamics model to determine the hamster rates of growth from the empirical data.

Overfitting The act of building models that are too complex for the data you have is known as "overfitting" the data[9]. In the model of hamster sizes, the model where we look at each life stage separately is an overfit model; We do not have the data to justify this complex of a model. The simpler model (ignoring the different stages) is superior.

Overfitting is unfortunately too common in model construction. Part of the reason is that the techniques people use to assess the accuracy of a model are often incorrect and inherently biased to cause overfitting. To see this, let's explore a simple example. Say we want to create a model to predict the heights of students in high schools (this is seemingly trivial, but bear with us). To build the model we have data from five hundred students at one high school.

We begin by averaging the heights of all the students in our data set and we find that the average student height is 5 feet 7 inches. That number by itself is a valid model for student height. It is a very simple model[10], but it is a model nonetheless: Simply predict 5 feet 7 inches for the height of any student.

We know we can make this model more accurate. To start, we decide to create a regression for height where gender is a variable. This gives us a new model

[9] The reverse – building models that are too simple – is called "underfitting". In practice, underfitting will be less of a problem as our natural tendency is to overfit.

[10] Statisticians would call this the "null" model, the simplest model possible.

which predicts women high-school students have a height of 5 feet 5 inches on average, while men have a height of 5 feet 9 inches on average. We calculated the R^2 for the model to be 0.21.

That's not bad, but for prediction purposes we can do better. We decide to include students' race as a predictor as we think that on average there might be differences in heights for different ethnicities. We complete this extended model including ethnic status as a predictor alongside gender and the R^2 fit of our model increases to 0.33.

We still think we can do better though, so we add age as a third predictor: We hypothesize that the older the students are, the taller they will be. The model including age as an additional linear variable is significantly improved with an R^2 of 0.56.

Once we have built this model, we realize that maybe we should not just have a linear relationship with age because as students grow older, their rate of growth will probably slow down. To account for this we decide to also include the square of age in our regression. With this added variable our fit improves to an R^2 of 0.59.

This is going pretty well, we might be on to something. But why stop with the square; what happens if we add higher order polynomial terms based on age? Why not go further and use the cube of age. The fit improves slightly again. We think we are on a roll and so we keep going. We add age taken to the fourth power, and then to the fifth power, and then to the sixth, and so on.

We get a little carried away and end up including 100 different powers of age and each time we add and new power our R^2 gets slightly better. We could keep going, but it's time to do a reality check.

Do really we think that including AGE^{100} made our model any better than when we only had 99 terms based on age? According to the R^2 metric it did (if only by a very small amount). However, we know intuitively it did not. Maybe the first few age variables helped, but once we get past a quadratic $(\text{AGE} + \text{AGE}^2)$ or cubic $(\text{AGE} + \text{AGE}^2 + \text{AGE}^3)$ relationship, we probably are not capturing any more real characteristics of how age affects a person's size.

Variables	R^2
Gender	0.21
Gender, Race	0.33
Gender, Race, Age	0.56
Gender, Race, Age^2	0.59
Gender, Race, $\text{Age}^2, \ldots, \text{Age}^{100}$	0.63
Gender, Race, $\text{Age}^2, \ldots, \text{Age}^{500}$	1.00

So why does our reported model accuracy – R^2 – keep getting better and better as we add these higher order power terms based on age to our regression?

This question is at the heart of overfitting. Let's imagine taking our exploitation of age to its logical conclusion. We could build a model with 500 different terms based on age ($\text{AGE} + \text{AGE}^2 + \text{AGE}^3 + ... + \text{AGE}^{500}$). The result of this regression would go through every single point in our population of five hundred students[11].This model would have a perfect R^2 of one (as it matches each point perfectly) but we know intuitively that it would be a horrible model.

Why is this model so bad? Imagine two students born a day apart one with a height of 6 feet 2 inches the other with a height of 5 feet 5 inches. Our model would indicate that a single day caused a 7-inch difference in height. Even more ridiculous, the model would predict a roller coaster ride for students as they aged. They would gain inches one day (according to the model) and lose them the next. Clearly this model is nonsensical. However, this nonsensical model has a perfect R^2, it is a paradox!

The key to unlocking the solution to the paradox and overcoming overfitting turns out to be surprisingly simple: *assess the accuracy of a model using data that were not used to build the model*.

The reason our overfit model for students looks so good using the R^2 error metric is that we measured the R^2 using the same data that we just used to build the model. This is an issue as we can force an arbitrarily high R^2 simply by continually increasing the complexity of our model. In this context the R^2 we are calculating turns out to be meaningless.

What we need to do is to find new data – new students – to test our model on. That will be a more reliable test of its accuracy. If we first built our model and then took it and applied it to a different high school and calculated the R^2 using this new data, we would obtain a truer measure of how good our model actually was.

Figure 4 illustrates the effect of overfitting using observation from 9 students. The top three graphs show plots of the heights and ages for these nine students. We fit three models to these data: a simple linear one, a quadratic polynomial, and an equation with nine terms so that it goes through each point exactly.

Below the three graphs we show the regular R^2 that most people use when fitting models, and also what the true R^2[12] would be if we applied the resulting model to new data. The regular R^2 always increases so if we used this naive metric we would always end up choosing the most complex model. As we can see, the true accuracy of the model decreases after we reach a certain

[11] Remember a polynomial equation with two terms can perfectly pass through two data points, an equation with three terms can perfectly pass through three points, and so on.

[12] You might have heard of R^2 variants such as the Adjusted R^2. The Adjusted R^2 is better than the regular R^2; however it is important to note that it is not the true R^2. Adjusted R^2 also has some issues with overfitting.

complexity. Therefore the middle model is really the better model in this case. When illustrated like this, this concept of overfitting should make a lot of sense; but, surprisingly, it is often overlooked in practice even by modeling experts.

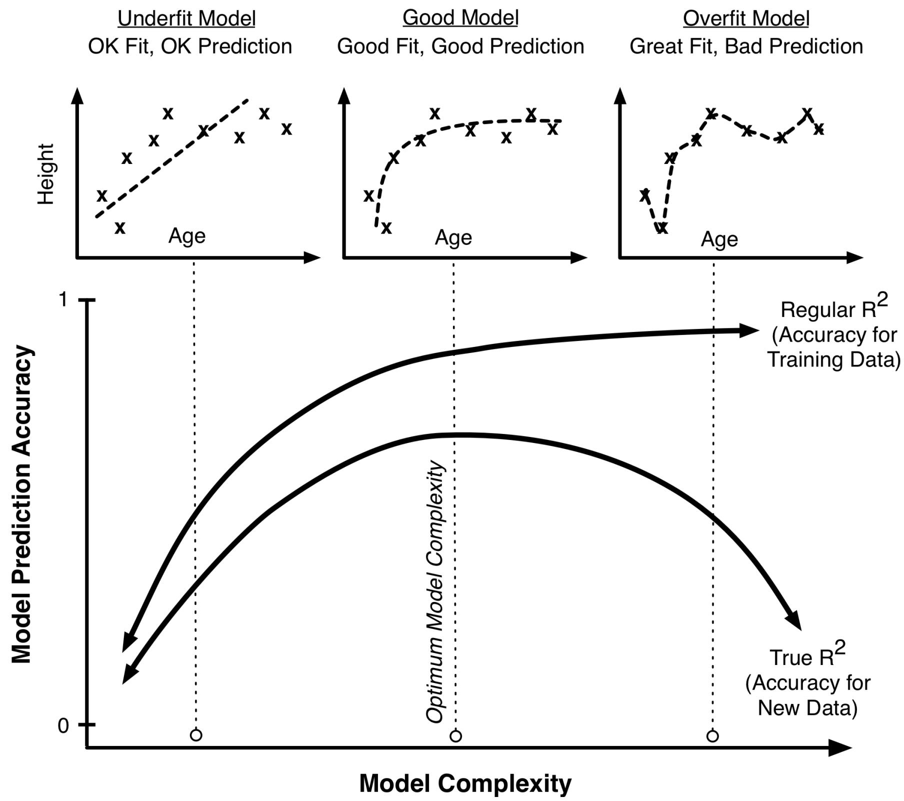


Figure 4. Illustration of overfitting. The best model is not necessarily the one that fits the data the closest.

In general, overfitting should be watched for carefully. If you do not have a good metric of model error, the inclination to add complexity to your model will be validated by misleadingly optimistic measures of error that make you think your model is getting better when it is actually getting worse. The optimization techniques we described earlier in this chapter are also susceptible to these problems as every time you add a new variable to be optimized the optimization error will always go down further (assuming the true optimal parameter configuration can be found). The more parameters you add the worse this effect will be.

How do we estimate the true error of the model fit? The simplest approach is to take your dataset and split it into two parts. Build the model with one half of the data and then measure the accuracy using the other half. So with our high-school students we would randomly assign each one to be used either to build the model or to assess the model's error. Advanced statistical techniques such as *cross-validation* or *bootstrapping* are other approaches and can be more effective given a finite amount of data. Unfortunately, we do not have space to discuss them here, but we would recommend the reader explore them on their own if they are interested in this topic.

No one ever got fired for saying, "Let's make this model more complex." After this chapter, we hope you understand why this advice, though safe to say, is often exactly the wrong advice.

Exercise 13-12

What is overfitting? What is underfitting?

Exercise 13-13

You have been asked to evaluate a model built by a consulting company. The company tells you that their model has an R^2 of 0.96 and is therefore a very accurate model.

Do you agree? What questions or tests do you need to do to determine if the model is good?

Chapter 14

Equilibria and Stability Analysis

This chapter extends on our mathematical analysis of models by introducing the concepts of points of equilibria and stability analysis. These types of analyses allow you to determine many behaviors of a system without needing to fully solving its differential equation model.

Although the trajectory for the state variables in differential equation models generally cannot be determined analytically, several key properties of models can often still be determined. These properties include:

- The location of equilibrium points
- The stability of the equilibrium points

An equilibrium point is defined as a set of state variable values that will cause the system to cease to change. Once the system enters an equilibrium configuration, it will not leave that configuration without an external stimulus. For instance, in our exponential growth model a single equilibrium point exists: that of zero people. If the population is empty, then the population will not grow and instead remain at 0 indefinitely.

In the exponential growth population model there is only one equilibrium point ($P = 0$). In other models you may have multiple equilibrium points. In a model of a highly infectious, incurable disease you can imagine a system where two equilibrium points exist: one where no one is infected and a second point where everyone is infected. As long as there were no infectious individuals, the population would remain healthy. If just a single infected individual were introduced into the population, the infection would, however, spread until everyone was infected and the population would then remain at that point (remember this hypothetical disease is incurable).

Multiple types of equilibria exist. Figure 1 illustrates what is known as the *stability* of equilibrium points. Each of the three panes in this figure show a different form of equilibrium for the ball. In all three the balls are in equilibrium:

if the no external forces come into play, the balls will not move. What differs in each of the three is what occurs if the balls are displaced a small amount.

Stable Equilibrium : In this type of equilibrium the ball will return to its original position if it is displaced. The structure of the system is such that the system is naturally attracted to the point of equilibrium. To use the physical metaphor, the equilibrium is at the bottom of a dip and the system naturally rolls into it.

Unstable Equilibrium : Here the ball will move further and further away from the point of equilibrium if it is displaced even a small amount. The equilibrium is unstable in that if we are just a small distance away from it, we move further away from it. To use the physical metaphor, the equilibrium is at the top of the hill and the system will move away from it unless it is placed at the exact point of equilibrium.

“Neutrally Stable Equilibrium” : This is a less common form of equilibrium and goes by several different names. In this case if the ball is moved it will stay fixed at its new location. It will not move closer to or further from the original equilibrium. Of the three types of equilibrium, this one is less interest or relevance in practice.

In the case of the highly infectious disease model, an equilibrium of everyone being healthy would be classified as an unstable equilibrium. The equilibrium would persist as long as no one brought the disease into the population (someone would not just spontaneously become ill), but if as little as a single sick person entered the population, the population would move further and further away from the equilibrium point of everyone being healthy and would never naturally return to it.

The equilibrium point of everyone being sick is, on the other hand, a stable equilibrium as no one recovers from the disease on their own. Even if you introduced healthy people into a population of sick individuals – moving the population away from the equilibrium – they too will eventually become sick restoring the population to the equilibrium of everyone being sick.

Exercise 14-1

Provide two examples each of situations where stable and unstable equilibria occur in nature. Describe these equilibria.

Answer Available

Equilibrium Points

Often, we can determine the equilibrium points for a system without fully needing to solve the trajectory for the state variables. Let’s implement the

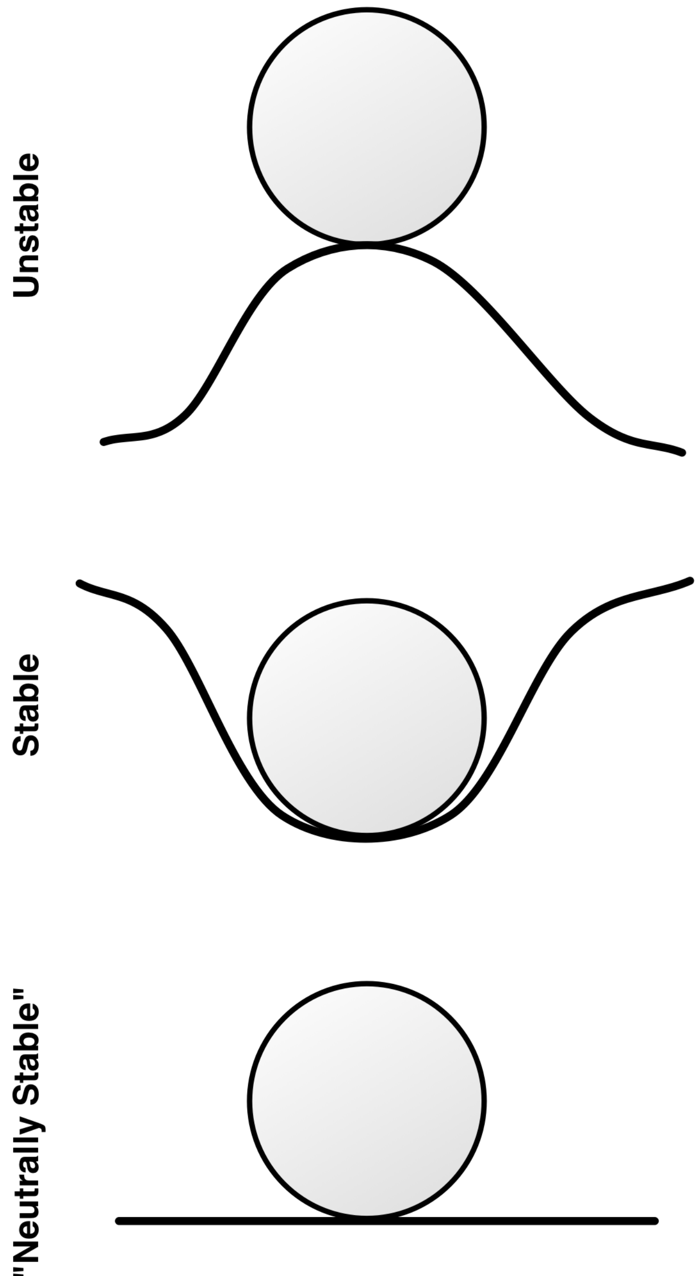


Figure 1. Three different types of stability.

simple disease model we've been discussing. We'll do so for both a differential equation model and a System Dynamics model, but we'll rely on differential equation version to do our analytic analysis.

One way to express the differential version of the model is to define two state variables: the number of healthy people (H) and the number of sick people (S). The rate of infection between sick and healthy people can be made a function of the number of people in each category. Clearly, if there are no sick people the infection rate is 0; but, just as clearly, if everyone is already sick then the infection rate will also be zero. One workable differential equation model to implement this behavior is shown below:

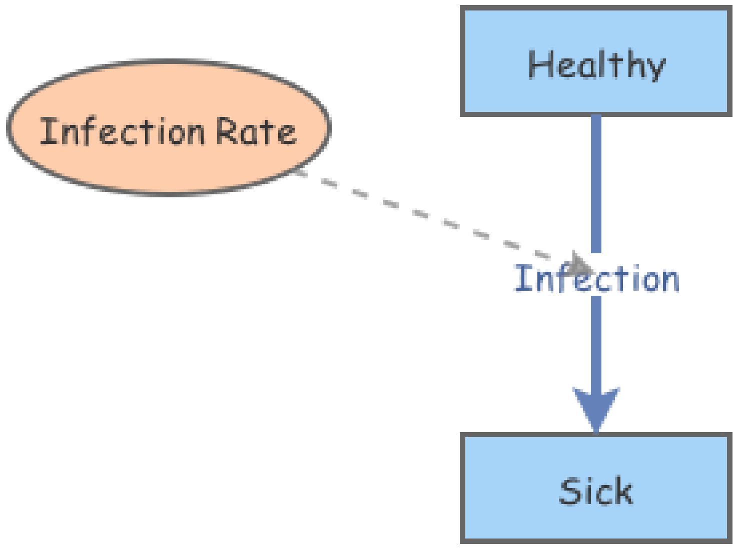
$$
\begin{aligned}
H(0) &= 100 \\
S(0) &= 1 \\
\frac{dH}{dt} &= -\alpha \times H \times S \\
\frac{dS}{dt} &= \alpha \times H \times S
\end{aligned}
$$

This model uses a single parameter (α) to control the infection rate. *alpha* is a non-zero positive value; the smaller α is, the slower the infection will progress and vice versa. This notation illustrates one of the clumsier aspects of implementing stock and flow models using differential equations. The flow values between two stocks have to be repeated twice once for each of the two connected state variable's derivatives.

Incurable Disease

This model illustrates stable and unstable equilibria using the scenario of an incurable disease in a population.

1. Create a new **Stock** named [**Healthy**].
2. Create a new **Stock** named [**Sick**].
3. Create a new **Flow** going from the primitive [**Healthy**] to the primitive [**Sick**]. Name that flow [**Infection**].
4. Create a new **Variable** named [**Infection Rate**].
5. Create a new **Link** going from the primitive [**Infection Rate**] to the primitive [**Infection**].
6. The model diagram should now look something like this:



7. This is the structure of our model. We have two stocks of people with people moving from the healthy stock to the sick stock as they become infected. Let's add the values and equations now.

8. Change the **Initial Value** property of the primitive [**Healthy**] to `100`.

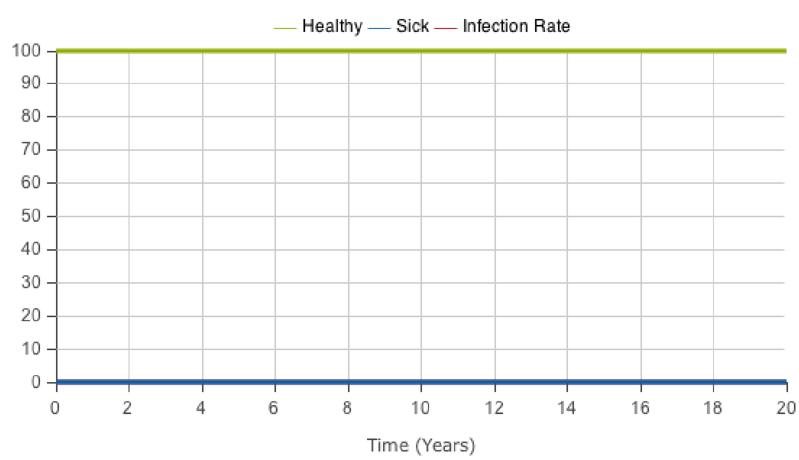
9. Change the **Initial Value** property of the primitive [**Sick**] to `0`.

10. Change the **Equation** property of the primitive [**Infection Rate**] to `0.01`.

11. Change the **Flow Rate** property of the primitive [**Infection**] to `[Infection Rate]*[Healthy]*[Sick]`.

12. There our model is fully setup. We've set it to start with everyone being healthy.

13. Run the model. Here are sample results:

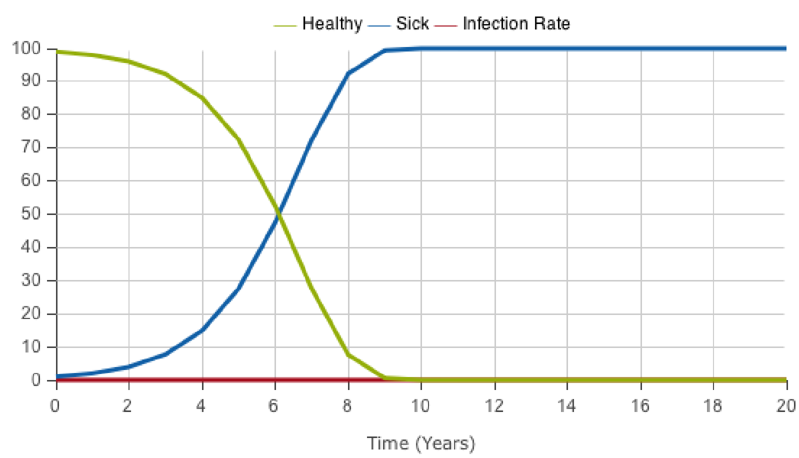


14. These results are quite stable. Everyone is healthy and no one gets sick. That indicates we have an equilibrium here. Let's now experiment by making a single person in the population sick.

15. Change the **Initial Value** property of the primitive [**Sick**] to `1`.

16. Change the **Initial Value** property of the primitive [**Healthy**] to `99`.

17. Run the model. Here are sample results:



18. That's more interesting! We can see that everyone being healthy is an unstable equilibrium as the system moves away from it if we deviate from it by even a small amount. We can also see that the second equilibrium (everyone being sick) is stable as the system moves towards it naturally.

Finding the equilibria for differential equation models is by-and-large straightforward analytically. We simply need to harness the definition of an equilibrium point: an equilibrium point is one where the state variables are constant and unchanging. Since the derivatives represent changes in the state variables, this statement is equivalent to saying the derivatives for the model are 0 at equilibrium points.

Based on this, in order to find the equilibrium points we simply need to set the derivatives in our model to 0 and solve the resulting equations. For the disease model we get:

$$
\begin{aligned}
H(0) &= 99 \\
S(0) &= 1 \\
0 &= -\alpha \times H \times S \\
0 &= \alpha \times H \times S
\end{aligned}
$$

The initial conditions will determine what equilibrium is arrived at but they do not affect the existence of the equilibria. Furthermore, the two equations we have set to 0 are equivalent[1] so we can simplify these equations to simply be:

$$0 = \alpha \times H \times S$$

Simple inspection reveals that this equation is true if and only if either $H = 0$, $S = 0$, or $\alpha = 0$. Thus we have mathematically shown that our equilibria are either when everyone is sick or everyone is healthy (or there is no infection whatsoever). Granted this is a trivial conclusion for this model and we stated it earlier. However, for more complex models this type of analysis can be very useful and will often reveal that equilibria are functions of the different parameter values in the model and they may enable you to explicitly determine how the equilibria changes as the model configuration changes.

Let's try a more complex example. Remember the predator prey model from earlier? We had the following set of equations to simulate the relationship between a moose and wolf population:

$$
\begin{aligned}
\frac{dM}{dt} &= \alpha \times M - \beta \times M \times W \\
\frac{dW}{dt} &= \gamma \times M \times W - \delta \times W
\end{aligned}
$$

Let's determine what the equilibrium values are for this model. As before, we start by setting the derivatives to 0:

$$
\begin{aligned}
0 &= \alpha \times M - \beta \times M \times W \\
0 &= \gamma \times M \times W - \delta \times W
\end{aligned}
$$

Solving this set of equations is more difficult than for the disease model. However a little bit of algebra reveals two solutions. One when $M = 0$ and $W = 0$ (there are no animals at all), and the second when $M = \delta/\gamma$ and $W = \alpha/\beta$. This is an example of where the equilibrium location depends on the values of the model parameters.

[1] Although we expressed this model as a function of two state variables H and S, it only has one independent state variable. Given the fixed population size, you know the value of H given S and vice versa.

Exercise 14-2

Find the equilibrium points for the system:

$$\frac{dX}{dt} = X^2 + X - 3$$

Answer Available

Exercise 14-3

Find the equilibrium points for the system:

$$\frac{dX}{dt} = \sin(X)$$

Answer Available

Exercise 14-4

Find the equilibrium points for the system:

$$\frac{dX}{dt} = 2 \times X + Y + 5$$
$$\frac{dY}{dt} = 3 \times X - 4 \times Y$$

Answer Available

Exercise 14-5

Find the equilibrium points for the system:

$$\frac{dX}{dt} = X^2 - Y$$
$$\frac{dY}{dt} = -2 \times X^2 - Y^2$$

Answer Available

Exercise 14-6

Do the locations of equilibria depend on the starting conditions? Does the system arriving at an equilibrium depend on the starting conditions?

Why or why not?

The Phase Plane

Up until now, when looking at model results we have been focused on time series plots and we have mainly been interested in the trajectory of the variables and stocks over time. For the mathematical analysis of differential equations, however, the primary graphical tool is not this time series plot; instead it is what is known as a phase plane plot.

Phase planes are almost like scatterplots. They show one of the state variables plotted against another of the state variables. A scatterplot could be used to show the path for these two variables over the course of a simulation. In the predator prey model the results of a scatterplot of the wolf and moose population will be an ellipsoid. The two populations will cycle continuously. A phase plane plot is similar to this, but rather than just showing one of these cycles for a given simulation run, the phase plane shows the trajectories for *all* combinations of moose and wolf population sizes.

Figure 2 illustrates a phase plane plot for the predator prey system. The trajectory for one set of parameter and state variable values is highlighted in red and, as expected, we see a continual oscillation. We can also see the trajectories for all the other combinations of state variables. We see that the system will always oscillate and the size of this oscillation depends on the initial conditions for the state variables. This illustration provides us with a good deal of information in a single graphic and the phase plane plot is a great way to summarize the behavior of a system with two state variables.

Let's quickly explore the phase plane plots for a simpler system than our predator prey model. Take a system consisting of two state variables[2] both of which grow (or decay) exponentially. These state variables will be assumed to be independent from each other so the value of one does not affect the value of the other:

[2] Just a helpful reminder if you are starting to get lost in some of this differential equation jargon. A "state variable" is just a stock. Return to the table at the beginning of this chapter to see how these terms relate to the system dynamics modeling terminology we have already learned.

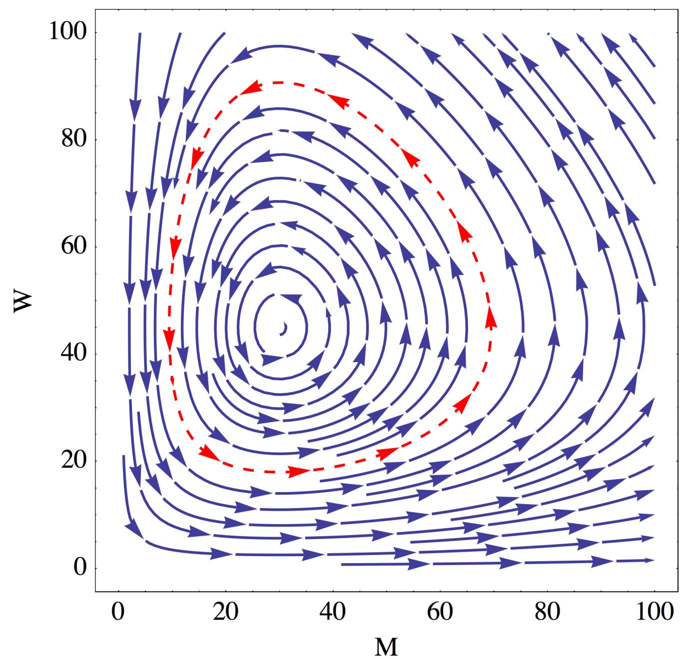


Figure 2. Predator prey phase plane plot. The trajectory for a single set of initial conditions is highlighted in red.

$$\frac{dX}{dt} = \alpha \times X$$
$$\frac{dY}{dt} = \beta \times Y$$

Clearly, there is an equilibrium point for this model at $X = 0$ and $Y = 0$. There are four general types of behavior around this equilibrium. One when $\alpha > 0$ and $\beta > 0$, one when $\alpha < 0$ and $\beta > 0$, one when $\alpha > 0$ and $\beta < 0$, and one when $\alpha < 0$ and $\beta < 0$. The phase planes for each of the four cases are shown in Figure 3.

From these plots we can visually determine how the stability of the equilibrium point at $X = 0, Y = 0$ changes as we change α and β. When $\alpha < 0$ and $\beta < 0$, we have a stable equilibrium; in all other cases we have an unstable equilibrium.

Exercise 14-7

Sketch out the phase plane for the differential equation model:

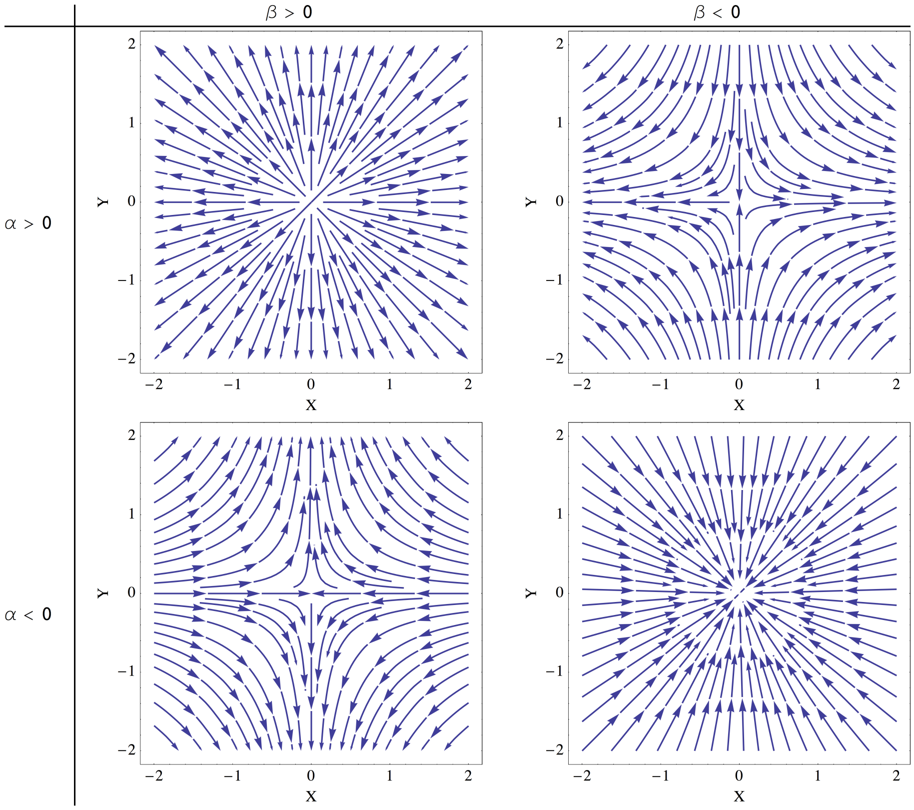


Figure 3. Phase planes for a simple two state variable exponential growth model.

$$\frac{dX}{dt} = -1$$
$$\frac{dY}{dt} = Y$$

Exercise 14-8

Sketch out the phase plane for the differential equation model:

$$\frac{dX}{dt} = X$$
$$\frac{dY}{dt} = Y^2$$

Stability Analysis

Now that we have learned how to analytically determine the location of equilibrium points, we may want to determine what type of stability occurs at these equilibria. As we stated earlier, for the incurable disease model it is trivial to arrive at the conclusion that the state of everyone being healthy is unstable while the state of everyone being sick is stable. In more complex models, it may be harder to draw conclusions or the stability of an equilibrium point may change as a function of the model's parameter values. Fortunately, there is a general way to determine the precise stability nature of the equilibrium points analytically.

The procedure to do this is relatively straightforward, but the theory behind it can be difficult to understand. The first key principle that must be understood is that of "linearization". To get a feel for linearization, let's take the curve in Figure 4. Clearly this curve is not linear. It has lots of bends and does not look at all like a line.

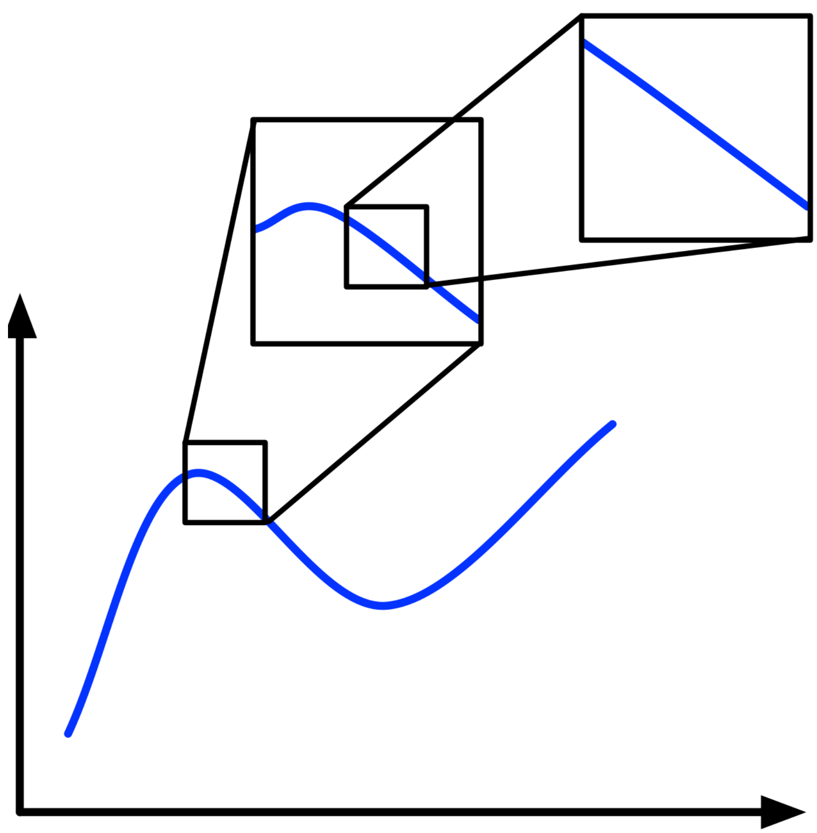


Figure 4. As we zoom in on a function it becomes more and more linear.

If we zoom in on any one part of the curve, however, the section we are zoomed in on starts to straighten out. If we keep zooming in, we will eventually reach a point where the section we are zoomed in on is effectively linear: basically a straight line. This is true for whatever part of the curve we zoom in on[3]. The more bendy parts of the curve will just take more zooming to convert them to a line.

We can conceptually do the same process for the equilibrium points in our phase planes. Even if the trajectories of the state variables in the phase planes are very curvy, if we zoom in enough on the equilibrium points, the trajectories at a point will eventually become effectively linear. The simple, two-state variable exponential growth model we illustrated with phase planes above are examples of a fully linear model. If we zoom in sufficiently on the equilibrium points for most models, the phase planes for the zoomed-in version of the model will eventually start to look like one of these linear cases.

Mathematically, we apply linearization to an arbitrary model by first calculating what is called the Jacobian matrix of the model. The Jacobian matrix is the matrix of partial derivatives of each of derivatives in the model with respect to each of the state variables:

$$\text{Jacobian} = \begin{bmatrix} \frac{\partial}{\partial X}X' & \cdots & \frac{\partial}{\partial Z}X' \\ \vdots & \ddots & \vdots \\ \frac{\partial}{\partial X}Z' & \cdots & \frac{\partial}{\partial Z}Z' \end{bmatrix}$$

The Jacobian is a linear approximation of our (potentially) non-linear model derivatives. Let's take the Jacobian matrix for the simple exponential growth model:

$$\frac{dX}{dt} = \alpha \times X$$
$$\frac{dY}{dt} = \beta \times Y$$

$$\text{Jacobian} = \begin{bmatrix} \frac{\partial}{\partial X}\alpha \times X & \frac{\partial}{\partial Y}\alpha \times X \\ \frac{\partial}{\partial X}\beta \times Y & \frac{\partial}{\partial Y}\beta \times Y \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

Exercise 14-9

Calculate the Jacobian matrix of the system:

[3] The one exception to this rule is if your curve is some sort of fractal. In this case no matter how much you zoom in on it, it will never become straight. In practice, however, this caveat is a non-issue.

$$\frac{dX}{dt} = X$$
$$\frac{dY}{dt} = Y^2$$

Answer Available

Exercise 14-10

Calculate the Jacobian matrix of the system:

$$\frac{dX}{dt} = X^2 - Y$$
$$\frac{dY}{dt} = -2 \times X^2 - Y^2$$

Answer Available

Exercise 14-11

Calculate the Jacobian matrix of the system:

$$\frac{dX}{dt} = X \times Y + \beta \times Y^2$$
$$\frac{dY}{dt} = \alpha \times X^3 + X^2 \times Y$$

Answer Available

This is complicated so don't worry if you don't completely understand it! Once you have the Jacobian, you calculate what are known as the eigenvalues of the Jacobian at the equilibrium points. This is also a bit complicated, so if your head is starting to spin, just skip forward in this chapter!

Nonetheless, eigenvalues and their sibling eigenvectors are an interesting subject. Given a square matrix (a matrix where the number of rows in equals the number of columns), an eigenvector is a vector which, when multiplied by the matrix, results is the original vector multiplied by some factor. This factor is known as an eigenvalue as is usually denoted λ. Given a matrix $\mathbf{A}$, an eigenvalue λ with associated eigenvector $\mathbf{V}$; the following equation will be true:

$$\mathbf{A} \times \mathbf{V} = \lambda \times \mathbf{V}$$

Let's look at an example for a 2×2 matrix:

$$\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \times \mathbf{V} = \lambda \times \mathbf{V}$$

What eigenvector and eigenvalue combinations satisfy this equation? It turns out there are two key ones:

$$\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 2 \times \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \end{bmatrix} = -1 \times \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Naturally, any multiple of an eigenvector will also be an eigenvector. For instance, in the case above, $[1, 0.5]$ and $[-2, 2]$ are also eigenvectors of the matrix.

We can interpret eigenvectors geometrically. Looking at the 2×2 matrix case, we can think of a vector as representing a coordinate in a two-dimensional plane: $[x, y]$. When we multiply our 2×2 matrix by the point, we transform the point into another point also in the two-dimensional plane. Due to the properties of eigenvectors, we know that when we transform an eigenvector, the transformed point will just be a multiple of the original point. Thus when a point that is on a matrix's eigenvector is transformed by that matrix, it will move inwards or outwards from the origin along the line defined by the matrix's eigenvector.

We can now relate the concept of eigenvalues and eigenvectors to our differential equation models. Take a look back at the phase planes for the exponential model example. For each of the phase planes, there are at least two straight lines of trajectories. In these cases the x- axis and the y-axis are the locations of these trajectories. If you have a system on the x- or y-axis in this example it will remain on that axis as it changes. This indicates that for this model, the eigenvectors are the two axises as a system on either of them does not change direction as it develops. That's the definition of of an eigenvector.

For our purposes though, we do not really care about the actual direction or angle for these eigenvectors. We instead care about whether the state variables move inwards or outwards along these vectors. We can determine this from the eigenvalues of the Jacobian matrix. If the eigenvalue for an eigenvector is negative, then the values move inwards along that eigenvector; while if the eigenvalue is positive, they move outward along the eigenvector.

These eigenvalues tell us all we need to know about the stability of the system. Returning to our illustration of stability as a ball on a hill, we can think of these eigenvalues as being the slopes of the hill around the equilibrium point. If the eigenvalues are negative, the ground slopes down towards the equilibrium point forming a cup (leading to a stable equilibrium). If the eigenvalues are positive, the ground slopes away from the equilibrium point creating a hill (leading to an unstable equilibrium).

Eigenvalues can be calculated straightforwardly for a given Jacobian matrix. Briefly, for the Jacobian matrix J, the eigenvalues λ are the values that satisfy the following equation where *det* is the matrix determinant and I is the identity matrix.

$$0 = det(J - \lambda \times I)$$

We can do a quick example of calculating the eigenvalues for the Jacobian matrix we derived for our two-state variable exponential growth model.

$$\begin{aligned} 0 &= det\left(\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} - \lambda \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \\ &= det\left(\begin{bmatrix} \alpha - \lambda & 0 \\ 0 & \beta - \lambda \end{bmatrix}\right) \\ &= (\alpha - \lambda) \times (\beta - \lambda) - 0 \times 0 \\ \lambda = \alpha, \lambda = \beta \end{aligned}$$

That is a fair amount of work to do. It's even more complicated if you have more than two state variables. However, once you have gone through the calculations and determined the linearized eigenvalues for your equilibrium points, you know everything you might want to know about the stability of the system.

Exercise 14-12

Find the eigenvalues of the following matrix:

$$\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$$

(Bonus: Determine the associated eigenvectors.)

Answer Available

Exercise 14-13

Find the eigenvalues of the following matrix:

$$\begin{bmatrix} 2 & 0 \\ 5 & 1 \end{bmatrix}$$

(Bonus: Determine the associated eigenvectors.)

Answer Available

Exercise 14-14

Find the eigenvalues of the following matrix:

$$\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$$

(Bonus: Determine the associated eigenvectors.)

Answer Available

Exercise 14-15

$$\begin{bmatrix} \alpha & \beta \\ 0 & \beta \end{bmatrix}$$

(Bonus: Determine the associated eigenvectors.)

Answer Available

In the exponential growth model we can see that when the eigenvalues are both negative we have a stable equilibrium (refer to the graphs we developed earlier), while if either one is positive (or they both are) we have an unstable equilibrium. This makes a lot of sense as if either one is positive it pushes the system away from the equilibrium making it unstable. While if they are both negative then they both push the system towards the equilibrium point. Visualize the ball sitting in the cup or on the hill.

Looking at it this way, we realize that *all we need in order to understand the stability of an equilibrium point are the eigenvalues of the Jacobian at* *the equilibrium point.* This is an incredibly powerful tool. It reduces the complex concept of stability, into an analytical procedure that can be applied straightforwardly.

Let's now look at some more examples.

First let's take our simple disease model from earlier. If you recall that model was:

$$\frac{dH}{dt} = -\alpha \times H \times S$$
$$\frac{dS}{dt} = \alpha \times H \times S$$

First let's calculate the Jacobian for this model. We take the partial derivatives of each of the two derivatives with respect to each of the two state variables to create a two-by-two matrix:

$$\text{Jacobian} = \begin{bmatrix} \frac{\partial}{\partial H} - \alpha \times H \times S & \frac{\partial}{\partial S} - \alpha \times H \times S \\ \frac{\partial}{\partial H} \alpha \times H \times S & \frac{\partial}{\partial S} \alpha \times H \times S \end{bmatrix} = \begin{bmatrix} -\alpha \times S & -\alpha \times H \\ \alpha \times S & \alpha \times H \end{bmatrix}$$

Next, we evaluate this Jacobian at one of our equilibrium points. Let's choose the one where the $S = 0$ (no one is sick) and $H = P$ (where P is the population size) so everyone is healthy:

$$\begin{bmatrix} 0 & -\alpha \times P \\ 0 & \alpha \times P \end{bmatrix}$$

We can now find the eigenvalues for this matrix. Once we go through the math we get two eigenvalues: 0 and $\alpha \times P$. What do these mean? Well, since one of the eigenvalues is positive, this indicates we have movement away from the equilibrium point along at least one of the eigenvectors. The other vector has no movement (0 as the eigenvalue), but this one positive value will ensure we have an unstable equilibrium. Again, think of the ball, the positive eigenvalue indicates the ground slopes downwards from the equilibrium point so a ball balanced on top of this hill will be very unstable.

Now let's do the second equilibrium. The one where $S = P$ and $H = 0$ (everyone is sick). Let's evaluate the Jacobian at this equilibrium:

$$\begin{bmatrix} -\alpha \times P & 0 \\ \alpha \times P & 0 \end{bmatrix}$$

Now let's find the eigenvalues for this matrix. Once we go through the math we get two eigenvalues: this time 0 and $-\alpha \times P$. Again, the 0 eigenvalue can be ignored as it does not cause growth or change. The second eigenvalue however is negative, indicating the system moves toward the equilibrium point again. Look back at our exponential growth phase planes. Negative coefficients indicate trajectories towards the equilibrium (create a cup for the ball). Thus this second equilibrium is a stable one.

It's time to look at a more complex example, we'll consider our predator prey model. First we calculate the Jacobian matrix for this model:

$$\text{Jacobian} = \begin{bmatrix} \frac{\partial}{\partial M}\alpha \times M - \beta \times M \times W & \frac{\partial}{\partial W}\alpha \times M - \beta \times M \times W \\ \frac{\partial}{\partial M}\gamma \times M \times W - \delta \times W & \frac{\partial}{\partial W}\gamma \times M \times W - \delta \times W \end{bmatrix} = \begin{bmatrix} \alpha - \beta \times W & -\beta \times M \\ \gamma \times W & \gamma \times M - \delta \end{bmatrix}$$

Now that we have the Jacobian, we'll evaluate it at the trivial equilibrium of $M = 0$ and $W = 0$. The resulting matrix is:

$$\begin{bmatrix} \alpha & 0 \\ 0 & -\delta \end{bmatrix}$$

The eigenvalues of this matrix are α and $-\delta$. Thus one of the eigenvectors approach the equilibrium and the other moves away from it. This means we have an unstable equilibrium, which is actually good news as it indicates that the two animal populations will not spontaneously go extinct.

Let's now evaluate the more complex equilibrium point we identified earlier of $M = \delta/\gamma$ and $W = \alpha/\beta$. First we calculate the Jacobian at this point:

$$\begin{bmatrix} 0 & \frac{-\beta \times \delta}{\gamma} \\ \frac{\gamma \times \alpha}{\beta} & 0 \end{bmatrix}$$

When we calculate the eigenvalues for this point we obtain $i\sqrt{\alpha \times \delta}$ and $-i\sqrt{\alpha \times \delta}$. Here the i indicates the imaginary number $\sqrt{-1}$. That's a little strange, so how do we interpret this? Well, it turns out that imaginary numbers in the eigenvalues indicate oscillations in the phase planes, thus this results means we have oscillations around the point of equilibrium. Since we have no real component in the eigenvalues, there is neither attraction towards the point of equilibrium or repulsion away from it so we have a stable oscillation around the equilibrium.

Of course we already knew that from our simulations, but this stability analysis allows us to mathematically determine this relationship, a capability that is a very powerful tool. The following table summarizes the different types of eigenvalues that can be found for a system with two state variables and their associated stabilities.

Real Parts	Imaginary Part?	Stability
Both Equal to 0	No	Neutrally Stable
Both Equal to 0	Yes	Stable Oscillations
Both greater than or equal to 0	No	Unstable
Both greater than or equal to 0	Yes	Unstable Oscillations
Both less than or equal to 0	No	Stable
Both less than or equal to 0	Yes	Damped Oscillations (Stable)

Exercise 14-16

A system's Jacobian matrix has two eigenvalues at an equilibrium point. Determine the stability of the system at this point for the following pairs of eigenvalues:

1. 0.5 and 4
2. -3 and 0.2
3. -3 and -1

Answer Available

Exercise 14-17

A system's Jacobian matrix has two eigenvalues at an equilibrium point. Determine the stability of the system at this point for the following pairs of eigenvalues:

1. $1 + 2i$ and $1 - 2i$
2. $-3 + 0.2i$ and $-3 - 0.2i$
3. $0.2i$ and $0.2i$

Answer Available

Exercise 14-18

A system's Jacobian matrix has a single eigenvalue at an equilibrium point. Determine the stability of the system at this point for the following eigenvalues:

1. 2.5
2. -1.2
3. 0.5

Answer Available

Analytical vs. Numerical Analysis

The majority of this book has been focused on the numerical analysis of models and the qualitative conclusions that can be drawn from these results. This chapter has introduced a set of analytical tools that can be used – for the most part – to analyze the same models we have presented elsewhere in the book. Now take a moment to reflect on these different forms of analysis and what each one can offer.

The great benefit of the analytical techniques we present here is that they can provide precise answers to the general behavior of the system. Most of these same answers can also be determined numerically (e.g. running the simulation many times and exploring the results), but those answers will be less precise and definite. If you manually attempt to explore the parameter space of your model, it is possible that you could miss some set of parameter values that will give you unexpected behavior. An analytical analysis may be fully comprehensive and can guarantee the completeness of your conclusions.

A weakness of analytical methods is that your model must be solvable analytically. This means that you will probably need to keep your model from growing too complex in order to keep it analytically tractable. Also, some common functions such as `IfThenElse` logic can make analytical work much more difficult. Further, some models may simply be impossible to analyze analytically and these insolvable models may in fact be very simple in practice. For example, any model containing both X and $\log(X)$ in the same equation will be intractable to many forms of analysis.

We think both analytical and numerical work has a lot of applicability in practice. We do worry, though, about some of the analytical models and work we see presented or published. Sometimes these models seem to us to be much too simple to adequately represent the system they are supposed to be modeling. True, analytically the results of the models appear elegant and clear, but if the model is too simple to be relevant these results have little use and may actually be very misleading in practice. We worry sometimes that a focus on analytical work[4] leads to modelers prioritizing analytical tractability over model utility in their decisions. We believe a focus on analytical results can lead to reductionist models with reduced practical utility and we caution modelers against becoming too focused on elegant solutions and the expense of relevance. Where available, more realistic models are preferable, even if they require numerical solutions than overly simplistic analytically solvable ones.

Exercise 14-19

What are the equilibrium points of the following system and their associated stabilities?

$$\frac{dX}{dt} = X \times Y + X^2$$
$$\frac{dY}{dt} = Y + 2$$

Answer Available

Exercise 14-20

What are the equilibrium points of the following system and their associated stabilities? α is a scalar number that may be positive or negative.

$$\frac{dQ}{dt} = -XQ \times R + R$$
$$\frac{dR}{dt} = \alpha - \alpha \times R^2$$

Answer Available

Exercise 14-21

You have a system dynamics model of a population of wolves. This model consists of a single stock [**Wolves**] (initial value 100), a single flow going into the stock [**Net Growth**], a parameter [**Growth Rate**] (value of 0.05), and a parameter [**Carrying Capacity**] (value of 6,000). The flow has the equation `[Growth Rate]*[Wolves]*(1-[Wolves]*[Carrying Capacity])`.

[4] And, rightly or wrongly, analytical work is generally considered more prestigious and "serious" than numerical work.

Build this model determine the location of the equilibria and their stability. Then prove these conclusions analytically.

Answer Available

Chapter 15

Exercise Answers

This section contains answers to selected exercises.

Chapter 7

Exercise 7-1

It would be better to build a statistical model in this case.

Exercise 7-2

It would be better to build a mechanistic model in this case.

Exercise 7-5

1. Prediction
2. Inference
3. Prediction
4. Narrative
5. Narrative
6. Inference

Chapter 8

Exercise 8-1

Minimum value: 0

Maximum value: 10,000,000 (this value is somewhat arbitrary but should be larger than the maximum size you expect this city to ever grow to)

Exercise 8-2

We use a standard deviation of 4 as we lack any information on what the dispersion should be.

```
Round(Rand(5, 15))
```

Exercise 8-3

```
Round(RandTriangular(0, 100, 20))
```

Exercise 8-4

```
Round(RandLogNormal(20, 4))
```

We use a standard deviation of 4 as we lack any information on what the dispersion should be.

Exercise 8-5

```
RandNormal(2.1, 0.3625)
```

Exercise 8-6

```
RandNormal(0.837, 0.106)
```

Chapter 10

Exercise 10-1

You can denote volume of water in the jar using the state variable J. Our equations will then be:

$$J(0) = 40$$
$$\frac{dJ}{dt} = -0.10 \times J$$

Exercise 10-2

You can denote the healthy stock using state variable H and the infected stock I. Our equations will then be:

$$H(0) = 100$$
$$I(0) = 1$$
$$\frac{dH}{dt} = -0.05 \times H \times I$$
$$\frac{dI}{dt} = 0.05 \times H \times I$$

Exercise 10-3

Approximately 8,865 animals.

Exercise 10-4

$$P = 10 - \alpha \times t$$

Exercise 10-5

$$P = 10 \times e^{0.05 \times t}$$

Exercise 10-6

$$P = \frac{20}{1 - 20 \times \beta \times t}$$

Exercise 10-7

20.0, 25.0, 29.0, 32.4, 35.5, 38.3

Exercise 10-8

20.0, 27.0, 37.5, 53.9, 78.3, 124.5

Exercise 10-9

20.0, 24.5, 28.3, 31.6, 34.6, 37.4

Exercise 10-10

20.0, 29.1, 44.7, 73.6, 131.5, 260.4

Chapter 11

Exercise 11-1

```
This <b>text is <i>italic</i> and bold.</b>
```

Exercise 11-2

Ordered list:

```
<ol>
    <li>Croatia</li>
    <li>Greece</li>
    <li>Peru</li>
</ol>
```

Unordered list:

```
<ul>
    <li>Croatia</li>
    <li>Greece</li>
    <li>Peru</li>
</ul>
```

Exercise 11-4

```
u {
    color: green;
}
```

Exercise 11-5

```
a {
    border: solid 2px red;
}
```

Exercise 11-6

```
var a = prompt("Enter the first number:");
var b = prompt("Enter the second number:");
var sum = a+b;

alert("There sum is: "+sum);
```

Exercise 11-7

```
h1 {
    text-decoration: underline;
}
```

Exercise 11-8

```
body {
    background-color: azure;
}
```

Exercise 11-10

```
input {
    background-color: yellow;
    color: navy;
}
```

Exercise 11-11

Change the alert to:

```
alert("Failed! You need "+(5000000000-pop)+" more people!");
```

Chapter 12

Exercise 12-2

1. Timeout trigger with value 10 days.
2. Probability trigger with value 20% (assuming time units of years).
3. Condition trigger. Value: `[Volume] > 5`

Exercise 12-4

```
Max(Filter(«2, 1.8, 1.9, 1.5», x < 1.95))
```

Exercise 12-5

```
Median(a^2)
```

Exercise 12-6

```
Min(Intersection(a, b))
```

Exercise 12-7

```
Sum(a)/Count(a)
```

Exercise 12-8

```
FindState(FindState([Population], [Infected]), [Female])
```

Exercise 12-9

```
Union(FindNotState([Population], [Infected]), FindState([Population], [Female]))
```

Exercise 12-10

```
Mean(Value(FindState([Population], [Male]), [Height]))-Mean(Value(FindState([Population], [Femal
```

Exercise 12-11

```
MoveTowards([Self], FindFurthest(FindState([Population, [Healthy]), [Self]), {1 Meter})
```

Exercise 12-12

```
range(x) <- max(x)-min(x)
```

or

```
Function Range(x)
    Max(x)-Min(x)
End Function
```

Exercise 12-13

```
Function Fib(n)
    If n = 1 or n = 2 Then
        1
    Else
        Fib(n-1) + Fib(n-2)
    End If
End Function
```

The 15th Fibonacci number is 610.

Chapter 13

Exercise 13-1

Squared error:

```
([Widgets]-[Historical Production])^2
```

Absolute value error:

```
Abs([Widgets]-[Historical Production])
```

Exercise 13-2

```
([Simulated]-[Historical])^4
```

Exercise 13-3

The optimizer can always minimize this simply making [**Simulated**] as small as possible. This will not result in a fit to the historical data.

Exercise 13-7

Change:

```
nullError += Math.pow(results.value(historical)[t] - average, 2);
simulatedError += Math.pow(results.value(historical)[t] - results.value(simulated)[t],
```

To:

```
nullError += Math.abs(results.value(historical)[t] - average);
simulatedError += Math.abs(results.value(historical)[t] - results.value(simulated)[t]);
```

Exercise 13-10

Example procedure:

1. Find the average hamster size at each time period by taking the mean of observations at that period.
2. Define two variables in the model: [**Infant Rate**] and [**Juvenile Rate**].
3. Define an error primitive [**Error**] the equation taking the absolute value of the difference between the simulated size and the average empirical size.
4. Run the optimizer to minimize this error term by adjusting the two rate variables.

Chapter 14

Exercise 14-1

Stable Equilibria: A piece of rubber that returns to its original shape after pulled, a forest where trees grow back once cut down.

Unstable Equilibria: A ball balanced on top of a sloped roof, a pole balanced perfectly on the floor.

Exercise 14-2

$X = -2.30$ and $X = 1.30$

Exercise 14-3

$X = 0, \pi, 2\pi, 3\pi, 4\pi, ...$

Exercise 14-4

$X = -1.82, Y = -1.36$

Exercise 14-5

$X = 0, Y = 0$ and $X = -1.41, Y = -2$ and $X = 1.41, Y = -2$

Exercise 14-9

$$\begin{bmatrix} 1 & 0\,0 & 2 \times Y \end{bmatrix}$$

Exercise 14-10

$$\begin{bmatrix} 2 \times X & -1 & -4 \times X & -2 \times Y \end{bmatrix}$$

Exercise 14-11

$$\begin{bmatrix} Y \times X & X + 2 \times \beta \times Y & 3 \times \alpha \times X^2 + 2 \times X \times Y & X^2 \end{bmatrix}$$

Exercise 14-12

Eigenvalue of 6 with eigenvector of $[1, 1]$. Eigenvalue of -2 with eigenvector of $[-1, 1]$.

Exercise 14-13

Eigenvalue of 2 with eigenvector of $[1, 5]$. Eigenvalue of 1 with eigenvector of $[0, 1]$.

Exercise 14-14

Eigenvalue of $\alpha - \beta$ with eigenvector of $[-1, 1]$. Eigenvalue of $\beta + \alpha$ with eigenvector of $[1, 1]$.

Exercise 14-15

Eigenvalue of α with eigenvector of $[1, 0]$. Eigenvalue of β with eigenvector of $\left[\frac{-\beta}{\alpha - \beta}, 1\right]$.

Exercise 14-16

1. Unstable
2. A saddle (unstable)
3. Stable

Exercise 14-17

1. Unstable oscillations
2. Damped oscillations (stable)
3. Stable oscillations

Exercise 14-18

1. Unstable
2. Stable
3. Unstable

Exercise 14-19

Equilibrium $X = 2, Y = -2$ is unstable.

$X = 0, Y = -2$ is an unstable saddle point.

Exercise 14-20

Equilibrium $Q = 1, R = 1$ is stable if $\alpha \geq 0$. Otherwise it is unstable.

$Q = 1, R = -1$ is unstable.

Exercise 14-21

The first equilibrium has no wolves and is unstable.

The second equilibrium is when the population size is equal to the carrying capacity. This equilibrium is stable.

Chapter 16

References