

[DRAFT] Beyond Connecting the Dots: Mastering the
Hidden Connections in Everything that Matters

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Chapter 1

Preface

Ludwig von Bertalanffy(1) first proposed, in 1937, that the same basic structures operated across all disciplines, and if one learned how these structures operated one could transfer much of their learning from one discipline to another. When moving from one discipline to another, one would simply have to learn the structures that were operating, and the labels on the elements of the structures. On first reading this may seem most profound, or maybe even preposterous.

However, if you think about it, maybe there is some truth to it after all. What follows is the introduction to a live Systems Thinking book presented from a cross discipline models perspective. Live in the sense that the models are presented in a form that allows you to actually interact with them.

von Bertalanffy wrote “Allegemein Systemlehre” which was translated into English as “General Systems Theory”(2) and I expect we’ve still not recovered from the translation error. What he intended was a “General Theory of Systems” or “General Systems Teaching,” a way to support learning about the structures which operated across all disciplines. Today there are a set of structures referred to as Systems Archetypes which I believe are just what Bertalanffy had in mind.

In the words of von Bertalanffy, “The student in ‘system science’ receives a technical training which makes systems theory – originally intended to overcome current overspecialization – into another of the hundreds of academic specialties”(1)

Systems Thinking is not a method though more of a way of looking at the world around us and understanding based not from understanding things though more from understanding relations and interactions between things. And while there are many who believe that Systems Thinking or a Systems Perspective provides the best foundation for creating effective approaches of dealing with challenges and shaping a better tomorrow. Yet even with that view, over the past 75 years it has not become widely adopted, even though during that period dozens of approaches have been developed with claim to embrace the Systems Thinking world view. I believe Pogo had it right when he said, “We have met

the enemy and he is us.” I have repeatedly commented to people that the greatest impediment to the adoption of Systems Thinking is Systems Thinkers.

This should provide you with a sense of why this book has to be different. Now let me offer you a view of how it will be different.

It is our intent to provide a basis for recovering from this overspecialization by offering an extensive series of models from everyday life that will show the value of looking at things though a different lense. We will then build on this to develop an understanding without all the terminology and complexity that typically drives people away from Systems Thinking.

References

- Davidson, Mark. 1983. Uncommon Sense: The Life and Thought of Ludwig von Bertalanffy <http://www.amazon.com/Uncommon-Sense-Thought-Bertalanffy-1901-1972/dp/087477165X/>

Chapter 2

Chapter 1 - It's The Pattern That Connects - v2 13.04.08

Notes to Self!

remove it and it's and I most everywhere.

If you give people the answers there is no need for them to practice insight and the purpose of BCTD is to encourage the practice of gaining insight into why systems behave the way they do.

Notes to Reviewers

v2 Modifications

- Ladder of Inference section removed so there is no Figure 4 at the moment.
- The embedded Insight Maker are more interactive because of an enhancement Scott has made to Insight Maker. Please run the models in the web page.
- The comparison structures model of Figure 23 has been completely overhauled.

Chapter Intent

Develop an awareness that the diverse world around us has a commonality that can be meaningfully represented by just a few interacting elements with rather simple attributes. The basic operation and interaction with embedded models must also be experienced and supporting aspects of Insight Maker explained.

Figure Captions

Each figure is followed by a sequenced figure caption line that starts with ** and these lines are also an internet link. These lines are inserted so I can easily get back to wherever that graphic originated should I need to create a revised version of it. These statements will be deleted by the post processor and replaced with figure captions which are embedded in the Markdown formatting.

Insight Maker References

I'm doing the best I can representing the version of Insight Maker I won't be able to see for a couple of months. The interactive Insight Maker models are embedded from Insight Maker and the model is owned by me. This means that when one looks at it in this chapter it doesn't look like it will look in the final book. Scott is creating a version of Insight Maker that will operate in a touch tablet environment. That version of Insight Maker will be embedded in the book and each book owner will own the models in the book. That means they will look different. As such I have to code something so I'll know what to include later and reviewers can look at and connect with the written words associated with it. Getting through this seems to be a tall order.

Macros

There are certain aspects of the text formatting we don't have figured out and have resigned ourselves to the fact that we won't have this figured out for some time. As such macros are being coded to be replaced in the content post processing phase. I sorry that it's likely to make the text a bit more difficult to read.

- *model attribute*
- *§*
- *equation*
- *model primitive*
- *ui reference*

Relation to Table of Contents

What follows was presented in the Table of Contents as three separate chapters though the writing seemed to get away from me. The may be split into several chapters or the table of contents may be corrected. Presently it's a bit difficult to tell.

What you learn, and your capacity to learn, serves as the basis for everything you do in your life. Yet, have you ever thought about how you really learn

about the world around you? Yes, there are some things you memorize early in life, like the times tables, and you learn to remember these, though is that really learning? Do you remember that if you put your hand on something very hot it will burn you, or is that something you learned? And if you learned that, how was it that that learning happened?

Consider the following

- I have a box that's about 3' wide, 3' deep and 6' high
- It's a rather heavy box
- The has a couple of doors on it
- When you open the doors it's cooler inside the box than outside
- One compartment is much colder than the other
- When you open the door a light comes on
- There's food inside the box
- The box is in the kitchen
- There are sticky notes all over the front of the box
- There's a collection of papers and stuff on top of the box
- If you move the box you'll probably find a lot of dust under it
- The box is plugged into an electrical outlet
- From time to time you can hear the box running

At some point in this sequence you probably became convinced that what was being described was a refrigerator. Now stop for a moment and ask yourself just how was it that you realized what was being described was a refrigerator? Yes it would have been easier if I had just shown you a picture of a refrigerator, though that would have spoiled it, wouldn't it.

As long as you knew beforehand what a refrigerator was, the statements could have been given to you in any order, and still at some point you would have finally realized what was being described. If you had never seen, nor heard about, a refrigerator before you would still be wondering what was being described and what to call it.

You have also most likely come to understand that all refrigerators are not identical. Some have one door with a separate compartment inside. Some have two doors and a drawer. Some are much smaller than others. Some can fit under a counter and some even fit on top of a counter. Some can be so large you can walk into them.

If you see any of these you quickly decide it's a refrigerator. How does that happen? Gregory Bateson, one of the great thinkers of our time, said, "It's the pattern that connects." If you reflect on this statement you should come to realize there are actually different ways to interpret what it means. In this particular case the pattern connects you to the following purpose



Figure 1. From the description you knew it was a refrigerator - but how?



Figure 2. Many kinds of refrigerators, or freezers - But how do you know?

- The box keeps food from readily spoiling by keeping it cold
- Part of the box is a freezer which keeps food from spoiling for even longer

and you understand it to be a refrigerator. Though now that we've arrived at this point we still haven't addressed the question of how you know. You probably were not actually taught that it's the above purpose that defines the essence of a refrigerator. Most people were not, though they have essentially learned it over time.

Models

Models are the way we look at, and understand the world around us. All we have are our models. They are the way we understand everything. This is so because we build our understanding based on what we already understand. The world around us simply has too much detail for us to pay attention to everything. A refrigerator has many pieces though how many do you really pay attention to? Probably not many unless you build or repair refrigerators. We filter out much of the detail around us so we don't become overloaded and we choose what to pay attention to. Sometimes we do this consciously and sometimes subconsciously. In the midst of what we choose to pay attention to there are patterns. Whether we realize it or not it is these patterns that we pay attention to and attempt to make sense of. We understand these patterns by linking them to extend patterns we already understand. And much of the world around us we simply ignore for if we didn't we would just become overwhelmed.

Remember

A model is a simplified version of some aspect of the world around us to help us understand something.

Learning

When we experience something that experience falls somewhere between complete novelty, meaning that we can't connect it with anything in our past experience, and complete confirmation, meaning that it represents something we perceive as already completely understood. The things we experience which lie somewhere between complete novelty and complete confirmation provide a basis for learning. They represent a basis for connecting to understood patterns, extending our understanding, and what results is learning. {Cite: Jantach, Eric. 1980. The Self-Organizing Universe: Scientific and Human Implications. Pergamon Press. <http://www.amazon.com/The-Self-Organizing-Universe-Implications-Innovations/dp/0080243118/>}

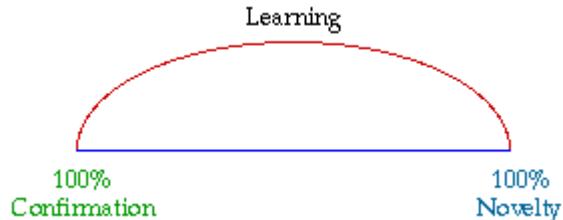


Figure 3. Experience between novelty and confirmation as a basis for learning

Consider running into a refrigerator that looks like no refrigerator you've never seen before. From an initial view you are likely not to perceive it as a refrigerator. As you inspect it to find it serves the purpose you've come to understand for refrigerators or if someone tells you it's a refrigerator you then expand or extend your awareness of the range of patterns that constitute a refrigerator.

A Basis for Flawed Learning

While reading the previous paragraphs did it dawn on you that much of this pattern recognition/connection/extension learning doesn't happen consciously? We connect with patterns and extend our knowledge at times without even being consciously aware that it is happening. And when it happens in an subconscious manner there isn't really any critical validation that happens

along with the learning. Because this ongoing learning happens without critical validation there are things we learn and come to believe which are actually incorrect. We have perceived patterns and extended our learning in a flawed manner. The really annoying thing is that we then act on these beliefs, and when we produce results that don't go the way we planned we wonder why. Or even worse, we don't actually learn from the results and correct our flawed models.

When we act on flawed beliefs when attempting to solve problems we typically create more problems than we fix. It has been said repeatedly that the majority of today's problems are the direct result of yesterday's solutions. Wouldn't this provide a sense that we might really benefit from a better way to think about the world around us, develop better understanding, and develop solutions that don't come back to haunt us in the future?

A Better Way

Based on the understanding I hope you've developed to this point it should be obvious that we could benefit from a better way to develop models of what we believe that are more likely to be correct as well as surface flaws in many of our current beliefs.

Ludwig von Bertalanffy first proposed, in 1937, that the same basic structures operated across all disciplines, and if one learned how these structures operated one could transfer much of their learning from one discipline to another.{Davidson, Mark. 1983. Uncommon Sense: The Life and Thought of Ludwig von Bertalanffy. J.P. Tarcher, Inc. <http://www.amazon.com/Uncommon-Sense-Thought-Bertalanffy-1901-1972/dp/087477165X/>} When moving from one discipline to another, one would simply have to learn the structures that were operating, and the labels on the elements of the structures. On first reading this may seem most profound, or maybe even preposterous. However, if you think about it, maybe there is some truth to it after all.

I'm not asking you to believe the previous statement just because it was provided here. Though if you give me a few minutes the experience that follows may allow you to arrive at a sensibility of the statement from your own perspective.

Consider the images in Figure 5 and ask yourself what it is that all these different items actually have in common.

Each of these items represents a collection of stuff. Admittedly each image represents different stuff though stuff just the same. Because in each case this stuff collected over time it's really more appropriate to refer to the the collections as accumulations. And as you will come to realize it is extremely important to remember that accumulations take time to accumulate, and often even longer to get rid of when you find out you don't want them.

The shorter term often used to refer to an accumulation is "stock." Just where this term originate I'm unsure and what you call an accumulation of stuff isn't

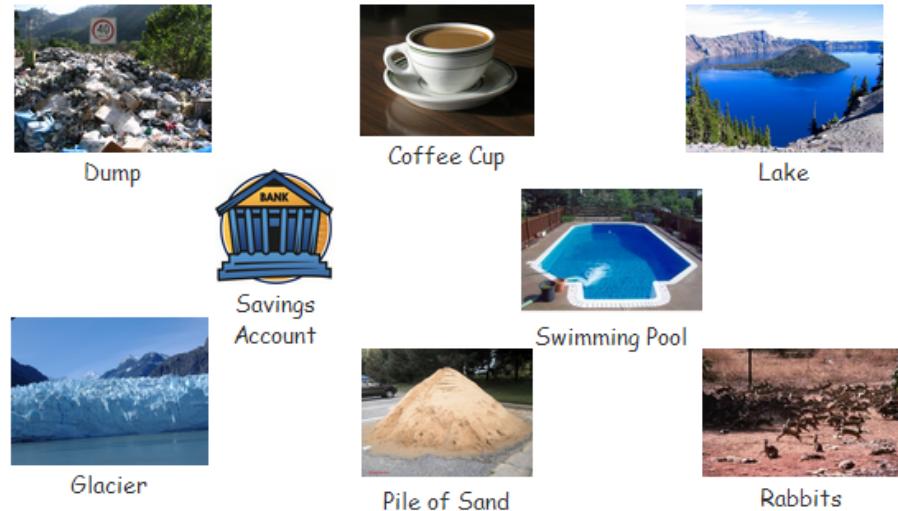


Figure 5. What do these items have in common?

nearly as important as remembering it's a bunch of stuff that collected over time. How much time is different for each one of the accumulations and now it's probably time to talk about how accumulations happen over time.

For each of the accumulations in Figure 5, how they change is a bit different, as are the time frames concerned. Time frame being the time it takes for some real noticeable change in the accumulation. Let me describe each in some detail.

Coffee Cup

You usually fill a coffee cup from a coffee pot and it takes a few seconds. Then you take a few minutes to drink the coffee as it's usually too hot to drink when you initially get it.

Dump

Generally a dump accumulates by the truckload after the garbage is picked up at houses or businesses in your community. If the dump were just getting started you'd probably notice it grow with each additional truck load. As it gets bigger and bigger it's gets more difficult to notice that it's growing, even though it is. While the dump is likely to grow almost every day we are probably more likely to think about the growth of the dump in months and years. And does it ever really go away? Usually when it gets to be too much a new dump is started somewhere else and the current dump is buried. Though when it's buried it doesn't really go away. It's still there and we'll probably talk more about dumps later on.

Glacier

A glacier is a long term accumulation of snow which packs down and turns to ice. Glaciers get bigger in the winter when snow falls and they get smaller in summer when some portion of the glacier melts. The time frame one usually uses to think about glaciers is years or even decades.

Lake

Lakes are bigger than a pond and smaller than an ocean and usually filled with fresh water, not salty that is. The lake is filled by rivers and streams that flow into it as well as rain water. One might think of this in terms of gallons per hour or gallons per minute in the case of a large inflow such as at Niagara Falls where the water flows into Lake Ontario in the USA. Water leaves the lake through rivers and streams as well as evaporation into the air. For a lake one might think about the water flowing into our out of the lake in hours though when considering the level of the lake itself the change might be considered over days or weeks. It sort of depends on what you're interested in.

Pile of Sand

The pile of sand probably showed up in a truck that dumped it right where it is. While it may have taken the truck a while to drive from the wherever it started it probably only took a couple of minutes to dump the truck once it arrived. And the sand is probably referred to in cubic yards, which is how much sand it takes to fill a box that's 1 yard wide, 1 yard deep, and 1 yard high. How long it takes for the sand to go away depends on how it's taken away. If you use a wheel barrow then you have to shovel the sand into the wheel barrow and take it to wherever you're going to use it. At this rate it may take days to move it. If you move it with a small piece of machinery, a Bobcat or a Backhoe, then will will probably only take a few minutes to an hour to get it moved.

Rabbits

A population of rabbits gets larger with new rabbit births and gets smaller with rabbit deaths. Have you ever heard the phrase "multiply like rabbits?" What it means is that it doesn't take very long for a few rabbits to become many rabbits, as long as there is a good food supply and not too many predators like wolves and coyotes. The time frame for considering a rabbit population is probably months to years.

Savings Account

A savings account is a bank account where if you put money and if you keep it there the bank will periodically give you money just for keeping it there. They won't give you very much, though some. If you keep putting money in your savings account every so often and never take it out one day you'll be rich. Yet,

for some reason that doesn't happen to too many people. We'll have to talk about that sometime later in the book. One generally thinks about the money associated with a savings account in dollars, the interest rate as a percentage, and the time frame in months and years.

Swimming Pool

Swimming pools usually hold thousands of gallons of water and you usually have a couple of options to fill one. You might use a garden hose, which will take days, or a hose from a fire hydrant, which will take a few hours, or from a tanker truck, which probably takes a few loads. In each case the water filling the pool is probably measured in gallons per hour. Once you fill the pool you lose a little water when people get in and out of the pool, thought not too much. Most of the water loss from a pool is through evaporation due to the sun and when you backwash the filter used to keep the pool clean. The change in amount of water is usually measured in gallons per hour.

Exercise

Take a few minutes and identify half a dozen situations you're familiar with where there are stocks that accumulate over time. What are the quantities for those stocks, e.g., gallons, pounds, kilograms, etc.. What are the flows that increase and decrease them and what are the time frames over which you think about the accumulation of that stock?

At this point you may be wondering why so much time was spent making you walk through all these examples for the accumulation of stuff. Since we said this was an interactive book you're probably wondering where the interaction is.

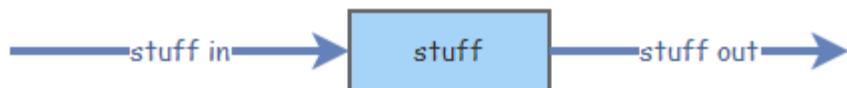


Figure 6. The Accumulation of Stuff

All the accumulations depicted in Figure 5 can be represented in a general form by the model in Figure 6. Remember we defined a model as a simplified version of some aspect of the world around us to help us understand something. It doesn't get much simpler than this does it?

Some amount of stuff flowing in causes stuff to increase over time and stuff flowing out causes stuff to decrease over time. With both of these happening at the same time stuff increases if stuff in is larger than stuff out. And if stuff out is greater than stuff in then the accumulation of stuff gets smaller. The most critical aspect of this to remember is that it takes time for stuff to increase or decrease. How fast the change happens depends on the amount of stuff in the flows.

Lets take a specific instance. Figure 7 represents Figure 6 in Insight Maker, an interactive modeling environment. We'll talk about how this was done shortly. Now suppose we have a swimming pool and we start filling it with a hose that fills at 50 gallons an hour. If we let the hose run for 24 hours how much water will be in the pool? Admittedly the math is pretty straight forward though the idea here is to show how you can use a model to show changes over time.

If I set up the model in Figure 7 with stuff = 0, stuff in = 50 and stuff out = 0, set the Time Settings for 24 hours, and then click the Run button, the model produces the graph in Figure 8.

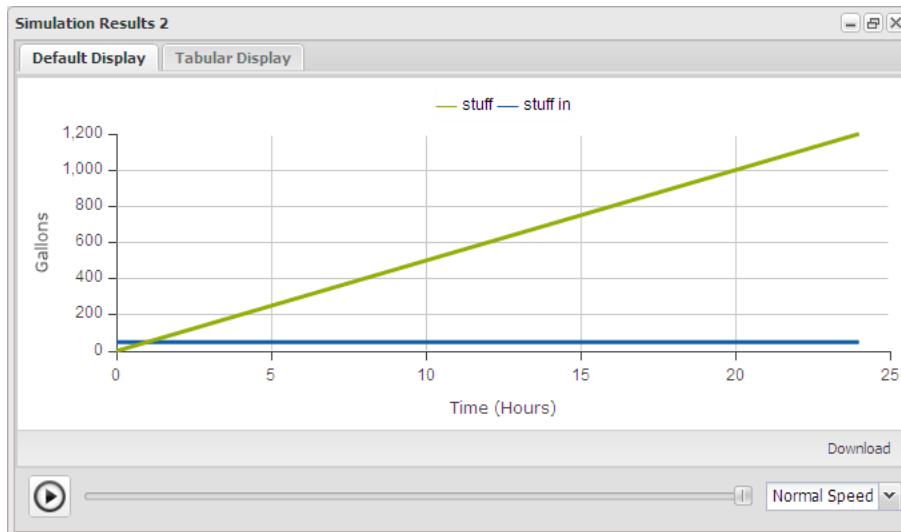


Figure 8. Adding water to the swimming pool

This graph indicates that after 24 hours the swimming pool will have 1,200 gallons of water in it. I know, it's about as interesting as watching paint dry. Actually, as you will come to find out, that's a good thing because this is really easy. A more interesting question might be, if the swimming pool holds 20,000 gallons of water how long will it take to fill with water at 50 gallons per hour? We'll get to this shortly.

Modeling Notes

As various models are developed and used I'll present aspects of the modeling environment that you'll need to do the exercises. I won't spend a lot of time on pieces you're not going to use immediately so please don't let any of the displays overwhelm you. I think it's far easier to remember things when you actually use them.

Canvas

The center area is the work area where you create models. This area may be scrolled if necessary. I'll talk about how to actually create models in the next chapter.

Stock

A rectangle indicates a quantity of something that accumulates, and accumulation takes time. Stocks don't change in the blink of an eye, well unless you blink for a long time.

Flow

A directed arrow representing the flow of something into or out of a stock. Remember that a stock can only be changed by a flow. Hand waving and magic don't work. The flow has to be explicit to cause a stock to change, and it takes time.

Toolbar

include another graphic to make the upper right arrow clear

Notice in the upper right corner there is a small down arrow. If you click on this arrow it will open the toolbar displayed in Figure 9. The toolbar contains all the tools you will use to build and modify models. Yes, you get to do everything on a single screen, with a few pop up windows of course.



Figure 9. Toolbar

Parameter Tab

Just below the arrow you clicked to open the top toolbar is a right pointing double caret. If you click this the parameter tab will close and the right pointing

double caret will now point left and can be used to open the parameter tab. This tab serves two different purposes.

If there are no elements of the model selected on the canvas the parameter tab will be similar to Figure 10 and contain the model description, tags, and parameter sliders used to set parameter values just before running the model.

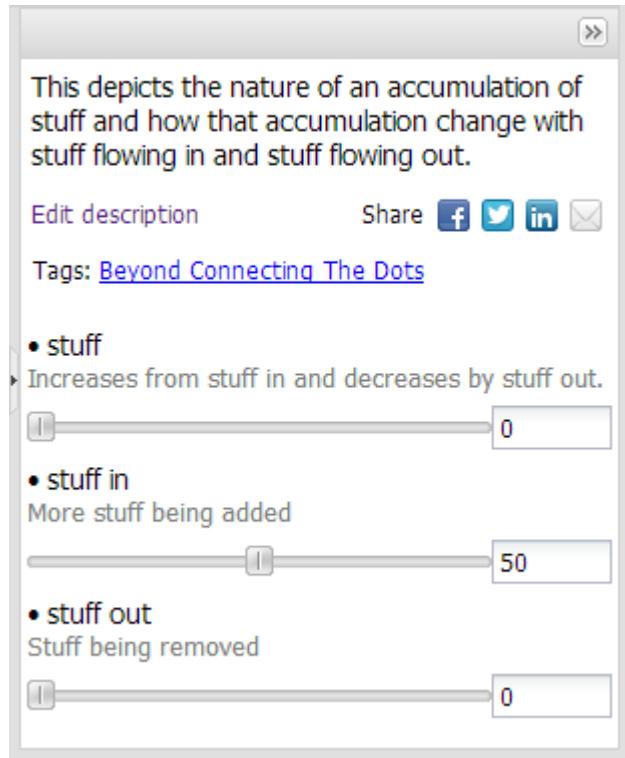


Figure 10. Configuration Panel

If there is a single element selected on the canvas then the parameter tab will present the list of parameters that can be set for that element. Figure 11 shows the parameters for the stuff element of the model. This is where I set the stuff in to 50 before running the model. Please don't be overwhelmed by this long list of parameters. We'll cover them one at a time as they are actually used in a model.

You should note in Figure 11 under the User Interface section it indicates that there should be a slider for stuff and it can be set for values from 0 to 100. Each element has some of the same parameters and some unique to it. Click one of the flows and see what its parameters are.

Just a couple more pieces and you can go interact with the model some more and get away from this boring description.

Stock

General	
(name)	stuff
Note	Increases from stuff in and decre...
Configuration	
Allow Negatives	Yes
Initial Value =	0
Behavior	
Delay	10
Stock Type	Store
User Interface	
Image	None
Show Value Slider	Yes
Slider Max	100
Slider Min	0
Validation	
Max Constraint	100
Max Constraint	No
Min Constraint	0
Min Constraint	No
Units	Unitless

? A stock stores a material or a resource. Lakes and Bank Accounts are both examples of stocks. One stores water while the other stores money. The Initial Value defines how much material is initially in the Stock.

Examples of valid Initial Values:

- Static Value
10
- Mathematical Equation
 *$\cos(2.78)+7*2$*
- Referencing Other Primitives
5+[My Variable]

Figure 11. Element Parameters

Time Settings

In Figure 8 I talked about the swimming pool filling for 24 hours. It's the Time Settings tool that allows you to define this for the model. Figure 12 shows the elements you can set before running a model.

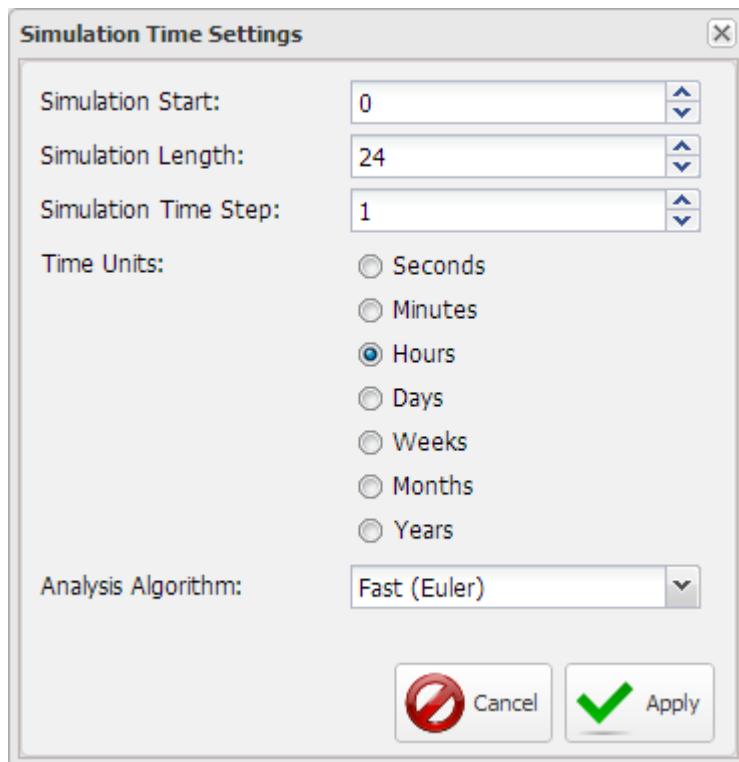


Figure 12. Time Settings

This is where I told the model to start at Time = 0 and runs for 24 time units. It steps one unit at a time and the unit is in Hours. Don't worry about Time Step for now. We'll get into that later.

Simulation Results

When you click the Run button the model is stepped through the defined time period and produces a display of the results. There are various options for the type of display and which elements are displayed as in Figure 13.

Configure Simulation Results

A default configuration is put together when the model is constructed on the canvas. If you click the Configure button in the upper right corner of the

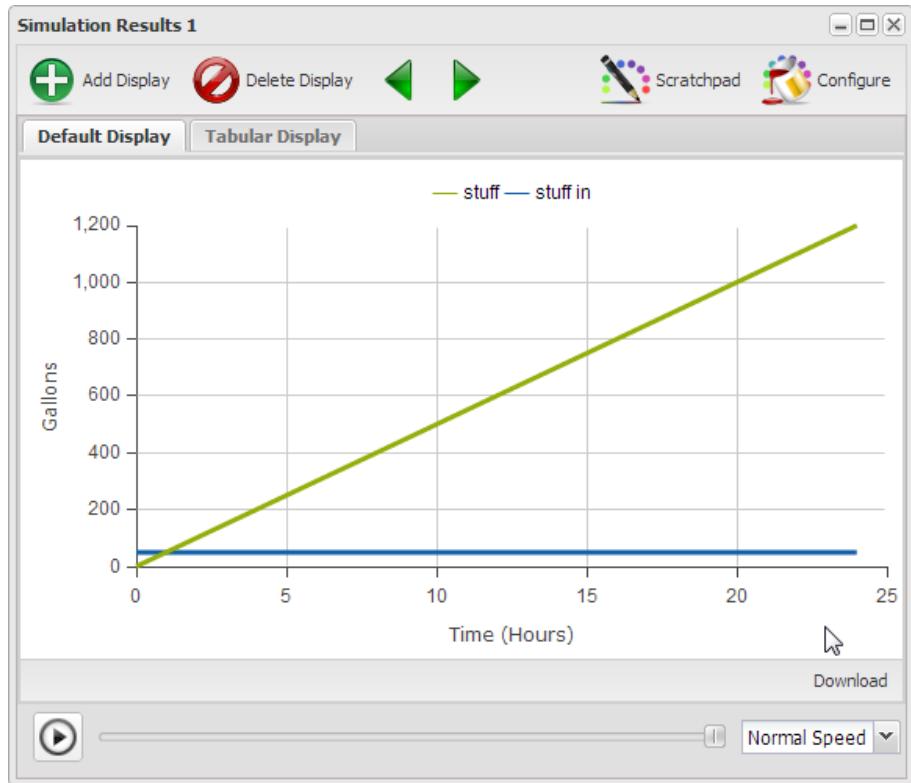


Figure 13. Simulation Results

Simulation Results window the Chart/Table Configuration window will open. It is in this window you indicate what type of display you want and which items of the model are to be displayed. The only part you need to be concerned about at the moment is the Y-Axis Label field. That's where I indicated that the items displayed were in Gallons. You will need to change this shortly in the next exercise.

Note that if you change items in the configuration they will be immediately reflected in the Simulation Results window when you click Apply. You don't need to run the model over again to see a different configuration of the data. This makes it very convenient when when you decide you need another display for one or two of the items.

I hope you haven't found this short introduction to the modeling environment too overwhelming. As I said I will try to introduce different parts of the environment just as you need them to interact with the models presented.

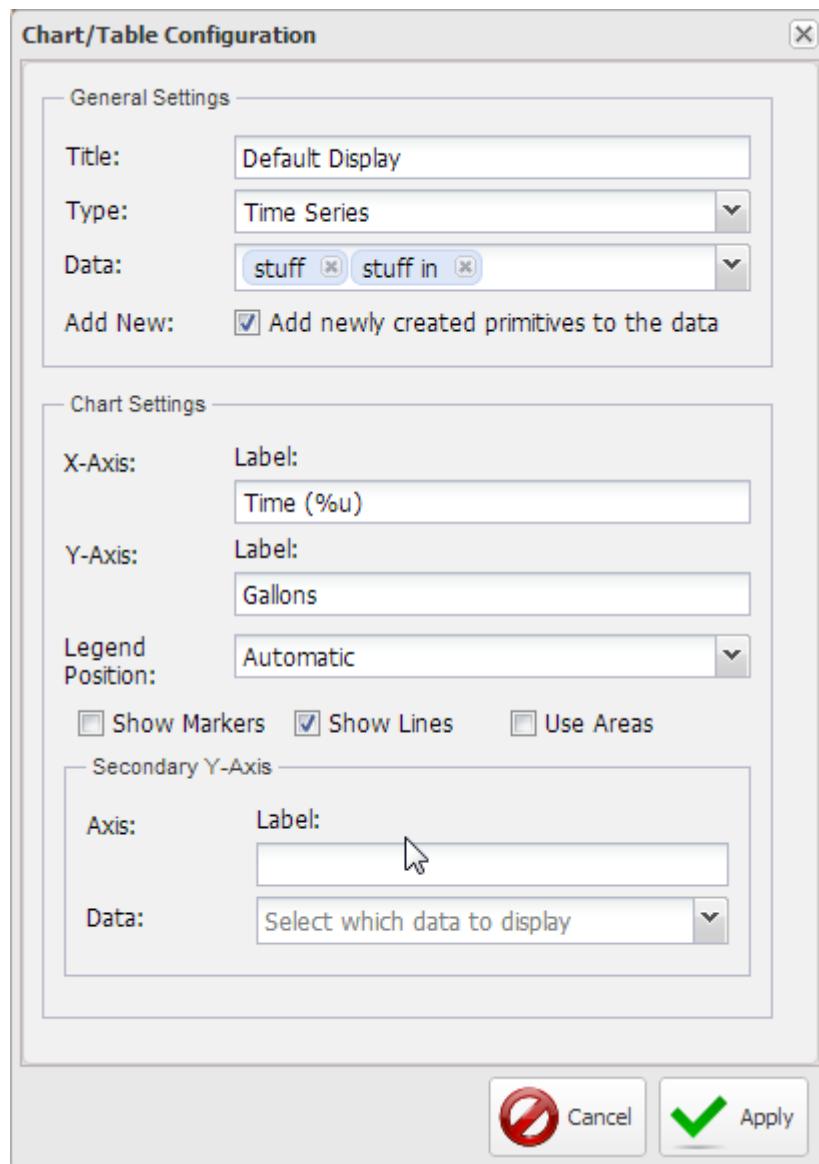


Figure 13. Chart/Table Configuration

Too much explaining and not enough hands on interaction gets to be real boring in a hurry. I encourage you to actually do the exercises presented. By interacting with the various aspects of the modeling environment you will develop a level of comfort and expertise which will serve you well throughout the rest of the book.

Exercise

Go back and consider the various pictures in Figure 5. Pick a couple of them to model. The only parts you need to set up are the Time Settings, how long will it run and the Time Units. You can also set the values for stuff, stuff in and stuff out on the Configuration Panel. After you run the model open the Chart/Table configuration window and set the Y-Axis Label appropriate for what you're modeling. I encourage you to be adventurous. Make new Displays, Table Displays, etc. You can't break anything, it's just an opportunity to become comfortable with the environment and learn.

Now that you've become intimately familiar with almost the simplest model possible lets go back and look at a couple of the pictures in Figure 5 and think about how the accumulations change in a bit more detail.

Rabbit Population Growth

If you modeled the accumulation of rabbits you may have already realized that the model of Figure 7 is missing something. Yes, if you add rabbits to rabbits you get even more rabbits. Though if you have more rabbits don't they create even more rabbits? Figure 14 is a model that reflects the the notion that rabbits create more rabbits.

Modeling Notes

There are a couple new pieces added into the model here and it's probably a good idea to explain the pieces before talking about how it works. The previous model had a stock, something that accumulates, and flows, the movement of stuff into or out of a stock. And the real important thing to remember is that accumulations take time to change. Stocks only change in the blink of an eye if you blink for a very long time.

Variable

A constant or equation used to influence some part of the model. Remember that a variable and a stock are different. A stock is an accumulation that changes over time as a result of one or more flows. A variable may change though it doesn't represent an accumulation. Rabbit Birth Rate is a variable, and in this model a constant value.

Link

A link is used to communicate a value of one element to another. The link doesn't actually represent something moving like a flow does.

= & i

If you mouse over the elements of the model you'll notice an = and an i appear. The i indicates there is additional info available. If you click the i a note window will open with a description of the element. This info was entered when the model was created. The = indicates there is a value or equation associated with the element. If you click the = it will open the *Equation Editor* window. We'll talk more about this when you start building a model.

Based on the previous modeling notes the model depicted in Figure 14 indicates that if you start with some population of Rabbits and each time period the current number of Rabbits times the Rabbit Birth Rate will result in a number of Births. This number of Births will then be added to the accumulation of Rabbits and figure into the calculation for the next period. If you mouse over the elements of the model and click on the = sign you can look at the definitions for the elements.

The Time Settings for the model were set up to run from 0 to 12 months. If you click the Run button you might be surprised when the model produces the graph in image in Figure 15.

The values in figure 14 are supposed to be 0 unless someone changed them.

Figure 15 really shouldn't be a surprise. If you look at the Configuration Panel you'll see that it indicates 0 Rabbits and 0 Rabbit Birth Rate. If there are no Rabbits how could anything happen? And if we had some Rabbits with the Rabbit Birth Rate was 0 what would you expect the result to be?

Suppose we start with 10 Rabbits, half of which are male and half of which are female. My research indicates that a female rabbit can give birth to between 18

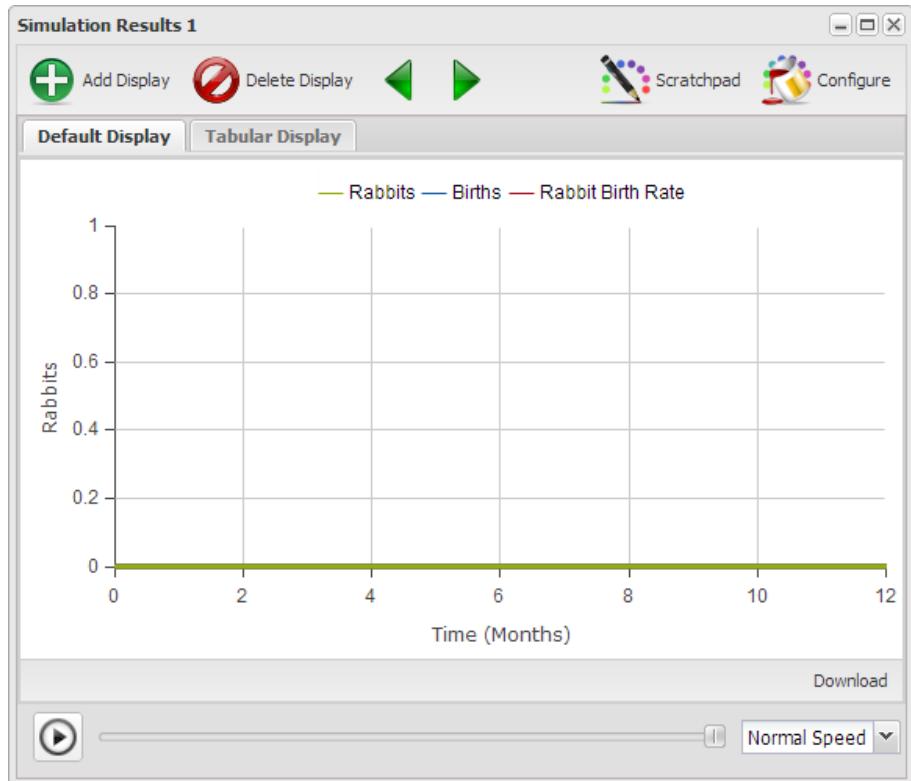


Figure 15. Rabbit Population Growth with No Rabbits

and 26 Rabbits a year. I'll average this out $(18 + 24) / 2 = 22$ and then I'll round this up to 24 just because it will make the math easier. If a female Rabbit can produce 24 Rabbits in a year, that's 2 per months, though it actually takes two Rabbits. With all these assumptions we get about 1 new Rabbit per month for each Rabbit. If you plug Rabbits = 10 and Rabbit Birth Rate = 1 into the model and run it you should get Figure 16.

Forty thousand Rabbits in a year? That seems a bit bizarre doesn't it? This result actually points out the real value of modeling, which is learning. You build a model based on what you think you understand. You then populate it with assumptions about the values and you run it. The result then either seems to make sense or seems really bizarre. In that case what the model is telling you is that either the structure is wrong, the assumptions are wrong, or both, because the world can't possibly be this bizarre. As a result you investigate the model and your assumptions and as you understand better the model gets better. At some point the model finally serves its purpose, to be a simplification of some aspect of the world which leads to a better understanding. I hope you come to find, as I have, that going round and round with a model can be a

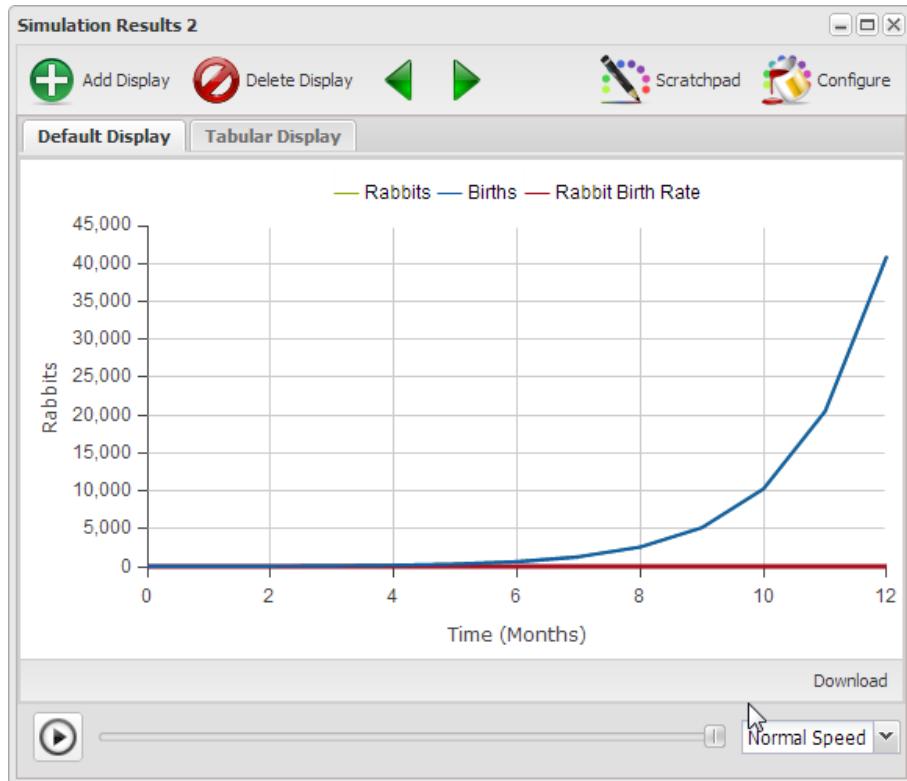


Figure 16. Rabbit Population Growth with 10 Rabbits

delightful learning process.

note the choppy nature of the graph. talk about why this is the result of the step size and we'll address this in chapter 2

After that sidetrack lets get back to our 40,000 Rabbits that can't possibly exist after a year. I'm pretty sure I can be certain how many Rabbits I started with at the beginning. And when I check my formula for Births = Rabbits * Rabbit Birth Rate it seems to be in order. This sort of means my assumption for Rabbit Birth Rate must be too big. And if you think about what the model is doing it's probably not too difficult to figure out that the model assumes that a Rabbit can be born this month and then give birth to another Rabbit next month. If a Rabbit has to mature for six months before it gives birth to Rabbits then the Rabbit Birth Rate might be something more like 20%. Using this estimate for Rabbit Birth Rate the model produces Figure 17.

Is this right? A good thing to remember at this point is that's actually the wrong question. A better question might be, "What have I learned, and is there

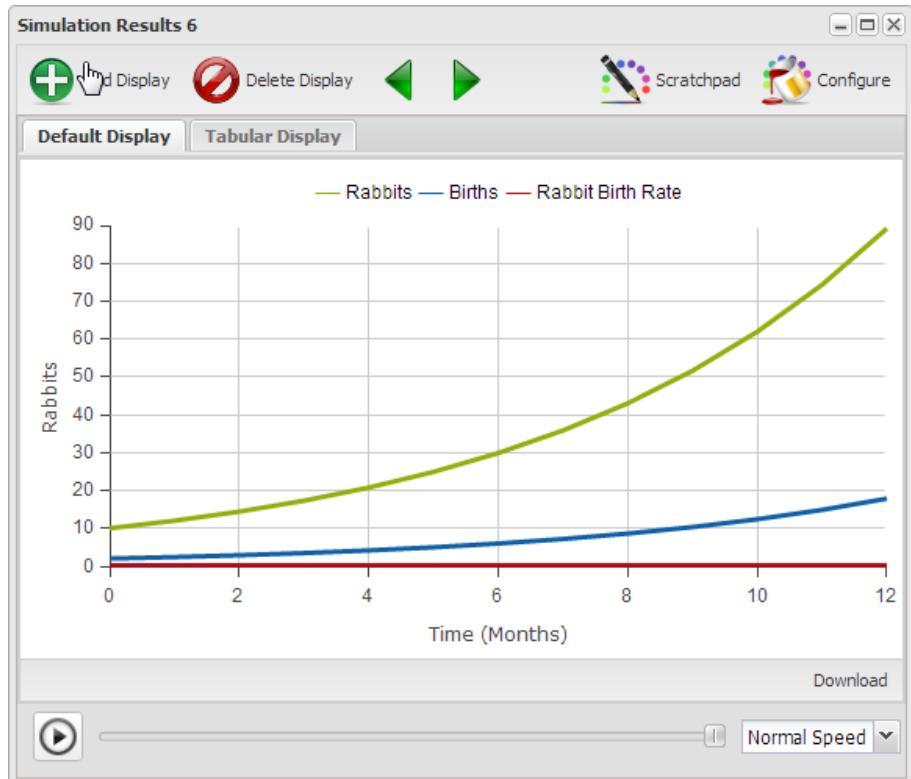


Figure 17. Rabbit Population Growth with 10 Rabbits and 20% Birth Rate

more I can learn?" The graph in Figure 17 sure seems more reasonable than what the model presented in Figure 16 though I don't think we have a high degree of confidence in the current Rabbit Birth Rate. And there are a number of other questions we could ask about our Rabbits. What is the Rabbit Death Rate? Do they have enough food to eat? Are they living out in the open where Coyotes and Foxes can get at them? Does their owner have a passion for Rabbit Stew? These might each be a basis for building a better model, though at this point we're going to leave the Rabbits alone and move on to something else.

The most important learning I hope you take away from this model is that when what flows into the accumulation increases as the accumulation increases the accumulation can get real big in a hurry. This is actually called exponential growth and we'll talk in more detail about this in due course.

Filling A Swimming Pool

Long long ago, meaning back in Figure 7 and Figure 8 I was talking about filling a swimming pool with a hose and how much water was in the pool after

a period of time. A more useful question might be, If the pool holds 20,000 gallons of water and the hose fills the pool at 50 gallons per hour, how long will it take to fill the pool. I know, you can do the math faster than it will take to build the model. Please bear with me a bit as there's another aspect of models right around the corner you will find very useful on an ongoing basis.

I begin with a Swimming Pool that needs to be filled with a hose. I know how many gallons of water it takes to fill the pool and I don't want to put too much water in the pool. I create a model where I compare the amount of the water in the Swimming Pool with the Full Level and use that to decide whether water is flowing in the hose or not. If you mouse over Hose and click the = sign you'll see the following equation.

$$\text{IfThenElse}([\text{Swimmng Pool}] < [\text{Full Level}], [\text{Full Level}]-[\text{Swimmng Pool}]), 0)$$

This says that if the Swimming Pool isn't full then I need to add enough water to fill the pool. And if the Swimming Pool is full then I add 0.

Modeling Note

Isn't it curious that the structure of this model looks just like the one for the Rabbit Population growth in Figure 14. We'll come back to this after we figure out how long it's going to take to fill the Swimming Pool.

With the Time Settings set for the model to run for 24 hours. Set the Swimming Pool to 0, meaning empty, and the Full Level to 20,000, on the Configuration go ahead and click the Run button. You should end up with the graph as shown in Figure 19.

note the choppy nature of the graph. talk about why this is the result of the step size and we'll address this in chapter 2

This is really great. We can fill the Swimming Pool in just 1 day, or can we? Either it's a really really big hose or we've done something wrong because it's probably not really possible fill the Swimming Pool with a Hose in one day if it takes 20,000 gallons or water.

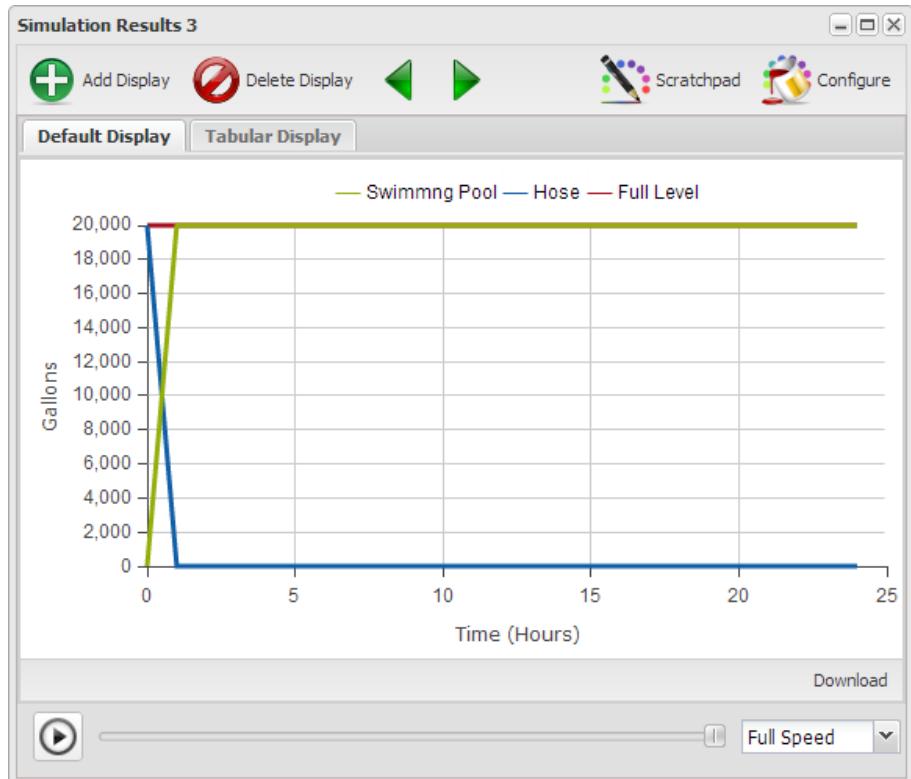


Figure 19. How long to fill the pool

Modeling Note

Hopefully you come to understand that when your models don't do what you expect them to do it's not a problem – it's an opportunity for learning. This is the real reason why we do modeling - to understand and learn. Just think of it as, the more things don't go the way you expect them too, the more opportunities you have to learn.

If you look back at the formula for the Hose, notice it didn't take into account the initial statement that the Hose could only deliver 50 gallons per hour. And, might it be useful if we could see what happened with different Hose capacities?

Figure 20 is a revised version of the model with Hose Capacity as a variable so you can set the capacity of the hose before you run the model.

The new formula for Hose takes into account both the current amount of water in the Swimming Pool, Full Level and Hose Capacity

```
IfThenElse([Swimmng Pool] < [Full Level], min([Full Level]-[Swimmng Pool],[Hose Capacity]), 0)
```

With Hose Capacity = 50 if you run the model it should produce Figure 21.

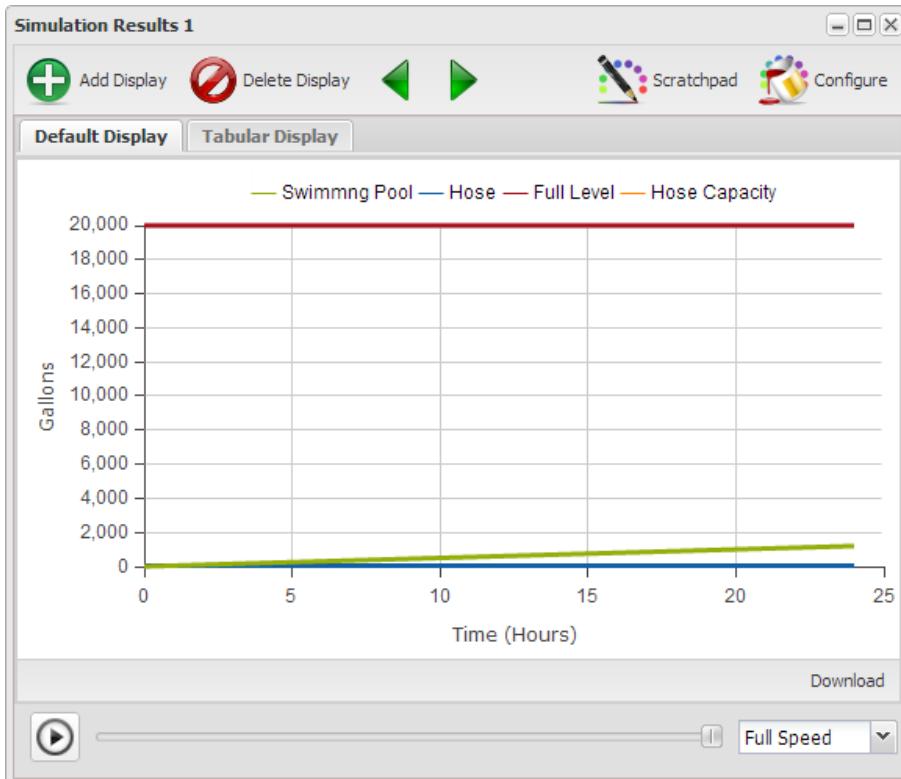


Figure 21. How long to fill the pool at 50 gallons per hour

Was this what you expected? Probably not. Over a period of 24 hours we've not even come close to filling the Swimming Pool.

Open the Time Settings and set the Simulation Length to 600 hours and Run the model again. Your run should produce the an equivalent of Figure 22.

Figure 22 indicates we need to wait 400 hours to fill the pool. That's a little over 16.5 days. I think we need a bigger hose.

While there are a number of things we could do to improve the model at this point I think we've gone far enough with this one.

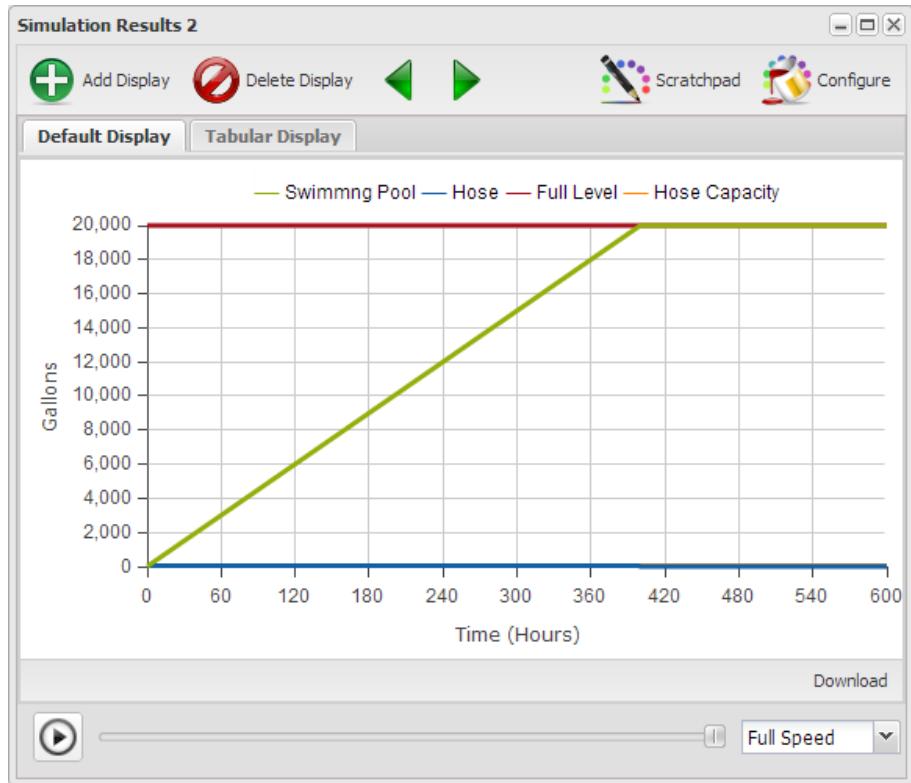


Figure 22. Filling the pool takes how long?

Exercise

Do a number of runs for the model in Figure 20 with different values for Full Level and Hose Capacity. Do you get a sense of how your choice of values impacts the results that appear over time?

Similar Structures / Different Behavior

If you compare Figures 7, 14 and 20 you should find them to be quite similar. And yet the behavior of the models are distinctly different.

Figure 23 presents the previous three models in a general form. This is so you can compare the different behavior of structures that are very similar. Flow Rate, Seeking Factor and Growth Factor are each factors which govern the rate of flow. Goal is a target value which the Growth model doesn't have. The

difference that makes a difference is what happens in the connection between the accumulation, or stock, and the flow.

The link between the stock and the flow provides information from one point to the other and is generally referred to as feedback, mostly likely because the information travels in the opposite direction as the flow.

Linear

In the Linear model the Flow simply depends on the Flow Rate variable, which is expected to be some constant value. This model is referred to as linear because the Accumulation of Stuff is a straight line as you can see in Figure 24.

Balancing

In the Goal Seeking model the State Change depends on the difference between the Goal and the Current State. This difference influences the State Change to increase the Current State until it reaches the Goal. The structure tries to bring about a balance between the Current State and the Goal so the difference is zero, and then there's no more State Change.

Reinforcing

In the Reinforcing Growth model Added depends on the value of Reinforcing Accumulation. This influences Added to increase the Reinforcing Accumulation which increases Added. One might consider a Reinforcing structure to be a Balancing structure that's out of control.

Would you believe that no matter how complicated a model may look it's really only some number of these structures connected together? In the next chapter you will begin actually building some models and investigating the implications of these structures.

Exercise

The values in the Figure 23 model elements were contrived so when you run the model it will produce the graph in Figure 24.

- Can you figure out why the values assigned are responsible for the curves produced?
- Alter the values for the parameters in the Configuration Panel and run the model to get a sense of the impact initial values have on the behavior of these structures.
- Can you explain to someone else the difference between Linear Growth, Goal Seeking and Reinforcing Growth in terms of why the structures produce the behavior they do?

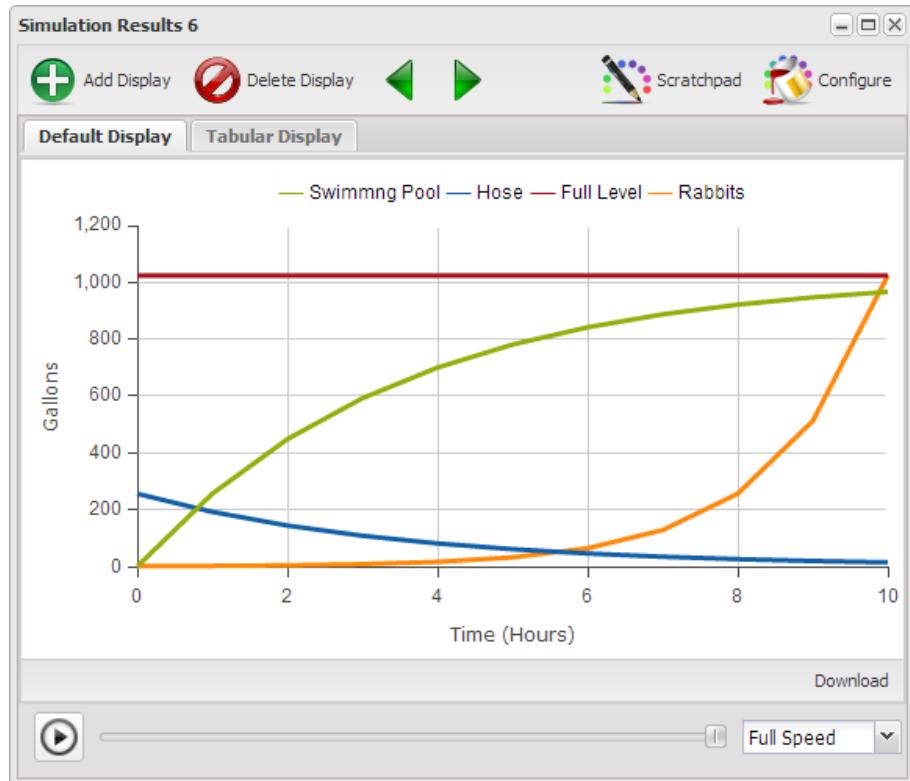


Figure 24. Balancing and Reinforcing Results

Summary

- Models are simplified versions of the world around us.
- We build models to help us understand and learn.
- We build simple models and add to them as we learn with them.
- Building models and learning is an iterative process.
- We learn as we go and seldom do we get models right the first time.
- Reinforcing and Balancing structures are the basic building blocks for all models.
- These building blocks can aid in understanding aspects of our interactions with the world around us.

Chapter 3

Chapter 2 - Dynamic Lego Blocks v1 13.04.24

Notes to Reviewers

This chapter has three objectives:

- **Modeling Environment.** Provide the reader with enough experience with the basic elements of the modeling environment so they can construct simple balancing and reinforcing loops. Supporting aspects of the environment necessary to run these models will also be described. This will build on the elements presented in the previous chapter. This is intended to be an introduction, not an exhaustive dissertation on modeling and simulation theory. The intent is to encourage the reader to develop and play with models, not chase them from the room screaming in fear never to return again.
- **Valid Structures.** Provide an understanding of which model element connections are valid and the reason for the allowed connections.
- **Basic Structures.** Develop further understanding of the Linear, Balancing and Reinforcing structures initially presented in Chapter 1.

Macros

There are certain aspects of the text formatting we don't have figured out and have resigned ourselves to the fact that we won't have this figured out for some time. As such macros are being coded to be replaced in the content post processing phase. I sorry that it's likely to make the text a bit more difficult to read.

- *model attribute*

- \S
 - *equation*
 - *model primitive*
 - *ui reference*
-

Figure 1 is the last model from the previous chapter. The intent of this chapter is for you to become comfortable creating these structures. In the process it is hoped that you will come to understand how it is that these three structures are essentially the building blocks for all the models you will ever build. Or at least almost.

The Blank Canvas

Some might look on the blank canvas and hesitate not sure where to start. You will be far better off if you look on the blank canvas as a gift of freedom which allows you to start anywhere. You will come to understand that if you approach modeling appropriately it won't matter where you start, you'll still end up with a meaningful model.

In the next few segments you will learn how to create on this canvas the three basic structures from which all models are constructed.

Notice in Figure 2 that similar tools are grouped on the *Toolbar*. Only a portion of the *Toolbar* is displayed though it's enough for what will be covered in this section.

To use any of the *Primitives* or *Connections* you click on the icon on the *Toolbar* to select it, then click on the canvas where you want it located, or used. For each tool there are a set of allowed uses. Once you place the item on the canvas it is named for what it is, with that name selected so you can type in the name you want. Names can contain any characters except braces "{}", brackets "[]", parentheses "()", and quotes '. If the label is not selected you can double-click it to select the label and then enter a new one, or you can enter the label in the *Configuration Panel* though we'll address that in a bit more detail later.

Exercise 2-1

Practice placing *Stock* and *Variable Primitives* on the blank canvas in Figure 2 and naming them. You can remove a *Primitive* by clicking on it to select it and then pressing the *Delete* key or clicking the *Delete* button in the *Actions* section of the *Toolbar*. Note that the *Save* option is disabled so you won't be able to save what you create. **Note:** This is only for the review copy. In the final copy you will be able to save what you create.

Stocks, Flows, Variables and Links

Stocks and *Variables* are connected using *Flows* and *Links* and there are very explicit rules associated with these connections. The allowed connections are depicted in Figure 3.

If *Use Links* or *Use Flows* is selected in the *Connections* segment of the *Toolbar* then when you mouse over an element of a model a little right pointing arrow shows up at the center of the element. You always draw a *Link* or a *flow* from one element to another and the arrow on the element points in the direction you draw the connection. If neither *Use Links* or *Use Flows* is selected then there will be no right pointing arrow when you mouse over the element.

Exercise 2-2

Click on the Set Up button on Figure 3, answer OK to both questions, and then repeatedly click Display to walk though a description of the valid connections between Stocks and Variables.

Hopefully the rules associated with the connections were easy to understand. Just remember that Flows represent the movement of stuff while Links only communicate the value of something from one location to another.

Valid Primitive Connections

The valid primitive connections of Figure 3 are described as follows.

Flow

A Flow adds stuff to a Stock, subtracts stuff from a Stock, or moves stuff from one Stock to another. The only way to change the quantity of stuff in a Stock is with a Flow.

- A flow out of a stock decreases it. If where the flow goes isn't relevant to the model then it just flows from the stock to the canvas. Select Flow from the toolbar and then click on the arrow that appears on the stock when you mouse over it and drag onto the canvas and release.

- A flow into a stock will increase it. If you don't care where the Flow is coming from then you first have to draw the Flow from the Stock to the canvas and click the Reverse button in the Connections section to get the Flow to come into the Stock from nowhere. It's just a quirk of the web implementation.
- A flow from one stock to another decreases the source and increases the destination. To get a flow between two Stocks draw the Stocks first and then draw the Flow from one Stock to the other.
- Flows can be bidirectional and we'll talk more about that the first time we use one in a model.

Link

A Link is used to communicate a value from one element to another. There is no flow of stuff through the link itself. The communication is considered to be instantaneous.

- You can use a Link from a Stock to a Variable to communicate the value of the Stock to be used in an equation. This does not change the Stock.
- You can use a Link to communicate the value of a Stock to a Flow to be used in the equation determining the value of the Flow in the next iteration. The Link does not change the value of the Stock.
- You can use a Link to communicate the value of a Flow to a Variable to be used in an equation. This does not change the value of the flow.
- You can use a Link to communicate the value of a Variable to a Flow to be used in the equation that defines the flow. This does not change the value of the Variable.
- You can use a Link to communicate the value of a Variable to another Variable so that value can be used in an equation in the destination variable. The link does not change the value of the source Variable.
- You can use a Link to communicate the value of a Variable to a Stock to be used as its Initial Value when the simulation begins. The value of the Variable is computed and assigned to the Stock as the simulation begins and it has no influence on the Stock during the simulation.

When you draw a link from one element to another it is created as a straight line. There are times when you would prefer that the connection be other than a straight line to make the diagram easier to follow. You can turn a straight line into a multiple segment line as follows.

- Click on the link to select it.
- Hold down the shift key and click somewhere in the middle of the link then release. This puts a little node on the line.
- Click on the node and move it as you wish to create a two segment link.

- You can create as many segments as you need, simply repeat the second step above.
 - If you wish to remove the segments select the head of the link, move it off the element it's connected to and then reconnect it. It will now be a straight link.
-

Exercise 2-3

Go back to Figure 2 and recreate Figure 3 for yourself. Actually making the connections helps develop a level of skill and comfort which will serve you well in the future.

Configuration Panel

Each of the four elements used to build a model has some of the same configuration options though because each has a different function there are some unique configuration options for each item. Some of the most frequently used options will be described in the following sections. The ones not described here will be described the first time they are used.

General

This section is where you can assign the (name) and Note for an item.

- **(name).** This is the label that you see on the item. You can double-click the item and edit this label on the item itself or change it here in the configuration panel.
- **Note.** Here you can enter a description of the item. You can enter short descriptions directly into the field. If you click the down arrow in right of the field it will open the **Note Editor** dialogue window which allows some formatting. The note that you enter here will pop up when you mouse over an item and click on the little-i that appears. If the element of the model is selected you can also open the Note Editor window by **CTRL+‘**. Adding comments to a model helps others to understand what you were thinking and when you go back to the model in the future the comments will help you understand what you were thinking. Yes, you completely understand now, though will you remember a year from now?

Configuration

This section is used to define how the element behaves during the simulation and is a little different for Stock, Flow and Variables, though quite similar. The behavior is essentially controlled by an equation which is defined in terms of the variables connected to it. This is an initial value for a Stock. You may enter a short value into the field though if you click the down arrow in the right of the field the **Equation Editor** window will open. In this window you can define the formula that defines the behavior of the element. You can also open the **Equation Editor** for an element by mousing over the element and clicking on the **equals (=)** sign that appears. All the built in functions on the tabs at the bottom of the window have descriptions associated descriptions and examples.

Additionally in this section you define whether stocks can have negative values and whether flows can flow in both directions. We'll talk more about these options the first time we use them.

User Interface

It is in this segment of the configuration panel that you define a slider for an element, if there is to be one. You can define a sliders for Stocks, to define it's initial value, for Flows and for Variables. Once you indicate there is to be a slider you then define the maximum and minimum values it may have, as well as the step size, how small are the variations you can define. If you leave the step size field blank then the slider can vary continuously.

An element may have a slider or a formula though not both. Sliders override equations. If you enter an equation and it disappears check to see if there was a slider defined and it hasn't been turned off.

Common Property # 1

To this point you've learned how to develop a static picture of a model. It is actually a model and provides a sense of the relationships between the various elements of the model. What it doesn't give you a sense of is the dynamic nature of these interactions over time. What are the implications of the relationships? In the next few sections you'll learn how to bring your model to life.

Look at the pictures in Figure 4 and ask yourself what it is that these images have in common. The images all represent very different kinds of things, some living, some not, though there is a characteristics they all have in common. Have you figured it out?

Maybe you notice the rabbits from the previous chapter? The things depicted in the various images all grow in one way or another, and some faster than others.



Figure 4. Common Property # 1

Constructing a Growth Structure

Lets use Figure 5 to construct a basic growth structure and in the process you'll learn about several of the parameters associated with the different elements of a model.

- Place a Stock on the canvas and label it Stuff.
- Now make sure the Stock is selected and take a look at the Configuration Panel on the right.
- Click on Use Flows to select that element.
- Mouse over the stock and click when the arrow appears at the center and drag onto the canvas somewhere outside the stock. Which direction doesn't make a difference though make sure you're a couple inches outside the stock before you release the mouse button.
- While the flow is still selected click on Reverse so you have a flow into the stock.
- Notice that the parameters in the Configuration Panel are different from those for the Stock.
- Click in the field to the right of Flow Rate = and change the zero to a 1.
- Click the Run Simulation button, which is on the “>>” drop down on the right end of the toolbar, and you've successfully created and run your first model. Admittedly it may not be very exciting though it is the first one, and one of many to follow.



Figure 6. Your First Model Output

Notice that the model ran for 20 years. That's because we used the default Time Settings.

Exercise 2-4

Open the Time Settings dialogue associated with Figure 5 and setup and run the model for different values of Simulation Length and Time Units. In what way do your changes alter the output?

Try creating a slider for your stock and flow. Set them for different values and run the model. The idea is just to develop your skill as well as a level of comfort in working with the tools.

Notice that in each case what you get is a rising slope for different time periods and at different angles. What you perceive in the graph is referred to a linear

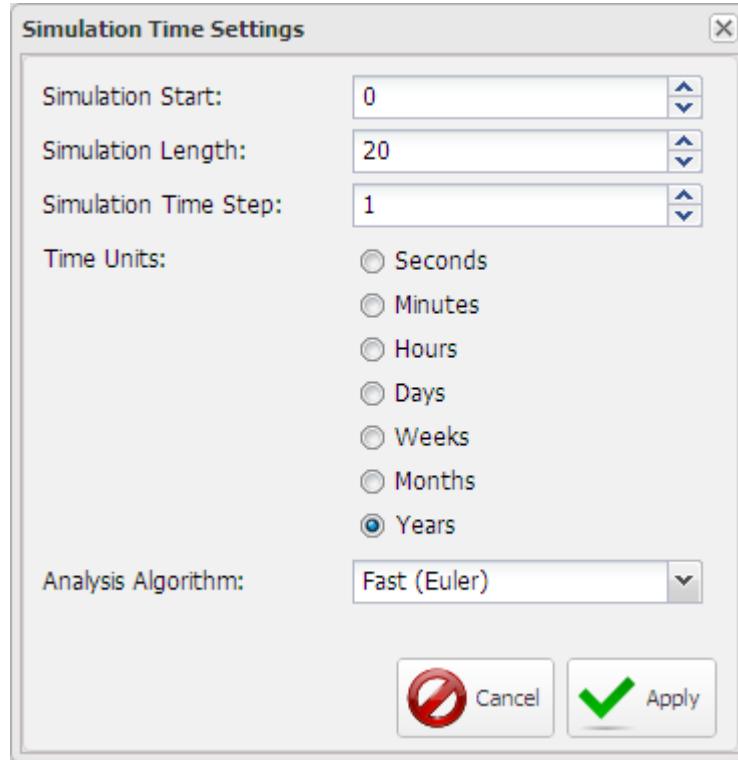


Figure 7. Simulation Time Settings

growth though this growth doesn't actually represent the growth associated with the images depicted in Figure 3. In those situations growth at each time is actually dependent on the size of the accumulation or stock at that time.

If we evolve the Figure 5 model into Figure 8 so the flow is dependent on the amount of stuff we find the growth to be very different.

Figure 8 represents only a couple of changes from Figure 5 as follows.

- Connect a link from stuff to Flow with a couple of handles so it can be reshaped to improve visibility.
- Mouse over the Flow and click on the = sign to open the *Equations Editor* and set the Flow to

stuff

as in Figure 9.

- Open *Time Settings* and set the *Simulation Length* to 10.
- Now Run the model. Note that because of the width of the embedded model you can't see the whole *Toolbar*. Clicking the » just to the right of

the *Tools* section of the *Toolbar* the rest of the options will drop down and you can select the *Run* option. You should now see the diagram in Figure 10.

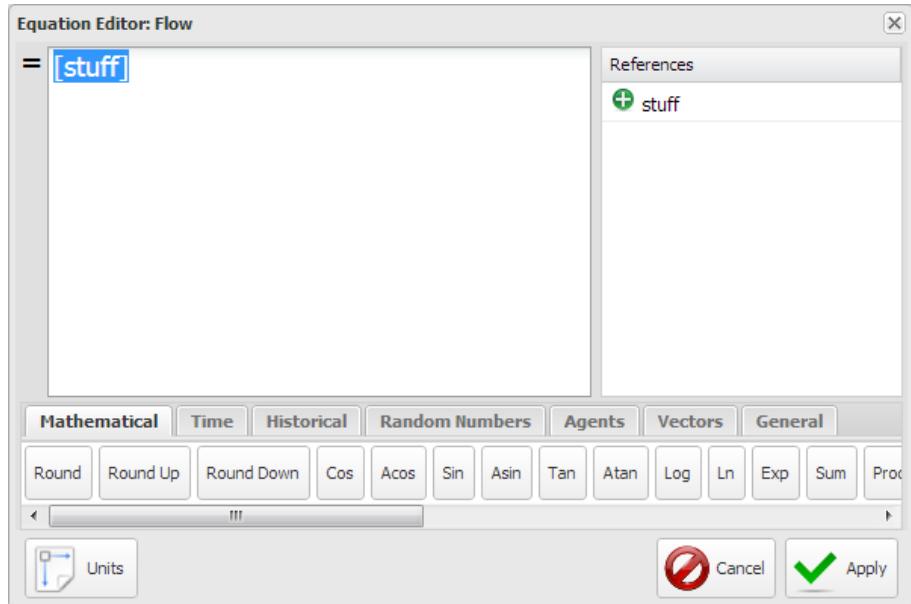


Figure 9. Equations Editor:Flow

The result of the run from the model in Figure 8 is depicted in Figure 9. The value after 10 Years is 1,024 which you should realize is just 2^{10} as expected because we started with a value of 1 and doubled it every year. This curve is referred to as an exponential growth curve.

Exercise 2-5

Notice that the curve in Figure 10 is a bit choppy where it turns up. Run the model in Figure 8 with a Time Step of .5, .25, .125, .0625 and compare the results. What questions are raised by the the results?

Time Units and Time Step Selection

The *Time Units* and *Time Step* selected for a model should be consistent with the time frame and level of detail of the model. You probably wouldn't develop

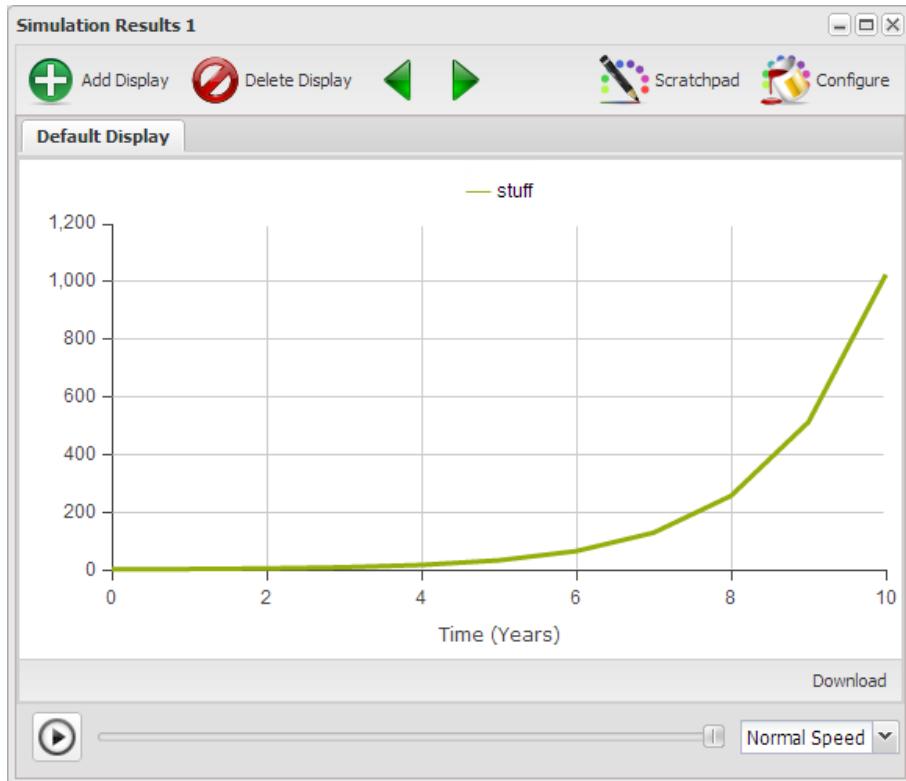


Figure 10. Growth of Stuff

a model about filling a bathtub with water and use *Time Units* of months. Minutes are probably more appropriate for this model. The *Time Step* is then selected to ensure none of the relevant transitions associated with the dynamic nature of the model are missed. A *Time Step* of .25, meaning 15 seconds, is probably sufficiently small to ensure there are no transitions missed.

Trial is actually the most appropriate approach to determine if you have an appropriate *Time Step* size. If you think .5 is appropriate then run the model with 1, .5, and .25 and if the results for 1 and .25 don't differ from .5 then you're probably OK. If .25 produced a different result then compare the .25 result with the .125 result. Once you get two runs where the values don't change then use the larger one.

Given this guidance how would you interpret the results you experienced in Exercise 2-5?

Exercise 2-6

Consider the images in Figure 4 and consider what *Time Units* and *Time Step* you would use in a model representing the growth in each of these areas.

The model in Figure 8 is for a Savings Account that is defined as compounding annually, i.e. calculating and adding interest once a year. This means that the most appropriate {Time Units} is years with a *Time Step* of 1. There are no other transitions in this model that need to be accommodated and running this model with any *Time Step* other than 1 will result in a less accurate result.

Exercise 2-7

Change the labels, values and simulation time settings for the model in Figure 8 for several of the growth situations presented in Figure 4 and then run the models. What becomes apparent from this exercise?

One aspect of trying to model the contexts of Figure 4 that should have become apparent is that there is a piece of the model that's missing.

The model in Figure 11 adds a factor, which is allowed to vary between 0 and 1, which is simply used to govern the flow. Mouse over the Flow and click the equal (=) sign to view the formula governing the flow.

$$\text{Flow} = [\text{stuff}] * [\text{factor}]$$

Exercise 2-8

Use the model in Figure 11 to implement the models in Exercise 2-7. Does this structure allow you to construct more realistic representations of the growth situations presented in Figure 4?

The model of Figure 11 is the standard reinforcing growth model depicted in Figure 1 at the beginning of this chapter. In the process of arriving this model the linear growth model of Figure 1 was developed first, and then evolved. Hopefully through the exercises to this point you have gained a deeper understanding of how this structure works and the breadth of it may be applied to many situations.

Common Property # 2

Look at the activities depicted by the images in Figure 12 and ask yourself what it is that these activities have in common. The images represent very different kinds of activities though there is a characteristic they all have in common. Have you figured it out?



Figure 12. Common Property # 2

Each activity depicted in Figure 12 represents the pursuit of some goal or objective. Admittedly the goals are very different and each is pursued in a very different manner.

Constructing a Goal Seeking Structure

As we have done repeatedly to this point we begin with a linear model consisting of a flow and a stock, along with a flow rate variable. To this we simply have

to add a goal and the appropriate feedback and we end up with the model in Figure 13.

When you look at Figure 13 admittedly we added Gap which we haven't addressed before. This was done so we could explicitly plot the difference between the Current value and the Goal. And the factor is simply a multiplier between 0 and 1 to govern the extent to which the Gap governs the change.

$$\text{Gap} = [\text{Goal}] - [\text{Current}]$$

$$\text{change} = [\text{Gap}] * [\text{factor}]$$

Take a look at the Time Settings for Figure 13 and you'll see that the model was set up to run from 0 to 10 with a time step of 1 and a units of hours. These were just selected to create a generic model where you could consider the Goal to be 100% and the other values as having values between 0 and 100%. This way we can consider the implications of the interactions without getting hung up on the actual values.

If you run this model you should get the result depicted in Figure 14. This shows how the value of Current approaches the goal as the value of Gap declines with a factor = 0.5.

Exercise 2-9

Run the model in Figure 13 with various values for factor. What do you notice about the relation between Current and Gap? And what do you notice about the curves as the factor gets larger and larger?

Under Time Units and Step Selection we talked about it being essential that the the Time Units were selected appropriate to what was being modeled. In this case since it's a generic model one Time Unit is pretty much as appropriate as any other. The Time Step is another matter though, or is it? We said one chooses a Time Step such that none of the relevant interactions are missed and the change from one Time Step to another doesn't change the result.

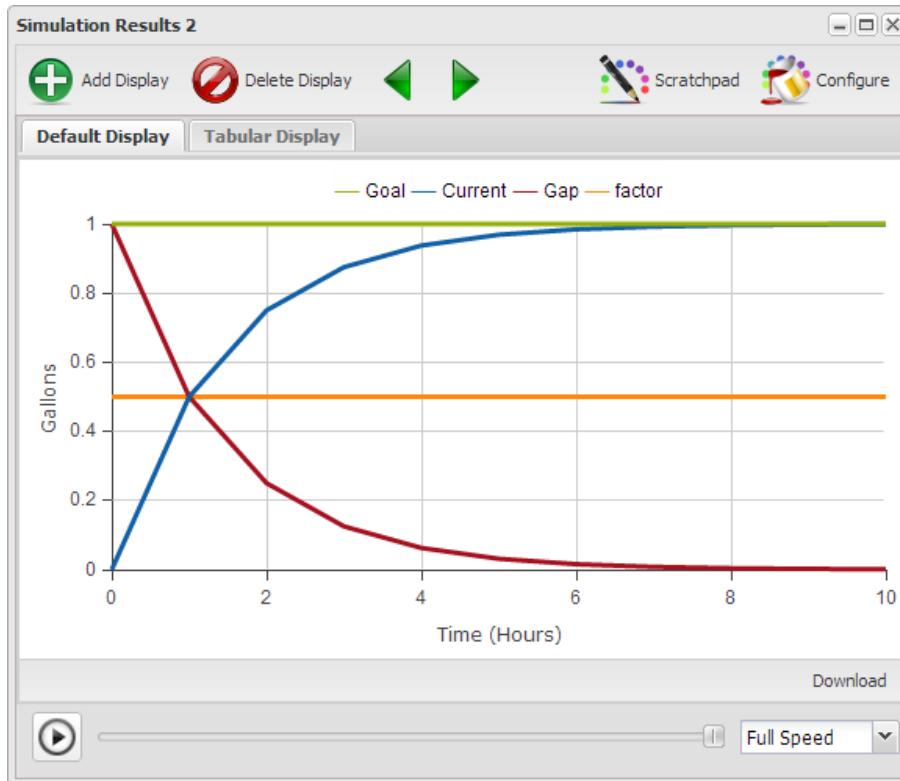


Figure 14. Goal Seeking Result

Exercise 2-10

Set up the model in Figure 13 to run with Current = 0, Goal = 1, and factor = .75. Now run the model with a Time Step of 1, .5. 25. .125. Does the result actually change? Look at the Tabular Display associated with the Simulation Result. As you make the Time Step smaller and smaller are the results more correct?

Considering that we don't know anything about a real environment being modeled it's a bit difficult to determine if the result is actually more correct as the Time Step used is smaller and smaller.

You might have also realized by this point that it would be quite difficult if we attempted to use this model to model any of the situations depicted in Figure 12. While progress toward the goal in the situations depicted is promoted by

the Gap between the Goal and Current the change in those situations isn't likely to be proportional to that Gap.

Figure 15 presents a modification to the model of Figure 13 where the factor has been replaced by a constraint. It looks like there have been lots of changes though they all cosmetic except the way Workers influence work on a daily basis.

If you run the model with Project Days Work = 60, Workers = 0 and Days Work Completed set at the default of zero and Time Step = 1 you should see the graph in Figure 16.

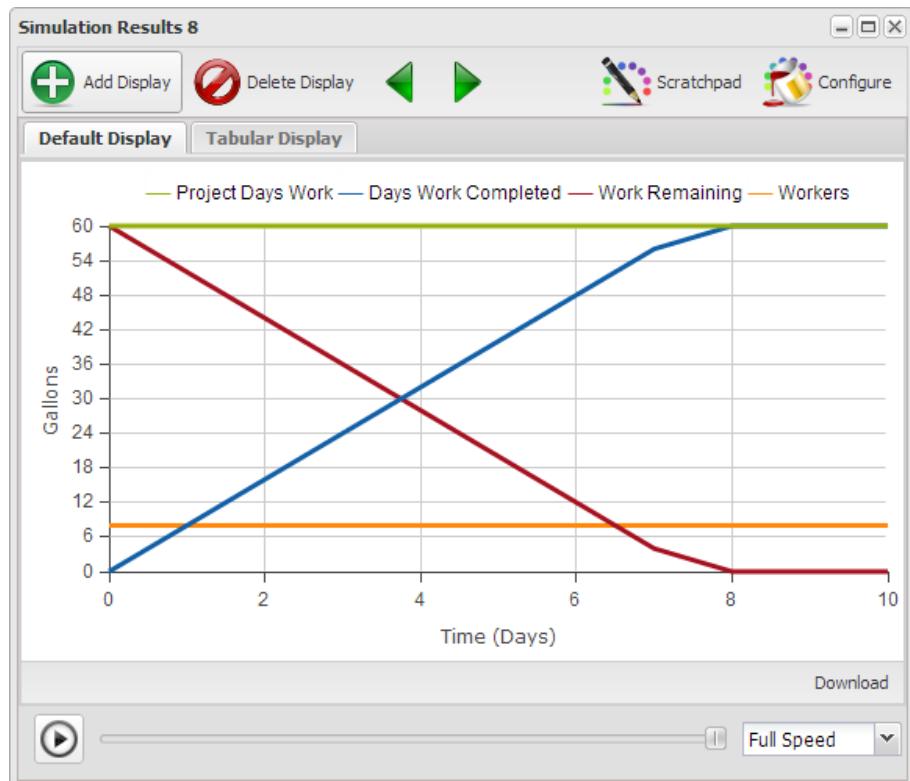


Figure 16. Goal Seeking with Constraint

The reason the graph looks like this is because of the constraint placed on the work because of the number of Workers available. This is accomplished by the formula embedded in the flow.

```
work = IfThenElse([Work Remaining] > [Workers],[Workers], [Work Remaining])
```

This says that if there is more Work Remaining than there are Workers available to do the work then the amount of work that day equals the number of Workers. This goes on for the first 7 days then on the 8th day there are only four days work required to finish the project which is represented by the different slope on the line on the 8th day. You can see this in detail if you look at the Tabular Display.

Exercise 2-11

Set up the model in Figure 16 to run with Time Step of .5. Compare the Tabular Display of this run with the results of the previous run above. By making the time step smaller have we improved the accuracy of result? Why?

Again the appropriate Time Step is one that captures the activity occurring within the model. In this case the Workers are in integers and Project Work days are in integers, and with the Time Units in days the appropriate Time Step is 1. If there were events which happened in the model on the order of hours then you would have to decide whether to alter the model to run in hours or reduce the Time Step to ensure it was small enough so no interactions in the model were missed.

Exercise 2-12

Use the model in Figure 15 and reconfigure it for a couple of the activities depicted in Figure 12. Note that for this exercise you will have to relabel the stock, flow, and variables accordingly. You will also have to decide on the most appropriate Time Units and Time Step to use.

Summary

Hopefully this chapter has helped you become more familiar with the modeling environment and four building blocks you will use most often.

- **Stock.** An accumulation of something that can only be changed by something flowing into or out of it.

- **Flow.** Something moving over time which adds to a stock or subtracts from a stock.
- **Variable.** Constant or equation computed each time the simulation steps.
- **Link.** Used to communicate a value from a Stock, Flow, or Variable, to a Stock, Flow or Variable. The source is not changed and a link to a stock can only be used to set it's initial value.

Because of the nature of the building blocks themselves there are only a small number of valid connections as depicted in Figure 3.

These valid connections are used to create only three different types of structures, linear growth, goal seeking and reinforcing growth. If you are comfortable with these you should be relieved to know that's all there are. Just three simple structures will be used for all the models you will ever build. Of course at times there may be quite a few of these connected together yet you should be confident that you know about the pieces. In the next chapter you'll find that some very interesting things happen when these structures are combined in certain ways.

Chapter 4

Models and Truth

All models are wrong, but some are useful – George E.P. Box

When considering the relationship between models and truth, it is useful to first take a step back and discuss different kinds of models. Modeling is a wide-ranging field with many distinctions made by modelers and mathematicians. Such distinctions are generally of little interest to us, as we believe focusing on them can often encourage a focus on jargon and formalism rather than the quality of a model. Furthermore, we will present our own classification scheme that once completed will clarify the core dichotomy that is at the heart of modeling. It can be useful, however, to briefly discuss the distinctions that are commonly made in order to obtain a deeper understanding of the choices underlying the development of a model.

Deterministic versus Stochastic Models

There are two polar opposite views of the world. One view says the fate of the universe is governed by strictly predictable laws of physics. In this view, the universe is in effect a giant machine and if its current state is known (down to each individual atomic particle) its future states through the rest of time are predetermined. Another view is that the universe is ruled by chance and randomness. Random quantum mechanical fluctuations merge and amplify leading to an infinite range of diverging possibilities.

Which of these two views the truth? We certainly do not know and it is possible that this will be one of the questions that physicists will never cease exploring. Albert Einstein had a particular viewpoint though. He was a strong partisan in favor of the deterministic view, famously remarking, “God does not play dice with the world.”

When creating a model of a system, we must make a choice about we treat chance. Do we build our model in deterministic way such that each time we run it we obtain the same results? Or do we conversely incorporate elements of

uncertainty so that each time the model is run we may see a different trajectory of outcomes?

Mechanistic versus Statistical Models

When beginning to build a model of a system, there are many questions that you should ask yourself. Two of them are:

1. Do I know (or have a hypothesis of) the mechanisms that drive the system?
2. Do I have data that describes the observed behavior of the system?

If the first question of these questions is answered in the affirmative, then you can build a mechanistic model that replicates your understanding (or hypothesis of) the true mechanisms in the system. If, on the other hand, the second question is answered in the affirmative, you can use statistical algorithms such as regressions to create a model of the system based purely on the data.

If neither question is answered affirmatively... well in that case there isn't much of anything you can build.

Aggregated versus Disaggregated

When building a model, the question of scale becomes very important. Imagine we are concerned about the affects of Global Climate Change on water resources. We may wish to examine the question of whether there will be sufficient water supplies given a rise in temperatures in the future.

At what scale do we build this model? The range of possible scales is very wide:

- At the most aggregate, we could simply estimate total worldwide water demands and supplies into the future.
- Maybe that is too coarse a scale, however, clearly having excess water in Norway has little impact on the situation in Egypt. We could instead create a finer resolution model that separately looked at water demand and consumption in each country.
- Possibly even that is still too coarse, maybe we should make our model even more granular to look at a specific cities and population clusters around the globe.
- At the extreme disaggregated level, we might even want to model individual people – all 7 billion of them – and their needs and movements around the world.

Clearly, there is no simple answer to this question of optimal scale. The best choice is highly context sensitive and depends on the needs of the specific model and application.

Prediction, Inference and Narrative

The three distinctions presented above can be used to classify models. We can even use them to classify the models we have discussed in this book. Most of our models would be classified as deterministic (random chance is generally not explicitly incorporated in these models), mechanistic (we generally assume mechanisms rather than estimating dependencies from data), and highly aggregated (the agent based models are an exception to this last point).

Outside of modelers, however, these distinctions are of little importance. Let's take off our modeler hats for a moment, and instead look at modeling from the perspective of a client (where client is a relative loose concept, it can include consulting arrangements but also work within an organization or in other contexts). As clients, we engage modeler to build a model to fulfill some specific purpose. The choices the modeler makes – aggregated versus disaggregated, stochastic versus deterministic – are just technical details. They are similar in importance to the choice of software or programming language used to build the model. It would be just as nonsensical to say a model was fundamentally bad because it was written in a relatively ancient programming language – like Pascal – as would be to say a model was fundamentally bad because it was constructed, for instance, to be deterministic.

What is of true importance is the success of the model in fulfilling the client's goals whatever they may be. Technical details matter – they can affect maintainability and other factors – but they are of secondary importance. Let's look back at Box's quote at the beginning of this chapter. We know all models are wrong, what we should really care about is their utility in meeting a specific task.

So instead of using these technical classifications to discuss models, we think it is much more useful to base our discussions of models on the models driving purpose. This allows us to leave behind relatively mundane technical and implementation details to focus on what we really care about. There are many different reasons for building models, but when they are examined they can be boiled down to three broad classes of model purposes: prediction, inference and narrative.

Prediction : Models used for prediction are the most straightforward. They attempt to forecast some outcome given information about variables that may influence that outcome. A weather forecast is an example of a model being used prediction. Likewise, when you apply for a credit card at a bank, they run a predictive model to determine your risk of default. When you apply for life insurance, similarly, the company has an actuarial model to predict how much they should charge you for a given payout. All these models take in data (the current temperature for the weather forecast, the amount of money in your bank account for your risk of default, your age for the life insurance application) and apply various forms of analysis to generate a prediction of the outcome.

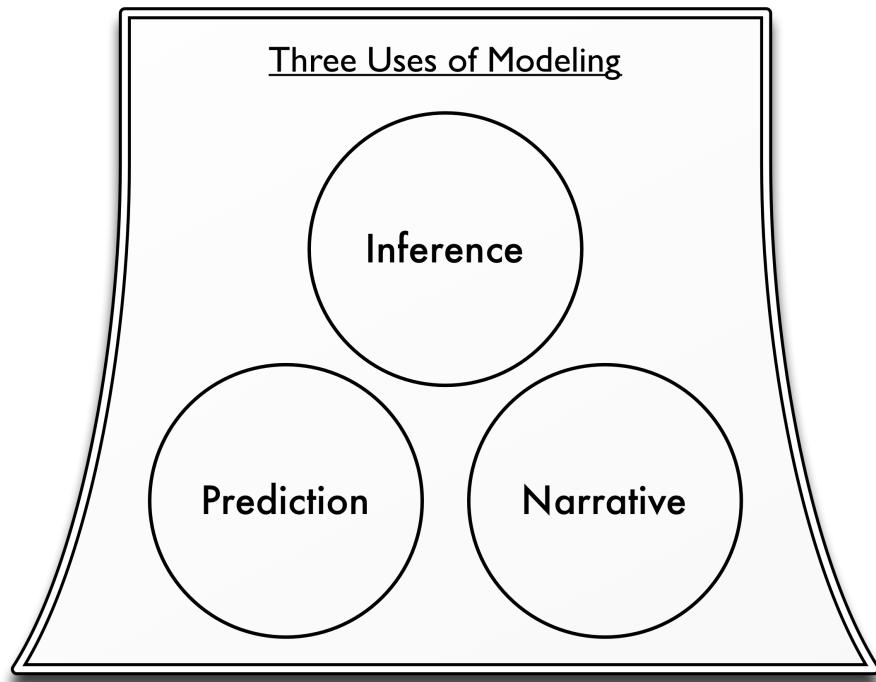


Figure 1. Three Usages of Models

Inference : Models used for inference are the most common in academic research. Often, academic research questions boil down to this simple template: “Does X affect Y ? ” These are inferential questions. As an example, a researcher may make a hypothesis statement such as, “The wealthier a high-school student’s family is, the higher the student’s test scores will be.” The researcher may then build a model to test the validity of this hypothesis and the model’s results will generally be phrased in terms of a p value indicating the significance of the evidence in support of the hypothesis.

Narrative : Models are often used to tell a persuasive story. When the Obama administration wanted to persuade lawmakers and the public to support their proposed stimulus, they famously published the graph shown in Figure 2. A lot of complex modeling and mathematics surely went into constructing this figure, however at its core its purpose was to tell the nation a story: things are going to be bad, but the recovery plan will make them less bad. These stories are at

the heart of narrative models and we will return to this figure later on.

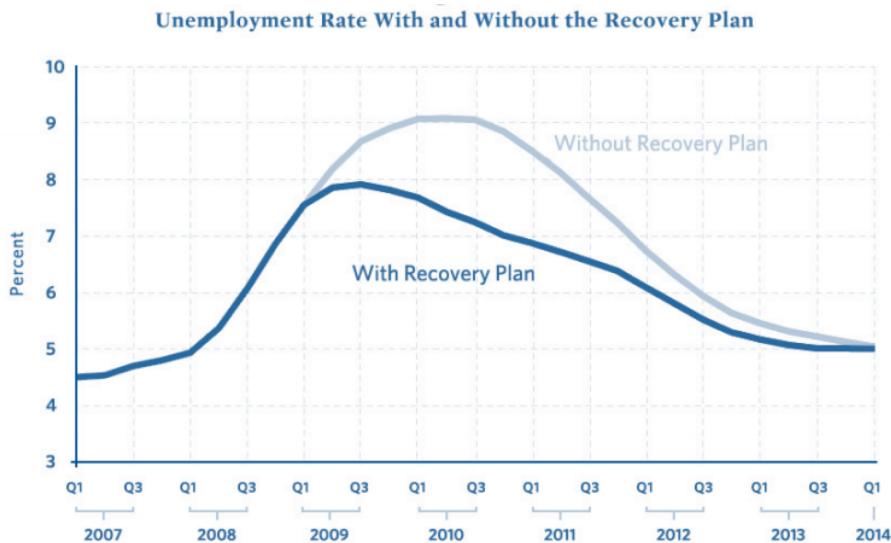


Figure 2. The Obama Administration's Predictions for the Effects of the Recovery Plan (Romer and Bernstein 2009)

All models can be classified in terms of these three primary purposes and it can be useful to discuss modeling projects in terms of them[¹And we strongly recommend doing so. It is important to clearly define the purpose at the start of a project. The techniques used and data that will be required depend significantly on the models overall purpose. Particularly, in many cases it is important to clarify whether your primary goal is to use a model for prediction or for narrative. Many clients and modelers expect to do both and end up with a model that does neither.]. Although this classification system is very useful, there is an even simpler classification system we can use, however, that once completed will clarify the core dichotomy that is at the heart of modeling.

The Strange Case of Inference

To help us get at this fundamental classification scheme, let's first talk for a moment about the process of inference. Take our earlier example of determining whether wealth results in increased high-school test scores. We phrased this hypothesis in a specific way: that increased wealth will always increase test scores. This illustrative statement, however, actually differs from what is often done in practice. In general, researchers simply asks the question "Does X affect Y ?" rather than "Does X increase Y ?" It's just a slight difference, but it is a more flexible question that allows for many forms of relationships. For our example, we would ask the question "Does wealth affect tests scores?"

The gold standard to answering questions like this is the controlled experiment. For our example, we could imagine an experiment where we took a sample of a thousand families from a school district. When these families' children enter high school we would randomly select half to be in a "poor" category and the other half to be in a "rich" category. Families in the rich category are given grants of \$500,000 a year to spend how they wish while the parents in the poor category are fired from their jobs and have their savings frozen for the duration of the experiment. Once the students graduate from high school, we could simply compare the average scores for the students in the poor and rich categories.

These controlled randomized experiments are the ideal approach to answering inferential questions like these as they allow you to truly determine the effect of what your variables, in this case wealth. For many types of questions, controlled experiments can be implemented (for instance does treatment with a novel drug help treat a disease). Unfortunately, in general complex social questions are simply impossible to answer with them. We can consider the testing procedure we just imagined to assess the effect of wealth on scores, but it would be impossible (and deeply unethical) to implement in a real community.

Traditional Model Based Inference

Given our general inability to do controlled experiments, how do we answer inferential questions? The standard way is to collect data and then construct a model enabling us to measure the statistical significance of our hypothesis given the data. Due to history and simplicity, linear regression models are by far the most commonly used type of model today. A linear regression predicts an outcome (Y) based on the multiplication of variables (X 's) by a set of coefficients determining the effect of the variables on the outcome (β 's):

$$Y = \beta_0 + \beta_1 \times X_1 + \beta_2 \times X_2 \dots$$

For the education example we could simple collect data on a number of students. We could measure their families' wealth (X_1 in the equation above) on the student's test scores (Y). We could then run the linear regression to determine the coefficient values (β_0 – the intercept – and β_1 – the effect of wealth on test scores). If we thought there were other factors that affected test scores, we could measure them and include them as addition X 's in the regression.

In addition to obtaining the values of these coefficients, we also obtain as a result from the regression the statistical significances or "p values" of these coefficients. Although p values are commonly used in statistics, they are ubiquitously misunderstood (These misunderstandings are not only made by on-the-ground practitioners and analysts, they are frequently shared, and propagated, by university-level statistics instructors; see, for instance, Haller and Krauss 2002.) so it is useful to briefly review them.

In short a p value measures the probability of seeing the measured data (or more extreme data) assuming the null hypothesis is true. Generally the null hypothesis will be that there is no relationship between the variables and the outcomes.

When looking at the significance of coefficients, a p value means the probability of seeing that value of a coefficient (or one even further from 0), assuming that the (unknown) truth is that the coefficient actually has a value of 0. In other words, it is the probability of seeing the observed non-zero value, assuming that the true value is in fact 0. Frequently, probabilities of 10%, 5% or 1% or smaller are taken as indicating statistical significance. These low values indicate that the coefficient value is so far from 0, and the probability of this occurring by chance so small, that we can reject the null hypothesis and accept the fact that the coefficient is not 0.

This is what a p value is. Now let's specifically mention what a p is not, as this is often misunderstood. p values do not represent the following commonly used interpretations:

- The probability that the null hypothesis is true (that the coefficient is 0)
- One minus the probability that the alternative hypothesis is true (that the coefficient is not 0)
- Any sort of “proof” that the null or alternative hypotheses are correct or incorrect
- The probability that you are making the correct or incorrect decision if you accept or reject the null or alternative hypothesis

Using the p we can do inference by using the statistical significance of the coefficients. If the probability of β_1 (the coefficient for the effect of wealth) occurring due to chance (given it is 0 in reality) is less than, say 5%, we can claim with reasonable strength that wealth does in fact affect test scores. This is the standard approach researchers take to model-based inference and is used ubiquitously.

A Troubled Sea of Assumptions

Let's stop for a second and consider what we have done here. In carrying out these logical steps to apply model based inference to determine whether wealth affects test scores, we have had to make one very large assumption: that the relationship between test scores and wealth is linear.

Our linear regression equation assumes that for every increase in one unit of wealth (X_1), test scores (Y) will increase on average by the amount of the coefficient (β_1). What if this were not in fact true? For instance, we could easily imagine the case where wealth initially helped test scores by providing students more resources and opportunities to learn. After a certain point, however,

wealth could theoretically negatively impact scores as very wealthy students might lack the pressure or motivation to study hard.

If we believed this was the case, then our linear regression model from earlier would be wrong as would the inferences we obtained from the model. We could correct our model and inferences by changing our regression formula to contain a squared term which could replicate this type of relationship:

$$Score = \beta_0 + \beta_1 \times \text{Wealth} + \beta_2 \times \text{Wealth}^2$$

Using this equation, at low values of wealth the $\beta_1 \times \text{Wealth}$ term will have the most effect on scores. Conversely, at high levels of wealth, the $\beta_2 \times \text{Wealth}^2$ term will have the most effect on scores. Thus by having a positive β_1 and a negative β_2 we can model wealth as having an initially beneficial and then detrimental effect. If our assumptions about the quadratic relationship are correct, then this model will yield accurate inferences. However, if they are wrong, our inferences will be wrong again.

What are we really doing when we assume regression forms like this? Now it might not be immediately obvious, but what we are in fact doing is telling a story. Using our first equation, we are telling the story that as wealth increases test scores will always increase. Bill Gate's children will perform amazingly well here! Using the second equation we are telling a different story: that as wealth increases test scores initially do as well but after a certain point increased wealth will hurt test scores. That picture isn't so rosy for Bill Gates!

And so we arrive at a very interesting conclusion. By choosing our equations to tell a story, our inferences are in fact based on narrative modeling approaches. True, these inferences build upon numerous calculations and very advanced theoretical underpinnings, but ultimately what governs our conclusions and inferences are the stories or narratives we tell our system. These are choices that we as a storyteller make and they are not determined by an objective truth or reality.

Predictive Inference

Is there an alternative approach to inference that does not rely on narrative? Can we accomplish it without assuming the relationships between variables? The answer is yes. Although they are not often used, alternative prediction-based approaches to inference are available. In these approaches, rather than calculating statistical significances as a function of an assumed model, we calculate significances as a function of the simple question: “Does knowing X help us to predict Y ?”. This question is effectively identical to our earlier question – “Does X affect Y ?”. – but it is structured in an explicitly predictive manner. If the answer to the question is true, then we can say that there is a relationship between X and Y .

The techniques to accomplish prediction-based inference are much newer than classic techniques as linear regression as they rely upon extensive computing power and would not be possible without modern technology. One of these approaches is the *A3* method (XXX Citation) which uses resampling based algorithms to obtain estimates of predictive accuracy and statistical significance. *A3* focuses purely on predictive accuracy of a model to determine whether a variable is significant and often requires the automatic exploration of hundreds or thousands of competing models to find the one that best describes the data. The results of these analyses are inferences that are founded in objective quality of model fits, not on subjective assumptions.

Predictive versus Narrative Modeling

As we can see, inferential techniques can be split into two categories: those based on narrative modeling methods and those based on predictive modeling methods. So from our original three categories of model purposes – prediction, inference, and narrative – we are left with just two fundamental types of modeling approaches: predictive modeling and narrative modeling.

This divide is not traditional used in the modeling field, but it is truly at the heart of modeling. Understanding the distinction between these two types of modeling will prove to be much more valuable than mastering fine technical details. The choice of whether to build a predictive or a narrative model is a fundamental one that shapes every aspect of a model and determines its ultimate utility for a specific purpose. The following sections will describe these two types of models in more detail.

Predictive Models

How do we define a predictive model? The naive answer is that a predictive model is one that makes predictions. If a model generates predictions for a future outcome or a given scenario, than it must be a predictive model. By this definition, a weather forecast is a predictive model as were the Obama administration's unemployment predictions we saw earlier.

Unfortunately, this straightforward definition is useless. Worse than being useless, it is actually quite dangerous.

Let us propose a model for next year's unemployment figures in the United States:

Generate a random number from 0 to 1. If the number is less than 0.1, unemployment will be 20%. If the number is greater than or equal to 0.1, unemployment will be 0%.

There, we have just constructed a model of unemployment. Furthermore, our model creates predictions. With just a few calculations we can forecast unemployment for the coming year. Isn't that convenient?

Of course this model is a joke. It is clearly a nonsensical tool to use to predict unemployment. However, using the naive definition of what it means to a predictive model, it would be classified as one.

What makes this simple model, such a poor model for prediction purposes?

There are several answers to this question. We might start by saying it is too *simple*. Clearly if we are trying to predict unemployment we should incorporate the current economic state and trends into our model. If the economy is improving, unemployment will probably drop and vice versa. This is a valid point. Let's address it by proposing an "improved" model:

Generate a random number from 0 to 1. If the number is less than the percentage change in GDP over the past year, unemployment will be 20% plus the current unemployment rate. If the number is greater than or equal to 0.1, unemployment will be the net change in the consumer price index over the past 8 years.

Is this a better model? Clearly, it is more complex than the previous one and it incorporates some relevant economic data and indicators. Equally as clear, however, is that it is also a joke no one should treat this as a real predictive model.

These toy economic models show that just generating predictions is not a useful criterion to define a predictive model. They also show that complexity and the use of relevant data is not a valid criterion. So how do we specify a predictive model? The answer is straightforward:

A predictive model is a model not only creates predictions but also must contain an *accurate assessment of prediction error*.

It is important to note that only the assessment of prediction error must be accurate, not the accuracy of the predictions themselves. Of course, ideally the predictions themselves will be accurate, however this is often not possible. Many systems are governed to a significant extent by chance and no model, no matter how good it is, will be able to create accurate predictions for the systems.

If you know the level of prediction error, however, you can contextualize poorly fitting models. You can determine how much to discount their predictions in your decision-making and analysis. Furthermore, and this is crucial, you can compare different predictive models. If your current model is insufficiently

accurate, you can develop another one and objectively test it to determine whether it is better than the current model.

Without measures of predictive accuracy, discussing predictions or comparing models that create predictions is an almost nonsensical endeavor. Such discussions will be governed by political concerns and partisanship as there is no objective foundation on which to base them.

Our two proposed models to estimate unemployment are clearly not predictive as we do not have any estimate of predictive error. Furthermore, we can also apply this definition to Obama's employment predictions we saw earlier. When we first presented the model, we called it a narrative model, which was probably slightly confusing as the model did generate predictions. However, using our definition of a predictive model we can see also that it is in fact not a predictive model. The model contains no estimate of prediction error (and one is not available in the original report) so it simply cannot be considered to be predictive.

If accurate estimates of prediction error are available, you can directly compare the prediction errors between different models to select the one with the lowest error. For instance, we could estimate prediction errors for the two joke models we proposed here along with the Obama administration's model to find the one with the lowest error. We would hope that the one the Obama administration presented to congress would be the most accurate. Before we test it however, we must not make the error of blindly accepting a model to be good based on who presented it to us or its complexity.

Why do we so rarely hear about the predictive accuracy of models? There are a number of reasons but they all boil down to three basic issues:

1. Assessing prediction error accurately is quite difficult.
2. Sharing prediction error may perversely decrease an audience's belief in a model.
3. Most models people use for prediction are actually narrative models and their predictive error is either abysmal or irrelevant.

Let's look at each of these points in detail. First consider the issue of the difficulty of assessing prediction error. In general, obtaining an accurate assessment of prediction error is much more difficult than developing the predictions themselves. Most commonly used approaches (for instance the standard R^2 from linear regression) have significant flaws. There are both theoretical and numerical methods that can be used to more accurate prediction errors in many cases (this will be discussed further in the section the Cost of Complexity; see also Fortmann-Roe (2012)). When dealing with time series data, however, like most of those explored in this book, it is often almost impossible to truly manage to accurately assess model prediction error. Recently, theoretical technique to approach these issues have just begun to be developed (e.g. He, Ionides, and

King (2009) or A. A. King et al. (2008)) but they are still impractical to apply in most cases.

If the challenge of measuring prediction error is overcome, there is an even more insidious barrier to its being published with the model. There is a perverse phenomena that the act of reporting prediction error will often *decrease* the credence an audience gives a model. An anecdote was relayed to us by a member of a team working on a model of disease spread. His team shared the predictions from the model with a group of policy makers. Everything was going great until the audience saw the error bars on the predictions. Where his audience had been content with the raw predictions, they were quite unhappy with the predictions when accompanied by their accurately estimated uncertainty. Why was this? Was the team's model particularly bad and these policy makers had a more predictive model at their disposal? Unfortunately, not. In a world where policy makers and clients are constantly shown models (like Obama's unemployment figures) with no measure of uncertainty (or even worse, poorly calculated, artificially low uncertainty), they come to have unrealistic expectations and often turn away good science in favor of magical guesstimates.

Finally, the most likely reason supposedly predictive models do not include prediction error is that they simply are not predictive. Generally models developed for a purportedly predictive purpose are actually narrative models in disguise. Why is this? Well lets look at the reason for most modeling projects. It is very rare that models are commissioned solely for the purpose of generating an accurate prediction. More often, models are part of some political process within an organization. Ultimately, the people funding the model expect it to prove a point to their benefit. In environments like these, it is to be expected that even the most purportedly predictive modeling efforts will become side tracked by political concerns and make significant compromises to fulfill these needs.

We can see the results of such influences in the predictions generated for unemployment presented earlier. Figure 3 shows the projections for the unemployment rates with and without the stimulus plan just as in Figure 2. Overlaid on this, however, are the true values of unemployment the occurred after the predictions were made. As is readily evident, the original modeling and predictions were well off the mark. Not only as reality worse than the projections assuming the stimulus was enacted (which it was) it is much worse than the projections for the economy assuming the stimulus had never been enacted at all! This is just a small example – one that is sadly replicated over and over again in business and policy making – of mistakenly treating a narrative model as a predictive one.

Narrative Models

In contrast to predictive models, a narrative model is one built to tell a story. When most people first hear the “narrative” terminology, they often have

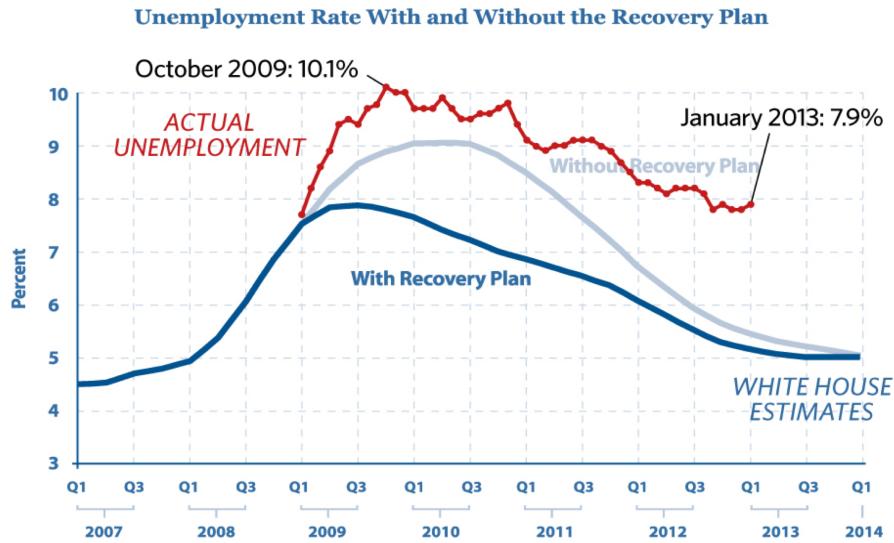


Figure 3. Unemployment projections versus reality (The Heritage Foundation 2013)

an instinctive negative reaction. We find this strange, as narratives are the fundamental human form of communication. We tell narratives to our friends and relatives. Politicians communicate their policies to us using narratives. Of course the vast majority of our entertainment is focused on narratives[⁷Even sports, a form of entertainment that innately contains no narrative, becomes wrapped in narrative as the announcers and commentators attempt to create stories to engage us.]. Business leaders and managers attempt to describe their strategies to us using narratives; and business books are in generally dominated by narrative anecdotes.

Generally we as a species do not view the world as a collection of numbers and probabilities; instead we see consequence and meaning. In short, narratives are how we see the world.

One critique of the term narrative is that it implies a lack of numbers or mathematics. This could not be further off the mark. There are many ways to construct narratives. Words are one, pictures are another, and music is a third. Numbers and mathematics are just another way of telling a story.

In fact, most statistical and mathematical models are in fact narrative models. We looked earlier at the case of linear regression as a tool to predict test scores as a function of wealth. Again the mathematical equation for this simple model was:

$$Score = \beta_0 + \beta_1 \times Wealth$$

This equation defines tells narrative. Translating this narrative into words, we would say:

Test scores are only determined by the wealth of a student's family. A child whose family is broke, will have a test score, on average, of β_0 . For every dollar of wealth a child's family accumulates, the child will score, on average, better on tests by β_1 .

You might or might not agree with this narrative (in our view it is a nonsensical and reductionist view of child achievement) but it shows the strict equivalence between this mathematical narrative and narrative prose. This process can be applied to all mathematical models. The mathematical definition of the model can be converted directly, with more or less lucidity, into a story describing how the system operates. The same can also be done in the reverse: we can take a descriptive narrative of a system and convert it into a mathematical description. As we have seen (will see? XXX) this is what tools like reference modes and pattern matching are designed to do efficiently: elicit a narrative from a subject in a way which can be reformulated quantitatively.

With predictive models, we can compare competing models based primarily on predictive accuracy[^Other criteria include ease of use, cost of filling data requirements, and computational requirements. But all those are generally secondary to prediction accuracy.]. But how do we evaluate and compare the quality of narrative models?

The basic criterion to assess a narrative models is that of its ability to be *persuasive*. Although persuasion is not a purely objective measure like prediction accuracy, we can decompose it into two components: believability and clarity. A persuasive model is one that is both highly believable and one that effectively communicates its message.

When building a narrative it is very important to use tools that are well suited to meeting these components. Again, as with most modeling work, statistical models like the wealth linear regression example are the most commonly used tools for the construction of narrative models. Unfortunately, they are quite poorly suited to this task in many ways. Most statistical models depend on numerous unrealistic and highly technical assumptions about the data. If these assumptions were enumerated in plain English, they would often conflict with people's understanding of a system discrediting the model. The alternative is to leave these assumptions hidden creating a black box model opaque to outside inspection.

This is often the road taken and in our view, it is a shame. It can be successful if the authority presenting the model is prestigious enough. But it will quickly crumble if any kind of rigorous scrutiny is applied to the model. Narrative models should never be given any credence if the operation of the model is not transparent. Most statistical models are built on assumptions that are never

made transparent to the audience. There is a reason for the saying, “there are lies, damned lies, and statistics.”

The modeling techniques presented in this book, on the other hand, are well suited for narrative modeling. The techniques we present are “clear box” modeling where the workings of the model are transparently evident and accessible. The models in the book have their structure explicitly described using an accessible modeling diagram showing the interactions between the different components in the model. The equations governing the evolution of a model are clear and readily available for each part of the model[^Admittedly, for complex models it may still require a significant investment on the part of an audience to fully understand the logic and equations in the model. But the opportunity is available.]. Furthermore, these modeling techniques used here make it straightforward to generate animated illustrations and displays to clearly communicate model results.

Confidence Building Steps for Narrative Models

When used correctly, the transparency of these modeling techniques results in models that are powerful persuasive tools. As with any model, however, there are concerns and questions people will invariably raise which could cause them to doubt the result of the modeling work. There are a number of techniques that you can use to help preemptively address these concerns and increase an audience’s confidence in your model.

The idea of building confidence in a model is closely tied to the standard concept of model verification and validation. We dislike this conceptual approach to assessing models as it seems to imply that a model can go through a process to get a big “VALID” or “VERIFIED” stamp on it. Returning to Pox’s quote at the beginning of this chapter, in reality all models are wrong and none of them are valid. Models can however be useful and for narrative models a key part of that utility is the confidence an audience has in them.

We favor the conceptual approach put forth by Forrester and Senge (1978), that there is not any single test or suite of tests that will verify or validate a model and that validity should instead be thought of as a function of confidence. This is a view that differs from that held by many modelers and laypeople. As Forrester and Senge note, “the notion of validity as equivalent to confidence conflicts with the view many seem to hold which equates validity with absolute truth.” We share their belief that model confidence is built up piece by piece from a variety of tests that, though they cannot prove anything, together comprise a persuasive case for the quality of a model.

There are three distinct areas where confidence needs to be developed:

1. That the model itself is well designed.
2. Given a design of the model, this design is implemented correctly.

3. The conclusions drawn from the model are accurate.

Model Design

Fundamentally the design of a narrative model is of utmost importance and needs to be justified to an audience[^This is different from predictive models where the results of the model are much more important than the design and the “proof is in the pudding” so to speak.]. There are two primary aspects to a model’s design: the structure of the model and the data used to parameterize the model.

Structure

Generally, the structure of the model should mirror that of the structure of the system being simulated. Depending on the complexity of the system, the model structure may need to carry out more or less aggregation and simplification of this reality. Nevertheless, all the primitives in the model should map on to reality in a way that is understandable and relatable to the audience. Furthermore, the model structure should include any components that an audience will think could be important drivers of the system. Missing a factor that the audience considers to be a key driver can fatally discredit a model in an audience’s mind irrespective of the performance or other qualities of the model. This is true even if the factor truly has a negligible effect. Generally, speaking it is much easier to include a factor an audience views as important than it is to convince the audience that the factor does not in actuality matter.

Data

The more a model uses real world data, the more confidence an audience will have in the model. Ideally, you will have empirical data to justify the value of every primitive in your model. In practice, however, such a goal is generally a pipe dream. For a complex model, obtaining data to parameterize every aspect of it is usually impossible[^Leading to the clichéd conclusion of many modeling studies: “We are unable to draw strong conclusions from this modeling work. Instead, our contribution has been to show where additional data needs to be collected.”]. When faced with model primitives that do not have empirical data to parameterize them, an approach must be taken to ensure that it does not appear that their values were chosen without justification or to arrive at a predetermined modeling conclusion. Sensitivity testing, as discussed later on, is one way to achieve this. Another is to carry out a survey of experts in the field in order to solicit a set of recommended parameter values that can then be aggregated or used to justify the ultimate parameterization.

Peer-Review

Going through a peer-review process can be extremely useful in establishing confidence in a model. Two general types of peer-review are available. In one,

the model may be incorporated into an academic journal article and submitted for publication. The article will then peer-reviewed by generally two or three anonymous academics in the field who will critique it judge whether or not it is worthy for publication. In the second type of peer-review, a peer-review committee may be hired to review a specific model and provide conclusions and recommendations to clients.

Peer-review can be very useful in establishing the credibility of a model. By engaging an independent group of experts to assess the model, their conclusions about its quality have the appearance of greater validity than those of the self-interested modelers[^When the peer review panel is hired by the client there is some conflict of interests, but the panel members should not be swayed by this.]. This can be especially useful when trying to meet some abstract standard such as that the model represents the “best available technology” or “best available science”.

A key risk of a peer-review is, of course, that the peer-review members will find a model lacking and criticize it. Good criticism can be very useful and help improve a model, however, much of criticism received in practice may be minor nitpicking details or actually detrimental advice that would make the model worse if followed.

Model Implementation

Although it is generally not as much a lightning rod as is the design of the model, the implementation of a model specification is a point at which there is significant potential for error to occur. Programming mistakes or mistyped equations can introduce bugs into a model that can be hard to identify. This is a particular problem in Black-box models but it is still an important point to consider for all types of models including those presented in this book. Fortunately, a number of steps can be made to ensure the model is implemented correctly.

Primitive Constraints

For many of the primitives in the model, there will be natural constraints. For instance, a stock representing the volume of water in a lake can never fall below 0. Similarly, if a variable represents the probability of an event occurring, it must be between 0 and 1.

Often these constraints are implicit without being formally specified in the model. Of course, a modeler may think, water volume can never become negative so why would I need to specify it? However, the existence of these constraints provides an opportunity to implement a level of automatic model checking. By specifying that a primitive can never go above or below a value (using the *Max Value* and *Min Value* properties in Insight Maker), you can create in effect a canary in the coal mine which can give you a warning if something is wrong in the model. If these constraints are ever violated an error

message can be given letting you know that you need to correct some aspect of your model.

Unit Specification

Since we introduced units in Chapter 3, we showed that they could be a useful tool in constructing models. Units can also be used to ensure that equations are entered correctly. If you fully specify the units in a model, many types of equation errors will result in invalid units, which will create an immediate error. By employing units in your model you can automatically catch a whole class of errors and mistyped equations.

Regression Tests

More general tests other than those specified above can be developed. For instance, the proper behavior of one part of the model may be determined and automated tests created to periodically confirm that the model continues to exhibit the correct behavior. Development of such tests are a common part of software engineering that we wish would see more use in model development. Insight Maker itself has a suite of over 1,000 individual regression tests that automatically test every aspect of its simulation engine.

In regards to regression testing it is important to ensure these tests truly are automated. It is not enough to examine a portion of the model, determine it is currently working correctly and leave it at that. The reason for this is future changes may break the existing functionality. Especially for complex models, a change in one part of the model may have an unexpected effect in another part. By implementing a set of automatic checks, you can protect your model against unintended changes and regressions.

A Second Pair of Eyes

That is not to say, however, that spot and point-in-time checks are not worthwhile. It can be very useful to have a second modeler review your models and check the equations. This can be useful not only to check simple mistakes but also to question and critique the fundamental structure and choices of the model.

The gold standard, however, in verifying that a model is implemented correctly according to specification is to have a second modeler completely reimplement the model according to this specification. Such reimplementation should ideally occur without access to the original model's code base to ensure that the second modeler does not simply copy bugs from the original model into the reimplementation. If the results from the two implementations concur, that is very strong evidence that the model has been implemented correctly. The result of the process will most likely identify numerous ambiguities in the specification, which could be valuable in and of itself.

Model Results

Given that the design of the model and its implementation are assumed to be correct, the burden still falls upon the modeler to transfer her confidence in the model's results to her audience. There are several different ways this can be done.

Expected Results

The first way is to simply demonstrate that the model generates expected results for normal inputs. For instance, if you had a model a reservoir, you would expect the volume of the reservoir to decline over time during the summer due to evaporation if no more water flowed into it. You can additionally test extreme scenarios and show that they generate the expected results. For example, if your reservoir was empty, you would expect the amount of water to evaporate from it to be zero. By enumerating these standard cases and showing the model results match the expected results you can help build confidence in the model.

Counterintuitive Results

Another attempt to increase confidence in a model is to show unexpected results that are justifiable. For instance, imagine a model that for a certain set of inputs would create what, at first glance, appeared to be the “wrong” behavior. Some lever in the model could lead to unexpected results. When first shown these results, they could decrease an audience’s confidence in the model. If the audience was then walked through the model step by step to show how those results were actually correct and in fact mirrored what was expected in reality, then that could actually greatly increase their confidence in the model results.

Forecasting

Possibly the most persuasive action to convince an audience of the effectiveness of a model is to forecast the future and then to show that this forecast is correct. Unfortunately, this is often hard to do in practice for multiple reasons including the fact that the scale of a model is often such that it could take several years or decades to generate data to test the model. Additionally, of course, most narrative models are poor predictors and should not be used for predictive purposes.

Sensitivity Testing

Sensitivity testing is a complex technique that has the potential to address many questions and doubts that may arise about a model. In general, the variables and numeric configuration values in a model will never be known with complete certainty. When the results from an election poll are published, the pollsters publish not only their predictions but also the uncertainty in the prediction (e.g., “the Democratic candidate will obtain $52\% \pm 3\%$ of the vote”). Similarly

when a building is constructed, the materials used will have certain properties – such as strength – which again are only known up to some errors or tolerance. It is the engineer and contractor's responsibilities to ensure that the materials are sufficient even given the uncertainty of their exact strengths.

The same occurs when modeling. Most primitive values in the model will have to be estimated by the modeler and there will be an error associated with these values. Of course the error will also be propagated through the model when it is simulated and affect the results output by the model. This error is one factor that can create doubt about a model and reduce an audience's confidence.

As a modeler, one approach to address this doubt would be to try to measure all the model's variables with great accuracy. You could search the available literature, carry out experiments, and survey experts to get as precise a set of parameter values as possible. If you were able to say with strong certainty that these values were so accurate and the errors so small that their effect on the results is negligible, than that would be one way of addressing the issue of uncertainty.

However, this is often impossible to do. When dealing with complex systems it is almost always the case that at least a couple variable values will never be able to be known with a significant degree of uncertainty. In this case, no matter how much research or experiments you do, you will never be able to pin down the precise values of these variables. How do we handle these cases?

The answer is straightforward: rather than trying to eliminate the uncertainty, we embrace it by explicitly including it in the model. If you can then show that the results of your model do not significantly change even given the uncertainty, you have a persuasive case for the validity of your results. Of course the results will always change when the uncertainty is introduced, but if the narrative of the model conclusions is persistent given this uncertainty it will greatly increase your audience's confidence in the results.

Uncertainty can be explicitly integrated into a model by replacing constant primitive values with a construct that represents the uncertainty in that value. For instance, imagine you had a simple population model of rabbits in a cage. You want to know how many rabbits you will have after two years. However, you don't know if you how many rabbits there initially are in the cage. You have been told that there are probably 12 rabbits, but the true number could range anywhere from 6 to 18.

If you model your population as a single stock, what should the initial value be? A naive model could be built where you the initial value of the rabbit stock was specified as 12. However, that does not incorporate the uncertainty and could be a source of criticism or doubt for the model. An alternative approach would be to specify that the initial value of the stock is a random number with a minimum value of 6 and a maximum value of 18. So each time you run the model you will get a different result. If you ran the model once, the initial value

might be chosen to be 7 and you would obtain one result. If you ran the model again, the initial value might be 13 and you would get another result.

If you run this stochastic model many times, you will get a range of results. These results can be automatically aggregated to show the range of outputs. For instance if you ran the model 100 times you could see what the maximum and minimum final populations were. This would give you a good feeling for how many rabbits you needed to prepare for after two years. In addition to the maximum and minimum you might be interested in the average of these 100 runs: the expected number of rabbits you would see. You could also plot the distribution of the final population sizes using a histogram to see how the results are distributed. This distribution would show how sensitive the outputs are to the uncertainty in the inputs: a form of sensitivity testing.

[XXX Embedded Demo]

There are four key distributions that are quite useful for specifying the uncertainty in a variable:

Uniform Distribution : The uniform distribution is defined by two parameters: a minimum and a maximum. Each number within these two boundaries has an equal probability of being sampled. The uniform distribution is useful when you know the boundaries on the values a variable can take on, but you do not have any information on the likelihood of the different values within this region. The uniform distribution can be used in Insight Maker using the function *Rand(Minimum, Maximum)*, the two parameters are optional and will default to 0 and 1 if *Rand()* is called without them.

Triangular Distribution : The triangular distribution is defined by three parameters: the minimum, the maximum, and the peak. Like the uniform distribution, the triangular distribution will only generate numbers between the minimum and maximum. Unlike the uniform distribution, the triangular distribution will not sample all numbers between these boundaries with equal likelihood. The value specified by the peak will have the most likelihood of being sampled with the likelihood falling off as you move away from the peak towards either the minimum or maximum boundary. The triangular distribution is useful when you know both the most likely value for a variable and you also know boundaries for the values a variable can take on. The triangular distribution can be used in Insight Maker using the function *RandTriangular(Minimum, Maximum, Peak)*.

Normal Distribution : The normal distribution is defined by two parameters: the mean of the distribution (generally denoted μ) and the standard deviation of the distribution (generally denoted σ). The most likely value to be sampled from the normal distribution is the mean. As you move away from the mean (in either a positive or negative direction), the likelihood of a number being sampled decreases. The standard deviation controls how fast this likelihood falls as you move away from the mean. Small standard deviations result in steep declines in the likelihood while large standard deviations result in more gradual

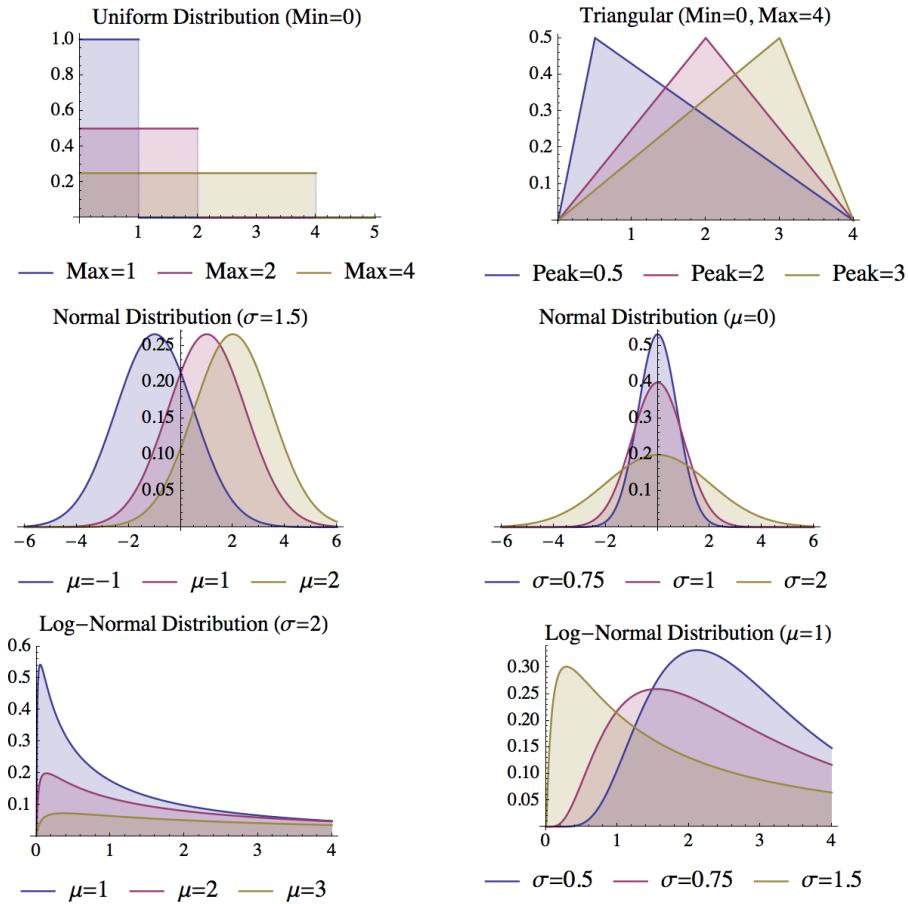


Figure 4. Common Distributions for Sensitivity Testing with Sample Parameter Values

declines. The normal distribution is useful when you do not have boundaries on the values for a variable but you do know what the most likely value for the variable should be (the mean). The normal distribution can be used in Insight Maker using the function *RandNormal(Mean, Standard Deviation)*.

Log-normal Distribution : The log-normal distribution is closely related to the normal distribution. In fact the logarithm of the values samples from a normal distribution will be log-normally distributed. Like the normal distribution, the log-normal distribution is defined by two parameters: the mean and standard deviation. Where the log-normal distribution differs from the normal distribution, is that negative values will never be generated by the log-normal distribution. Thus it is useful when you have a variable which you know cannot be negative but for which you do not have an upper bound.

The log-normal distribution can be used in Insight Maker using the function *RandLogNormal(Mean, Standard Deviation)*. The log-normal distribution can also be used to represent other types of one-sided boundaries. For instance, the following equation could be used to represent a variable whose number was always less than 5: $5 - \text{RandLogNormal}(2, 1)$

There are many other forms of probability distributions. Some notable ones are the Binomial Distribution (*RandBinomial(Count, Probability)*), the Negative Binomial Distribution (*RandNegativeBinomial(Successes, Probability)*), the Poisson Distribution (*RandPoisson(Lambda)*), the Exponential Distribution (*RandExp(Lambda)*) and the Gamma Distribution (*RandGamma(Alpha, Beta)*). These distributions can be used to address very specific modeling usage cases and needs (for instance, the Poisson distribution can be used to model the number of arrivals over time), however, the four distributions described in detail above should generally be sufficient for most sensitivity testing needs.

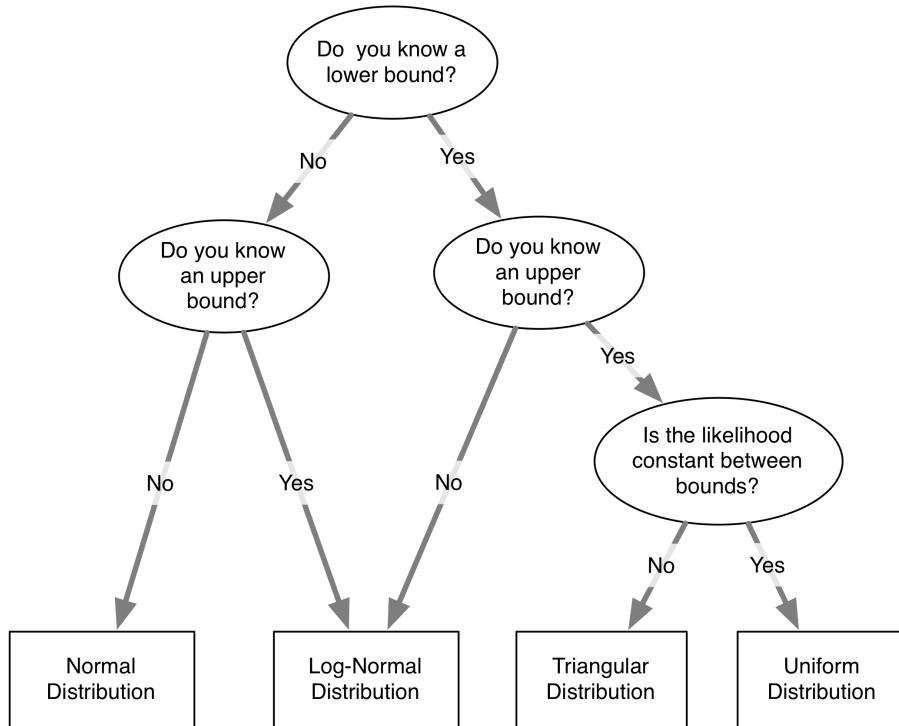


Figure 5. Choices in Selecting a Distribution for a Variable's Value

The astute reader will notice that our discussion up to this has failed to address an important point: how do we determine the uncertainty of a variable? It is very easy to say that we do not know the precise value of a variable, but it is much harder to define the uncertainty of it. One case where we can precisely define uncertainty is when you take a random sample of measurements. For

instance, suppose our model included the height of the average American man as a variable. We could randomly select a hundred men and measure their heights. In this case our uncertainty would be normally distributed with a mean equal to the mean of our sample of one hundred men and a standard deviation equal to the standard error of our sample of one hundred men[^ Please note that this contradicts slightly what we said earlier. Clearly, a person cannot have a negative height while the normal distribution will sometimes generate negative values. So wouldn't a log-normal distribution be better than a normal distribution? Mechanistically, it would, however statistically we can show that due to the Central Limit Theorem the normal distribution does asymptotically precisely model our uncertainty. Given a large enough sample size (100 is more than enough in this case), the standard deviations for uncertainty will be so small that the chances of seeing a negative number (or even one far from the mean) are effectively none.]. For any random sample of n values from a population, the same should hold true: you will be able to model your uncertainty using a normal distribution with:

$$\mu = \frac{\text{Value}_1 + \text{Value}_2 + \text{Value}_3 + \dots + \text{Value}_n}{n}$$

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (\text{Value}_i - \mu)^2}$$

However, in most applied cases you will not be able to apply this normality assumption. Generally you will not have a nice random sample, or you might have no data at all and instead have some abstract variable you need to specify a value for. In these cases, it is up to you to make a judgment call on the uncertainty. Choose one of the four distributions detailed above and use whatever expert knowledge available to you to place an estimate on the parameterization of uncertainty. One rule of thumb, however, is that it is better to overestimate uncertainty than underestimate it. It is better to err on the side of overestimating your lack of knowledge than it is to obtain undue confidence in model results due to an underestimation of uncertainty.

Chapter 5

Model Testing

This is some **Markdown**.

fjdshkfjds

fdshfjsd

What is “Time Step”?

You do not have to know calculus to use Insight Maker. You just need to know some algebra. There are two details about calculus that Insight Maker asks for your help in selecting the best option. These are in the “Time Settings” dialog box where you configure the time settings.

The goal is to have the incremental step change as small as possible without using too much computer memory and processing time. When computers were first used for calculus they did not have sufficient memory and processing speed to use extremely small incremental step change values.

Open the “Time Settings” diaglog box. The simulation start and length are for you to decide how long to run your simulation. Then there is the “Simulation Time Step”. First, the units for this are NOT time. Using the word “time” here is a misnomer. “Time Step” means the change in value of the horizontal axis (or x-axis) as the value of the vertical axis changes.

In calculus this value is used to calculate an exact answer by taking the limit as the change in the horizontal axis value approaches zero. OK, that was complex, here is what you need to know:

1. Pick a value for “Time Step” that is 0.125 or less for your first run.
2. Observe the shape of the graph and record the exact value of the last value calculated for the Y-axis and X-axis (time).
3. Pick a new value for “Time Step” that is half of the previous value.

4. Observe the shape of the graph and compare the exact value of the last calculated value to those in step 2 above.
5. If there is no change then you are done. If there is a change in the shape of the graph or if the values are different then repeat step 3 until there is no change out to the appropriate number of significant digits for your values.

The second item of concern is the “Analysis Algorithm”. Again, when computers were slow and had less memory, the integration method was important. The choice of using “Fast (Euler)” is not necessary in the 21st Century. You should always use “Accurate (RK4)”.

RK4 means Runge-Kutta, fourth-order method, an accurate method for finding approximation solutions to ordinary differential equations.

Model: Test Model

```
{“create”: “Stock,”geometry“:{”x“:400,”y“:140,”width“:100,”height“:40},”name“:”Healthy“}
{“create”: “Stock,”geometry“:{”x“:400,”y“:280,”width“:100,”height“:40},”name“:”Infected“}
{“target”: “Infected,”geometry“:{”x“:400,”y“:280,”width“:100,”height“:40}}
```

DIAGRAM

```
{“create”: “Flow,”geometry“:{”x“:0,”y“:0,”width“:100,”height“:100},”alpha“:”Healthy,
“omega”: “Infected,”name“:”Infection“}
```

DIAGRAM

The basic model structure has been laid-out, we can start to define parameter values and equations. We’ll start with a very simple model containing a population of 100 people and where 2 people becoming sick each year.

```
{“attribute”: “InitialValue,”target“:”Healthy, “value”: “100”}
{“attribute”: “FlowRate,”target“:”Infection, “value”: “2”}
```

RESULTS

The results are as we would expect. However this is not a model of an infectious disease as the infection rate does not depend on the presence of infected individuals. Let’s change the infection rate so it depends on the contact rates between sick and healthy people.

```
{“attribute”: “FlowRate,”target“:”Infection, “value”: “0.006/[Healthy]/[Infected]”}
```

RESULTS

That’s strange. No one ever gets sick. Why is that? It turns out it is because we start the simulation with no infected people in the model. Since we’re modeling an infectious disease, this means there is no one to start the epidemic! Let’s change that we’ll add a single infected person to get the epidemic started.

```
{“attribute”: “InitialValue,”target“:”Infected, “value”: “1”}
```

RESULTS

That looks about right. Before moving on, let’s spend a moment to improve our model structure. Right now if we wanted to edit the infection rate, we would have to dig down in the equations to find the right number. Let’s make our model more modular, without changing any results, by separating the infection rate into its own variable.

```
{“create”: “Variable,”geometry“:{”x“:230,”y“:145,”width“:120,”height“:50},”name“:”Infection Rate”}
```

```
{“create”: “Link,”geometry“:{”x“:0,”y“:0,”width“:100,”height“:100},”alpha“:”Infection Rate, “omega”: “Infection”}
```

DIAGRAM

```
{“attribute”: “Equation,”target“:”Infection Rate, “value”: “0.006”}
```

```
{“attribute”: “FlowRate,”target“:”Infection, “value”: “[Infection Rate]/[Healthy][Infected]”}
```

RESULTS

We can hide the display of the infection rate by configuring the display.

```
{“attribute”: “Primitives,”target“:”DISPLAY, “value”:[“Healthy”, “Infected”]}
```

RESULTS

Now that we have our basic model working, let’s extend it by adding the phenomena of people recovering from the disease. We’ll model something like the Chicken Pox where people become immune to the disease after they recover.

```
{“create”: “Stock,”geometry“:{”x“:400,”y“:400,”width“:100,”height“:40},”name“:”Immune”}
```

```
{“target”: “Immune,”geometry“:{”x“:400,”y“:400,”width“:100,”height“:40}}
```

```
{“create”: “Flow,”geometry“:{”x“:0,”y“:0,”width“:100,”height“:100},”alpha“:”Infected, “omega”: “Immune,”name“:”Recovery”}
```

```
{“create”: “Variable,”geometry“:{”x“:610,”y“:290,”width“:120,”height“:50},”name“:”Recovery Rate”}
```

```
{“create”: “Link,”geometry“:{”x“:0,”y“:0,”width“:100,”height“:100},”alpha“:”Recovery Rate, “omega”: “Recovery”}
```

DIAGRAM

```
{“attribute”: “Equation,”target“:”Recovery Rate, “value”: “0.1”}
```

```
{“attribute”: “FlowRate,”target“:”Recovery, “value”: “[Recovery Rate]*[Infected]”}
```

```
{“attribute”: “Equation,”target“:”Infection Rate, “value”: “0.008”}
```

```
{“attribute”: “Primitives,”target“:”DISPLAY, “value”:[“Healthy”, “Infected”, “Immune”]}
```

RESULTS

Fantastic! Now we have a working disease simulation. You can experiment with different population sizes, infection rates and recovery rates to see how they change the results.

This is what is known as the *SIR Model* (Susceptible-Infected-Recovered) in the modeling community.

Chapter 6

End Model

Some more text...

Chapter 7

Model: Another Model

testing...

Chapter 8

End Model

This is the end of the chapter.

Chapter 9

References

- Forrester, Jay Wright, and Peter M. Senge. 1978. "Tests for building confidence in system dynamics models."
- Fortmann-Roe, Scott. 2012. "Accurately Measuring Model Prediction Error" (apr). <http://scott.fortmann-roe.com/docs/MeasuringError.html>.
- Haller, H., and S. Krauss. 2002. "Misinterpretations of Significance: A Problem Students Share with Their Teachers." *Methods of Psychological Research Online* 7 (1): 1–20.
- He, D., E. L. Ionides, and A. A. King. 2009. "Plug-and-play inference for disease dynamics: measles in large and small populations as a case study." *Journal of The Royal Society Interface* 7 (43) (dec): 271–283.
- King, Aaron A., Edward L. Ionides, Mercedes Pascual, and Menno J. Bouma. 2008. "Inapparent infections and cholera dynamics." *Nature* 454 (7206) (aug): 877–880.
- Romer, Christina, and Jared Bernstein. 2009. "The job impact of the American recovery and reinvestment plan."
- The Heritage Foundation. 2013. "Unemployment Rate January 2013" (feb). <http://www.heritage.org/multimedia/infographic/2013/02/unemployment-rate-january-2013>.