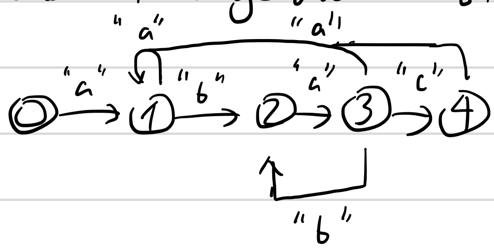
* let | = "such th"

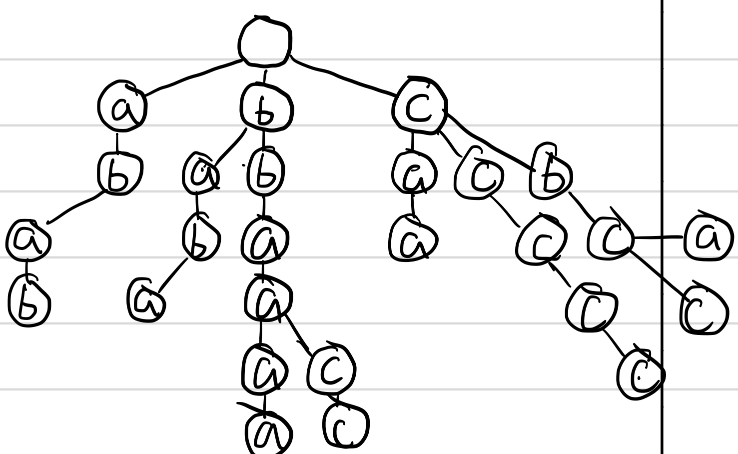
1.)

a.) $\Sigma = \{a, b, c\}$

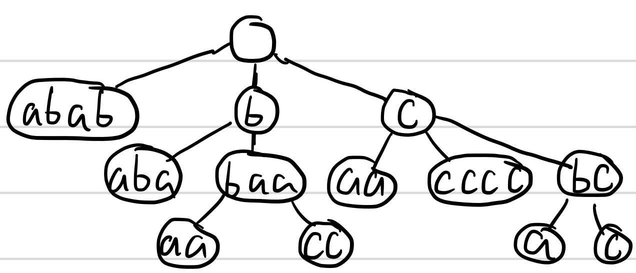
* cases that go back to state "0" aren't represented to reduce clutter



b.)



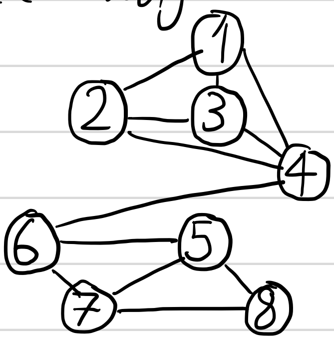
c.)



2.)

a.)

* let adj. = adjacency



b.) use an edge list to minimize space for $E = O(V)$, b/c if we say $E = V$.

comparing space requirements

Edge list: space = E, adj list: space = $E + V$ = $V + V$

and Adj. Matrix: space = V^2

we can see edge list with $\Theta(E)$ is the best. Especially for, $E = O(V^2)$ assuming again asymptotic variables are equal

$E = V^2$ edge list: space = V^2

adj. list: space = $2V^2$

adj matrix: V^2.

b/c they have the same space of V^2

Edge list or adj. matrix is the best

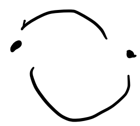
c.)

	a	b	c	d
a	0	0	0	0
b	1	0	0	0
c	1	0	0	0
d	1	0	0	0

explanation on 2 pages from this page →

In terms of access time adj. matrix is the best b/c it can be done in worst case constant time. Edge list is worst case adj. time = $\Theta(E)$.

Adjacency list is Θ V



3.)

* constant $k \mid k \in \mathbb{N} \wedge k \neq 0$
* graph coloring: no edge connect same color v
* Chromatic #: minimum # color, color a graph.

a.) $n = 2k$.

b.) $n = k \rightarrow$ n can be k, b/c star graphs always have chromatic # 2

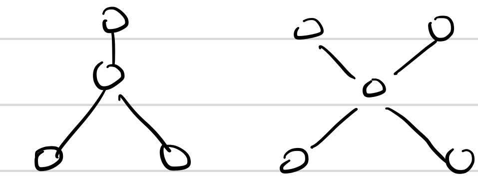
c.) $n = 2k+1$. n is odd b/c the "+1" is the central node which is it's own color. all the other nodes are diffrent color from central node b/c central node connects to all other nodes. the other nodes besides the edgese from central node are connected in a wheel type formation so they must have as mentioned in 3b.) chromatic #2 w/ 2k verties. that's when 2k is from in $n=2k+1$. Therefore the central node color (1) plus the chromatic behavior of the cycle type graph in W = chromatic #3 | $n=2k+1$

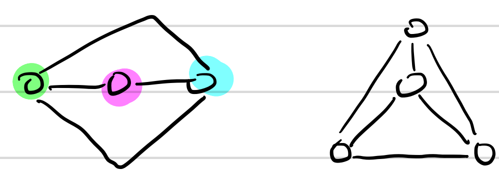
d.) chromatic #3 is false. See rest of answer on NEXT PAGE.

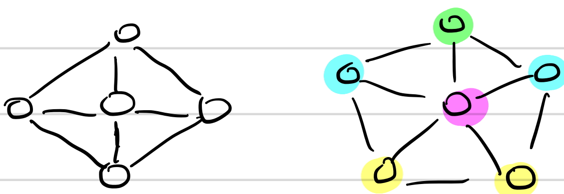
e.) $K_{n,n}$ cant be chromatic #3, b/c $K_{n,m}$ of $n+m \mid (n \text{ and } m) \in \mathbb{N}$ and $(n \text{ and } m) \neq 0$ always chromatic #2. It's always 2, b/c complete nature makes all v touch. However, the bipartite nature makes two distinct group & each v in each group cant touch each other. That NEXT PAGE →

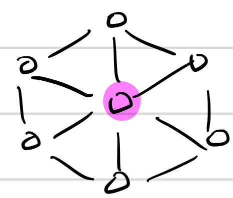
cyclese graphs have to always have $V=2k$ b/c the colors must alternate from start to finish. If, $V=2k+1$ it wouldn't work b/c it would be always possible for end vertex → to be adjacent to same color (end vertex −1).

b/c the central node can be one color and all others another (others don't touch).



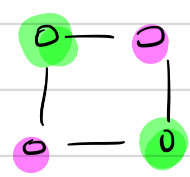






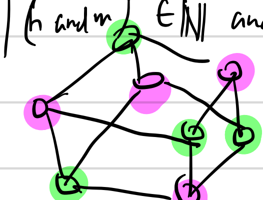
Q_1

Q_2 =

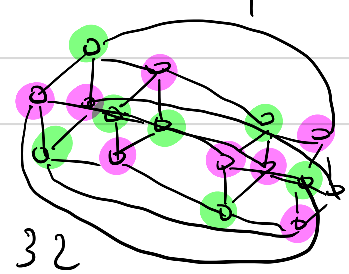


Q_3

- $V = 8$
- $E = (2^{3-1})(3) = 12$
- $n = 3$, edge touch each vertex.



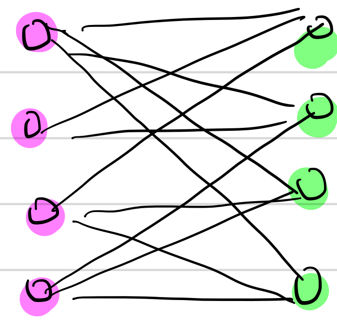
2^{n-1} point



Q_4

$V = 2^4 = 16$

$E = (2^{4-1})(4) = 32$



d.) the chromatic # I believe after testing up to Q_4 is always chromatic # = 2. chromatic #3 is impossible. the hypercube is always chromatic #2 b/c the vertices can always be divided into two distinct sets each 2^{n-1}. As well you can get chromatic #2 b/c the graph is bipartite (Q_3 see example)

e.) means all v inside a group are same color. the only other vertices that a vertex can touch are all those in the other group that are only one color. Therefore K_{nn} must be actually chromatic # 2 b/c all $K_{n,m}$ are chromatic #2.

2.) c.) In G | relationship of vertice a to its edge between itself and b represented as {a, b}. also given {a, b} = 0 a has in degree from b, and {a, b} = 1 a has an out-degree to b. To check for universal sink create a multidimensional array of n x n size where n = # of vertiese.

implement the following Psuedo code.

set count_deg = 0
set univ_sink = false
set matrix [n][n]
itterate through all matrix[0][n]
IF a ellem in range matrix [0][n] equal to 0 Ø
then check IF all ellements under ellem that was equal to zero are "1" For i = 1 to n
check IF matrix [i][column ellem equal to Ø]
= 1 (keep a count) If count $\boxed{\text{same letter}}$
= n-1 then $\boxed{\text{universal sink}}$

Also, assume for (,) case having no loopback equals 0. ex {a, a}