Protocol Verification

A Brief Introduction to Model Checking and Temporal Logic

Andrés Goens (U. of Amsterdam)

SPLV 2024 @ Strathclyde

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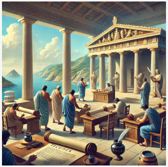
Motivation

Protocol Verification?

Protocols

Examples of protocols

- Distributed systems (e.g. paxos) 

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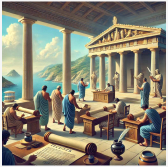
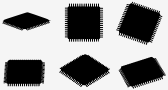
- Distributed systems (e.g. paxos) 
- Hardware (e.g. cache coherence) 

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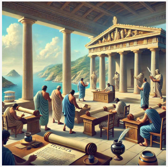
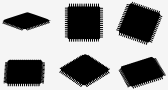
- Distributed systems (e.g. paxos) 
- Hardware (e.g. cache coherence) 
- Cryptographic protocols (e.g. TLS) 

Verification

Examples of properties

- Fairness 

Verification

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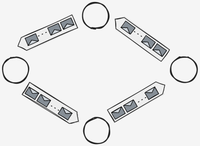
- Deadlock-freedom 

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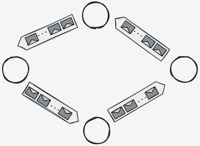
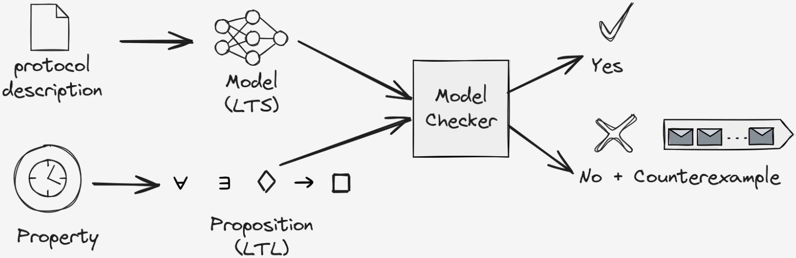
- Deadlock-freedom 
- Safety 

What this course is about



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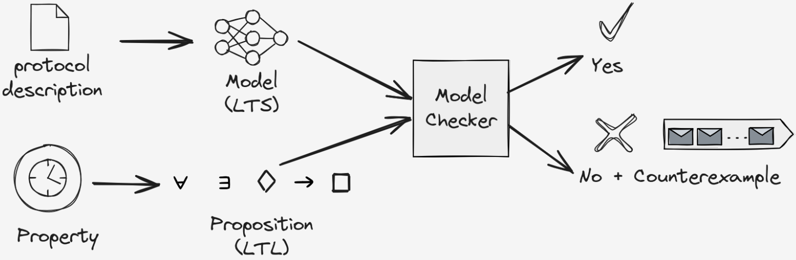

This is what we'll cover

Overview of the course

What you *will* (hopefully) know by the end

- Labeled transition systems (LTS)
- Modeling languages (promela)
- (Propositional) Linear Temporal Logic (LTL)
- Examples!

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What you will *not* (necessarily) know by the end

- Other logics (e.g. CTL*, μ calculus)
- How model checking works internally (decision procedures)

Modelling Protocols

Labeled Transition Systems

Definition (Labeled Transition Systems)

A labeled transition system is a tuple of the form $(S, \mathrm{Act}, \rightarrow, S_0, \mathrm{AP}, L)$, where S is a set of states, $S_0 \subseteq S$ a subset of initial states, Act is a set (of actions), $\rightarrow \subseteq \mathrm{Act} \times S \times S$ is a (transition) relation, AP is a set (of atomic propositions) and $L : S \rightarrow \mathrm{Pow}(\mathrm{AP})$ is a (labeling) function.

Example: Traffic Light

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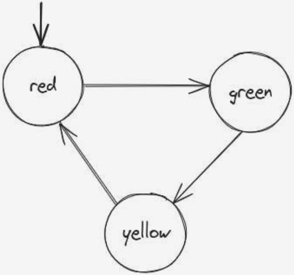

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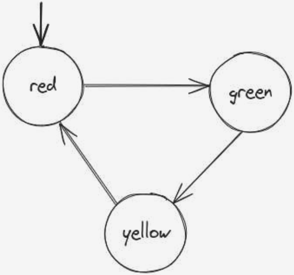
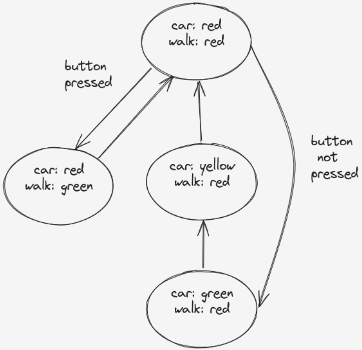


- $S = \{\text{red}, \text{green}, \text{yellow}\},\ S_0 = \text{red}$
- $\text{Act} = \{*\}$
- $\rightarrow = \{(*, \text{red}, \text{green}), (*, \text{green}, \text{yellow}), (*, \text{yellow}, \text{red})\}$
- $\text{AP} = L = \varnothing.$

Two Traffic Lights



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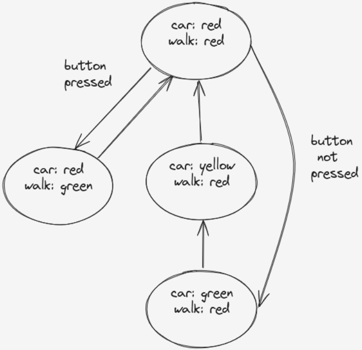

- $\text{Act} = \{\epsilon, \text{button pressed}, \text{no button pressed}\}$
- $\text{AP} = \{\text{Pedestrians can go}, \text{Cars can go}\}$
- $L = \text{cars: red, walk: green} \mapsto \{\text{Pedestrians can go}\}, \ldots$

Interleaving

Two traffic lights $\nleftrightarrow$ One LTS

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Definition (Interleaving)

Let $TS_i = (S_i, \mathrm{Act}_i, \rightarrow_i, S_{0,i}, \mathrm{AP}_i, L_i), i = 1, 2$ be two transition systems. We define the transition system $TS_1 \,|||\, TS_2 :=$ $(S_1 \times S_2, \mathrm{Act}_1 \times \mathrm{Act}_2, \rightarrow, S_{0,1} \times S_{0,2}, \mathrm{AP}_1 \cup \mathrm{AP}_2, L_1 \times L_2)$, where $L_1 \times L_2 : S_1 \times S_2 \rightarrow \mathrm{Pow}(\mathrm{AP}_1 \cup \mathrm{AP}_2)$ is defined as $(L_1 \times L_2)(s_1, s_2) = L_1(s_1) \cup L_2(s_2)$ and $\rightarrow$ is defined by

$$\frac{s_1 \rightarrow_1^\alpha s_1'}{(s_1, s_2) \rightarrow^\alpha (s_1', s_2)} \qquad \frac{s_2 \rightarrow_2^\alpha s_2'}{(s_1, s_2) \rightarrow^\alpha (s_1, s_2')} \; .$$

We call this construction the *interleaving* of TS_1 and TS_2.

Interleaving

Two traffic lights $\not\leftrightarrow$ One LTS

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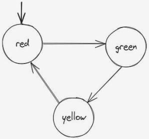
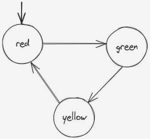
Note that this means the two TS are *independent*

Example: Intearleaving



$|||$



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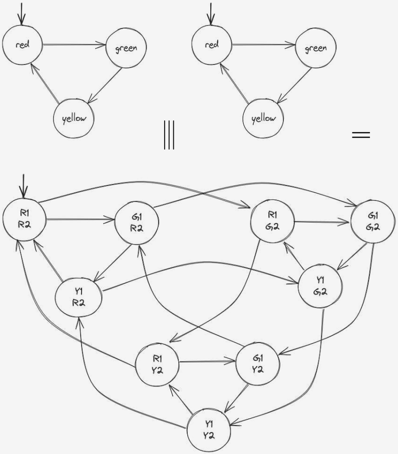


Parallel Composition

Definition (Handshake)

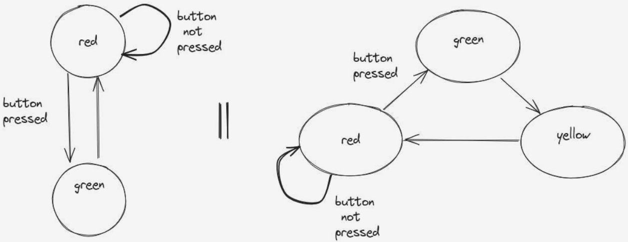
Let $TS_i = (S_i, \mathrm{Act}_i, \rightarrow_i, S_{0,i}, \mathrm{AP}_i, L_i), i = 1, 2$ be two transition systems and $H \subseteq \mathrm{Act}_1 \cap \mathrm{Act}_2$. We define the transition system $TS_1 \parallel_H TS_2 := (S_1 \times S_2, \mathrm{Act}_1 \times \mathrm{Act}_2, \rightarrow$ $, S_{0,1} \times S_{0,2}, \mathrm{AP}_1 \cup \mathrm{AP}_2, L_1 \times L_2)$, where $\rightarrow$ is defined by:

$$\frac{s_1 \rightarrow_1^\alpha s_1' \quad \alpha \notin H}{(s_1, s_2) \rightarrow^\alpha (s_1', s_2)} \qquad \frac{s_2 \rightarrow_1^\alpha s_2' \quad \alpha \notin H}{(s_1, s_2) \rightarrow^\alpha (s_1, s_2')}$$

$$\frac{s_1 \rightarrow_1^\alpha s_1' \quad s_1 \rightarrow_1^\alpha s_1' \quad \alpha \in H}{(s_1, s_2) \rightarrow^\alpha (s_1', s_2')}$$

We call this the *parallel composition with handshake H*. When $H = \mathrm{Act}_1 \cap \mathrm{Act}_2$, we omit H.

Two Traffic Lights, revisited



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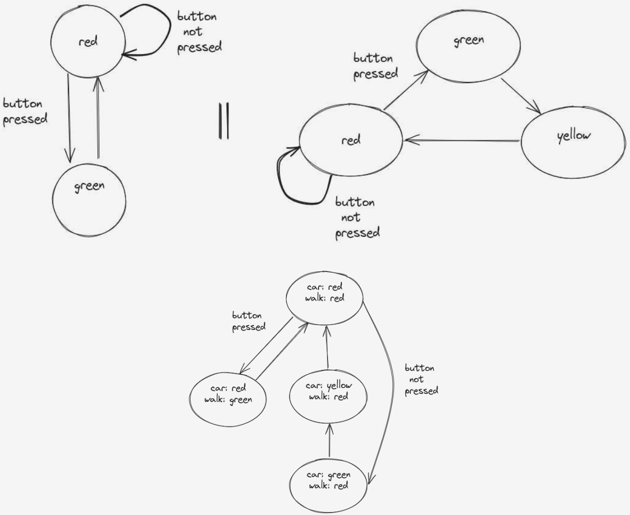
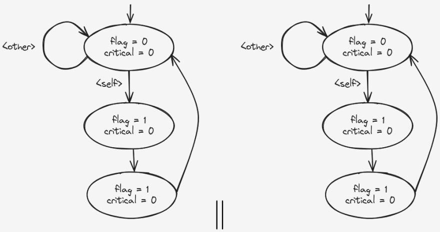


Concurrency: Message Passing



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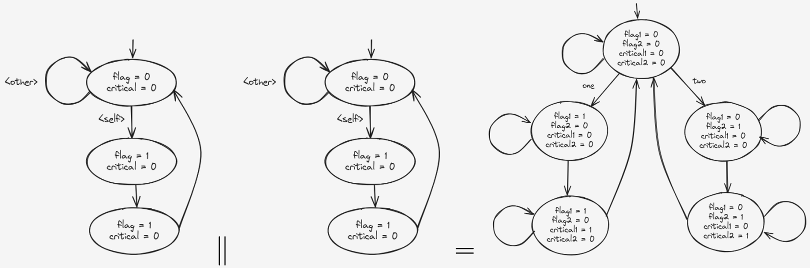

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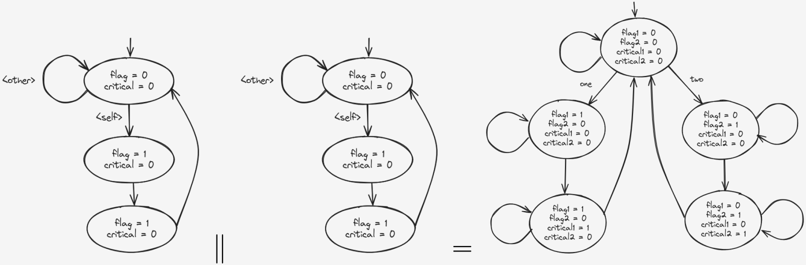
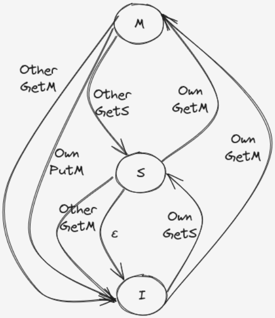


Assumption: atomicity of read-modify-writes here. Reasonable?

MSI Cache Coherency Protocol



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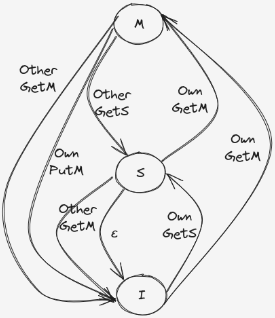

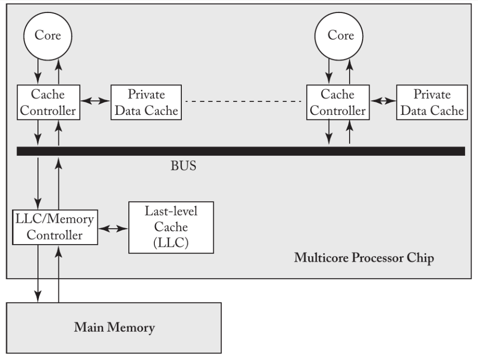

Source: Nagarajan, Vijay, et al. A primer on memory consistency and cache coherence. Springer Nature, 2020.

State Graph

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- Visualization (graphs): very useful!

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Definition (Predecessors/Successors)

Let $TS = (S, \mathrm{Act}, \rightarrow, S_0, \mathrm{AP}, L)$ be a transition system. For $s \in S, \alpha \in \mathrm{Act}$, we define $\mathrm{Post}(s, \alpha) := \{s' \in S \mid s \rightarrow^{\alpha} s'\}, \mathrm{Post}(s) := \bigcup_{\alpha \in \mathrm{Act}} \mathrm{Post}(s, \alpha)$ as the successors of s, and similarly Pre for the predecessors.

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Definition (State Graph)

Let $TS = (S, \mathrm{Act}, \rightarrow, S_0, \mathrm{AP}, L)$ be a transition system. We call the directed graph $G(TS) = (S, E)$ the state graph of TS, where $E = \{s, s' \in S \times S \mid s \in S, s' \in \mathrm{Post}(s)\}$

Path Fragments

Definition (Path fragments)

Let $TS = (S, \mathrm{Act}, \rightarrow, S_0, \mathrm{AP}, L)$ be a transition system. A sequence $\pi = \pi_0\pi_1\pi_2 \ldots \in (S)_{\mathbb{N}}$ is called a *path fragment* if $\pi_{i+1} \in \mathrm{Post}(\pi_i) \forall i \in \mathbb{N}$. It is called *finite* if it is a finite sequence $(\pi_i)_{i=0}^{N}$instead.
For a path fragment π, we denote the i-th element by $\pi[i]$ and similarly the sub-sequence $(\pi_k)_{k=i}^{j}$ by $\pi[i..j]$

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Sequences of transitions = path framgents through the state graph

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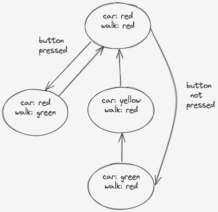
Definition (Path)

A path fragment π is called a *path* if it is initial and maximal.

Example: Paths in Traffic Light



A Typical Traffic Light in the UK?

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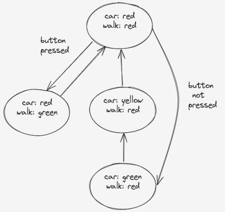

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Non-example

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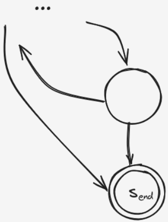

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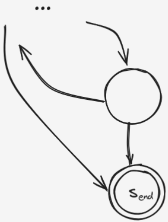
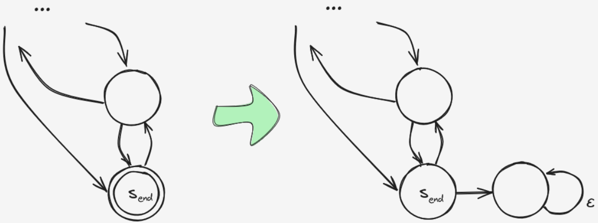


$$\text{Post}(s) = \varnothing$$

End States

Modeling end states with infinite paths



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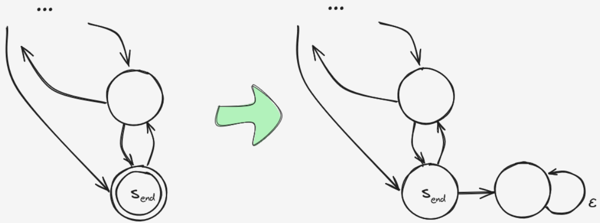

Assumption

For the rest of this course we assume no end states s with $\mathrm{Post}(s) = \emptyset$.