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MSDS 460
Homework Assignment 1
10/2/2022

Question 1:

SteelCo manufactures three types of steel at two different steel mills. During a given month, Mill 1 has 200 hours of blast furnace time available, whereas Mill 2 has 300 hours. Because of differences in the furnaces at each mill, the time and cost to produce a ton of steel differs for each mill and are shown in the following table. Each month, SteelCo must manufacture a total of at least 400 tons of Steel 1, 500 tons of Steel 2, and 300 tons of Steel 3 to meet demand; however, the total amount of Steel 2 manufactured should not exceed the combined amount of Steel 1 and Steel 3. Also, in order to maintain a roughly uniform usage of the two mills, management's policy is that the percentage of available blast furnace capacity (time) used at each mill should be the same. Clearly formulate a linear program (LP) to minimize the cost of manufacturing the desired steel.

Table 1

Mill	Steel 1		Steel 2		Steel 3	
	Cost ($)	Time (Min)	Cost ($)	Time (Min)	Cost ($)	Time (Min)
Mill 1	10	20	11	22	14	28
Mill 2	12	24	9	18	10	30

<u>Solution:</u>
Variables:

M1S1 = Steel 1 manufactured at Mill 1
M1S2 = Steel 2 manufactured at Mill 1
M1S3 = Steel 3 manufactured at Mill 1
M2S1 = Steel 1 manufactured at Mill 2
M2S2 = Steel 2 manufactured at Mill 2
M2S3 = Steel 3 manufactured at Mill 2

Objective function:

Min Z = 10*M1S1 + 11*M1S2 + 14*M1S3 + 12*M2S1 + 9*M2S2 + 10*M2S3

Subject to:

20*M1S1 + 22*M1S2 + 28*M1S3 <= 12,000

24*M2S1 + 18*M2S2 + 30*M2S3 <= 18,000

M1S1 + M2S1 >= 400

M1S2 + M2S2 >= 500

M1S3 + M2S3 >= 300

M1S1 + M2S1 + M1S3 +M2S3 – M1S2 – M2S2 >= 0

(20*M1S1 + 22*M1S2 + 28*M1S3)/12,000 – (24*M2S1 + 18*M2S2 + 30*M2S3)/18,000 = 0

M1S1, M2S1, M1S2, M2S2, M1S3, M2S3 >= 0

Question 2:

Consider the following linear program:

Max $Z = -4x_1 + 2x_2$
Subject To
$-2x_1 + 2x_2 \leq 7$
$x_1 \geq 2$
$x_1 - 4x_2 \leq 0$
$2x_1 + 2x_2 \geq 10$
$x_1, x_2 \geq 0$

Part A: Write the LP in standard equality form.

Solution:
Objective Function:

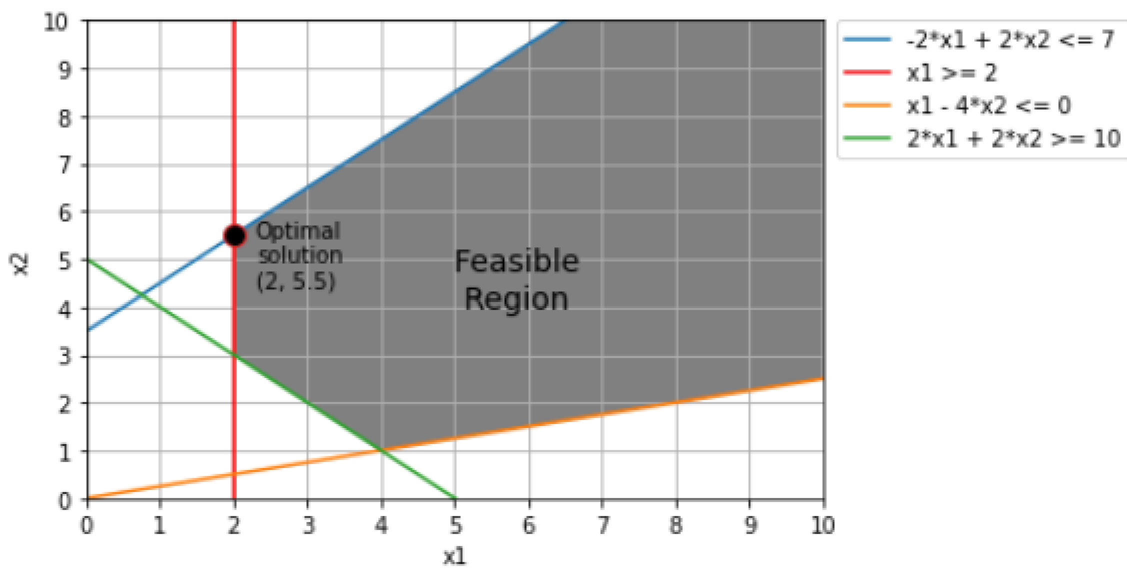
$$\text{Max } Z = -4x_1 + 2x_2$$

Subject to:

$$-2x_1 + 2x_2 + s_1 = 7$$
$$x_1 - s_2 = 2$$
$$x_1 - 4x_2 + s_3 = 0$$
$$2x_1 + 2x_2 - s_4 = 10$$
$$x_1, x_2, s_1, s_2, s_3, s_4 >= 0$$

Part B: Solve the original LP graphically (to scale). Clearly identify the feasible region and, if one or more exist, the optimal solution(s) (provide exact values for x_1, x_2, and Z).

Solution:



$x_1 = 2$
$x_2 = 5.5$
$Z = 3$

Question 3:

At the beginning of month 1, Finco has $400 in cash. At the beginning of months 1, 2, 3, and 4, Finco receives certain revenues, after which it pays bills (see Table 2 below). Any money left over may be invested for one month at the interest rate of 0.1% per month; for two months at 0.5% per month; for three months at 1% per month; or for four months at 2% per month. Use linear programming to determine an investment strategy that maximizes cash on hand at the beginning of month 5. Formulate an LP to maximize Finco's profit.

Table 2

Month	Revenues ($)	Bills ($)
1	400	600
2	800	500
3	300	500
4	300	250

<u>Solution:</u>

Variables:

X_{11} = amount invested in month 1 and matures at the end of month 1
X_{12} = amount invested in month 1 and matures at the end of month 2
X_{13} = amount invested in month 1 and matures at the end of month 3
X_{14} = amount invested in month 1 and matures at the end of month 4
X_{22} = amount invested in month 2 and matures at the end of month 2
X_{23} = amount invested in month 2 and matures at the end of month 3
X_{24} = amount invested in month 2 and matures at the end of month 4
X_{33} = amount invested in month 3 and matures at the end of month 3
X_{34} = amount invested in month 3 and matures at the end of month 4
X_{44} = amount invested in month 4 and matures at the end of month 4

Objective Function:

$$\text{Max } Z = (4)1.02X_{14} + (3)1.01X_{24} + (2)1.005X_{34} + (1)1.001X_{44}$$
$$= 1.08X_{14} + 1.03X_{24} + 1.01X_{34} + 1.001X_{44}$$

Subject to:

$$X_{11} + X_{12} + X_{13} + X_{14} + 600 = 800$$
$$X_{22} + X_{23} + X_{24} + 500 = 800 + 1.001X_{11}$$
$$X_{33} + X_{34} + 500 = 300 + 1.01X_{12} + 1.001X_{22}$$
$$X_{44} + 250 = 300 + 1.03X_{13} + 1.01X_{23} + 1.001X_{33}$$
$$X_{11}, X_{12}, X_{13}, X_{14}, X_{22}, X_{23}, X_{24}, X_{33}, X_{34}, X_{44} >= 0$$

Question 4:

Turkeyco produces two types of turkey cutlets for sale to fast-food restaurants. Each type of cutlet consists of white meat and dark meat. Cutlet 1 sells for $4/lb and must consist of at least 70% white meat. Cutlet 2 sells for $3/lb and must consist of at least 60% white meat. At most, 50 lb of cutlet 1 and 30 lb of cutlet 2 can be sold. The two types of turkey used to manufacture the cutlets are purchased from the GobbleGobble Turkey Farm. Each type 1 turkey costs $10 and yields 5 lb of white meat and 2 lb of dark meat. Each type 2 turkey costs $8 and yields 3 lb of white meat and 3 lb of dark meat.

Part A: Formulate an LP to maximize Turkeyco's profit.

<u>Solution:</u>

Variables:

C1W = Cutlet 1 white meat
C1D = Cutlet 1 dark meat
C2W = Cutlet 2 white meat
C2D = Cutlet 2 dark meat
T1 = Type 1 Chicken
T2 = Type 2 Chicken

Objective Function:

Max Z = 4(C1W + C1D) + 3(C2W + C2D) - 10T1 - 8T2

Subject to:

.7*(C1W + C1D) <= C1W
.6*(C2W + C2D) <= C2W
C1W + C1D <= 50
C2W + C2D <= 30
C1W + C2W <= 5T1 + 3T2
C1D + C2D <= 2T1 + 3T2
C1W, C1D, C2W, C2D, T1, T2 <= 0

Part B: Solve the LP (provide exact values for all variables and the optimal objective function).

<u>Solution:</u>

```
Cutlet_1_dark = 15.0
Cutlet_1_white = 35.0
Cutlet_2_dark = 12.0
Cutlet_2_white = 18.0
Type_1_Chicken = 8.6666667
Type_2_Chicken = 3.2222222
Objective= 177.55555539999997
```

Question 5:

A company wants to plan production for the ensuing year to minimize the combined cost of production and inventory costs. In each quarter of the year, demand is anticipated to be 130, 160, 250, and 150 units, respectively. The plant can produce a maximum of 200 units each quarter. The product can be manufactured at a cost of $15 per unit during the first quarter, however the manufacturing cost is expected to rise by $1 per quarter. Excess production can be stored from one quarter to the next at a cost of $1.50 per unit, but the storage facility can hold a maximum of 60 units. How should the production be scheduled so as to minimize the total costs?

Part A: Formulate an LP model to minimize costs.

Solution:

Variables:

- Q1P = Q1 Production
- Q2P = Q2 Production
- Q3P = Q3 Production
- Q4P = Q4 Production
- Q1S = Q1 Storage
- Q2S = Q2 Storage
- Q3S = Q3 Storage
- Q4S = Q4 Storage

Objective function:

Min Z = 15Q1P + 16Q2P + 17Q3P + 18Q4P + 1.5(Q1S + Q2S + Q3S + Q4S)

Subject to:

Q1P, Q2P, Q3P, Q4P <= 200
Q1S, Q2S, Q3S, Q4S <= 60
Q1P >= 130
Q1P – 130 = Q1S
Q2P + Q1S >= 160
Q2P + Q1S - 160 = Q2S
Q3P + Q2S >= 250
Q3P + Q2S -250 = Q3S
Q4P + Q3S >= 150
Q4P + Q3S- 150= Q4S
Q1P, Q2P, Q3P, Q4P, Q1S, Q2S, Q3S, Q4S >= 0

Part B: Solve the LP (provide exact values for all variables and the optimal objective function).

Solution:

```
Q1_Production = 140.0
Q1_Storage = 10.0
Q2_Production = 200.0
Q2_Storage = 50.0
Q3_Production = 200.0
Q3_Storage = 0.0
Q4_Production = 150.0
Q4_Storage = 0.0
Objective= 11490.0
```