NU Industries’ Linear Optimized Profit Maximization

MSDS460 - Final Project

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Abstract

Profit maximization for companies that generate sales via the manufacturing of products is influenced by a number of variables. Often, these variables include labor, raw material, customer acquisition, storage, and transportation costs. Given the nuance and complexity of an individual company's production environment, a straightforward path or intuitive strategy toward profit maximization can be exceedingly elusive. The research presented in this paper centers around NU Industries' two production plants, three products, and the company's challenge of determining the product mix, marketing strategy, and allocation of resources that will maximize profits over the coming five periods.

Linear programming and the simplex method were successfully used to model the factors relevant to the challenge facing NU Industries and solve for an optimal solution. A total of 120 variables and 44 constraints were included in the model which was solved using the Python package Pulp and a sensitivity report was created using GLPK.

The proposed product mix and use of resources resulting from the solved model generated a profit of $7,132,135 over a five sales cycle time period. It was determined that the availability of raw materials was the main constraint limiting production and preventing the company from fully leveraging its marketing budget and generating further profits. Deeper analysis and business recommendations are offered within the paper.

Linear programming proved to be an efficient method for addressing the complexities of NU Industries' maximization problem. It provided interpretable results that could be used to better understand all factors driving profit margins and improve production strategy.

Keywords

1. Linear Programming
2. Production
3. Supply Chain
4. Profit Maximization

Introduction

NU Industries (NU) is a manufacturer with two plants: Plant A and Plant B. NU produces three products: Widgets, Gadgets, and Flugels. The company is preparing its production plan for the upcoming five sale periods. To adequately plan, NU needs to consider how to allocate labor, materials, and production costs across the two plants and three products while meeting pre-established demand. The allocation of these resources is critical as it will determine NU Industries' success in meeting its contract obligations and maximizing its profits.

In a perfect world, NU would simply produce the exact amount of units in each period required to meet demand for each product. This production strategy would allow for costs to be controlled as much as possible, reducing the overall impact to the company's bottom line. However, there are a variety of factors that add complexity to NU's production environment and necessitate a more robust strategy.

For example, labor costs, which vary between plants, are expected to increase after two of the five periods have passed. The number of hours required to produce a unit of each product are different per product per plant. The number of units that can be stored at the end of a period are limited and raw materials available to the company are constrained and sold to the plants at different rates. In addition to the aforementioned cost considerations, it is also possible for the company to increase demand through a fixed marketing budget. NU's marketing department has noted that dollars allocated to marketing efforts for a specific product can be expected to translate into demand in the subsequent period. However, the per unit acquisition costs of the three products are different. Given these factors, and others to be explored later in this paper, the company faces a complex challenge in maximizing its profits over the next five sales cycles.

Given the current worldwide economic challenges many companies are facing strong headwinds, such as the increasing costs of materials, supply chain constraints and disruptions, and increasing fuel and energy costs, this report found NU Industries profit maximization challenge to be particularly relevant (Goodman 2022). Though simplified in comparison to many real-world cases, this problem allowed the team to build a real world optimization model that takes into account various real world considerations. NU Industries' fact pattern includes considerations for marketing, transportation, storage, and costs of production. The costs of production needed to be modeled in the problem include variable labor and raw material costs. All companies strive to create the highest profit margins possible but often the path to achieving this goal is not readily available or intuitive and this problem serves as a great example.

This report elected to model NU's profit maximization problem utilizing linear optimization to make use of the simplex method and find the optimal mix of products and resources. In order to utilize the advantages of the simplex method a linear model needs to be built that reflects the challenge facing NU Industries. It's important to outline all of the variables and constraints that need to be incorporated into the model in order to properly capture the product mix and production location that provides the optimal operational strategy. These factors are detailed in Appendix A.

Literature Review

Linear Optimization is a modeling technique, widely used in Operational Research. Since its introduction in the 1950's, it has been regarded as one the most important advances in the sciences. It has become a ubiquitous tool commonly used for resource allocation. One of the main reasons that linear programming has remained such a popular method for solving decision related problems is that its use of the simplex method allows for extremely large problems to be solved efficiently. Linear programming breaks down problems into a set of variables, constraints, and an objective function. The objective function is the linear formula that is being optimized either as a maximization or a minimization model. The objective function is built of many variables and is subject to linear constraints the model must adhere to. Once the model has been established the simplex method can be applied to find the optimal solution within the feasible space outlined by the constraints (Hillier and Lieberman 2001, 25 - 79).

Currently, linear programming is being used by companies to better navigate challenging times, to deal with the “Great Supply Chain Disruption”, and to attempt to reduce costs while increasing revenue. As noted in a 2022 New York Times article, “[t]he breadth and persistence of supply chain troubles in part result from how the coronavirus pandemic has accelerated trends that have been unfolding for decades, especially the growth of e-commerce.” As a result of this disruption, lead times have increased, storage capacities have been stretched, products remain aboard ships in ports for weeks, and prices along the supply chain have risen significantly (Goodman 2022).

A study published in the May 2022 issue of the IOSR Journal of Engineering explored the use of linear programming for the production, distribution, and inventory systems of a bottling company. In this study, the bottling company desired to limit operating costs while increasing sales and revenues of soft drink products. Demand, capacity limits, and budget constraints all played into achieving this goal. Further, the company looked to optimize what items should be produced, their production levels, and their production locations. The bottling company used a commercial solver to create a linear model that had three variables and five constraints. The discovered solution was able to increase the companies’ profit margin by approximately $4.5 million (Ibinab 2022, 20). The scope of this problem and factors needed to be considered in it were deemed to be similar to the one facing NU Industries’ and a contemporary example of optimization through linear programming.

In 2021, linear programming was used to optimize the production capacity of a manufacturing company. This process was overviewed in the NIPES Journal of Science and Technology Research and notes that a linear programming problem was formulated and solved using LINDO software. The challenge presented in this case study centered around a manufacturer that produced ten types of plastic products. The problem was formulated so that the company would be able to identify the mix of products that would “maximize the total profit after satisfying a set of constraints.” These constraints included factors such as raw materials, labor hour requirements, and machine hour requirements (Amosun and Muhammed 2021, 119-122). This study’s problem description and implementation of linear programming was relevant to the challenges facing NU industries and further supported the decision to use linear programming.

Model Development

As NU Industries operates two plants, it is important to model the drivers specific to each plant as the optimal solution will need to consider when and at which facility to produce the optimal product mix. Factors applicable to both plants are considered in the overall model construction. For example, the selling price of each product, marketing budget, raw material availability, and overall demand are shared by the plants. All aspects of the individual plants and the overall manufacturing process have been modeled to fit the linear optimization formulation.

As noted prior, the company's profits are generated through the sales of three products: Widgets, Gadgets, and Flugels. The sales price, in dollars, for each product can be seen in Table 1. The sales price for each product is fixed for all five periods.

Table 1

Sales Price by Product			
	Widget	Gadget	Flugel
Price	$2,490.00	$1,990.00	$2,970.00

NU Industries has forecasted a demand that must be met for each of the five sales periods. These forecasts capture the demand for each product in each period and are expected to be the same as actual demand. Demand generated via marketing efforts that have taken place in previous periods are not factored into these estimates. Additional demand generated via marketing will be determined via the model's optimization process. The fixed demand for each product and period, in units, can be seen in Table 2.

Table 2

Demand Before Use of Marketing Budget					
	Period 1	Period 2	Period 3	Period 4	Period 5
Widgets	70	125	185	190	200
Gadgets	200	300	295	245	240
Flugels	140	175	205	235	230

NU Industries has allocated a fixed budget for the upcoming five periods in which marketing efforts can be taken to increase the demand for a product. The marketing team has noted that marketing efforts made in a given period generate demand in the subsequent period. In Period 1, additional demand cannot be generated, but marketing campaigns in Period 1 will generate demand in Period 2 for the given product marketed. The total marketing budget for the entire five period cycle is $70,000. The marketing department has identified a per unit marketing cost that will guarantee a sale in the following period. These costs can be identified as the minimal cost that guarantees a sale of one additional unit. The per unit marketing costs, in dollars, for each product are included in Table 3.

Table 3

Marketing			
	Widget	Gadget	Flugel
Cost	$160.00	$120.00	$180.00

The demand for each product must be met with a manufactured unit. NU Industries has several factors to consider in the costs of production. Widgets and Flugles require a mixture of two types of raw materials with an amount of labor, while Gadgets only require one type of raw material and an associated amount of labor. The amount of raw materials available for use in production across the two plants is fixed for each period. This amount remains constant across all five periods. For any given period, NU can use 140,000 pounds (70 tons) of Raw Material 1 and 5,000 pounds (2.5 tons) of Raw Material 2. There is a transportation cost from the raw material manufacturers unique to each material and each plant combination. The distribution cost of Raw Material 1 costs $1.25 per pound to Plant A and $1.45 per pound to Plant B. The distribution cost of Raw Material 2 costs $2.65 per pound to Plant A and $2.90 per pound to Plant B. Due to limited storage of raw materials, each plant can only purchase the amount used for production within the given period.

Each product must be manufactured from the raw materials using some form of labor. Labor is categorized as either regular or overtime. Regular labor hours are the standard rate employees are paid and are limited differently at each plant. Overtime labor hours are not limited but legally require that employees are paid at a higher rate per hour compared to regular hours. With continual economic headwinds, NU Industries predicts that labor costs will increase during the five periods. Periods 1 and 2 will have lower regular and overtime costs than Periods 3, 4, and 5 as percentage increases are expected to occur at the start of Period 3. Each plant's labor costs, in dollars, and regular hour limits by period can be seen in Table 4.

Table 4

Labor Costs by Plant per Period						
	Period 1	Period 2	Period 3	Period 4	Period 5	Hour Limit
	Plant A					
Regular	$11.00	$11.00	$11.55	$11.55	$11.55	2500
Overtime	$16.50	$16.50	$17.33	$17.33	$17.33	N/A
	Plant B					
Regular	$11.00	$11.00	$12.10	$12.10	$12.10	3800
Overtime	$16.50	$16.50	$18.15	$18.15	$18.15	N/A

The manufacturing process at each plant, and for each product, requires varying amounts of raw material and labor. As a result, this may make it more favorable for the company to produce certain products at a particular plant. This may also result in fractions of units to be built in both plants to be joined as one unit in the Distribution Center in order to meet demand. The breakdown of each plant's manufacturing requirements is captured in Table 5. Note that the raw material requirements are in pounds and the labor requirements in hours.

Table 5

Manufacturing Requirements			
	Widget	**Gadget**	**Flugel**
	Plant A		
Raw Material 1 (pounds)	194	230	178
Raw Material 2 (pounds)	8.6	0	11.6
Labor Hour (hours)	9.5	7.1	11.1
	Plant B		
Raw Material 1 (pounds)	188	225	170
Raw Material 2 (pounds)	9.2	0	10.8
Labor Hour (hours)	9.1	7.8	10.6

The production of the products at each plant must then be used for sales to meet demand in each period. NU Industries has identified that each plant can store units of products at the plant location. These units can only be stored for a single period. Products manufactured and stored in Period 1, for example, must be used to meet sales demands in Period 2. Though the cost for storing a unit of the products is fixed for all five periods, the storage capacity for each plant is limited. Plant A can store up to 70 units per period while Plant B can store up to 50 units per period. One unit of storage at either plant can store one unit of any one of the three products. For example, storing 10 units of Widgets is the same as storing 10 units of Gadgets from the perspective of physical storage capacity.

The final component of the manufacturing process to consider is the transportation of the products from both plants to NU Industries' Distribution Center. The transportation costs of these products is fixed across all five periods but varies by product and plant location. Transportation of finished products, raw materials, and labor between Plant A and Plant B is not permitted. Fixed storage and transportation costs are outlined in Table 6.

Table 6

Storage and Transportation Costs			
	Widget	**Gadget**	**Flugel**
		Plant A	
Storage	$7.50	$5.50	$6.50
Transportation	$6.30	$4.60	$5.50
		Plant B	
Storage	$7.80	$5.70	$7.00
Transportation	$6.50	$5.00	$5.70

Modeling NU Industries' manufacturing process as a linear optimization problem is a challenging exercise. To encapsulate the relevant factors, a lengthy objective function was established that factored in the resource management constraints outlined above. The objective function and constraints capture all of the nuances between products created, the allocation between plants, the usage of regular versus overtime hours, and the additional demand generated through the consumed marketing budget.

Capturing all of these nuances in a linear programming model required a significant number of variables. Crucial to linear model construction is the linear relationship between all of the variables. Linear modeling does not restrict variables from being treated as whole integer units and for the NU Industries problem partial units were considered acceptable.

The model employed 108 variables to capture all of the units of each product produced with 12 variables capturing the number of units associated with demand generated from marketing. The model accounted for 44 constraints. All marketing costs must fall under the marketing budget of $70,000. Any excess funds do not get counted towards NU Industries' profit, which separates any excess from the objective function calculation.

The objective function and constraints are captured below with abbreviations to condense the presentation. The full objective function and constraints in python format can be seen in the iPython notebook submitted with this report.

The following abbreviations represent each item in the problem formulation below.

- Time Period = T in (1, 2, 3, 4, 5)
- Product = P in (Widgets, Gadget, Flugel)
- Plant = N in (Plant A, Plant B)
- Labor Type = L in (Regular, Overtime)
- Storage Type = D in (Stored, Not Stored)

Maximize

(Sales Price_{P} * Units_{TPNLD}) – (Raw Material 1 Cost_{PN} * Units_{TPNLD}) – (Raw Material 2 Cost_{PN} * Units_{TPNLD}) – (Labor Cost_{TNL} * Units_{TPNLD}) – (Storage Cost_{PND} * Units_{TPNLD}) – (Transportation Cost_{PN} * Units $_{TPNLD}$)

Subject To

Units_{TPNLD} == Forecasted Demand_{T} + Marketing Demand_{T}

Units_{TPNLD} <= Raw Material 1_{T}

Units_{TPNLD} <= Raw Material 2_{T}

Units_{TPNLD} <= Regular Labor Hours_{TN}

Units_{TPNLD} <= Storage Capacity_{TN}

$\sum$ Marketing Expense_{TP} <= 70,000

Units_{TPNLD} >= 0, Marketing Expense_{TP} >= 0

Implementation

In the world of operational research there are many linear optimization platforms that can optimize models of varying sizes. To solve the NU Industries problem the team applied the simplex implementation provided in the Python package PuLP. Using Google's Collab allowed the coast to coast team to develop the model concurrently and to ensure every team member had the latest version. After the model was fully developed and established as stable, the team extended the implementation to an offline cycle utilizing the Python package GLPK to perform sensitivity analysis reports.

Optimization Results

NU Industries' linear model can be optimized so that NU Industries makes a stronger profit over the five periods than that which would be generated with a less robust approach. The model optimizes so that the best mix of resources are utilized and allocated between the plants and products in a way that minimizes costs while meeting demand. The model also increases profits by considering profit margins in the use of marketing campaigns. The breakdown of the number of products to be produced and stored, for each period and via regular and overtime hours, for Plant A is shown in Table 7. The results for Plant B can be seen in Table 8.

Table 7

	Plant A											
	Produced						Stored					
	Widget		Gadget		Flugel		Widget		Gadget		Flugel	
	Reg	Over	Reg	Over	Reg	Over	Reg	Over	Reg	Over	Reg	Over
Period 1	**61.37**	0.00	**200.00**	0.00	0.00	0.00	0.00	0.00	**70.00**	0.00	0.00	0.00
Period 2	0.00	0.00	**180.00**	0.00	0.00	0.00	0.00	0.00	**63.04**	0.00	0.00	0.00
Period 3	**89.79**	0.00	**231.96**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Period 4	**80.05**	0.00	**245.00**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Period 5	**83.79**	0.00	**240.00**	0.00	0.00	0.00						

Table 8

	Plant B											
	Produced						Stored					
	Widget		Gadget		Flugel		Widget		Gadget		Flugel	
	Reg	Over	Reg	Over	Reg	Over	Reg	Over	Reg	Over	Reg	Over
Period 1	**8.63**	0.00	0.00	0.00	**140.00**	0.00	0.00	0.00	**50.00**	0.00	0.00	0.00
Period 2	**2.34**	**122.66**	0.00	0.00	**356.48**	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Period 3	**65.87**	**29.33**	0.00	0.00	**301.94**	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Period 4	**109.95**	0.00	0.00	0.00	**264.10**	**15.01**	0.00	0.00	0.00	0.00	0.00	0.00
Period 5	**116.21**	0.00	0.00	0.00	**258.72**	**15.96**						

This report treated the demand constraints as equality constraints to ensure that enough units were produced to meet the current demand plus any additional units generated via advertising. Due to the desire to have no excess inventory by the end of Period 5, overproduction for each period was avoided. The model's production planning solution satisfies the equality constraint for the current baseline demand for each product and for all periods. The additional demand for Flugels created via advertising for Periods 2, 3, 4, and 5 were also met. The demand for each period, by product, can be seen in Table 9.

Table 9

	Demand Units			Demand w/ Marketing			Sold Units		
	Widget	Gadget	Flugel	Widget	Gadget	Flugel	Widget	Gadget	Flugel
Period 1	70	200	140	70.00	200.00	140.00	70.00	200.00	140.00
Period 2	125	300	175	125.00	300.00	356.48	125.00	300.00	356.48
Period 3	185	295	205	185.00	295.00	301.94	185.00	295.00	301.94
Period 4	190	245	235	190.00	245.00	279.12	190.00	245.00	279.12
Period 5	200	240	230	200.00	240.00	274.69	200.00	240.00	274.69

The advertising budget constraint has an upper limit of $70,000. The solution consumes $66,100.42 of the budget to advertise and increase demand for Flugels. The solution only advertises for Flugels as this product has the highest profit margin (see Figure A and Figure B). This increased demand for Flugels (gray) maximizes profits.

Figure A.

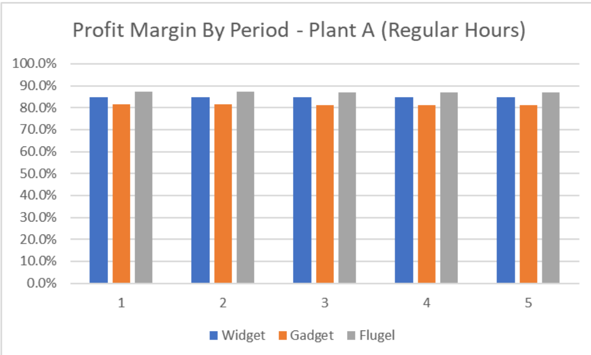
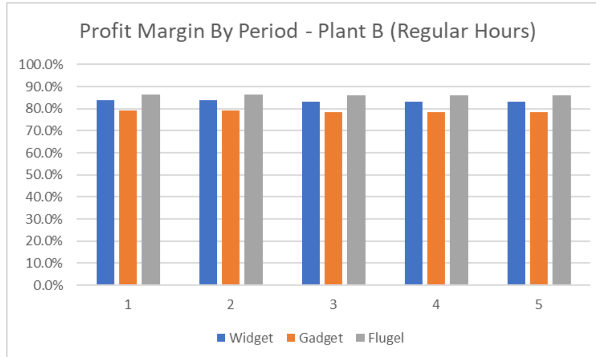


Figure B.



	Profit Margin - Plant A		
	Widget	Gadget	Flugel
1	84.9%	81.4%	87.2%
2	84.9%	81.4%	87.2%
3	84.7%	81.2%	87.0%
4	84.7%	81.2%	87.0%
5	84.7%	81.2%	87.0%

	Profit Margin - Plant B		
	Widget	Gadget	Flugel
1	83.7%	79.0%	86.5%
2	83.7%	79.0%	86.5%
3	83.3%	78.6%	86.1%
4	83.3%	78.6%	86.1%
5	83.3%	78.6%	86.1%

However, the model was unable to use the full advertising budget as it reached the upper limit constraint for Raw Material 1 for periods 2, 3, 4, and 5. This prevents NU from manufacturing any additional units of any product in these periods. As a result, there is no reason to increase demand via advertising. The only period where the Raw Material 1 constraint is not maxed out is in Period 1, but NU is not able to increase demand through marketing for this period. As a result, there remains $3,899.58 of unused advertising budget and this constraint effectively has a shadow price of $0. Therefore, it is most favorable for NU Industries to first allocate all available resources through cost efficient optimization planning to produce enough products to meet the baseline demand. After meeting the forecasted demand, any remaining resources can be allocated to increase the demand and maximize the production of Flugels within the given production constraints. The allocation of marketing demand by units can be seen in Table 10.

Table 10

Marketing					
	Widget	Gadget	Flugel		
Period 1	0.00	0.00	**181.48**	181.48	
Period 2	0.00	0.00	**96.94**	96.94	
Period 3	0.00	0.00	**44.12**	44.12	
Period 4	0.00	0.00	**44.69**	44.69	
Period 5				0.00	
Total Units	0.00	0.00	367.22		367.22

Labor constraints were treated as inequality constraints since the amount of regular labor hours that could be used at each plant were limited to 2,500 hours at Plant A and 3,800 hours at Plant B. Thus, regular labor hours were constrained to be less than or equal to these limits. There were no restrictions on the number of overtime hours that could be used. As a result, the labor constraints only considered regular labor hours. The solution created via the model adequately limited regular labor hours to or below the available limits.

Plant A maxed out regular labor hours in all periods except for Period 2. In this period, the mixture of products and units produced maxed out the available pounds of Raw Material 1 required for production before all regular labor hours could be utilized. Consequently the utilization of all of Raw Material 2 was never reached during this period. The usage of raw materials by each plant, for each period, can be seen in Table 11.

Table 11

Raw Material Usage by Plant and Period

	Raw Material 1 - Plant A				Raw Material 1 - Plant B				Total Usage
	Widget	Gadget	Flugel	Total	Widget	Gadget	Flugel	Total	
Period 1	11,905.47	62,100.00	0.00	74,005.47	1,622.74	11,250.00	23,800.00	36,672.74	110,678.21
Period 2	0.00	55,898.15	0.00	55,898.15	23,500.00	0.00	60,601.85	84,101.85	140,000.00
Period 3	17,420.21	53,351.85	0.00	70,772.06	17,898.56	0.00	51,329.38	69,227.94	140,000.00
Period 4	15,530.21	56,350.00	0.00	71,880.21	20,670.11	0.00	47,449.68	68,119.79	140,000.00
Period 5	16,255.16	55,200.00	0.00	71,455.16	21,847.58	0.00	46,697.26	68,544.84	140,000.00

	Raw Material 2 - Plant A				Raw Material 2 - Plant B				Total Usage
	Widget	Gadget	Flugel	Total	Widget	Gadget	Flugel	Total	
Period 1	527.77	0.00	0.00	527.77	79.41	0.00	1,512.00	1,591.41	2,119.18
Period 2	0.00	0.00	0.00	0.00	1,150.00	0.00	3,850.00	5,000.00	5,000.00
Period 3	772.24	0.00	0.00	772.24	875.89	0.00	3,260.93	4,136.81	4,909.05
Period 4	688.45	0.00	0.00	688.45	1,011.52	0.00	3,014.45	4,025.97	4,714.42
Period 5	720.59	0.00	0.00	720.59	1,069.14	0.00	2,966.65	4,035.79	4,756.38

Plant B maxed out regular labor hours in all periods except for Period 1. In this period, the number of units produced, based on both forecasted demand and marketing demand, did not require the use of all regular labor hours at Plant B. In periods when marketing can generate more demand, Plant B can utilize the full capacity of regular labor hours.

Overtime hours were not utilized at Plant A in any of the periods. However, overtime hours were utilized in Periods 2 and 3 to make Widgets at Plant B and in Periods 4 and 5 and to make Flugels at Plant B. The utilization of overtime labor hours is primarily used for the products with the highest profit margin. With all of the overtime being consumed at Plant B the number of overtime hours never exceeds 30% of the regular labor capacity. Period 2 took advantage of the lower labor costs, the large marketing campaigns in Period 1, and the ability to store products for Period 3 to maximize the number of units produced. Period 2 had the largest overtime to regular capacity usage with 29.38%. Period 3 had a smaller usage with 7.02% and dropped further for Period 4 and 5 with 4.19% and 4.45% respectively. The labor usage can be seen in Table 12.

Table 12

Labor Hours by Plant								
	Regular - Plant A				Regular - Plant B			
	Widget	Gadget	Flugel	Total	Widget	Gadget	Flugel	Total
Period 1	583.00	1,917.00	0.00	2,500.00	78.55	390.00	1,484.00	1,952.55
Period 2	0.00	1,725.55	0.00	1,725.55	21.30	0.00	3,778.70	3,800.00
Period 3	853.05	1,646.95	0.00	2,500.00	599.46	0.00	3,200.54	3,800.00
Period 4	760.50	1,739.50	0.00	2,500.00	1,000.52	0.00	2,799.48	3,800.00
Period 5	796.00	1,704.00	0.00	2,500.00	1,057.52	0.00	2,742.48	3,800.00
	Overtime - Plant A				Overtime - Plant B			
	Widget	Gadget	Flugel	Total	Widget	Gadget	Flugel	Total
Period 1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Period 2	0.00	0.00	0.00	0.00	1,116.20	0.00	0.00	1,116.20
Period 3	0.00	0.00	0.00	0.00	266.90	0.00	0.00	266.90
Period 4	0.00	0.00	0.00	0.00	0.00	0.00	159.15	159.15
Period 5	0.00	0.00	0.00	0.00	0.00	0.00	169.23	169.23

The utilization of regular and overtime labor in earlier periods can reduce production costs which increase NU Industries profits. In order to take advantage of the cheaper labor costs, the model utilizes storage capacity at both plants A and B. Period 1 had much lower contract requirements for all products, so leftover labor hours were used to create items that would be stored for sales in the later periods. For Plant A in Periods 1 and 2, regular labor hours were set aside to build and store Gadgets. For Period 1, all 70 units of storage space were used for Gadgets, while in Period 2 only 63.04 units of storage space were used for Gadgets. For Plant B, only Period 1 stored any products, completely filling all 50 available storage units with Gadgets. The breakdown of storage applications for both plants can be seen in Tables 7 & 8.

The Gadgets stored in Period 1 in both plants, were used to meet the rather high Period 2 contract requirements for Gadgets. Similarly, the Gadgets stored in Plant A during Period 2 were used to meet demand requirements for Period 3. After the first two periods, each plant was able to meet the demand for all products within the same period, confirming that the cost of storage was more expensive than the increase in labor price from Period 3 onward, even when being forced to utilize overtime.

The maximization of the linear model allows NU Industries to be profitable over the five period sales cycles. The simplex solver maximizes the sales of each of the three products for all five periods. The model demonstrates that demand can be generated via marketing funds to increase unit sales. The greater demand in Flugels, the highest profit margin product, was maximized to increase the overall sales per period.The breakdown of each product's sales by period can be seen in Table 13.

Table 13

Sales					
Price	**2490**	**1990**	**2970**		
	Widget	**Gadget**	**Flugel**		
Period 1	$174,300.00	$398,000.00	$415,800.00	$988,100.00	
Period 2	$311,250.01	$597,000.00	$1,058,750.00	$1,967,000.00	
Period 3	$460,650.00	$587,049.99	$896,754.43	$1,944,454.43	
Period 4	$473,100.00	$487,550.00	$828,973.90	$1,789,623.91	
Period 5	$498,000.01	$477,600.00	$815,828.67	$1,791,428.68	
	$1,917,300.02	$2,547,199.99	$4,016,107.00		$8,480,607.02
				$8,480,607.02	

The sale of the three products, across the five periods, generated $8,480,607 for NU Industries in sales. The optimized model balances the maximization of sales while also minimizing the overall costs of production. The four areas for the cost of production include: raw materials to make the products, labor hours to produce the products, storage of the products at each plant, and the transportation costs to get the products from the plants to the store for sale.

The breakdown of all the costs of production can be seen in Table 14. The optimized model has outlined that raw materials contribute the most to the overall cost with $965,351 (RM1: $903,681 & RM2: 61,670) followed by labor costs at $363,545 (R: $334,324 & O: $29,221). Storage and transportation costs contributed to a smaller portion of the overall costs with $1,016 and $18,557. The overall costs of production for the five time periods comes to a total of $1,348,471. The results of the model highlight that NU Industries can generate a net profit of $7,132,135.

Table 14

Sales and Costs	
Material Cost	-$965,351.10
Labor Cost	-$363,545.78
Storage Cost	-$1,016.69
Transporatiion Cost	-$18,557.68
Total Cost	-$1,348,471.26
Total Sales	$8,480,607.02
Total Cost	-$1,348,471.26
Optimized Profit	**$7,132,135.76**

Sensitivity Analysis

After finding the optimal profit, a sensitivity report was used to identify opportunities to improve the objective function value. It was discovered that production in Period 1 was limited due to hitting the maximum storage limits for Plant A and Plant B in Period 1. This results in Raw Material 1 and Raw Material 2 not being fully used in Period 1, as the contracted demand has been met alongside hitting the maximum storage, resulting in 29,321.79 and 2,880.82 pounds remaining, respectively. NU Industries could increase profit by increasing the available storage capacity in Period 1 for both Plant A and Plant B. The shadow prices for one additional unit at Plant A and one additional unit at Plant B are $3,346.20 and $3,321.41, respectively. In order to increase production in the remaining periods, NU Industries would require additional Raw Material 1 in Periods 3, 4, and 5 and both Raw Material 1 and Raw Material 2 in Period 2. Per the sensitivity analysis report, the shadow price for one additional pound of Raw Material 1 in Periods 3, 4, and 5 is $14.67. The shadow prices for one additional pound of Raw Material 1 and one additional pound of Raw Material 2 in Period 2 are $14.67 and $1.65, respectively. Due to the production limits above, NU Industries is not able to fully utilize the available marketing budget, so there is no need to increase advertising budget. The sensitivity analysis confirms this as it shows a $0 shadow price for the marketing budget constraint variable.

The decision variables from the sensitivity report can be seen in Table 15.

Table 15

Plant A Decision Variables											
Variable	**Activity**	**Slack**	**Marginal**	**Variable**	**Activity**	**Slack**	**Marginal**	**Variable**	**Activity**	**Slack**	**Marginal**
W_1_A_O_N		2061.66	-22.46	G_1_A_O_N		1580.75	-16.79	F_1_A_O_N		2528.11	-41.77
W_1_A_O_S		2054.16	-553.11	G_1_A_O_S		1575.25	-16.79	F_1_A_O_S		2521.61	-824.59
W_1_A_R_N	61.37	2113.91		G_1_A_R_N	200.00	1619.80		F_1_A_R_N		2589.16	-15.53
W_1_A_R_S		2106.41	-530.65	G_1_A_R_S	70.00	1614.30		F_1_A_R_S		2582.66	-798.34
W_2_A_O_N		2061.66	-59.44	G_2_A_O_N		1580.75	-39.05	F_2_A_O_N		2528.11	-102.14
W_2_A_O_S		2054.16	-66.75	G_2_A_O_S		1575.25	-39.05	F_2_A_O_S		2521.61	-108.64
W_2_A_R_N		2113.91	-7.19	G_2_A_R_N	180.00	1619.80		F_2_A_R_N		2589.16	-41.09
W_2_A_R_S		2106.41	-14.50	G_2_A_R_S	63.04	1614.30		F_2_A_R_S		2582.66	-47.59
W_3_A_O_N		2053.82	-53.26	G_3_A_O_N		1574.89	-39.80	F_3_A_O_N		2518.95	-92.51
W_3_A_O_S		2046.32	-60.76	G_3_A_O_S		1569.39	-45.30	F_3_A_O_S		2512.45	-99.01
W_3_A_R_N	89.79	2108.69		G_3_A_R_N	231.96	1615.90		F_3_A_R_N		2583.06	-30.28
W_3_A_R_S		2101.19	-7.50	G_3_A_R_S		1610.40	-5.50	F_3_A_R_S		2576.56	-36.78
W_4_A_O_N		2053.82	-53.26	G_4_A_O_N		1574.89	-39.80	F_4_A_O_N		2518.95	-92.51
W_4_A_O_S		2046.32	-60.76	G_4_A_O_S		1569.39	-45.30	F_4_A_O_S		2512.45	-99.01
W_4_A_R_N	80.05	2108.69		G_4_A_R_N	245.00	1615.90		F_4_A_R_N		2583.06	-30.28
W_4_A_R_S		2101.19	-7.50	G_4_A_R_S		1610.40	-5.50	F_4_A_R_S		2576.56	-36.78
W_5_A_O_N		2053.82	-53.26	G_5_A_O_N		1574.89	-39.80	F_5_A_O_N		2518.95	-92.51
W_5_A_R_N	83.79	2108.69		G_5_A_R_N	240.00	1615.90		F_5_A_R_N		2583.06	-30.28

Plant B Decision Variables											
Variable	**Activity**	**Slack**	**Marginal**	**Variable**	**Activity**	**Slack**	**Marginal**	**Variable**	**Activity**	**Slack**	**Marginal**
W_1_B_O_N		2034.07	-50.05	G_1_B_O_N		1530.05	-67.49	F_1_B_O_N		2511.58	-58.30
W_1_B_O_S		2026.27	-556.22	G_1_B_O_S		1524.35	-42.90	F_1_B_O_S		2504.58	-816.83
W_1_B_R_N	8.63	2084.12		G_1_B_R_N		1572.95	-24.59	F_1_B_R_N	140.00	2569.88	
W_1_B_R_S		2076.32	-506.17	G_1_B_R_S	50.00	1567.25		F_1_B_R_S		2562.88	-758.53
W_2_B_O_N		2034.07		G_2_B_O_N		1530.05	-16.40	F_2_B_O_N	105.30	2511.58	
W_2_B_O_S		2026.27	-7.61	G_2_B_O_S		1524.35	-16.60	F_2_B_O_S		2504.58	-7.00
W_2_B_R_N	125.00	2084.12		G_2_B_R_N		1572.95	-16.40	F_2_B_R_N	251.18	2569.88	
W_2_B_R_S		2076.32	-7.61	G_2_B_R_S		1567.25	-16.60	F_2_B_R_S		2562.88	-7.00
W_3_B_O_N		2019.06		G_3_B_O_N		1517.18	-24.16	F_3_B_O_N	25.18	2494.09	
W_3_B_O_S		2011.26	-7.80	G_3_B_O_S		1511.48	-29.86	F_3_B_O_S		2487.09	-7.00
W_3_B_R_N	95.21	2074.11		G_3_B_R_N		1564.37	-24.16	F_3_B_R_N	276.76	2558.22	
W_3_B_R_S		2066.31	-7.80	G_3_B_R_S		1558.67	-29.86	F_3_B_R_S		2551.22	-7.00
W_4_B_O_N		2019.06		G_4_B_O_N		1517.18	-24.16	F_4_B_O_N	15.01	2494.09	
W_4_B_O_S		2011.26	-7.80	G_4_B_O_S		1511.48	-29.86	F_4_B_O_S		2487.09	-7.00
W_4_B_R_N	109.95	2074.11		G_4_B_R_N		1564.37	-24.16	F_4_B_R_N	264.10	2558.22	
W_4_B_R_S		2066.31	-7.80	G_4_B_R_S		1558.67	-29.86	F_4_B_R_S		2551.22	-7.00
W_5_B_O_N		2019.06		G_5_B_O_N		1517.18	-24.16	F_5_B_O_N	15.96	2494.09	
W_5_B_R_N	116.21	2074.11		G_5_B_R_N		1564.37	-24.16	F_5_B_R_N	258.72	2558.22	

Marketing Decision Variables											
Variable	**Activity**	**Slack**	**Marginal**	**Variable**	**Activity**	**Slack**	**Marginal**	**Variable**	**Activity**	**Slack**	**Marginal**
W_1_K			-738.92	G_1_K			-1754.16	F_1_K	181.48		
W_2_K			-739.12	G_2_K			-1759.66	F_2_K	96.94		
W_3_K			-739.12	G_3_K			-1759.66	F_3_K	44.12		
W_4_K			-739.12	G_4_K			-1759.66	F_4_K	44.69		

Business Recommendations

To maximize profit NU Industries should aim to sell as many Flugel units as possible as they have the highest profit margin. To achieve this, NU Industries should allocate $66,100.42 out of the $70,000 marketing budget for advertising of Flugels. To further increase profit, NU Industries should explore the following scenarios. If possible, the company should increase the upper limits for Period 1 storage capacity in order to use up the remaining unused raw materials at no more than $3,346.20 for one additional unit at Plant A and no more than $3,321.41 for one additional unit at Plant B. Also, NU Industries should look to increase the upper limits for Raw Material 1 in periods 3, 4, and 5 at no more than $14.67 for one additional pound. Lastly, NU Industries should find opportunities to increase the upper limits for Raw Material 1 and Raw Material 2 in Period 2 at no more than $14.67 and $1.65 for one additional pound, respectively. Increasing these upper limits would allow NU Industries to produce more units. In turn, this would create additional opportunities for marketing to increase demand for Flugels, effectively increasing NU Industries' profit.

Integer Model Considerations

When attempting to develop an integer model, outright restricting each variable to integers in the program results in calculations that require an excessively large amount of time to complete, continuing to run for well over 24 hours. At the point in time of this paper, both Google Colab and offline external devices could not compute a simplex method for these numbers of variables.The solver used is GLPK_CMD, which is robust yet very slow compared to commercial solvers like Gurobi. However, from a recommendation standpoint, it is unlikely that the solution would be materially different if using an integer model. Since Flugels will still have the highest profit margin in this model, it is still favorable for NU to advertise for only Flugels in order to create additional demand. The solution would still recommend producing only enough to meet the baseline demand and allocate any remaining resources to produce excess Flugels to meet the additional demand from advertising. This will impact NU's bottom line as without partial units revenue will decrease slightly.

Conclusion

With recent economic struggles many companies have renewed their focus on maximizing sales while minimizing the costs of production. NU Industries faces these same challenges. This report outlines a recommendation that would allow NU to achieve a net profit of $7,132,135 at the end of the coming five sales cycles. In order to achieve this goal, this report established a linear model that encapsulated the objective function and all of the constraints NU Industries will face. Using a simplex solver, implemented in Python, the model was optimized to find the best mix of generated demand and use of resources. Through post optimization analysis and the review of sensitivity reports, this approach has been verified as being the optimal solution for NU industries. Future work in relation to this problem could involve refining the storage assumption, applying different integer problem solving techniques to find a mixed integer problem solution, and testing different what-if analyses based on changing demands, constraints, and variables.

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Appendix A: Detailed Project Description

Selling Prices

Widgets	Gadgets	Flugels
$2,490	$1,990	$2,970

Demand Before Use of Marketing Budget

Product	Period 1	Period 2	Period 3	Period 4	Period 5
Widgets	70	125	185	190	200
Gadgets	200	300	295	245	240
Flugels	140	175	205	235	230

Marketing Budget

- $70,000 can be used to increase demand
- Every $160 spent on widgets advertising is expected to increase demand by 1 unit in the following period
- Every $120 spent on Gadget advertising is expected to increase demand by 1 unit in the following period
- Every $180 spent on Flugel advertising is expected to increase demand by 1 unit in the following period

Raw Material Available

Material	Tons Available Per Period
Material 1	70 (140,000pounds)
Material 2	2.5 (5,000 pounds)

Raw Material Costs to Each Plant

Material	Plant A	Plant B
Material 1	$1.25/pounds	$1.45/pound
Material 2	$2.65/pound	$2.90/pound

**Each plant only purchases raw material that can be used within a given period due to limited storage*

Labor Costs

Plant A

Labor Type	Period 1	Period 2	Period 3	Period 4	Period 5
Regular	$11/hour	$11/hour	$11.55/hour	$11.55/hour	$11.55/hour
Overtime	$16.5/hour	$16.5/hour	$17.33/hour	$17.33/hour	$17.33/hour

**Regular time labor availability is limited to 2500 hours in each period, but overtime can be scheduled in any amount if necessary.*

Plant B

Labor Type	Period 1	Period 2	Period 3	Period 4	Period 5
Regular	$11/hour	$11/hour	$12.10/hour	$12.10/hour	$12.10/hour
Overtime	$16.5/hour	$16.5/hour	$18.15/hour	$18.15/hour	$18.15/hour

**Regular time labor availability is limited to 3800 hours in each period, but overtime can be scheduled in any amount if necessary.*

Plant B

Labor Type	Period 1	Period 2	Period 3	Period 4	Period 5
Regular	$11/hour	$11/hour	$12.10/hour	$12.10/hour	$12.10/hour
Overtime	$16.5/hour	$16.5/hour	$18.15/hour	$18.15/hour	$18.15/hour

**Regular time labor availability is limited to 3800 hours in each period, but overtime can be scheduled in any amount if necessary.*

Raw Materials Required for Production

Material 1

Plant	Widgets	Gadgets	Flugels
Plant A	194 pounds	230 pounds	178 pounds
Plant B	188 Pounds	225 pounds	170 pounds

Material 2

Plant	**Widgets**	**Gadgets**	**Flugels**
Plant A	8.6 pounds	1 pound	11.6 pounds
Plant B	9.2 Pounds	0.0 pounds	10.8 pounds

Labor Requirements

Plant A	**Widgets**	**Gadgets**	**Flugels**
Hours/Unit	9.5	7.1	11.1

Plant B	**Widgets**	**Gadgets**	**Flugels**
Hours/Unit	9.1	7.8	10.6

Storage and Costs

Plant A	**Widgets**	**Gadgets**	**Flugels**
Costs/Unit/Period	$7.50/unit	$5.50/unit	$6.50/unit

**The product inventory area can store a combined maximum of 70 units*

Plant B	**Widgets**	**Gadgets**	**Flugels**
Costs/Unit/Period	$7.80/unit	$5.70/unit	$7.00/unit

**The product inventory area can store a combined maximum of 50 units*

Transportation Costs

Product	**Plant A**	**Plant B**
Widgets	$6.30/unit	$6.50/unit
Gadgets	$4.60/unit	$5.00/unit
Flugels	$5.50/unit	$5.70/unit

Unit Cost

	Plant A			Plant B		
	Widgets	**Gadget**	**Flugel**	**Widgets**	**Gadget**	**Flugel**
	Regular Hours					
Period 1&2	$376.09	$370.20	$380.84	$405.88	$417.05	$400.12
Period 3, 4 & 5	$381.32	$374.11	$386.95	$415.89	$425.63	$411.78
	Overtime Hours					
Period 1&2	$428.34	$409.22	$441.89	$455.93	$459.95	$458.42
Period 3, 4 & 5	$436.18	$415.11	$451.05	$470.95	$472.82	$475.91