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MSDS 460: Homework Assignment 2
10/16/2022

Homework Assignment (Problem Set) 2:

Question 1:
1A. Write the general dual problem associated with the given LP.
(Do not transform or rewrite the primal problem before writing the general dual)

Maximize $-4x_1 + 2x_2$
Subject To
$4x_1 + x_2 + x_3 = 20$
$2x_1 - x_2 \geq 6$
$x_1 - x_2 + 5x_3 \geq -5$
$-3x_1 + 2x_2 + x_3 \leq 4$
$x_1 \leq 0$, $x_2 \geq 0$, x_3 unrestricted

Solution:
Dual Formulation:
Min $Z = 20\pi_1 + 6\pi_2 - 5\pi_3 + 4\pi_4$
Subject To
$4\pi_1 + 2\pi_2 + \pi_3 - 3\pi_4 <= 4$
$\pi_1 - \pi_2 - \pi_3 + 2\pi_4 >= 2$
$\pi_1 + 5\pi_3 + \pi_4 = 0$
π_1 unrestricted, $\pi_2, \pi_3 <= 0$, $\pi_4 >= 0$

1B. Given the following information for a product-mix problem with three products and three resources.
Primal Decision Variables: x_1 = number of unit 1 produced; x_2 = # of unit 2 produced; x_3 = # of unit 3 produced

Primal Formulation:

Max Z (Rev.) =	$25x_1$	$+ 30x_2$	$+ 20x_3$		
Subject To	$8x_1$	$+ 6x_2$	$+ x_3$	≤ 50	(Res. 1 constraint)
	$4x_1$	$+ 2x_2$	$+ 3x_3$	≤ 20	(Res. 2 constraint)
	$2x_1$	$+ x_2$	$+ 2x_3$	≤ 25	(Res. 3 constraint)
		x_1, x_2, x_3		≥ 0	(Nonnegativity)

Dual Formulation:

Min W =	$50\pi_1$	$+ 20\pi_2$	$+25\pi_3$	
Subject To	$8\pi_1$	$+ 4\pi_2$	$+2\pi_3$	≥ 25
	$6\pi_1$	$+ 2\pi_2$	$+\pi_3$	≥ 30
	π_1	$+ 3\pi_2$	$+2\pi_3$	≥ 20
	π_1, π_2, π_3			≥ 0

Optimal Solution:
Optimal Z = Revenue = $268.75

$x_1 = 0$ (Number of unit 1)	Dual Var. Optimal Value = 22.5 (Surplus variable in 1st dual constraint)
$x_2 = 8.125$ (Number of unit 2)	Dual Var. Optimal Value = 0 (Surplus variable in 2nd dual constraint)
$x_3 = 1.25$ (Number of unit 3)	Dual Var. Optimal Value = 0 (Surplus variable in 3rd dual constraint)
Resource Constraints:	
Resource 1 = 0 leftover units	Dual Var. Optimal Value = 3.125 = π_1
Resource 2 = 0 leftover units	Dual Var. Optimal Value = 5.625 = π_2
Resource 3 = 14.375 leftover units	Dual Var. Optimal Value = 0 = π_3

1Bi. What is the fair-market price for one unit of Resource 3?
The fair-market price or shadow price of one unit of Resource 3 is $0, because Resource 3 is a non-binding constraint.

1Bii. What is the meaning of the surplus variable value of 22.5 in the 1st dual constraint with respect to the primal problem?
The surplus variable value of 22.5 represents the reduced cost of product x1. This indicates how much the objective function coefficient for x1 must be improved before the value of x1 will be included in the optimal solution for the primal problem.

Question 2:

Carco manufactures cars and trucks. Each car contributes $300 to profit and each truck, $400; these profits do not consider machine rental. The resources required to manufacture a car and a truck are shown below. Each day Carco can rent up to 98 Type 1 machines at a cost of $50 per machine. The company now has 73 Type 2 machines and 260 tons of steel available. Marketing considerations dictate that at least 88 cars and at least 26 trucks be produced.

Part A: Formulate the problem as a Linear Program.

Solution:

Variables: C = cars

T = Trucks

Objective function: Maximize Z = 260C + 350T

Subject To:

C >= 88
T >= 26
0.8C + T <= 98
0.6C + 0.7T <= 73
2C + 3T <= 260
C, T $\geq$ 0

Part B: Solve the LP (provide exact values for all variables and the optimal objective function).
Hint: The optimal objective function value is $32540
[Note, I am providing this hint because having the optimal solution is necessary to do Part C.]

C = 88
T = 27.6
Objective function value = $32,540

Decision Variables		Changing Cells				
		C	T			
		88	27.6			
	Revenue	$ 300.00	$ 400.00			
	Production Cost	$ 40.00	$ 50.00			
	Shipping Cost					
Objective				Objective Cell		
	Profit Coefficients	$ 260.00	$ 350.00	$ 32,540.00		
Constraints	Car supply	1		88	>=	88
	Truck Supply		1	27.6	>=	26
	Type 1 Machine capacity	0.8	1	98	<=	98
	Type 2 Machine capacity	0.6	0.7	72.12	<=	73
	Steel supply	2	3	258.8	<=	260
				0	<=	98
	Constraint Name					

Part C: Answer the following questions from your output. *(Note, do not simply rerun the model – use the Linear Programming output and Sensitivity Analysis to explain your answers.)*

i) If cars contributed $310 to profit, what would be the new optimal solution to the problem?

The optimal solution would remain the same since the $10 increase is within the allowable increase of 20.

ii) What is the most that Carco should be willing to pay to rent an additional Type 1 machine for 1 day?

The most the Carco should be willing to pay to rent an additional Type 1 machine for 1 day is $350.

iii) What is the most that Carco should be willing to pay for an extra ton of steel?

At the current solution there is unused steel and therefore it is a non-binding constraint. Carco should pay $0 for an extra ton of steel

iv) If Carco were required to produce at least 86 cars, what would Carco's profit become?

Since the shadow price is -20, producing 86 cars would result in a $40 profit increase to $32,580.

v) Carco is considering producing jeeps. A jeep contributes $600 to profit and requires 1.2 days on machine 1, 2 days on machine 2, and 4 tons of steel. Should Carco produce any jeeps?

Yes, Carco should produce Jeeps. Since the rent cost for Type 1 machine is fixed at $50, Jeeps will have a higher profit margin than trucks. This increase in profit is higher than the increase in days on Type 1 machine 1 and increase in steel requirement. Since there is an excess of trucks being produced in the current solution, it would be more profitable to decrease the amount of trucks being produced and allocate resources to produce Jeeps.

Table:

Vehicle Type	Days on Machine 1	Days on Machine 2	Tons of Steel
Car	0.8	0.6	2
Truck	1	0.7	3

Question 3:

A catering company must have the following number of clean napkins available at the beginning of each of the next four days: day 1: 15, day 2: 12, day 3: 18, and day 4: 6. After being used, a napkin can be cleaned by one of two methods: fast service or slow service. Fast service costs $0.10 per napkin, and a napkin cleaned via fast service is available for use the day after it is last used. Slow service costs $0.06 per napkin, and a napkin cleaned via slow service is available two days after they were last used. New napkins can be purchased for a cost of $0.20 per napkin.

Part A: Formulate the problem as a minimum cost transportation problem.

<u>Solution:</u>

Xij – the number of napkins from node i to be ready for node j

Variables: X11 = day 1 new napkins

X12 = day 1 napkins fast cleaned

X13 = day 1 napkins slow cleaned

X22 = day 2 new napkins

X23 = day 2 napkins fast cleaned

X24 = day 2 napkins slow cleaned

X33 = day 3 new napkins

X34 = day 3 napkins fast cleaned

X44 = day 4 new napkins

Objective function: Minimize Z = .2(X11+X22+X33+X44) + .1(X12+X23+X34) + .06(X13+D24)

Subject To:

$X11 = 15$
$X12 + X22 = 12$
$X13 + X23 + X33 = 18$
$X24 + X34 + X44 = 6$
$X12 + X13 – X11 <= 0$
$X23 + X24 – X12 – X22 <= 0$
$X34 - X13 - X23 - X33 <= 0$
$Xij \geq 0$

Part B: Solve the problem (provide exact values for all variables and the optimal objective function).

X11 = 15
X12 = 9
X13 = 6
X22 = 3
X23 = 12
X24 = 0
X33 = 0
X34 = 6
X44 = 0

Objective function value = $6.66

		Changing Cells											
		x11	x12	x13	x22	x23	x24	x33	x34	x44			
Decision Variables		15	9	6	3	12	0	0	6	0			
	Revenue												
	Production Cost	$ 0.20	$ 0.10	$ 0.06	$ 0.20	$ 0.10	$ 0.06	$ 0.20	$ 0.10	$ 0.20			
	Shipping Cost												
Objective	Cost Coefficients	$ 0.20	$ 0.10	$ 0.06	$ 0.20	$ 0.10	$ 0.06	$ 0.20	$ 0.10	$ 0.20	Objective Cell $ 6.66		
Constraints	day 1 req	1									15	=	15
	day 2 req		1		1						12	=	12
	day 3 req			1		1		1			18	=	18
	day 4 req						1		1	1	6	=	6
	day 1 clean max	-1	1	1							0	<=	0
	day 2 clean max		-1		-1	1	1				0	<=	0
	day 3 clean max			-1		-1		-1	1		-12	<=	0
	Constraint Name												

Question 4:
A university has three professors who each teach four courses per year. Each year, four sections of marketing, finance, and production must be offered. At least one section of each class must be offered during each semester (fall and spring). Each professor's time preferences and preference for teaching various courses are given below.

The total satisfaction a professor earns teaching a class is the sum of the semester satisfaction and the course satisfaction. Thus, professor 1 derives a satisfaction of 3 + 6 = 9 from teaching marketing during the fall semester.

Part A: Formulate the problem as a minimum cost network flow problem that can be used to assign professors to courses so as to maximize the total satisfaction of the three professors. Draw the network and identify the nodes and arcs.

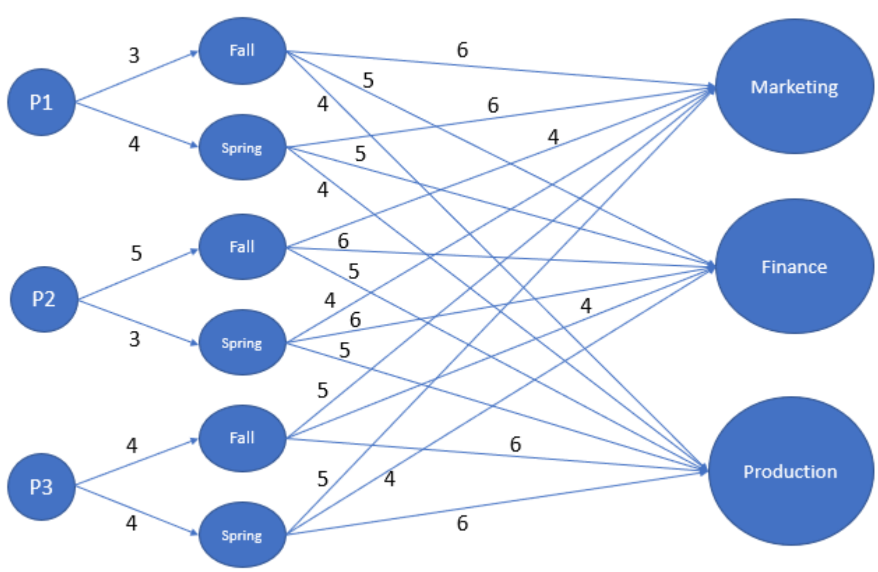
Solution:

X_{ijk} – the number of classes from node i (professor node) to node j (semester node) to node k (class node)

Variables: XP1FM = Professor 1 Fall Marketing classes
XP1FF = Professor 1 Fall Finance classes
XP1FP = Professor 1 Fall Production classes
XP2FM = Professor 2 Fall Marketing classes
XP2FF = Professor 2 Fall Finance classes
XP2FP = Professor 2 Fall Production classes
XP3FM = Professor 3 Fall Marketing classes
XP3FF = Professor 3 Fall Finance classes
XP3FP = Professor 3 Fall Production classes
XP1SM = Professor 1 Spring Marketing classes
XP1SF = Professor 1 Spring Finance classes
XP1SP = Professor 1 Spring Production classes
XP2SM = Professor 2 Spring Marketing classes
XP2SF = Professor 2 Spring Finance classes
XP2SP = Professor 2 Spring Production classes
XP3SM = Professor 3 Spring Marketing classes
XP3SF = Professor 3 Spring Finance classes
XP3SP = Professor 3 Spring Production classes

Objective function: Maximize Z = 9XP1FM + 8XP1FF + 7XP1FP + 9XP2FM + 11XP2FF + 10XP2FP + 9XP3FM + 8P3FF + 6XP3FP + 10XP1SM + 9XP1SF + 8XP1SP + 7XP2SM + 9XP2SF + 8XP2SP + 9XP3SM + 8XP3SF + 6XP3SP

Subject To:
XP1FM + XP2FM + XP3FM + XP1SM + XP2SM + XP3SM = 4
XP1FF + XP2FF + XP3FF + XP1SF + XP2SF + XP3SF = 4
XP1FP + XP2FP + XP3FP + XP1SP + XP2SP + XP3SP = 4
XP1FM + XP2FM + XP3FM >= 1
XP1SM + XP2SM + XP3SM >= 1
XP1FF + XP2FF + XP3FF >= 1
XP1SF + XP2SF + XP3SF >= 1
XP1FP + XP2FP + XP3FP >= 1
XP1SP + XP2SP + XP3SP >= 1
XP1FM + XP1FF + XP1FP + XP1SM + XP1SF + XP1SP = 4
XP2FM + XP2FF + XP2FP + XP2SM + XP2SF + XP2SP = 4
XP3FM + XP3FF + XP3FP + XP3SM + XP3SF + XP3SP = 4



Part B: Solve the problem (provide exact values for all variables and the optimal objective function).

Table:

	Professor 1	Professor 2	Professor 3
Fall Preference	3	5	4
Spring Preference	4	3	4
Marketing	6	4	5
Finance	5	6	4
Production	4	5	6

Solution:

XP1FM = 0
XP1FF = 0
XP1FP = 0
XP2FM = 0
XP2FF = 1
XP2FP = 3
XP3FM =1
XP3FF = 0
XP3FP = 0
XP1SM = 3
XP1SF = 0
XP1SP = 1
XP2SM = 0
XP2SF = 0
XP2SP = 0
XP3SM = 0
XP3SF = 3
XP3SP = 0

Objective function value = 112

		Changing Cells																				
		XP1FM	XP1FF	XP1FP	XP2FM	XP2FF	XP2FP	XP3FM	XP3FF	XP3FP	XP1SM	XP1SF	XP1SP	XP2SM	XP2SF	XP2SP	XP3SM	XP3SF	XP3SP			
Decision Variables		0	0	0	0	1	3	1	0	0	3	0	1	0	0	0	0	3	0			
	Satisfaction	9	8	7	9	11	10	9	8	6	10	9	8	7	9	8	9	8	6			
Objective																				Objective Cell		
	Cost Coefficients	9	8	7	9	11	10	9	8	6	10	9	8	7	9	8	9	8	6	112		
	Marketing total req	1			1			1			1			1			1			4	=	4
	Finance class totall req		1			1			1			1			1			1		4	=	4
	Production class total req			1			1			1			1			1			1	4	=	4
	Marketing class fall req	1			1			1												1	>=	1
	Finance class fall req		1			1			1											1	>=	1
Constraints	Production class fall req			1			1			1										3	>=	1
	Marketing class spring req										1			1			1			3	>=	1
	Finance class spring req											1			1			1		3	>=	1
	Production class spring req												1			1			1	1	>=	1
	P1 course total	1	1	1							1	1	1							4	=	4
	P2 course total				1	1	1							1	1	1				4	=	4
	P3 course total							1	1	1							1	1	1	4	=	4