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# Analyzing Cryptocurrency Returns

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## Abstract

1 Our project explores forecasting and analyzing relationships among cryptocurrency  
2 returns. We show that a VAR model forecasting multiple cryptocurrencies jointly  
3 outperforms more powerful non-linear univariate models. We also show multiple  
4 results regarding structure analysis on the VAR model, comparing the orthogonal-  
5 ized impulse responses and generalized forecast error variance decomposition for  
6 each cryptocurrency.

## 7 1 Introduction

8 Since Bitcoin's inception in the year 2009, popularity of cryptocurrencies has exploded. Once a  
9 niche, relatively-unknown topic, cryptocurrencies are now prominently featured throughout the  
10 mainstream media. Similar to the Gold Rush and the Dot-Com boom of the early 2000s, most view  
11 cryptocurrencies as a way to strike it rich. Bitcoin's frequent price surges have been well-documented,  
12 as just in the last year the price has risen by over \$50,000 dollars. As cryptocurrencies continue to  
13 produce profits and flood the markets, it is a very lucrative question to ask if one can understand their  
14 behavior.  
15 This project aims to tackle two main questions: how can we forecast cryptocurrency returns and  
16 what are the dependence relationships among them? As mentioned before, it is easy to imagine why  
17 we might want to be able to forecast returns. But in order to do this, we can leverage the pair-wise  
18 correlations between cryptocurrencies. Better understanding this network of relationships has two  
19 large benefits. Firstly, it can provide more information for our forecasting models to utilize. Secondly,  
20 it can help us better understand the risk and diversification of a portfolio containing these assets.

## 21 2 Background

22 Most prior work either compares more traditional time series methods to machine learning models  
23 or solely look at using ML-based models to forecast prices and returns. ML-based models have  
24 mostly been shown to outperform them. "...Independent of the period under analysis... and method,  
25 ML models present high levels of accuracy and improve the predictability of prices and returns of  
26 cryptocurrencies, outperforming competing models such as [ARIMA] and [EMA]" [3]. Persson  
27 et. al. investigated [2] using a Residual Recurrent Neural Network (R2N2) that showed significant  
28 improvements over ARIMA-type models.  
29 As for analyzing the relationships between cryptocurrencies, Qiang et. al. [1] investigated several  
30 properties related to the six largest cryptocurrencies by market capitalization at the time (Bitcoin,  
31 Ethereum, Ripple, Litecoin, Stellar, and Dash). Their primary contribution was quantifying spillover  
32 effects across these currencies. They conclude that Litecoin along with Bitcoin are at the center of

33 returns, and the second-largest cryptocurrency Ethereum is only a recipient of spillovers effects (as  
34 opposed to a contributor such as Litecoin).

35 The most recent data from their dataset was February 22, 2018, while the earliest data in our dataset  
36 starts on January 1st, 2018. As the market surrounding cryptocurrencies has vastly changed in just  
37 a few short years, it is possible that the relationship dynamics have also changed, especially when  
38 considering the influx of many new competing cryptocurrencies. They primarily used the generalized  
39 forecast-error variance decomposition (FEVD) to quantify their metrics of connectedness.

### 40 **3 Methods**

#### 41 **3.1 Models**

42 The univariate modeling methods used were primarily based on the ARIMA-type models covered  
43 in class. We used a VAR model to forecast the cryptocurrency returns jointly and a large number  
44 of different linear and nonlinear univariate model structures to forecast individual cryptocurrency  
45 returns.

##### 46 **3.1.1 Self-Exciting Transition Autoregressive (SETAR)**

47 In financial markets, stock index has been observed to exhibit different characteristics under different  
48 states of economy, (e.g. during high volatility during recession). Therefore, adopting a model that can  
49 switch smoothly from one scenario to another could theoretically improve performance. Self-Exciting  
50 Transition Autoregressive (SETAR) models find a threshold variable  $Z_t$  which determines which  
51 fitted linear model to use:

$$X_{t+m} = \begin{cases} \phi_1 + \phi_{10}X_t + \dots + \phi_{1p}X_{t-p} + \epsilon_{t+s} & Z_t \leq c \\ \phi_2 + \phi_{20}X_t + \dots + \phi_{2p}X_{t-p} + \epsilon_{t+s} & Z_t > c \end{cases} \quad (1)$$

52 Thus SETAR models can potentially explain nonlinear dynamics of financial time series, although  
53 empirical studies provide mixed results for these types of models [4]. We also used the generalized  
54 version Logistic Smooth Transition Autoregressive (LSTAR), which uses a logistic function to  
55 weight the use of the two models depending on the value of the threshold variable.

##### 56 **3.1.2 Additive Autoregressive Model (AAR)**

The non-parametric Additive Autoregressive Model (AAR) model (or GAM) took the form

$$x_{t+m} = \mu + \sum_{i=1}^p s_i(x_{t-i-1})$$

57 where  $s_i$  are smooth functions represented by penalized cubic regression splines.

##### 58 **3.1.3 Neural Network**

The neural network model used was very basic compared to deep, more complicated models that would be more effective. We used a simple version implemented in the nnet R package for ease of use. The forecasts are defined as:

$$x_{t+s} = \theta_0 + \sum_{j=1}^D g(\gamma_{0j} + \sum_{i=1}^m \gamma_{ij} x_{t-i-1})$$

59 where  $D$  is the number of hidden units, and  $g$  is the activation function.

### 60 **3.2 Structural Analysis**

For the structural analysis, we first fit a VAR model to the data. Explicitly this is modeled as

$$Y_t = A + B_1 Y_{t-1} + B_2 Y_{t-2} + \dots + A_p Y_{t-p} + u_t$$

61 where  $p = \#$  of lags,  $k = \#$  of time series,  $A \in R^k$ ,  $B_j \in R^{k \times k}$ , and  $u_t \in R^k \sim WN(0, \Sigma_\omega)$ .  
62 We can then perform structure analysis on the model. We are interested in this because, unlike in the  
63 univariate case, there is a bidirectional relationship between all of the variables (i.e. the prediction  
64 of variable  $x$  depends on variables whose values also depend on  $x$ ). With this interconnectedness  
65 comes a price though. Individual coefficient estimates provide limited information on the reaction of  
66 a system to a change since all the variables depend on one another. This motivates us to look at the  
67 impulse response function (IRF), which measures the effects of a shock to a variable on a variable.

When we transform our VAR(p) process into its infinite moving average process, we can derive the values for this function as  $\Phi_i$  where

$$Y_t = \mu + \sum_{i=0}^{\infty} \Phi_i u_{t-i}$$

The only issued posed by this formulation is that we are unable to reason about instantaneous relationships between variables. One solution is to compute the orthogonalized impulse responses (oIR), where we decompose the error covariance matrix  $\Sigma_\omega$  into a lower triangular matrix  $L$  with positive diagonal elements. We accomplished this by taking the Choleski decomposition.

$$\Sigma_\omega = LL^T$$

68 We can also examine the Forecast Error Variance Decomposition (FEVD), which decomposes the  
69 variance of the forecast error into the contributions from specific external shocks. This allows us  
70 to see how important a shock is in explaining the variations of the variables in the model. Here we  
71 calculate the generalized FEVD as discussed in Qiang et. al. [1].

## 72 4 Experiments

73 We computed basic summary statistics for each cryptocurrency to compare and contrast them.

|          | Binance Coin | Bitcoin       | Bitcoin Cash  | Cardano      | Dogecoin      | EOS.IO        | Ethereum     | Ethereum Classic | IOTA          | Litecoin     | Maker        | Monero        | Stellar      | TRON         |  |
|----------|--------------|---------------|---------------|--------------|---------------|---------------|--------------|------------------|---------------|--------------|--------------|---------------|--------------|--------------|--|
| Mean     | 3.183935e-05 | -1.559993e-06 | -4.636348e-06 | 4.236218e-06 | -1.235018e-06 | -1.569049e-06 | 3.745601e-05 | 1.619966e-05     | -1.381606e-05 | 2.475019e-06 | 7.652283e-06 | -1.020357e-05 | 9.131856e-06 |              |  |
| Variance | 2.972191e-05 | 4.055206e-06  | 4.196183e-05  | 2.034111e-05 | 6.596995e-05  | 2.329647e-05  | 6.227767e-06 | 8.051281e-05     | 6.432256e-05  | 1.262420e-05 | 3.596191e-05 | 4.426824e-05  | 2.715928e-05 | 2.422917e-05 |  |
| Skewness | 1.138876e+00 | 1.429658e+00  | 3.423190e+00  | 2.409741e-01 | 2.296246e+00  | 8.170746e-01  | 6.970623e-01 | 1.270059e+00     | 3.821637e-01  | 1.436972e+00 | 7.120396e-01 | 2.978419e-01  | 1.092518e+00 | 1.188936e+00 |  |
| Kurtosis | 1.072900e+02 | 6.883791e+01  | 3.849582e+02  | 3.358373e+01 | 1.058000e+02  | 4.709867e+01  | 7.885892e+01 | 1.299667e+02     | 2.990081e+01  | 6.674963e+01 | 3.038574e+01 | 6.413195e+01  | 4.284071e+01 | 5.935696e+01 |  |

74  
75 As we would expect, all of the cryptocurrencies meet the standard definition of being "fat-tailed".  
76 Every cryptocurrency's kurtosis is greater than 1, indicating that it has a wide tail. Most of the  
77 currencies are also heavily skewed, indicating by their large skewness (third moment) values.

### 78 4.1 Univariate Model

79 We first looked at univariate models for each currency to determine what type of SARIMA model  
80 is most appropriate. Shown here is our analysis on Bitcoin returns. To decide on a model, we first  
81 checked that the dependent variable was roughly stationary (which it was since the target is already  
82 the log-transformed returns). We then inspected the ACF and PACF plots, as shown below.  
83 The cyclical structure of the PACF plot indicates a seasonal component to the series. Inspecting the  
84 ACF plot, we see that it cuts off at lag 1s ( $s = \text{season} = 15$ ). The PACF tails off at lags 1s, 2s, ... .  
85 This implies an SMA(1),  $P = 0$ ,  $Q = 1$ , in the season ( $s = 15$ ).  
86 We then look at the non-seasonal component. The ACF tails off but the PACF cuts off at lag 3 or 4.  
87 We tried fitting both an ARIMA(3,0,0) x (0,0,1)\_15 and ARIMA(4,0,0) x (0,0,1)\_15 and forecasted  
88 the next 30 observations on a held-out test set.  
89 After fitting both models, we found that all of the coefficients are statistically significant (but this  
90 could easily be due to the immense size of our data). The standardized residuals looked approximately  
91 like white noise. The ACF of the residuals exhibited no large values or obvious pattern. The only  
92 suboptimal diagnostic is that the QQ-plot showed a large departure from expected normality. The  
93 AR(3) model performs slightly better than the AR(4) model in all of the AICc, AIC, and BIC metrics

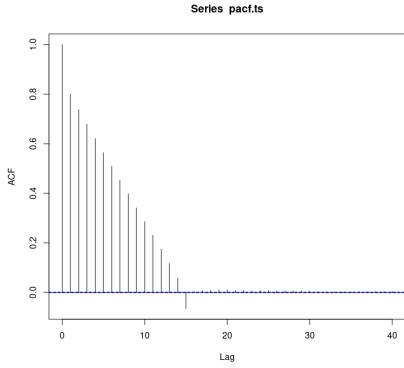


Figure 1: ACF plot for Bitcoin returns

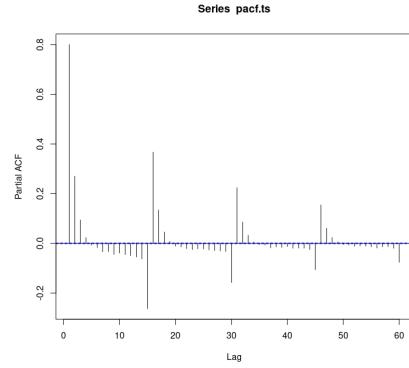


Figure 2: PACF plot for Bitcoin returns

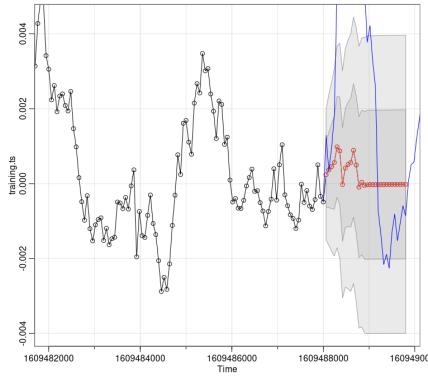


Figure 3: Seasonal AR(3) model for Bitcoin returns

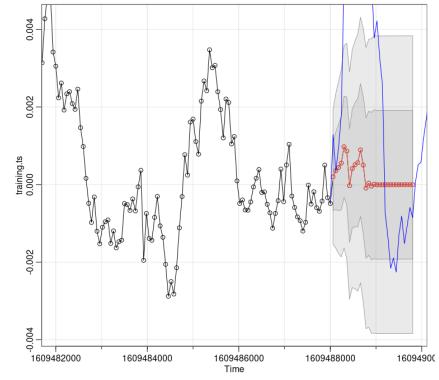


Figure 4: Seasonal AR(4) model for Bitcoin returns

## 94 4.2 Causality Analysis

95 We next fitted a VAR model using multiple cryptocurrencies to predict the target returns. There were  
 96 several different configurations of the data to test different aspects. We ran analyses on datasets with  
 97 samples from only three times per day and samples from every five minutes. We also split the data  
 98 into two subsamples - data before the creation of Dogecoin (pre-Doge) and data after the creation of  
 99 Dogecoin (post-Doge). Our sample sizes were large enough that we felt there was little loss in splitting  
 100 the data yet we could analyze if Dogecoin had any effects on the dynamics of the cryptocurrency  
 101 market. Across both datasets we included Cardano, Stellar, Ethereum, Bitcoin, Binance Coin, and  
 102 Litecoin. In order to draw conclusions about causality between the cryptocurrencies, we utilized two  
 103 common methods: Impulse Response (IR) Analysis, and Forecast Error Variance Decomposition  
 104 (FEVD).

### 105 4.2.1 Impulse Response Analysis

106 We first inspect the analysis of the 3-daily dataset. Since Orthogonal Impulse Response Analysis  
 107 (oIRA) depends on the ordering of the predictors, we ran the analysis on two different orderings. The  
 108 first was ordered by market cap, with Ethereum and Bitcoin being the last cryptocurrencies under the  
 109 hypothesis that they would have the most effect on all other cryptocurrencies. The second ordering  
 110 was chosen to have Ethereum and Bitcoin in the middle with Litecoin and Binance Coin at the end.  
 111 All responses were computed on both the pre-Doge and post-Doge subsamples.

```

> ortho_no_doge
      Cardano      Stellar  Binance.Coin      Litecoin      Ethereum      Bitcoin
Cardano  5.698473e-03 0.000000e+00 0.000000e+00 0.000000e+00 0.0000000000 0.0000000000
Stellar  1.759604e-03 6.367694e-03 0.000000e+00 0.000000e+00 0.0000000000 0.0000000000
Binance.Coin 1.659010e-03 1.056639e-03 4.971020e-03 0.000000e+00 0.0000000000 0.0000000000
Litecoin -1.407675e-04 5.812176e-05 -2.448732e-04 3.584929e-03 0.0000000000 0.0000000000
Ethereum -3.782010e-04 -2.895360e-04 -1.675385e-04 -3.044961e-05 0.0026298979 0.0000000000
Bitcoin -9.537067e-05 1.735227e-04 -1.722155e-05 9.860124e-05 -0.0001382203 0.001687873
> ortho_doge
      Cardano      Stellar  Binance.Coin      Litecoin      Ethereum      Bitcoin      Dogecoin
Cardano  4.929620e-03 0.000000e+00 0.000000e+00 0.0000000000 0.0000000000 0.0000000000 0.0000000000
Stellar  1.158150e-03 4.915792e-03 0.000000e+00 0.0000000000 0.0000000000 0.0000000000 0.0000000000
Binance.Coin 4.361932e-04 5.179125e-06 4.652516e-03 0.0000000000 0.0000000000 0.0000000000 0.0000000000
Litecoin -3.095929e-05 -8.446236e-05 -1.533696e-04 0.0036401640 0.0000000000 0.0000000000 0.0000000000
Ethereum -1.969467e-04 -2.848951e-04 -1.635131e-04 0.0002908909 0.0023593197 0.0000000000 0.0000000000
Bitcoin -4.188793e-04 -2.312512e-04 3.420225e-05 0.0001635994 0.0002571024 0.0022549321 0.0000000000
Dogecoin -1.626570e-04 8.058125e-05 -1.948394e-04 -0.0012403999 -0.0012597760 -0.0005813688 0.007126421

```

113 Comparing the instantaneous responses pre-Doge and post-Doge, in every currency except Bitcoin  
 114 the impulse response onto itself either stayed about the same or weakened. For example, when  
 115 looking at the response of Ethereum to a shock in Ethereum, the response decreased by 10%. Cardano  
 116 decreased by 13% and Stellar by 23%. The only currency this didn't hold for was Bitcoin, which saw  
 117 a 25% increase.

118 One would think that the introduction of a new currency would create larger forces within the network,  
 119 causing each currency to become less dependent on itself. A shock to Ethereum has less impact on  
 120 Ethereum's future state with the introduction of Dogecoin. Why Bitcoin did not follow this pattern  
 121 may be because of the unique nature of Bitcoin's changing status in the market. To this day, Bitcoin is  
 122 arguably the only cryptocurrency well-known in the mainstream media. It arguably started becoming  
 123 "mainstream" around our cutoff date of when Dogecoin was introduced in December 2019. Another  
 124 possibility is that Bitcoin's dependency on itself was so low pre-Doge that it increased just by the  
 125 nature of cryptocurrencies. Pre-doge Bitcoin had the lowest value at .00169, where the second-lowest  
 126 was Ethereum at .00263. Post-Doge this grew to .00225, which was much closer to Ethereum's new  
 127 value.

128 We can also see that after the instantaneous effect there was very little response to a shock, as the  
 129 impulse response was near zero for any forecast two steps out or greater.

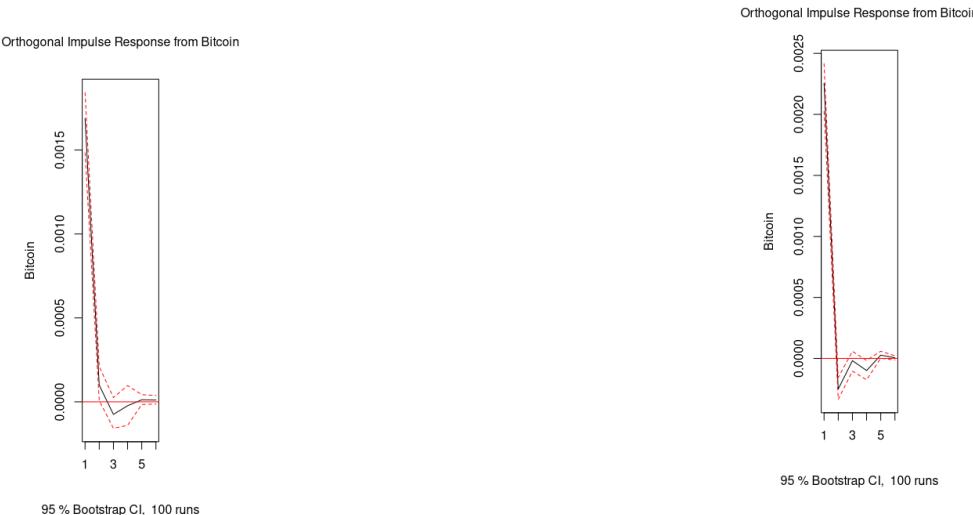


Figure 5: Bitcoin-Bitcoin orthogonal IR pre-Doge, 3-daily

Figure 6: Bitcoin-Bitcoin orthogonal IR post-Doge, 3-daily

130 We also calculated the orthogonal impulse responses with different orderings, but found them to have  
 131 negligible effects

```

Cardano      Stellar      Ethereum      Bitcoin      Binance.Coin      Litecoin
Cardano  5.698473e-03  0.000000e+00  0.000000e+00  0.000000e+00  0.0000000000  0.0000000000
Stellar   1.759604e-03  6.367694e-03  0.000000e+00  0.000000e+00  0.0000000000  0.0000000000
Ethereum  -3.782010e-04 -2.895360e-04  2.635405e-03  0.000000e+00  0.0000000000  0.0000000000
Bitcoin   -9.537067e-05  1.735227e-04 -1.379759e-04  1.690859e-03  0.0000000000  0.0000000000
Binance.Coin 1.659010e-03  1.056639e-03 -3.160186e-04 -7.641777e-05  0.0049603767  0.0000000000
Litecoin  -1.407675e-04  5.812176e-05 -2.585334e-05  2.094370e-04 -0.0002438192  0.003578785
> ortho_doge
Cardano      Stellar      Ethereum      Bitcoin      Binance.Coin      Litecoin      Dogecoin
Cardano  4.929620e-03  0.000000e+00  0.0000000000  0.0000000000  0.0000000000  0.0000000000  0.0000000000
Stellar   1.158150e-03  4.915792e-03  0.0000000000  0.0000000000  0.0000000000  0.0000000000  0.0000000000
Ethereum  -1.969467e-04 -2.848951e-04  0.0023828016  0.0000000000  0.0000000000  0.0000000000  0.0000000000
Bitcoin   -4.188793e-04 -2.312512e-04  0.0002721938  0.0022593507  0.0000000000  0.0000000000  0.0000000000
Binance.Coin 4.361932e-04  5.179125e-06 -0.0003192658  0.0001088935  0.0046402708  0.0000000000  0.0000000000
Litecoin  -3.095929e-05 -8.446236e-05  0.0004549135  0.0002064569 -0.0001273197  0.003606735  0.0000000000
Dogecoin  -1.626570e-04  8.058125e-05 -0.0013854182 -0.0006494473 -0.0002754342 -0.001041418  0.007126421

```

132 When we conducted the analysis on the five-minute dataset, almost all of the directions of the impulse  
 133 responses stayed the same as in the 3-daily dataset; e.g. a shock to Cardano had a posive response to  
 134 Cardano, a shock to Stellar had a negative response to Ethereum in both datasets.

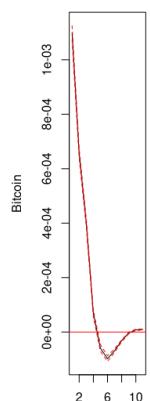
```

> ortho_no_doge
Cardano      Stellar      Binance.Coin      Litecoin      Ethereum      Bitcoin
Cardano  0.0038176467  0.0000000e+00  0.0000000000  0.0000000e+00  0.0000000000  0.0000000000
Stellar   0.0011046273  4.823952e-03  0.0000000000  0.0000000e+00  0.0000000000  0.0000000000
Binance.Coin 0.0007200358  8.530049e-04  0.0039513573  0.0000000e+00  0.000000e+00  0.0000000000
Litecoin  -0.0002633288 -9.922313e-05 -0.0002033944  1.994071e-03  0.000000e+00  0.0000000000
Ethereum  -0.0002378396 -7.915954e-05 -0.0001656710  3.056640e-05  1.534364e-03  0.0000000000
Bitcoin   -0.0001195707  3.282631e-06 -0.0001581579  6.628559e-05 -7.316185e-05  0.001098722
> ortho_doge
Cardano      Stellar      Binance.Coin      Litecoin      Ethereum      Bitcoin      Dogecoin
Cardano  2.944557e-03  0.000000e+00  0.0000000e+00  0.0000000e+00  0.0000000000  0.0000000000
Stellar   7.882955e-04  3.102283e-03  0.0000000e+00  0.0000000e+00  0.0000000000  0.0000000000
Binance.Coin 1.405117e-04  3.295464e-04  2.747128e-03  0.0000000e+00  0.0000000000  0.0000000000
Litecoin  -4.062874e-05  1.577642e-04 -5.531385e-05  2.332057e-03  0.0000000e+00  0.0000000000  0.0000000000
Ethereum  -1.598007e-04 -1.295050e-04 -1.648295e-04  7.015462e-05  1.685505e-03  0.0000000000  0.0000000000
Bitcoin   -9.612673e-05 -3.066337e-05 -4.515277e-05 -5.808547e-05 -9.731103e-05  0.0013312309  0.0000000000
Dogecoin  5.546563e-04  3.789994e-04  2.314022e-04 -3.888928e-04 -3.044318e-04  0.0003316661  0.003907869
>

```

136  
 137 However, we note two interesting changes. First, in the five-minute dataset there are statistically sig-  
 138 nificant non-zero impulse responses after the first time step consistent across all the cryptocurrencies.  
 139 As an example, here is the graph of the response to Bitcoin from a shock from Bitcoin (below). There  
 140 is a clear negative response to a positive shock from 20 to 50 minutes out.

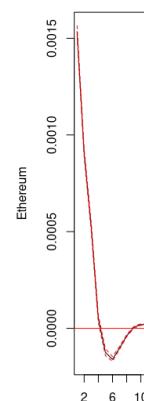
Orthogonal Impulse Response from Bitcoin



95 % Bootstrap CI, 100 runs

Figure 7: Bitcoin orthogonal IR, 5-minute

Orthogonal Impulse Response from Ethereum



95 % Bootstrap CI, 100 runs

Figure 8: Ethereum orthogonal IR, 5-minute

141 Secondly, the responses onto itself were all considerably smaller. For example, the response of  
 142 Cardano to a shock in Cardano was 33% smaller when looking at five-minute intervals.

143 **4.2.2 Forecast Error Variance Decomposition (FEVD)**

144 We also calculated the H-step-ahead generalized FEVD for each cryptocurrency, pre and post-Doge  
 145 every five minutes (table of errors included in Appendix). We note three interesting observations.  
 146 First, generally the smaller (in market cap) cryptocurrencies were more dependent on the larger  
 147 cryptocurrencies. For example, 88% of the variance in Cardano's forecasts could be explained by a  
 148 shock to Cardano itself, with 12% coming from the other five cryptocurrencies. However, 95% of the  
 149 variance in Bitcoin's forecasts could be explained by a shock to Bitcoin itself.

150 Secondly, in general we see that every cryptocurrency saw their self FEVD percentage increase in  
 151 the post-Doge dataset; e.g. the variance in a cryptocurrency's forecast contributed by a shock to that  
 152 same cryptocurrency grew. For example, Cardano increased from 88% to 91%, Litecoin and Bitcoin  
 153 from 95% to 98%, etc. We also note the substantial change in Binance Coin's decomposition. It went  
 154 from 87% to 96%, by far the largest increase.

155 Lastly, we note that these percentages slightly decreased over time, demonstrating a larger dependency  
 156 on other cryptocurrencies as time moves forward, across both pre-Doge and post-Doge. For example,  
 157 at the first step forecast Bitcoin is at 95% and drops to 92% by the 10th-step forecast in the pre-Doge  
 158 dataset. In the post-Doge dataset it drops from 98% to 97% by the 10th-step forecast.

159 **4.3 Forecasting Methods**

160 We also tried different model architectures to forecast target returns. These models were all applied  
 161 to univariate time series for each currency. The following analysis was conducted on the Bitcoin time  
 162 series.

163 We found that a simple VAR(3) model outperformed the more powerful non-linear univariate models  
 164 in all of Mean Error (ME), Root Mean-Squared Error (RMSE), and Mean Absolute Error (MAE). We  
 165 also found that out of all models, the Generalized Additive Autoregressive Model (AAR) with six  
 166 penalized cubic splines performed the best. We see in the below example that the AAR model was  
 167 able to better predict the increase in returns in the first two steps and decrease in returns at the 15th  
 168 step.

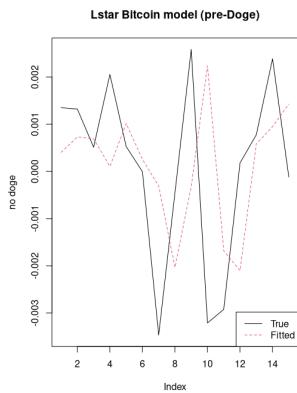


Figure 9: Lstar model

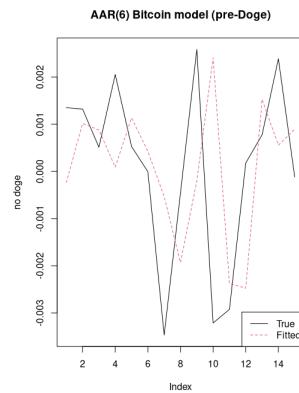


Figure 10: Additive Ar Model

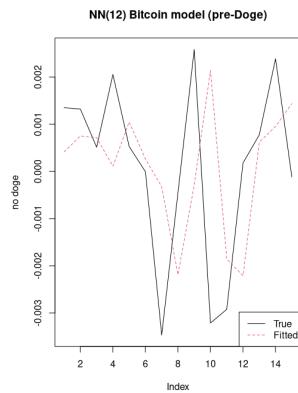


Figure 11: NN forecast

169 The table of the model test statistics are below.

|      | AR(3)     | VAR(3)   | Setar(1)  | Setar(3)  | Lstar(3)  | Aar(3)    | Aar(6)    | NN(3)     |
|------|-----------|----------|-----------|-----------|-----------|-----------|-----------|-----------|
| ME   | -2.95E-05 | 1.17E-05 | -2.80E-05 | -5.51E-05 | -4.54E-05 | -3.31E-05 | -2.50E-05 | -2.93E-05 |
| RMSE | 1.95E-03  | 5.70E-04 | 1.99E-03  | 1.96E-03  | 1.96E-03  | 1.96E-03  | 1.93E-03  | 1.95E-03  |
| MAE  | 1.36E-03  | 3.46E-04 | 1.38E-03  | 1.36E-03  | 1.35E-03  | 1.35E-03  | 1.33E-03  | 1.36E-03  |

170

171 **5 Conclusions**

172 This study analyzes the results of non-linear modeling applied to cryptocurrency returns and quantifies  
173 network effects between the largest cryptocurrencies on the market. We show that there are many  
174 complex relationships between the several cryptocurrencies analyzed and that these can be utilized  
175 to better forecast individual currencies. Our simple VAR(3) model using all six cryptocurrencies  
176 performed the best out of the more powerful nonlinear, univariate models.

177 Furthermore, we provide evidence for a number of interesting conclusions about the relationships  
178 between cryptocurrencies. The orthogonal impulse responses showed larger instantaneous responses  
179 to shocks in smaller cryptocurrencies than larger. They also showed a consistent negative relationship  
180 between 20 to 50 minutes before settling to zero. Lastly the magnitude of the instantaneous orthogonal  
181 impulse responses were shown to be smaller in the post-Doge dataset, suggesting that the introduction  
182 of another cryptocurrency damped the impact of a single shock to an individual cryptocurrency on  
183 itself.

184 The generalized FEVD showed a larger proportion of forecasting error could be explained by other  
185 cryptocurrencies in smaller cryptocurrencies. It also showed that this proportion slightly decreases  
186 with time, yet consistently remained high (over 90% across almost all cryptocurrencies) after all  
187 forecasts within 10 steps ahead.

188 **6 Future Work**

189 There are several aspects of the study that could be improved. For example, it is hard to conduct  
190 inference on the orthogonal impulse responses without knowing their dependency on the order, their  
191 distributional properties, and their repeatability. Is there a way to account for every possible ordering  
192 combination of the predictors to come to a general statistic conveying the information? What is the  
193 statistical significance of the computed values? And how much does our calculated values depend on  
194 the sample taken. If we were to compute these statistics on each half of the dataset, would they be  
195 strongly correlated?

196 There is also a lot more that could be done with the forecasting. A primary limitation was that all of  
197 the non-linear methods used could only be applied to univariate time series.

198 Finally, we did not explore using additional features to improve forecasting performance, which  
199 would be very useful.

200 **References**

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## 218 7 Appendix

| \$Litecoin  | Cardano  | Stellar | Binance.Coin | Litecoin | Ethereum | Bitcoin | Cardano | Stellar | Binance.Coin | Litecoin | Ethereum | Bitcoin |
|---|--|---------|--------------|----------|----------|---------|---------|---------|--------------|----------|----------|---------|
| [1,] 0.01612270 0.005621967 0.01602438 0.9525639 0.002592635 0.007074383    | [1,] 0.8833825 0.04401279 0.02716550 0.01495176 0.02043392 0.010053549     |         |              |          |          |         |         |         |              |          |          |         |
| [2,] 0.01724565 0.004816927 0.01572011 0.9521605 0.002760488 0.007296371    | [2,] 0.8757015 0.05192785 0.03183123 0.01390651 0.01804048 0.008592405     |         |              |          |          |         |         |         |              |          |          |         |
| [3,] 0.01716423 0.004607327 0.01544531 0.9521856 0.002762952 0.007834590    | [3,] 0.8736766 0.05581987 0.03162611 0.01356821 0.01725551 0.008053693     |         |              |          |          |         |         |         |              |          |          |         |
| [4,] 0.01719132 0.004919020 0.01564820 0.9514696 0.002804339 0.007967547    | [4,] 0.8722504 0.05735525 0.03160160 0.01352523 0.01717357 0.008093940     |         |              |          |          |         |         |         |              |          |          |         |
| [5,] 0.01750169 0.005320369 0.01619117 0.9502982 0.002784943 0.007903638    | [5,] 0.8719593 0.05731244 0.03162645 0.01376639 0.01716461 0.008170838     |         |              |          |          |         |         |         |              |          |          |         |
| [6,] 0.01773228 0.005589267 0.01656188 0.9495179 0.002760114 0.007838597    | [6,] 0.8720213 0.05704023 0.03161264 0.01396229 0.01711420 0.008249300     |         |              |          |          |         |         |         |              |          |          |         |
| [7,] 0.01779858 0.005644673 0.016662800 0.9493378 0.002752381 0.007838542   | [7,] 0.8720685 0.05696443 0.03159035 0.01399955 0.01708280 0.008294346     |         |              |          |          |         |         |         |              |          |          |         |
| [8,] 0.01779963 0.005645643 0.016662527 0.9493206 0.002752277 0.007836585   | [8,] 0.8720682 0.05697805 0.03157987 0.01399491 0.01707647 0.008032512     |         |              |          |          |         |         |         |              |          |          |         |
| [9,] 0.01780396 0.005650869 0.016664583 0.9492815 0.002753201 0.007864607   | [9,] 0.8720495 0.05698856 0.03158027 0.01400285 0.01707609 0.008302741     |         |              |          |          |         |         |         |              |          |          |         |
| [10,] 0.01781152 0.005658498 0.01666771 0.9492438 0.002753643 0.007864839   | [10,] 0.8720350 0.05698670 0.03158037 0.01401389 0.01707499 0.008309085    |         |              |          |          |         |         |         |              |          |          |         |
| \$Ethereum  | Cardano  | Stellar | Binance.Coin | Litecoin | Ethereum | Bitcoin | Cardano | Stellar | Binance.Coin | Litecoin | Ethereum | Bitcoin |
| [1,] 0.02196375 0.005687160 0.01838199 0.002584347 0.9495187 0.0009640568   | [1,] 0.04424555 0.8916672 0.05262558 0.006185822 0.0003916309              |         |              |          |          |         |         |         |              |          |          |         |
| [2,] 0.02491817 0.007113994 0.02092666 0.001992894 0.9435562 0.00222120966  | [2,] 0.06353723 0.8566031 0.07034138 0.004011141 0.004958108 0.0005490787  |         |              |          |          |         |         |         |              |          |          |         |
| [3,] 0.02594649 0.007670171 0.020101040 0.001845937 0.9407942 0.0027328171  | [3,] 0.06785530 0.8500929 0.07326618 0.003866205 0.004286879 0.0006325224  |         |              |          |          |         |         |         |              |          |          |         |
| [4,] 0.02609466 0.007731451 0.02100832 0.001942069 0.9402968 0.0029267492   | [4,] 0.07048041 0.8466510 0.07411079 0.003784101 0.004350262 0.0006171328  |         |              |          |          |         |         |         |              |          |          |         |
| [5,] 0.02608601 0.007703585 0.02111508 0.001958228 0.9402287 0.0029138249   | [5,] 0.07082443 0.8459325 0.07396224 0.003855380 0.004553339 0.0008721277  |         |              |          |          |         |         |         |              |          |          |         |
| [6,] 0.02625595 0.007708680 0.02120723 0.001943287 0.9399319 0.0029492714   | [6,] 0.07076780 0.8452937 0.07403026 0.003980489 0.004629575 0.0012963322  |         |              |          |          |         |         |         |              |          |          |         |
| [7,] 0.02640599 0.007725260 0.02124095 0.001951365 0.9395449 0.0030215107   | [7,] 0.07073488 0.8448399 0.07417398 0.004019260 0.004631640 0.0016003415  |         |              |          |          |         |         |         |              |          |          |         |
| [8,] 0.02644917 0.007734798 0.02123892 0.001964128 0.9395563 0.0030567152   | [8,] 0.07073963 0.8446824 0.07422741 0.004022071 0.004631160 0.0016972937  |         |              |          |          |         |         |         |              |          |          |         |
| [9,] 0.02644953 0.007735687 0.02123996 0.001968010 0.9395449 0.0030619633   | [9,] 0.07074888 0.8446535 0.07423184 0.004022354 0.004638030 0.0017054586  |         |              |          |          |         |         |         |              |          |          |         |
| [10,] 0.02645215 0.007734875 0.02124384 0.001967801 0.9395394 0.0030619231  | [10,] 0.07075086 0.8446432 0.07422891 0.004026025 0.004643231 0.0017077246 |         |              |          |          |         |         |         |              |          |          |         |
| \$Bitcoin   | Cardano  | Stellar | Binance.Coin | Litecoin | Ethereum | Bitcoin | Cardano | Stellar | Binance.Coin | Litecoin | Ethereum | Bitcoin |
| [1,] 0.01090167 0.0004207304 0.02268597 0.0007114049 0.00097453 0.9579050   | [1,] 0.02676365 0.05082046 0.8703149 0.01464076 0.01684866 0.02061158      |         |              |          |          |         |         |         |              |          |          |         |
| [2,] 0.01568310 0.0013219105 0.02661093 0.008217448 0.0012348627 0.9469318  | [2,] 0.02901224 0.05689492 0.8661599 0.01376622 0.01533736 0.01882938      |         |              |          |          |         |         |         |              |          |          |         |
| [3,] 0.01797798 0.002499097 0.0279604 0.0009328653 0.0018820129 0.9403518   | [3,] 0.02846162 0.05771006 0.8676730 0.01366549 0.01455396 0.01793585      |         |              |          |          |         |         |         |              |          |          |         |
| [4,] 0.01909599 0.0029107300 0.02813997 0.009777299 0.0020571440 0.9380189  | [4,] 0.02846185 0.05846039 0.8668408 0.01364769 0.01467645 0.01791279      |         |              |          |          |         |         |         |              |          |          |         |
| [5,] 0.01915004 0.0029546748 0.02815038 0.0009804592 0.0020695008 0.9378708 | [5,] 0.02836199 0.05819871 0.8668913 0.01376093 0.01495747 0.01782959      |         |              |          |          |         |         |         |              |          |          |         |
| [6,] 0.01906669 0.0029564655 0.02823520 0.009772129 0.0020844435 0.9377946  | [6,] 0.02819603 0.05781393 0.8673627 0.01385462 0.01506054 0.01771215      |         |              |          |          |         |         |         |              |          |          |         |
| [7,] 0.01911055 0.0030068744 0.02848401 0.009813509 0.0021335106 0.9374515  | [7,] 0.0281281 0.05767722 0.8675921 0.01387353 0.01505325 0.01767572       |         |              |          |          |         |         |         |              |          |          |         |
| [8,] 0.01917596 0.0030541212 0.02853412 0.009865444 0.0021730064 0.9371982  | [8,] 0.02812684 0.05766197 0.8676131 0.01387165 0.01504996 0.01767644      |         |              |          |          |         |         |         |              |          |          |         |
| [9,] 0.01920590 0.0030710849 0.02853594 0.009883534 0.0021838842 0.9371197  | [9,] 0.02813176 0.05765956 0.8675968 0.01387468 0.01506078 0.01767645      |         |              |          |          |         |         |         |              |          |          |         |
| [10,] 0.01920938 0.0030731039 0.02853741 0.009884577 0.0021841434 0.9371114 | [10,] 0.02812927 0.05765221 0.8675971 0.01387883 0.01506828 0.01767431     |         |              |          |          |         |         |         |              |          |          |         |

Figure 12: FEVD pre-Doge

| \$Litecoin   | Cardano  | Stellar | Binance.Coin | Litecoin | Ethereum | Bitcoin | Dogecoin | Cardano | Stellar | Binance.Coin | Litecoin | Ethereum | Bitcoin | Dogecoin |
|--|--|---------|--------------|----------|----------|---------|----------|---------|---------|--------------|----------|----------|---------|----------|
| [1,] 0.002969668 0.003673673 0.0002617476 0.9837318 0.00155833 0.007914535 0.008685484       | [1,] 0.911934 0.05531384 0.023460307 0.0002753106 0.00798441 0.004688936 0.01739402    |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [2,] 0.005228195 0.0036331087 0.0020353123 0.9795154 0.001549308 0.0079317841 0.008931844    | [2,] 0.9025503 0.05636277 0.004216143 0.007657372 0.008876905 0.005959711 0.02123241   |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [3,] 0.00564909 0.003673673 0.0002617476 0.9795154 0.001549308 0.0079317841 0.008931844      | [3,] 0.8967347 0.05752434 0.004635298 0.007061675 0.008797675 0.006758979 0.02478882   |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [4,] 0.0063720996 0.007876326 0.0005158454 0.9738151 0.002320944 0.01670385                  | [4,] 0.8937773 0.05786861 0.005612604 0.009928850 0.008788283 0.006772695 0.02618800   |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [5,] 0.006852530 0.007972876 0.0005219664 0.9727703 0.002602345 0.01620658                   | [5,] 0.8933769 0.05747927 0.00569028 0.0010201874 0.008905315 0.006860155 0.02619788   |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [6,] 0.006467509 0.007971033 0.0006151912 0.9725482 0.002658576 0.002579745 0.013717128      | [6,] 0.8933189 0.05760377 0.006074403 0.001243017 0.009348504 0.007162479 0.02600767   |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [7,] 0.00791105 0.0030068744 0.009813509 0.0021335106 0.9374515                              | [7,] 0.8930674 0.05696276 0.006056066 0.0010208506 0.009422647 0.00733239 0.02613206   |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [8,] 0.005083057 0.007974901 0.00800810793 0.9719734 0.002705838 0.002831311 0.013234741     | [8,] 0.8927999 0.05698007 0.006069163 0.0010215732 0.009418635 0.007364652 0.02634581  |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [9,] 0.005081567 0.007975087 0.00800884808 0.9717419 0.002723557 0.0028241818 0.013347711    | [9,] 0.8926453 0.05698224 0.006117615 0.0010239504 0.009419171 0.007364862 0.02643413  |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [10,] 0.005194633 0.00797203 0.00800813241 0.9716407 0.002792102 0.002845883                 | [10,] 0.8925777 0.05696851 0.006149896 0.0010248274 0.009455417 0.007384383 0.02643931 |         |              |          |          |         |          |         |         |              |          |          |         |          |
| \$Ethereum   | Cardano  | Stellar | Binance.Coin | Litecoin | Ethereum | Bitcoin | Dogecoin | Cardano | Stellar | Binance.Coin | Litecoin | Ethereum | Bitcoin | Dogecoin |
| [1,] 0.005711105 0.0089810764 0.01145782 0.001511468 0.0056799 0.00344053 0.01042631         | [1,] 0.05477098 0.9030429 0.014744669 0.00372347 0.008419976 0.001432401 0.014221673   |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [2,] 0.007972063 0.011192858 0.01454572 0.001139445 0.00544863 0.00344052 0.01042634         | [2,] 0.06184172 0.8784947 0.01906578 0.006917405 0.006560018 0.001668642 0.02545175    |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [3,] 0.00756889 0.01217923 0.01714080 0.001474270 0.005357700 0.008085731 0.0173177          | [3,] 0.06288554 0.8711510 0.020286449 0.00719959 0.006130216 0.002220653 0.03012316    |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [4,] 0.00742184 0.012274414 0.01771110 0.001573126 0.00344117 0.009295607 0.01732101         | [4,] 0.06301005 0.8684719 0.02153026 0.00768498 0.00612655 0.002240465 0.03093965      |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [5,] 0.006462500 0.005683933 0.0204251107 0.00487059 0.002307367 0.00235785 0.005545914      | [5,] 0.06300049 0.8682637 0.02165966 0.007608072 0.006133696 0.002261166 0.03102046    |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [6,] 0.005069858 0.005927262 0.025474641 0.004909494 0.002393473 0.00273007 0.005605957      | [6,] 0.06310034 0.8673558 0.02147036 0.007592919 0.006106742 0.002380455 0.03193327    |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [7,] 0.00578441 0.0062774464 0.02539355 0.004897562 0.002995458 0.002722501 0.005964615      | [7,] 0.06311105 0.8671844 0.02139034 0.007592919 0.006106742 0.002380455 0.03193327    |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [8,] 0.00578441 0.0062774464 0.02539355 0.004897562 0.002995458 0.002722501 0.005964615      | [8,] 0.06312435 0.8659333 0.02143399 0.007608977 0.006072499 0.002443319 0.03322969    |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [9,] 0.005084841 0.006554524 0.002659578 0.004983832 0.003080101 0.00915939 0.005619399      | [9,] 0.06320843 0.8658451 0.02149950 0.007619089 0.006075181 0.002447158 0.03330555    |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [10,] 0.005084841 0.006554524 0.002659578 0.004983832 0.00308014460 0.009159209 0.0056195297 | [10,] 0.06320843 0.8657807 0.02152934 0.007618447 0.006076767 0.002459234 0.03333245   |         |              |          |          |         |          |         |         |              |          |          |         |          |
| \$Dogecoin   | Cardano  | Stellar | Binance.Coin | Litecoin | Ethereum | Bitcoin | Dogecoin | Cardano | Stellar | Binance.Coin | Litecoin | Ethereum | Bitcoin | Dogecoin |
| [1,] 0.01788713 0.01476466 0.005326516 0.008283607 0.01023176 0.00561974 0.0278476           | [1,] 0.02447726 0.05127375 0.9630057 0.0002562329 0.011545655 0.001520227 0.005469402  |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [2,] 0.0195477 0.02074372 0.013778913 0.005786492 0.01153318 0.007056794 0.02046461          | [2,] 0.03729103 0.02013233 0.9483307 0.0002857545 0.01085136 0.001541290 0.015128604   |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [3,] 0.01942718 0.02094594 0.014820770 0.006015650 0.01038052 0.016126424 0.0122835          | [3,] 0.00432394 0.02174080 0.9469250 0.0002857830 0.01046964 0.002458175 0.013813612   |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [4,] 0.01938272 0.02140785 0.016537296 0.005134733 0.01052138 0.019969091 0.00604073         | [4,] 0.04463975 0.02229185 0.9451949 0.000909799 0.01047629 0.002494813 0.013992396    |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [5,] 0.01939635 0.02127181 0.016554613 0.005004746 0.01171141 0.021360625 0.00933834         | [5,] 0.004680657 0.02228102 0.9447403 0.0012401128 0.01044252 0.02251362 0.014083818   |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [6,] 0.01939635 0.02127181 0.016554613 0.005004746 0.01171141 0.021360625 0.00933834         | [6,] 0.004647631 0.02215044 0.9445915 0.0015510246 0.01036612 0.022714815 0.013978487  |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [7,] 0.01915668 0.02120362 0.016423512 0.005655252 0.01121919 0.021528228 0.0092809          | [7,] 0.004645074 0.02213107 0.9446047 0.001560606 0.01033816 0.02272309 0.013944278    |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [8,] 0.01913105 0.02099739 0.016501614 0.006546237 0.01132203 0.02273204 0.00932285          | [8,] 0.004647765 0.02217087 0.9444612 0.0015649198 0.01033810 0.02285297 0.013934621   |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [9,] 0.01912306 0.02098900 0.016555067 0.006560505 0.01137592 0.022568555 0.00928278         | [9,] 0.004687331 0.02218422 0.9443935 0.0006007042 0.01034443 0.022855763 0.013934078  |         |              |          |          |         |          |         |         |              |          |          |         |          |
| [10,] 0.0191182 0.02098381 0.0165558647 0.006577055 0.01140466 0.022602513 0.00927551        | [10,] 0.004688316 0.02218529 0.9443675 0.0006  |         |              |          |          |         |          |         |         |              |          |          |         |          |