# Chapter 8.9.1 Order 3 B-Trees

### Introduction to B-trees

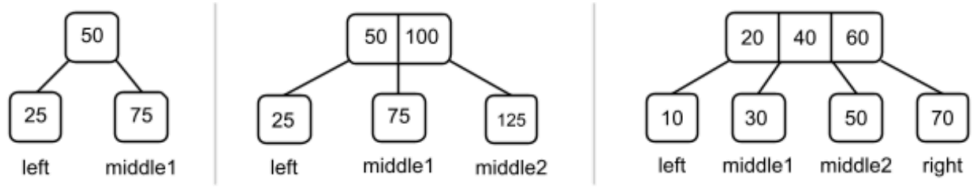
In a binary tree, each node has one key and up to two children. A ***B-tree*** with order K is a tree where nodes can have up to K-1 keys and up to K children. The ***order*** is the maximum number of children a node can have. Ex: In a B-tree with order 4, a node can have 1, 2, or 3 keys, and up to 4 children. B-trees have the following properties:

- All keys in a B-tree must be distinct.
- All leaf nodes must be at the same level.
- An internal node with N keys must have N+1 children.
- Keys in a node are stored in sorted order from smallest to largest.
- Each key in a B-tree internal node has one left subtree and one right subtree. All left subtree keys are < that key, and all right subtree keys are > that key.

## 2-3-4 Tree Node Labels

- The keys in a 2-3-4 tree node are labeled as A, B, and C
- The child nodes of a 2-3-4 tree internal node are labeled as left, middle1, middle2, and right.
- If a node contains 1 key, then keys B and C, as well as children middle2 and right, are not used.
- If a node contains 2 keys, then key C, as well as the right child, are not used.
- A 2-3-4 tree node containing exactly 3 keys is said to be full, and uses all keys and children

Figure 8.9.1: 2-3-4 child subtree labels.

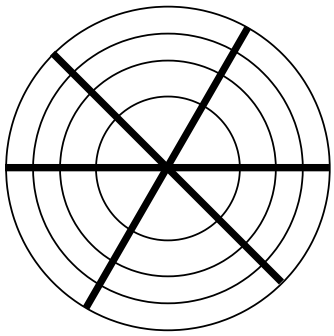



## 2-3-4 Tree Search Algorithm

- Given a key, a search algorithm returns the first node found matching that key, or returns null if a matching node is not found.
- Searching a 2-3-4 tree is a recursive process that starts with the root node.
- If the search key equals any of the keys in the node, then the node is returned.
- Otherwise, a recursive call is made on the appropriate child node.
- Which child node to use depends on the value of the search key in comparison to the node's keys.

| Condition | Child node to search |
|---|---|
| key < node's A key | left |
| node has only 1 key or key < node's B key | middle1 |
| node has only 2 keys or key < node's C key | middle2 |
| none of the above | right |

```
BTreeSearch(node, key) {
   if (node is not null) {
      if (node has key) {
         return node
      }
      if (key < node -->A) {
         return BTreeSearch(node -->left, key)
      }
      else if (node -->B is null || key < node -->B) {
         return BTreeSearch(node -->middle1, key)
      }
      else if (node -->C is null || key < node -->C) {
         return BTreeSearch(node -->middle2, key)
      }
      else {
         return BTreeSearch(node -->right, key)
      }
   }
   return null
}
```

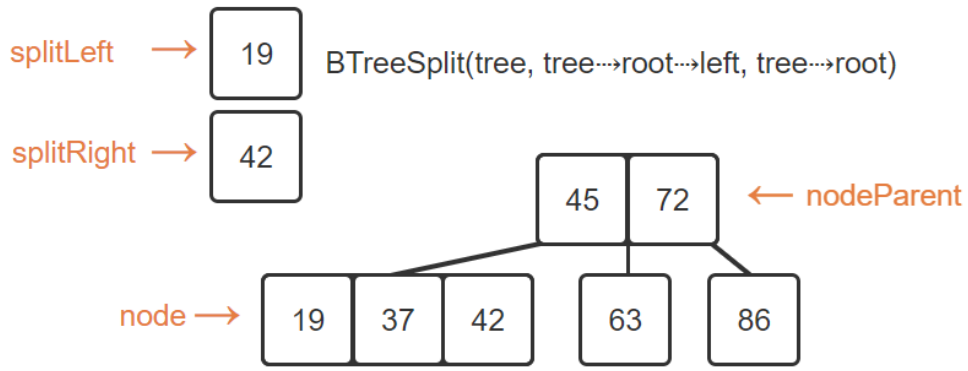


```
//BTreeSplit
BTreeSplit(tree, node, nodeParent){

    if (node is not null){
        return null;
    }

    splitLeft = new BTreeNode(node-->A, node-->left, node-->middle);
    splitRight = new BTreeNode(node-->C, node-->middle2, node-->right);

    if ( nodeParent is not null){
        BTreeInsertKeyWithChildren(nodeParent, node-->B, splitLeft, splitRight);

    }
    else {
        nodeParent = new BTreeNode(node-->B, splitLeft, splitRight);
        tree-->root = nodeParent;
    }
    return nodeParent;

}
```

45 72

37

```
BTreeSplit(tree, node, nodeParent) {
    if (node is not full) {
        return null
    }

    splitLeft = new BTreeNode(node⇢A, node⇢left, node⇢middle1)
    splitRight = new BTreeNode(node⇢C, node⇢middle2, node⇢right)
    if (nodeParent is not null) {
        BTreeInsertKeyWithChildren(nodeParent, node⇢B, splitLeft, splitRight)
    }
    else {
        nodeParent = new BTreeNode(node⇢B, splitLeft, splitRight)
        tree⇢root = nodeParent
    }
    return nodeParent
}
```



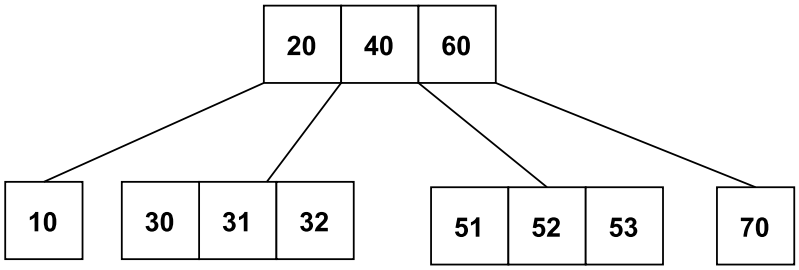

Since nodeParent is not null, the key 37 moves from node into nodeParent and the two newly allocated children are attached to nodeParent as well.

```
//BTreeInsertKeyWithChildren
BTreeInsertKeyWithChildren (parent, key, leftChild, rightChild){

    if( key < parent-->A){
        parent--> C = parent-->B;
        parent--> B = parent-->A;
        parent--> A = key;
        parent--> right = parent -->middle2;
        parent--> middle2 = parent-->middle1;
        parent--> middle1 = rightChild;
        parent--> left = leftChild;
    }
    else if (parent-->B is null || key < parent--> B){
        parent--> C = parent --> B;
        parent--> B = key;
        parent--> right = parent--> middle2;
        parent--> middle2 = rightChild;
        parent--> middle1 = leftChild;
    }
    else{
        parent--> C = key;
        parent--> right = rightChild;
        parent--> middle2 = leftChild;
    }
}
```

# Inserting a key into a leaf node

- A new key is always inserted into a non-full leaf node.
- The table below describes the 4 possible cases for inserting a new key into a non-full leaf node.

| Condition | Outcome |
|---|---|
| New key equals an existing key in node | No insertion takes place, and the node is not altered. |
| New key is < node's first key | Existing keys in node are shifted right, and the new key becomes node's first key. |
| Node has only 1 key or new key is < node's middle key | Node's middle key , if present, becomes last key, and new key becomes node's middle key. |
| None of the above | New key becomes node's last key. |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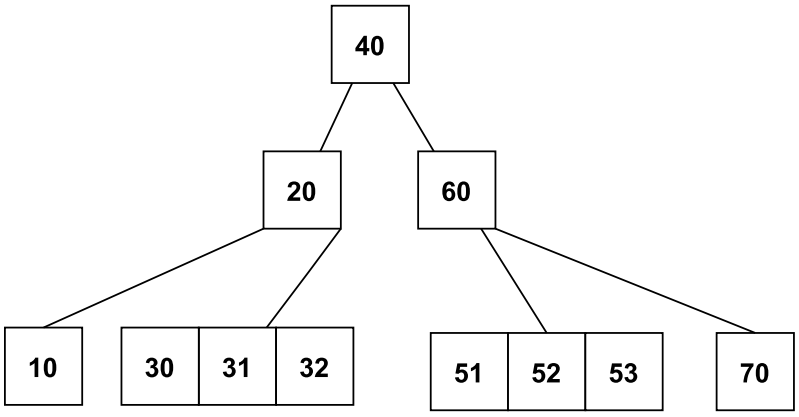
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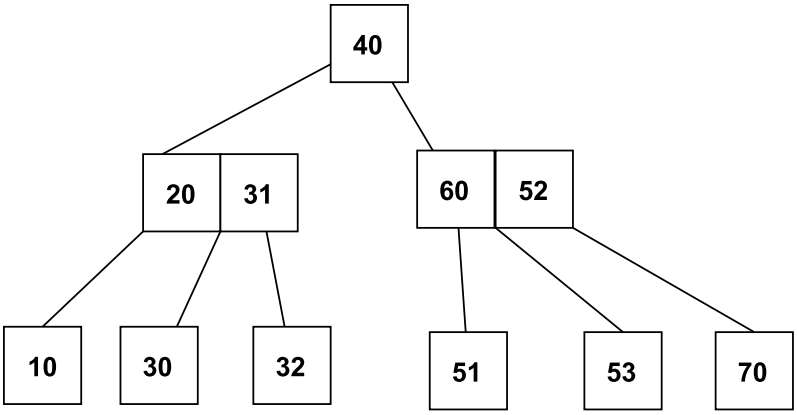
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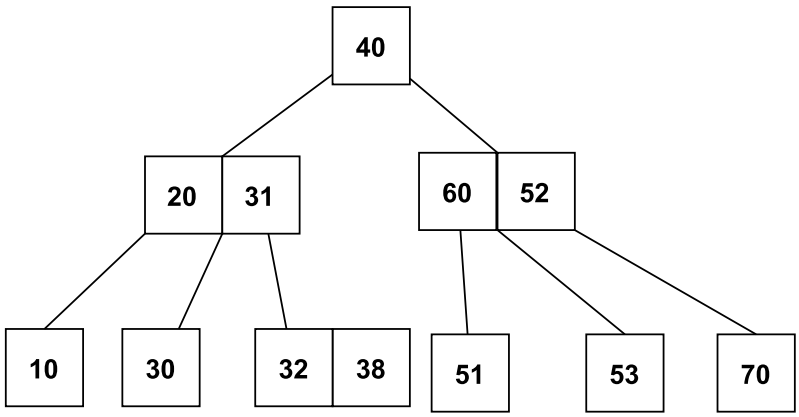
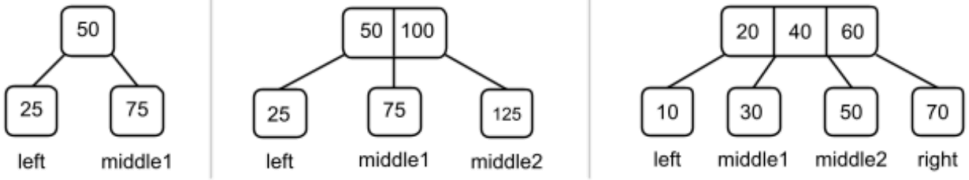
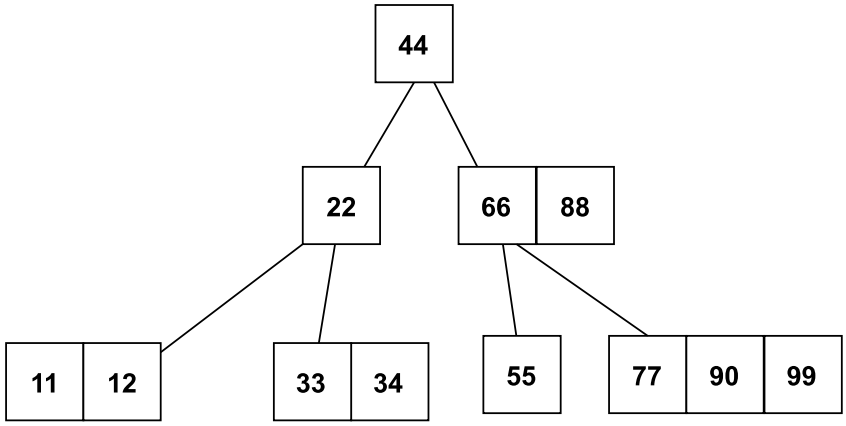
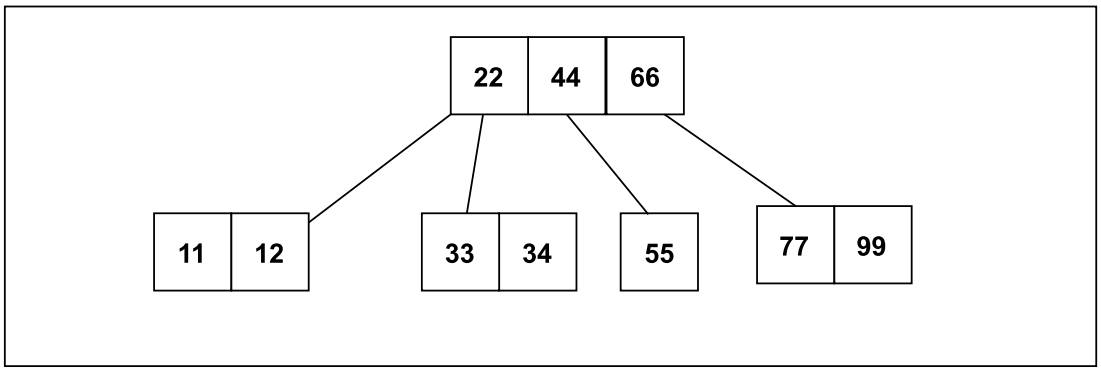


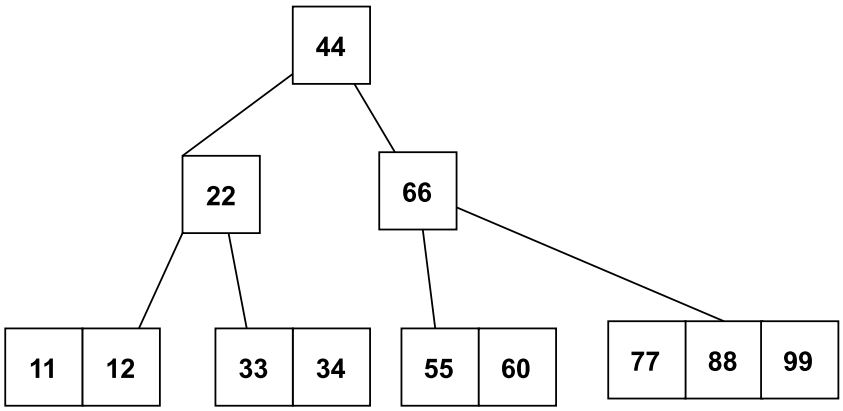

Figure 8.9.1: 2-3-4 child subtree labels.



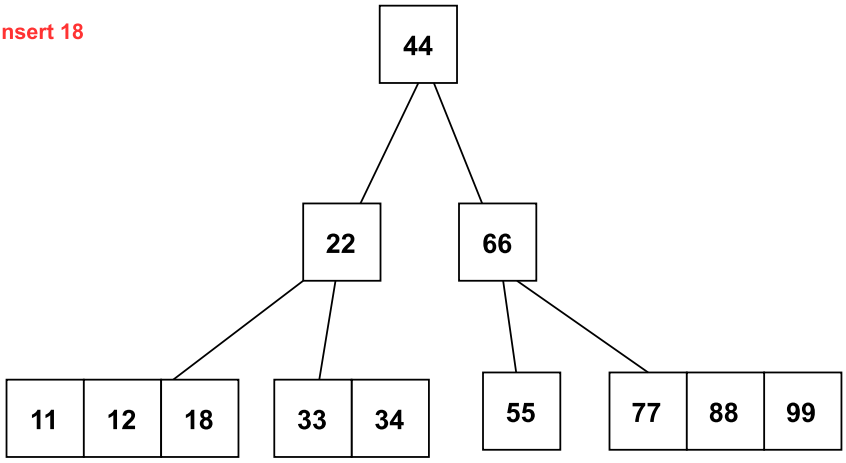


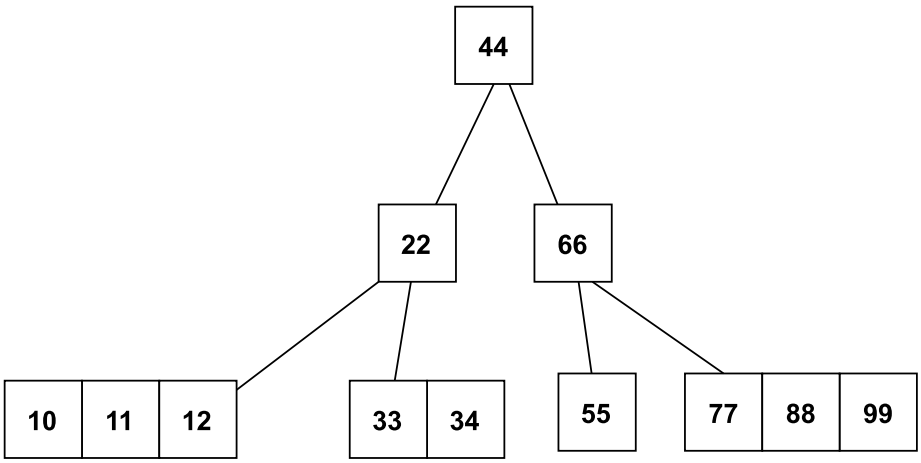

Insert 60



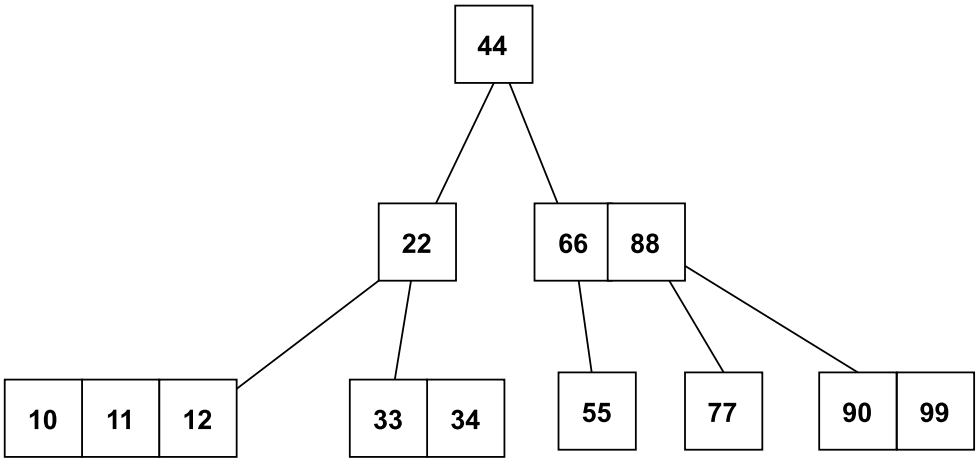

Insert 18



Insert 10




Insert 90




# Merge Algorithm

- A B-Tree merge operates on a node with 1 key and increases the node's keys to 2 or 3 using either a rotation or fusion
- A node's 2 adjacent siblings are checked first during a merge, and if either has 2 or more keys, a key is transferred via a rotation
- Such a rotation increases the number of keys in the merged node from 1 to 2.
- If all adjacent siblings of the node being merged have 1 key, then fusion is used to increase the number of keys in the node from 1 to 3.
- The merge operation can be performed on any node that has 1 key and a non-null parent node with at least 2 keys

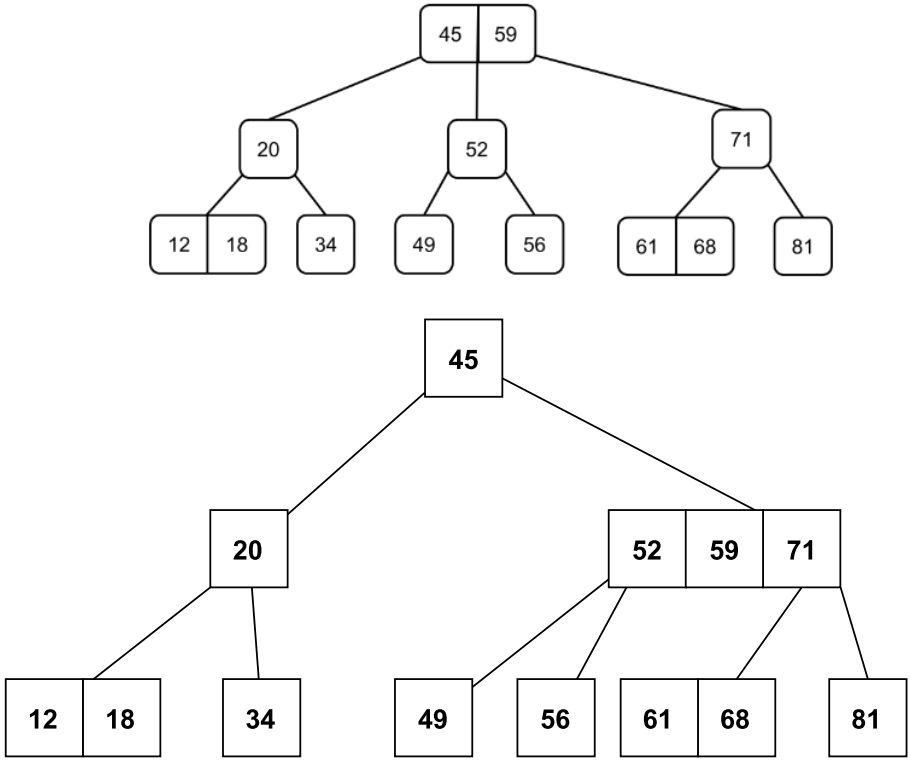
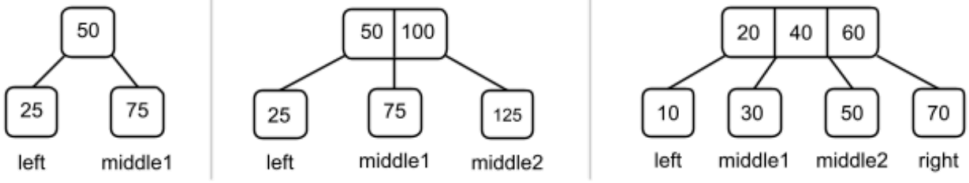
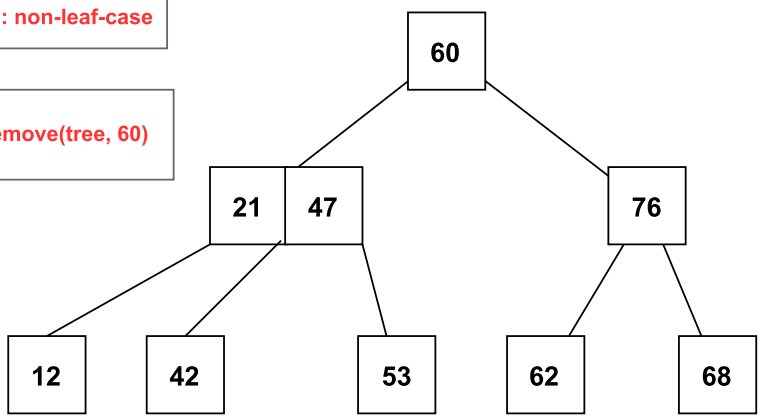



Figure 8.9.1: 2-3-4 child subtree labels.

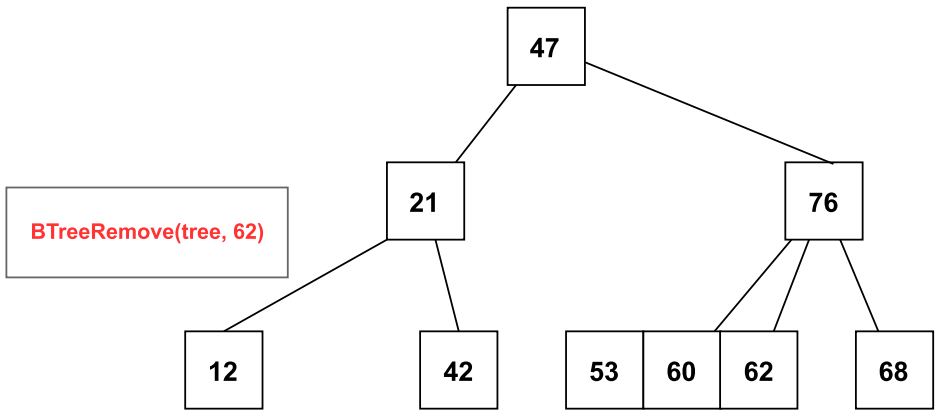



**8.13.5 BTree Remove Algorithm: non-leaf-case**

**BTreeRemove(tree, 60)**




- When deleting 60, the process is more complex due to the key being found in an internal node
- The key 62 is a suitable replacement for 60, but 62 must be recursively removed before the swap.
- After the recursive removal completes, 60 is replaced with 62


BTreeRemove(tree, 62)
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76
12
42
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60
62
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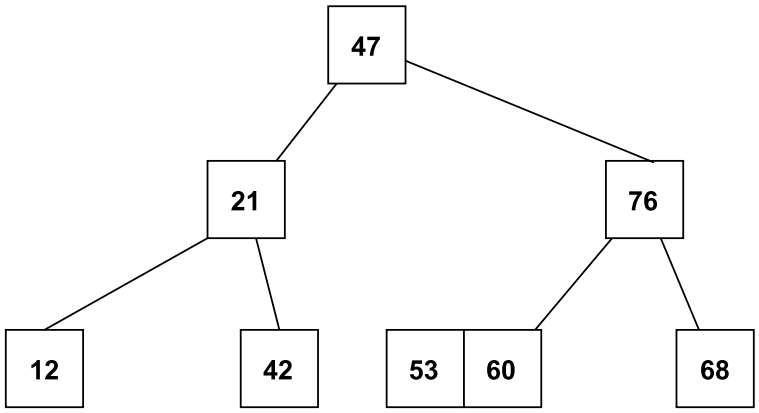
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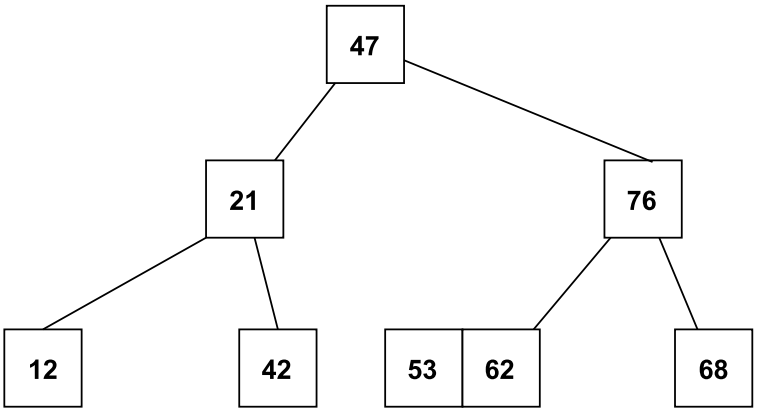
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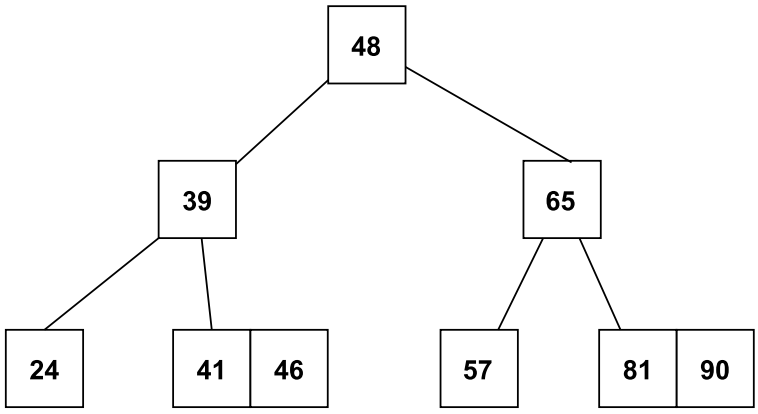
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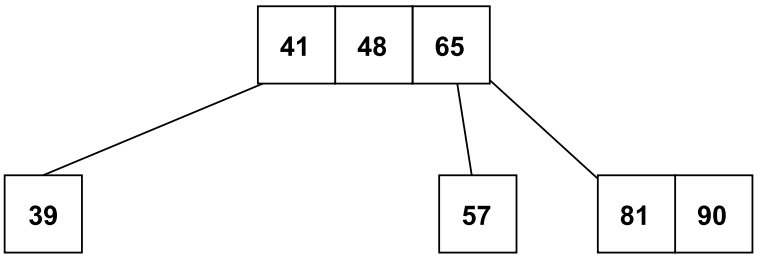
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