MTRN3020: Modelling & Control of Mechatronic Systems

# Inverted Pendulum Experiment


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I verify that the contents of this report are my own work


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# Contents

# 1 Introduction

Control systems are vitally important in the modern world due to the demand for intelligent systems that are stable and respond quickly to an input.

An inverted pendulum mounted on a cart will naturally come to rest in the downward position if no force is applied to the cart. In order for the pendulum to remain upright, the cart needs to constantly adjust its position in order to keep the position of the pendulum vertical. This is an almost impossible task to achieve manually. However, implementing a digital controller to shift the carts position is an effective way to overcome this manual limitation. The state of the system can be measured, with the carts position and the angular position of the pendulum. These states can be fed back into the controller to calculate the force required to move the cart to a new position which will maintain the pendulum in the upright position.

The design of a controller is of particular interest, as it can determine whether the system is unstable, responds quickly, has oscillations and can influence many other outcomes. An approach for optimising the controller is pole placement. The desired poles will help to determine the required gain of the controller, which can then be implemented.

This experiment is designed to see if a digital controller for the system described above can be designed and implemented such that the inverted pendulum can remain upright for the duration of the experiment.

# 2 Aim

The outcome of this experiment is to gain experience in designing a digital control system. This will be done by modelling an inverted pendulum mounted on a Quanser High Fidelity Linear Cart System and continually updating the carts position to ensure the pendulum remains upright.

In order to achieve this, the derivation of desired poles of the system and subsequent calculation of a suitable gain matrix will be learned. This controller will then be applied and the results recorded and analysed to examine the differences between simulated and experimental results.

# 3 Experiment

## 3.1 Procedure

The designed controller was connected to the Quanser High Fidelity Linear Cart System with a pendulum attached. The controller was started and run for 49 seconds. After the controller was started, the pendulum was moved to the upright position and released. The cart and controller were allowed to run for the remainder of the 49 seconds. The cart's position and the pendulum's angular position were recorded and stored in a MATLAB data structure for future processing. The experimental setup is replicated in *Figure 1.*

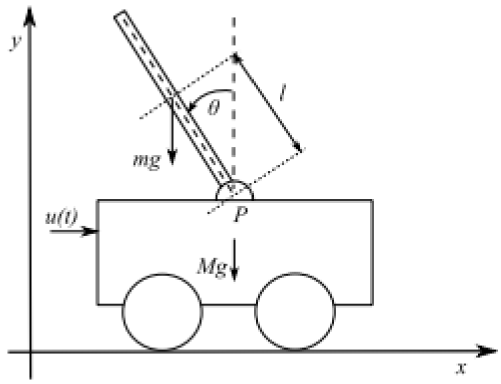


Figure 1: Diagram of experimental cart setup [1]

### 3.2 Dynamic Equations using Newton-Euler Method

In order to obtain a dynamic equation for the system shown in *Figure 1*, the Newton-Euler method will be employed.

First, the sum of the forces in the x and y directions will be added to equal the acceleration in their respective directions according to Newton's second law.

$$u - P_x = M\ddot{x} \tag{1}$$

$$F_N - P_y - Mg = 0 \tag{2}$$

Then Newton's second law can be used again for the pendulum, with the assumption that its total mass is concentrated at its centre of gravity.

$$P_x = m\ddot{x}_p \tag{3}$$

$$P_y - mg = m\ddot{y}_p \tag{4}$$

The acceleration of the pendulum's centre of mass can be calculated using its original position.

$$x_p = x - l\sin\theta$$

$$y_p = l\cos\theta$$

$$\dot{x}_p = \dot{x} - l\dot{\theta}\cos\theta$$

$$\dot{y}_p = -l\dot{\theta}\sin\theta$$

$$\ddot{x}_p = \ddot{x} + l\dot{\theta}^2\sin\theta - l\ddot{\theta}\cos\theta \tag{5}$$

$$\ddot{y}_p = -l\ddot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta \tag{6}$$

The equations for the acceleration of the pendulum's centre of mass, (5) and (6), can be substituted into Newton's second law for the pendulum's centre of mass, (3) and (4).

$$P_x = m(\ddot{x} + l\dot{\theta}^2\sin\theta - l\ddot{\theta}\cos\theta) \tag{7}$$

$$P_y - mg = m(-l\ddot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta) \tag{8}$$

Equation (7) can be directly substituted into (1).

$$u - m(\ddot{x} + l\dot{\theta}^2\sin\theta - l\ddot{\theta}\cos\theta) = M\ddot{x}$$

$$u = (M + m)\ddot{x} + ml\dot{\theta}^2 \sin\theta - ml\ddot{\theta}\cos\theta \tag{9}$$

If we consider the pendulum again, we can see that the reaction force P can be converted to its x and y components using the angle of the pendulum.

$$P_x = P\sin\theta \tag{10}$$

$$P_y = -P\cos\theta \tag{11}$$

Equations (7) and (8) can be rewritten and multiplied by cos and sin.

$$P\sin\theta = m(\ddot{x} + l\dot{\theta}^2\sin\theta - l\ddot{\theta}\cos\theta)$$

$$-P\cos\theta - mg = m(-l\ddot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta)$$

$$P\sin\theta\cos\theta = m\cos\theta(\ddot{x} + l\dot{\theta}^2\sin\theta - l\ddot{\theta}\cos\theta) \tag{12}$$

$$-P\cos\theta\sin\theta - mg\sin\theta = m\sin\theta(-l\ddot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta) \tag{13}$$

Equations (12) and (13) can then be added together and simplified.

$$P\sin\theta\cos\theta - P\cos\theta\sin\theta - mg\sin\theta = m\cos\theta(\ddot{x} + l\dot{\theta}^2\sin\theta - l\ddot{\theta}\cos\theta) + m\sin\theta(-l\ddot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta)$$

$$-mg\sin\theta = m\ddot{x}\cos\theta + ml\dot{\theta}^2\sin\theta\cos\theta - ml\dot{\theta}^2\sin\theta\cos\theta - ml\ddot{\theta}\cos\theta^2 - ml\ddot{\theta}\sin\theta^2$$

$$-g\sin\theta = \ddot{x}\cos\theta - l\ddot{\theta}(\cos\theta^2 + \sin\theta^2)$$

$$l\ddot{\theta} - g\sin\theta - \ddot{x}\cos\theta = 0 \tag{14}$$

Hence the final dynamic equations for this pendulum system are equations (9) and (14).

$$u = (M + m)\ddot{x} + ml\dot{\theta}^2 \sin\theta - ml\ddot{\theta}\cos\theta$$

$$l\ddot{\theta} - g\sin\theta - \ddot{x}\cos\theta = 0$$

## 3.3 MATLAB Script for Gain Vector

In order to determine the required gain vector, we need to calculate certain pole positions that produce the desired characteristics of the system. In this report, the percentage overshoot was required to be 5.50097542652545 and the settling time of the system as 1.118917967 seconds. From these parameters, (15), (16) and (17) were used to define key characteristics of the system.

$$\zeta = \frac{-\ln(OS\%)}{\sqrt{\Pi^2 + \ln(OS\%)^2}} \tag{15}$$

$$\omega_n = \frac{-\ln(0.02\sqrt{1-\zeta^2})}{\zeta T_s} \tag{16}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{17}$$

$$s^2 + 2\omega_n\zeta s + \omega_n^2 \tag{18}$$

From these and the characteristic equation of the system (18), the dominant poles can be worked out from (19).

$$p_{1,2} = -\zeta\omega_n \pm j\omega_d \tag{19}$$

The non dominant poles were required to be slower in response than the poles shown above as well as show no oscillations. The values for this were provided. Finally, the fifth pole influenced the integral gain of the position error. This was also provided and set close to the origin to remove oscillations

Now that all the poles were determined, the MATLAB place function [2] was used. The state space system model along with the desired poles are passed in to the function and the gain matrix is calculated so that the eigenvalues of $A$ - $KB$ are equivalent to the poles.

Please see *z5165456.m* attached to this submission.

# 4 Results

In the experiment, both the cart position and the pendulum angular position were measured and displayed in this report.

## 4.1 Cart Position

The captured measurements for the desired, simulated and experimental cart positions are shown in *Figure 2*. The experiment was run for a total of 49 seconds.

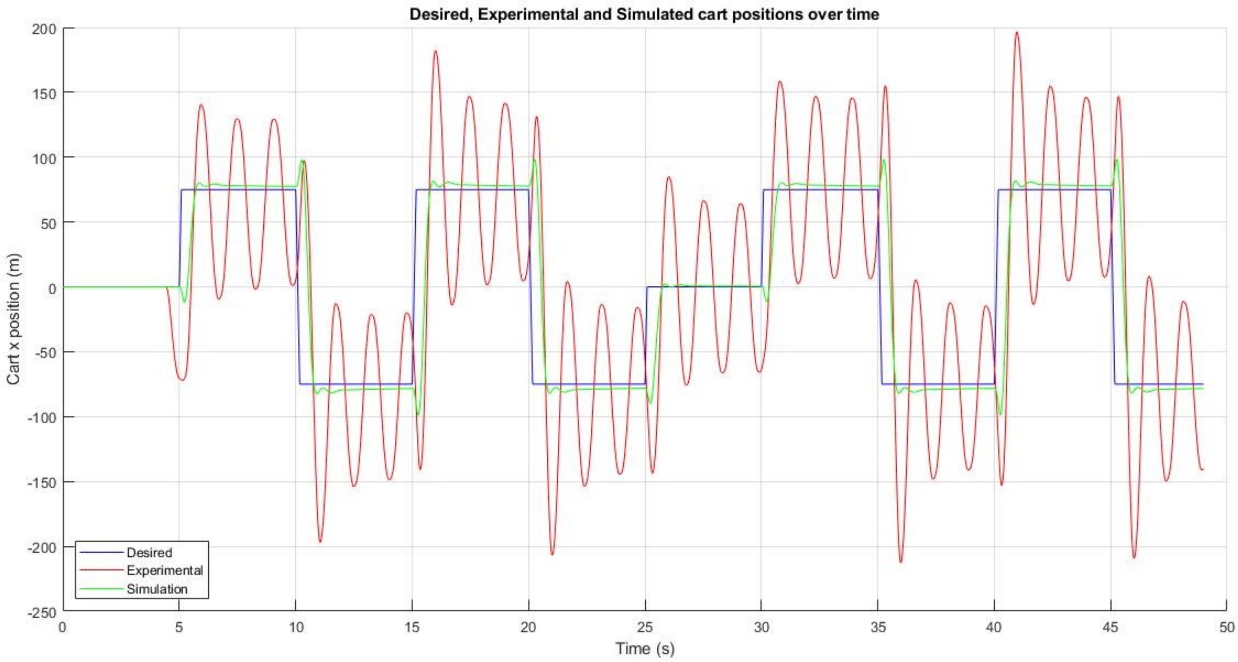


Figure 2: MATLAB graphical representation of desired (blue), experimental (red) and simulated (green) cart positions over time

## 4.2 Pendulum Position

The captured measurements for the simulated and experimental pendulum angular positions are shown in *Figure 3.* In the experimental results, the pendulum started in the downward -180 degree position and was moved to the upright position before t = 5 seconds.

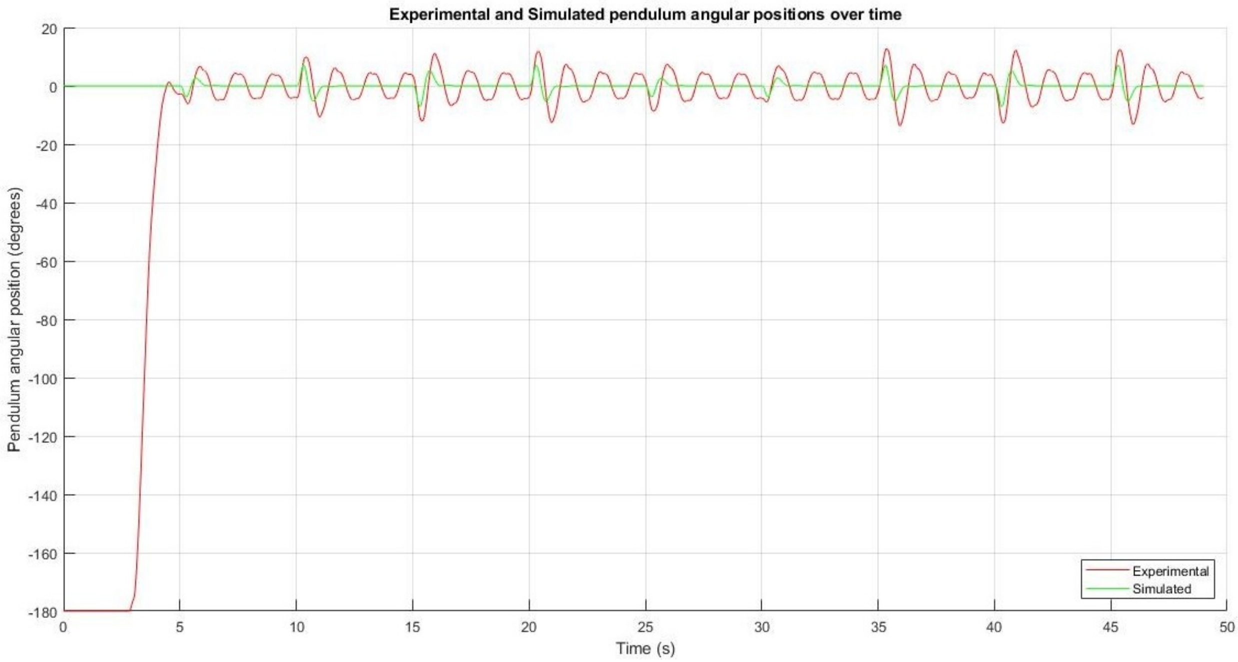


Figure 3: MATLAB graphical representation of experimental (red) and simulated (green) pendulum angular positions over time

# 5 Discussion

The experimental results displayed in *Figure 2* and *Figure 3* show that there is significant deviation of the experimental results from the simulated results. Both sets of experimental results show much more oscillatory behaviour compared to the simulated case.

In this experiment there are two main sources of error that are contributing to this difference between the experimental and simulation results.

In a physical system with movement, friction is always going to impact on the outcome of the system. In this inverted pendulum system, there are a number of frictional forces acting on the cart. Firstly, the cables which provide the measured data to the computer are connected directly to the cart and slide across the surface when moved, introducing some friction. Another source of friction is the cart sliding along the rails. These frictional forces resist the carts motion and hence the required force to move the cart is underestimated as the friction is not modelled into the control system model. This cause the cart to not move far enough to compensate for the pendulum angle and so in the experimental results the cart moves further than the simulated result.

Another source of error is the computational delay between the measurements received from the sensor and the transfer of these measurements to the computational control system through the aforementioned cables. This delay means that the real life system has changed state by the time that the model has received the now outdated measurements and so there is error in the output of the control system.

There are other minor sources of error int his system. For example, there would be a small amount of rounding error in the control systems calculations, there is some human error introduced while positioning the cart and pendulum and also there may be a small amount of air resistance for the cart moving.

# 6 References


[1] J. Katupitiya, “Modeling and control of an inverted pendulum on a cart,” 2021.

[2] Mathworks, “Place,” 2021. [Online]. Available: `https://au.mathworks.com/help/control/ref/place.html`.