MTRN3020: Modelling & Control of Mechatronic Systems

Position Control Experiment

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Faculty of Engineering

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I verify that the contents of this report are my own work

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1 Introduction

There are many physical systems that are required to respond in a predictable way to inputs provided by operators. For example, a shaft that needs to rotate to a certain position. In this case, simply passing the desired position to the system may produce undesirable behaviour such as oscillations, slow responses or even unstable systems. In order to remove this unpredictability, control systems can be designed with certain specifications in mind such as the desired time constant or steady state error. Thus, in the shaft example, if the shaft was to be directly moved to the set position with no oscillations about this set point, this could be achieved with a controller.

In order to achieve a controller that achieves the desired specifications, there are a multitude of methods. The Direct Analytical Method (Ragazzini's Method) and Root Locus Method are two direct methods for achieving the desired controller. They are called direct as they are performed purely in the z-domain with no conversion necessary between the s and z domains.

At the end of this experiment, a controller should be designed that can produce the desired output, in terms of time constant and steady state error, in response to inputs into the system it was designed around. In this case this will be an extendable arm attached to a motor shaft similar to the system described in the first paragraph. Furthermore, the designed controller should also be robust insofar that if the arm length of the experimental rig is adjusted, the system still settles at the desired position.

2 Aim

The purpose of this experiment is to first design a controller for a position control rig. This will involve using a Direct Analytical Method to obtain controller coefficients within the internal velocity loop and use of the Root Locus Method to obtain the gain value for the feedback loop. Then this controller will be implemented in a practical experimental rig to analyse the discrepancies between the experimental and simulated results.

3 Experimental Procedure

In this experiment, a vertically mounted motor is affixed to the ground at one end and attached to a gearbox at the other end. The gearbox is connected to a horizontal arm that rotates about the vertical axis. An encoder located above the gearbox is used to measure the rotations of the arm. There is another encoder to measure the motor shaft position which is located below the motor. The arms length can be adjusted in order to test the controllers robustness against different conditions. Figure 1 shows a schematic of the setup.

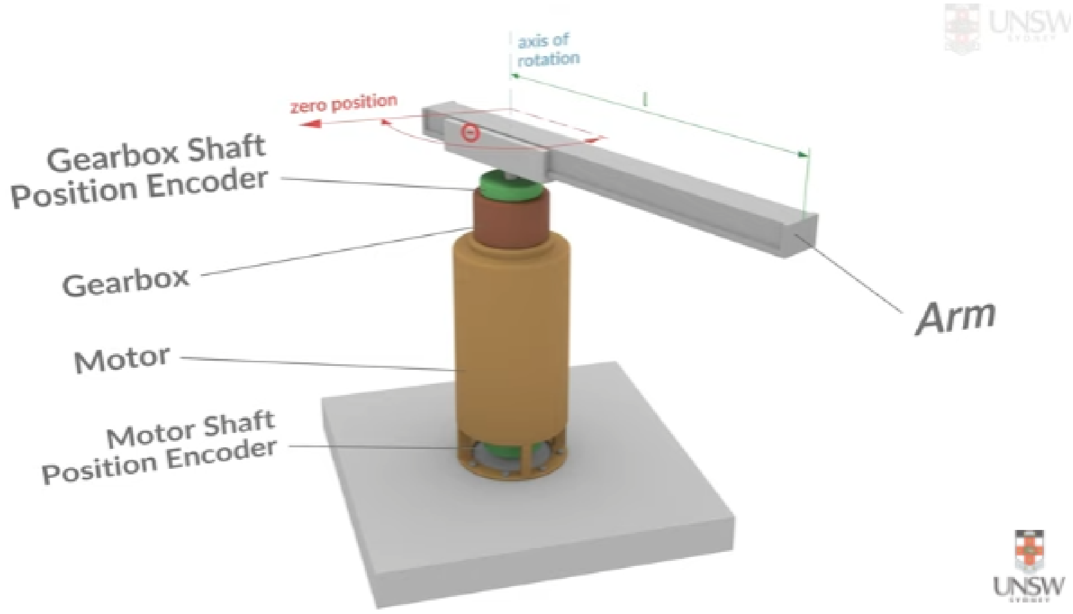

Figure 1: Diagram of experimental position control experiment [1]

A control computer is connected to the motor via an amplifier and can send drive signals to the motor which will in turn move the gears and arm.

The designed control algorithm will run on the control computer to control the signal being sent to the motor.

The feedback that is sent to the control computer is from the gearbox positioning, which represents the arms position.

4 Controller Design Calculations

The design of the controller using the Direct Analytical Method and the Root Locus Method will follow the steps outlined in [2].

In order to find the system transfer function, a curve if fitted to experimental data of the system. In Figure 2, the speed of the output shaft in response to a 24V (normalised to 1V) input is shown in blue while the curve fitted response is shown in red.

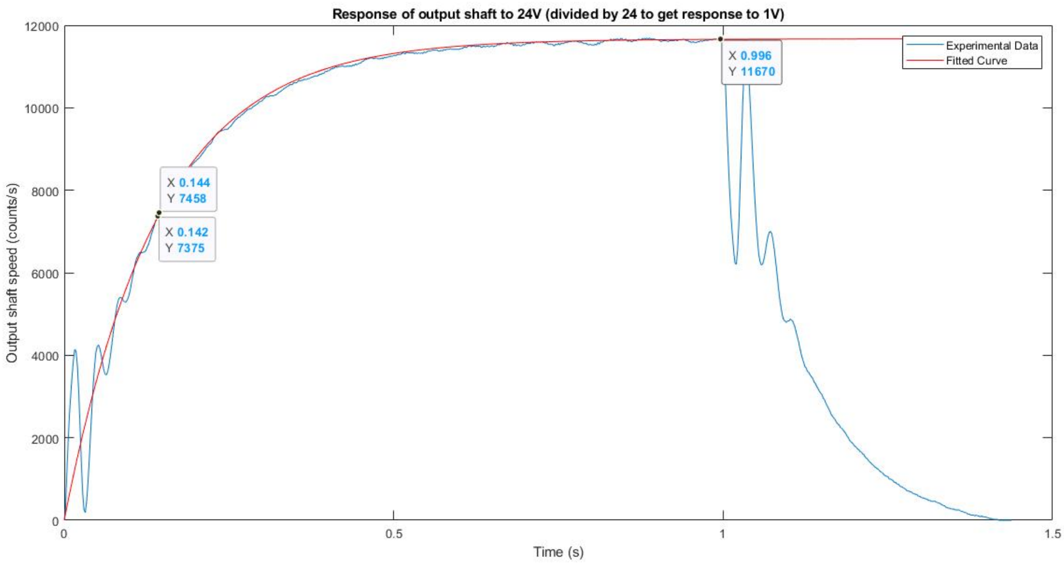

Figure 2: MATLAB plot of motor experimental (blue) and curve fitted (red) responses to a 24 volt input which has been normalised to a 1V input [3]

The equation used for the curve fitting is seen in Equation (1).

$$y = A(1 - e^{\frac{-t}{B}}) \tag{1}$$

The values of A and B are 11670 and 0.143 respectively, from Figure 2.

$$y = 11670(1 - e^{\frac{-t}{0.143}}) \tag{2}$$

This can be translated to a transfer function that relates the voltage to the shaft position as shown in Equation 3.

$$G_p(s) = \frac{11667}{s(1 + 0.143s)} \frac{2\pi}{8192} \tag{3}$$

$$G_p(s) = \frac{8.9482}{s(1 + 0.143s)} \tag{4}$$

To determine the discrete time (Z transform) form of this equation, the MATLAB function 'c2dm' can be used with the sampling time of 0.008 seconds. The result is shown in Equation 5.

$$G_p(z) = \frac{0.0019655702(z + 0.98153)}{(z - 1)(z - 0.94559)} \frac{(z - 1)}{0.008z} \tag{5}$$

$$G_p(z) = \frac{0.24570(z + 0.98153)}{z(z - 0.94559)} \tag{6}$$

It is given that the desired time constant is 0.041 seconds. From this we can calculate the desired pole in the s-plane.

$$s = \frac{-1}{0.041}$$

$$s = -24.39024$$

Now, the desired pole in the z domain can be calculated.

$$z_p = e^{sT}$$

$$z_p = 0.82273$$

Now the F(z) equation can be derived by considering the pole calculated above and the zero in the plant transfer function that needs to be absorbed in the controller to avoid ringing. Furthermore to satisfy the causality constraint, a delay must also be introduced to F(z). Thus, F(z) is shown in Equation 7.

$$F(z) = \frac{b_0(z + 0.98153)}{z(z - 0.82273)} \tag{7}$$

To determine the value of b_0, the zero steady state error can be applied. The final expression for F(z) can then be seen in Equation 10.

$$F(1) = 1 = \frac{b_0(z + 0.98153)}{z(z - 0.82273)} \tag{8}$$

$$b_0 = 0.08946 \tag{9}$$

$$F(z) = \frac{0.08946(z + 0.98153)}{z(z - 0.82273)} \tag{10}$$

Then the transfer function of the controller can be obtained using the following expression, arriving at Equation 12.

$$G_c(z) = \frac{1}{G_p(z)} \frac{F(z)}{1 - F(z)} \tag{11}$$

$$G_c(z) = \frac{0.36410z^2 - 0.34429z}{z^2 - 0.91219z - 0.08781} \tag{12}$$

Therefore the transfer function can be written as shown in Equation 13.

$$\frac{M(z)}{E(z)} = \frac{0.36410 - 0.34429z^{-1}}{1 - 0.91219z^{-1} - 0.08781z^{-2}} \tag{13}$$

This can be put into a difference equation form to obtain the coefficients of the controller which match very closely the provided coefficients for the same controller.

$$m(k) = 0.91219m(k-1) + 0.08781m(k-2) + 0.36410e(k) - 0.34429e(k-1) \tag{14}$$

The equation for both the controller and the plant can be written as follows in Equation 15 and 16.

$$G_c(z) = \frac{0.36410z(z - 0.94559)}{(z-1)(z + 0.08781)} \tag{15}$$

$$G_p(z) = \frac{0.24570(z + 0.98153)}{z(z - 0.94559)} \tag{16}$$

The combination of these two equations forms Equation 17.

$$G(z) = \frac{0.08946(z + 0.98153)}{(z-1)(z + 0.08781)} \tag{17}$$

The closed loop transfer function is then represented in Equation 18.

$$G(z) = \frac{0.08946(z + 0.98153)}{z(z - 0.822273)} \tag{18}$$

The combination of the integrator, the gear ratio and gain function then produces Equation 19.

$$G(z) = \frac{0.00003728K(z + 0.9815)}{(z - 1)(z - 0.82273)} \tag{19}$$

The root locus can be plotted as shown in Figure 3.

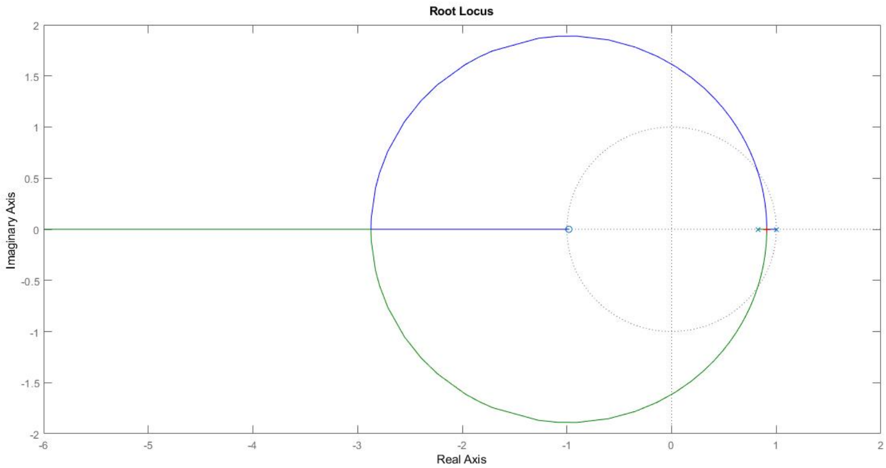

Figure 3: MATLAB rlocus plot with desired position highlighted by a red cross [3]

Using MATLAB's rlocfind function, it is determined that the desired gain value corresponds to a K of 111.39. Hence the final open loop transfer function is shown in Equation 20.

$$G(z) = \frac{0.00415262(z + 0.9815)}{(z - 1)(z - 0.82273)} \tag{20}$$

5 Simulink Block Diagram

The block diagram shown in Figure 4 is a simplified model for the system described in the Experimental Procedure.

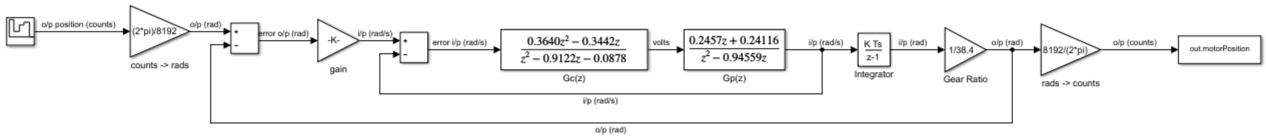

Figure 4: MATLAB Simulink Block Diagram of Position Controller Experiment [3]

The desired output shaft position is provided to the model in counts and is converted into radians. The error between the shafts desired position and the measured output is calculated and multiplied by a gain that was calculated in the above section to provide the input shaft velocity in radians per second. The error in this value is also calculated via a feedback loop, which is then passed into the controller to determine the voltage needed by the motor to achieve this state. This voltage is passed into the system and the output position of the input shaft in radians is differentiated to obtain the input shaft velocity in radians per second for the feedback loop (the differentiation is not explicitly shown in the diagram as it is already include within the plant). This value is also integrated again to obtain the input shaft position in radians. A gear ratio of 38.4 is used between the input and output shaft and so to obtain the final measured position of the output shaft the input shaft value needs to be divided by 38.4. This value can then be fed back and examined to view the response of the system.

6 Part A

In this part, the experimental results using an arm length of 445mm were compared to the simulated results with the controller and plant determined using the ROT445 data. The comparison is shown in 5.

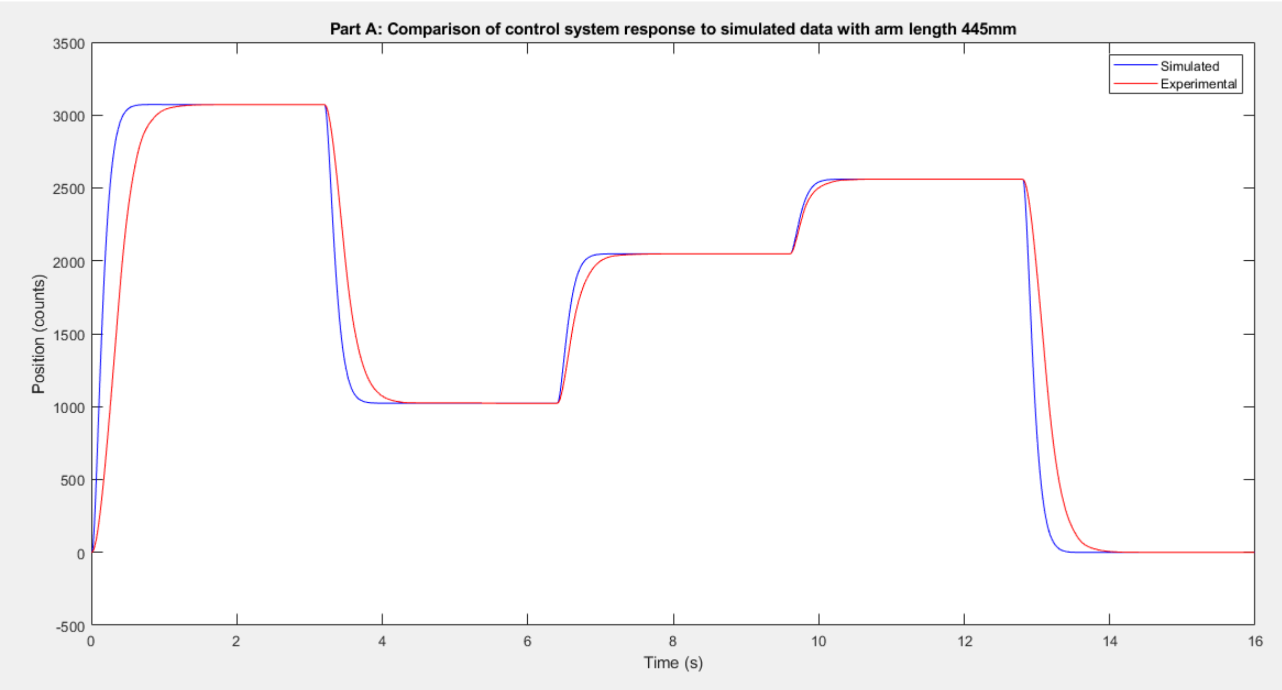

Figure 5: MATLAB plot of temporal experimental(red) and simulated (blue) output shaft position speed with designed controller and arm length of 445mm

It is apparent that there are some significant differences between the experimental and simulated data, particularly with regards to the speed of the response. The steady state error is virtually nonexistent, so the controller performs well for this specification. However, the speed of the simulated response is much quicker than that of the experimental data. There are a number of sources where this error could be coming from.

One source of error is the rounding that takes place during the calculation of the plant, controller and gain values. In most cases in this report, the values obtained were rounded to 4 decimal places. This means that there exists some difference between the values input into the system model and the physical system which could cause the controller to apply less voltage than actually required, thus slowing down the speed of the response as shown in Figure 5.

Another significant source of error in this experiment would be the backlash in the extendable arm. Backlash occurs when two gears are not exactly meshed together which allows some movement in the gears. In this case, the backlash has not noticeable affected the steady state output, but could have possibly contributed to the speed of the response discrepancy. As the voltage from the controller is applied, the play between the gears needs to be removed first which does not actually move the arm. Hence, the movement in the arm would be slightly delayed by this which is reflected in Figure 5.

There is also some computational delay between the control computer and the physical system. This should mean that the measurements from the physical system are slightly outdated when they reach the control computer and therefore the output of the control computer is outdated as well. There exists also delay between the output of the control computer back to the physical system which exacerbates this error. Hence the experimental data may show somewhat of a delay relative to the simulated data, as shown in Figure 5.

Then there are the errors in the encoder measurements of the shaft. These measurements are fed into the control computer but cannot be assumed to be completely accurate. In this case, the encoder may be overestimating the position of the arm and hence the voltage applied to the motor is underestimated producing the delayed experimental response shown in 5.

These reasons could all contribute to the fact that the experimental response is slower than the simulated response. However, overall, the controller performs well in terms of accuracy in particular.

7 Part B

In this part, the experimental data was changed to using an an arm length of 335mm instead of 445mm on the experimental rig. The plants transfer function was therefore changed to Equation 21.

$$G(z) = \frac{0.3375(z + 0.9630)}{z(z - 0.9253)} \tag{21}$$

The comparison is shown in Figure 6.

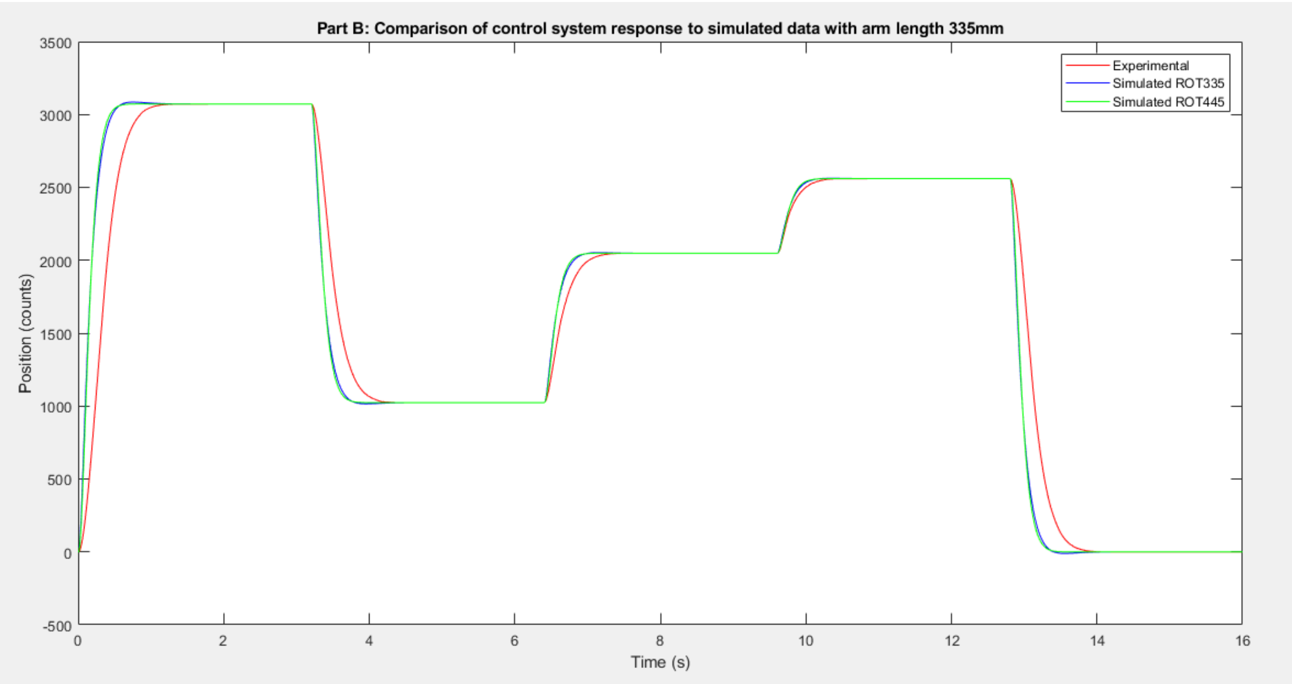

Figure 6: MATLAB plot of temporal experimental (red), simulated 335mm arm length (blue) and simulated 445mm arm length (green) output shaft position speed with designed controller

It is evident from this plot that the designed controller still performs well even with the change in plant. In particular, the steady state error is very minimal for the experimental data. However, as in the previous part, the speed of response is still quite different between the experimental and simulated responses. The reasons for this can be explained through the reasons mentioned in the previous section, primarily rounding errors, computational delay, backlash and measurement errors.

In the simulated data, there is some overshoot of the response relative to the response under the 'correct' plant transfer function. This is expected as, in the previous section, our controller was designed for that particular plant. Now that the plant has been adjusted to account for the change in arm length, the plant transfer function has been changed due to the weight redistribution and other physical properties. The controller has remained the same, and so is therefore not optimised for this new plant transfer function. Hence, some undesirable characteristics such as the marginal overshoot have been introduced. This difference in systems is made more obvious when comparing the experimental datasets from each section. The response of the system using 335mm arm length plant responds slightly quicker. The arm for ROT335 was shorter and therefore required less force to move, hence the system could respond quicker.

Overall, the steady state error for this changed system is very minimal and so the controller is quite robust considering the circumstances. However, its is evident that a change in the plant could introduce some changes to the system response and therefore this state should be carefully monitored if any changes to the plant occur.

8 Conclusion

This report showed the practical application of controller design using Direct Analytical methods. However, it also displayed that discrepancies are introduced between practical and simulated systems due to calculation errors and physical restrictions such as backlash. It was also shown that a designed controller can be robust when a minor change is introduced into the physical system, for example with the change in arm length. However, this case needs to be closely monitored as undesirable response characteristics, such as oscillations and overshoot may be introduced with this change.

An example of a system that might need to employ this controller design concept are solar panels. Often it is beneficial to redirect solar panels directly towards the sun to optimise sunlight capture and power output. Whether this is done manually or is reprogrammed, the movement of the solar panel should be predictable and manageable. Hence the direct analytical methods such as the ones outlined in this report could be used to ensure the desired response of the system.

9 References

[1] (2019). "Mtrn3020 - introduction to the position control experiment," UNSW, [Online]. Available: https://www.youtube.com/watch?v=KWMdLsWpd44.

[2] J. Katupitiya, "Lab iii : Position controller design guide," 2021.

[3] Mathworks, "Matlab," 2021. [Online]. Available: https://au.mathworks.com/.