

Step	Algorithm:
1a	
4	where
2	
3	while do
2,3	$\wedge$
5a	where
6	
8	
5b	
7	
2	
	endwhile
2,3	$\wedge \neg( \quad )$
1b	

Step	<b>Algorithm:</b> $[C] := \text{SYRK2\_UNB\_VAR1}(A, B, C)$
1a	$C = \widehat{C}$
4	$A \rightarrow \begin{pmatrix} A_T \\ A_B \end{pmatrix}, B \rightarrow \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \rightarrow \begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix}$ <b>where</b> $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$
2	$\begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \widehat{C}_{BL} & \widehat{C}_{BR} \end{pmatrix}$
3	<b>while</b> $m(A_T) < m(A)$ <b>do</b>
2,3	$\begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \widehat{C}_{BL} & \widehat{C}_{BR} \end{pmatrix} \wedge m(A_T) < m(A)$
5a	$\begin{pmatrix} A_T \\ A_B \end{pmatrix} \rightarrow \begin{pmatrix} A_0 \\ a_1^T \\ A_2 \end{pmatrix}, \begin{pmatrix} B_T \\ B_B \end{pmatrix} \rightarrow \begin{pmatrix} B_0 \\ b_1^T \\ B_2 \end{pmatrix}, \begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix}$ <b>where</b> $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$
6	$\begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & \widehat{c}_{01} & \widehat{C}_{02} \\ \widehat{c}_{10}^T & \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{pmatrix}$
8	$c_{01} = A_0 \beta_1 + B_0 \alpha_1 + c_{01}$ $c_{10}^T = c_{01}^T$ $\gamma_{11} = \alpha_1^T \beta_1 + \beta_1^T \alpha_1 + \gamma_{11}$
5b	$\begin{pmatrix} A_T \\ A_B \end{pmatrix} \leftarrow \begin{pmatrix} A_0 \\ a_1^T \\ A_2 \end{pmatrix}, \begin{pmatrix} B_T \\ B_B \end{pmatrix} \leftarrow \begin{pmatrix} B_0 \\ b_1^T \\ B_2 \end{pmatrix}, \begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix}$
7	$\begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & A_0 \beta_1 + B_0 \alpha_1 + \widehat{c}_{01} & \widehat{C}_{02} \\ \alpha_1^T B_0^T + \beta_1^T A_0^T + \widehat{c}_{10}^T & \alpha_1^T \beta_1 + \beta_1^T \alpha_1 + \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{pmatrix}$
2	$\begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \widehat{C}_{BL} & \widehat{C}_{BR} \end{pmatrix}$
	<b>endwhile</b>
2,3	$\begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \widehat{C}_{BL} & \widehat{C}_{BR} \end{pmatrix} \wedge \neg(m(A_T) < m(A))$
1b	$[C] = \text{syrk2}(A, B, \widehat{C})$

**Algorithm:**  $[C] := \text{SYRK2\_UNB\_VAR1}(A, B, C)$

$$A \rightarrow \begin{pmatrix} A_T \\ \hline A_B \end{pmatrix}, B \rightarrow \begin{pmatrix} B_T \\ \hline B_B \end{pmatrix}, C \rightarrow \begin{pmatrix} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{pmatrix}$$

**where**  $A_T$  has 0 rows,  $B_T$  has 0 rows,  $C_{TL}$  is  $0 \times 0$

**while**  $m(A_T) < m(A)$  **do**

$$\begin{pmatrix} A_T \\ \hline A_B \end{pmatrix} \rightarrow \begin{pmatrix} A_0 \\ \hline a_1^T \\ \hline A_2 \end{pmatrix}, \begin{pmatrix} B_T \\ \hline B_B \end{pmatrix} \rightarrow \begin{pmatrix} B_0 \\ \hline b_1^T \\ \hline B_2 \end{pmatrix}, \begin{pmatrix} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{pmatrix}$$

**where**  $a_1$  has 1 row,  $b_1$  has 1 row,  $\gamma_{11}$  is  $1 \times 1$

$$c_{01} = A_0 \beta_1 + B_0 \alpha_1 + c_{01} c_{10}^T = c_{01}^T \gamma_{11} = \alpha_1^T \beta_1 + \beta_1^T \alpha_1 + \gamma_{11}$$

$$\begin{pmatrix} A_T \\ \hline A_B \end{pmatrix} \leftarrow \begin{pmatrix} A_0 \\ \hline a_1^T \\ \hline A_2 \end{pmatrix}, \begin{pmatrix} B_T \\ \hline B_B \end{pmatrix} \leftarrow \begin{pmatrix} B_0 \\ \hline b_1^T \\ \hline B_2 \end{pmatrix}, \begin{pmatrix} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{pmatrix}$$

**endwhile**

Step	Algorithm: $[C] := \text{SYRK2\_UNB\_VAR1}(A, B, C)$
1a	$C = \widehat{C}$
4	where
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3	while do
2,3	$\wedge$
5a	where
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	endwhile
2,3	$\wedge \neg( \quad )$
1b	$[C] = \text{syrk2}(A, B, \widehat{C})$

Step	Algorithm: $[C] := \text{SYRK2\_UNB\_VAR1}(A, B, C)$
1a	$C = \hat{C}$
4	
	where
2	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{C}_{TL} + A_T B_T^T + B_T A_T^T & \hat{C}_{TR} \\ \hline \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right)$
3	while do
2,3	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{C}_{TL} + A_T B_T^T + B_T A_T^T & \hat{C}_{TR} \\ \hline \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge$
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	where
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2	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{C}_{TL} + A_T B_T^T + B_T A_T^T & \hat{C}_{TR} \\ \hline \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right)$
	endwhile
2	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{C}_{TL} + A_T B_T^T + B_T A_T^T & \hat{C}_{TR} \\ \hline \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge \neg( \quad )$
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4	where
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3	while $m(A_T) < m(A)$ do
2,3	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{C}_{TL} + A_T B_T^T + B_T A_T^T & \hat{C}_{TR} \\ \hline \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge m(A_T) < m(A)$
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2	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{C}_{TL} + A_T B_T^T + B_T A_T^T & \hat{C}_{TR} \\ \hline \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right)$
	endwhile
2,3	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{C}_{TL} + A_T B_T^T + B_T A_T^T & \hat{C}_{TR} \\ \hline \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge \neg(m(A_T) < m(A))$
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Step	<b>Algorithm:</b> $[C] := \text{SYRK2\_UNB\_VAR1}(A, B, C)$
1a	$C = \hat{C}$
4	$A \rightarrow \begin{pmatrix} A_T \\ A_B \end{pmatrix}, B \rightarrow \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \rightarrow \begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix}$ <b>where</b> $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$
2	$\begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} = \begin{pmatrix} \hat{C}_{TL} + A_T B_T^T + B_T A_T^T & \hat{C}_{TR} \\ \hat{C}_{BL} & \hat{C}_{BR} \end{pmatrix}$
3	<b>while</b> $m(A_T) < m(A)$ <b>do</b>
2,3	$\begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} = \begin{pmatrix} \hat{C}_{TL} + A_T B_T^T + B_T A_T^T & \hat{C}_{TR} \\ \hat{C}_{BL} & \hat{C}_{BR} \end{pmatrix} \wedge m(A_T) < m(A)$
5a	<b>where</b>
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5b	
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2	$\begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} = \begin{pmatrix} \hat{C}_{TL} + A_T B_T^T + B_T A_T^T & \hat{C}_{TR} \\ \hat{C}_{BL} & \hat{C}_{BR} \end{pmatrix}$
	<b>endwhile</b>
2,3	$\begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} = \begin{pmatrix} \hat{C}_{TL} + A_T B_T^T + B_T A_T^T & \hat{C}_{TR} \\ \hat{C}_{BL} & \hat{C}_{BR} \end{pmatrix} \wedge \neg(m(A_T) < m(A))$
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Step	Algorithm: $[C] := \text{SYRK2\_UNB\_VAR1}(A, B, C)$
1a	$C = \widehat{C}$
4	$A \rightarrow \left( \begin{array}{c c} A_T \\ \hline A_B \end{array} \right), B \rightarrow \left( \begin{array}{c c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$ where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$
2	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$
3	while $m(A_T) < m(A)$ do
2,3	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(A_T) < m(A)$
5a	$\left( \begin{array}{c c} A_T \\ \hline A_B \end{array} \right) \rightarrow \left( \begin{array}{c c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array} \right), \left( \begin{array}{c c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right)$ where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$
6	
8	
5b	$\left( \begin{array}{c c} A_T \\ \hline A_B \end{array} \right) \leftarrow \left( \begin{array}{c c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array} \right), \left( \begin{array}{c c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right)$
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2	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$
	endwhile
2,3	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \neg(m(A_T) < m(A))$
1b	$[C] = \text{syrk2}(A, B, \widehat{C})$



Step	Algorithm: $[C] := \text{SYRK2\_UNB\_VAR1}(A, B, C)$
1a	$C = \widehat{C}$
4	$A \rightarrow \left( \begin{array}{c c} A_T \\ \hline A_B \end{array} \right), B \rightarrow \left( \begin{array}{c c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$ where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$
2	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$
3	while $m(A_T) < m(A)$ do
2,3	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(A_T) < m(A)$
5a	$\left( \begin{array}{c c} A_T \\ \hline A_B \end{array} \right) \rightarrow \left( \begin{array}{c c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array} \right), \left( \begin{array}{c c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right)$ where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$
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5b	$\left( \begin{array}{c c} A_T \\ \hline A_B \end{array} \right) \leftarrow \left( \begin{array}{c c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array} \right), \left( \begin{array}{c c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right)$
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2	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$
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2,3	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \neg(m(A_T) < m(A))$
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3	while $m(A_T) < m(A)$ do
2,3	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(A_T) < m(A)$
5a	$\left( \begin{array}{c} A_T \\ \hline A_B \end{array} \right) \rightarrow \left( \begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right)$ where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$
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8	
5b	$\left( \begin{array}{c} A_T \\ \hline A_B \end{array} \right) \leftarrow \left( \begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right)$
7	$\left( \begin{array}{c c c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right) = \left( \begin{array}{c c c} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & A_0 \beta_1 + B_0 \alpha_1 + \widehat{c}_{01} & \widehat{C}_{02} \\ \hline \alpha_1^T B_0^T + \beta_1^T A_0^T + \widehat{c}_{10}^T & \alpha_1^T \beta_1 + \beta_1^T \alpha_1 + \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \hline \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{array} \right)$
2	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$
	endwhile
2	$\left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \neg(m(A_T) < m(A))$
1b	$[C] = \text{syrk2}(A, B, \widehat{C})$

Step	<b>Algorithm:</b> $[C] := \text{SYRK2\_UNB\_VAR1}(A, B, C)$
1a	$C = \widehat{C}$
4	$A \rightarrow \begin{pmatrix} A_T \\ A_B \end{pmatrix}, B \rightarrow \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \rightarrow \begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix}$ <b>where</b> $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$
2	$\begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \widehat{C}_{BL} & \widehat{C}_{BR} \end{pmatrix}$
3	<b>while</b> $m(A_T) < m(A)$ <b>do</b>
2,3	$\begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \widehat{C}_{BL} & \widehat{C}_{BR} \end{pmatrix} \wedge m(A_T) < m(A)$
5a	$\begin{pmatrix} A_T \\ A_B \end{pmatrix} \rightarrow \begin{pmatrix} A_0 \\ a_1^T \\ A_2 \end{pmatrix}, \begin{pmatrix} B_T \\ B_B \end{pmatrix} \rightarrow \begin{pmatrix} B_0 \\ b_1^T \\ B_2 \end{pmatrix}, \begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix}$ <b>where</b> $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$
6	$\begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & \widehat{c}_{01} & \widehat{C}_{02} \\ \widehat{c}_{10}^T & \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{pmatrix}$
8	$c_{01} = A_0 \beta_1 + B_0 \alpha_1 + c_{01}$ $c_{10}^T = c_{01}^T$ $\gamma_{11} = \alpha_1^T \beta_1 + \beta_1^T \alpha_1 + \gamma_{11}$
5b	$\begin{pmatrix} A_T \\ A_B \end{pmatrix} \leftarrow \begin{pmatrix} A_0 \\ a_1^T \\ A_2 \end{pmatrix}, \begin{pmatrix} B_T \\ B_B \end{pmatrix} \leftarrow \begin{pmatrix} B_0 \\ b_1^T \\ B_2 \end{pmatrix}, \begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix}$
7	$\begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & A_0 \beta_1 + B_0 \alpha_1 + \widehat{c}_{01} & \widehat{C}_{02} \\ \alpha_1^T B_0^T + \beta_1^T A_0^T + \widehat{c}_{10}^T & \alpha_1^T \beta_1 + \beta_1^T \alpha_1 + \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{pmatrix}$
2	$\begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \widehat{C}_{BL} & \widehat{C}_{BR} \end{pmatrix}$
	<b>endwhile</b>
2,3	$\begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \widehat{C}_{BL} & \widehat{C}_{BR} \end{pmatrix} \wedge \neg(m(A_T) < m(A))$
1b	$[C] = \text{syrk2}(A, B, \widehat{C})$

Step	<b>Algorithm:</b> $[C] := \text{SYRK2\_UNB\_VAR1}(A, B, C)$
	$A \rightarrow \left( \begin{array}{c} A_T \\ A_B \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ B_B \end{array} \right), C \rightarrow \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$ <p>where <math>A_T</math> has 0 rows, <math>B_T</math> has 0 rows, <math>C_{TL}</math> is <math>0 \times 0</math></p>
	while $m(A_T) < m(A)$ do
	$\left( \begin{array}{c} A_T \\ A_B \end{array} \right) \rightarrow \left( \begin{array}{c} A_0 \\ \frac{a_1^T}{A_2} \end{array} \right), \left( \begin{array}{c} B_T \\ B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \frac{b_1^T}{B_2} \end{array} \right), \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right)$ <p>where <math>a_1</math> has 1 row, <math>b_1</math> has 1 row, <math>\gamma_{11}</math> is <math>1 \times 1</math></p>
	$\begin{aligned} c_{01} &= A_0 \beta_1 + B_0 \alpha_1 + c_{01} \\ c_{10}^T &= c_{01}^T \\ \gamma_{11} &= \alpha_1^T \beta_1 + \beta_1^T \alpha_1 + \gamma_{11} \end{aligned}$
	$\left( \begin{array}{c} A_T \\ A_B \end{array} \right) \leftarrow \left( \begin{array}{c} A_0 \\ \frac{a_1^T}{A_2} \end{array} \right), \left( \begin{array}{c} B_T \\ B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \frac{b_1^T}{B_2} \end{array} \right), \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right)$
	endwhile

**Algorithm:**  $[C] := \text{SYRK2\_UNB\_VAR1}(A, B, C)$

$$A \rightarrow \begin{pmatrix} A_T \\ \hline A_B \end{pmatrix}, B \rightarrow \begin{pmatrix} B_T \\ \hline B_B \end{pmatrix}, C \rightarrow \begin{pmatrix} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{pmatrix}$$

**where**  $A_T$  has 0 rows,  $B_T$  has 0 rows,  $C_{TL}$  is  $0 \times 0$

**while**  $m(A_T) < m(A)$  **do**

$$\begin{pmatrix} A_T \\ \hline A_B \end{pmatrix} \rightarrow \begin{pmatrix} A_0 \\ \hline a_1^T \\ \hline A_2 \end{pmatrix}, \begin{pmatrix} B_T \\ \hline B_B \end{pmatrix} \rightarrow \begin{pmatrix} B_0 \\ \hline b_1^T \\ \hline B_2 \end{pmatrix}, \begin{pmatrix} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{pmatrix}$$

**where**  $a_1$  has 1 row,  $b_1$  has 1 row,  $\gamma_{11}$  is  $1 \times 1$

$$c_{01} = A_0 \beta_1 + B_0 \alpha_1 + c_{01} c_{10}^T = c_{01}^T \gamma_{11} = \alpha_1^T \beta_1 + \beta_1^T \alpha_1 + \gamma_{11}$$

$$\begin{pmatrix} A_T \\ \hline A_B \end{pmatrix} \leftarrow \begin{pmatrix} A_0 \\ \hline a_1^T \\ \hline A_2 \end{pmatrix}, \begin{pmatrix} B_T \\ \hline B_B \end{pmatrix} \leftarrow \begin{pmatrix} B_0 \\ \hline b_1^T \\ \hline B_2 \end{pmatrix}, \begin{pmatrix} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{pmatrix}$$

**endwhile**