Step	Algorithm:
1a	
4	where
2	
3	while do
2,3	$\wedge$
5a	where
6	
8	
5b	
7	
2	
	endwhile
2,3	$\wedge \neg($ $)$
1b	

<table>
<tr><th>Step</th><th>Algorithm: $[C] := \mathrm{SYRK2\_UNB\_VAR1}(A, B, C)$</th></tr>
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Algorithm: $[C] := \textsc{syrk2\_unb\_var1}(A, B, C)$

$$A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), C \rightarrow \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$$

where A_T has 0 rows, B_T has 0 rows, C_{TL} is 0×0

while $m(A_T) < m(A)$ do

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$$

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$\mathrm{c}_{01} = A_0\beta_1 + B_0\alpha_1 + c_{01}c_{10}^T = c_{01}^T\gamma_{11} = \alpha_1^T\beta_1 + \beta_1^T\alpha_1 + \gamma_{11}$

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endwhile

Step	Algorithm: $[C] := \text{SYRK2\_UNB\_VAR1}(A, B, C)$
1a	$C = \widehat{C}$
4	where
2	
3	while do
2,3	$\wedge$
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8	
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3	while do
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6	
8	
5b	
7	
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<tr><td>5a</td><td>$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$, $\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$
where a_1 has 1 row, b_1 has 1 row, γ_{11} is 1×1</td></tr>
<tr><td>6</td><td>$\left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{c|c|c} A_0 B_0^T + B_0 A_0^T + \widehat{C_{00}} & \widehat{c_{01}} & \widehat{C_{02}} \\ \hline \widehat{c_{10}^T} & \widehat{\gamma_{11}} & \widehat{c_{12}^T} \\ \hline \widehat{C_{20}} & \widehat{c_{21}} & \widehat{C_{22}} \end{array}\right)$</td></tr>
<tr><td>8</td><td></td></tr>
<tr><td>5b</td><td>$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$, $\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$</td></tr>
<tr><td>7</td><td>$\left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{c|c|c} A_0 B_0^T + B_0 A_0^T + \widehat{C_{00}} & A_0 \beta_1 + B_0 \alpha_1 + \widehat{c_{01}} & \widehat{C_{02}} \\ \hline \alpha_1^T B_0^T + \beta_1^T A_0^T + \widehat{c_{10}^T} & \alpha_1^T \beta_1 + \beta_1^T \alpha_1 + \widehat{\gamma_{11}} & \widehat{c_{12}^T} \\ \hline \widehat{C_{20}} & \widehat{c_{21}} & \widehat{C_{22}} \end{array}\right)$</td></tr>
<tr><td>2</td><td>$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right)$</td></tr>
<tr><td></td><td>endwhile</td></tr>
<tr><td>2</td><td>$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(m(A_T) < m(A))$</td></tr>
<tr><td>1b</td><td>$[C] = \text{syrk2}(A, B, \widehat{C})$</td></tr>
</table>

Step	Algorithm: $[C] := \text{SYRK2\_UNB\_VAR1}(A, B, C)$
1a	$C = \widehat{C}$
4	$A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$, $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$ where A_T has 0 rows, B_T has 0 rows, C_{TL} is 0×0
2	$\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right)$
3	while $m(A_T) < m(A)$ do
2,3	$\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge m(A_T) < m(A)$
5a	$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$, $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$ where a_1 has 1 row, b_1 has 1 row, γ_{11} is 1×1
6	$\left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{c\|c\|c} A_0 B_0^T + B_0 A_0^T + \widehat{C_{00}} & \widehat{c_{01}} & \widehat{C_{02}} \\ \hline \widehat{c_{10}^T} & \widehat{\gamma_{11}} & \widehat{c_{12}^T} \\ \hline \widehat{C_{20}} & \widehat{c_{21}} & \widehat{C_{22}} \end{array}\right)$
8	$c_{01} = A_0 \beta_1 + B_0 \alpha_1 + c_{01}$ $c_{10}^T = c_{01}^T$ $\gamma_{11} = \alpha_1^T \beta_1 + \beta_1^T \alpha_1 + \gamma_{11}$
5b	$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$, $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$
7	$\left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{c\|c\|c} A_0 B_0^T + B_0 A_0^T + \widehat{C_{00}} & A_0 \beta_1 + B_0 \alpha_1 + \widehat{c_{01}} & \widehat{C_{02}} \\ \hline \alpha_1^T B_0^T + \beta_1^T A_0^T + \widehat{c_{10}^T} & \alpha_1^T \beta_1 + \beta_1^T \alpha_1 + \widehat{\gamma_{11}} & \widehat{c_{12}^T} \\ \hline \widehat{C_{20}} & \widehat{c_{21}} & \widehat{C_{22}} \end{array}\right)$
2	$\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right)$
	endwhile
2,3	$\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(m(A_T) < m(A))$
1b	$[C] = \text{syrk2}(A, B, \widehat{C})$

Step	Algorithm: $[C] := \text{SYRK2\_UNB\_VAR1}(A, B, C)$
	$A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$, $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$ where A_T has 0 rows, B_T has 0 rows, C_{TL} is 0×0
	while $m(A_T) < m(A)$ do
	$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$, $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$ where a_1 has 1 row, b_1 has 1 row, γ_{11} is 1×1
	$c_{01} = A_0\beta_1 + B_0\alpha_1 + c_{01}$ $c_{10}^T = c_{01}^T$ $\gamma_{11} = \alpha_1^T\beta_1 + \beta_1^T\alpha_1 + \gamma_{11}$
	$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ a_1^T \\ \hline A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right)$, $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$
	endwhile

Algorithm: $[C] := \textsc{syrk2\_unb\_var1}(A, B, C)$

$$A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), C \rightarrow \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$$

where A_T has 0 rows, B_T has 0 rows, C_{TL} is 0×0

while $m(A_T) < m(A)$ **do**

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$$

where a_1 has 1 row, b_1 has 1 row, γ_{11} is 1×1

$$\mathrm{c}_{01} = A_0\beta_1 + B_0\alpha_1 + c_{01}c_{10}^T = c_{01}^T\gamma_{11} = \alpha_1^T\beta_1 + \beta_1^T\alpha_1 + \gamma_{11}$$

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ a_1^T \\ \hline A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$$

endwhile