Step	Algorithm:
1a	
4	where
2	
3	while do
2,3	$\wedge$
5a	where
6	
8	
5b	
7	
2	
	endwhile
2,3	$\wedge \neg($ $)$
1b	

<table>
<tr><td>Step</td><td>Algorithm: $[C] := \text{SYR2K\_UNB\_VAR2}(A, B, C)$</td></tr>
<tr><td>1a</td><td>$C = \widehat{C}$</td></tr>
<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), C \rightarrow \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$
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<tr><td>2</td><td>$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} + A_T B_B^T + B_T A_B^T \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right)$</td></tr>
<tr><td>3</td><td>while $m(A_T) < m(A)$ do</td></tr>
<tr><td>2,3</td><td>$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} + A_T B_B^T + B_T A_B^T \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge m(A_T) < m(A)$</td></tr>
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:
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Algorithm: $[C] := \textsc{syr2k\_unb\_var2}(A, B, C)$

$A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$, $C \rightarrow \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$

where A_T has 0 rows, B_T has 0 rows, C_{TL} is 0×0

while $m(A_T) < m(A)$ **do**

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$$

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update line 1

:

update line n

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ a_1^T \\ \hline A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$$

endwhile

Step	Algorithm: $[C] := \text{SYR2K\_UNB\_VAR2}(A, B, C)$
1a	$C = \widehat{C}$
4	where
2	
3	while do
2,3	$\wedge$
5a	where
6	
8	
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7	
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	endwhile
2,3	$\wedge \neg($ $)$
1b	$[C] = \text{syr2k}(A, B, \widehat{C})$

Step	Algorithm: $[C] := \text{SYR2K\_UNB\_VAR2}(A, B, C)$
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3	while do
2,3	$\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} + A_T B_B^T + B_T A_B^T \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge$
5a	where
6	
8	
5b	
7	
2	$\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_{TL} + A_T B_T^T + B_T A_T^T & \widehat{C}_{TR} + A_T B_B^T + B_T A_B^T \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right)$
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8	
5b	
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1b	$[C] = \text{syr2k}(A, B, \widehat{C})$

Step	Algorithm: $[C] := \textsc{syr2k\_unb\_var2}(A, B, C)$
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5a	$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$ where a_1 has 1 row, b_1 has 1 row, γ_{11} is 1×1
6	
8	
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Step	Algorithm: $[C] := \textsc{syr2k\_unb\_var2}(A, B, C)$
	$A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$, $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$ where A_T has 0 rows, B_T has 0 rows, C_{TL} is 0×0
	while $m(A_T) < m(A)$ do
	$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$, $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$ where a_1 has 1 row, b_1 has 1 row, γ_{11} is 1×1
	update line 1 : update line n
	$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ a_1^T \\ \hline A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right)$, $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$
	endwhile

Algorithm: $[C] := \textsc{syr2k\_unb\_var2}(A, B, C)$

$A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$, $C \rightarrow \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$

where A_T has 0 rows, B_T has 0 rows, C_{TL} is 0×0

while $m(A_T) < m(A)$ **do**

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$$

where a_1 has 1 row, b_1 has 1 row, γ_{11} is 1×1

---

update line 1

:

update line n

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ a_1^T \\ \hline A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$$

endwhile