Step	Algorithm:		
1a			
4			
	where		
2			
3	while do		
2,3		٨	
5a			
	where		
6			
8			
5b			
7			
2			
	endwhile		
2,3		^ ¬()
1b			

Step	Algorithm: $[y] := \text{SYM_AXPY_UNB_VAR1}(A, x, y)$
1a	$y = \widehat{y}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, x \to \begin{pmatrix} x_T \\ x_B \end{pmatrix}, y \to \begin{pmatrix} y_T \\ y_B \end{pmatrix}$ where A_{TL} is 0×0 , x_T has 0 rows, y_T has 0 rows
2	$\left(rac{y_T}{y_B} ight) = \left(rac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B} ight)$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \wedge m(A_{TL}) < m(A)$
5a	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array}\right) $ where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row
6	$\left(\frac{y_0}{\underline{\psi_1}}\right) = \left(\frac{A_{00}x_0 + \widehat{y_0}}{\widehat{\psi_1}}\right)$
8	$\left(\frac{y_0}{\psi_1}\right) := \left(\frac{y_0}{\psi_1}\right) + \left(\frac{(a_{10}^T)^T \chi_1}{a_{10}^T x_0 + a_{11} \chi_1}\right)$
5b	$ \left(\frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BR} \end{vmatrix} \leftarrow \left(\frac{A_{00}}{a_{01}} \begin{vmatrix} a_{01} \\ a_{11} \end{vmatrix} a_{12}^{T} \\ A_{20} \begin{vmatrix} a_{21} \\ a_{21} \end{vmatrix} A_{22}\right), \left(\frac{x_{T}}{x_{B}}\right) \leftarrow \left(\frac{x_{0}}{\chi_{1}} \\ x_{2}\right), \left(\frac{y_{T}}{y_{B}}\right) \leftarrow \left(\frac{y_{0}}{\psi_{1}} \\ y_{2}\right) $
7	$ \left(\frac{y_0}{\psi_1}\right) = \left(\frac{A_{00}x_0 + (a_{10}^T)^T \chi_1 + \widehat{y}_0}{a_{10}^T x_0 + a_{11}\chi_1 + \widehat{\psi}_1}\right) \\ \frac{\widehat{y}_2}{\widehat{y}_2} $
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right)$
	endwhile
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \land \neg (m(A_{TL}) < m(A))$
1b	[y] = Ax + y

Algorithm: $[y] := \text{SYM_AXPY_UNB_VAR1}(A, x, y)$

$$A \to \left(\frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BL} \end{vmatrix} \right), x \to \left(\frac{x_T}{x_B}\right), y \to \left(\frac{y_T}{y_B}\right)$$

where A_{TL} is 0×0 , x_T has 0 rows, y_T has 0 rows

while $m(A_{TL}) < m(A)$ do

$$\left(\frac{A_{TL} | A_{TR}}{A_{BL} | A_{BR}}\right) \to \left(\frac{A_{00} | a_{01} | A_{02}}{a_{10}^T | a_{11} | a_{12}^T}\right), \left(\frac{x_T}{x_B}\right) \to \left(\frac{x_0}{\chi_1}\right), \left(\frac{y_T}{y_B}\right) \to \left(\frac{y_0}{\psi_1}\right)$$

where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row

$$\left(\frac{y_0}{\frac{\psi_1}{y_2}}\right) := \left(\frac{y_0}{\frac{\psi_1}{y_2}}\right) + \left(\frac{(a_{10}^T)^T \chi_1}{a_{10}^T x_0 + a_{11} \chi_1}\right)$$

$$\left(\frac{A_{TL} | A_{TR}}{A_{BL} | A_{BR}}\right) \leftarrow \left(\frac{A_{00} | a_{01} | A_{02}}{a_{10}^T | a_{11} | a_{12}^T}\right), \left(\frac{x_T}{x_B}\right) \leftarrow \left(\frac{x_0}{\chi_1}\right), \left(\frac{y_T}{y_B}\right) \leftarrow \left(\frac{y_0}{\psi_1}\right)$$

endwhile

Step	Algorithm: $[y] := \text{SYM_AXPY_UNB_VAR1}(A, x, y)$
1a	$y=\widehat{y}$
4	where
2	
3	while do
2,3	\wedge
5a	
	where
6	
8	
5b	
7	
2	
	endwhile
2,3	^¬()
1b	[y] = Ax + y

la $y = \hat{y}$ 4 where 2 $\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right)$ 3 while do 2.3 $\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \wedge$ 5a where 6 8 5b 7 2 $\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right)$ endwhile 2 $\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \wedge \neg ($ 1b $ y = Ax + y$	Step	Algorithm: $[y] := \text{SYM_AXPY_UNB_VAR1}(A, x, y)$
where $ \begin{array}{ccccccccccccccccccccccccccccccccccc$	1a	$y = \widehat{y}$
where $ \begin{array}{ccccccccccccccccccccccccccccccccccc$	4	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		
$ \begin{array}{ccc} 2,3 & \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \land \\ 5a & & & \\ &$	2	 - =
$ \begin{array}{cccc} $	3	while do
where $ \begin{array}{c} 6 \\ 8 \\ 5b \\ 7 \\ 2 \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \\ \text{endwhile} \\ 2 \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \land \neg () \end{array} $	2,3	$\left(rac{y_T}{y_B} ight) = \left(rac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B} ight) \wedge$
where $ \begin{array}{c} 6 \\ 8 \\ 5b \\ 7 \\ 2 \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \\ \text{endwhile} \\ 2 \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \land \neg () \end{array} $		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	5a	
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8 5b 7 2 $\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right)$ endwhile 2 $\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \land \neg ($	6	
5b $ \begin{array}{cccccccccccccccccccccccccccccccccc$	O	
5b $ \begin{array}{cccccccccccccccccccccccccccccccccc$		
5b $ \begin{array}{cccccccccccccccccccccccccccccccccc$	8	
$ \begin{array}{ccc} 7 & & \\ 2 & \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \\ & \text{endwhile} \\ 2 & \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \land \neg () \\ \end{array} $		
$ \begin{array}{ccc} 7 & & \\ 2 & \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \\ & \text{endwhile} \\ 2 & \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \land \neg () \\ \end{array} $		
$ \begin{array}{ccc} 7 & & \\ 2 & \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \\ & \text{endwhile} \\ 2 & \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \land \neg () \\ \end{array} $	5b	
$ 2 \qquad \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) $ endwhile $ 2 \qquad \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \land \neg (\qquad) $		
$ 2 \qquad \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) $ endwhile $ 2 \qquad \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \land \neg (\qquad) $		
$ \frac{2}{y_B} = \frac{\widehat{y}_B}{\widehat{y}_B} $ endwhile $ 2 \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \land \neg (\qquad) $	7	
$ \frac{2}{y_B} = \frac{\widehat{y}_B}{\widehat{y}_B} $ endwhile $ 2 \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \land \neg (\qquad) $		
$ \frac{2}{y_B} = \frac{\widehat{y}_B}{\widehat{y}_B} $ endwhile $ 2 \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \land \neg (\qquad) $		$\left(y_T\right) \left(A_{TL}X_T + \hat{y}_T\right)$
endwhile $2 \left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \hat{y}_T}{\hat{y}_B}\right) \land \neg ($	2	
$ \frac{1}{2} \left(\frac{1}{y_B} \right) = \left(\frac{1}{\widehat{y}_B} \right) \wedge \neg () $		
	2	
	1b	

Step	Algorithm: $[y] := \text{SYM_AXPY_UNB_VAR1}(A, x, y)$
1a	$y = \widehat{y}$
4	
	1
	where $\left(y_T\right) = \left(A_{TL}X_T + \widehat{y}_T\right)$
2	$\left(\frac{g_I}{y_B}\right) = \left(\frac{11_L 21_I + g_I}{\widehat{y}_B}\right)$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \wedge m(A_{TL}) < m(A)$
5a	
	where
6	
8	
5h	
5b	
7	
	$\left(\frac{y_T}{y_T} \right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{y_T} \right)$
2	$\left(\frac{\overline{y}}{y_B}\right) = \left(\frac{\overline{y}}{\widehat{y}_B}\right)$
	endwhile
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \wedge \neg (m(A_{TL}) < m(A))$
1b	[y] = Ax + y

Step	Algorithm: $[y] := \text{SYM_AXPY_UNB_VAR1}(A, x, y)$
1a	$y = \widehat{y}$
4	$A \to \left(\frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BR} \end{vmatrix}, x \to \left(\frac{x_T}{x_B}\right), y \to \left(\frac{y_T}{y_B}\right)$ where A_{TL} is 0×0 , x_T has 0 rows, y_T has 0 rows
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right)$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \wedge m(A_{TL}) < m(A)$
5a	
	where
6	
8	
0	
5b	
7	
2	$\left(rac{y_T}{y_B} ight) = \left(rac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B} ight)$
	endwhile
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \land \neg (m(A_{TL}) < m(A))$
1b	[y] = Ax + y

Step	Algorithm: $[y] := \text{SYM_AXPY_UNB_VAR1}(A, x, y)$
1a	$y = \widehat{y}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, x \to \begin{pmatrix} x_T \\ x_B \end{pmatrix}, y \to \begin{pmatrix} y_T \\ y_B \end{pmatrix}$ where A_{TL} is 0×0 , x_T has 0 rows, y_T has 0 rows
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right)$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \wedge m(A_{TL}) < m(A)$
5a	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array}\right) $ where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row
6	
8	
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right) $
7	
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right)$
	endwhile
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \land \neg (m(A_{TL}) < m(A))$
1b	[y] = Ax + y

Step	Algorithm: $[y] := \text{SYM_AXPY_UNB_VAR1}(A, x, y)$
1a	$y = \widehat{y}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, x \to \begin{pmatrix} x_T \\ x_B \end{pmatrix}, y \to \begin{pmatrix} y_T \\ y_B \end{pmatrix}$ where A_{TL} is 0×0 , x_T has 0 rows, y_T has 0 rows
2	$ \left(\frac{y_T}{y_B} \right) = \left(\frac{A_{TL} X_T + \widehat{y}_T}{\widehat{y}_B} \right) $
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \wedge m(A_{TL}) < m(A)$
5a	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array}\right) $ where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row
6	$\begin{pmatrix} \frac{y_0}{\psi_1} \\ \frac{y_2}{\psi_2} \end{pmatrix} = \begin{pmatrix} \frac{A_{00}x_0 + \widehat{y}_0}{\widehat{\psi}_1} \\ \hline \widehat{y}_2 \end{pmatrix}$
8	
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right) $
7	
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right)$
	endwhile
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \land \neg (m(A_{TL}) < m(A))$
1b	[y] = Ax + y

Step	Algorithm: $[y] := \text{SYM_AXPY_UNB_VAR1}(A, x, y)$
1a	$y = \widehat{y}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, x \to \begin{pmatrix} x_T \\ x_B \end{pmatrix}, y \to \begin{pmatrix} y_T \\ y_B \end{pmatrix}$ where A_{TL} is 0×0 , x_T has 0 rows, y_T has 0 rows
2	$\left(rac{y_T}{y_B} ight) = \left(rac{A_{TL} X_T + \widehat{y}_T}{\widehat{y}_B} ight)$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \wedge m(A_{TL}) < m(A)$
5a	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array}\right) $ where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row
6	$ \left(\frac{y_0}{\psi_1}\right) = \left(\frac{A_{00}x_0 + \widehat{y}_0}{\widehat{\psi}_1}\right) $ $ \left(\frac{y_0}{\psi_1}\right) = \left(\frac{A_{00}x_0 + \widehat{y}_0}{\widehat{y}_2}\right) $
8	
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right) $
7	$ \left(\frac{y_0}{\psi_1}\right) = \left(\frac{A_{00}x_0 + (a_{10}^T)^T \chi_1 + \widehat{y}_0}{a_{10}^T x_0 + a_{11}\chi_1 + \widehat{\psi}_1}\right) \\ \widehat{y}_2 $
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right)$
	endwhile
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \land \neg (m(A_{TL}) < m(A))$
1b	[y] = Ax + y

Step	Algorithm: $[y] := \text{SYM_AXPY_UNB_VAR1}(A, x, y)$
1a	$y = \widehat{y}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, x \to \begin{pmatrix} x_T \\ x_B \end{pmatrix}, y \to \begin{pmatrix} y_T \\ y_B \end{pmatrix}$ where A_{TL} is 0×0 , x_T has 0 rows, y_T has 0 rows
2	$ \left(\frac{y_T}{y_B} \right) = \left(\frac{A_{TL} X_T + \widehat{y}_T}{\widehat{y}_B} \right) $
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \wedge m(A_{TL}) < m(A)$
5a	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array}\right) $ where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row
6	$\begin{pmatrix} \frac{y_0}{\psi_1} \\ \frac{1}{y_2} \end{pmatrix} = \begin{pmatrix} \frac{A_{00}x_0 + \widehat{y}_0}{\widehat{\psi}_1} \\ \frac{1}{\widehat{y}_2} \end{pmatrix}$
8	$\left(\frac{y_0}{\psi_1}\right) := \left(\frac{y_0}{\psi_1}\right) + \left(\frac{(a_{10}^T)^T \chi_1}{a_{10}^T x_0 + a_{11} \chi_1}\right)$
5b	$ \left(\frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BR} \end{vmatrix} \leftarrow \left(\frac{A_{00}}{a_{01}} \begin{vmatrix} a_{01} \\ a_{11} \end{vmatrix} a_{12} \\ A_{20} \begin{vmatrix} a_{21} \\ a_{21} \end{vmatrix} A_{22} \right), \left(\frac{x_T}{x_B}\right) \leftarrow \left(\frac{x_0}{\chi_1} \\ x_2\right), \left(\frac{y_T}{y_B}\right) \leftarrow \left(\frac{y_0}{\psi_1} \\ y_2\right) $
7	$ \left(\frac{y_0}{\psi_1}\right) = \left(\frac{A_{00}x_0 + (a_{10}^T)^T \chi_1 + \widehat{y}_0}{a_{10}^T x_0 + a_{11}\chi_1 + \widehat{\psi}_1}\right) \\ \frac{\widehat{y}_2}{\widehat{y}_2} $
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right)$
	endwhile
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}X_T + \widehat{y}_T}{\widehat{y}_B}\right) \land \neg (m(A_{TL}) < m(A))$
1b	[y] = Ax + y

Algorithm: $[y] := \text{SYM_AXPY_UNB_VAR1}(A, x, y)$
$A \to \left(\frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BL} \end{vmatrix}, x \to \left(\frac{x_T}{x_B}\right), y \to \left(\frac{y_T}{y_B}\right)$ where A_{TL} is 0×0 , x_T has 0 rows, y_T has 0 rows
while $m(A_{TL}) < m(A)$ do
$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array}\right) $ where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row
$ \left(\frac{y_0}{\psi_1}\right) := \left(\frac{y_0}{\psi_1}\right) + \left(\frac{(a_{10}^T)^T \chi_1}{a_{10}^T x_0 + a_{11} \chi_1}\right) $
$ \left(\frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BL} \end{vmatrix} \leftarrow \left(\frac{A_{00}}{a_{01}} \begin{vmatrix} a_{01} \\ a_{10} \end{vmatrix} a_{11} \begin{vmatrix} a_{12} \\ a_{12} \end{vmatrix}, \left(\frac{x_T}{x_B}\right) \leftarrow \left(\frac{x_0}{\chi_1} \\ x_2 \end{vmatrix}, \left(\frac{y_T}{y_B}\right) \leftarrow \left(\frac{y_0}{\psi_1} \\ y_2 \end{vmatrix}\right) $
endwhile

Algorithm: $[y] := \text{SYM_AXPY_UNB_VAR1}(A, x, y)$

$$A \to \left(\frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BL} \end{vmatrix}, x \to \left(\frac{x_T}{x_B}\right), y \to \left(\frac{y_T}{y_B}\right)$$

where A_{TL} is 0×0 , x_T has 0 rows, y_T has 0 rows

while $m(A_{TL}) < m(A)$ do

$$\left(\frac{A_{TL} | A_{TR}}{A_{BL} | A_{BR}}\right) \to \left(\frac{A_{00} | a_{01} | A_{02}}{a_{10}^T | a_{11} | a_{12}^T}\right), \left(\frac{x_T}{x_B}\right) \to \left(\frac{x_0}{\chi_1}\right), \left(\frac{y_T}{y_B}\right) \to \left(\frac{y_0}{\psi_1}\right)$$

where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row

$$\left(\frac{y_0}{\frac{\psi_1}{y_2}}\right) := \left(\frac{y_0}{\frac{\psi_1}{y_2}}\right) + \left(\frac{(a_{10}^T)^T \chi_1}{a_{10}^T x_0 + a_{11} \chi_1}\right)$$

$$\left(\frac{A_{TL} | A_{TR}}{A_{BL} | A_{BR}}\right) \leftarrow \left(\frac{A_{00} | a_{01} | A_{02}}{a_{10}^T | a_{11} | a_{12}^T}\right), \left(\frac{x_T}{x_B}\right) \leftarrow \left(\frac{x_0}{\chi_1}\right), \left(\frac{y_T}{y_B}\right) \leftarrow \left(\frac{y_0}{\psi_1}\right)$$

endwhile