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| Step | Algorithm: $[y] := \text{SYMV_L_UNB_VAR5}(A, x, y)$ |
| 1a | $y = \hat{y}$ |
| 4 | $A \rightarrow \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right), y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right)$ where A_{BR} is 0×0 , x_B has 0 rows, y_B has 0 rows |
| 2 | $\left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} \hat{y}_T \\ \hline \hat{y}_B + A_{BR}x_B \end{array} \right)$ |
| 3 | while $m(A_{BR}) < m(A)$ do |
| 2,3 | $\left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} \hat{y}_T \\ \hline \hat{y}_B + A_{BR}x_B \end{array} \right) \wedge m(A_{BR}) < m(A)$ |
| 5a | $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$ where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row |
| 6 | $\left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right) = \left(\begin{array}{c} \hat{y}_0 \\ \hline \hat{\psi}_1 \\ \hline \hat{y}_2 + A_{22}x_2 \end{array} \right)$ |
| 8 | $\psi_1 := \psi_1 + a_{11}\chi_1 + a_{21}^T x_2$ $y_2 := y_2 + a_{21}\chi_1$ |
| 5b | $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$ |
| 7 | $\left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right) = \left(\begin{array}{c} \hat{y}_0 \\ \hline \hat{\psi}_1 + a_{11}\chi_1 + a_{21}^T x_2 \\ \hline \hat{y}_2 + a_{21}\chi_1 + A_{22}x_2 \end{array} \right)$ |
| 2 | $\left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} \hat{y}_T \\ \hline \hat{y}_B + A_{BR}x_B \end{array} \right)$ |
| | endwhile |
| 2,3 | $\left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} \hat{y}_T \\ \hline \hat{y}_B + A_{BR}x_B \end{array} \right) \wedge \neg(m(A_{BR}) < m(A))$ |
| 1b | $[y] = \text{symv.l}(A, x, \hat{y}) = Ax + \hat{y}$ |

Algorithm: $[y] := \text{SYMV_L_UNB_VAR5}(A, x, y)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right), y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right)$$

where A_{BR} is 0×0 , x_B has 0 rows, y_B has 0 rows

while $m(A_{BR}) < m(A)$ **do**

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$$

where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row

$$\psi_1 := \psi_1 + a_{11}\chi_1 + a_{21}^T x_2 \quad y_2 := y_2 + a_{21}\chi_1$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$$

endwhile

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| Step | Algorithm: $[y] := \text{SYMV_L_UNB_VAR5}(A, x, y)$ |
| 1a | $y = \hat{y}$ |
| 4 | where |
| 2 | |
| 3 | while do |
| 2,3 | \wedge |
| 5a | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | |
| | endwhile |
| 2,3 | $\wedge \neg(\quad)$ |
| 1b | $[y] = \text{symv.l}(A, x, \hat{y}) = Ax + \hat{y}$ |

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| | where |
| 2 | $\left(\frac{y_T}{y_B} \right) = \left(\frac{\hat{y}_T}{\hat{y}_B + A_{BR}x_B} \right)$ |
| 3 | while do |
| 2,3 | $\left(\frac{y_T}{y_B} \right) = \left(\frac{\hat{y}_T}{\hat{y}_B + A_{BR}x_B} \right) \wedge$ |
| 5a | |
| | where |
| 6 | |
| 8 | |
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| 7 | |
| 2 | $\left(\frac{y_T}{y_B} \right) = \left(\frac{\hat{y}_T}{\hat{y}_B + A_{BR}x_B} \right)$ |
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| 3 | while $m(A_{BR}) < m(A)$ do |
| 2,3 | $\left(\frac{y_T}{y_B} \right) = \left(\frac{\hat{y}_T}{\hat{y}_B + A_{BR}x_B} \right) \wedge m(A_{BR}) < m(A)$ |
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| | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | $\left(\frac{y_T}{y_B} \right) = \left(\frac{\hat{y}_T}{\hat{y}_B + A_{BR}x_B} \right)$ |
| | endwhile |
| 2,3 | $\left(\frac{y_T}{y_B} \right) = \left(\frac{\hat{y}_T}{\hat{y}_B + A_{BR}x_B} \right) \wedge \neg(m(A_{BR}) < m(A))$ |
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| 2 | $\left(\begin{array}{c} y_T \\ y_B \end{array} \right) = \left(\begin{array}{c} \hat{y}_T \\ \hat{y}_B + A_{BR}x_B \end{array} \right)$ |
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| | endwhile |
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| 5a | $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$ where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row |
| 6 | $\left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right) = \left(\begin{array}{c} \hat{y}_0 \\ \hline \hat{\psi}_1 \\ \hline \hat{y}_2 + A_{22}x_2 \end{array} \right)$ |
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| 6 | $\left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right) = \left(\begin{array}{c} \hat{y}_0 \\ \hline \hat{\psi}_1 \\ \hline \hat{y}_2 + A_{22}x_2 \end{array} \right)$ |
| 8 | $\psi_1 := \psi_1 + a_{11}\chi_1 + a_{21}^T x_2$ $y_2 := y_2 + a_{21}\chi_1$ |
| 5b | $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$ |
| 7 | $\left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right) = \left(\begin{array}{c} \hat{y}_0 \\ \hline \hat{\psi}_1 + a_{11}\chi_1 + a_{21}^T x_2 \\ \hline \hat{y}_2 + a_{21}\chi_1 + A_{22}x_2 \end{array} \right)$ |
| 2 | $\left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} \hat{y}_T \\ \hline \hat{y}_B + A_{BR}x_B \end{array} \right)$ |
| | endwhile |
| 2,3 | $\left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} \hat{y}_T \\ \hline \hat{y}_B + A_{BR}x_B \end{array} \right) \wedge \neg(m(A_{BR}) < m(A))$ |
| 1b | $[y] = \text{symv.l}(A, x, \hat{y}) = Ax + \hat{y}$ |

| | |
|------|---|
| Step | Algorithm: $[y] := \text{SYMV_L_UNB_VAR5}(A, x, y)$ |
| | |
| | $A \rightarrow \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right), y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right)$ <p>where A_{BR} is 0×0, x_B has 0 rows, y_B has 0 rows</p> |
| | |
| | while $m(A_{BR}) < m(A)$ do |
| | |
| | $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$ <p>where α_{11} is 1×1, χ_1 has 1 row, ψ_1 has 1 row</p> |
| | |
| | $\psi_1 := \psi_1 + a_{11}\chi_1 + a_{21}^T x_2$ $y_2 := y_2 + a_{21}\chi_1$ |
| | $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$ |
| | |
| | |
| | endwhile |
| | |
| | |

Algorithm: $[y] := \text{SYMV_L_UNB_VAR5}(A, x, y)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right), y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right)$$

where A_{BR} is 0×0 , x_B has 0 rows, y_B has 0 rows

while $m(A_{BR}) < m(A)$ **do**

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$$

where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row

$$\psi_1 := \psi_1 + a_{11}\chi_1 + a_{21}^T x_2 \quad y_2 := y_2 + a_{21}\chi_1$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$$

endwhile