Step	Algorithm:		
1a			
4			
	where		
2			
3	while do		
2,3		٨	
5a			
	where		
6			
8			
5b			
7			
2			
	endwhile		
2,3		^ ¬(	)
1b			

Step	Algorithm: $[y] := \text{SYM\_AXPY\_UNB\_VAR1}(A, x, y)$
1a	$y=\widehat{y}$
4	
4	
	where
2	
3	while do
2,3	$\wedge$
,	
5a	
	where
6	
Ü	
0	
8	
F1_	
5b	
7	
2	
	endwhile
2,3	$\wedge \neg ($ )
2,9	
1b	$[y] = \operatorname{sym\_axpy}(A, x, \widehat{y})$

Step	Algorithm: $[y] := \text{SYM\_AXPY\_UNB\_VAR1}(A, x, y)$
1a	$y = \widehat{y}$
4	
_	
	where $\left( \frac{1}{2} \right) = \left( \frac{1}{2} \right)$
2	$\left(rac{y_T}{y_B} ight) = \left(rac{\widehat{y}_T}{\widehat{y}_B} ight)$
3	while do
2,3	$\left(rac{y_T}{y_B} ight) = \left(rac{\widehat{y}_T}{\widehat{y}_B} ight) \wedge$
5a	
	where
	where
6	
8	
~1	
5b	
7	
1	
	$\left( \widehat{\eta}_{lr} \right) \left( \widehat{\eta}_{lr} \right)$
2	$\left(rac{y_T}{y_B} ight) = \left(rac{\widehat{y}_T}{\widehat{y}_B} ight)$
	endwhile
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \land \neg ($
1b	$[y] = \operatorname{sym\_axpy}(A, x, \widehat{y})$

Step	Algorithm: $[y] := \text{SYM\_AXPY\_UNB\_VAR1}(A, x, y)$
1a	$y = \widehat{y}$
4	where
2	$ \left( \frac{y_T}{y_B} \right) = \left( \frac{\widehat{y}_T}{\widehat{y}_B} \right) $
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \wedge m(A_{TL}) < m(A)$
5a	
	where
6	
8	
5b	
7	
2	$\left( rac{y_T}{y_B}  ight) = \left( rac{\widehat{y}_T}{\widehat{y}_B}  ight)$
	endwhile
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \land \neg (m(A_{TL}) < m(A))$
1b	$[y] = \operatorname{sym}_{-}\operatorname{axpy}(A, x, \widehat{y})$

Step	Algorithm: $[y] := \text{SYM\_AXPY\_UNB\_VAR1}(A, x, y)$
1a	$y = \hat{y}$
4	$A \to \left(\frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BL} \end{vmatrix}, x \to \left(\frac{x_T}{x_B}\right), y \to \left(\frac{y_T}{y_B}\right)$ where $A_{TL}$ is $0 \times 0$ , $x_T$ has 0 rows, $y_T$ has 0 rows
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right)$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \wedge m(A_{TL}) < m(A)$
5a	
	where
6	
8	
5b	
7	
2	$\left( rac{y_T}{y_B}  ight) = \left( rac{\widehat{y}_T}{\widehat{y}_B}  ight)$
	endwhile
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \land \neg (m(A_{TL}) < m(A))$
1b	$[y] = \operatorname{sym\_axpy}(A, x, \widehat{y})$

Step	Algorithm: $[y] := \text{SYM\_AXPY\_UNB\_VAR1}(A, x, y)$
1a	$y = \hat{y}$
4	$A \to \left(\frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BR} \end{vmatrix}, x \to \left(\frac{x_T}{x_B}\right), y \to \left(\frac{y_T}{y_B}\right)$ where $A_{TL}$ is $0 \times 0$ , $x_T$ has 0 rows, $y_T$ has 0 rows
2	$ \left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) $
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \wedge m(A_{TL}) < m(A)$
5a	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array}\right) $ where $\alpha_{11}$ is $1 \times 1$ , $\chi_1$ has 1 row, $\psi_1$ has 1 row
6	
8	
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right) $
7	
2	$\left( rac{y_T}{y_B}  ight) = \left( rac{\widehat{y}_T}{\widehat{y}_B}  ight)$
	endwhile
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \land \neg (m(A_{TL}) < m(A))$
1b	$[y] = \operatorname{sym\_axpy}(A, x, \widehat{y})$

Step	Algorithm: $[y] := \text{SYM\_AXPY\_UNB\_VAR1}(A, x, y)$
1a	$y=\widehat{y}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, x \to \begin{pmatrix} x_T \\ x_B \end{pmatrix}, y \to \begin{pmatrix} y_T \\ y_B \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $x_T$ has 0 rows, $y_T$ has 0 rows
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right)$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \wedge m(A_{TL}) < m(A)$
5a	$ \left(\frac{A_{TL}}{A_{BL}}\begin{vmatrix} A_{TR} \\ A_{BR} \end{vmatrix}\right) \to \left(\frac{A_{00}}{a_{01}}\begin{vmatrix} a_{01} \\ a_{10} \end{vmatrix} a_{11}\begin{vmatrix} a_{12} \\ A_{20} \end{vmatrix} \right), \left(\frac{x_T}{x_B}\right) \to \left(\frac{x_0}{\chi_1}\right), \left(\frac{y_T}{y_B}\right) \to \left(\frac{y_0}{\psi_1}\right) $ where $a_{11}$ is the standard form $a_{12}$ and $a_{13}$ and $a_{14}$ are the standard form.
6	where $\alpha_{11}$ is $1 \times 1$ , $\chi_1$ has 1 row, $\psi_1$ has 1 row $\begin{pmatrix} \underline{y_0} \\ \underline{\psi_1} \\ \underline{y_2} \end{pmatrix}$
8	
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right) $
7	
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right)$
	endwhile
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \land \neg (m(A_{TL}) < m(A))$
1b	$[y] = \operatorname{sym\_axpy}(A, x, \widehat{y})$

Step	Algorithm: $[y] := \text{SYM\_AXPY\_UNB\_VAR1}(A, x, y)$
1a	$y = \hat{y}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, x \to \begin{pmatrix} x_T \\ x_B \end{pmatrix}, y \to \begin{pmatrix} y_T \\ y_B \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $x_T$ has 0 rows, $y_T$ has 0 rows
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right)$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \wedge m(A_{TL}) < m(A)$
5a	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array}\right) $ where $\alpha_{11}$ is $1 \times 1$ , $\chi_1$ has 1 row, $\psi_1$ has 1 row
6	$\begin{pmatrix} \frac{y_0}{\psi_1} \\ \frac{y_2}{y_2} \end{pmatrix}$
8	
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right) $
7	$\left( rac{y_0}{\psi_1}  ight) $
2	$\left(rac{y_T}{y_B} ight) = \left(rac{\widehat{y}_T}{\widehat{y}_B} ight)$
	endwhile
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \land \neg (m(A_{TL}) < m(A))$
1b	$[y] = \operatorname{sym}_{-}\operatorname{axpy}(A, x, \widehat{y})$

Step	Algorithm: $[y] := \text{SYM\_AXPY\_UNB\_VAR1}(A, x, y)$
1a	$y = \widehat{y}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, x \to \begin{pmatrix} x_T \\ x_B \end{pmatrix}, y \to \begin{pmatrix} y_T \\ y_B \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $x_T$ has 0 rows, $y_T$ has 0 rows
2	$\left(rac{y_T}{y_B} ight) = \left(rac{\widehat{y}_T}{\widehat{y}_B} ight)$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \wedge m(A_{TL}) < m(A)$
5a	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array}\right) $ where $\alpha_{11}$ is $1 \times 1$ , $\chi_1$ has 1 row, $\psi_1$ has 1 row
6	$\left( rac{y_0}{\psi_1}  ight)$
8	update line 1 : update line n
5b	$ \left(\frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BR} \end{vmatrix} \leftarrow \left(\frac{A_{00}}{a_{01}} \begin{vmatrix} a_{01} \\ a_{11} \end{vmatrix} a_{12}^{T} \\ A_{20} \begin{vmatrix} a_{21} \\ A_{22} \end{vmatrix} \right), \left(\frac{x_{T}}{x_{B}}\right) \leftarrow \left(\frac{x_{0}}{\chi_{1}} \\ x_{2}\right), \left(\frac{y_{T}}{y_{B}}\right) \leftarrow \left(\frac{y_{0}}{\psi_{1}} \\ y_{2}\right) $
7	$\left( egin{array}{c} y_0 \ \hline \psi_1 \ \hline y_2 \end{array}  ight)$
2	$\left( \overline{\frac{y_T}{y_B}} \right) = \left( \overline{\widehat{y}_T} \right)$
	endwhile
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \land \neg (m(A_{TL}) < m(A))$
1b	$[y] = \operatorname{sym\_axpy}(A, x, \widehat{y})$

Step	Algorithm: $[y] := \text{SYM\_AXPY\_UNB\_VAR1}(A, x, y)$
	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, x \to \begin{pmatrix} x_T \\ x_B \end{pmatrix}, y \to \begin{pmatrix} y_T \\ y_B \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $x_T$ has 0 rows, $y_T$ has 0 rows
	while $m(A_{TL}) < m(A)$ do
	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array}\right) $ where $\alpha_{11}$ is $1 \times 1$ , $\chi_1$ has 1 row, $\psi_1$ has 1 row
	update line 1 : update line n
	$ \left(\frac{A_{TL} A_{TR}}{A_{BL} A_{BR}}\right) \leftarrow \left(\frac{A_{00} a_{01} A_{02}}{a_{10}^{T} \alpha_{11} a_{12}^{T}}\right), \left(\frac{x_{T}}{x_{B}}\right) \leftarrow \left(\frac{x_{0}}{\chi_{1}}\right), \left(\frac{y_{T}}{y_{B}}\right) \leftarrow \left(\frac{y_{0}}{\psi_{1}}\right) $
	endwhile

Step	Algorithm: $[y] := \text{SYM\_AXPY\_UNB\_VAR1}(A, x, y)$
1a	$y = \widehat{y}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, x \to \begin{pmatrix} x_T \\ x_B \end{pmatrix}, y \to \begin{pmatrix} y_T \\ y_B \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $x_T$ has 0 rows, $y_T$ has 0 rows
2	$\left(rac{y_T}{y_B} ight) = \left(rac{\widehat{y}_T}{\widehat{y}_B} ight)$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \wedge m(A_{TL}) < m(A)$
5a	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array}\right) $ where $\alpha_{11}$ is $1 \times 1$ , $\chi_1$ has 1 row, $\psi_1$ has 1 row
6	$\left(\frac{y_0}{\frac{\psi_1}{y_2}}\right)$
	update line 1
8	:
	update line n
5b	$ \left(\frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BR} \end{vmatrix}\right) \leftarrow \left(\frac{A_{00}}{a_{01}} \begin{vmatrix} a_{01} \\ a_{10} \end{vmatrix} a_{11} \begin{vmatrix} a_{12} \\ a_{20} \end{vmatrix} \right), \left(\frac{x_T}{x_B}\right) \leftarrow \left(\frac{x_0}{\chi_1} \\ x_2\right), \left(\frac{y_T}{y_B}\right) \leftarrow \left(\frac{y_0}{\psi_1} \\ y_2\right) $
7	$\left( egin{array}{c} y_0 \ \hline \psi_1 \ \hline y_2 \end{array}  ight)$
2	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right)$
	endwhile
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{\widehat{y}_B}\right) \land \neg (m(A_{TL}) < m(A))$
1b	$[y] = \operatorname{sym}_{-}\operatorname{axpy}(A, x, \widehat{y})$

Algorithm:  $[y] := \text{SYM\_AXPY\_UNB\_VAR1}(A, x, y)$ 

$$A \to \left(\frac{A_{TL} A_{TR}}{A_{BL} A_{BR}}\right), x \to \left(\frac{x_T}{x_B}\right), y \to \left(\frac{y_T}{y_B}\right)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $x_T$  has 0 rows,  $y_T$  has 0 rows

while  $m(A_{TL}) < m(A)$  do

$$\left(\frac{A_{TL} | A_{TR}}{A_{BL} | A_{BR}}\right) \to \left(\frac{A_{00} | a_{01} | A_{02}}{a_{10}^T | a_{11} | a_{12}^T}\right), \left(\frac{x_T}{x_B}\right) \to \left(\frac{x_0}{\chi_1}\right), \left(\frac{y_T}{y_B}\right) \to \left(\frac{y_0}{\psi_1}\right)$$

where  $\alpha_{11}$  is  $1 \times 1$ ,  $\chi_1$  has 1 row,  $\psi_1$  has 1 row

update line 1

•

update line n

$$\left(\frac{A_{TL} | A_{TR}}{A_{BL} | A_{BR}}\right) \leftarrow \left(\frac{A_{00} | a_{01} | A_{02}}{a_{10}^T | a_{11} | a_{12}^T}\right), \left(\frac{x_T}{x_B}\right) \leftarrow \left(\frac{x_0}{\chi_1}\right), \left(\frac{y_T}{y_B}\right) \leftarrow \left(\frac{y_0}{\psi_1}\right)$$

endwhile