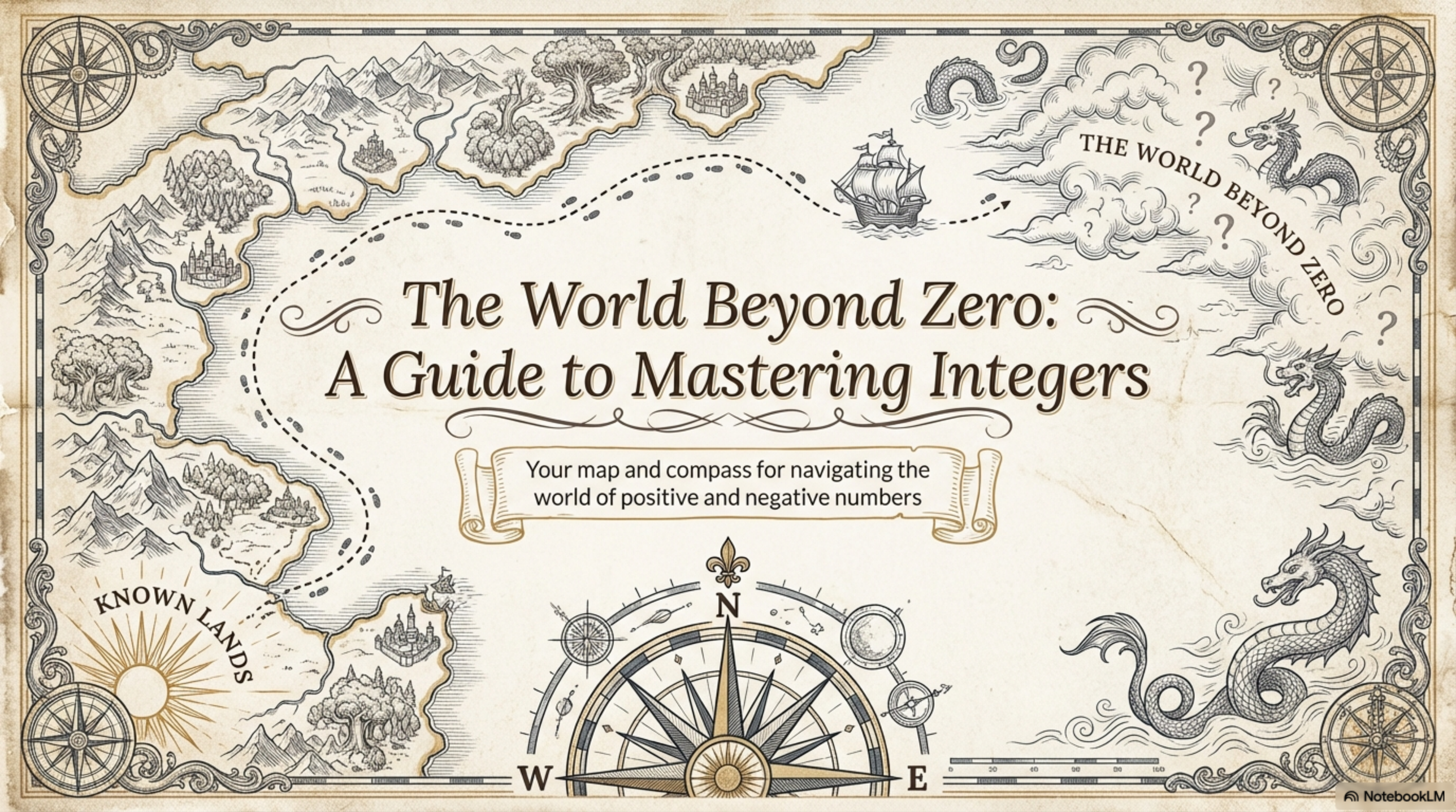
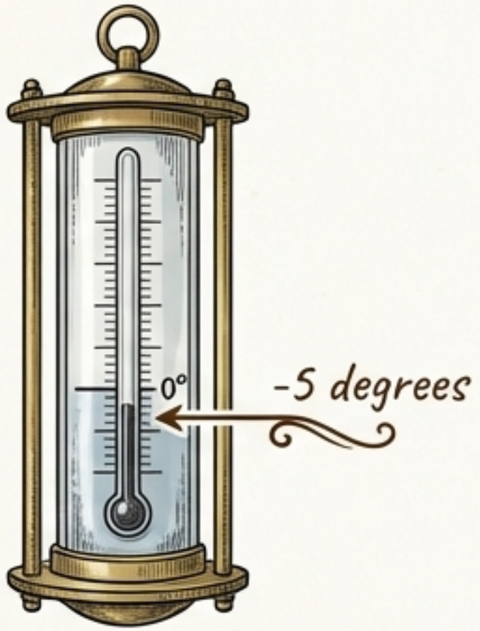


The World Beyond Zero: A Guide to Mastering Integers

Your map and compass for navigating the world of positive and negative numbers

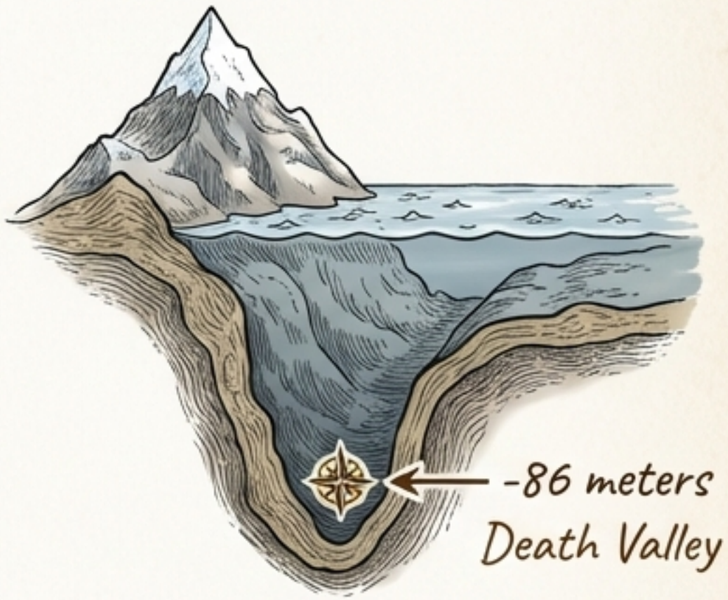
Our Old Number Map is Incomplete.

We use numbers to describe the world, but what happens when...



The temperature drops below freezing?

You owe money or have a debt?



You're exploring below sea level?

How do we measure things that are *less than nothing*?
To describe our world fully, we need a new kind of number.

Welcome to the World of Integers.

Definition: "An integer is a member of the set of all whole numbers and their opposites."

In Simple Terms: "Integers are all positive and negative whole numbers, including zero."

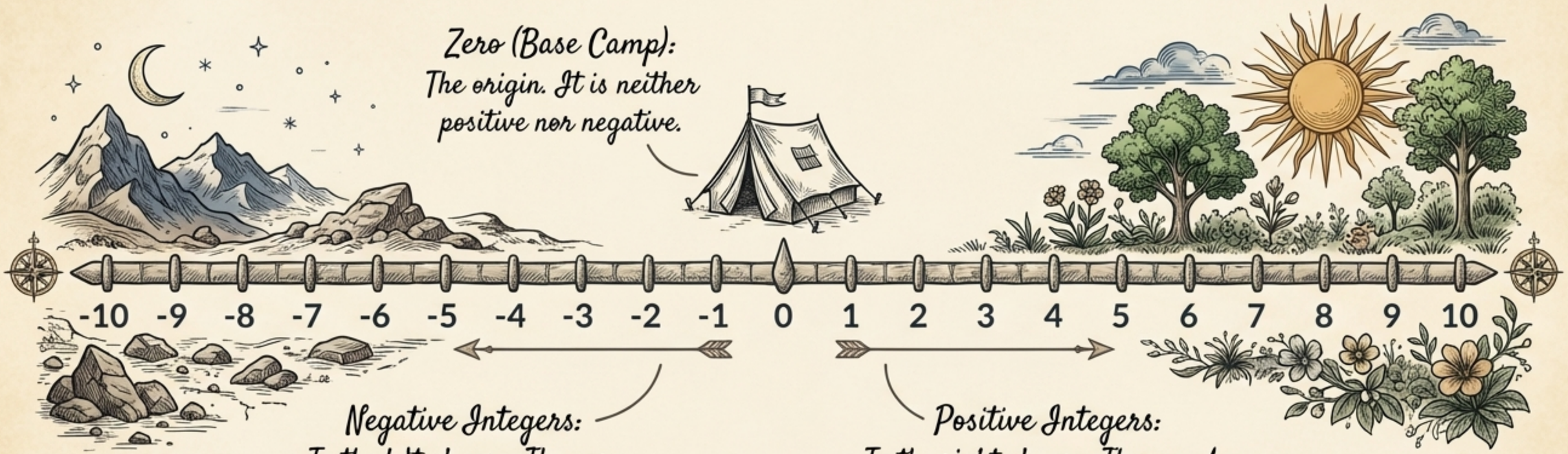
The Golden Rule:
"There are **no fractions or decimals** allowed."

Quick Check – "Am I an integer?"

5 (✓ Yes) -15 (✓ Yes) 2.5 (✗ No) -95 (✓ Yes) -4.75 (✗ No)

Key Takeaway: "Zero is a special integer. It is not positive or negative, but it is a whole number."

The Great Path: Charting Integers on the Number Line



The farther to the right a number is, the greater its value. The farther to the left, the less its value. So, -7 is less than -4.

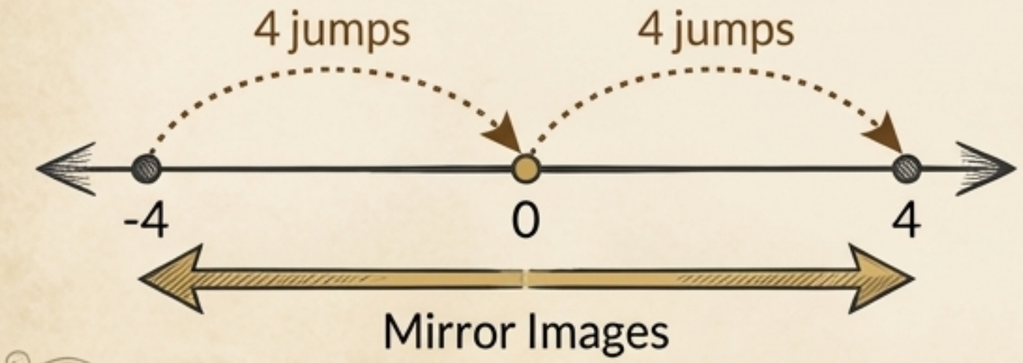
Direction and Distance: Opposites and Absolute Value

Opposites: A Question of Direction

Concept: Two numbers are opposites if they are the same distance from zero but on different sides. Think of them as mirror images.

Example:
The opposite of 4 is -4.
The opposite of -(-3) is 3.

Special Case: Zero is its own opposite.

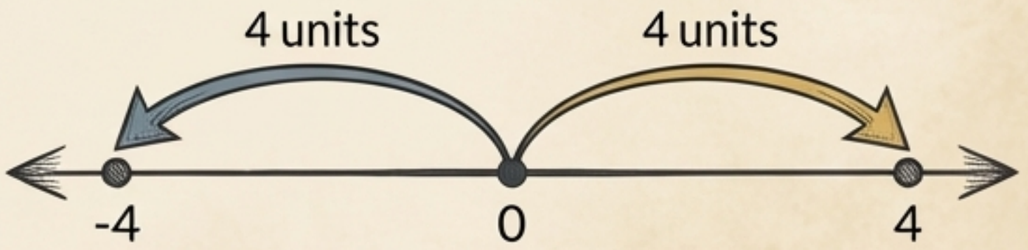


Absolute Value: A Measure of Distance

Concept: Absolute Value is the distance between a number and zero on the number line. Distance is always positive.

Notation: Written as |a|

Examples:
| -5 | = 5 (because -5 is 5 units from zero).
| 5 | = 5 (because 5 is 5 units from zero).



Part 3: Mastering Movement
Learning to Navigate with Your Explorer's Toolkit.

Tool 1: The Number Line Journey
We can think of adding and subtracting as taking a journey along the number line. The first number is your starting point, and the second number tells you how far and in which direction to move.

Tool 2: Two-Color Counters & the "Zero Pair"
A powerful tool for understanding *why* the rules work. A positive counter and a negative counter pair up to make zero. This "zero pair" is the key to simplifying problems.

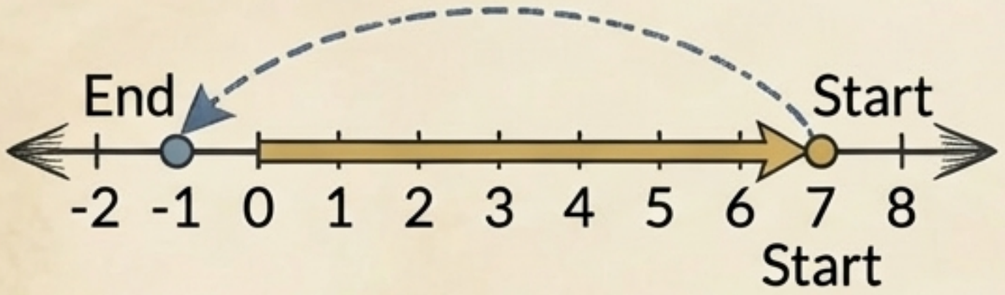
1 + (-1) = 0

Adding Integers is a Journey on the Number Line.

The Process

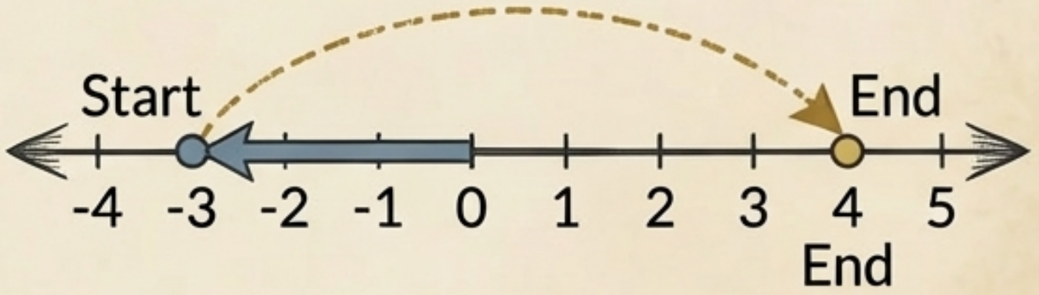
1. Start at the first number.
2. Adding a **positive** number means moving to the **right**.
3. Adding a **negative** number means moving to the **left**.

Example 1: 7 + (–8)



Start at 7. Move 8 steps to the left.
You land on -1.

Example 2: (–3) + 7



Start at -3. Move 7 steps to the right.
You land on 4.

The Rules of Addition

Rule 1: Adding with the SAME Sign

How: Add their absolute values.

Result: Keep the same sign.

$(-12) + (-6) = -18$ → *You're just getting more negative.*

Rule 2: Adding with DIFFERENT Signs

How: Subtract the smaller absolute value from the larger absolute value.

Result: Keep the sign of the number with the greater absolute value.

$(-9) + 2 = -7$ → *The negative number is "stronger".*

$5 + (-2) = 3$ → *The positive number is "stronger".*

The Smartest Shortcut: Subtraction is Just Adding the Opposite.

The "Change, Change" Rule: Subtracting an integer is the SAME as adding its opposite.

1. Keep the first number the same.
2. Change the subtraction sign to an addition sign.
3. Change the sign of the second number to its opposite.
4. Now, just follow the rules for addition!

Example: -17 – 18 – (-35)

Step 1 (First two numbers): -17 + (-18). Same signs, so add and keep the negative. Result: -35.
Step 2 (Bring down the rest): -35 – (-35).
Step 3 (Apply the rule): -35 + (+35). Opposites, so the result is 0.

Problem to try: 2 – (-9) – 8

(Answer: 3)

Part 4: Unlocking the Patterns

Discovering the “Universal Laws” of Multiplication

From Concrete to Abstract

- Positive × Positive = Positive *(This is familiar: 3 × 4 = 12)*
- Positive × Negative = Negative *(Think of it as repeated addition: 4 × (-3) is (-3) + (-3) + (-3) + (-3) = -12)*
- Negative × Negative = Positive *(This is the trickiest, but follows the pattern. We can think of it as taking away groups of debt.)*

The Rules in a Table

Same Signs	Different Signs
+ ● + = +	+ ● - = -
- ● - = +	- ● + = -
Answer is POSITIVE	Answer is NEGATIVE

Same Signs = Positive Answer.
Different Signs = Negative Answer.

The Laws of Division Mirror Multiplication

The same logic applies: The sign rules for dividing integers are identical to the rules for multiplying them.

Same Signs	Different Signs
+ ÷ + = +	+ ÷ − = −
− ÷ − = +	− ÷ + = −
Answer is POSITIVE	**Answer is NEGATIVE**

Same Signs = Positive Answer. Different Signs = Negative Answer.

Special Cases with Zero

- The quotient of an integer and zero is **zero**. (e.g., 0 ÷ 3 = 0)
- Division by zero is **undefined**. (e.g., − 6 ÷ 0 = Undefined)

You've Reached the Summit: An Integer Mastery Checklist.

You've mastered integers when you can confidently...

- ☐ **Solve** multi-step problems involving all four operations with integers (e.g., −12 − (− 5) − (− 125) + 270°).
- ☐ **Apply** the correct sign rules for multiplying and dividing any two integers.
- ☐ **Rewrite** any subtraction problem as an addition problem and solve it.
- ☐ **Add** integers with the same and different signs.
- ☐ **Explain** the difference between an integer's opposite and its absolute value.
- ☐ **Place** any integer on a number line and compare its value to another.
- ☐ **Define** an integer and identify non-examples (like fractions).