Dummit & Foote Ch. 3.3: The Isomorphism Theorems

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Let G be a group.

1. (10/20/23)

Let F be a finite field of order q and let $n \in \mathbb{Z}^+$. Prove that $|GL_n(F) : SL_n(F)| = q - 1$.

Proof. Define a map $\varphi : GL_n(F) \to F^\times$ by $\varphi(A) = \det A$ for all $A \in GL_n(F)$. From Ch. 3.1, Exercise 35., φ is a surjective homomorphism with $\ker \varphi = SL_n(F)$.

From Corollary 17, we have:

$$|GL_n(F) : \ker \varphi| = |\varphi(GL_n(F))|, \text{ which implies that}$$
$$|GL_n(F) : SL_n(F)| = \underbrace{|F^\times|}_{\varphi \text{ is surjective}} = q - 1,$$

as desired. □