Dummit & Foote Ch. 2.5: The Lattice of Subgroups of a Group

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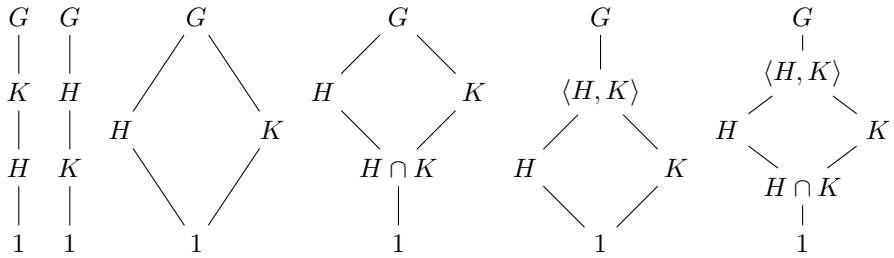
1. (8/11/23)

Let H and K be subgroups of G. Exhibit all possible sublattices which show only G, 1, H, K, and their joins and intersections. What distinguishes the different drawings?



The left two lattices show the group structure when either $H \leq K$ or $K \leq H$ (they omit any subgroups of the smaller of the two, as well as any containing subgroups between the larger and G).

The next lattice shows the group structure when H and K are not comparable, their intersection consists only of the identity, and their join is all of G. The final three lattices show the cases where $H \cap K$ is a subgroup not equal to the identity, where $\langle H, K \rangle$ is a subgroup not equal to G, and where both of these occur.