Dummit & Foote Ch. 7.1: Introduction to Rings

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Let R be a ring with 1.

1. (7/1/24)

Show that $(-1)^2 = 1$ in $\mathbb{R}$.

Proof. We have:

$$(-1) + (-1)^2 = \underbrace{(-1)(1)}_{\text{identity}} + (-1)(-1) = \underbrace{(-1)(1 + (-1))}_{\text{distribution}} = (-1) \underbrace{(0)}_{\text{inverses}} = 0,$$

and therefore, since $(-1) + (-1)^2 = 0$, $(-1)^2 = 1$. $\square$

2. (7/1/24)

Prove that if u is a unit in R then so is $-u$.

Proof. Recall that u is a unit in R if there exists some $v \in R$ such that $uv = vu = 1$.

Now:

$$\begin{aligned}(-u)(v) &= -(uv) = -1, \text{ which implies that} \\ (-u)(v)(-1) &= (-1)^2 = 1, \text{ so} \\ (-u)(-v) &= 1,\end{aligned}$$

which implies that $-u$ is also a unit in R. $\square$