Dummit & Foote Ch. 1: Groups

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2022

1. (11/14/22)

Let G be a group. Determine which of the following binary operations are associative:

a) The operation $\star$ on $\mathbb{Z}$ defined by $a \star b = a - b$:
Not associative. $3 \star (2 \star 1) = 3 - 1 = 2$ but $(3 \star 2) \star 1 = 3 - 2 = 1$.

b) The operation $\star$ on $\mathbb{R}$ defined by $a \star b = a + b + ab$:
Associative.

$$a \star (b \star c) = a \star (b + c + bc) = a + b + c + bc + ab + ac + abc = \\ (a + b + ab) \star c = (a \star b) \star c$$

c) The operation $\star$ on $\mathbb{Q}$ defined by $a \star b = \frac{a+b}{5}$:
Not associative. $0 \star (1 \star 1) = 0 + 2/5 = 2/5$ but $(0 \star 1) \star 1 = 1/5 \star 1 = 6/5 * 1/5 = 6/25$.

d) The operation $\star$ on $\mathbb{Z} \times \mathbb{Z}$ defined by $(a, b) \star (c, d) = (ad + bc, bd)$