

Dummit & Foote Ch. 1.2: Dihedral Groups

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1. (1/23/23)

Compute the order of each of the elements in the following groups:

(a) D_6

- r, r^2 : 3
- s, sr, sr^2 : 2

(b) D_8

- r : 4
- r^2 : 2
- r^3 : 4
- s, sr, sr^2, sr^3 : 2

(c) D_{10}

- r, r^2, r^3, r^4 : 5
- s, sr, sr^2, sr^3, sr^4 : 2

2. (1/23/23)

Use the generators and relations of $D_{2n} = \langle r, s \mid r^n = s^2 = 1, rs = sr^{-1} \rangle$ to show that if x is any element of D_{2n} which is not a power of r , then $rx = xr^{-1}$.

Proof. Let $x \in D_{2n}$ such that $x \neq r^k$ for all $k \in \mathbb{Z}$. Then, since all elements of D_{2n} can be written as a product of generators s and r , we must have $x = sr^k$ for some $k \in \{1, 2, \dots, n-1\}$. Therefore:

$$rx = rsr^k = sr^{-1}r^k = sr^{k-1} = sr^k r^{-1} = xr^{-1},$$

as desired. □