Munkres Ch. 2: Topological Spaces and Continuous Functions

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§12. Topological Spaces
§13. Basis for a Topology

1. (7/30/24)

Let X be a topological space; let A be a subset of X. Suppose that for each $x \in A$ there is an open set U containing x such that $U \subset A$. Show that A is open in X.

Proof. From the definition of an open set in a topological space, we know that an arbitrary union of open sets is again an open set. We will show that A is a union of open sets in X, and is therefore open.

For each $x \in A$, there exists an open set U_x containing x that is a subset of A. We claim that $A = \bigcup_{x \in A} U_x$.

Let $a \in A$. It is given that there exists a $U_a \subset A$ with $a \in U_x$. Therefore $a \in \bigcap_{x \in A} U_x$.

Conversely, let $a \in \bigcap_{x \in A} U_x$. Then a lies in some U_x such that $U_x \subset A$, and so $a \in A$. Thus $A = \bigcup_{x \in A} U_x$. $\square$