1 Background and General Info for Cubic Galileon and the Project

The Cubic Galileon has an action given by equation 1

$$S = \int d^4x\sqrt{-g}[\frac{R}{2} - \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - \frac{1}{\Lambda^2}\phi_{,\mu}\phi^{,\mu}\Box\phi] + S_m[\tilde{g_{\mu\nu}}],$$
$$\tilde{g_{\mu\nu}} = e^{2\alpha\phi}g_{\mu\nu} \tag{1}$$

Equation 1 results in the equations of motion for the field given by equation 2

$$\Box\phi + \frac{2}{\Lambda^2}[(\Box\phi)^2 - \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi] = 8\pi\alpha\rho \tag{2}$$

First, we will solve equation 2 for a static and spherically symmetric case, giving us the scalar field for a nonrelativistic, stationary mass. We will then take this solution and perturb it temporally, which will give us a small perturbation to the field that will tell us about the possible effects of scalar radiation.

1.1 Spherically Symmetric and Static Assumptions

Assuming spherical symmetry and a static metric, all derivatives besides the radial derivative will vanish, resulting in an simplifications given in equation 3

$$\Box\phi = \phi_{,r,r} + \frac{2}{r}\phi_{,r}$$
$$\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi = \phi^2_{,r,r} + \frac{2}{r}\phi^2_{,r} \tag{3}$$

These simplifications result in the equation of motion for the field in equation 4.

$$\phi_[, r, r] + \frac{2}{r}\phi_{,r} + \frac{8}{\Lambda^2 r}\phi_{,r} + \frac{4}{\Lambda^2 r^2}\phi^2_{,r} = 8\pi\alpha\rho \tag{4}$$

The solution to this equation will set the stage for our perturbed system, giving us a background solution to overlay our time dependent perturbation. The solution was determined by the shooting method, implemented in Mathematica, and using the initial conditions shown in equation 5

$$\phi_{,r}(0) = 0$$
$$\phi_\infty = \phi_{cosmological}$$
$$\phi_<(r = R) = \phi_>(r = R)$$
$$\phi'_<(r = R) = \phi'_>(r = R) \tag{5}$$

where $\phi_{cosmological}$ is the value of the field in vacuum far from any sources, inspired by cosmological measurements.

1.2 Time Perturbation

The field and source were then perturbed temporally, resulting in a new field and source given in equation 6

$$\begin{gathered}\phi \Rightarrow \phi_0 + \delta\phi \\ \rho \Rightarrow \rho_0 + \delta\rho sin(\omega t)\end{gathered} \tag{6}$$

Incorporating time in the equations of motion for the field result in new operators defined in equation 7

$$\begin{gathered}\Box\phi = \phi_{,r,r} + \frac{2}{r}\phi_{,r} - \phi_{,t,t} \\ \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi = \phi^2_{,r,r} + \frac{2}{r}\phi^2_{,r}\end{gathered} \tag{7}$$

Equation7 gives a new equation of motion, shown in equation 8

$$\phi_{,r,r} + \frac{2}{r}\phi_{,r} - \phi_{,t,t} + \frac{2}{\Lambda^2}[\frac{4}{r}\phi_{,r}\phi_{r,r} - \phi_{r,r}\phi_{,t,t} - \frac{4}{r}\phi, t, t\phi_{,r} + \frac{2}{r^2}\phi^2_{,r}] = 8\pi\alpha\rho(r,t) \tag{8}$$

Because we are still assuming flat and static space (ie $\eta_{\mu\nu}$), the Christoffel symbols for the system are still time independent, resulting only in additional terms $\propto \phi_{,t,t}$. Inserting the definitions of ϕ and ρ from equation 6 into equation 8, then expanding and dropping terms $\sim \delta\phi^2$ gives us the equation of motion for the perturbation to the field:

$$\delta\phi_{,r,r}+\frac{2}{r}\delta\phi_{,r}-\delta\phi_{,t,t}+\frac{2}{\Lambda^2}[\frac{4}{r}\delta\phi_{,r}\phi_{r,r}+\frac{4}{r}\delta\phi_{,r,r}\phi_{,r}-\phi_{r,r}\delta\phi_{,t,t}-\frac{4}{r}\delta\phi_{,t,t}\phi_{,r}+\frac{2}{r^2}\delta\phi^2_{,r}] = 8\pi\alpha\delta\rho sin(\omega t) \tag{9}$$

1.3 Computation