1 Background and General Info for Cubic Galileon and the Project

The Cubic Galileon has an action given by equation 1

$$S = \int d^4x\sqrt{-g}[\frac{R}{2} - \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - \frac{1}{\Lambda^2}\phi_{,\mu}\phi^{,\mu}\Box\phi] + S_m[\tilde{g_{\mu\nu}}], \qquad \tilde{g_{\mu\nu}} = e^{2\alpha\phi}g_{\mu\nu} \tag{1}$$

Equation 1 results in the equations of motion for the field given by equation 2

$$\Box\phi + \frac{2}{\Lambda^2}[(\Box\phi)^2 - \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi] = 8\pi\alpha\rho \tag{2}$$

First, we will solve equation 2 for a static and spherically symmetric case, giving us the scalar field for a nonrelativistic, stationary mass. We will then take this solution and perturb it temporally, which will give us a small perturbation to the field that will tell us about the possible effects of scalar radiation.

1.1 Spherically Symmetric and Static Assumptions

Assuming spherical symmetry and a static metric, all derivatives besides the radial derivative will vanish, resulting in an simplifications given in equation 3

$$\Box\phi = \phi_{,r,r} + \frac{2}{r}\phi_{,r} \qquad \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi = \phi_{,r,r}^2 + \frac{2}{r}\phi_{,r}^2 \tag{3}$$

These simplifications result in the equation of motion for the field in equation 4.

$$\phi_[, r, r] + \frac{2}{r}\phi_{,r} + \frac{8}{\Lambda^2 r}\phi_{,r} + \frac{4}{\Lambda^2 r^2}\phi_{,r}^2 = 8\pi\alpha\rho \tag{4}$$

The solution to this equation will set the stage for our perturbed system, giving us a background solution to overlay our time dependent perturbation. The solution was determined by the shooting method, implemented in Mathematica, and using the initial conditions shown in equation 5

$$\begin{aligned} &\phi_{,r}(0) = 0 \\ &\phi_\infty = \phi_{cosmological} \\ &\phi_<(r = R) = \phi_>(r = R) \\ &\phi'_<(r = R) = \phi'_>(r = R) \end{aligned} \tag{5}$$

where $\phi_{cosmological}$ is the value of the field in vacuum far from any sources, inspired by cosmological measurements.

1.2 Time Perturbation

The field and source were then perturbed temporally, resulting in new solutions given in equation 6

$$
\begin{aligned}
\phi &\Rightarrow \phi_0 + \delta\phi \\
\rho &\Rightarrow \rho_0 + \delta\rho Sin(\omega t)
\end{aligned}
\tag{6}
$$