If G is a connected looped multigraph the *Feynmann loop number* $L(G)$ is the number of 1-vertices plus the number 2-vertices plus the number of holes (i.e. rank of the fundamental group= $|E| - |V| + 1 = \chi + 1$).

$$L(G) = |V_1| + |V_2| + |E| - |V| + 1$$

Observe that if an edge $e = \{vw\}$ with is added to G then

$$L(G \cup \{e\}) = \begin{cases} L(G) - 1 & \text{if } \deg(v) = \deg(w) = 2 \text{ and } v \neq w \\ L(G) & \text{if } v = w \text{ and } \deg(v) \in \{1, 2\} \\ & \text{ or if } \deg(v) = 2 \text{ and } \deg(w) \neq 2 \\ L(G) + 1 & \text{else} \end{cases}$$

If G is a looped multigraph its simplification $S(G)$ is the simple graph obtained from G by deleting all loops and collapsing multiedges. Observe that G can be obtained from $S(G)$ by adding loops and multiedges and if $L(G) \leq n$ then $S(G)$ has at most n holes and $2(n-1)$ vertices as

$$\begin{aligned} n &\geq |V_1| + |V_2| + \frac{1}{2}\sum_k k|V_k| - |V| + 1 \\ &= \frac{1}{2}|V_1| + |V_2| + \sum_{k \geq 3} \left(\frac{k}{2} - 1\right)|V_k| + 1 \\ &\geq \frac{1}{2}|V| + \frac{1}{2}|V_2| + 1 \\ &\geq \frac{1}{2}|V| + 1 \end{aligned}$$

These bound is tight. If $n > 2$ then the looped graph with a single vertex and n loops achieves the bound on the number of holes. The trivalent graph with $2(n-1)$ vertices arranged in a cycle with every other edge a 2-multiedge has achieves a loop number n.

All Feynmann diagrams with Feynmann loop number at most n can thus be generated by first generating all trees with at most $2(n-1)$ vertices then iteratively adding edges between pairs of vertices. Every edge added also adds an additional hole. If looped multigraph G has more than $n - |V_1| - |V_2|$ holes it cannot be the subgraph of a Feynmann diagram with Feynmann loop number at most n. Thus contributing diagrams can be computed by brute force exhaustion via iteratively adding edges.