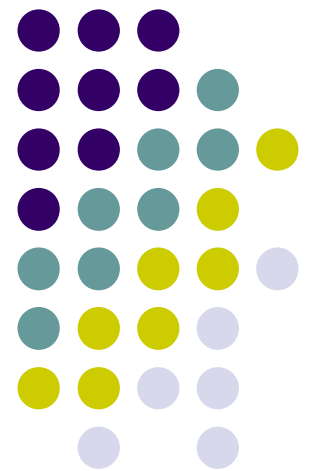


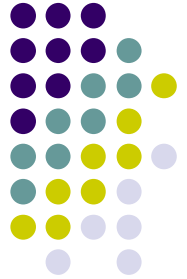
# Math 1711

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## Section 2.3: Matrices and Multiplication

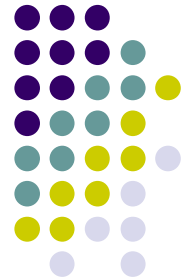


# Matrices



A *matrix* is a rectangular array of numbers:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & \dots & a_{mn} \end{bmatrix} = [a_{ij}]$$



# Matrix terminology

- **Dimension:** (# of rows) x (# of columns)

Denoted as  $m \times n$



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Denoted as  $m \times n$

- **Column matrix (or vector):** has dimension  $m \times 1$

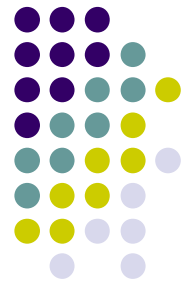


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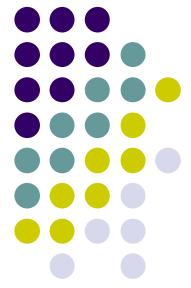


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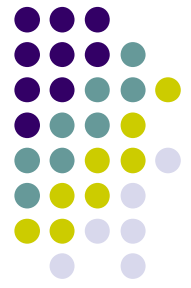


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- **Zero matrix:** all entries are  $= 0$

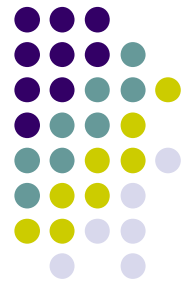


# Matrix Arithmetic

- **Matrix Addition:** if  $A$  and  $B$  both have dimension  $m \times n$ , then:

$$A \pm B = [a_{ij}] \pm [b_{ij}] = [a_{ij} \pm b_{ij}]$$





# Matrix Arithmetic

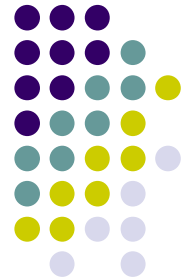
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- **Scalar multiplication:** if  $A$  is an  $m \times n$  matrix and  $c$  is any real number, then:

$$cA = c[a_{ij}] = [ca_{ij}]$$

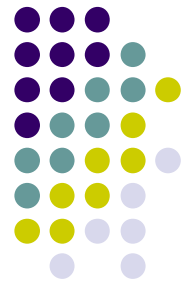
## Example 1: Evaluate the expression



$$5 \begin{bmatrix} 2 & -4 & 3 \\ -1 & 0 & 4 \end{bmatrix} - 3 \begin{bmatrix} 4 & -1 & -5 \\ -1 & 3 & 0 \end{bmatrix}$$

- |  |   |
|--|---|
| 1) $\begin{bmatrix} 22 & -23 & 0 \\ -8 & 9 & 20 \end{bmatrix}$   | 3) $\begin{bmatrix} 25 & -17 & 3 \\ -5 & -9 & 23 \end{bmatrix}$ |
| 2) $\begin{bmatrix} -2 & -17 & 30 \\ -2 & -9 & 20 \end{bmatrix}$ | 4) None of these  |

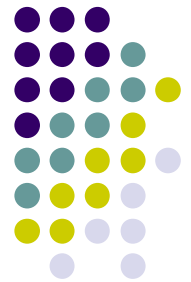




# Matrix Multiplication

Let  $A$  be an  $m \times r$  matrix, and  $B$  be an  $r \times n$  matrix. Then the product  $AB$  is defined by:

$[ab]_{ij}$  = product of  $i^{\text{th}}$  row of  $A$  with  $j^{\text{th}}$  column of  $B$

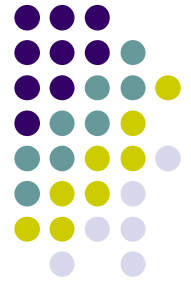


## Example 2

Given the matrices  $A$  and  $B$  below, find  $AB$  and  $BA$ .

$$A = \begin{bmatrix} -1 & 4 \\ 3 & -3 \\ 5 & 0 \end{bmatrix}, B = \begin{bmatrix} 7 & -1 & -1 \\ 0 & 4 & -2 \end{bmatrix}$$

## Example 3: Evaluate the matrix product.



$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & -2 \\ 1 & 1 \end{bmatrix}$$

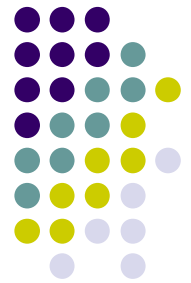
1)  $\begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix}$

3)  $\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$

2)  $\begin{bmatrix} 3 & 1 & -5 \\ -1 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$

4) None of these

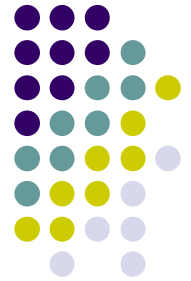




# Properties of Matrix Arithmetic

- **Associative:**  $A(BC) = (AB)C$
- **Commutative:**  $A + B = B + A$
- **Distributive:**  $A(B+C) = AB + AC$
- **Zero:**  $A + \mathbf{0} = \mathbf{0} + A = A$  and  $A\mathbf{0} = \mathbf{0}A = \mathbf{0}$
- **NO COMMUTATIVE PROPERTY FOR MATRIX MULTIPLICATION!**

The difficulty of today's lecture on a scale of 1-5 (5 being hardest) is:



1. 1
2. 2
3. 3
4. 4
5. 5

**Have a great afternoon!**

