Math 1711

Sections 2.1 and 2.2:
Gauss-Jordan Elimination

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Matrices

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- An *augmented matrix* represents a system of linear equations.

General System/Augmented Matrix

The system with m equations, n unknowns :

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = y_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = y_2$$

$$\ldots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = y_m$$

has augmented matrix :

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} & y_1 \\ a_{21} & a_{22} & \ldots & a_{2n} & y_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} & y_m \end{bmatrix}$$

Solution

A *solution* to the system is a point

$$\left(x_1, x_2, \ldots, x_n\right)$$

that satisfies every equation.

We can have a *unique solution*, *infinitely many solutions*, or *no solution*.

Reduced Row Echelon Form (RREF)

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RREF (cont)

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- If there are rows entirely of 0’s, they must be on the bottom.

RREF (cont)

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- The leading 1 in any row is to the right of the leading 1 in the row above (i.e., leading 1's go down and to the right).
- If there are rows entirely of 0's, they must be on the bottom.
- All other entries in a column with a leading 1 must be equal to 0.

Example 1: Which matrices are in RREF?

$$A = \begin{bmatrix} 1 & 0 & -1 & | & 3 \\ 0 & 1 & 2 & | & -5 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -5 & 0 & 0 & | & 0 \\ 0 & 0 & 1 & 0 & | & -2 \\ 0 & 0 & 0 & 1 & | & 7 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & -7 & 4 & | & -3 \\ 0 & 1 & 0 & 3 & | & 5 \\ 0 & 0 & 0 & 1 & | & -3 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

1. Matrix A
2. Matrix B
3. Matrix C
4. Matrices A and B
5. Matrices A and C
6. Matrices B and C
7. All of the above
8. None of the above

Row Operations

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3. Add a multiple of any row to any other row.

$$R_i = R_i + aR_j$$

Pivoting

When we pivot about a number in a matrix, we:

1. Use a row operation to make that number equal to 1, and
2. Use row operations to get zeros for all other numbers in that *column*.

Gauss-Jordan Elimination

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2. Using row operations, make the entry in the first row, first column into a “1”. (NOTE: if all entries in the first column are 0’s, go to the first row, second column instead.)

Gauss-Jordan, continued

3. Using row operations, get 0's in all the other entries in the column.

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4. Move down and to the right, and continue steps 2 and 3 on the new column. (This process is called *pivoting*.)

Gauss-Jordan, continued

3. Using row operations, get 0's in all the other entries in the column.

4. Move down and to the right, and continue steps 2 and 3 on the new column. (This process is called *pivoting*.)

5. When you can no longer move down and to the right, stop and write the system of equations.

Gauss-Jordan, continued

6. Write the solution, solving for every original values. If a variable can take on any number, then write "any number" as its value.

Example 2

Use the Gauss-Jordan elimination method to solve the system of equations:

$$2x + 4y - 2z = 2$$
$$4x + 9y - 3z = 8$$
$$-2x - 3y + 7z = 10$$

Example 3: Write out the solution.

$$\begin{bmatrix} 1 & 0 & -2 & 0 & | & 5 \\ 0 & 1 & 4 & 0 & | & 7 \\ 0 & 0 & 0 & 1 & | & -3 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

1. x = -1, y =19, z = -3, w = 0
2. x = 5 + 2z, y = 7 – 4z, z = any #, w = -3
3. x = 5, y = 7, z = 0, w = -3
4. x = -2z + 5, y = 4z + 7, z = t, w = 0
5. No solution

Example 4: Find the solution.

$$\begin{bmatrix} 1 & 0 & 0 & -3 & | & 4 \\ 0 & 1 & 0 & 2 & | & 2 \\ 0 & 0 & 1 & -7 & | & 5 \\ 0 & 0 & 0 & 0 & | & 4 \end{bmatrix}$$

1. x = 4, y = 2, z = 5, w = 4
2. x = 4 + 3w, y = 2 – 2w, z = 5 + 7w, w = any #
3. x = 16, y = -6, z = 33, w = 4
4. No solution

Example 5: Find the solution.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 3 \\ 0 & 0 & 1 & -4 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

1. x = 3 + 2z
   y = -2 + 4z
   z = any #

2. x = 3 + 2w
   y = 0
   z = -2 + 4w
   w = any #

3. x = 3 + 2w
   y = any #
   z = -2 + 4w
   w = any #

4. No solution

Example 6

Solve the system of equations using the Gauss-Jordan elimination method.

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 3 \\ 2x_1 - x_2 - 4x_3 - x_4 = -3 \end{cases}$$

Example 7: Which statement(s) is (are) true?

1. A system with the same number of equations and variables always has a unique solution.
2. A system with more variables than equations cannot have a unique solution.
3. A system with more equations than variables cannot have a unique solution.
4. Statements 1 and 2.
5. Statements 2 and 3.
6. Statements 1 and 3.
7. All of the above.
8. None of the above.

Example 8:

A corporation wants to lease a fleet of 12 airplanes with a combined carrying capacity of 220 passengers. The three types of available planes carry 10, 15, and 20 passengers, respectively.

How many of each type of plane should be leased by the corporation?

Use Gauss-Jordan elimination to solve.