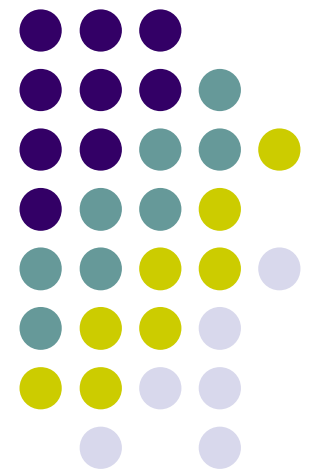
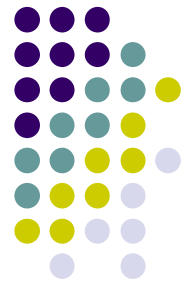


Math 1711

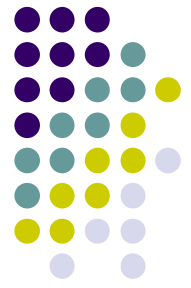
Sections 2.4, 2.5:
Matrix Inverses





Identity Matrix

- The identity matrix, I , is a square matrix with 1's along the main diagonal, and 0's everywhere else.

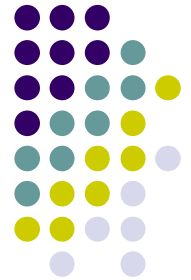


Identity Matrix

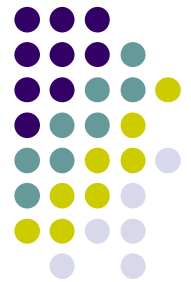
- The identity matrix, I , is a **square** matrix with 1's along the main diagonal, and 0's everywhere else.
- Some identity matrices:

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, I_n = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & 1 \end{bmatrix}$$

Identity Properties

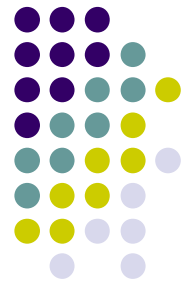


- For any matrix A , $AI=A$ and $IA=A$.



Identity Properties

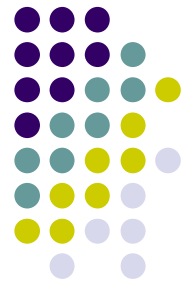
- For any matrix A , $AI=A$ and $IA=A$.
- A and B are called inverses if $AB=I$ and $BA=I$.



Identity Properties

- For any matrix A , $AI=A$ and $IA=A$.
- A and B are called inverses if $AB=I$ and $BA=I$.

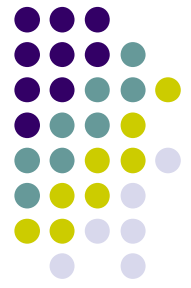
Note: If A and B are inverses, they must both be square matrices with the same dimensions ($n \times n$).



Which statement(s) is/are true?

1. If A and B are inverses, then $AB=BA$.
2. If A has an inverse, then A is a square matrix.
3. If $AB=I$, then $BA=I$ for any matrices A and B .
4. Statements 1 and 2 only.
5. Statements 1 and 3 only.
6. Statements 2 and 3 only.
7. Statements 1, 2, and 3.





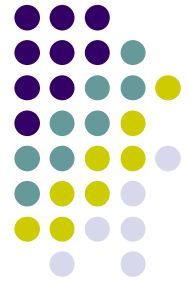
Inverse of a 2x2 Matrix

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, and $ad - bc \neq 0$,

$$\text{then } A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Example 1: Find the inverse of the matrix.

$$A = \begin{bmatrix} -4 & 3 \\ -2 & 1 \end{bmatrix}$$



$$(1) \begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix}$$

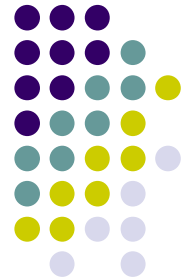
$$(3) \begin{bmatrix} 1 & -3 \\ 2 & 2 \\ 1 & -2 \end{bmatrix}$$

$$(2) \begin{bmatrix} -2 & -\frac{3}{2} \\ 1 & \frac{1}{2} \end{bmatrix}$$

(4) None of these

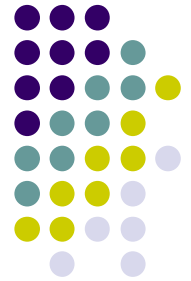


Gauss-Jordan Inverse Method for any $n \times n$ matrix



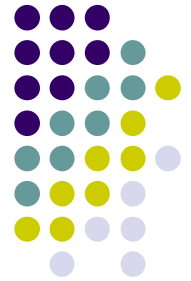
1. Write the augmented matrix $[A \mid I]$.

Gauss-Jordan Inverse Method for any $n \times n$ matrix

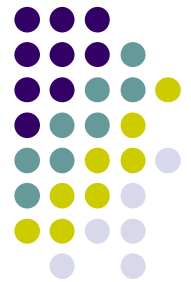


1. Write the augmented matrix $[A \mid I]$.
2. Row reduce to RREF.

Gauss-Jordan Inverse Method for any $n \times n$ matrix



1. Write the augmented matrix $[A \mid I]$.
2. Row reduce to RREF.
3. If the resulting matrix has the form $[I \mid B]$, then B is the inverse of A . Otherwise, A is not invertible.

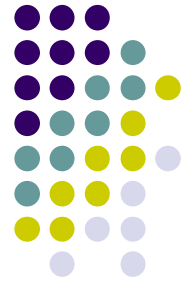


Example 2

Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 4 & 2 \\ 0 & 2 & 1 \\ 3 & 5 & 3 \end{bmatrix}$$

Writing a Matrix Equation



$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Coefficient matrix

We will define
three matrices,
 A , X , B , so that
 $AX=B$:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix}$$

variable matrix

$$B = \begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_n \end{bmatrix}$$

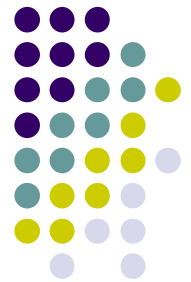
"equals to" matrix

Solving $AX=B$



If A is invertible, then:

$$X = A^{-1}B$$



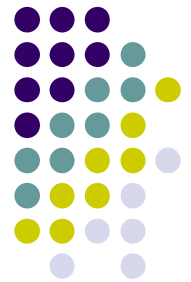
Example 3

Find the matrix equation for:

$$\begin{cases} 3x + 4y = -2 \\ 2x - 5y = 3 \end{cases}$$

$$\begin{array}{ll} 1) \begin{bmatrix} 3 & 2 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} & 3) \begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \\ 2) \begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix} & 4) \begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{array}$$

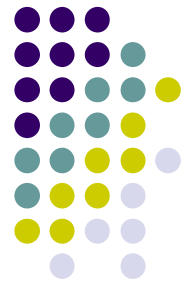




Example 3 (continued)

Now use the method of inverses to solve the system of equations:

$$\begin{cases} 3x + 4y = -2 \\ 2x - 5y = 3 \end{cases}$$



Example 4

Solve the two systems simultaneously:

$$\left\{ \begin{array}{l} x + 4y + 2z = 3 \\ 2y + z = 3 \\ 3x + 5y + 3z = -2 \end{array} \right. \quad \left\{ \begin{array}{l} x + 4y + 2z = -1 \\ 2y + z = 1 \\ 3x + 5y + 3z = -3 \end{array} \right.$$

The difficulty of today's lecture on a scale of 1-5 (5 being hardest) is:



1. 1
2. 2
3. 3
4. 4
5. 5

Have a great afternoon!

