

NAME:

SECTION:

Laplace Transform \mathcal{L}

Time	Frequency	Time	Frequency
$f(t)$	$\mathcal{L}[f](s)$	$\mathcal{L}^{-1}[F](t)$	$F(s)$
$f + g$	$\mathcal{L}[f] + \mathcal{L}[g]$	cf	$c\mathcal{L}[f]$
f'	$s\mathcal{L}[f] - f(0)$	$f^{(n)}$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$
$tf(t)$	$-\frac{d}{ds}F(s)$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$f(t) = f(t + T)$	$\frac{\int_0^T f(t)e^{-st}dt}{1 - e^{-sT}}$	$f * g(t) = \int_0^t f(t - \tau)g(\tau)d\tau$	$\mathcal{L}[f]\mathcal{L}[g]$
$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$	$\frac{1}{a}f\left(\frac{t}{a}\right)$	$F(as)$
1	$\frac{1}{s}$	$\delta(t - c)$	e^{-cs}
$e^{\lambda t}$	$\frac{1}{s - \lambda}$	$e^{\lambda t}f(t)$	$F(s - \lambda)$
t^n	$\frac{n!}{s^{n+1}}$	t^p for $p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$\sinh at$	$\frac{1}{s^2 - a^2}$	$\cosh at$	$\frac{s}{s^2 - a^2}$
$u(t - c)$	$\frac{e^{-cs}}{s}$	$u(t - c)f(t - c)$	$e^{-cs}F(s)$

Heaviside unit step function u , Dirac delta δ , gamma function Γ

Quiz 11: Find the inverse Laplace transform of the frequency function

$$F(s) = \frac{3 - s}{s^2 + 4s + 5}$$

$$\begin{aligned}
 \mathcal{L}^{-1}[F](t) &= \mathcal{L}^{-1}\left[\frac{5 - (s + 2)}{(s + 2)^2 + 1^2}\right](t) \\
 &= e^{-2t}\mathcal{L}^{-1}\left[\frac{5 - s}{s^2 + 1^2}\right](t) \\
 &= e^{-2t}\left(5\mathcal{L}^{-1}\left[\frac{1}{s^2 + 1^2}\right](t) - \mathcal{L}^{-1}\left[\frac{s}{s^2 + 1^2}\right](t)\right) \\
 &= e^{-2t}(5\cos t - \sin t)
 \end{aligned}$$