NAME:

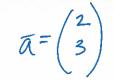
SECTION:

**Quiz** 7: Suppose two populations x and y evolve according to the equation

$$\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x(5-x-y) \\ y(7-2x-y) \end{pmatrix}$$

1. (3 pts) Find the *coexistence* equilibrium point a where both populations are nonzero.

$$x+y=5$$
 $2x+y=7$ 



2. (3 pts) What linear system best approximates the differential equation near the equilibrium point a? 
$$\frac{7}{3} = \begin{pmatrix} -2 & -2 \\ -6 & -3 \end{pmatrix}$$

$$\frac{1}{3} = \begin{pmatrix} -2 & -2 \\ -6 & -3 \end{pmatrix} \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} -2$$

$$\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ -6 & -3 \end{pmatrix} \begin{pmatrix} x-2 \\ y-3 \end{pmatrix}$$

3. (3 pts) Is the euilibrium point a stable, unstable, or semistable?

$$\det \begin{vmatrix} -2-\lambda & -2 \\ -6 & -3-\lambda \end{vmatrix} = \lambda^2 + 5\lambda - 6 = (\lambda - 1)(\lambda + 6)$$

(2) is semistable since the eigenvalue have mixed signs

Penadic solutions to Orbits may be stable, unstable, or semistable Polar Coordinates X=F(X) have Y = X + 4 2 period T>0 if  $x = ras\theta$ X(t+T) = X(t)y= rsiNe for all t. Find all the periodic solutions and the stability of their orbits: 在=|-cos(#r) 提=| 2) #= r(3-r) = 1 Trom a phase portrait. 3)  $\frac{dr}{dt} = r(1-r)(2-r)(3-r)$  What happens if  $\theta$  charge with r?

Afterpot a phase portraint showing  $\frac{d\theta}{dt} = r - \frac{3}{2}$  the periodic solutions and stability.  $\frac{dx}{dt} = y + \frac{xf(r)}{r}$   $\frac{dy}{dt} = -x + \frac{yf(r)}{r}$ has periodic solutions corresponding to their zeros of f(r).

What is the direction of notion on the closed trajectories.

in the phase plane? 1) Show that Try  $f(r) = r(r-6)^2(r^2-6r+5)$