Math 1711

Sections 2.4, 2.5:

Matrix Inverses

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Identity Matrix

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- Some identity matrices:

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, I_n = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & 1 \end{bmatrix}$$

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Note: If A and B are inverses, they must both be square matrices with the same dimensions (nxn).

Which statement(s) is/are true?

1. If A and B are inverses, then $AB=BA$.
2. If A has an inverse, then A is a square matrix.
3. If $AB=I$, then $BA=I$ for any matrices A and B.
4. Statements 1 and 2 only.
5. Statements 1 and 3 only.
6. Statements 2 and 3 only.
7. Statements 1, 2, and 3.

Inverse of a 2x2 Matrix

$$\text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ and } ad - bc \neq 0,$$

$$\text{then } A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

<u>Example 1</u>: Find the inverse of the matrix.

$$A = \begin{bmatrix} -4 & 3 \\ -2 & 1 \end{bmatrix}$$

$(1)\begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix}$

$(2)\begin{bmatrix} -2 & -\frac{3}{2} \\ 1 & \frac{1}{2} \end{bmatrix}$

$(3)\begin{bmatrix} \frac{1}{2} & \frac{-3}{2} \\ 1 & -2 \end{bmatrix}$

(4) None of these

Gauss-Jordan Inverse Method for any *nxn* matrix

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3. If the resulting matrix has the form [I | B], then B is the inverse of A. Otherwise, A is not invertible.

Example 2

Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 4 & 2 \\ 0 & 2 & 1 \\ 3 & 5 & 3 \end{bmatrix}$$

Writing a Matrix Equation

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \text{ Coefficient matrix}$$

We will define three matrices, *A*, *X*, *B*, so that *AX*=*B*:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} \text{ variable matrix}$$

$$B = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{bmatrix} \text{ "equals to" matrix}$$

Solving AX=B

If *A* is invertible, then:

$$\boxed{X = A^{-1}B}$$

Example 3

Find the matrix equation for:

$$\begin{cases} 3x + 4y = -2 \\ 2x - 5y = 3 \end{cases}$$

1) $\begin{bmatrix} 3 & 2 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

3) $\begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

2) $\begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

4) $\begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Example 3 (continued)

Now use the method of inverses to solve the system of equations:

$$\begin{cases} 3x + 4y = -2 \\ 2x - 5y = 3 \end{cases}$$

Example 4

Solve the two systems simultaneously:

$$\begin{cases} x+4y+2z=3 \\ 2y+z=3 \\ 3x+5y+3z=-2 \end{cases} \qquad \begin{cases} x+4y+2z=-1 \\ 2y+z=1 \\ 3x+5y+3z=-3 \end{cases}$$

The difficulty of today's lecture on a scale of 1-5 (5 being hardest) is:

1. 1
2. 2
3. 3
4. 4
5. 5

Have a great afternoon!