Section 4.1 : Extreme Values of Functions

Chapter 4 : Applications of Derivatives

Math 1551, Differential Calculus

Section 4.1 Extreme Values of Functions

Topics

1. local and absolute extreme values of a function
2. critical points and how to locate them

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Identify critical points and extreme values of a function.
2. Give an example, or sketch a function whose critical points, or local extrema, or global extreme values are given.

Absolute Extrema

Definitions

If $f(x)$ is any function defined on domain D. The function has an **absolute maximum** of $f(x)$ on D at a point $x = c$, if

$$f(c) \geq f(x), \text{ for all } x \in D.$$

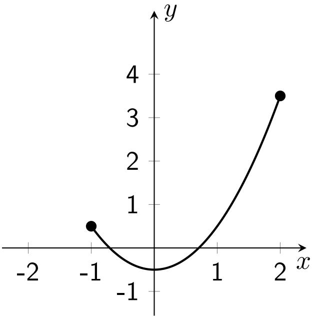
The function has an **absolute minimum** on D at $x = c$ if

$$f(c) \leq f(x), \text{ for all } x \in D.$$

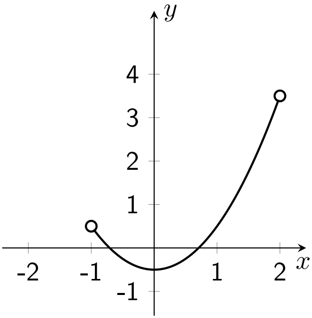
The absolute minimum and maximum values of $f(x)$ are also called the **extreme values**, or the **global** min and max of f.

Example

What are the absolute minimum and maximum values of $f = x^2 - \frac{1}{2}$ on domain D?



$D = [-1, 2]$



$D = (-1, 2)$

Participation Activity: Index Card

- Please work in groups of two or three
- Each group submits **one** completed card
- Print full names at the top of your card
- Every student in a group gets the same grade
- Grading scheme per question:
  - 0 marks for no work or for students working by themselves
  - 1 mark for starting the problem or for a final answer with insufficient justification
  - 2 marks for a complete solution

The activity consists of one or two of the examples in this lecture. Your instructor will pass out index cards.

Local Extrema

Definitions

A function has a **local maximum** at $x = c$ if $f(x) \leq f(c)$ for all x in an open interval containing c.

A function has a **local minimum** at $x = c$ if $f(x) \geq f(c)$ for all x in an open interval containing c.

The absolute minimum and maximum values of $f(x)$ are also local extrema.

Example 1

If possible, sketch a function, $f(x)$, that satisfies the following criteria, and sketch its derivative. If it is not possible to do so, state why.

a) $f(x)$ is continuous, even, local minima at $x = 0$ and $x = 2$, local maximum at $x = 1$, absolute maximum at $x = 3$.

b) $f(x)$ is continuous, odd, local minimums at $x = 1$ and $x = 3$, f has no local maximums.

Example 2

Identify the absolute minimum and maximum values of $y = 6x^2 - x^3$ on the domain $x \in [-3, 5]$.

Critical Points

Theorem

If $f(x)$ has a local minimum or maximum at $x = c$, and c is in the domain of f, then $f'(c) = 0$.

Definition

An interior point of the domain of $f(x)$ where $f'(x) = 0$ or where f' is undefined is a **critical point** of f.

To identify the absolute extrema of a function $f(x)$:

1) identify all critical points
2) evaluate f at the critical points and at endpoints of the domain

The largest and smallest values are the absolute extrema.

Additional Examples (as time permits)

1. Give a formula for a function that whose domain is $\mathbb{R}$, has an absolute maximum at $x = 1$, and no absolute minimum.
2. Identify the absolute minimum and maximum values of $y = 2 - |x|$ on the domain $x \in [-3, 5]$.
3. If possible, sketch a continuous and even function whose domain is $\mathbb{R}$, has a local maximum at $x = 1$, and has a local minimum at $x = -1$. If it is not possible, state why.