**NAME:** **SECTION:**

**Quiz 4:** Consider the matrix $A = \begin{pmatrix} -3 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 2 & -3 \end{pmatrix}$ for the differential equation $\frac{d}{dt}\mathbf{x} = A\mathbf{x}$. The matrix $A$ has eigenvalue $-3$ with algebraic multiplicity 3, but geometric multiplicity 1.

1. (3 pts) Find an eigenvector $v$ for $A$ and give a corresponding solution to $\frac{d}{dt}\mathbf{x} = A\mathbf{x}$.

2. (3 pts) Find a rank 2 generalized eigenvector $\mathbf{w}_2$ for $A$ (so that $(A - \lambda I)\mathbf{w}_2 = \mathbf{v}$) and give a corresponding solution to $\frac{d}{dt}\mathbf{x} = A\mathbf{x}$.

3. (3 pts) Find a rank 3 generalized eigenvector $\mathbf{w}_3$ for $A$ (so that $(A - \lambda I)\mathbf{w}_3 = \mathbf{w}_2$ ) and give a corresponding solution to $\frac{d}{dt}\mathbf{x} = A\mathbf{x}$.

4. (3 pts) Compute the matrix exponential $e^{At}$.