Worksheet 1, Math 1551, Spring 2017

Sections from Thomas 13^{th} Edition: 1.1, 1.2, 1.3

Welcome to Math 1551 Recitation!

Recitations are meant for students to complete additional exercises, with a TA and other students, on topics recently covered in lecture. Recitations are meant to be

- active: students should be solving exercises themselves during recitations
- collaborative: students are encouraged to work together during recitation, to present their solutions on the board, and to ask for help from the TA

Each worksheet provides exercises for one recitation.

- There may not be time in recitation to complete all exercises.
- Students are expected to be able to solve all worksheet exercises.

Students are also encouraged to seek help from TAs, their instructor, and other students if they have any questions on any of the worksheet exercises. All questions are numbered, so students can more easily refer to them outside of recitation, in Piazza, in office hours, and so on.

Background Review

- *Function* $y = f(x)$: a rule that assigns each value of x to exactly one value of y.
- *Domain of a function* $f(x)$: all the values that x can have.
- *Range of a function* $f(x)$: all the values that $f(x)$ can have.
- *Composite function*: we define $(f \circ g)(x) = f(g(x))$.

You should be able to graph the functions below.

$y = mx + b$	$y = x^2$	$y = x^3$	$y = x^n$, n any integer
$y = \lvert x \rvert$	$y = \sqrt{x}$	$\sqrt{1-x^2}$	$\frac{1}{x}$
$\sin x$	$\cos x$	$\tan x$	$\csc x$
$\sec x$	$\cot x$		

You should also be able to graph transformations of the above functions. Recall the following basic transformations:

- *Vertical shift*: $y = f(x) + c$ shifts up or down c units
- *Horizontal shift*: $y = f(x - c)$ shifts left or right c units.
- *Reflections*: $y = f(x)$ reflects about the x-axis and $y = f(x)$ reflects about the y-axis.
- *Stretching/Compressing*: $y = af(x)$ stretches when $|a| > 1$ and compresses when $0 < |a| < 1$; $y = f(bx)$ stretches when $0 < |b| < 1$ and compresses when $|b| > 1$.

Trigonometric Identities

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1, \qquad 1 + \tan^2 x = \sec^2 x, \qquad 1 + \cot^2 x = \csc^2 x$$

Half-Angle Formulas

$$\sin^2 x = \frac{1}{2}[1 - \cos(2x)], \qquad \cos^2 x = \frac{1}{2}[1 + \cos(2x)]$$

Double-Angle Formulas

$$\sin(2x) = 2\sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

Angle Sum and Difference

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin y \sin x$$

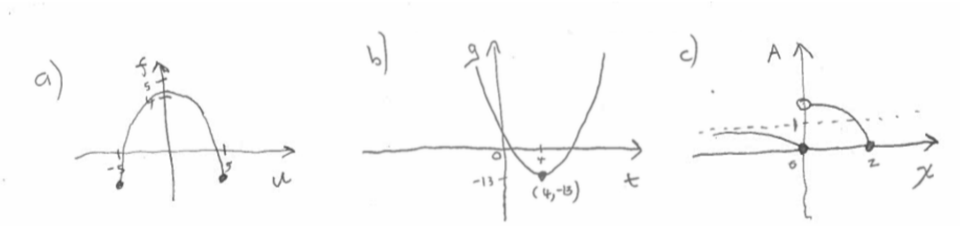
Exercises

(1) Identify the domain and range of the following functions and sketch them.

(a) $f(u) = \sqrt{25 - u^2} - 1$
Solution: We need $25 - u^2 \geq 0$, so the domain is $u \in [-5, 5]$. For the range, $\sqrt{25 - u^2}$ is between 0 and 5, so $f(u)$ is between -1 and 4. The range is the interval $[-1, 4]$.

(b) $g(t) = t^2 - 8t + 3$
Solution: Complete the square: $t^2 - 8t + 3 = t^2 - 8t + 16 - 16 + 3 = (t-4)^2 - 13$. The domain is the set of real numbers, and the range is the interval $[-13, \infty)$.

(c) $A(x) = \begin{cases} 1 + \dfrac{1}{x-1} & x \leq 0 \\ \sqrt{4 - x^2} & 0 < x \leq 2 \end{cases}$
Solution: $1 + \frac{1}{x-1}$ is defined for $x \leq 0$ and $\sqrt{4 - x^2}$ is defined on $(0, 2]$. So the domain of A is the interval $(-\infty, 2]$. Range: $[0, 2)$.
Solution Graphs:



(2) Given that $\sec\theta = -\sqrt{2}$, and that $\theta \in [\pi, 3\pi/2]$, compute the values of the other five trig functions.
Solution: Since $\sec\theta = -\sqrt{2}$, then $\cos\theta = -1/\sqrt{2}$, so $\theta = 5\pi/4$.

$$\begin{aligned}
\cos 5\pi/4 &= -1/\sqrt{2} \\
\sin 5\pi/4 &= -1/\sqrt{2} \\
\csc 5\pi/4 &= -\sqrt{2} \\
\tan 5\pi/4 &= 1 \\
\cot 5\pi/4 &= 1
\end{aligned}$$

(3) If possible, give at least one example of a function for each of the following cases.
(a) A function that is decreasing for all values of x and whose range is the interval $(0, \infty)$.
(b) A non-constant even function whose range is the interval $[0, 1]$.
(c) A function, $f(x)$, so that $f \circ g = (x+3)(x-3)$, where $g = x^2 - 10$.
Solution: The purpose of this question to help students become familiar with properties of elementary and rational functions.
(a) 2^{-x}, or a^{-x} for any $a > 0$.
(b) $|\cos(x)|$, or $|\cos(ax)|$ for any non-zero a.
(c) $f(x) = x + 1$.

(4) Express $y(t)$ as a single sine function $y(t) = \frac{\sqrt{3}}{2}\cos(t) + \frac{1}{2}\sin(t)$
Solution: Use the identity $\sin(x+y) = \cos x \sin y + \cos y \sin x$.

$$\begin{aligned}
y &= \frac{\sqrt{3}}{2}\cos(t) + \frac{1}{2}\sin(t) \\
&= \cos t \sin\frac{\pi}{3} + \cos\frac{\pi}{3}\sin(t) \\
&= \sin(t + \pi/3)
\end{aligned}$$

(5) Identify all values of θ that satisfy $\sin 2\theta - \cos\theta = 0$.
Solution:

$$\begin{aligned}
0 &= \sin 2\theta - \cos\theta \\
&= 2\cos\theta\sin\theta - \cos\theta \\
&= \cos\theta(2\sin\theta - 1)
\end{aligned}$$

So either $\cos\theta = 0$ which implies $\theta = \pi/2 + n\pi$, or $\sin\theta = 1/2$, which implies $\theta = \pi/6 + 2n\pi$ and $\theta = 5\pi/6 + 2n\pi$, n is any integer.