

Quiz 5

✓ calculators or notes.

Name

Key

Section

1) The matrix $A = \begin{pmatrix} 6 & -3 & 0 \\ 2 & -1 & 2 \\ -2 & 1 & 4 \end{pmatrix}$ has eigenvalues 6, 3, and 0.

a) (3pts) Find ~~an~~^{an} eigenvector for eigenvalue 0.

$$\text{ref } A = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{null } A = \text{span} \left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \right\}$$

b) (3pts) $A \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ -6 \end{pmatrix}$ and $A \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$. Give the general solution to $\dot{X} = AX$.

$$X(t) = c_1 e^{6t} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + c_2 e^{3t} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} e^{6t} & 0 & 0 \\ 0 & e^{3t} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

for any $c_1, c_2, c_3 \in \mathbb{R}$

c) (3pts) Sketch a phase portrait with solution trajectories for $\dot{X} = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}^{-1} X$

Stable equilibrium.
Solutions tend parallel to the dominant eigendirection $\text{span} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

