NAME: Key SECTION:

Quiz 7: Suppose two populations x and y evolve according to the equation

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x(5 - x - y) \\ y(7 - 2x - y) \end{pmatrix}$$

1. (3 pts) Find the *coexistence* equilibrium point **a** where both populations are nonzero.

$x + y = 5$
$2x + y = 7$

$$\left|\begin{matrix} 1 & 1 & 5 \\ 2 & 1 & 7 \end{matrix}\right| \xrightarrow{rref} \left|\begin{matrix} 1 & 0 & 2 \\ 0 & +1 & +3 \end{matrix}\right|$$

$$\boxed{\bar{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}}$$

2. (3 pts) What linear system best approximates the differential equation near the equilibrium point **a**?

$$F'\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ -6 & -3 \end{pmatrix}$$

$$F'\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 - 2x - y & -x \\ -2y & 7 - 2x - 2y \end{pmatrix}$$

$$\boxed{\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ -6 & -3 \end{pmatrix}\begin{pmatrix} x - 2 \\ y - 3 \end{pmatrix}}$$

3. (3 pts) Is the euilibrium point **a** stable, unstable, or semistable?

$$\det\left|\begin{matrix} -2 - \lambda & -2 \\ -6 & -3 - \lambda \end{matrix}\right| = \lambda^2 + 5\lambda - 6 = (\lambda - 1)(\lambda + 6)$$

$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is semistable since the eigenvalues have mixed signs

Polar Coordinates

$r^2 = x^2+y^2$

$x = r\cos\theta$

$y = r\sin\theta$

Periodic solutions to $\dot{\mathbf{x}} = F(\mathbf{x})$ have period $T>0$ if

$\mathbf{x}(t+T) = \mathbf{x}(t)$

for all t.

Orbits may be stable, unstable, or semistable

1) Find all the periodic solutions and the stability of their orbits:

$$\frac{dr}{dt} = 1-\cos(\pi r) \qquad \frac{d\theta}{dt} = 1$$

(2) $\frac{dr}{dt} = r(3-r)^2 \quad \frac{d\theta}{dt} = 1$ Draw a phase portrait.

(3) $$\frac{dr}{dt} = r(1-r)(2-r)(3-r)$$

$$\frac{d\theta}{dt} = r - 3/2$$

What happens if $\dot{\theta}$ changes with r? Attempt a phase portrait showing the periodic solutions and stability.

(4) Show that

$$\frac{dx}{dt} = y + \frac{x f(r)}{r} \qquad \frac{dy}{dt} = -x + \frac{y f(r)}{r}$$

has periodic solutions corresponding to the zeros of $f(r)$.

What is the direction of motion on the closed trajectories in the phase plane?

Try $f(r) = r(r-6)^2(r^2-6r+5)$