Math 1711

Section 2.3:

Matrices and Multiplication

Matrices

A *matrix* is a rectangular array of numbers:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & \cdots & a_{mn} \end{bmatrix} = \left[a_{ij}\right]$$

Matrix terminology

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- **Square matrix**: has dimension n x n
- **Zero matrix**: all entries are = 0

Matrix Arithmetic

- **Matrix Addition**: if *A* and *B* both have dimension m x n, then:

$$A \pm B = [a_{ij}] \pm [b_{ij}] = [a_{ij} \pm b_{ij}]$$

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- **Scalar multiplication**: if *A* is an m x n matrix and *c* is any real number, then:

$$cA = c[a_{ij}] = [ca_{ij}]$$

Example 1: Evaluate the expression

$$5\begin{bmatrix} 2 & -4 & 3 \\ -1 & 0 & 4 \end{bmatrix} - 3\begin{bmatrix} 4 & -1 & -5 \\ -1 & 3 & 0 \end{bmatrix}$$

1) $\begin{bmatrix} 22 & -23 & 0 \\ -8 & 9 & 20 \end{bmatrix}$

3) $\begin{bmatrix} 25 & -17 & 3 \\ -5 & -9 & 23 \end{bmatrix}$

2) $\begin{bmatrix} -2 & -17 & 30 \\ -2 & -9 & 20 \end{bmatrix}$

4) None of these

Matrix Multiplication

Let A be an $m \times r$ matrix, and B be an $r \times n$ matrix. Then the product AB is defined by:

$[ab]_{ij}$ = product of i^{th} row of A with j^{th} column of B

Example 2

Given the matrices *A* and *B* below, find *AB* and *BA*.

$$A = \begin{bmatrix} -1 & 4 \\ 3 & -3 \\ 5 & 0 \end{bmatrix}, B = \begin{bmatrix} 7 & -1 & -1 \\ 0 & 4 & -2 \end{bmatrix}$$

Example 3: Evaluate the matrix product.

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & -2 \\ 1 & 1 \end{bmatrix}$$

1) $\begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix}$

3) $\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$

2) $\begin{bmatrix} 3 & 1 & -5 \\ -1 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$

4) None of these

Properties of Matrix Arithmetic

- **Associative**: $A(BC) = (AB)C$
- **Commutative**: $A + B = B + A$
- **Distributive**: $A(B+C) = AB + AC$
- **Zero**: $A + \mathbf{0} = \mathbf{0} + A = A$ *and* $A\mathbf{0} = \mathbf{0}A = \mathbf{0}$
- *NO COMMUTATIVE PROPERTY FOR MATRIX MULTIPLICATION*!

The difficulty of today's lecture on a scale of 1-5 (5 being hardest) is:

1. 1
2. 2
3. 3
4. 4
5. 5

Have a great afternoon!