NAME: SECTION:

Quiz 4: No calculators. Justify all answers. No partial credit is given for an unexplained and incorrect answer.

The matrix

$$A = \begin{pmatrix} 8 & -3 \\ 18 & -7 \end{pmatrix}$$

has eigenvectors $v_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

1. (3 pts) What is the eigenvalue of A associated to eigenvector v_1?

$$\begin{pmatrix} 8 & -3 \\ 18 & -7 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} = -1 \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

eigenvalue -1

2. (3 pts) What is the eigenvalue of A associated to eigenvector v_2?

$$\begin{pmatrix} 8 & -3 \\ 18 & -7 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix} = 2 \cdot \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

eigenvalue 2

3. (3 pts) Give the general solution to the differential equation

$$\frac{d}{dt}\mathbf{x} = \begin{pmatrix} 8 & -3 \\ 18 & -7 \end{pmatrix} \mathbf{x}$$

$$X(t) = c_1 e^{-t} \begin{pmatrix} 1 \\ 3 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{2t} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$