Math 1551 Learning Goals

Learning goals articulate what students are **expected to be able to do** in a course that can be measured. This course has **course-level** learning goals that pertain to the entire course, and **section-level** learning goals that pertain to sections of the course.

Course-Level Learning Goals

By the end of this course, it is expected that students will be able to do the following.

A) Construct mathematical expressions and graphs involving functions and their derivatives.

B) Compute mathematical quantities using differential calculus and interpret their meaning.

C) Analyze mathematical statements and expressions (for example, to assess whether a particular statement is accurate).

D) Write logical progressions of precise statements to justify and communicate mathematical reasoning.

E) Apply calculus concepts to solve real-world problems such as optimization and related rate problems.

For example, students will be expected to construct related rate equations to compute the value of a variable and interpret the resulting value in the context of the given problem.

Section-Level Learning Goals

Section-level learning goals are connected with particular sections of this course.

1 Functions

1.1 Functions and Their Graphs

1. Determine whether a graph is a function
2. Characterize functions using domain and range, symmetry

1.2 Combining Functions; Shifting and Scaling Graphs

1. sketch functions using shifting, scaling, reflections
2. compose functions, find the domain and range of the composition

1.3 Trigonometric Functions

1. Apply arc length formulas.
2. Sketch trigonometric functions.
3. Apply identities to simplify and evaluate expressions involving trigonometric functions.

1.5 Exponential Functions

1. Sketch functions.
2. Determine whether a graph is a function.
3. Characterize functions using domain and range, symmetry, intervals of increasing/decreasing.

1.6 Inverse Functions and Logarithms

1. Determine whether a function has an inverse, and if it does, find the inverse.
2. Evaluate and simplify expressions involving logarithms and inverse sine and cosine functions.
3. Find the domain and range, and sketch composite functions that incorporate logarithms and inverse sine and cosine functions.

2 Limits and Continuity

2.1 Rates of Change and Tangents to Curves

1. Characterize motion of a falling object using Galileo's law to estimate distance travelled and speed.
2. Estimate and compute the rate of change of a function.
3. Construct the equation of a tangent line to a function at a point.

2.2 Limit of a Function and Limit Laws

1. Determine whether limits exist, where they exist, and if they do, evaluate them.
2. Evaluate limits using the Sandwich Theorem.

2.4 One-Sided Limits

1. Determine whether limits and one-sided limits exist, where they exist, and if they do, evaluate them.
2. Evaluate limits using identities involving $\frac{\sin\theta}{\theta}$ and $\frac{\cos\theta - 1}{\theta}$.

2.5 Continuity

1. Determine whether, and where, a function is continuous.
2. Extend functions to be continuous at a point.
3. Apply the intermediate value theorem to characterize functions.

2.6 Limits Involving Infinity; Asymptotes of Graphs

1. Evaluate infinite limits.
2. Identify and sketch horizontal, oblique, and vertical asymptotes of functions.

3 Differentiation

3.1 Tangents and the Derivative at a Point

1. Compute and interpret derivatives of functions of a single variable.

3.2 The Derivative as a Function

1. Compute the derivative of a function
2. Construct the equation of a tangent line at a point.
3. Sketch the derivative of a function over an interval, or sketch a function given a graph of its derivative.
4. Identify where functions are differentiable.

3.3 Differentiation Rules

1. Compute the derivative of a function using derivative rules.
2. Solve equations involving derivatives (for example, to locate points on a graph where the tangent line has a particular slope).

3.4 The Derivative as a Rate of Change

1. Compute the velocity, speed, and acceleration of a moving object, given its position as a function of time.
2. Give examples of expressions and draw graphs that represent the motion of a moving object.
3. Interpret equations and graphs that represent the motion of a moving object.

3.5 Derivatives of Trigonometric Functions

1. Differentiate trigonometric functions.

Please memorize the derivatives of the six trigonometric functions that were stated in the lecture for this section.

3.6 The Chain Rule

1. Differentiate composite functions using the Chain Rule.

3.7 Implicit Differentiation

1. Use implicit differentiation to calculate derivatives.
2. Construct the equation of a normal line to a curve.

3.8 Derivatives of Inverse Functions and Logarithms

1. Differentiate an inverse function
2. Apply logarithmic differentiation to differentiate functions.
3. Differentiate logarithmic and exponential functions.

3.9 Inverse Trigonometric Functions

1. Differentiate inverse trigonometric functions.

Students are not expected to memorize the derivatives of the **inverse** trigonometric functions. They'll be given on the cover page of quizzes/midterms/exams.

3.10 Related Rates

1. Solve related rate problems.

3.11 Linearization and Differentials

1. Construct differentials, and linearizations centered on a point.
2. Use differentials and linearizations to approximate function values, and to characterize how functions are changing near a given point.
3. Characterize the error made in a linear approximation.

Note that the textbook also explores sensitivity to change, which we won't have time to cover and students are not expected to know.

4 Applications of Derivatives

4.1 Extreme Values of Functions

1. Identify critical points and extreme values of a function.
2. Give an example, or sketch a function whose critical points, or local extrema, or global extreme values are given.

4.2 The Mean Value Theorem

1. Determine whether Rolle's Theorem and the Mean Value Theorem can be applied to a given function and interval.
2. Apply Rolle's theorem and the Mean Value Theorem to characterize the roots, or the rate of change of a function (for example, to identify where the derivative of a function is equal to a particular value).
3. Give examples of functions whose derivatives meet certain criteria by using the Mean Value Theorem.

4.3 Monotonic Functions, the First Derivative Test

1. Determine where a function is increasing or decreasing.
2. Classify critical points using the first derivative test.
3. Sketch functions using the first derivative and the first derivative test.

4.4 Concavity and Curve Sketching

1. Determine where a function is concave up or concave down.
2. Classify critical points using the second derivative test.
3. Sketch functions using characteristics such as concavity, intervals of increasing/decreasing, extrema, symmetry, intercepts, asymptotes, domain and range.

4.6 Applied Optimization

1. Solve optimization problems.

4.7 Newton's Method

1. Given a differentiable function $f(x)$ and an initial estimate x_0, apply one or two steps of Newton's method to estimate a solution to an equation.

4.8 Antiderivatives

1. Construct antiderivatives and indefinite integrals of functions.
2. Apply indefinite integrals to solve differential equations and initial value problems.