Section 2.3 : The Precise Definition of a Limit

Chapter 2 : Limits and Continuity

Math 1551, Differential Calculus

Section 2.3 The Precise Definition of a Limit

Topics

We will cover these topics in this section.

1. The precise definition of limit.

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Confirm that a given limit exists using the precise definition of limit.

Example

Suppose $y = \sqrt{x-1}$.

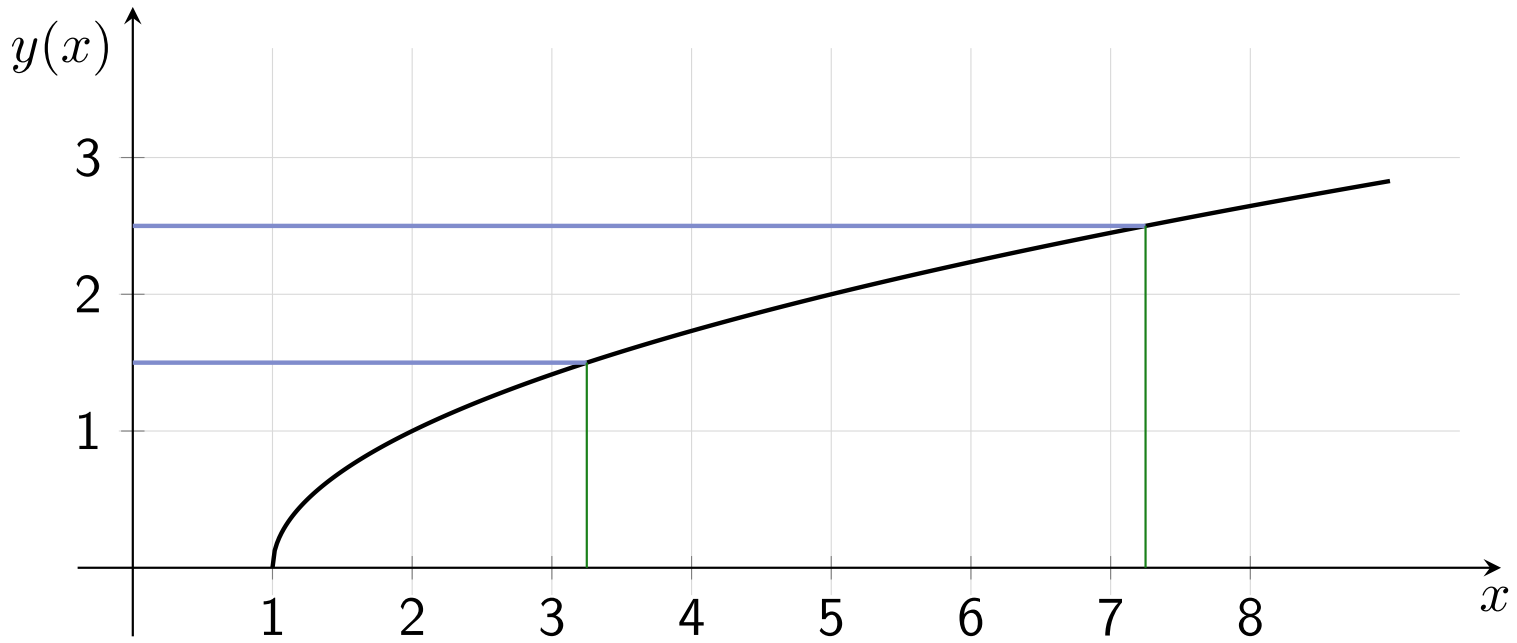
a) Sketch $y(x)$.

b) Evaluate $\lim_{x \to 5} y(x)$.

c) For what values of x is $|y-2| < \frac{1}{2}$?

Example

Given $y = \sqrt{x-1}$, for what values of x is $|y-2| < \frac{1}{2}$?



If $x \in [3.25, 7.25]$, then $|y-2| < \frac{1}{2}$.

Participation Activity: Worksheet

(if time permits)

- Please solve worksheet problems in groups of 2 or 3 students
- Each group submits **one** completed worksheet
- **Clearly print** full names at the top of your sheet
- Every student in a group gets the same grade
- Grading scheme per question:
  - 0 marks for no work or for students working by themselves
  - 1 mark for starting the problem or for a final answer with insufficient justification
  - 2 marks for a complete solution

Example 2

Suppose $y = 5x - 7$.

a) Evaluate $\lim_{x \to 2} y(x)$.

b) For what values of x is $|y - 3| < \frac{1}{10}$?

c) For what values of x is $|y - 3| < \frac{1}{100}$?

d) For what values of x is $|y - 3| < \frac{1}{1000}$?

e) For what values of x is $|y - 3| < \epsilon$? Assume that ϵ is a very small and positive real number.

f) Find a $\delta > 0$ such that for all x, $0 < |x - 2| < \delta$, implies that $|y - 3| < \epsilon$.

Formal Definition of Limit

Definition

Suppose $f(x)$ is defined on some open interval about $x = c$, except possibly at c. The limit of $f(x)$ as x approaches c is the number L, and

$$\lim_{x \to c} f(x) = L$$

if, for every $\epsilon > 0$, there exists a corresponding δ such that

$$0 < |x - c| < \delta \quad \Rightarrow \quad |f(x) - L| < \epsilon$$

Example

Let $f(x) = 3 + 2x$.

a) Find an interval about $x = 4$, for which $|f(x) - 11| < \epsilon$.

b) Use the formal definition of limit to show that $\lim_{x \to 4}(3 + 2x) = 11$.