Quiz 9:

Find a particular solution to the inhomogenous equation

$$y''' - 2y'' = -36t + 22$$

Solution: The characteristic polynomial is $\lambda^3 - 2\lambda^2$. Since 0 is a multiplicity 2 root and the inhomogeneity is a first degree polynomial, we will guess a solution of the form

$$y_p(t) = (at + b)\, t^2$$

where a and b are constants to be determined. Then

$$y_p''' - 2y_p'' = 6a - 2(6at + 2b) = -12at + 6a - 4b = -36t + 22$$

Since this is true for all t we have $-12a = -36$ and $6a - 4b = 22$ so $a = 3$ and $b = -1$. So a particular solution is given by

$$y_p(t) = 3t^3 - t^2$$

The full general solution is

$$y(t) = c_0 + c_1 t - t^2 + 3t^3 + c_2 e^{2t}$$