

# Iterative blind deconvolution method and its applications

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A simple iterative technique has been developed for blind deconvolution of two convolved functions. The method is described, and a number of results obtained from a computational implementation are presented. Some further possible applications are indicated.

The convolution  $c(\mathbf{x})$  of two functions,  $f(\mathbf{x})$  and  $g(\mathbf{x})$ , can be expressed mathematically by the integral equation

$$c(\mathbf{x}) = \int_{-\infty}^{+\infty} f(\mathbf{x}_1)g(\mathbf{x} - \mathbf{x}_1)d\mathbf{x}_1. \quad (1)$$

If the Fourier transforms of these functions are represented by their corresponding uppercase letters, then the Fourier-transform representation of Eq. (1) becomes

$$C(\mathbf{u}) = F(\mathbf{u})G(\mathbf{u}). \quad (2)$$

The process of convolution arises frequently in optics,<sup>1</sup> and if one of the functions  $f$  or  $g$  is known, methods such as Weiner filtering<sup>2</sup> and iterative restoration<sup>3</sup> can recover the other function.

The problem of deconvolution becomes more difficult if neither of the functions  $f(\mathbf{x})$  and  $g(\mathbf{x})$  is known, i.e., only the output signal,  $c(\mathbf{x})$ , is available. The problem is now termed blind deconvolution.<sup>4</sup>

The purpose of this Letter is to describe briefly a simple method for realizing blind deconvolution that has produced some promising results. The method is analogous in concept to various iterative image-processing techniques.<sup>5-7</sup>

Lane and Bates<sup>8,9</sup> have presented two possible solutions to the blind deconvolution problem, both of which show promise. One of these<sup>9</sup> is also iterative in nature, but, in contrast to the technique described here, the Fourier phase of the convolution is disregarded.

Starting with complete, although possibly noisy, knowledge of the convolution function  $c(\mathbf{x})$ , the present technique uses some general *a priori* information concerning the functions  $f(\mathbf{x})$  and  $g(\mathbf{x})$  (for example, the functions may be known to be nonnegative everywhere) and attempts to deconvolve the two functions. The general algorithm, designed to be implemented on a digital computer using fast-Fourier-transform algorithms, is shown in Fig. 1.

As an example, a conceptually simple deconvolution is now explained with reference to Fig. 1. Consider trying to deconvolve two nonnegative, real functions  $f(\mathbf{x})$  and  $g(\mathbf{x})$  from their convolution  $c(\mathbf{x})$ , expressed by Eq. (1).

The basic deconvolution method consists of the following steps. First, a nonnegative-valued initial estimate  $\tilde{f}_0(\mathbf{x})$  is input into the iterative scheme. This function is Fourier transformed to yield  $\tilde{F}_0(\mathbf{u})$  (step 1),

which is then inverted to form an inverse filter and multiplied by  $C(\mathbf{u})$  to form a first estimate of the second function's spectrum  $G_0(\mathbf{u})$  (step 2). This estimated Fourier spectrum is inverse transformed to give  $g_0(\mathbf{x})$  (step 3). The image-domain constraint of non-negativity is now imposed by putting to zero all points of the function  $g_0(\mathbf{x})$  that have a negative value (step 4). A positive constrained estimate  $\tilde{g}_0(\mathbf{x})$  is consequently formed that is Fourier transformed to give the spectrum  $\tilde{G}_0(\mathbf{u})$  (step 5). This is inverted to form another inverse filter and multiplied by  $C(\mathbf{u})$  to give the next spectrum estimate  $F_1(\mathbf{u})$  (step 6). A single iterative loop is completed by inverse Fourier transforming  $F_1(\mathbf{u})$  to give  $f_1(\mathbf{x})$  (step 7) and by constraining this function to be nonnegative, yielding the next function estimate  $\tilde{f}_1(\mathbf{x})$  (step 8). The iterative loop is repeated until two positive functions with the required convolution,  $c(\mathbf{x})$ , have been found.

Unfortunately, two major problems exist:

(1) The inverse filter has associated problems because of the need to invert a function that possesses regions of low value. Defining the filter in such regions is difficult.

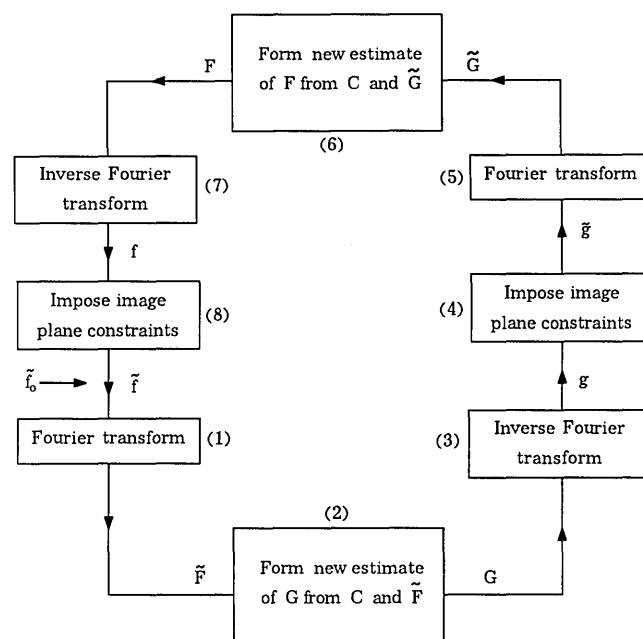


Fig. 1. General deconvolution algorithm.

(2) Zeros at particular spatial frequencies in either of the functions  $F(\mathbf{u})$  or  $G(\mathbf{u})$  result in no information at that spatial frequency being present in the convolution.

The various steps of the basic deconvolution algorithm are now further explained, and the approaches used for dealing with problems (1) and (2) are described.

The image-domain constraint of nonnegativity is commonly used in iterative algorithms associated with optical processing owing to the nonnegativity property of intensity distributions. The complete image-domain constraint used in this research not only forces the function estimate to be positive but also conserves energy at each iteration. The latter condition is realized by uniformly redistributing the sum of the function's negative values over the function estimate. These processing steps can be represented by

$$\begin{aligned}\tilde{f}_i(\mathbf{x}) &= f_i(\mathbf{x}), & f_i(\mathbf{x}) &\geq 0, \\ \tilde{f}_i(\mathbf{x}) &= 0, & \text{otherwise,} \\ E &= \int_{-\infty}^{+\infty} [f_i(\mathbf{x}) - \tilde{f}_i(\mathbf{x})] d\mathbf{x},\end{aligned}$$

where  $E$  is the sum of the function's negative values, and the energy redistribution

$$\tilde{f}_i(\mathbf{x}) = \tilde{f}_i(\mathbf{x}) + E/N, \quad (3)$$

where  $N$  is the number of pixels in the image data array when the processing is performed on a digital computer. If the function estimate still contains negative regions, then the processing is repeated. Eventually a nonnegative constrained function is formed with the total energy being conserved. It has been found that this method of energy conservation results in increased convergence compared with that obtained by using a renormalization procedure in Fourier space.

The Fourier-domain constraint may be described as constraining the Fourier product of the two function spectrum estimates to be equal to the convolution spectrum, in agreement with Eq. (2). In order to impose the Fourier-domain constraint, problems (1) and (2) need to be dealt with. It should be noted that, at the  $i$ th iteration, two estimates for each Fourier spectrum are available, for example, the function  $\tilde{F}_i(\mathbf{u})$  and the estimate  $C(\mathbf{u})/\tilde{G}_i(\mathbf{u})$  obtained on imposing the Fourier-domain constraint. Both of these estimates have associated properties in common with the desired deconvolved solution. The function  $\tilde{F}_i(\mathbf{u})$  has a non-negative inverse transform, and the second estimate obviously satisfies the Fourier-domain constraint. Therefore at each iteration the two estimates are averaged to form a composite new estimate; see Eq. (4b) below. This averaging is not essential for convergence; however, the convergent rate is dependent on  $\beta$ , and a method of selecting the optimum value of  $\beta$  has not been found. Small, confined regions of low or zero value (lower than the noise level) present in the convolution are dealt with by using only the estimate  $\tilde{F}_i(\mathbf{u})$ . The estimate obtained by imposing the Fourier constraint contributes no information to the new estimate; see Eq. (4a).

Now the problem of  $\tilde{G}_i(\mathbf{u})$  [or equivalently of  $\tilde{F}_i(\mathbf{u})$ ] having a small modulus is considered. If the modulus of  $\tilde{G}_i(\mathbf{u})$  is less than the modulus of the convolution spectrum, then instead of performing the linear averaging previously described, the inverses of the two function spectrum estimates are averaged; see Eq. (4c). The new composite estimate is now the inverse of this average. The rationale behind this averaging is simply that large function estimate values, obtained when the inverse filter function has a large value, are prevented from dominating in the average. This is intuitively reasonable, as large inverse filter values are a consequence of small values of  $\tilde{G}_i(\mathbf{u})$ , which are likely to be noisy. The Fourier-domain constraint can be summarized as follows:

if  $|C(\mathbf{u})| < \text{noise level}$ ,

$$F_{i+1}(\mathbf{u}) = \tilde{F}_i(\mathbf{u}); \quad (4a)$$

if  $|\tilde{G}_i(\mathbf{u})| \geq |C(\mathbf{u})|$ ,

$$F_{i+1}(\mathbf{u}) = (1 - \beta)\tilde{F}_i(\mathbf{u}) + \beta \frac{C(\mathbf{u})}{\tilde{G}_i(\mathbf{u})}, \quad (4b)$$

if  $|\tilde{G}_i(\mathbf{u})| < |C(\mathbf{u})|$ ,

$$\frac{1}{F_{i+1}(\mathbf{u})} = \frac{(1 - \beta)}{\tilde{F}_i(\mathbf{u})} + \beta \frac{\tilde{G}_i(\mathbf{u})}{C(\mathbf{u})}, \quad (4c)$$

where  $0 \leq \beta \leq 1$ . The constant  $\beta$  is set before the algorithm is run.

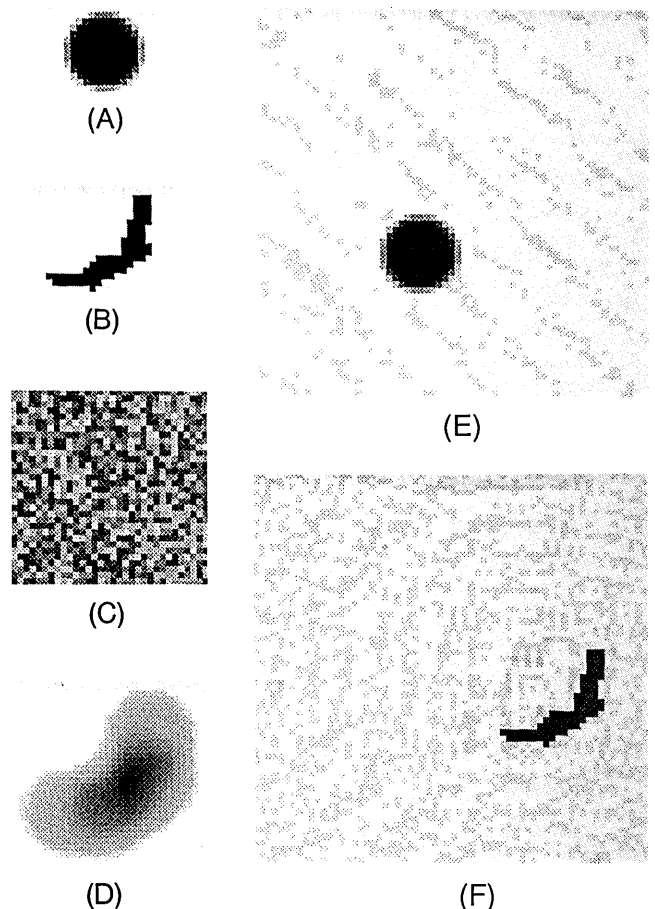


Fig. 2. Simple example of deconvolution algorithm results.

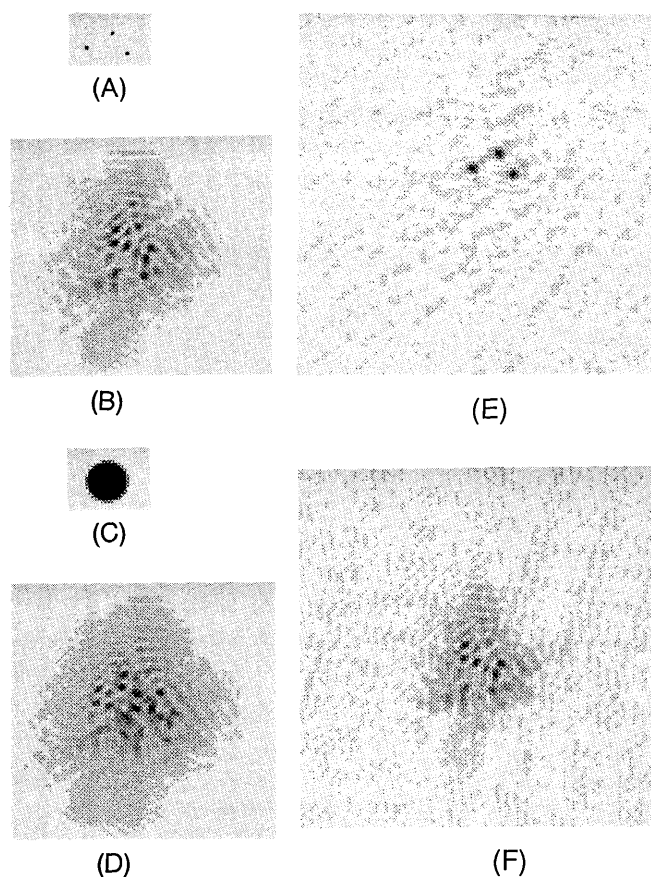


Fig. 3. Example of deconvolving a computer-generated speckle image.

The problem of extended regions of low or zero value present in the convolution spectrum at high spatial frequencies is now considered. This problem generally arises either because of a natural falling off of the high-frequency content of many Fourier spectra of interest or, as in the case of a band-limited imaging system, because the Fourier spectrum of one of the functions forming the convolution is explicitly zero for all spatial frequencies beyond a certain cutoff. The approach to solving this problem that has been taken here consists of introducing a few straightforward additional steps to the iterative scheme. A weighting or apodization function,  $W(u)$ , is defined that is greater than zero for all spatial frequencies up to some band limit and zero beyond, the band limit being determined by the convolution spectrum. The weight function additionally is required to have a nonnegative inverse Fourier transform so that the nonnegativity constraint is not invalidated. The incoherent transfer function of a circular-aperture lens is a particularly simple example and the one used in the reconstruction examples presented here. The weight function is used in the following manner. First, at each iteration when the new estimate of a Fourier spectrum is formed by the averaging process of Eqs. (4), then the new estimate is multiplied by  $W(u)$ . Second, after subsequent imposition of the image domain constraints the updated spectrum estimate is divided by the weight function.

Figures 2 and 3 show some examples of results obtained on implementing the described algorithm. The residual noise level in the deconvolved results is less than 5%. Figures 2(E) and 2(F) show the results obtained on using the deconvolution algorithm for the convolution shown in Fig. 2(D) of the two functions in Figs. 2(A) and 2(B). The results were obtained after 200 iterations starting with the estimate [Fig. 2(C)] and  $\beta = 0.9$ . The use of a weighting function was not found necessary for this example. Figure 3 shows a second, more interesting example. The convolution in this instance is a computer simulation of the sort of speckle image obtained on imaging through atmospheric turbulence. The figure layout is the same as in Fig. 1. This result was obtained after 500 iterations with  $\beta = 0.9$ . A weighting function was used in this example that has resulted in the reconstructed object in Fig. 2(E) being approximately the original function [Fig. 2(A)] convolved with the inverse Fourier transform of the weighting function.

Although the examples of the use of the blind deconvolution algorithm presented are naturally quite specific, the algorithm depicted in Fig. 1 is general. Various image- and Fourier-domain constraints can easily be incorporated, for example, constraining both reconstructed functions in image space to be nonzero only within the extent of the convolution. Products of Fourier spectra occurring in many branches of image processing can potentially be deconvolved by using a suitable form of the general algorithm. Work is currently being carried out to apply the deconvolution method to the image-restoration problems arising from the speckle-processing techniques known as triple correlation analysis,<sup>10</sup> the method of Knox and Thompson,<sup>11</sup> and speckle interferometry.<sup>12</sup>

Finally, it should be stressed that the uniqueness and convergence properties of the deconvolution algorithm are uncertain and that the effect of various amounts of noise existing in the convolution data is at present unknown.

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