Calculating the Infinite Strain Axis of Olivine Crystals from an Imposed External Velocity Field

ISA direction

The Infinite Strain Axis (ISA) of olivine crystals is an approximation of olivine's Lattice Preferred Orientation (LPO) of acoustic wave speed originally formulated in Kaminksi and Ribe (2002). It is thought that ISA calculations are a good approximate for the direction of mantle flow, but Conrad et al. (2007) discusses the limitations of ISA's relationship to both the global mantle flow field and at plate boundaries. A detailed example of ISA's failure to approximate the complex flow fields at subduction zones is shown in Jadamec and Billen (2010). This document will outline all the steps to calculating the ISA of an olivine crystal from an imposed external velocity field as well as include a companion MATLAB script (ISA.m) that calculates the ISA from an analytical corner flow solution in 2-D (with no time dependence). The format line (line xxx) refers to the starting line in which the value is calculated in the companion MATLAB script.

The deformation of olivine crystals in a grain aggregate is described by the velocity gradient tensor (line 110):

$$L_{ij} = \frac{\partial v_i}{\partial x_j} = E_{ij} - \varepsilon_{ijk}\Omega_k \tag{1}$$

Where E_{ij} is the strain rate tensor (line 41), ε_{ijk} is the permutation symbol (line 93), and Ω_k is the local vorticity (line 100).

The deformation gradient tensor is then used to calculate the deformation gradient tensor (line 151) at t_{max} (line 127):

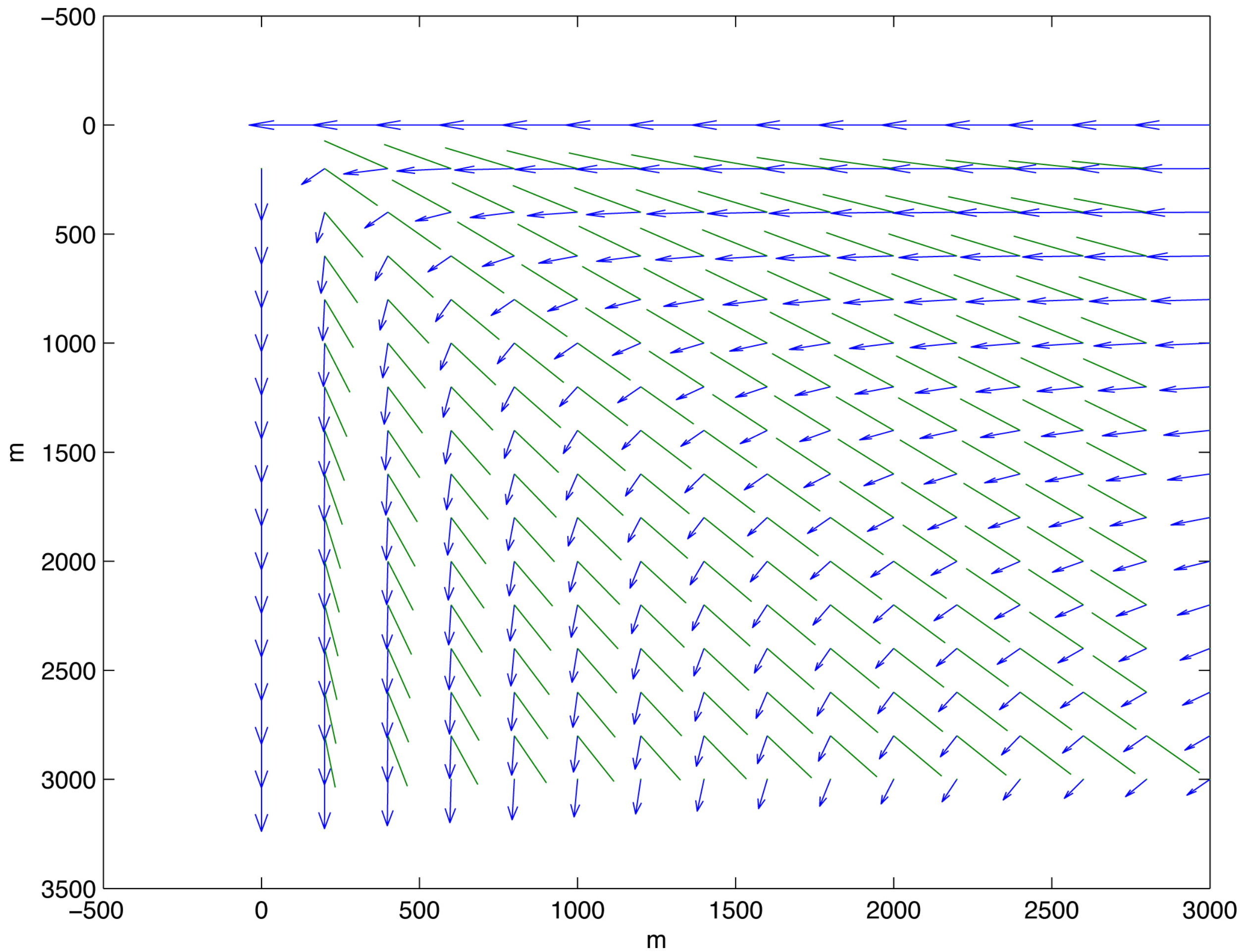
$$F = \exp(Lt) = I + Lt + \frac{L^2}{2!}t^2 + \frac{L^3}{3!}t^3 + \cdots \tag{2}$$

$$\tau_{max} = t_{max}\,\dot{\varepsilon} \tag{3}$$

Where $\dot{\varepsilon}$ (line 127) is the absolute value of the largest eigen value of the strain rate tensor. Kaminski and Ribe (2002) show in appendix A that $\tau_{max} \approx 75$, so solving for t_{max} using that value will be used for t in equation 2. Given the deformation gradient tensor, the left stretch tensor can be calculated (line 165):

$$U = FF^T \tag{4}$$

It's important to note that Conrad et al. (2007) pointed out that the original calculation presented in appendix A of Kaminski and Ribe (2002) is incorrect, and that equation 4 is the proper way to calculate the left stretch tensor. From the left stretch tensor, we can calculate the **direction of the ISA** (line 176) by finding the eigen vector that corresponds to the largest eigen value of the left stretch tensor. The figure below is a calculation of the ISA axis (green bars) in a velocity field (blue vectors):



The Lag Parameter

The Lag parameter is a measure of how long it takes the ISA to adjust to the change in flow direction. If the change in flow direction is too fast, ISA (and the LPO approximated by it) no longer becomes a good proxy for mantle flow. The Lag Parameter (line 233):

$$\Pi = \frac{1}{\dot{\varepsilon}}\left|\frac{D\Theta}{Dt}\right| \tag{5}$$

Where $\frac{D}{Dt}$ is the material derivative given by:

$$\frac{D\Theta}{Dt} = \frac{\partial\Theta}{\partial t} + u \cdot \nabla\Theta \tag{6}$$

Where u is the local velocity and Θ is (line 219):

$$\Theta = \cos^{-1}(\hat{u} \cdot \hat{e}) \quad (7)$$

Where $\hat{u}$ is the local flow direction and $\hat{e}$ is the local ISA.

References

Conrad, C. P., Behn, M. D., & Silver, P. G. (2007). Global mantle flow and the development of seismic anisotropy: Differences between the oceanic and continental upper mantle. *Journal of Geophysical Research*, *112*(B7), B07317. doi:10.1029/2006JB004608

Jadamec, M. a, & Billen, M. I. (2010). Reconciling surface plate motions with rapid three-dimensional mantle flow around a slab edge. *Nature*, *465*(7296), 338–41. doi:10.1038/nature09053

Kaminski, É., & Ribe, N. M. (2002). Timescales for the evolution of seismic anisotropy in mantle flow. *Geochemistry, Geophysics, Geosystems*, *3*(8), 1–17. doi:10.1029/2001GC000222