

Inference and hypothesis testing

EDS 222

Tamma Carleton

Fall 2022

Announcements/check-in

- Assignment 04 will cover the next two weeks of material (due 11/22, 9am)

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- Change in OH this week (individual slots)

Today

Thinking about uncertainty

Sampling distributions

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Hypothesis testing: conceptual foundations

Null hypotheses, alternative hypotheses, rejecting the null

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The Central Limit Theorem, standard errors, Z-scores, p-values

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Confidence

Confidence intervals

Thinking about uncertainty

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- We have a *randomly drawn sample* and are trying to learn about the population from our sample
- But our sample statistics would have been different had we randomly drawn a different set of observations!
- This is **natural variability** and it means that all our sample statistics are uncertain estimates of population parameters, even if they are unbiased (e.g., no convenience sampling, no systematic non-response, etc.)

Why does uncertainty matter?

Key question: Is our estimate indicating anything more than sampling variability or "noise"?

- This is the question **statistical inference** and **hypothesis testing** are trying to answer

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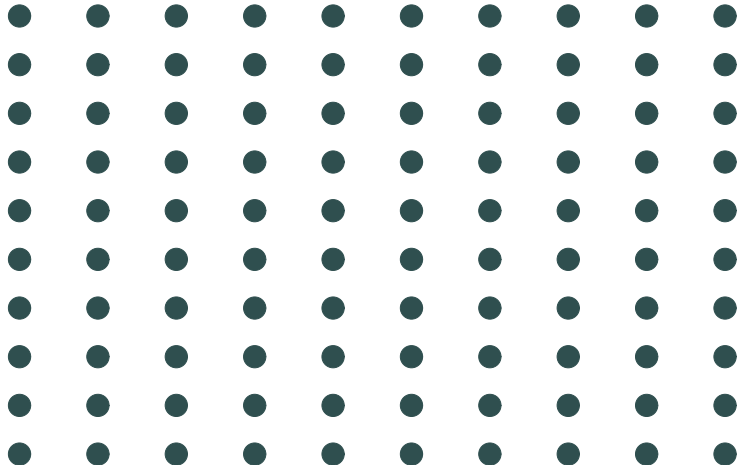
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- Does this mean there is a gender wage gap, or did we just *happen* to get a few high-earning men and a few low-earning women in this group?

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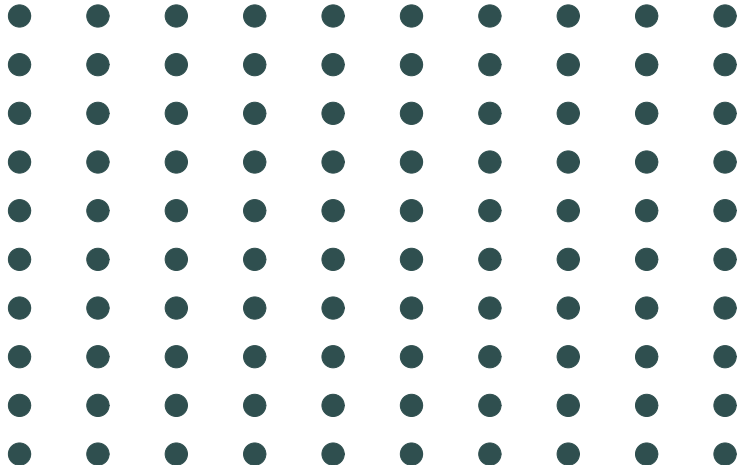
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- Does this mean there is a gender wage gap, or did we just *happen* to get a few high-earning men and a few low-earning women in this group?
- If we collected another independent sample of 100, would the gap be the same?

Population vs. sample

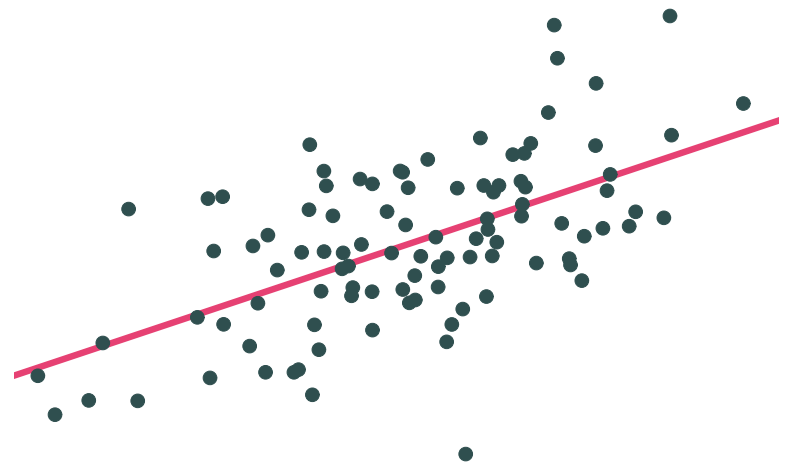


Population

Population vs. sample



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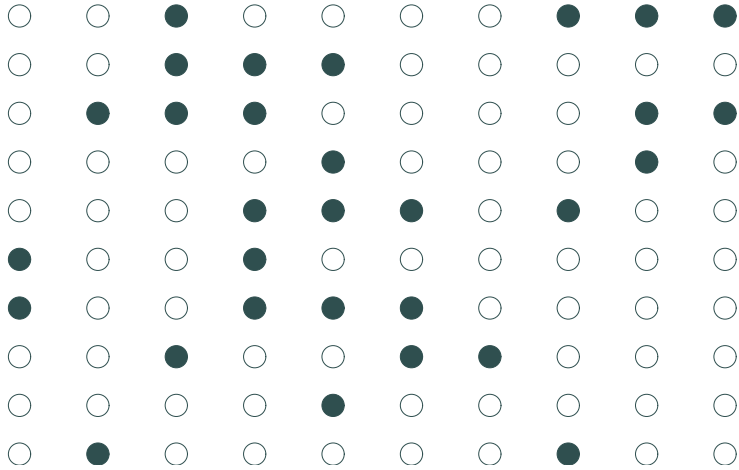


Population relationship

$$y_i = 2.53 + 0.57x_i + u_i$$

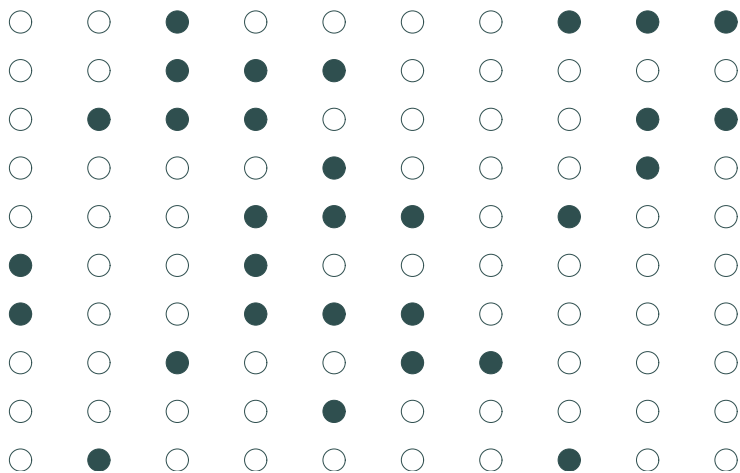
$$y_i = \beta_0 + \beta_1 x_i + u_i$$

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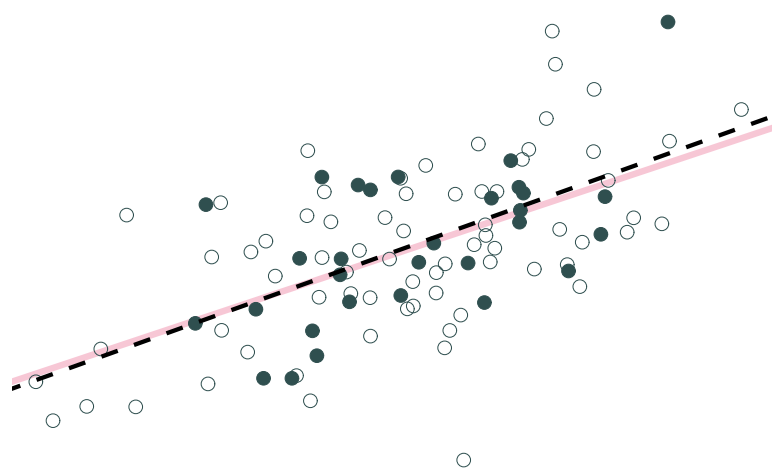


Sample 1: 30 random individuals

Population vs. sample



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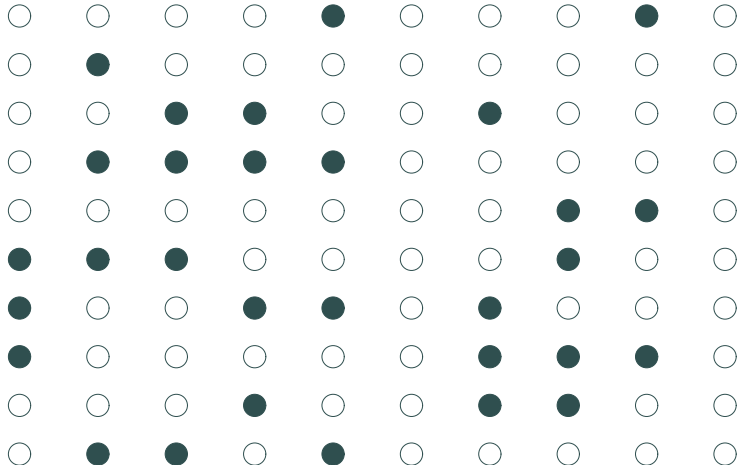
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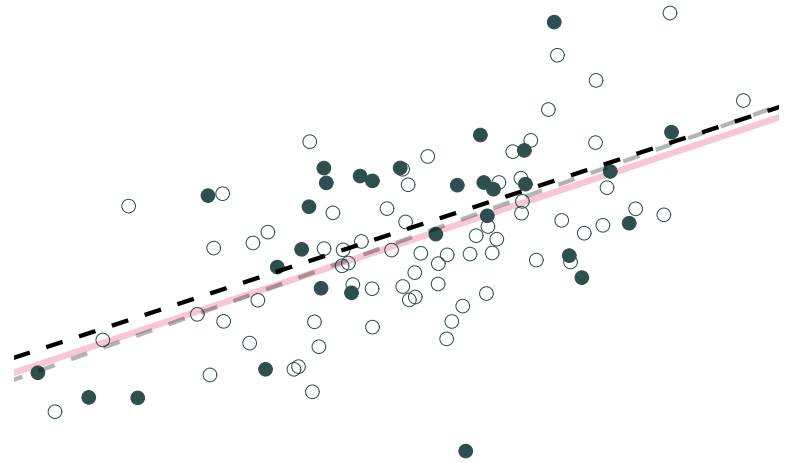
Sample relationship

$$\hat{y}_i = 2.36 + 0.61x_i$$

Population vs. sample



Sample 2: 30 random individuals



Population relationship

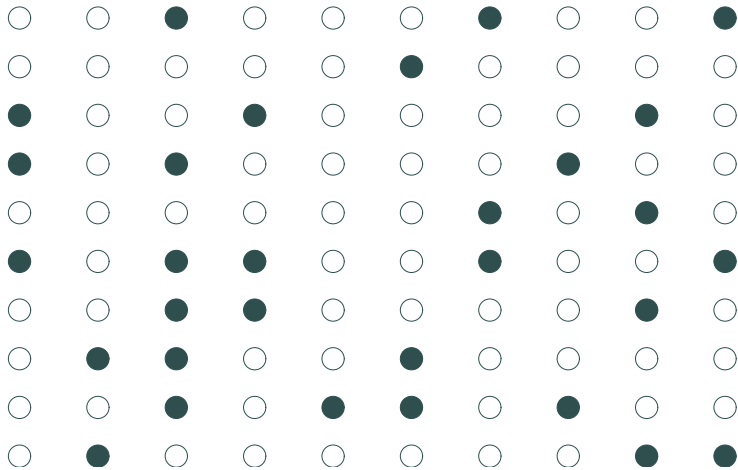
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Sample relationship

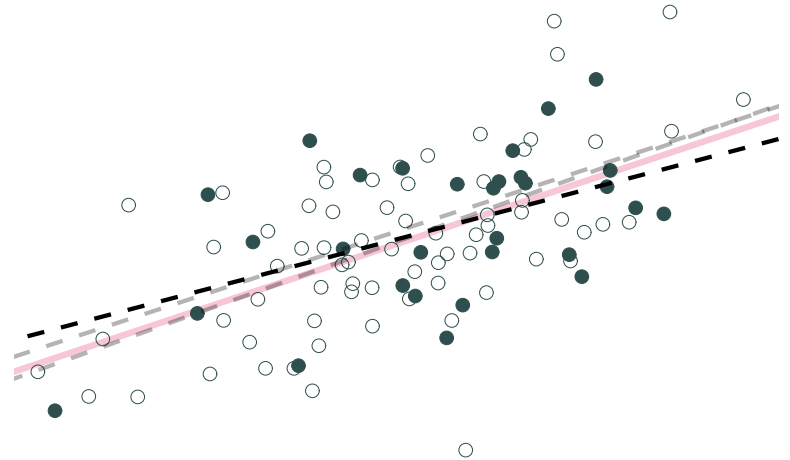
$$\hat{y}_i = 2.79 + 0.56x_i$$

Population vs. sample

count: false



Sample 3: 30 random individuals



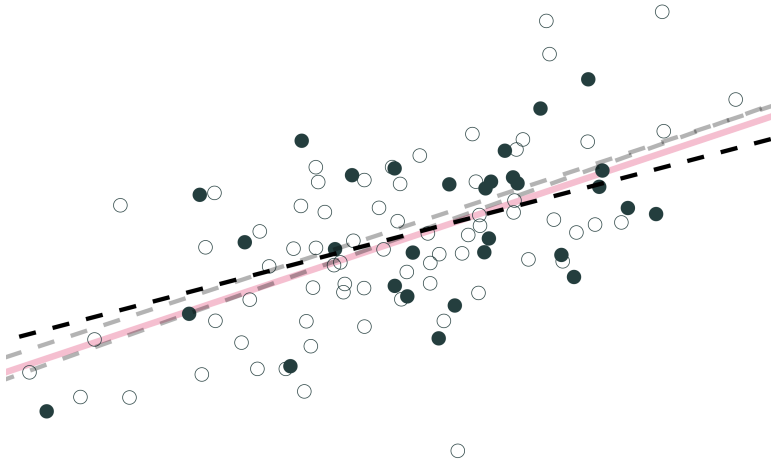
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Sample relationship

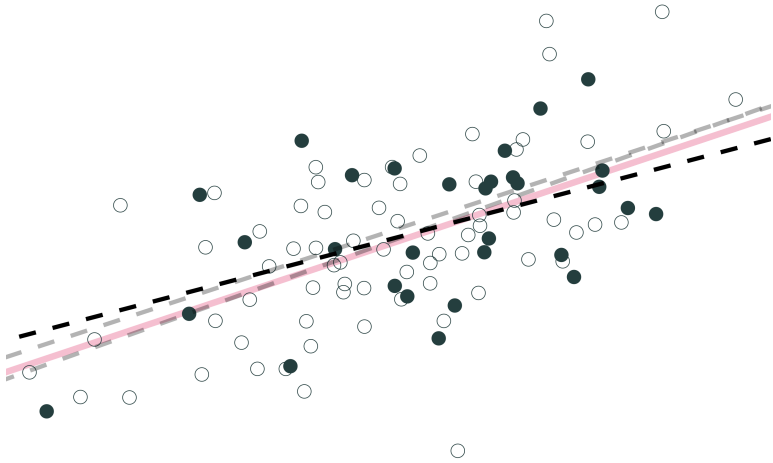
$$\hat{y}_i = 3.21 + 0.45x_i$$

Population vs. sample



- On **average**, our regression lines match the population line very nicely.
- However, **individual lines** (samples) can really miss the mark.
- Differences between individual samples and the population lead to **uncertainty** for the statistician

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Keeping track of uncertainty allows us to test hypotheses about the population using just our sample

Hypothesis testing: conceptual foundations

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- H_A : **Alternative hypothesis** A hypothesis that the measured quantity is nonzero. In other words: whatever I recovered in my sample is due to true population differences or effects.

Hypothesis testing: Example

Example: Are runners getting slower or faster over time?

Random sample of 100 runners from the 2017 Cherry Blossom 10-mile race.

- Mean 2006 finish time for all runners: 93.29 minutes
- Mean 2017 finish time: 98.78 minutes
- Standard deviation 2017 finish time: 16.59 minutes
- H_0 : The average time was the same in the two years. That is,
$$\mu_{2017} - \mu_{2006} = 0$$
- H_A : The average time was not the same in the two years. That is,
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Here, the difference in mean finish time between 2017 sample and the 2006 census of runners is:

- $98.78 - 93.29 = 5.49$

Hypothesis test asks if this point estimate is actually different from zero once we account for sampling variability

Hypothesis testing: Rejecting the null

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Rejecting the null involves both a **point estimate** and a measure of **uncertainty** or spread in your data

Hypothesis testing: in practice

Hypothesis testing in five steps

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5. Based on #4, either **reject** or **fail to reject** the null

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We already know all about these two steps.

- Null and alternative hypotheses will depend entirely on the statistical question of interest.
- Data collection and point estimates (e.g., means, regression parameters, variances, etc.) we have studied at length in this class

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Let's turn to some definitions.

Sampling distribution

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A **sampling distribution** is the distribution of all possible values of a sample statistic from samples of a given size from a given population.

- The sampling distribution describes how sample statistics (e.g., mean, regression parameters) vary from one sample (or study) to the next
- This is *not* the same as the **data distribution**!
 - Distribution of your data = distribution within one sample (e.g., gives you *one* sample mean)
 - Sampling distribution = distribution across samples (e.g., gives you *many* sample means)

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For example, recall our regression above, where the population model is:

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- A regression using one sample gives us *one* set of coefficients, called the **point estimates**: $\hat{\beta}_0 = 2.36$ and $\hat{\beta}_1 = 0.61$
- If we could collect 1000 samples and run that regression 1,000 times, we would recover the **sampling distribution** for each coefficient

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But how do we obtain one of these?

You only have one sample of data! Where does the sampling distribution come from?

--

We derive the sampling distribution from applying the Central Limit Theorem

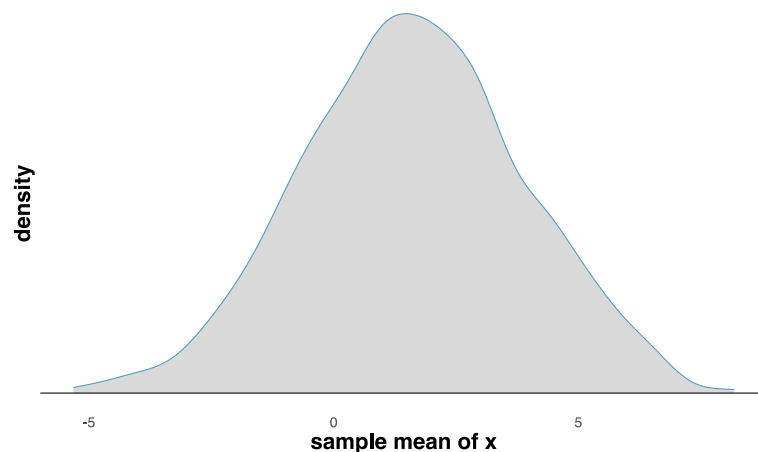
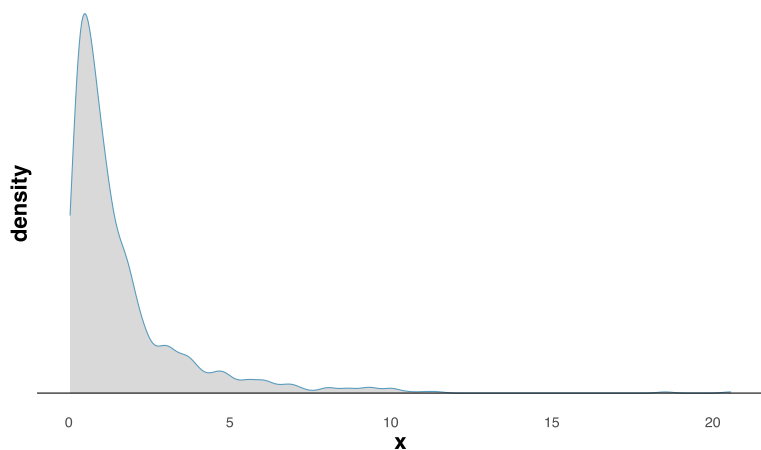
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This is true *even if* the underlying data are not normally distributed!



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Question: How big does our sample need to be?

Answer: Rule of thumb is $n \geq 30$

But this is not a hard and fast rule! Be cautious about hypothesis testing and inference with small sample sizes.

Standard errors

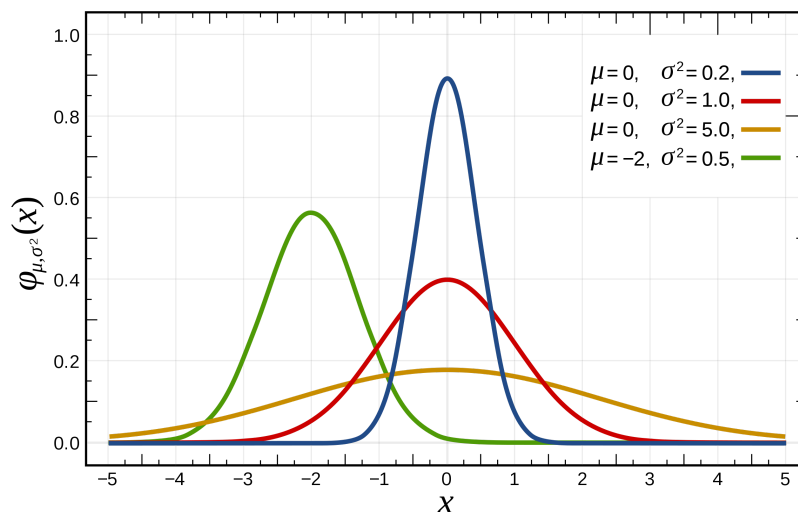
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but there are so many normal distributions!

We need to know the μ and σ of our sampling distribution in order to fully model the variability of our statistic.



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$$\mu_{2017} - \mu_{2006} = 0$$

- H_0 : Male and female wages have a mean *difference* of zero.

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- H_0 : There is *no effect* of neonicotinoid use on colony collapse disorder.

$$\beta_1 = 0. \text{ (Note that linear regression parameters are conditional means)}$$

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For example, if we are interested in a sample mean, our friend the **Central Limit Theorem** tells us that:

$$SE = \frac{s^2}{\sqrt{n}}$$

Q: What happens to the standard error as sample size increases?
Why?

Standard error for regression slope

The standard deviation of your sampling distribution is called the **standard error**

How you calculate the standard error depends on the research question.

For example, if we are interested in a regression slope, the CLT plus some algebra tell us that:

$$SE = \sqrt{\text{var}(\hat{\beta}_1)} = \sqrt{\frac{s^2}{\sum_i (x_i - \bar{x})^2}}$$

Q: What happens to the standard error as sample size increases?
Why?

SE for comparing two means

The standard deviation of your sampling distribution is called the **standard error**

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For example, if we are interested in the difference between two means, the CLT plus some algebra tell us that:

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

In all these cases, the SE is the standard deviation of the sampling distribution!

Summary: Standard errors

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Standard error measures how dispersed our sample statistic is around the population parameter of interest (highly dispersed = large SE = a lot of uncertainty about the population parameter from our one sample)

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Hypothesis testing in five steps

Step 4: quantify the probability that your sample statistic differs from the null by the observed amount, if the null were true

- I know how that my sample statistic is drawn from a normal distribution with mean μ and an estimated standard deviation given by SE .

Hypothesis testing in five steps

Step 4: quantify the probability that your sample statistic differs from the null by the observed amount, if the null were true

- I know how that my sample statistic is drawn from a normal distribution with mean μ and an estimated standard deviation given by SE .
- This should tell me something about **how unlikely it was** that I happened to draw my point estimate if the null were true, right?

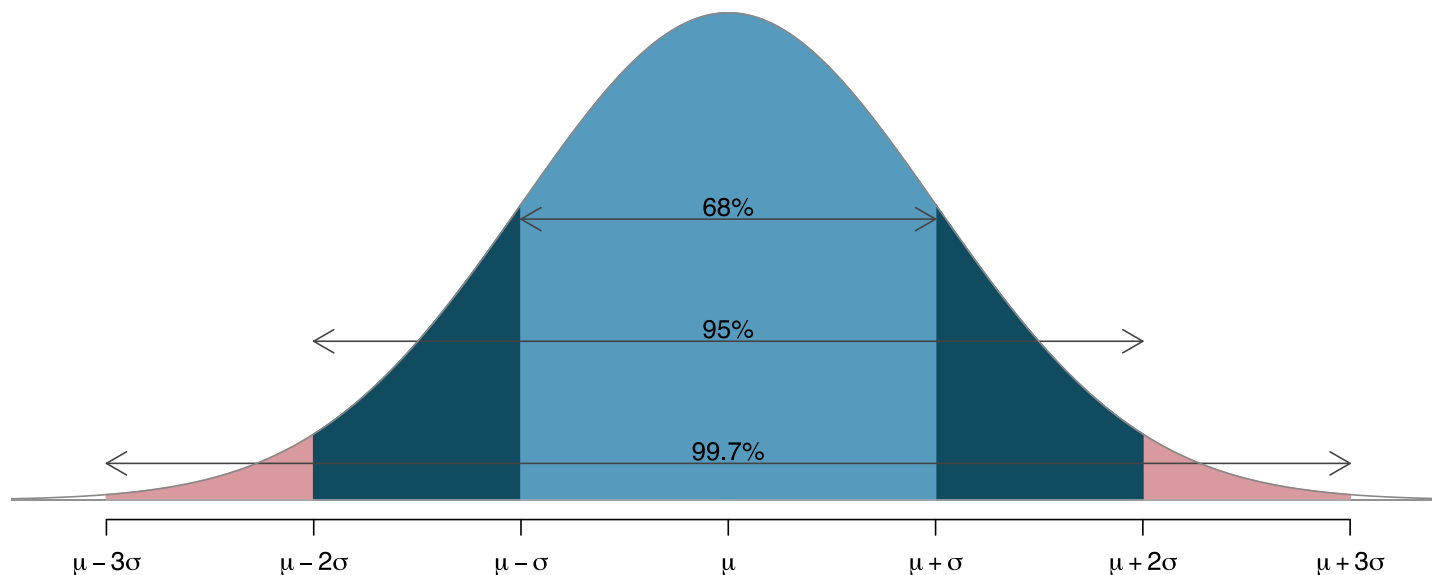
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- I know how that my sample statistic is drawn from a normal distribution with mean μ and an estimated standard deviation given by SE .
- This should tell me something about **how unlikely it was** that I happened to draw my point estimate if the null were true, right?
- Yep! But we need a couple more definitions to get all the way there.

The 68-95-99.7 rule

For a normal distribution:



Probabilities for falling within 1, 2, and 3 standard deviations of the mean in a normal distribution.

Z-score

- The 68-95-99.7 rule is helpful if your point estimate (sample statistic) is exactly 1, 2, or 3 standard deviations from the mean.

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Z-score: How many standard deviations is a value from the mean?

$$z = \frac{x_i - \mu}{\sigma}$$

Z-score for hypothesis testing

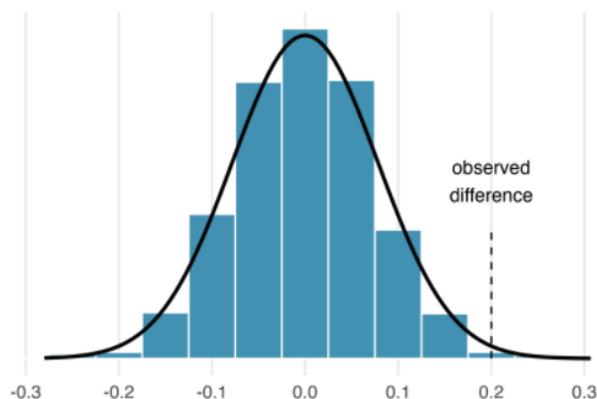
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Z-score for hypothesis testing

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Z-score for hypothesis testing: How many standard deviations is a point estimate from the null?

$$z = \frac{\text{point estimate} - \text{null value}}{SE}$$



Quantifying probabilities: p -value

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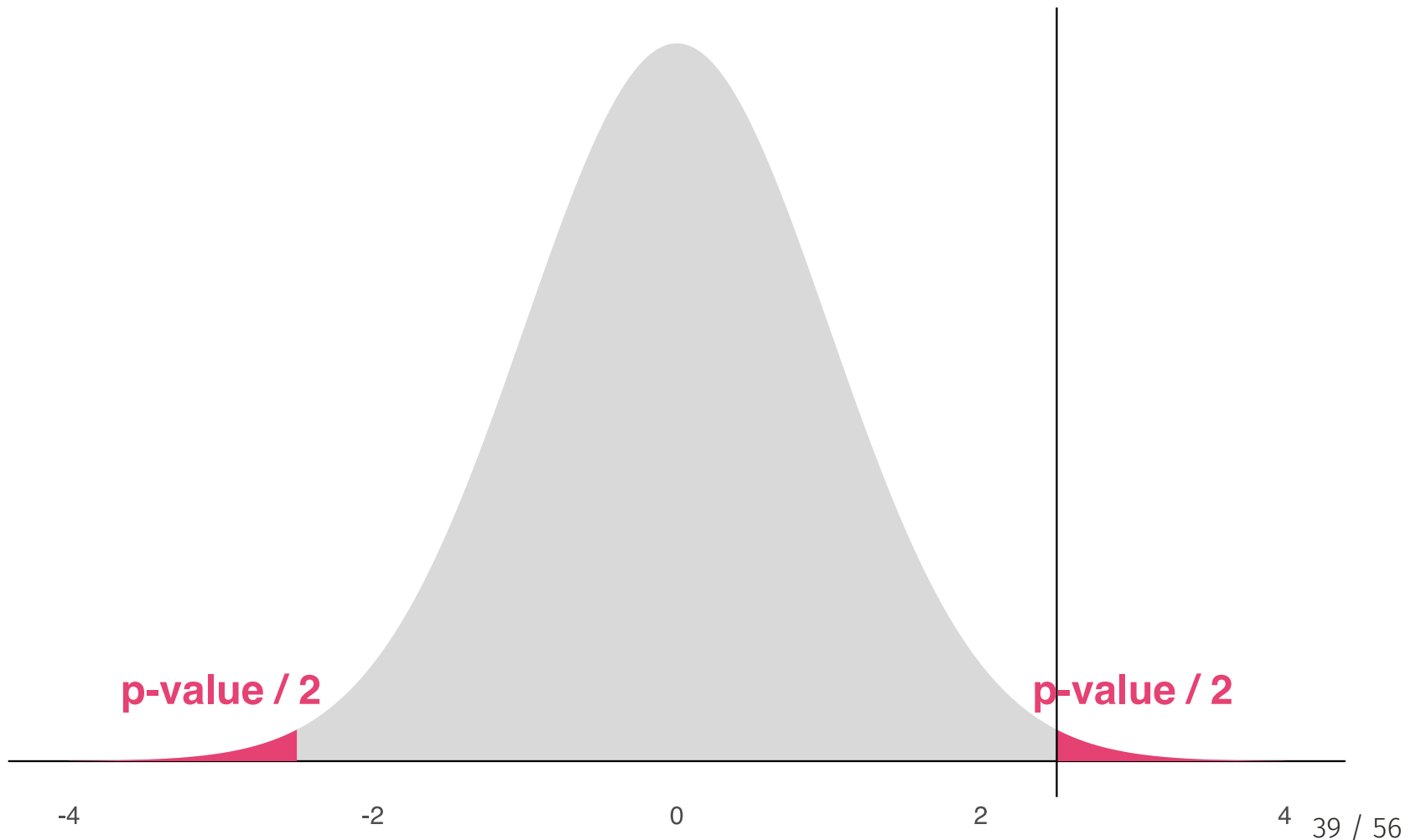
- The Z-score is also called the **test statistic**
- The Z-score/test statistic allows us to compute the **p -value**:

p -value is the probability of getting a point estimate *at least as extreme* as ours **if the null hypothesis were true**.

$$p - value = Pr(Z < -|z| \text{ or } Z > |z|) = 2 * Pr(Z > |z|)$$

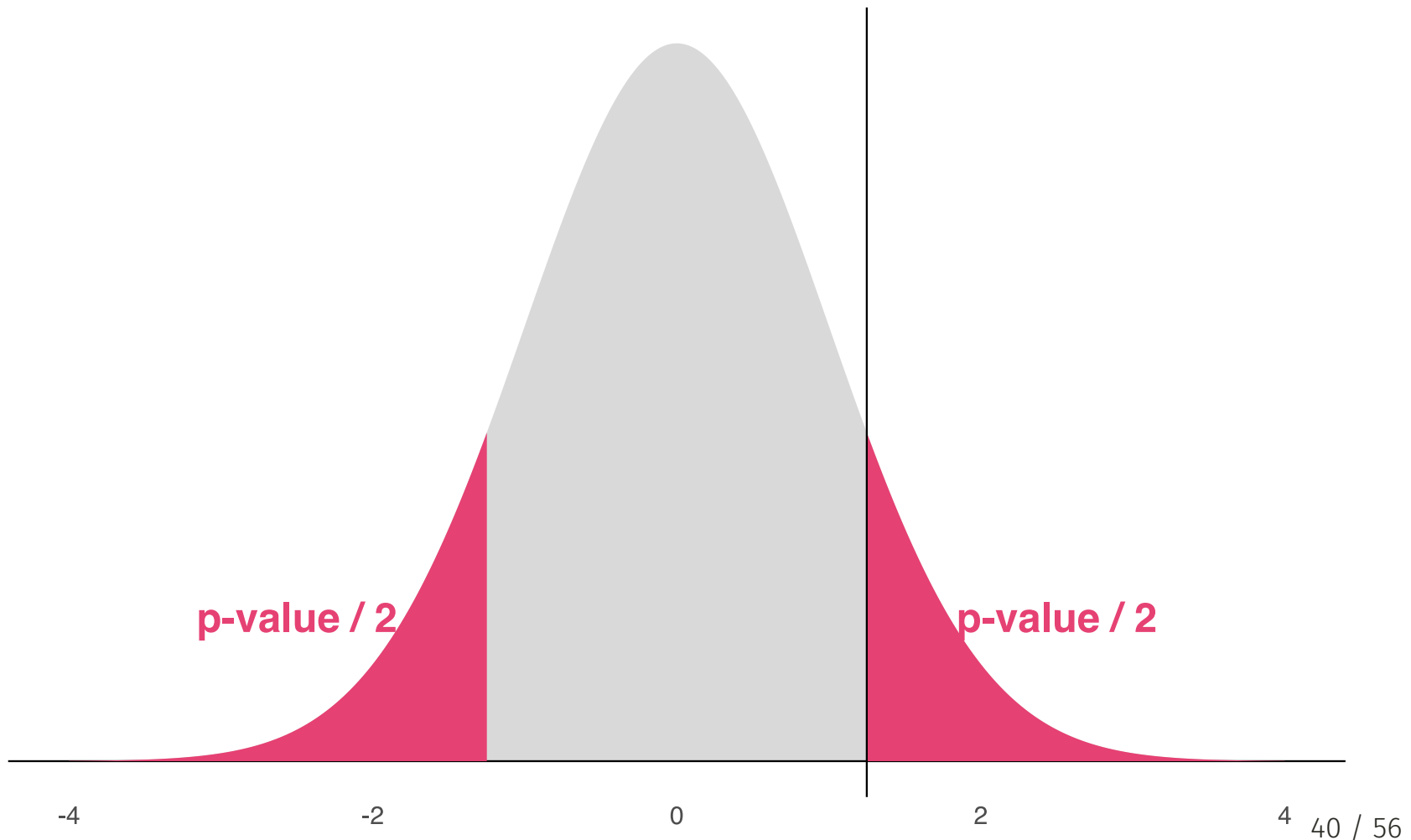
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Hypothesis testing with p -values

- p -value is the probability of observing a point estimate as extreme as yours if the null were true

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- p -value is the probability of observing a point estimate as extreme as yours if the null were true
- p -value is the area under the sampling distribution to the right and to the left of the absolute value of your test statistic (z-score, z)

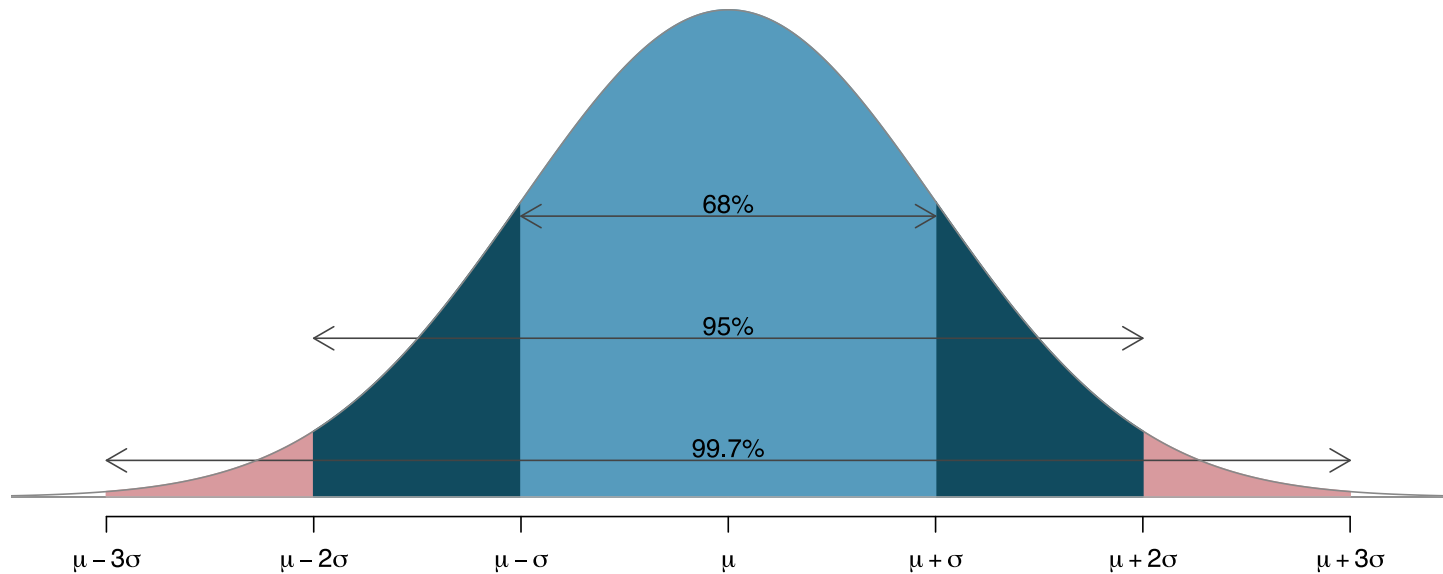
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How do I compute a p -value from a test statistic?

- **In math:** Integrate the sampling distribution's probability density function between $-\infty$ and $-|z|$; multiply by 2
- **In R:** `pnorm()`, `t.test()`, `summary(lm())`, ...

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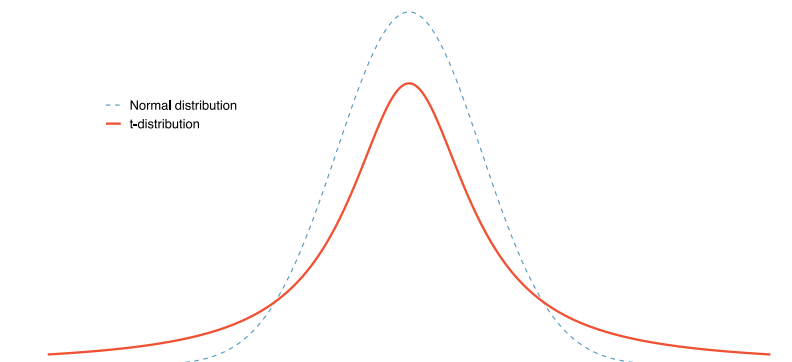
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2. We have a **large enough sample** (at the very least $n \geq 30$)

When n is relatively small, we can still proceed, we just need to use a t -distribution (and T-score -- use `pt()` in `R`) instead of a normal distribution (and Z-score)



Hypothesis testing in five steps

The general framework for implementing a hypothesis test is:

1. **Define the null** and alternative hypotheses
2. Collect data and compute the **point estimate of the statistic**
3. Model the **variability of the statistic**
4. Given this variability, **quantify the probability that your sample statistic differs from the null** by the observed amount, if the null were true
5. Based on #4, either **reject** or **fail to reject** the null

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 - Reject the null if $p < 0.05$ and $\alpha = 0.05$
- In general, reject the null if $p < \alpha$.
 - Other common α s: 0.01, 0.1

Statistical significance

We say a point estimate is "statistically significant" when:

$$p < \alpha$$

For example:

"[W]e find a **statistically-significant** effect whereby increases in surface UV intensity lowers subsequent COVID-19 growth rates...we estimate that a 1 kJm⁻²hr⁻¹ increase in local UV reduces local COVID-19 growth rates by .09 (±.04, $p = .01$) percentage points over the ensuing 17 days." --- *Carleton et al., 2021*

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- Failing to reject the null tells us we do not have sufficient evidence to prove there is an effect or a difference

Constructing confidence intervals

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- Effect sizes are important! We care not just about whether a treatment effects an outcome, but by *how much*

A **confidence interval** is a range of plausible values where we may find the true population value.

- It tells us something about the magnitude of the parameter of interest, as well as our uncertainty around our estimate

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When the sampling distribution of a point estimate can be modeled as normal, the point estimate we observe will be within 1.96 standard errors of the true value of interest about 95% of the time (think back to the 68-95-99.7 rule).

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Thus, a 95% confidence interval for such a point estimate can be constructed:

$$\text{point estimate} \pm 1.96 * SE$$

We can be 95% confident this interval captures the true value.

Also can see this as: `2*pnorm(-1.96) = .05`

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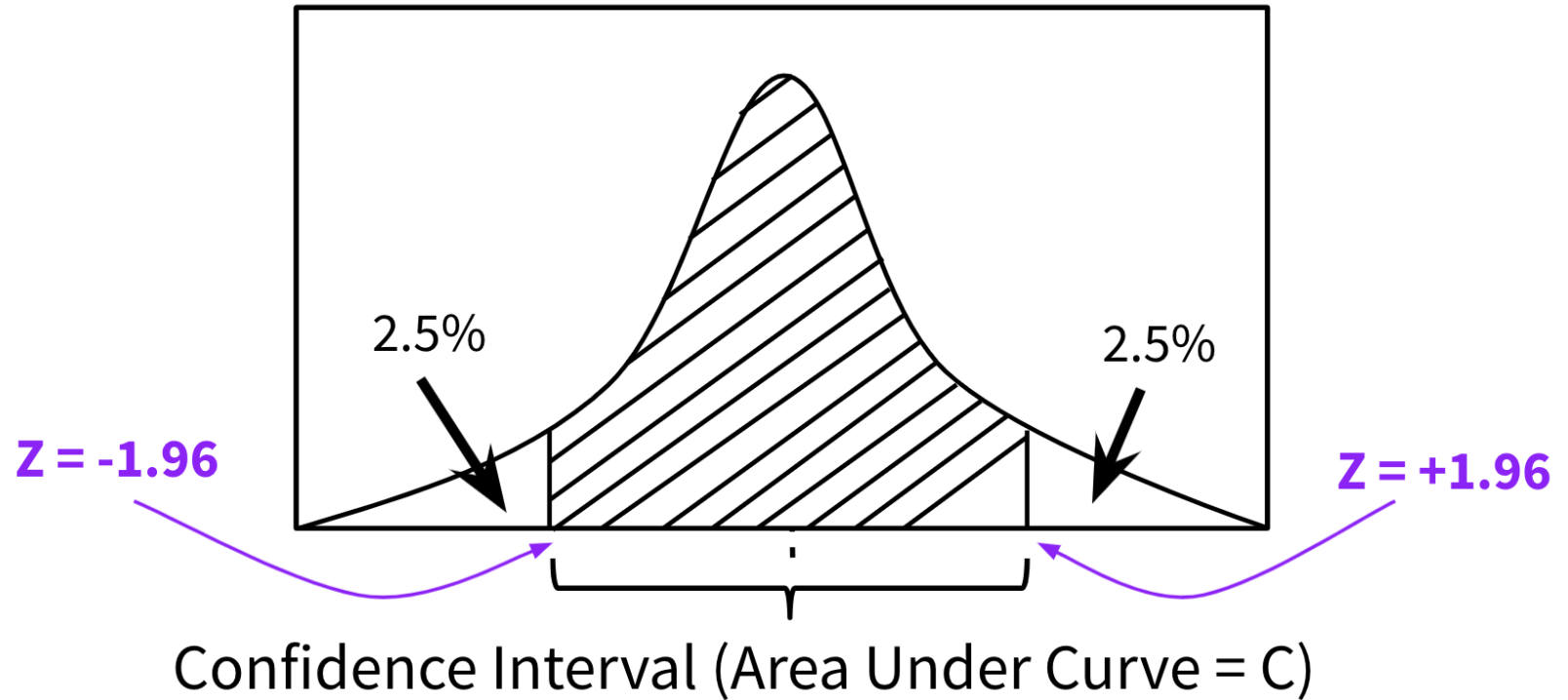
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Why do the z-scores get larger for higher confidence intervals?

A 95% confidence interval visual



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A 95% confidence interval tells you there is a **95% chance that your interval includes the true population parameter**.

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Why is this a big deal?

The population parameter *is not random*. So it either **is or is not** inside your CI.

Slides created via the R package **xaringan**.

Some slide components were borrowed from **Ed Rubin's** awesome course materials.